Solution

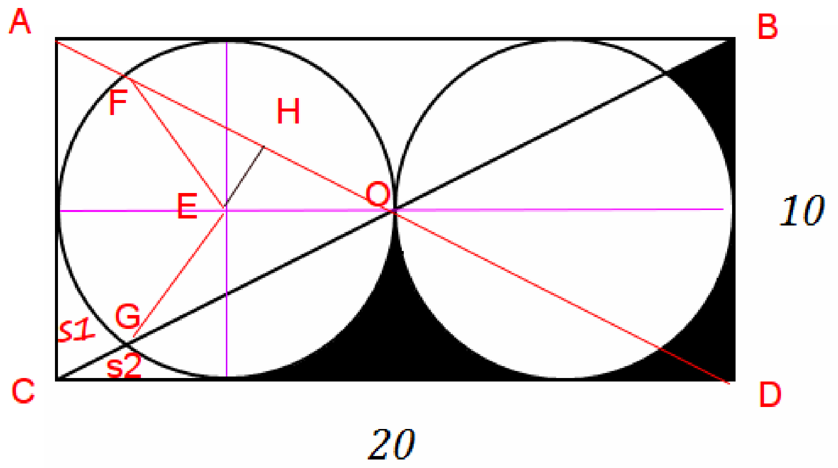
Solution by Owen Lee

2016 年 8 月 13 日

Info

Question from:https://www.quora.com/What-is-the-area-of-the-shaded-part

For clear demonstration,I am sorry that i have to mess up the nice graph.



Uppercase letters in the messed graphic stand for points among which E represents the left circle center , Q represents the two circle's tangency point,the foot of E on line $\overline{FQ}$ is H.Whats more, $S_1 and S_2$ represents the irregular part area where the are marked respectively.

OK,Now lets begin!

We can see that the graph is symmetric with respect to line $\overline{CB}$.If we add S_2 to shaded part,we will have the shaed part's area is $\frac{1}{2}(S_{ABCD} - S_{cirlces})$,whiche

$S_{cirlces}$ represetns the sum of the two circles' s area. If we could get the area of S_2, then we will have really shaded part' s area. Yes, we could get the area of S_2.

- We could get the area of $S_1 + S_2$.

$$S_1 + S_2 = \frac{1}{8}(S_{ABCD} - S_{circles}) = \frac{1}{8}(\cdot 10 \cdot 20 - 2 \cdot \pi \cdot 5^2) = 25 - \frac{25}{4}\pi$$

- 2. Now lets get the area of S_{FEO} and $S_{O\widehat{FG}}$.

$$S_{AOC} = \frac{1}{2} \cdot 10 \cdot 10 = 50$$

Let $\alpha = \angle CAD$, so we have

$$\sin\alpha = \frac{\overline{AC}}{\overline{AD}} = \frac{1}{\sqrt{5}}, \cos\alpha = \frac{\overline{CD}}{\overline{AD}} = \frac{2}{\sqrt{5}}$$

, so we have that

$$S_{FEO} = \overline{EH} \cdot \overline{OH} = \overline{OE} \cdot \sin\alpha \cdot \overline{OE} \cdot \cos\alpha = 5 \cdot \frac{1}{\sqrt{5}} \cdot 5 \cdot \frac{2}{\sqrt{5}} = 10$$

and we also have the sector

$$S_{E\widehat{FG}} = \frac{1}{2} \cdot 5^2 \cdot \alpha = 50 \arcsin\frac{1}{\sqrt{5}}$$

.

- 3. Now we get the area of S_1

$$S_1 = \frac{1}{2}(S_{AOC} - 2S_{FEO} - S_{E\widehat{FG}}) = \frac{1}{2}(50 - 2 \cdot 10 - 50 \arcsin\frac{1}{\sqrt{5}}) = 15 - 25 \arcsin\frac{1}{\sqrt{5}}$$

- 4. Then we get the area of

$$S_2 = 25 - \frac{25}{4}\pi - (15 - 25 \arcsin\frac{1}{\sqrt{5}}) = 10 - \frac{25}{4}\pi + 25 \arcsin 1\sqrt{5}$$

- 5. We have the really shaded part' s area:

$$S = \frac{1}{2}(S_{ABCD} - S_{cirlces}) - S_2 = \frac{1}{2}(10 \cdot 20 - 2\pi \cdot 5^2) - (10 - \frac{25}{4}\pi + 25 \arcsin\frac{1}{\sqrt{5}}) = 90 - \frac{75}{4}\pi - 25 \arcsin\frac{1}{\sqrt{5}}$$