Ising model

1 Theory

Let $\Lambda = (V, E)$ be a graph, $|V| = N$ is the number of sites. Let $\mathcal{X} = \{\pm 1\}^V$ be the state space or the space of configuration . Hamiltonian $H : \mathcal{X} \to \mathbb{R}$ is

$$H(\sigma) := -J \sum_{(v,w)\in E} \sigma_v \sigma_w.$$

where $J > 0$ is a constant, the positivity implies the ferromagnetic property. Magnetization $M : \mathcal{X} \to \mathbb{R}$ is

$$M(\sigma) := \frac{1}{N} \sum_{v \in V} \sigma_v.$$

For a fixed parameter $\beta > 0$, Gibbs measure is

$$\mu_\beta(\{\sigma\}) := \frac{1}{Z(\beta)} e^{-\beta H(\sigma)},$$

where $Z(\beta)$ is the partition function

$$Z(\beta) := \sum_{\sigma \in \mathcal{X}} e^{-\beta H(\sigma)}.$$

Note that μ depends on β and N. We want to compute the limit of $M_* \mu$ as $N \to \infty$. But we see for each fixed β and N.

single-flip dynamics Glauber vs Metropolis Transition probability

$$p_G(\sigma \to \sigma') = \frac{e^{-\beta(H(\sigma')-H(\sigma))}}{1 + e^{-\beta(H(\sigma')-H(\sigma))}}$$

$$p_M(\sigma \to \sigma') = \min\{1, e^{-\beta(H(\sigma')-H(\sigma))}\}$$

Peierls argument Ising 1D Onsager 2D

Two-dimensional Euler equation