Classical Physics

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5.1 Maxwell equations

We use the mostly minus convention and the Einstien summation convention. Let $M := \mathbb{R}^{1,3}$ be the Minkowski space. Consider a line bundle L over M and take an open subset U on which the bundle is trivialized.

A section of L describes...? Why is the external current J in $\Omega^3(U, \mathfrak{g})$?

A connection of L describes a photon field.

The Maxwell equation is the equation of motion of electromagnetic potential A and electromagnetic field F, and the inhomogeneous version is written as

$$d * F = \mu_0 J \qquad : \qquad \partial_\nu F^{\mu\nu} = \mu_0 J^\mu,$$

where $F \in \Omega^2(U, \mathfrak{g})$ and $J \in \Omega^3(U, \mathfrak{g})$ such that

$$F := dA + A \wedge A \qquad : \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

where $A \in \Omega^1(U, \mathfrak{g})$. Note that we always have $A \wedge A = 0$ because $\mathfrak{g}$ is abelian.

$$\begin{aligned} F = dA &= d(A_\mu\, dx^\mu) \\ &= dA_\mu \wedge dx^\mu + A_\mu d^2 x^\mu \\ &= \partial_\nu A_\mu\, dx^\nu \wedge dx^\mu \\ &= \partial_\nu A_\mu\, (dx^\nu \otimes dx^\mu - dx^\mu \otimes dx^\nu) \\ &= (\partial_\mu A_\nu - \partial_\nu A_\mu)\, dx^\mu \otimes dx^\nu. \end{aligned}$$

or

$$\begin{aligned} F(X, Y) &= \partial_\nu A_\mu\, (dx^\nu \wedge dx^\mu)(X, Y) \\ &= \partial_\nu A_\mu (X^\nu Y^\mu - Y^\nu X^\mu) \\ &= \partial_\nu A_\mu X^\nu Y^\mu - \partial_\nu A_\mu\, Y^\nu X^\mu \\ &= \partial_\mu A_\nu X^\mu Y^\nu - \partial_\nu A_\mu X^\mu Y^\nu \\ &= (\partial_\mu A_\nu - \partial_\nu A_\mu) X^\mu Y^\nu. \end{aligned}$$

The Maxwell equation is given by the Yang-Mills action

$$S[A] = \int_M \mathrm{tr}\left(-\frac{1}{\mu_0} F \wedge *F - A \wedge J\right)? \qquad : \qquad S = \int d^4x \left[-\frac{1}{\mu_0} F_{\mu\nu} F^{\mu\nu} - A_\mu J^\mu\right],$$

where J is given as an external current.

The anti-symmetry of F implies that $\partial_\mu \partial_\nu F^{\mu\nu} = 0$, so we have the charge conservation $\partial_\mu J^\mu = 0$. quantum fluctuation of continuous field is interpreted as a particle

- Poincaré symmetry
- Gauge symmetry
- locality

$$A_\mu \mapsto A'_\mu = A_\mu + \partial_\mu \alpha$$

For a local gauge transform $g \in \Omega^0(U, G)$, by taking the logarithm suitably, we can identify g to $\alpha \in \Omega^0(U, \mathfrak{g})$ which satisfies $\exp \alpha = g$, and we have $d\alpha \in \Omega^1(U, \mathfrak{g})$.

Since $A \in \Omega^1(U, \mathfrak{g})$ is in fact a connection form $\omega \in \Omega^1(P|_U, \mathfrak{g})$ such that ... , the gauge action of $g \in \Omega^0(U, G)$ is given by

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connection as a section?

6.1 (Maxwell equations by action)**.**

$$\mathcal{L} := -\frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu} - A_\mu J^\mu.$$

Since

$$\frac{\partial\mathcal{L}}{\partial A_\kappa} = -\frac{\partial(A_\mu J_\mu)}{\partial A_\kappa} = -\frac{\partial A_\mu}{\partial A_\kappa}J_\mu = -\delta^\kappa_\mu J_\mu = -J^\kappa.$$

and

$$\frac{\partial\mathcal{L}}{\partial(\partial_\kappa A_\lambda)} = -\frac{1}{4\mu_0}\frac{\partial(F_{\mu\nu}F^{\mu\nu})}{\partial(\partial_\kappa A_\lambda)} = -\frac{1}{2\mu_0}\frac{\partial(\partial_\mu A_\nu\partial^\mu A^\nu - \partial_\mu A_\nu\partial^\nu A^\mu)}{\partial(\partial_\kappa A_\lambda)} = -\frac{1}{\mu_0}(\partial^\kappa A^\lambda - \partial^\lambda A^\kappa) = -\frac{1}{\mu_0}F^{\kappa\lambda}$$

because

$$\begin{aligned}F_{\mu\nu}F^{\mu\nu} &= (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)\\ &= \partial_\mu A_\nu\partial^\mu A^\nu - \partial_\mu A_\nu\partial^\nu A^\mu - \partial_\nu A_\mu\partial^\mu A^\nu + \partial_\nu A_\mu\partial^\nu A^\mu\\ &= 2(\partial_\mu A_\nu\partial^\mu A^\nu - \partial_\mu A_\nu\partial^\nu A^\mu)\end{aligned}$$

and

$$\begin{aligned}\frac{\partial(\partial_\mu A_\nu\partial^\mu A^\nu)}{\partial(\partial_\kappa A_\lambda)} &= \frac{\partial(\partial_\mu A_\nu)}{\partial(\partial_\kappa A_\lambda)}\partial^\mu A^\nu + \eta^\rho_\mu\eta^\sigma_\nu\partial_\mu A_\nu\frac{\partial(\partial_\rho A_\sigma)}{\partial(\partial_\kappa A_\lambda)}\\ &= \delta^\kappa_\mu\delta^\lambda_\nu\partial^\mu A^\nu + \eta^\rho_\mu\eta^\sigma_\nu\partial_\mu A_\nu\delta^\kappa_\rho\delta^\lambda_\sigma = 2\partial^\kappa A^\lambda\end{aligned}$$

similarly with

$$\frac{\partial(\partial_\mu A_\nu\partial^\nu A^\mu)}{\partial(\partial_\kappa A_\lambda)} = 2\partial^\lambda A^\kappa,$$

the Euler-Lagrange equation is given by

$$0 = \frac{\partial\mathcal{L}}{\partial A_\kappa} - \partial_\kappa\frac{\partial\mathcal{L}}{\partial(\partial_\kappa A_\lambda)} = -J^\kappa + \frac{1}{\mu_0}\partial_\kappa F^{\kappa\lambda}.$$

6.2 (Noether theorem for classical fields).

$$\mathcal{L} := -\partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi$$

Under the translation symmetry $\phi(x) \to \phi'(x) = \phi(x - \varepsilon)$ with infinitesimal transform parameter vector ε, since we have $\delta\phi = -\varepsilon^\mu \partial_\mu \phi$ from

$$\phi' = \phi - \varepsilon^\mu \partial_\mu \phi + O(\varepsilon^2)$$

and $\delta\mathcal{L} =$ from

$$\mathcal{L}' = \mathcal{L},$$

we can write

$$\begin{aligned}\delta\mathcal{L} &= \frac{\partial \mathcal{L}}{\partial \phi}\delta\phi + \frac{\partial \mathcal{L}}{\partial \phi^*}\delta\phi^* + \frac{\partial \mathcal{L}}{\partial(\partial\phi)}\delta\partial\phi + \frac{\partial \mathcal{L}}{\partial(\partial\phi^*)}\delta\partial\phi^* \\ &= \end{aligned}$$