

Contents

I	Algebra	3
1	Equations	4
1.1	Polynomials	4
1.2	Simultaneous equations	4
1.3	Real solutions	4
1.4	Integer solutions	4
2	Inequalities	5
2.1	Symmetry	5
2.2	Homogeneity	5
3	Functions	6
3.1	Properties of functions	6
3.2	Functions over \mathbb{R}	6
3.3	Other domains	6
II	Combinatorics	7
4	Counting	8
4.1	Orbits	8
4.2	Generating functions	8
5	Algorithms	9
5.1	Invariants	9
5.2	Games	9
6	Graphs	10
6.1	Double counting	10
6.2	Non-constructive existence	10
III	Geometry	11
7	Plane geometry	12
7.1	Angle chasing	12
7.2	Length ratios	12
7.3	Triangle centers	12
7.4	Conics	12

8	Analytic methods	13
8.1	Trigonometry	13
8.2	Complex variables	13
8.3	Barycentric coordinates	13
9	Transformations	14
9.1	Similarity	14
9.2	Inversion	14
9.3	Projectivity	14
IV	Collegiate courses	15
10	Calculus	16
10.1	Asymptotics	16
10.2	Infinite series	16
10.3	Indefinite integrals	16
10.4	Integral inequalities	16
11	Linear algebra	17
11.1	Determinants	17
11.2	Spectrum	17
11.3	Commuting matrices	17
11.4	Positive definiteness	17
12	General physics	18
12.1	Mechanics	18
12.2	Waves	18
12.3	Thermodynamics	18
12.4	Electromagnetism	18
12.5	Optics	18
12.6	Atoms	18

Part I

Algebra

Chapter 1

Equations

1.1 Polynomials

1.2 Simultaneous equations

1.3 Real solutions

factorization discriminant image of square(polynomial) intermediate value

1.4 Integer solutions

factorization root identity image of square(polynomial)

Chapter 2

Inequalities

2.1 Symmetry

2.2 Homogeneity

Chapter 3

Functions

3.1 Properties of functions

3.2 Functions over \mathbb{R}

3.3 Other domains

Part II

Combinatorics

Chapter 4

Counting

4.1 Orbits

4.2 Generating functions

Chapter 5

Algorithms

5.1 Invariants

5.2 Games

Chapter 6

Graphs

6.1 Double counting

6.2 Non-constructive existence

Pigeonhole principle, Probabilistic methods, Extremal theory

Part III

Geometry

Chapter 7

Plane geometry

7.1 Angle chasing

Cyclic quadrilaterals

7.2 Length ratios

menelous and ceva

7.3 Triangle centers

7.4 Conics

Chapter 8

Analytic methods

8.1 Trigonometry

8.2 Complex variables

8.3 Barycentric coordinates

Chapter 9

Transformations

9.1 Similarity

spiral homothety

9.2 Inversion

9.3 Projectivity

Part IV

Collegiate courses

Chapter 10

Calculus

10.1 Asymptotics

10.2 Infinite series

Let a_n be a real sequence and $S_n := a_1 + \cdots + a_n$ be the partial sum.

1. Show that if $a_n \downarrow 0$ and $S_n \leq 1 + na_n$, then $S_n \leq 1$.

10.3 Indefinite integrals

10.4 Integral inequalities

Chapter 11

Linear algebra

11.1 Determinants

11.2 Spectrum

canonical forms

11.3 Commuting matrices

two by two matrices

11.4 Positive definiteness

Chapter 12

General physics

12.1 Mechanics

Let e and g be the coefficient of restitution and gravitational acceleration.

1. At the time a particle at the origin is thrown with an initial speed v_0 , another particle at (a, b) with $b > 0$ begins a free fall. Find the minimum value of v_0 such that two particles collide in the region $y \geq 0$.
2. A particle at the ground is projected with an initial speed v_0 at an angle θ , towards a vertical wall as far away as L . Find v_0 such that it bounces back to its original position after striking the wall and ground only once.
3. A particle is released at height h from a plane inclined at an angle θ to the horizontal. Find the length l from the first point of collision to the point at which the particle begins to slide down.

12.2 Waves

12.3 Thermodynamics

12.4 Electromagnetism

12.5 Optics

12.6 Atoms