

Algebraic Structures

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Part I

Groups

Chapter 1

Subgroups

subgroups homomorphisms, image, kernel, inverse images normality, quotient, coset
counting direct sum, direct product

Chapter 2

Group actions

2.1 Orbits and stabilizers

Invariants on orbit space. The size and number of orbits.

2.1 (Transitive actions). stabilizer of an action is well defined

2.2 (Free actions). no fixed point, trivial stabilizer for any point, every orbit has 1-1 correspondence to group

2.2 Action by conjugation

2.3 Action by left multiplication

Chapter 3

Symmetry groups

elements by order elements by conjugacy class subgroups by conjugacy class

3.1 Cyclic groups

3.2 Symmetric groups

3.3 Matrix groups

dihedral groups

Exercises

3.1. Let G be a finite group. If $G/Z(G)$ is cyclic, then G is abelian.

3.2. Let G be a finite group. If $x \mapsto x^3$ is a surjective endomorphism, then G is abelian.

Part II

Rings

Chapter 4

Ideals

Chapter 5

Integral domains

Chapter 6

Polynomial rings

6.1 Irreducible polynomials

relation to maximal ideals Irreducibles over several fields

Part III

Modules

Chapter 7

Exact sequences

free modules inj, proj

Chapter 8

Hom functor and tensor products

hom and duality tensor product algebras?

Chapter 9

Modules over a principal ideal domain

invariant factors and elementary divisors

Part IV

Vector spaces

Chapter 10

Multilinear forms

Duality Adjoints Inner product

Chapter 11

Normal forms

11.1 Finitely generated $\mathbb{F}[x]$ -modules

cyclic subspaces

11.2 Similarity

GL, SL, PSL?

11.3 Spectral theorems

Exercises

Chapter 12

Tensor algebras

Exterior algebras Symmetric algebras