

Problem set

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Part I

Algebra

Chapter 1

Equations

1.1 Polynomials

1.2 Simultaneous equations

1.3 Real solutions

1.1 (Factorization).

1.2 (Discriminant).

1.3 (Image of square function).

1.4 (Intermediate value theorem).

1.4 Integer solutions

1.5 (Factorization).

1.6 (Square roots).

1.7 (Gaps between perfect squares).

Chapter 2

Inequalities

2.1 Symmetry

2.2 Homogeneity

Chapter 3

Functions

3.1 Properties of functions

3.2 Functions over \mathbb{R}

3.3 Other domains

Part II

Combinatorics

Chapter 4

Counting

4.1 Orbits

4.2 Generating functions

Chapter 5

Algorithms

5.1 Invariants

5.2 Games

Chapter 6

Graphs

6.1 Double counting

6.2 Non-constructive existence

Pigeonhole principle, Probabilistic methods, Extremal theory

Part III

Geometry

Chapter 7

Plane geometry

7.1 Angle chasing

Cyclic quadrilaterals

7.2 Length ratios

menelous and ceva

7.3 Triangle centers

7.4 Conics

Chapter 8

Analytic methods

8.1 Trigonometry

8.2 Complex variables

8.3 Barycentric coordinates

Chapter 9

Transformations

9.1 Similarity

spiral homothety

9.2 Inversion

9.3 Projectivity

Part IV

Collegiate courses

Chapter 10

Calculus

10.1 Asymptotics

10.2 Infinite series

10.1. Let a_n be a real sequence and $S_n := a_1 + \cdots + a_n$ be its partial sum.

(a) Show that if $a_n \downarrow 0$ and $S_n \leq 1 + na_n$, then $S_n \leq 1$.

10.3 Indefinite integrals

10.4 Integral inequalities

Chapter 11

Linear algebra

11.1 Determinants

11.2 Spectrum

canonical forms

11.3 Commuting matrices

two by two matrices

11.4 Positive definiteness