Probability Theory

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Random variables

Chapter 1

Measure theory for probability

1.1 Uniqueness of measures

1.1 (Dynkin's π-λ theorem)**.** Let $\mathcal{P}$ be a π-system and $\mathcal{L}$ a λ-system respectively. Denote by $\ell(\mathcal{P})$ the smallest λ-system containing $\mathcal{P}$.

(a) If $A \in \ell(\mathcal{P})$, then $\mathcal{G}_A := \{B : A \cap B \in \ell(\mathcal{P})\}$ is a λ-system.

(b) $\ell(\mathcal{P})$ is a π-system.

(c) If a λ-system is a π-system, then it is a σ-algebra.

(d) If $\mathcal{P} \subset \mathcal{L}$, then $\sigma(\mathcal{P}) \subset \mathcal{L}$.

1.2.

1.2 Kolmogorov extension theorem

Chapter 2

Probability distributions

sample space, events random variable, distributions, expectation

sample space of an "experiment"

equally likely outcomes coin toss dice roll ball drawing number permutation life time of a light bulb

discrete vs continuous joint, conditional, expectation

Chapter 3

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