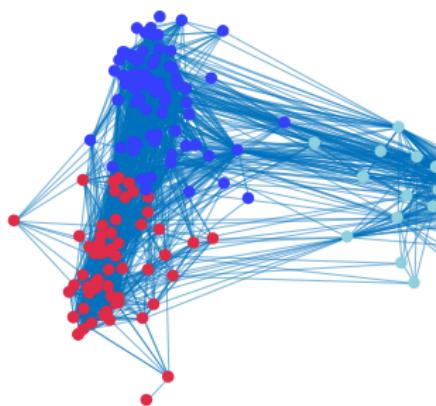


# Evolutionary Dynamics on Evolving Graphs



Barbara Ikica

Faculty of Mathematics and Physics  
University of Ljubljana

18 December 2019

# Evolutionary Dynamics on Evolving Graphs

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## Motivation

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### Evolutionary Dynamics on Graphs

The Modified  
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### Evolving Graphs

Co-evolution of the  
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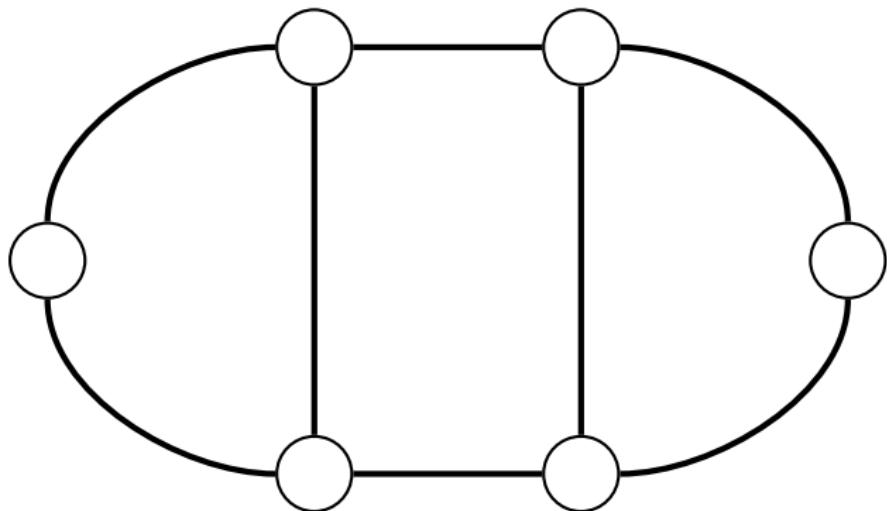
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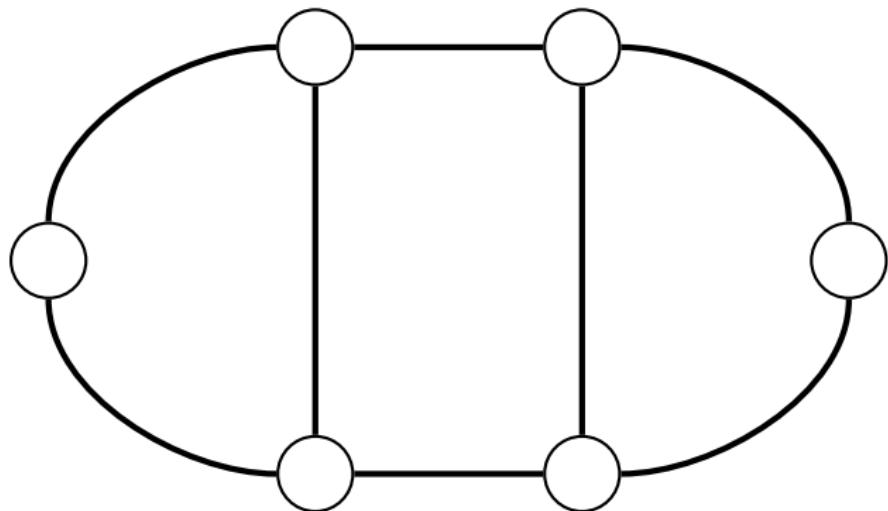
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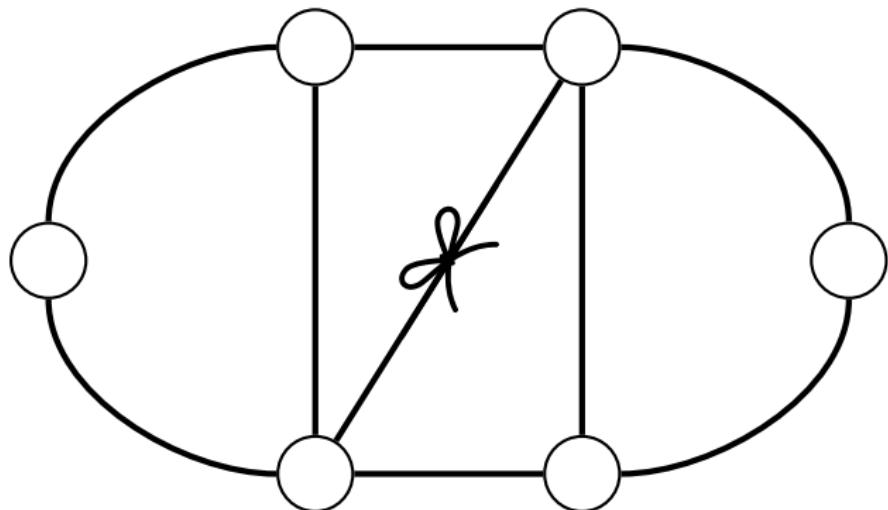
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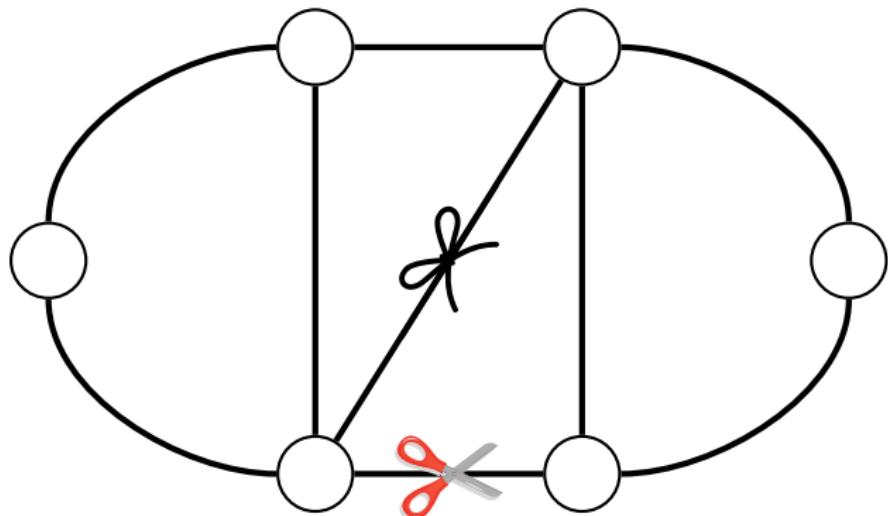
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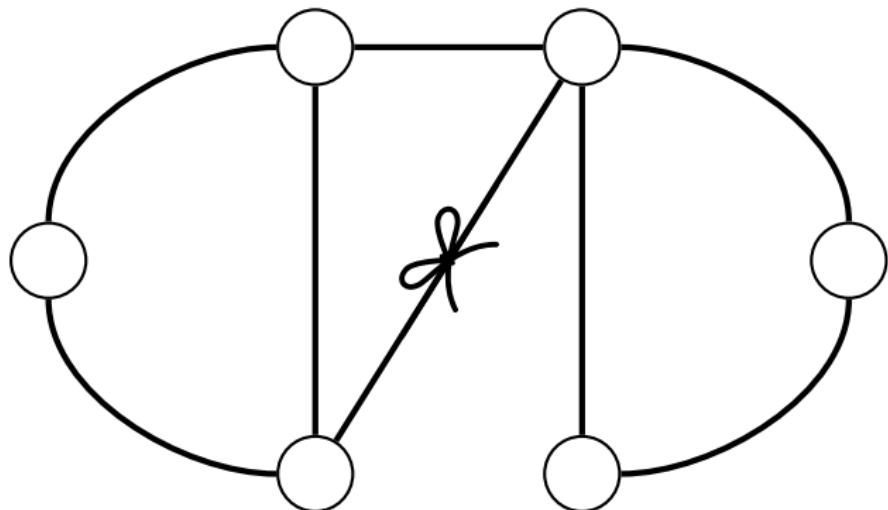
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# Evolutionary Dynamics on Evolving Graphs

