Problem set

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10.1. Let a_n be a real sequence and $S_n := a_1 + \cdots + a_n$ be its partial sum.

(a) Show that if $a_n \downarrow 0$ and $S_n \leq 1 + na_n$, then $S_n \leq 1$.

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