Differential Equations

Ikhan Choi

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Linear ordinary differential equations

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1.2 Complex roots

1.3 Repeated roots

Chapter 2

Variable coefficient equations

2.1 Series solution

2.2 Fuch's theorem

2.3 Orthogonal polynomials

2.4 Sturm-Liouville theory

2.5 The Frobenius method

Fuch's theorem

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Inhomogeneous equations

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3.2 Variation of parameters

3.3 Damped oscillation

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discontinuous data gluing

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Boussinesq equation
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Chapter 12

Nonlinear waves and diffusion

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Nonlinear diffusion equation