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**Players/agents**

$$\mathcal{P} = \left\{ \text{Icon 1}, \text{Icon 2}, \text{Icon 3}, \text{Icon 4}, \text{Icon 5}, \text{Icon 6} \right\}$$

# Evolutionary Dynamics on Evolving Graphs

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**Players/agents**  $\mathcal{P} = \left\{ \begin{array}{c} \text{Icon 1} \\ \text{Icon 2} \\ \text{Icon 3} \\ \text{Icon 4} \\ \text{Icon 5} \\ \text{Icon 6} \end{array} \right\}$

**Strategies/states**  $S_i = \left\{ \begin{array}{c} \text{Icon 1} \\ \text{Icon 2} \\ \text{Icon 3} \end{array} \right\}$

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**Players/agents**  $\mathcal{P} = \left\{ \begin{array}{c} \text{Icon 1} \\ \text{Icon 2} \\ \text{Icon 3} \\ \text{Icon 4} \\ \text{Icon 5} \\ \text{Icon 6} \end{array} \right\}$

**Strategies/states**  $S_i = \left\{ \begin{array}{c} \text{Icon 1} \\ \text{Icon 2} \\ \text{Icon 3} \end{array} \right\}$

**Payoff**  $\pi_i : \left\{ \left( \begin{array}{c} \text{Icon 1} \\ \text{Icon 2} \\ \text{Icon 3} \\ \text{Icon 4} \\ \text{Icon 5} \\ \text{Icon 6} \end{array} \right) \rightarrow \text{Icon 4} \right\}$

# Evolutionary Dynamics on Evolving Graphs

**Players/agents**  $\mathcal{P} = \left\{ \begin{array}{c} \text{Cup icon} \\ \text{Female icon} \\ \text{Male icon} \\ \text{Female icon} \\ \text{Male icon} \\ \text{Female icon} \end{array} \right\}$

**Strategies/states**  $S_i = \left\{ \begin{array}{c} \text{Rock icon} \\ \text{Scissors icon} \\ \text{Paper icon} \end{array} \right\}$

**Payoff**  $\pi_i : \left\{ \left( \begin{array}{c} \text{Cup icon} \\ \text{Female icon} \\ \text{Male icon} \\ \text{Female icon} \\ \text{Male icon} \\ \text{Female icon} \end{array}, \begin{array}{c} \text{Rock icon} \\ \text{Scissors icon} \\ \text{Paper icon} \end{array} \right) \right\} \rightarrow \text{Money bag icon with } \text{€}$

