Quantum Field Theory

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November 21, 2023

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$\mathrm{Spin}(1,3) \cong \mathrm{SL}(2,\mathbb{C})$ Dirac and Weyl representations helicity and chirality positive energy representation

Chapter 3

Conformal fields

more general than Lorentz symmetry, locally SO(2,4) for Minkowski

Local operators are not generally fields. Conformal group is not compact so that the representations may not be given as the direct sum of finite dimensional irreducible representations, which implies that there are no particles in CFT.

3.1

3.1. Let S be the Polyakov action

$$S = \frac{1}{2\pi\alpha'} \int d^2z\, \partial X^\mu \overline{\partial} X_\mu.$$

The path integral of the total derivate is zero,

$$0 = \int [dX] \frac{\delta}{\delta X_\mu(\sigma)} [e^{-S} \mathcal{F}[X]].$$

We will use this frequently. Note that

$$\langle \mathcal{F}[X] \rangle = \int [dX] e^{-S} \mathcal{F}[X].$$

(a) By letting $\mathcal{F}[X] = 1$, we obtain the equation of motion from the Polyakov action is $\partial\overline{\partial} X^\mu(\sigma) = 0$. So X^μ span a Riemann surface in the D-dimensional space-time.

(b) By letting $\mathcal{F}[X] = X^\nu$, we obtain

$$\frac{1}{\pi\alpha'} [\partial\overline{\partial} X^\mu(\sigma)] X^\nu(\sigma') = -\eta^{\mu\nu} \delta^2(\sigma - \sigma').$$

We define the *conformal normal ordering*, which differs from the *creation-annihilation normal ordering*, such that

$$: X^\mu(\sigma) := X^\mu(\sigma), \qquad : X^\mu(\sigma_1) X^\nu(\sigma_2) := X^\mu(\sigma_1) X^\nu(\sigma_2) + \frac{\alpha'}{2} \eta^{\mu\nu} \ln |z_1 - z_2|^2.$$

3.2 (Operator product expansion)**.**

$$A_i(\sigma') A_j(\sigma) = \sum_k c_{ij}^k(\sigma' - \sigma) A_k.$$

3.3. We introduce an unknown j^a such that the arbitrary local coordinate transform at σ_0

$$\phi'_\alpha(\sigma) = \phi_\alpha(\sigma) + \rho(a)\delta\phi_\alpha(\sigma), \qquad \delta \sim \varepsilon$$

leads to the first variation term of the functional measure

$$\begin{aligned} [d\phi']e^{-S[\phi']} &= [d\phi]e^{-S[\phi]}\left[1 + \frac{i\varepsilon}{2\pi}\int d^2\sigma\sqrt{g}\,j^a(\sigma)\partial\rho(\sigma) + O(\varepsilon^2)\right] \\ &= [d\phi]e^{-S[\phi]}\left[1 - \frac{i\varepsilon}{2\pi}\int d^2\sigma\sqrt{g}\nabla_a j^a(\sigma)\rho(\sigma) + O(\varepsilon^2)\right] \end{aligned}$$

and leads to the first variation term of the local operator

$$A'(\sigma_0) = A(\sigma_0) + \rho(a)\delta A(\sigma_0).$$

By the integration by parts, in order for the above transformation for arbitrary ρ to be a symmetry, we have $\nabla_a j^a = 0$, where a is the two-dimensional index. Considering $\rho = \mathbf{1}_R$ for some local region R, containing σ_0, and

$$[d\phi']e^{-S[\phi']}A'(\sigma_0)\cdots = [d\phi]e^{-S[\phi]}\left[1 - \frac{i\varepsilon}{2\pi}\int d^2\sigma\sqrt{g}\nabla_a j^a(\sigma)\rho(\sigma) + O(\varepsilon^2)\right](A(\sigma_0)+\rho(a)\delta A(\sigma_0))\cdots,$$

where the insertion is outside the region R so that the integration by parts is doable, we have the Ward identity by cancelling out $O(\varepsilon^2)$ or

$$\delta A = \frac{i\varepsilon}{2\pi}\int d^2\sigma\sqrt{g}\nabla_a j^a(\sigma)\rho(\sigma)A.$$

Using the divergence and the residue theorem, we have another form of the Ward identity

We apply this on space-time translation, the world-sheet translation, and the local conformal transformation(a kind of world-sheet rotation).

If we consider the world-sheet translation, the Noether current is described by T_{ab}. T_{ab} is trace-less and diagonal in the complex coordinates, we can describe the tensor as a holomorphic function $T(z)$.

Conformal invariance puts strong constraints on the form of OPE involving $T(z)$.

Part II

Gauge theory

Chapter 4

Yang-Mills theory

Why spin 1?: vector-like particles can be interpreted as the field in (1/2,1/2) representation of Lorentz group.

4.1 Interacting fields

pair production(1941)

lagrangian of standard model mass, charge, superselection sectors mass can be defined as the coefficient of potential term in the Lagrangian.

4.2 Higgs mechanism

4.3 Quantum electrodynamics

Chapter 5

Supersymmetry

Chapter 6

Geometric quantization

GB, BRST, BV formalisms

Part III

Chapter 7

Phase transition, Ginzburg Landau theory, Thermal states, Phonon, Quantum Hall effect,

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Chapter 10

Open bosonic string

String field: quantum theory for world-sheet, classical theory for space-time.

Free theory construction

We first consider the boundary CFT, whose Fock space is constructed by bosonic operators a_n^μ and fermionic ghost operators b_n, c_n

Using the BRST operator BPZ inner product is a symmetric bilinear form

The nilpotency $Q_B^2 = 0$, which is equivalent to $D = 26$, implies the gauge invariance of the action $S := -\frac{1}{2}\langle \Psi, Q_B \Psi \rangle_{BPZ}$.

CFT explanation