

# Abstract Harmonic Analysis

Ikhan Choi

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## **Part I**

# **Fourier analysis**

# Chapter 1

## Locally compact groups

### 1.1 Topological groups

### 1.2 Haar measures

1.1 (Riesz-Markov-Kakutani representation theorem).

Why is the break of  $\sigma$ -finiteness not serious?

### 1.3 Group algebra

1.2 (Modular functions).

1.3 (Convolution).

### 1.4 Structure theorems

## Chapter 2

# Pontryagin duality

### 2.1 Fourier inversion

2.1 (Positive definite functions).

2.2 (Bochner's theorem).

2.3 (Fourier inversion theorem).

2.4 (Plancherel's theorem).

## Chapter 3

# Spectral synthesis

### 3.1 Closed ideals of the colvolution algebra

## **Part II**

# **Representation theory**

## Chapter 4

# Unitary representations

### 4.1

4.1 (Schur's lemma).

### 4.2 Group $C^*$ -algebras

4.2 (Operator-value Fourier transform).

### 4.3 Functions of positive type

4.3 (Functions of positive type).

4.4 (Fourier-Stieltjes algebra).

4.5 (GNS construction for locally compact groups). Let  $G$  be a locally compact group. By a state of  $C^*(G)$ , we could construct the GNS representation of  $G$ . An analog of GNS construction for  $L^1(G)$  without completion is doable, when given a function of positive type on  $G$ , instead of a state.



## Chapter 5

# Compact groups

5.1 Peter-Weyl theorem

5.2 Tannaka-Krein duality

5.3 Example of compact Lie groups

## Chapter 6

# Mackey machine

### 6.1 Example of non-compact Lie groups

Wigner classification

**Part III**

**Kac algebras**

## **Part IV**

# **Topological quantum groups**

## **Chapter 7**

# **Compact quantum groups**

## **Chapter 8**

# **Locally compact quantum groups**

### **8.1 Multiplicative unitaries**