**Update rule**  $p_{i \rightarrow j} = \left( 1 + e^{-\omega[\pi_i - \pi_j]} \right)^{-1} \text{ (Fermi rule)}$

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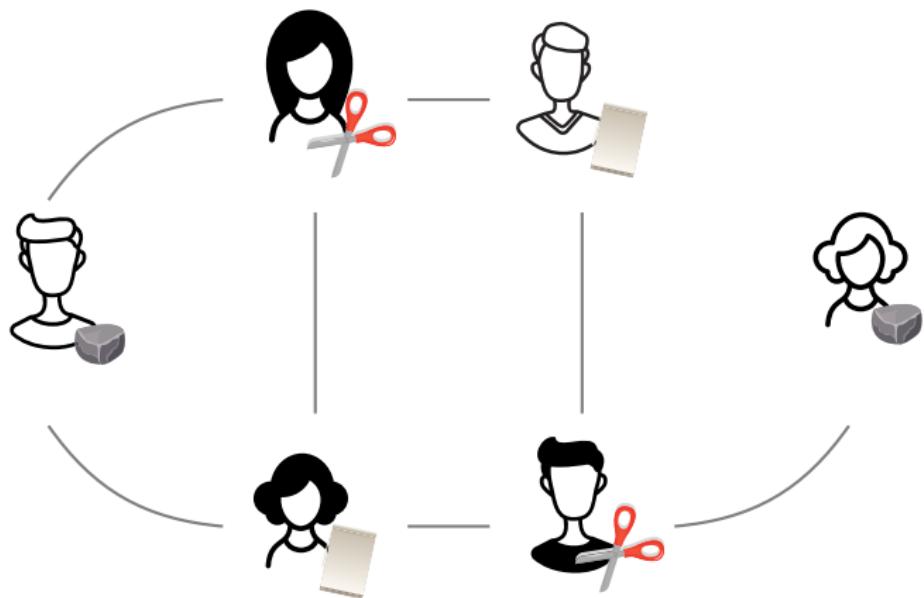
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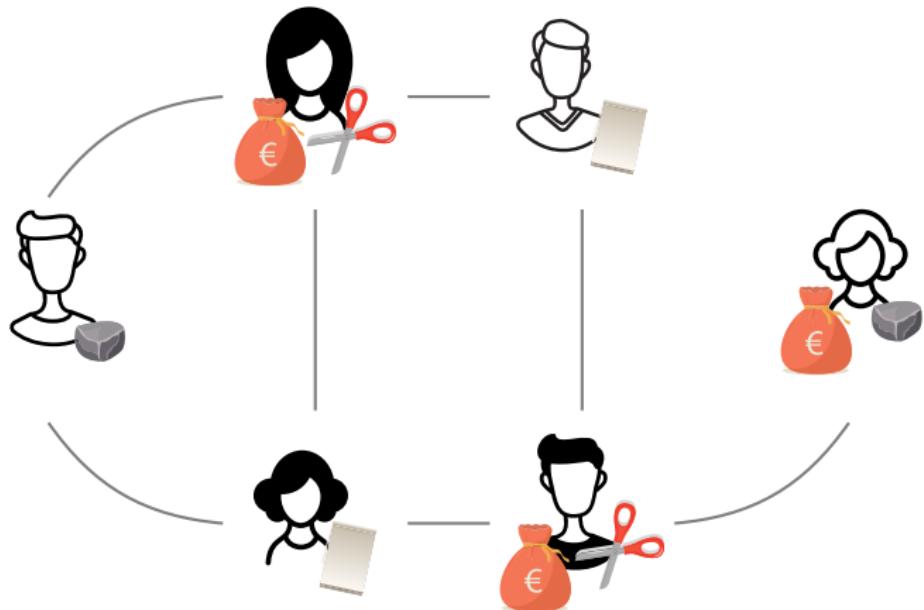
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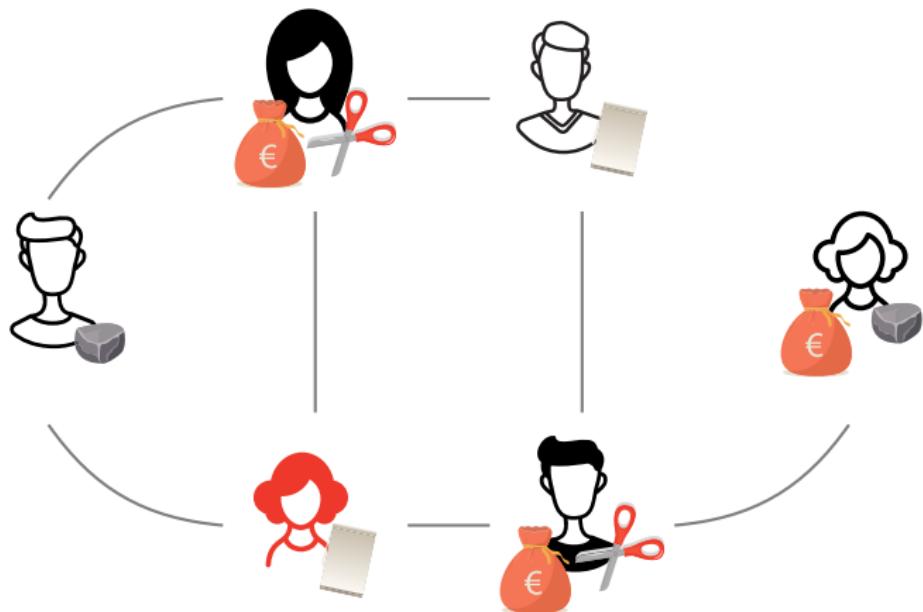
