ELECTRE Method Using Interval-Valued Intuitionistic Fuzzy Sets and Possibility Theory for Multi-Criteria Decision Making Problem Resolution

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***Abstract*—In this work we propose an approach of multi-criteria decision making (MCDM) using Elimination Et Choice Transiting Reality (ELECTRE) methods, interval-valued intuitionistic fuzzy (IVIF) sets and possibility theory. The proposition concerns the computation of concordance sets and discordance sets using possibility measures. The proposed approach is applied to select the best investment projects decision problem from literature. Therefore results are compared and concluding remarks are given.**

***Index Terms*—Interval-valued intuitionistic fuzzy sets, possibility measure, multi-criteria decision making, concordance matrix, discordance matrix, ELECTRE I, ELECTRE II .**

I. INTRODUCTION

We are interested to the technique for Elimination Et Choice Transiting Reality [1]: ELECTRE I and ELECTRE II which have been widely applied numerous domains. The first version deals with crisp values. When information are incomplete, imprecise and uncertain information using fuzzy sets or one of its generalization are employed to define other versions of ELECTRE. Boran et al. [2] applied ELECRE method with intuitionistic fuzzy sets, to select appropriate supplier in group decision making environment (IFSs) [3]. Hashemi et al. [4] presented ELECTRE III method for solving MCDM problems described with IVIF data. Ermatita et al. [5] applied ELECTRE to detect genes that cause cancer. In the same way, San Cristobal Meteo [6] used ELECTRE III to rank alternatives of Biomass plants data set. Possibility theory [7] has been used successfully in MCDM problem resolution. Chen [8] extended ELECTRE and TOPSIS methods using likelihood based on IVIF sets (IVIFSs) to select an appropriate bridge construction. Zhang and Xu [9] used ELECTRE based on interval numbers and possibility theory to compute the concordance and discordance index for the selection of leather manufacture alternatives. Chen [10] used score function to determine the concordance and discordance indices. Our aims is to compute concordance sets and discordance sets using possibility measures under interval-valued intuitionistic fuzzy environment using ELECTRE I and ECLECTRE II.
This paper is organized as follows: In section 2, some preliminaries about IVIFS and possibility theory are presented. In section 3, MCDM method is presented. In section 4, an approach using ELECTRE I and ELECTRE II methods, IVIF sets and possibility theory is proposed. In section 5, the proposed approach is applied to investment projects and in section 6 a conclusion is deduced.

II. PRELIMINARIES

In this section, we present basic definitions of IVIFSs and possibility theory.

A. Definition of IVIFSs

An IVIFS $\tilde{A}$ is defined by Atanassov as [11]:

$$\tilde{A} = \left\{ \left\langle x_i, \left[\mu_{\tilde{A}}^L(x_i), \mu_{\tilde{A}}^U(x_i)\right], \left[\nu_{\tilde{A}}^L(x_i), \nu_{\tilde{A}}^U(x_i)\right] \right\rangle | x_i \in X \right\} \quad (1)$$

where $\left[\mu_{\tilde{A}}^L(x_i), \mu_{\tilde{A}}^U(x_i)\right]$ and $\left[\nu_{\tilde{A}}^L(x_i), \nu_{\tilde{A}}^U(x_i)\right]$ denote the intervals of membership and non membership degrees of an element $x_i \in \tilde{A}$ satisfying:
$0 \leq \mu_{\tilde{A}}^U(x_i) + \nu_{\tilde{A}}^U(x_i) \leq 1$, $0 \leq \nu_{\tilde{A}}^L(x_i) \leq \nu_{\tilde{A}}^U(x_i) \leq 1$ and $\mu_{\tilde{A}}^L(x_i) \leq \mu_{\tilde{A}}^U(x_i)$. The hesitation degree of an interval-valued intuitionistic fuzzy number x_i to set $\tilde{A}$ is defined as: $\pi_{\tilde{A}}(x_i) = [1 - \mu_{\tilde{A}}^U(x_i) - \nu_{\tilde{A}}^U(x_i), 1 - \mu_{\tilde{A}}^L(x_i) - \nu_{\tilde{A}}^L(x_i)]$.
If $\mu_{\tilde{A}}^L(x_i) = \mu_{\tilde{A}}^U(x_i)$ and $\nu_{\tilde{A}}^L(x_i) = \nu_{\tilde{A}}^U(x_i)$ then $\tilde{A}$ is reduced to an IFS.

