Algebraic Structures

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Contents

Part I

Groups

Chapter 1

Subgroups

subgroups homomorphisms, image, kernel, inverse images normality, quotient, coset counting direct sum, direct product

Chapter 2

Group actions

2.1 Orbits and stabilizers

Invariants on orbit space. The size and number of orbits.

2.1 (Transitive actions)**.** stabilizer of an action is well defined

2.2 (Free actions)**.** no fixed point, trivial stabilizer for any point, every orbit has 1-1 correspondence to group

2.2 Action by conjugation

2.3 Action by left multiplication

Chapter 3

Symmetry groups

elements by order elements by conjugacy class subgroups by conjugacy class

3.1 Cyclic groups

3.2 Symmetric groups

3.3 Matrix groups

dihedral groups

Exercises

3.1. Let G be a finite group. If $G/Z(G)$ is cylic, then G is abelian.

3.2. Let G be a finite group. If $x \mapsto x^3$ is a surjective endomorhpism, then G is abelian.

Part II

Rings

Chapter 4

Ideals

Chapter 5

Integral domains

Chapter 6

Polynomial rings

6.1 Irreducible polynomials

relation to maximal ideals Irreducibles over several fields

Part III

Modules

Chapter 7

Exact sequences

free modules inj, proj

Chapter 8

Hom functor and tensor products

hom and duality tensor product algebras?

Chapter 9

Modules over a principal ideal domain

invariant factors and elementary divisors

Part IV

Vector spaces

Chapter 10

Multilinear forms

Duality Adjoints Inner product

Chapter 11

Normal forms

11.1 Finitely generated $\mathbb{F}[x]$-modules

cyclic subspaces

11.2 Similarity

GL, SL, PSL?

11.3 Spectral theorems

Exercises

Chapter 12

Tensor algebras

Exterior algebras Symmetric algebras