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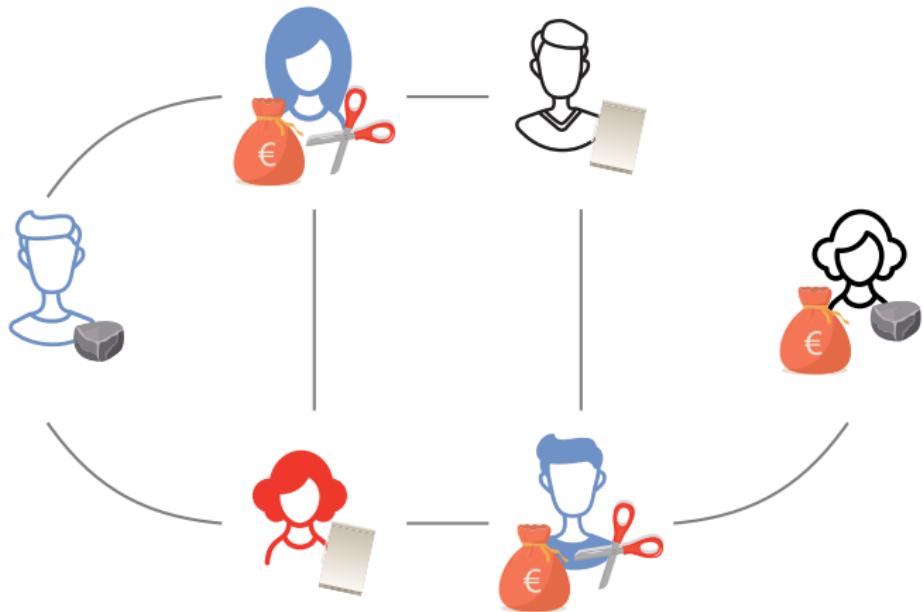
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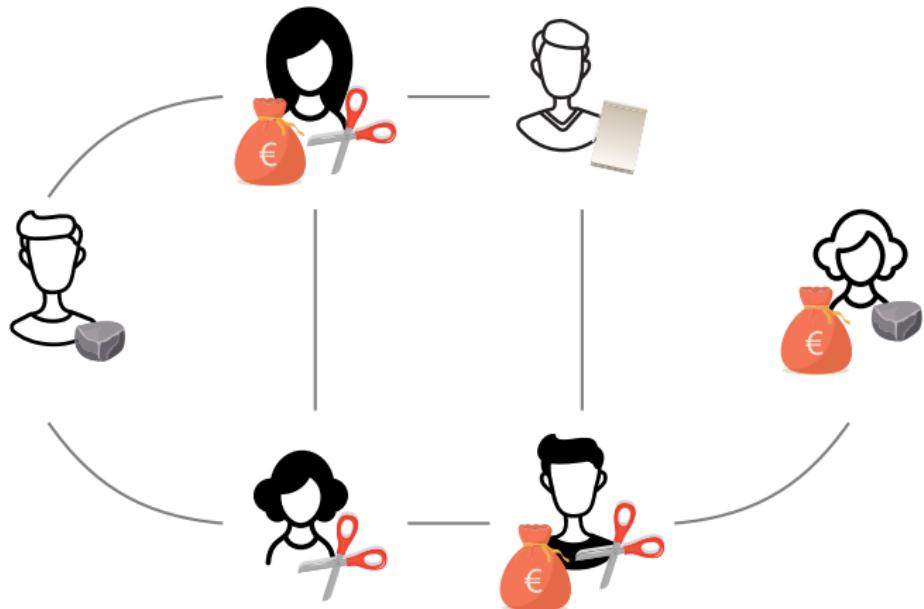
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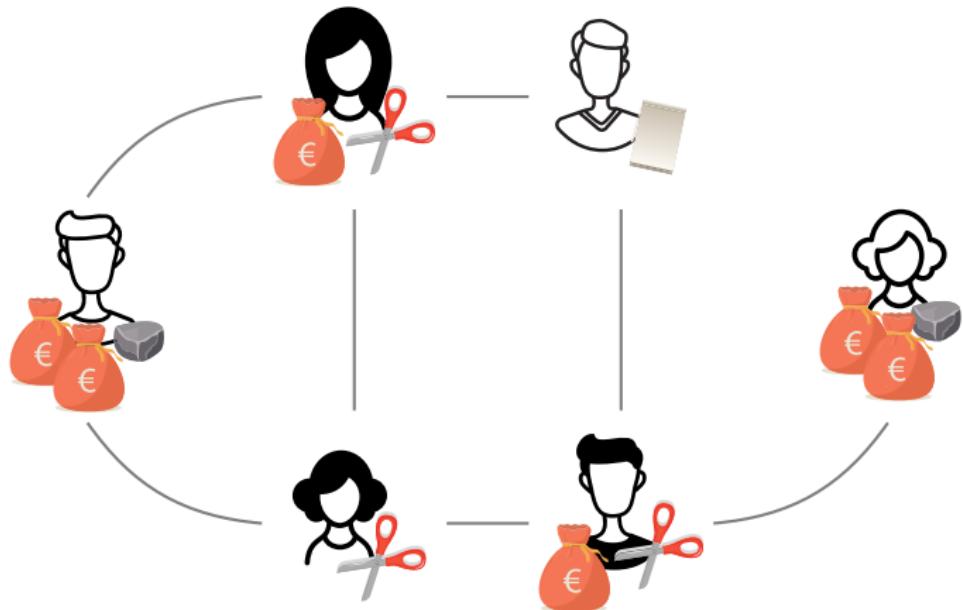
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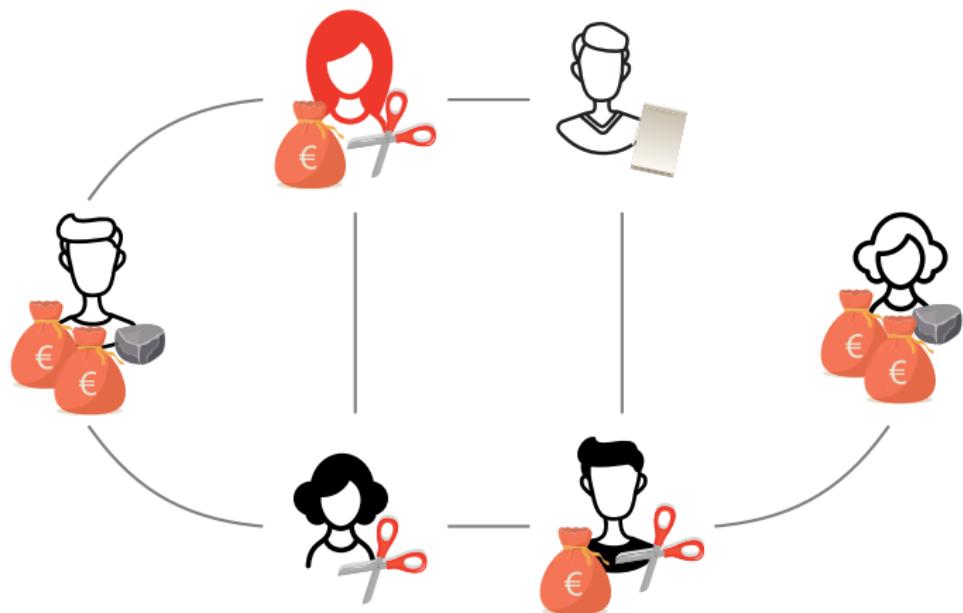