B. Possibility Measures

The possibility theory, investigated by Zadeh and Dubois, defines a pair of measures: possibility and necessity [7], [12]. Let A a subset of states S and $\prod$ a possibility distribution, which maps the set of interpretations Ω to the binary set $\{0, 1\}$.

- The possibility measure π of A is defined in $[0, 1]$.

$$\pi(A) = \sup_{s \in A} \prod(s) \quad (2)$$

  $\pi(A)$ evaluates to what extent A is consistent with $\prod$ and reflects the most normal situation in which A is true.
- The degree of necessity (certainty) quantifies the certainty of A.

$$N(A) = 1 - \pi(A^c) = \inf_{s \notin A} (1 - \prod(s)) \quad (3)$$

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where A^c is the complement of A and the Necessity measure $N(A)$ evaluates to what extent A is certainly implied by $\prod$ and reflects the more normal situation where A is false.
Note that if $N(A) = 1$: A is certainly true, and $N(A) = 0$: A is not certain (A still be possible).

III. MCDM Based Interval valued intuitionistic Fuzzy Numbers

For a multi-criteria decision making problem, let $A = \{A_1, A_2, \ldots, A_m\}$ the set of alternatives, $C = \{C_1, C_2, \ldots, C_n\}$ the set of criteria and $W = (\omega_1, \omega_2, \ldots, \omega_n)^T$ the set of weights of criteria, where $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$.
If the values of criteria are represented by interval valued intuitionistic fuzzy numbers $\tilde{\alpha}_{ij} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}])$, where $[a_{ij}, b_{ij}]$ are the fuzzy membership degree of the alternative A_i over criteria C_j and $[c_{ij}, d_{ij}]$ represents the fuzzy non-membership degree of the alternative A_i over criteria C_j, then the decision matrix $\tilde{D}_{m\times n}(a_{ij})$ is obtained as.

$$\tilde{D} = \begin{pmatrix} & X_1 & X_2 & \ldots & X_n \\ A_1 & ([a_{11}, b_{11}], [c_{11}, d_{11}]) & ([a_{12}, b_{12}], [c_{12}, d_{12}]) & \cdots & ([a_{1n}, b_{1n}], [c_{1n}, d_{1n}]) \\ A_2 & ([a_{21}, b_{21}], [c_{21}, d_{21}]) & ([a_{22}, b_{22}], [c_{22}, d_{22}]) & \cdots & ([a_{2n}, b_{2n}], [c_{2n}, d_{2n}]) \\ & \vdots & \vdots & \vdots & \vdots \\ A_m & ([a_{m1}, b_{m1}], [c_{m1}, d_{m1}]) & ([a_{m2}, b_{m2}], [c_{m2}, d_{m2}]) & \ldots & ([a_{mn}, b_{mn}], [c_{mn}, d_{mn}]) \end{pmatrix} \tag{4}$$

IV. Presentation of the Different Version of ELECTRE

The first version of ELECTRE is ELECTRE I [13]. There are other versions of ELECTRE: ELECTRE II, III, IV and ELECTRE TRI. All these versions are based on the same fundamental concepts but they differ on some operations and on the type of the decision problem [14]. Specifically, ELECTRE I is used for selecting the problems, ELECTRE TRI for the assignment of those problems and ELECTRE II, III and IV for ranking the problems. The latter uses pairwise comparison by using concordance and discordance indexes. The concordance index indicates that the alternative A is better than the alternative B in term of sum of weights. A discordance index is opposite to the concordance index. The decision maker uses concordance and discordance indices to analyse outranking relations among different alternatives and to choose the best alternative. Next $A = \{A_1, A_2, \ldots, A_m\}$ represents a finite set of alternatives and $X = \{x_1, x_2, \ldots, x_n\}$ represent n criteria and the decision matrix (4) is given in section III. The preferences are increasing with value x_j, if x_j is the greatest value then A is the best alternative. The outranking relation of $a \longrightarrow b$ (denoted as aSb) means "a is at least as good as b". To validate a statement aSb, some important conditions concordance which are given by (8) and discordance index (9) should be verified.

