Computational Mathematics

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Part I

Numerical analysis

Chapter 1

Ordinary differential equations

1.1 Polynomial interpolations

1.2 Differentiation and integration

1.3 Runge-Kutta methods

1.4 Multi-step methods

Chapter 2

Numerical linear algebra

Chapter 3

Finite difference methods

3.1 Elliptic equations

3.1 (1D Poisson equation)**.** Consider the following boundary value problem:

$$\begin{cases} -u''(x) = f(x), & \text{in } (0,1), \\ u(0) = u(1) = 0. \end{cases}$$

We discretize it by $(u_j)_{j=0}^N$ such that $hN = 1$ and

$$\begin{cases} -\dfrac{u_{j+1} - 2u_j + u_{j-1}}{h^2} = f_j, & \text{for } j = 1, \cdots, N-1, \\ u_0 = u_N = 0. \end{cases}$$

$$\frac{1}{h^2} \begin{pmatrix} 2 & -1 & & 0 \\ -1 & 2 & \ddots & \\ & \ddots & \ddots & -1 \\ 0 & & -1 & 2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_{N-1} \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_{N-1} \end{pmatrix}$$

eigenvalue problems

3.2 Parabolic equations

3.3 Hyperbolic equations

CFD

Chapter 4

Finite element methods

Chapter 5

Optimization

5.1 Convex optimization

5.2 Optimal control

5.3 Operations research

theory of decision making

5.4 Mathematical programming

Chapter 6

Monte Carlo method

Part II

Information theory

Chapter 7

Communication theory

shannon's theory

Chapter 8

Coding theory

Chapter 9

Cryptography

Part III

Mathematical statistics

Chapter 10

Statistical models

Chapter 11

Statistical inference

estimation, testing hypothesis, ranking, selection

11.1 Parametric inference

11.2 Non-parametric inference

Chapter 12