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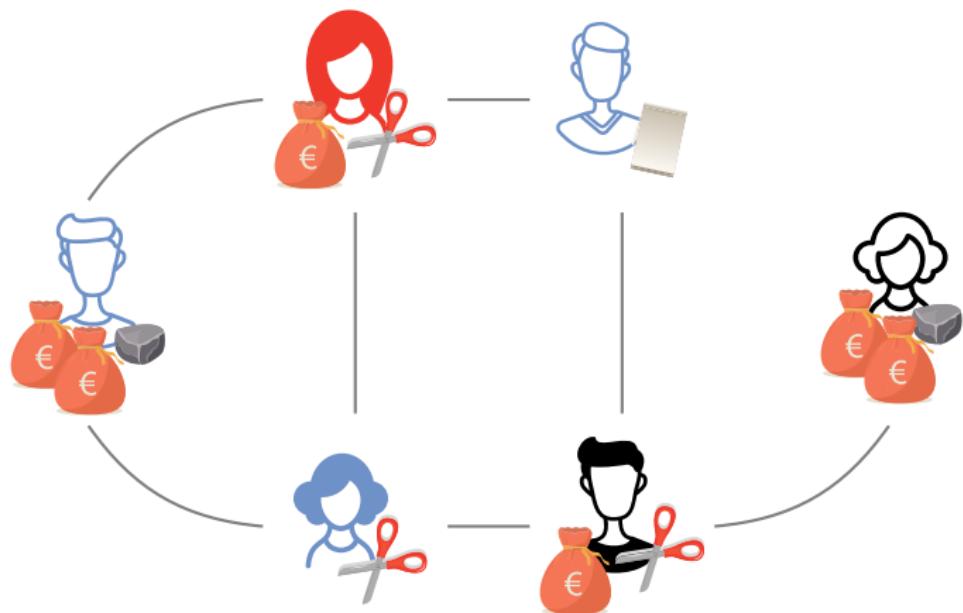
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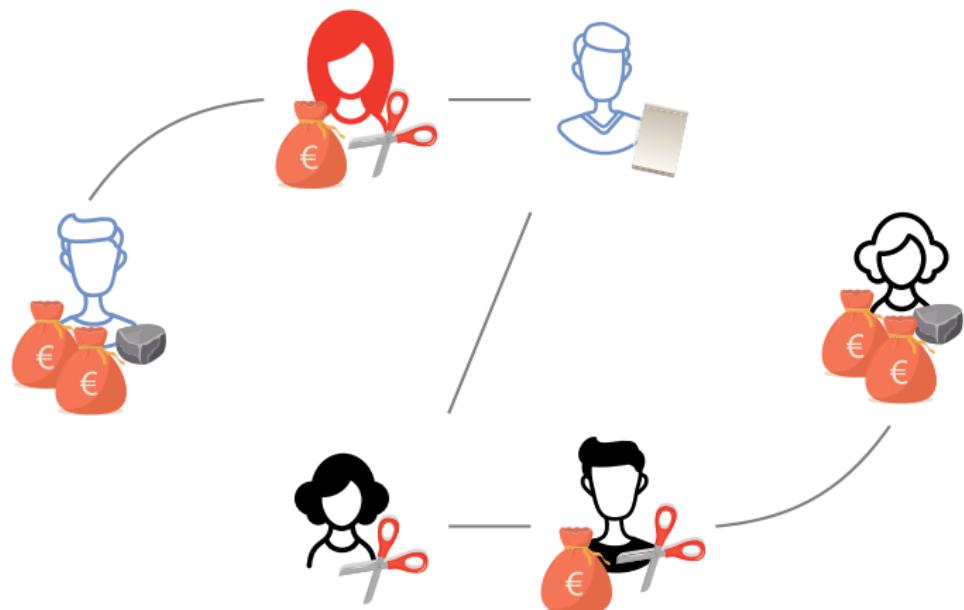
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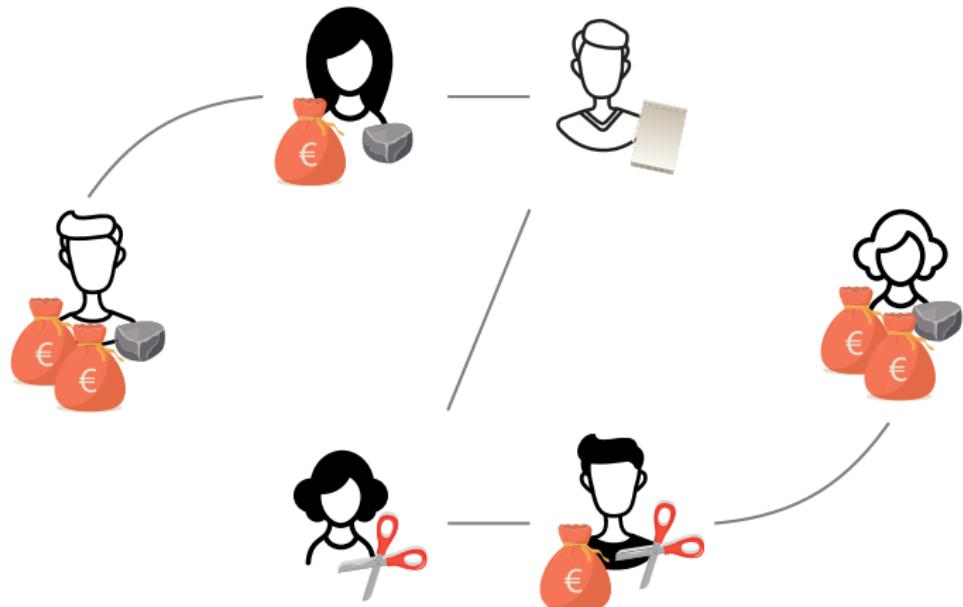
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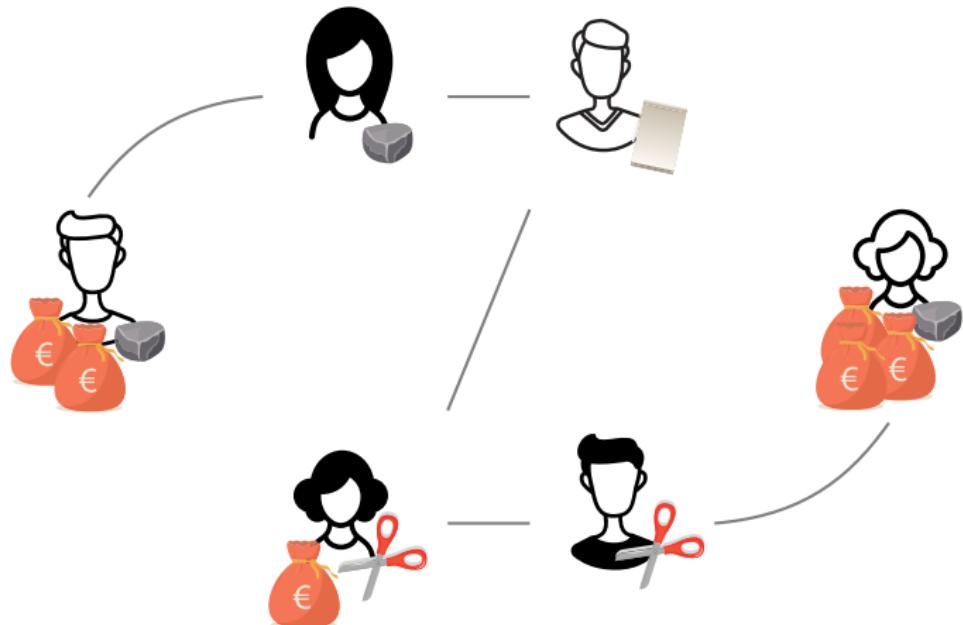
# Evolutionary Dynamics on Evolving Graphs



