

Algebraic Structures

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Part I

Groups

cyclic, symmetric, matrix direct sum, direct product free, presentation, generators

Chapter 1

1.1

1.2

Chapter 2

Subgroups

subgroups homomorphisms, image, kernel, inverse images normality, quotient, coset counting

Chapter 3

Group actions

3.1 Orbits and stabilizers

3.2 Action by conjugation

3.3 Action by left multiplication

Exercises

3.1. Let G be a finite group. If $G/Z(G)$ is cyclic, then G is abelian.

3.2. Let G be a finite group. If $x \mapsto x^3$ is a surjective endomorphism, then G is abelian.

Part II

Rings

Chapter 4

Ideals

Chapter 5

Integral domains

Chapter 6

Polynomial rings

Part III

Modules

exact seq free modules inj, proj hom and duality tensor product
modules over pid, abelian groups algebras

Chapter 7

Exact sequences

Chapter 8

Hom functor and tensor products

Chapter 9

Modules over a principal ideal domain

Part IV

Vector spaces

Chapter 10

Canonical forms

spectral theorems determinants

Chapter 11

Multilinear forms

Duality Adjoints Inner product

Chapter 12

Tensor algebras

Exterior algebras Symmetric algebras