

Commutative Algebra

Ikhan Choi

October 3, 2023

Contents

I	Affine algebraic geometry	2
1	Nullstellensatz	3
1.1	Radicals	3
1.2	Affine varieties	3
2	Primary decomposition	4
2.1	Primary ideals	4
2.2	Uniqueness theorems	4
2.3	Gröbner basis	4
3	Localization	5
3.1	Valuation	5
II	Dimension theory	6
4		7

Part I

Affine algebraic geometry

Chapter 1

Nullstellensatz

1.1 Radicals

1.2 Affine varieties

1.1 (Weak nullstellensatz).

1.2 (Noether normalization theorem).

$$\operatorname{Spec}(\mathbb{C} + (x^2 - 1)\mathbb{C}[x]) = \{0, \mathbb{C}\}.$$

Chapter 2

Primary decomposition

2.1 Primary ideals

2.2 Uniqueness theorems

Noether-Lasker

2.3 Gröbner basis

2.1 (Buchberger's algorithm).

Exercises

primary vs prime powers primary vs prime radical

Chapter 3

Localization

local ring extension of ideals

3.1 Valuation

DVR dedekind domains

Part II

Dimension theory

Chapter 4

4.1 (Krull's Hauptidealsatz).