# Evolutionary Dynamics on Evolving Graphs



# Evolutionary Dynamics on Evolving Graphs



# *Evolutionary Dynamics on Evolving Graphs*

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Barbara Ikica

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# Evolutionary Dynamics on Evolving Graphs

$t_D$     } := time scale of the { dynamics  
 $t_G$     } graph



# Evolutionary Dynamics on Evolving Graphs

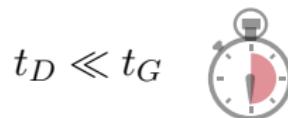
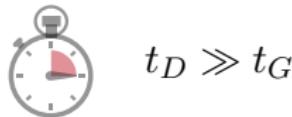
$t_D$     } := time scale of the { dynamics  
 $t_G$     graph



$$t_D \gg t_G$$

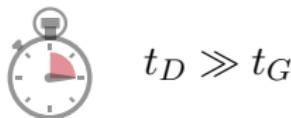
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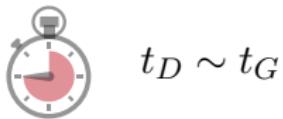


# Evolutionary Dynamics on Evolving Graphs

$t_D$     } := time scale of the { dynamics  
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$$t_D \ll t_G$$



# Evolutionary Dynamics on Evolving Graphs

$$t_D \gg t_G$$

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# Evolutionary Dynamics on Evolving Graphs