A. ELECTRE I Method From Literature

The ELECTRE method consists of the following steps:

- *Step 1:* Compute the normalized of decision matrix:

$$R_{ij} = \frac{X_{ij}}{\sqrt{\sum_{i=1}^{m}(X_{ij}^2)}} \tag{5}$$

- *Step 2:* Compute the weights of criteria using the IVIF entropy method [15],[16] as follows:

$$H_j = \frac{1}{m}\sum_{i=1}^{m}(\sin\frac{\pi\times[1+\mu_{ij}^L+p(\mu_{ij}^U-\mu_{ij}^L)-\nu_{ij}^L-q(\nu_{ij}^U-\nu_{ij}^L)]}{4} + \sin\frac{\pi\times[1-\mu_{ij}^L-p(\mu_{ij}^U-\mu_{ij}^L)+\nu_{ij}^L+q(\nu_{ij}^U-\nu_{ij}^L)]}{4} - 1)\times\frac{1}{\sqrt{(2)}-1} \tag{6}$$

- *Step 3:* Compute the weighted decision matrix using weights of criteria (7)

$$V_{ij} = w_j R_{ij} \tag{7}$$

- *Step 4:* Determine the concordance index and the discordance index for each pair assessment of the relation ranking. Each pair of alternatives is defined by A_k and A_l where $k, l = 1, 2, \ldots, m$ and the criteria (j) is divided into two parts.
  - The set of concordance index is defined by $\{c_{kl}\}$ shows the sum of weights of criteria for which A_k alternative is better than A_l alternative.

$$\{c_{kl}\} = \{j/V_{kj} \geq V_{ij}\} \tag{8}$$

    with $j = 1, \ldots, n$
  - The set of discordance index is defined by $\{d_{kl}\}$ given by

$$\{d_{kl}\} = \{j/V_{kj} \leq V_{ij}\} \tag{9}$$

    with $j = 1, \ldots, n$
- *Step 5:* Determine the matrix of concordance (C) which contains elements of the computed concordance index and associated with attribute weights:

$$C_{kl} = \sum_{j\in c_{kl}} w_j \tag{10}$$

- *Step 6:* Determine the matrix of discordance (D) which is formed from the discordance indexes calculated by (9).

$$D_{Kl} = \frac{\max\{v_{kj} - v_{ij}\}_{j\in d_{kl}}}{\max\{v_{kj} - v_{ij}\}_{j\in v_j}} \tag{11}$$

where $\max\{v_{kj} - v_{ij}\}$ determines the maximum difference on any criterion. Compute the threshold $\underline{c}$ using the matrix of concordance (C).

$$\underline{c} = \frac{\sum_{k=1}^{m}\sum_{l=1}^{m} C_{kl}}{m(m-1)} \tag{12}$$

A alternative A_k dominates alternative A_l if $c_{kl} \geq \underline{c}$.

- *Step 7:* Determine the matrix F: the elements of the matrix F are determined as the dominant concordance index:

$$F_{kl} = \begin{cases} 1, & C_{kl} \geq \underline{c} \\ 0, & C_{kl} < \underline{c} \end{cases}$$

Similarly for dominant discordance matrix G with threshold $\underline{d}$.

$$\underline{d} = \frac{\sum_{k=1}^{m}\sum_{l=1}^{m} d_{kl}}{m(m-1)} \tag{13}$$

The elements G are determined as the dominant discordance:

$$G_{kl} = \begin{cases} 1 & d_{kl} \geq \underline{d} \\ 0 & d_{kl} < \underline{d} \end{cases}$$

- *Step 8:* Calculate the aggregation of the dominant matrix (E) gives a partial preference order of alternatives as follows:

$$E_{kl} = F_{kl} \times G_{kl} \tag{14}$$

if $E_{kl} = 1$ then the alternative A_k is the best among alternative A_l.

B. Proposition of IVIF ELECTRE I using Possibility Theory

The IVIF ELECTRE I using possibility theory method consists of the following steps:

- *Step 1:* We extended the normalized of decision matrix with interval weights used in [17] to interval valued intuitionistic normalisation of decision matrix.
  - For benefit criteria

$$R_{\mu_{ij}^L} = \frac{\mu_{ij}^L}{\sqrt{\sum_{i=1}^{m}(\mu_{ij}^L)^2) + (\mu_{ij}^U)^2}} \tag{15}$$

$$R_{\mu_{ij}^U} = \frac{\mu_{ij}^U}{\sqrt{\sum_{i=1}^{m}(\mu_{ij}^L)^2 + (\mu_{ij}^U)^2}} \tag{16}$$

$$R_{\nu_{ij}^L} = \frac{\nu_{ij}^U}{\sqrt{\sum_{i=1}^{m}(\nu_{ij}^L)^2 + (\nu_{ij}^U)^2}} \tag{17}$$

$$R_{\nu_{ij}^U} = \frac{\nu_{ij}^L}{\sqrt{\sum_{i=1}^{m}(\nu_{ij}^L)^2 + (\nu_{ij}^U)^2}} \tag{18}$$

