

Mathematical Physics

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Part I

Mechanics

Chapter 1

Lagrangian mechanics

1.1 Newtonian mechanics

1.1 (Laws of motion). Galilean structure, Galilean group

1.2 (Conservation laws).

1.2 Calculus of variations

1.3 (Euler-Lagrange equation).

1.4 (Closed system). $\frac{\partial \mathcal{L}}{\partial t} = 0$

1.5 (Definition of generalized momentum). $\frac{\partial \mathcal{L}}{\partial q} = 0$

1.6 (Equivalence to Newtonian mechanics).

1.3 Rigid bodies

1.7 (Inertia tensor).

1.8 (Eulerian angle).

1.9 (Lagrangian top).

Exercises

Oscillation

1.10 (Harmonic oscillator).

1.11 (Damped oscillation).

1.12 (Pendulum).

1.13 (Lissajous curve).

1.14 (Coupled oscillation).

Central forces

1.15 (Polar coordinates).

1.16 (Effective potential).

1.17 (Kepler's problem).

1.18 (Rutherford scattering).

System of particles

1.19 (Closed systems).

1.20 (Collisions).

1.21 (Two-body problem).

1.22 (Three-body problem).

Euler-Lagrange equations

1.23 (Brachistochrone).

1.24 (Geodesic on the sphere).

1.25 (Dido's isoperimetric problem).

1.26 (Pendulum with moving support). A rheonomic system

1.27 (Sliding beads on a rim).

1.28 (Double pulley system).

Chapter 2

Hamiltonian mechanics

Exercises

Chapter 3

Continuum mechanics

3.1 Conservation laws

3.2 Fluid mechanics

3.3 Solid mechanics

plasticity, elasticity?

Part II

Thermal physics

Chapter 4

Thermodynamics

4.1 Laws of thermodynamics

Equation of states Maxwell's relations

4.2 Thermal processes

Chapter 5

Kinetic theory

ergodic hypothesis Boltzmann statistics Boltzmann equation, chapman enskog BBGKY hierarchy stochastic processes linear response

Chapter 6

Statistical mechanics

6.1 Ensembles

ensembles microcanonical, canonical, grand canonical classical gas Boltzmann distribution

6.2 Quantum statistics

Two statistics Fermi sea Bose-Einstein condensation

Part III

Classical field theory

Chapter 7

Relativity

7.1 Special relativity

7.2 General relativity

7.3 Einstein field equation

7.4 Black holes

Chapter 8

Electromagnetism

8.1 Maxwell equations

gauge transform

8.2 Optics

Chapter 9

Lagrangian field theory

Part IV

Quantum mechanics

Chapter 10

Historical backgrounds

10.1 Wave-particle duality

10.1 (Black body radiation). (1901)

10.2 (Photon interaction).

Photoelectric effect(1905)

Compton scattering(1923)

10.3 (Atom model). (a) Rutherford scattering(1911)

(b) Bohr model

(c) Franck-Hertz experiment(1914)

(d) De Broglie waves(1924)

10.4 (Electron diffraction). (a) Davisson-Germer(1927)

(b) George Pagit Thompson(1928)

10.2 Nuclear physics

neutrino

Chapter 11

Quantization

11.1 Interpretations of quantum mechanics

11.1 (Wave function). Hilbert space, Dirac notation

11.2 (Pictures).

11.3 (Copenhagen interpretation). POVM and measurement observables and self-adjoint operators

11.4 (Hidden variable theory). EPR paradox, Bell's inequality, CHSH inequality

11.2 Canonical quantization

11.5 (Canonical commutation relation).

11.6 (Weyl quantization).

11.7 (Stone-von Neumann theorem).

11.3 Spin

11.8 (Projective representations).

Chapter 12

Schrödinger equation

12.1 Time-independent potentials

12.1 (Infinite well).

12.2 (Harmonic oscillator).

12.3 (Free particle).

12.4 (Hydrogen atom).

12.2 Approximation methods

WKB approximation

12.3 Relativistic Schrödinger equation

fine structure Klein Gordon equation

12.4 Scattering theory