Abstract Harmonic Analysis

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Contents

Ι	Fourier analysis on groups	2
1	Locally compact groups	3
	1.1 Haar measures	3
	1.2 Group algebras	3
	1.3 Pontryagin duality	4
	1.4 Structure theorems	4
	1.5 Spectral synthesis	4
2	Representation theory	5
3	Compact groups	6
	3.1 Peter-Weyl theorem	6
	3.2 Tannaka-Krein duality	6
	3.3 Example of compact Lie groups	6
4	Mackey machine	7
	4.1 Example of non-compact Lie groups	7
5	Kac algebras	8
II	Topological quantum groups	9
6	Compact quantum groups	10
7	Locally compact quantum groups	11
	7.1 Multiplicative unitaries	11
III	Tensor categories	12

Part I Fourier analysis on groups

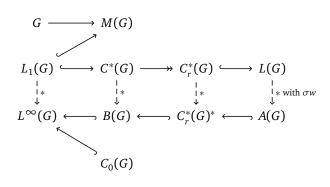
Locally compact groups

1.1 Haar measures

- **1.1** (Non- σ -finite measures). Following technical issues are important
 - (a) The Fubini theorem
 - (b) The Radon-Nikodym theorem
 - (c) The dual space of L^1 space
- 1.2 (Existence of the Haar measure).

1.2 Group algebras

- 1.3 (Modular functions).
- 1.4 (Convolution).
- 1.5 (Positive definite functions). Bochner theorem
- 1.6 (Fourier-Stieltjes algebra).
- **1.7** (GNS construction for locally compact groups). Let G be a locally compact group. By a state of $C^*(G)$, we could construct the GNS representation of G. An analog of GNS construction for $L^1(G)$ without completion is doable, when given a function of positive type on G, instead of a state.



1.8 (Uniformly continuous functions). G acts on $C_{lu}(G)$ and $L^1(G)$ continuously with respect to the point-norm topology. A function on G is left uniformly continuous if and only if it is written as f * x for some $f \in L^1(G)$ and $x \in L^{\infty}(G)$.

1.3 Pontryagin duality

- **1.9** (Dual group).
- 1.10 (Fourier inversion theorem).
- 1.11 (Plancherel's theorem).

1.4 Structure theorems

1.5 Spectral synthesis

Representation theory

- 2.1 (Schur's lemma).
- 2.2 (Operator-valued Fourier transform).

Since it is not easy to introduce the quantum dual of G for now, we cannot discuss $L^1(G)$ as the Fourier algebra, the predual of the quantum group von Neumann algebra. $(A(G) = L(G)_* = L^1(\widehat{G})$ and also is the closed linear span of matrix coefficients of the left regular representation.)

Compact groups

- 3.1 Peter-Weyl theorem
- 3.2 Tannaka-Krein duality
- 3.3 Example of compact Lie groups

Mackey machine

4.1 Example of non-compact Lie groups

Wigner classification

Kac algebras

Part II Topological quantum groups

Compact quantum groups

Locally compact quantum groups

7.1 Multiplicative unitaries

Part III Tensor categories