Contents

Part I

Traditional topics

Chapter 1

Geometry of Banach spaces

dentability, Bishop-Phelps, Diestel's book https://gowers.wordpress.com/2009/02/07/a-remarkable-recent-result-in-banach-space-theory/

Chapter 2

Spectral theory

Chapter 3

Hardy spaces

Part II

Operator algebra

Chapter 4

Classification of C*-algebras

4.1 K-theory and approximately finite algebras

Elliott conjecture: amenable simple separable C^*-algerbas are classified by K-theory.

4.2 Crossed products and C*-dynamical systems

4.3 Abstract harmonic analysis

Group C^* algebras

Chapter 5

Classification of factors

Direct integral of factors.

Type I factors. It possess a minimal projection. It is isomorphic to the whole $B(H)$ for some Hilbert space. Therefore, it is classified by the cardinality of H.

Type II factors. No minimal projection, but there are non-zero finite projections so that every projection can be “halved” by two Murray-von Neumann equivalent projections.

In type II_1 factors, the identity is a finite projection Also, Murray and von Neumann showed there is a unique finite tracial state and the set of traces of projections is $[0, 1]$. Free probability theory attacks the free groups factors, which are type II_1.

In type II_∞ factors There is a unique semifinite tracial state up to rescaling and the set of traces of projections is $[0, \infty]$.

In type III factors no non-zero finite projections exists. Classified the $\lambda \in [0, 1]$ appeared in its Connes spectrum, they are denoted by III_λ. Tomita-Takesaki theory. It is represented as the crossed product of a type II_∞ factor and $\mathbb{R}$.

Amenability, equivalently hyperfiniteness is a very nice condition in von Neumann algebra theory. Group-measure space construction can construct them. There are unique hyperfinite type II_1 and II_∞ factors, and their property is well-known. Fundamental groups of type II factors, discrete group theory, Kazhdan’s property (T) are used.

Tensor product facctors such as Araki-Woods factors and Powers factors.

5.1 Hyperfinite factors

weight, trace, state.

finite trace=tracial state.

5.1 (Uniformly hyperfinite algebras)**.** Let $\mathcal{A}$ be a uniformly hyperfinite algebra.

(a) Every matrix algebra admits a unique finite trace.

(b) Every UHF algebra admits a unique finite trace.

(c) Every hyperfinite

5.2 (Classification of UHF algebras)**.**

Chapter 6

Subfactor theory

The way how quantum systems are decomposed. Quite combinatorial! And has Galois analogy.

6.1 (Jones index theorem)**.** A *subfactor* of a factor M is a factor N containing 1_M.

Tensor categories and topological invariants of 3-folds. Ergodic flows.

Part III

Mathematical quantum theory

Chapter 7

Quantum statistical physics

CCR and CAR representation problems KMS state

Chapter 8

Algebraic quantum field theory

8.1 Wightman axioms

8.2 Nets of algebras

Doplicher-Haag-Roberts theory defines the concept of “tensor product” of net of algebras by considering composition of endomorphisms between nets of algebras. The commutativity, up to unitary equivalence, of this tensor product follows from the locality axiom. It is figured by Longo that the DHR theory has an analogy with subfactor theory of Jones.

A net of algebras on the 1+1 Minkowski space is decomposed as a tensor product of two nets of algebras on the 1-dimensional space, which are called chirl nets. Due to the disconnectivity of the completion of open sets in 1 dimensional space, the unitary mapping that connects “tensor products” of nets of algebras brings a braiding; their representations produce a baided tensor category.

Chapter 9

Conformal field theory

Chapter 10

Strict deformation quantization

Chapter 11

Quantum information theory

11.1 Operator spaces