  - For cost criteria

$$R_{\mu_{ij}^L} = \frac{1/\mu_{ij}^U}{\sqrt{\sum_{i=1}^{m}(1/\mu_{ij}^L)^2}} \tag{19}$$

$$R_{\mu_{ij}^U} = \frac{1/\mu^L ij}{\sqrt{\sum_{i=1}^{m}(1/\mu_{ij}^U)^2}} \tag{20}$$

$$R_{\nu_{ij}^L} = \frac{1/\nu_{ij}^U}{\sqrt{\sum_{i=1}^{m}(1/\nu_{ij}^L)^2}} \tag{21}$$

$$R_{\nu_{ij}^U} = \frac{1/\nu_{ij}^L}{\sqrt{\sum_{i=1}^{m}(1/\nu_{ij}^U)^2}} \tag{22}$$

- *Step 2:* Compute the weighted normalized decision matrix:

$$v_{ij} = w_j R_{ij} \tag{23}$$

- *Step 3:* Determine the concordance set C_{ij} and the discordance set D_{ij}, therefore, calculate the possibility matrix of each alternative under each attribute using formula (25).
- *Step 4:* Calculate the ranking vectors using formula as follows:

$$\upsilon_i = \frac{1}{m(m-1)}(\sum_{j=1}^{m} P_{ij} + \frac{m}{2} - 1), i \in M \tag{24}$$

Where $p_{ij} = p(\tilde{\alpha}_{ij} \geq \tilde{\alpha}'_{ij})$ [18] and more details of possibility measures are given in [19].

$$\begin{aligned} p(\tilde{\alpha}_{ij} \geq \tilde{\alpha}'_{ij}) &= \gamma \min \\ &(\max(\frac{b_{ij} - a'_{ij}}{b_{ij} - a_{ij} + b'_{ij} - a'_{ij}}, 0), 1) + (1-\gamma) \\ &\min(\max(\frac{d'_{ij} - c'_{ij}}{d_{ij} - c_{ij} + d'_{ij} - c'_{ij}}, 0), 1) \end{aligned} \tag{25}$$

where $\gamma \in [0, 1]$ gives the decision makers' preference on membership degree or non-membership degree. When the decision maker is optimal, $\gamma \geq 0.5$ and when the decision maker is pessimistic, $\gamma < 0.5$.

- *Step 5:* Determine the rank of alternatives under each attribute using (24)
- *Step 6:* According to the achieved rank of vectors and alternatives, determine the concordance set c_{kl} and the discordance set d_{kl} respectively:

$$c_{kl} = \{j | \upsilon_{kj} \geq \upsilon_{lj}\} \tag{26}$$

$$d_{kl} = \{j | \upsilon_{kj} < \upsilon_{lj}\} \tag{27}$$

- *Step 7:* Determine the matrix of concordance (C) which contains elements of the computed concordance index and associated with attribute weights:

$$C_{kl} = \sum_{j \in c_{kl}} w_j \tag{28}$$

- *Step 8:* Determine the matrix of discordance (D) which is formed from the discordance indexes calculated by (9). D is associated with values of attributes.

$$D_{Kl} = \frac{\max_{j \in D_{kl}} w_D * dis(X_{kj}, X_{lj})}{\max_{j \in J} dis(X_{kj}, X_{lj})} \tag{29}$$

where w_D is the sum of weights associated with criteria.

$$w_D = \sum_{j \in D_{kl}} w_j \tag{30}$$

and

$$dis(X_{kj}, X_{lj}) = \sqrt{\frac{1}{4}(\mu^L_{kj}-\mu^L_{lj})^2+(\mu^U_{kj}-\mu^U_{lj})^2+(\nu^L_{kj}-\nu^L_{lj})^2)+(\nu^U_{kj}-\nu^U_{lj})^2)} \tag{31}$$

- *Step 9:* Compute the threshold $\underline{c}$ using the matrix of concordance (C).

$$\underline{c} = \frac{\sum_{k=1}^{m}\sum_{l=1}^{m} C_{kl}}{m \times (m-1)} \tag{32}$$

The alternative A_k dominates the alternative A_l, if $c_{kl} \geq \underline{c}$.

