

# Mathematical Physics

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**Part I**

**Mechanics**

# Chapter 1

## Lagrangian mechanics

### 1.1 Newtonian mechanics

1.1 (Laws of motion). Galilean structure, Galilean group

1.2 (Conservation laws).

### 1.2 Calculus of variations

1.3 (Euler-Lagrange equation).

1.4 (Closed system).  $\frac{\partial \mathcal{L}}{\partial t} = 0$

1.5 (Definition of generalized momentum).  $\frac{\partial \mathcal{L}}{\partial q} = 0$

1.6 (Equivalence to Newtonian mechanics).

### 1.3 Rigid bodies

1.7 (Inertia tensor).

1.8 (Eulerian angle).

1.9 (Lagrangian top).

## Exercises

### Oscillation

1.10 (Harmonic oscillator).

1.11 (Damped oscillation).

1.12 (Pendulum).

1.13 (Lissajous curve).

1.14 (Coupled oscillation).

## **Central forces**

1.15 (Polar coordinates).

1.16 (Effective potential).

1.17 (Kepler's problem).

1.18 (Rutherford scattering).

## **System of particles**

1.19 (Closed systems).

1.20 (Collisions).

1.21 (Two-body problem).

1.22 (Three-body problem).

## **Euler-Lagrange equations**

1.23 (Brachistochrone).

1.24 (Geodesic on the sphere).

1.25 (Dido's isoperimetric problem).

1.26 (Pendulum with moving support). A rheonomic system

1.27 (Sliding beads on a rim).

1.28 (Double pulley system).

## **Chapter 2**

# **Hamiltonian mechanics**

### **Exercises**

## **Chapter 3**

# **Continuum mechanics**

### **3.1 Conservation laws**

### **3.2 Fluid mechanics**

### **3.3 Solid mechanics**

plasticity, elasticity?



## **Part II**

# **Thermal physics**

## Chapter 4

# Thermodynamics

### 4.1 Laws of thermodynamics

Equation of states Maxwell's relations

### 4.2 Thermal processes

## Chapter 5

# Kinetic theory

ergodic hypothesis Boltzmann statistics Boltzmann equation, chapman enskog BBGKY hierarchy stochastic processes linear response

## Chapter 6

# Statistical mechanics

### 6.1 Ensembles

ensembles microcanonical, canonical, grand canonical classical gas Boltzmann distribution

### 6.2 Quantum statistics

Two statistics Fermi sea Bose-Einstein condensation

## **Part III**

# **Classical field theory**

## **Chapter 7**

# **Relativity**

**7.1 Special relativity**

**7.2 General relativity**

**7.3 Einstein field equation**

**7.4 Black holes**

## Chapter 8

# Electromagnetism

### 8.1 Maxwell equations

gauge transform

### 8.2 Optics

## **Chapter 9**

# **Lagrangian field theory**



## **Part IV**

# **Quantum mechanics**

# Chapter 10

## Historical backgrounds

### 10.1 Wave-particle duality

10.1 (Black body radiation). (1901)

10.2 (Photon interaction).

Photoelectric effect(1905)

Compton scattering(1923)

10.3 (Atom model). (a) Rutherford scattering(1911)

(b) Bohr model

(c) Franck-Hertz experiment(1914)

(d) De Broglie waves(1924)

10.4 (Electron diffraction). (a) Davisson-Germer(1927)

(b) George Pagit Thompson(1928)

### 10.2 Nuclear physics

neutrino

# Chapter 11

## Quantization

### 11.1 Interpretations of quantum mechanics

11.1 (Wave function). Hilbert space, Dirac notation

11.2 (Pictures).

11.3 (Copenhagen interpretation). POVM and measurement observables and self-adjoint operators

11.4 (Hidden variable theory). EPR paradox, Bell's inequality, CHSH inequality

### 11.2 Canonical quantization

11.5 (Canonical commutation relation).

11.6 (Weyl quantization).

11.7 (Stone-von Neumann theorem).

### 11.3 Spin

11.8 (Projective representations).

$\text{Spin}(3) \cong \text{SU}(2)$  spin representation Clebsch-Gordon, singlet and triplet

## Chapter 12

# Schrödinger equation

### 12.1 Time-independent potentials

12.1 (Schrödinger operators).

12.2 (Infinite well).

12.3 (Harmonic oscillator).

12.4 (Free particle).

12.5 (Hydrogen atom).

### 12.2 Approximation methods

WKB approximation

### 12.3 Relativistic Schrödinger equation

fine structure Klein Gordon equation

### 12.4 Scattering theory