Operator Algebra

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Contents

Part I

C^*-algebras

Chapter 1

1.1 Multiplier algebra

1.1 (Multiplier algebra)**.** Let $\mathcal{A}$ be a C*-algebra. A *double centralizer* of $\mathcal{A}$ is a pair (L, R) of bounded linear maps on $\mathcal{A}$ such that $aL(b) = R(a)b$ for all $a, b \in \mathcal{A}$. The *multiplier algebra* $M(\mathcal{A})$ of $\mathcal{A}$ is defined to be the set of all double centralizers of $\mathcal{A}$.

1.2 (Essential ideals)**.** (a) Hilbert C*-module description

1.3 (Examples of multiplier algebras)**.** (a) $M(K(H)) \cong B(H)$.

(b) $M(C_0(\Omega)) \cong C_b(\Omega)$.

Proof. (a)

(b) First we claim $C_0(\Omega)$ is an essential ideal of $C_b(\Omega)$. Since $C_b(\Omega) \cong C(\beta\Omega)$, and since closed ideals of $C(\beta\Omega)$ are corresponded to open subsets of $\beta\Omega$, $C_0(\Omega) \cap J$ is not trivial for every closed ideal J of $C_b(\Omega)$.

Now we have an injective *-homomorphism $C_b(\Omega) \to M(C_0(\Omega))$, for which we want to show the surjectivity. Let $g \in M(C_0(\Omega))^+$. □

1.4 (Strict topology)**.**

1.2 Hereditary C*-subalgebras

1.5 (Hereditary C*-subalgebra and state embedding)**.**

Part II

Von Neumann algebras

Chapter 2

2.1 (Semi-finite traces)**.** Let M be a von Neumann algebra and τ is a trace. For a trace τ

(a) τ is semi-finite if and only if $x \in M^+$ has a net $x_\alpha \in L^1(M,\tau)^+$ such that $x_\alpha \uparrow x$ strongly.

(b) Let τ be normal and faithful. Then, τ is semi-finite if and only if

$$\tau(x) = \sup\{\,\tau(y) : y \le x,\ y \in L^1(M,\tau)^+\,\} \quad \text{for} \quad x \in M^+.$$

Part III

Operator K-theory

Chapter 3

Brown-Douglas-Fillmore theory

3.1 (Haagerup property)**.**

Baum-Connes conjecture Non-commutative geometry Elliott theorem

Part IV

Subfactor theory