

Problem set

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November 10, 2022

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Part I

Algebra

Chapter 1

Equations

1.1 Polynomials

1.2 Simultaneous equations

1.3 Real solutions

1.1 (Factorization).

1.2 (Discriminant).

1.3 (Image of square function).

1.4 (Intermediate value theorem).

1.4 Integer solutions

1.5 (Factorization).

1.6 (Square roots).

1.7 (Gaps between perfect squares).

Chapter 2

Inequalities

2.1 Symmetry

2.2 Homogeneity

Chapter 3

Functions

3.1 Properties of functions

3.2 Functions over \mathbb{R}

3.3 Other domains

Part II

Combinatorics

Chapter 4

Counting

4.1 Orbits

4.2 Generating functions

Chapter 5

Algorithms

5.1 Invariants

5.2 Games

Chapter 6

Graphs

6.1 Double counting

6.2 Non-constructive existence

Pigeonhole principle, Probabilistic methods, Extremal theory

Part III

Geometry

Chapter 7

Plane geometry

7.1 Angle chasing

Cyclic quadrilaterals

7.2 Length ratios

menelous and ceva

7.3 Triangle centers

7.4 Conics

Chapter 8

Analytic methods

8.1 Trigonometry

8.2 Complex variables

8.3 Barycentric coordinates

Chapter 9

Transformations

9.1 Similarity

spiral homothety

9.2 Inversion

9.3 Projectivity

Part IV

College math

Chapter 10

Calculus

10.1 Asymptotics

10.2 Infinite series

Let a_n be a real sequence and $S_n := a_1 + \cdots + a_n$ be the partial sum.

1. Show that if $a_n \downarrow 0$ and $S_n \leq 1 + na_n$, then $S_n \leq 1$.

10.3 Indefinite integrals

10.4 Integral inequalities

Chapter 11

Linear algebra

11.1 Determinants

11.2 Spectrum

canonical forms

11.3 Commuting matrices

two by two matrices

11.4 Positive definiteness

Part V

General physics

Chapter 1

Mechanics

1.1 Equation of motion

1.1 (Projectile motion). Let e and g be the coefficient of restitution and gravitational acceleration.

- (a) At the time a particle at the origin is thrown with an initial speed v_0 , another particle at (a, b) with $b > 0$ begins a free fall. Find the minimum value of v_0 such that two particles collide in the region $y \geq 0$.
- (b) A particle at the ground is projected with an initial speed v_0 at an angle θ , towards a vertical wall as far away as L . Find v_0 such that it bounces back to its original position after striking the wall and ground only once.
- (c) A particle is released at height h from a plane inclined at an angle θ to the horizontal. Find the length l from the first point of collision to the point at which the particle begins to slide down.

Solution. (b) $R := v_0^2 \sin^2 \theta / 2g$

$$L + e(R - L) + eR = 2L.$$

□

1.2 (Normal force).

1.3 (Tension).

1.4 (Pulley).

1.5 (Friction).

1.2 Rigid body

1.6 (Equilibrium).

1.7 (Rolling disks).

1.8 (Parallel axis).

1.3 Conservation laws

1.9 (Gravitational energy).

1.10 (Elastic energy).

1.11 (Elastic collision).

1.12 (Inelastic collision).

1.13 (Angular momentum).

1.4 Centripetal force

1.14 (Circular motion).

1.15 (Oscillation).

1.16 (Central force). effective potential

1.17 (Fictitious force).

1.5 Fluids

Chapter 2

Waves

2.1 Waves on a string

2.1 (Boundary conditions).

2.2 (Superposition).

2.3 (Standing waves).

2.2 Sound waves

2.4 (Doppler effect).

beat, Helmholtz resonator, supersonic waves, shock,

Chapter 3

Thermodynamics

3.1 First law

Equation of states?

3.1 (Kinetic theory).

3.2 (Quasi-static processes).

3.3 (Composite systems).

3.2 Second law

3.4 (Entropy).

3.5 (Free energies).

Chapter 4

Electromagnetism

4.1 Electrostatics

4.2 Magnetostatics

4.3 Electromagnetic induction

4.4 Circuits

Chapter 5

Optics

5.1 Geometric optics

5.2 Interference

5.3 Diffraction

Chapter 6

Modern physics

6.1 Special relativity

6.2 Atoms

6.3 Nuclear physics