- *Step 10:* Determine the matrix F of elements determined as dominant concordance indexes:

$$F_{kl} = \begin{cases} 1, & C_{kl} \geq \underline{c} \\ 0, & C_{kl} < \underline{c} \end{cases}$$

The same is done for dominant discordance matrix G with $\underline{d}$ threshold.

$$\underline{d} = \frac{\sum_{k=1}^{m}\sum_{l=1}^{m} D_{kl}}{m(m-1)} \tag{33}$$

The elements of the matrix G are determined as:

$$G_{kl} = \begin{cases} 1 & D_{kl} \geq \underline{d} \\ 0 & D_{kl} < \underline{d} \end{cases}$$

- *Step 11:* calculate the aggregation of the dominant matrix E) gives a partial preference order of alternatives, given as follows:

$$E_{kl} = F_{kl} \times G_{kl} \tag{34}$$

if $e_{kl} = 1$ than the alternative A_k is the best to be selected from the alternative A_l.

C. Proposition of IVIF ELECTRE II Using Possibility Theory

ELECTRE II [20] uses the same concordance discordance indexes as ELECTRE I, but it is necessary to calculate the pure value of these indexes [21].

- Step 1–8: Same as steps of ELECTRE I, section IV-B.
- Step 9: Compute the pure concordance and the pure discordance indices as follows:
  - Pure value of the concordance

$$c_k = \sum_{i=1, i\neq k}^{n} c_{ki} - \sum_{i=1, i\neq k}^{n} c_{ik} \tag{35}$$

  - Pure value of the discordance

$$d_k = \sum_{i=1, i\neq k}^{n} d_{ki} - \sum_{i=1, i\neq k}^{n} d_{ik} \tag{36}$$

The alternatives are ranked according to the highest average rank.

V. Application of ELECTRE Methods

A. Description of data sets

The data set is borrowed from [22]. A management team should evaluate each investment to select the best among the investment projects considering four criteria. The aggregated matrix of experts preferences presented as follows.

$$\begin{pmatrix} [0.6452,0.7382],[0.0974,0.2129] & [0.6347,0.732],[0,0.2155] & [0.6724,0.7662],[0,0.1838] & [0.6718,0.7703],[0,0.1764] \\ [0.5590,0.6560],[0.1687,0.2927] & [0.6997,0.7984][0,0.1498] & [0.5148,0.6388],[0,0.2934] & [0.6724,0.7662],[0,0.1838] \\ [0.2000,0.3492],[0.3845,0.5721] & [0.3891,0.4880],[0.3233,0.4581] & [0.2959,0.4056],[0.3845,0.5345] & [0.6347,0.7320],[0,0.2155] \\ [0.6974,0.7912],[0,0.1587] & [0.6880,0.7796],[0,0.1719] & [0.6880,0.77960],[0,0.1719] & [0.7632,0.8184],[0,0.1501] \end{pmatrix} \tag{37}$$

B. Application of IVIF ELECTRE I Method

We apply IVIFS ELECTRE I to rank investment projects presented in section V-A.

- We apply (15), (16), (17) and (18) to determine the normalized decision matrix as follows:

$$\begin{pmatrix} [0.6581,0.7529],[0.4160,0.9094] & [0.4605,0.5311],[0,0.6772] & [0.3922,0.4469],[0,0.5002] & [0.3366,0.3859],[0,0.4328] \\ [0.6486,0.7611],[0.4994,0.8664] & [0.5117,0.5839],[0,0.4053] & [0.3228,0.4006],[0,0.6218] & [0.3553,0.4048],[0,0.3630] \\ [0.4970,0.8678],[0.5578,0.8300] & [0.5240,0.6571],[0.3639,0.5156] & [0.3301,0.4525],[0.3477,0.4833] & [0.4809,0.5546],[0,0.1913] \\ [0.6612,0.7502],[0,1.0000] & [0.4645,0.5264],[0,0.7348] & [0.3802,0.4308],[0,0.5921] & [0.3587,0.3847],[0,0.4593] \end{pmatrix} \tag{38}$$

- We compute the weight of criteria using (6), achieved results are: $w = \{0.2132, 0.2551, 0.2235, 0.3082\}$
- Compute the weighted normalized decision matrix V using the weight vector w.

