Complex Analysis

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Contents

Chapter 1

Holomorphic functions

1.1 Holomorphic functions

1.1. Let $p \in \mathbb{C}[z]$ with $p(z) = \sum_{k=0}^{n} a_k z^k$. There is $R > 0$ such that $\frac{|a_n|}{2}|z|^n \leq |p(z)| \leq \frac{3|a_n|}{2}|z|^n$ for $|z| \geq R$.

Proof. Since $z \neq 0$, let $w(z) := p(z)/z^n - a_n$. If we choose $R > 0$ such that

$$\max_{0 \leq k \leq n} \frac{|a_k|}{R^{n-k}} \leq \frac{|a_n|}{2n}$$

so that we have for $|z| \geq R$

$$|w(z)| \leq \sum_{k=0}^{n-1} \frac{|a_k|}{|z|^{n-k}} \leq \frac{|a_n|}{2},$$

hence we get

$$|p(z)| = |a_n + w(z)||z|^n \geq \frac{|a_n|}{2}|z|^n.$$

$\square$

1.2. Let $f : \Omega \to \mathbb{C}$ be a holomorphic function on a domain. Then, $f\bar{(}z) = f(\bar{z})$ if and only if $f(z) \in \mathbb{R}$ for $z \in \Omega \cap \mathbb{R}$.