Abstract Harmonic Analysis

Ikhan Choi

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1.1 (Non-σ-finite measures)**.** Four technical issues

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(b) The Fubini theorem

(c) The Radon-Nikodym theorem

(d) The dual space of L^1 space

1.2 (Radon measures)**.** Let Ω be a locally compact Hausdorff space. The Riesz-Markov-Kakutani representation theorem states that positive bounded linear functionals on $C_0(\Omega)$ are corresponded to finite regular Borel measures on Ω. A Radon measure is a generalization of a regular Borel measure, introduced in order to extend this theorem for *unbounded* but positive linear functionals.

1.3 (Riesz-Markov-Kakutani representation theorem for C_c)**.** Let Ω be a locally compact Hausdorff space. We are concerned with positive linear functionals on $C_c(\Omega)$.

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4.3 (Functions of positive type).

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