$$t_D \ll t_G$$

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$$t_D \sim t_G$$

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$$t_D \sim t_G$$

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*News providers*



# Evolutionary Dynamics on Evolving Graphs

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$$t_D \sim t_G$$

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*News providers*



*News consumers*

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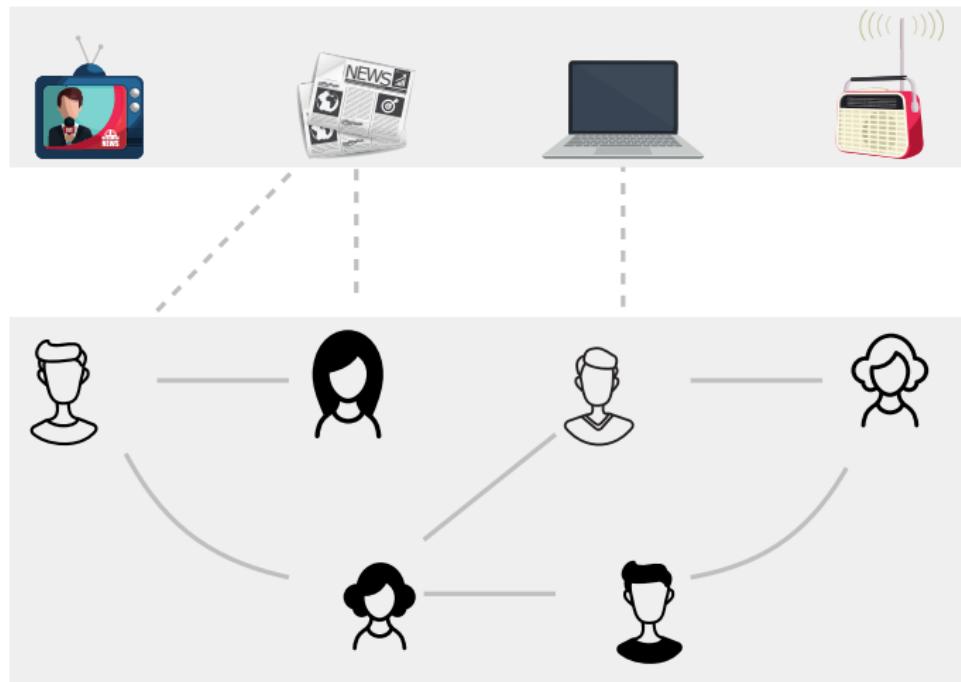
