

Probability Theory

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Part I

Random variables

Chapter 1

Measure theory for probability

1.1 Uniqueness of measures

1.1 (Dynkin's π - λ theorem). Let \mathcal{P} and \mathcal{L} be a π -system and a λ -system respectively, and $\ell(\mathcal{P})$ the smallest λ -system containing \mathcal{P} .

- (a) If $A \in \ell(\mathcal{P})$, then $\mathcal{G}_A := \{B : A \cap B \in \ell(\mathcal{P})\}$ is a λ -system.
- (b) $\ell(\mathcal{P})$ is a π -system.
- (c) If a λ -system is a π -system, then it is a σ -algebra.
- (d) If $\mathcal{P} \subset \mathcal{L}$, then $\sigma(\mathcal{P}) \subset \mathcal{L}$.

1.2.

1.2 Kolmogorov extension theorem

Chapter 2

Probability distributions

sample space, events random variable, distributions, expectation

sample space of an "experiment"

equally likely outcomes coin toss dice roll ball drawing number permutation life

time of a light bulb

discrete vs continuous joint, conditional, expectation

Chapter 3

Independence

Part II

Limit theorems

Chapter 4

Laws of large numbers

Chapter 5

Central limit theorems

Chapter 6

Part III

Stochastic processes

Chapter 7

Martingales

Chapter 8

Markov chains

Chapter 9

Wiener process

Part IV

Stochastic calculus