Representation Theory

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Contents

Part I

Finite group representations

Chapter 1

Character theory

1.1 (Definition of group representations).

1.2 (Interwining maps).

1.3 (Irreducible representations). indecomposable and irreducible

1.4 (Maschke's theorem).

1.5 (Space of interwining maps and inner product). $\mathrm{Hom}_G(V, W)$ dimension is equal to the inner product of characters

direct sum of rep -> sum of char

injectivity proof Suppose two characters p and r are equal. Maschke: all characters are sum of irreducible characters Schur: orthogonality, so the coefficients are all equal irreducible-factor-wisely construct an isomoprhism.

irreducible characters form an ONB of the space of class functions proof: irred number counting group algebra double counting? surjectivity desciption nonnegative integral linear combination of irreducible characters

character table: computation of matrix elements by character table abelian group, 1dim rep lifting

1.6 (Modules and representations). ring <-> group module <-> representation finitely generated <-> finite dimensional

1.7 (Group algebra). or group ring, regular representation $k[G]$-module and G-representation correspondence

1.8 (Wedderburn's theorem). central idempotents dimension computation any irrep is a summand of CG, and the dimension arg implies CG is dsum of all irrep.

tensoring, complex, real symmetric, exterior

the dual inner product: conjugacy check relation to normal subgroups center of rep

algebraic integer dim of irrep divides group order burnside pq theorem

Chapter 2

Computation of irreducible representations

2.1 Symmetric groups

young tableux

2.2 Linear groups over finite fields

GL2 and SL2 over finite fields

2.3 Induced representations

induction and restriction of reps (from and to subgroup) frobenius reciprocity, mackey theory

Chapter 3

Brauer theory

Part II

Lie groups

Chapter 4

Lie correspondence

Lie's three theorems Baker-Campbell-Hausdorff formula

Chapter 5

Classical groups

SO, SU

Chapter 6

Representations of compact groups

unitary representation fundamental group obstruction infinite dimension: Peter Weyl projective representations

Part III

Lie algebras

Chapter 7

Semisimplicity

killing forms, cartan subalgebra

Chapter 8

Root systems

dynkin digram real forms

Chapter 9

Representations of Lie algebras

universal enveloping algebra, pbw theorem, verma module highest weight theorem

Part IV

Quantum groups

Chapter 10

Hopf algebras

Chapter 11