$$\begin{pmatrix} [0.1403,0.1605],[0.0887,0.1939] & [0.1175,0.1355],[0,0.1728] & [0.0877,0.0999],[0,0.1118] & [0.1037,0.1189],[0,0.1334] \\ [0.1383,0.1623],[0.1065,0.1847] & [0.1305,0.1490],[0,0.1034] & [0.0721,0.0895],[0,0.1390] & [0.1095,0.1248],[0,0.1119] \\ [0.1060,0.1850],[0.1189,0.1770] & [0.1337,0.1676],[0.0928,0.1315] & [0.0738,0.1011],[0.0777,0.1080] & [0.1482,0.1709],[0,0.0590] \\ [0.1410,0.1599],[0,0.2132] & [0.1185,0.1343],[0,0.1874] & [0.0850,0.0963],[0,0.1323], & [0.1106,0.1186],[0,0.1416] \end{pmatrix} \tag{39}$$

- Determine the concordance set C_{ij} and discordance set D_{ij}, first, we calculate the possibility matrix of the five alternatives under each attribute with formula (25), then we compute the concordance matrix (C) using equation (28). The obtained results are presented as follows:

$$P_1 = \begin{bmatrix} 0.5000 & 0.5129 & 0.5451 & 0.4449 \\ 0.4871 & 0.5000 & 0.5319 & 0.4313 \\ 0.4681 & 0.4549 & 0.5000 & 0.3985 \\ 0.6015 & 0.5687 & 0.5551 & 0.5000 \end{bmatrix} \quad (40)$$

$\upsilon_1 = (0.2502, 0.2459, 0.2351, 0.2688)$

$$P_2 = \begin{bmatrix} 0.5000 & 0.2557 & 0.3282 & 0.5116 \\ 0.7443 & 0.5000 & 0.6087 & 0.7668 \\ 0.3913 & 0.6718 & 0.5000 & 0.7032 \\ 0.2968 & 0.2332 & 0.4884 & 0.5000 \end{bmatrix} \quad (41)$$

$\upsilon_2 = (0.2163, 0.3017, 0.2722, 0.2099)$

$$P_3 = \begin{bmatrix} 0.5000 & 0.7467 & 0.7104 & 0.5880 \\ 0.2533 & 0.5000 & 0.4946 & 0.3222 \\ 0.5054 & 0.2896 & 0.5000 & 0.3764 \\ 0.6236 & 0.6778 & 0.4120 & 0.5000 \end{bmatrix} \quad (42)$$

$\upsilon_3 = (0.2954, 0.2142, 0.2226, 0.2678)$

$$P_4 = \begin{bmatrix} 0.5000 & 0.3822 & 0.1533 & 0.4363 \\ 0.6178 & 0.5000 & 0.1726 & 0.5840 \\ 0.8274 & 0.8467 & 0.5000 & 0.8529 \\ 0.1471 & 0.4160 & 0.5637 & 0.5000 \end{bmatrix} \quad (43)$$

$\upsilon_4 = (0.2060, 0.2395, 0.3356, 0.2189)$

- The ranking problem of interval valued intuitionistic fuzzy numbers can be transformed to a problem of solving the ranking vector of possibility degree matrix.
  $\upsilon_1 = (0.2502, 0.2459, 0.2351, 0.2688)$
  $\Rightarrow A4 > A1 > A2 > A3$
  $\upsilon_2 = (0.2163, 0.3017, 0.2722, 0.2099)$
  $\Rightarrow A2 > A3 > A1 > A4$
  $\upsilon_3 = (0.2954, 0.2142, 0.2226, 0.2678)$
  $\Rightarrow A1 > A4 > A3 > A2$
  $\upsilon_4 = (0.2060, 0.2395, 0.3356, 0.2189)$
  $\Rightarrow A3 > A2 > A4 > A1$
  According to the ranking vectors and alternatives ranking have got, we can get C_{ij} and D_{ij}, $i, j = 1, \ldots, 4$
  $C_{12} = \{1,3\}, C_{13} = \{1,3\}, C_{14} = \{2,3\}$
  $C_{21} = \{2,4\}, C_{23} = \{1,2\}, C_{24} = \{2,4\}$
  $C_{31} = \{2,4\}, C_{32} = \{3,4\}, C_{34} = \{2,4\},$
  $C_{41} = \{1,4\}, C_{42} = \{1,3\}, C_{43} = \{1,3\}$
  $D_{12} = \{2,4\}, D_{13} = \{2,4\}, D_{14} = \{1,4\}$
  $D_{21} = \{1,3\}, D_{23} = \{3,4\}, D_{24} = \{1,3\}$
  $D_{31} = \{1,3\}, D_{32} = \{1,2\}, D_{34} = \{1,3\}$
  $D_{41} = \{2,3\}, D_{42} = \{2,4\}, D_{43} = \{2,4\}$
- Calculate the concordance index using (28) as:

