Problem set

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November 10, 2022

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Let a_n be a real sequence and $S_n := a_1 + \cdots + a_n$ be the partial sum.

1. Show that if $a_n \downarrow 0$ and $S_n \leq 1 + na_n$, then $S_n \leq 1$.

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(a) At the time a particle at the origin is thrown with an initial speed v_0, another particle at (a, b) with $b > 0$ begins a free fall. Find the minimum value of v_0 such that two particles collide in the region $y \geq 0$.

(b) A particle at the ground is projected with an initial speed v_0 at an angle θ, towards a vertical wall as far away as L. Find v_0 such that it bounces back to its original position after striking the wall and ground only once.

(c) A particle is released at height h from a plane inclined at an angle θ to the horizontal. Find the length l from the first point of collision to the point at which the particle begins to slide down.

Solution. (b) $R := v_0^2 \sin^2\theta / 2g$

$$L + e(R - L) + eR = 2L.$$

□

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