Algebraic quantum field theory

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Contents

Chapter 1

Axiomatic: Osterwalder-Schrader, Wightman, Haag-Kastler

CFT

Statistical physics: Gibbs state by DLR equation, Lieb-Robinson bound, quantum theory

1.1

1.1 (Wightman axioms)**.** Let $\mathbb{R}^{1+d}$ be the Minkowski space and $\mathcal{P}_+^\uparrow$ the Poincaré group. A *Wightman field* is a linear map $\phi : \mathcal{S}(\mathbb{R}^{1+d}) \to \mathrm{End}(D)$ together with a group homomorphism $U : \mathcal{P}_+^\uparrow \to U(D)$ and a specified vector Ω in a inner product space D such that

(i) Locality: if $\operatorname{supp} f$ and $\operatorname{supp} g$ are space-like separated, then $[\phi(f), \phi(g)] = 0$ on D,

(ii) Covariance: $\mathrm{Ad}\, U(\gamma)\phi(f) = \phi(\gamma^* f)$,

(iii) Positivity of energy:

(iv) Vacuum: U fixes Ω and the span of $\{\phi(f_1)\cdots\phi(f_n)\Omega\}$ is dense in D.