$$C = \begin{bmatrix} 0 & 0.4367 & 0.4367 & 0.4786 \\ 0.5633 & 0 & 0.4683 & 0.5633 \\ 0.5633 & 0.5317 & 0 & 0.5633 \\ 0.5214 & 0.4367 & 0.4367 & 0 \end{bmatrix} \quad (44)$$

- Calculate the discordance index using (29) as:

$$D = \begin{bmatrix} 0 & 0.0603 & 0.0871 & 0.4973 \\ 0.5633 & 0 & 0.0875 & 0.5633 \\ 0.5623 & 0.6381 & 0 & 0.5075 \\ 0.1394 & 0.2746 & 0.1460 & 0 \end{bmatrix} \quad (45)$$

- Compute the concordance dominance matrix F. The threshold value $\bar{c}$ can be calculated as follows:
  $\bar{c} = \sum_{i=1}^{4} \sum_{j=1}^{4} C_{ij} = 0.5000$
  Through, we compare the elements of concordance matrix with the value $\bar{c}$. We achieve the concordance dominance matrix F as follows:

$$F = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad (46)$$

- Compute the discordance dominance matrix G using the threshold value $\bar{d}$ (33) as follows:
  $\bar{d} = \sum_{i=1}^{4} \sum_{j=1}^{4} D_{ij} = 0.3439$

$$G = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad (47)$$

- Calculate the aggregation of the matrix E using (34):

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad (48)$$

The obtained results are computed using (34) showing that the alternative A4 dominate A1, revealing that the best alternative is A2 which ranks first.

C. Application of ELECTRE II

- Determine the net superior values (NSV) and the net inferior values (NIV) of each alternative, Using the concordance index (44) and discordance matrix (45) to determine the final average rank (FAR). Table I compares the performance of each project of investment with the net superior and net inferior values. The computation results of the net superior values show that the best alternative that ranks first is A3 and the worst alternative is A1. However, the results are shown in table I revealing that the best alternative is A3 which ranks first and A4 ranked

TABLE I
ALTERNATIVE RANKING ORDER USING ELECTRE II

Alternatives	NSV	Rank	NIV	Rank	FAR
A1	-0.2960	4	-0.6203	3	3
A2	0.1898	2	0.2411	2	2
A3	0.3166	1	1.3872	1	1
A4	-0.2104	3	-1.0080	4	3

last, based on the net inferior values. According to the ELECTRE I method A4 dominate A1 but luck information for the rest of alternatives. Therefore ELECTRE I is used for selecting the best project of investment.

Based in the average rank presented in I shows that A3 is the best alternative and ranks first.

D. Comparison Results with other Methods from Literature

TABLE II
COMPARISON OF POSSIBILSTIC IVIF-ELECTREII RESULTS WITH THE DATA YIELDED BY DIFFERENT METHODS

Alternatives	A1	A2	A3	A4
Possibilistic IVIF ELECTRE II	3	2	1	3
IVIF ELECTRE III	1	2	3	4
TOPSIS [23]	1	2	4	3
TOPSIS [24]	1	2	3	4
COPRAS [25]	1	2	4	3

Table II shows that the best alternative is A3 using possibilistic IVIF ELECTRE II. However using IVIF ELECTRE III, TOPSIS and COPRAS method the best alternative is A1. It should be noted that the difference between the modified method and other methods presented in table II can be caused by the impact of possibility theory on the ELECTRE II.

VI. CONCLUSION

In this study, we detailed IVIFS ELECTRE I from literature. An approach of possibilistic ELECTRE is developed for MCDM problems with IVIFS using possibility measure for computation concordance and discordance sets. In the presented method, the decision maker select an ideal alternative with possibility measure. IVIFS method using possibility theory can efficiently help the decision-maker for making decisions. The approach is applied to rank investment projects. In future work, we will combine possibility measure with PROMETHEE method using IVIFS.

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