Abstract Harmonic Analysis

Ikhan Choi

June 16, 2022

Contents

Part I

Fourier analysis

Chapter 1

Locally compact groups

1.1 Topological groups

1.2 Haar measures

1.1 (Riesz-Markov-Kakutani representation theorem).

Why is the break of σ-finiteness not serious?

1.3 Group algebra

1.2 (Modular functions).

1.3 (Convolution).

1.4 Structure theorems

Chapter 2

Fourier inversion

2.1 Fourier inversion

2.1 (Positive definite functions).

2.2 (Bochner's theorem).

2.3 (Fourier inversion theorem).

2.4 (Plancherel's theorem).

2.2 Pontryagin duality

proofs

Chapter 3

Spectral synthesis

3.1 Closed ideals of the colvolution algebra

Part II

Representation theory

Chapter 4

Unitary representations

4.1

4.1 (Schur's lemma).

4.2 Group C*-algerbas

4.2 (Operator-value Fourier transform).

4.3 Functions of positive type

4.3 (Functions of positive type).

4.4 (Fourier-Stieltjes algebra).

4.5 (GNS construction for locally compact groups). Let G be a locally compact group. By a state of $C^*(G)$, we could construct the GNS representation of G. An analog of GNS construction for $L^1(G)$ without completion is doable, when given a function of positive type on G, instead of a state.

Chapter 5

Compact groups

5.1 Peter-Weyl theorem

5.2 Tannaka-Krein duality

5.3 Example of compact Lie groups

Chapter 6

Mackey machine

6.1 Example of non-compact Lie groups

Wigner classification

Part III

Kac algebras

Part IV

Locally compact quantum groups