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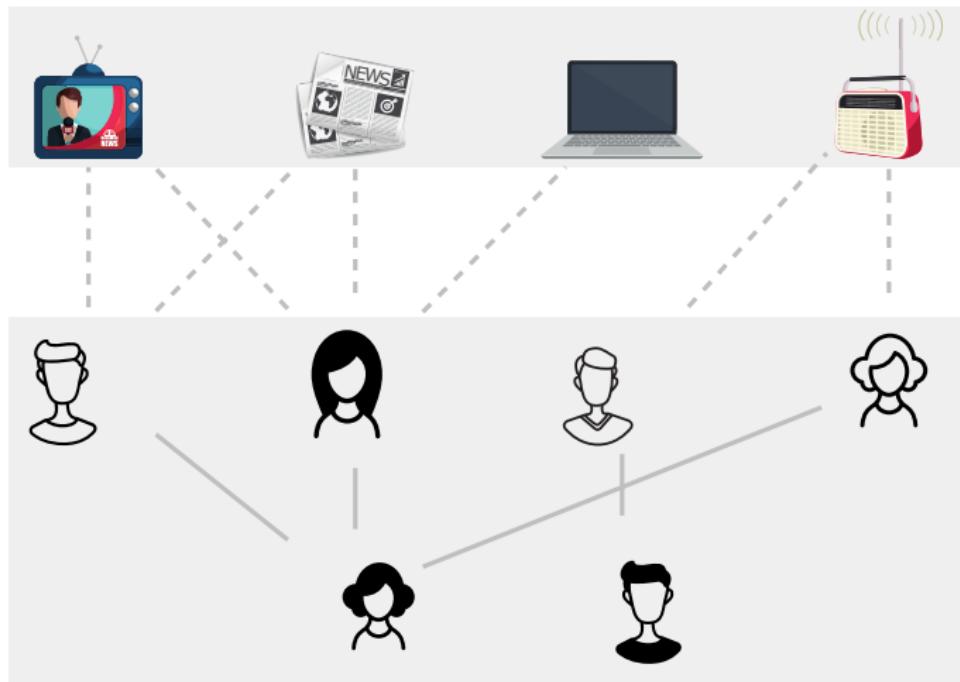
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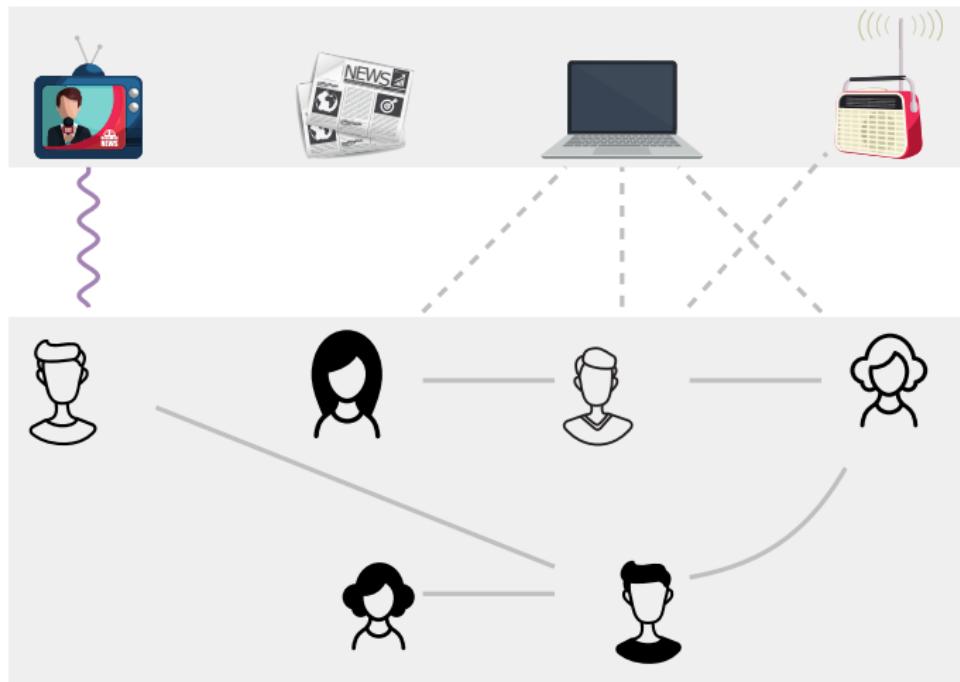
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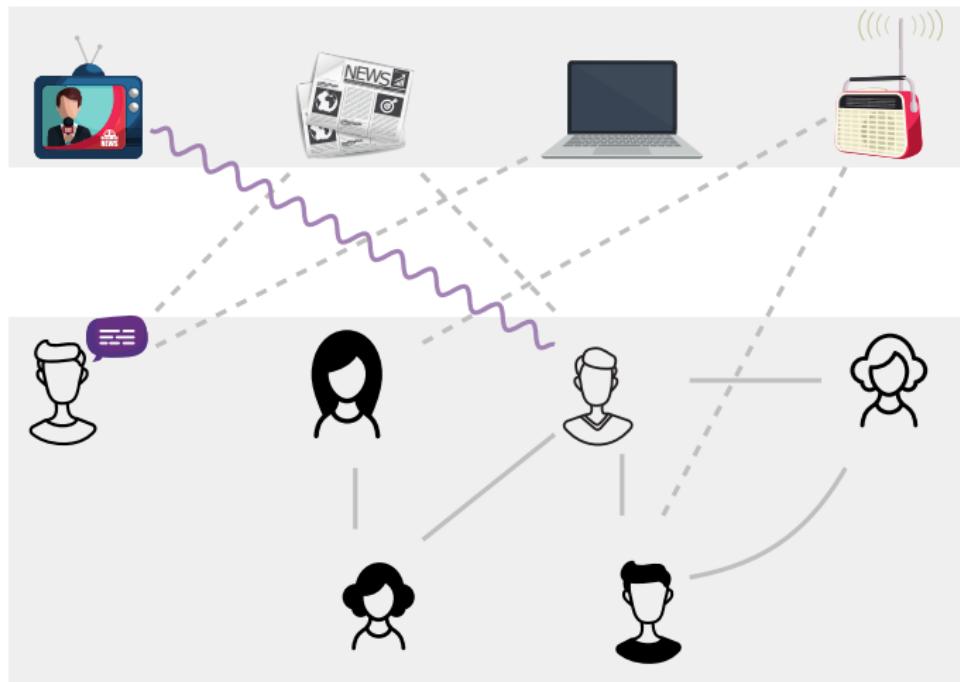
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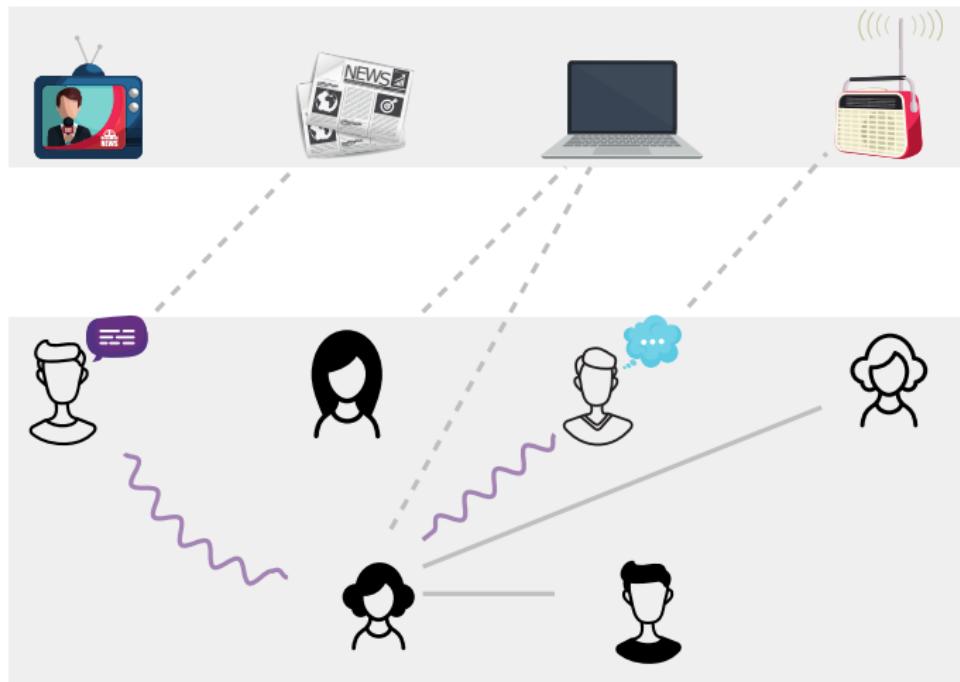
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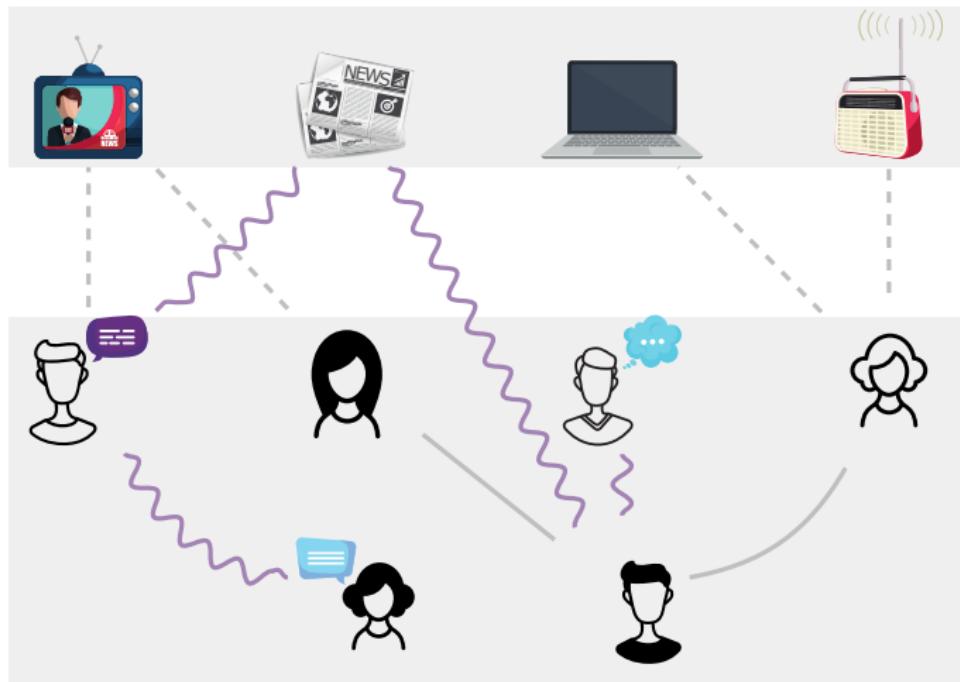
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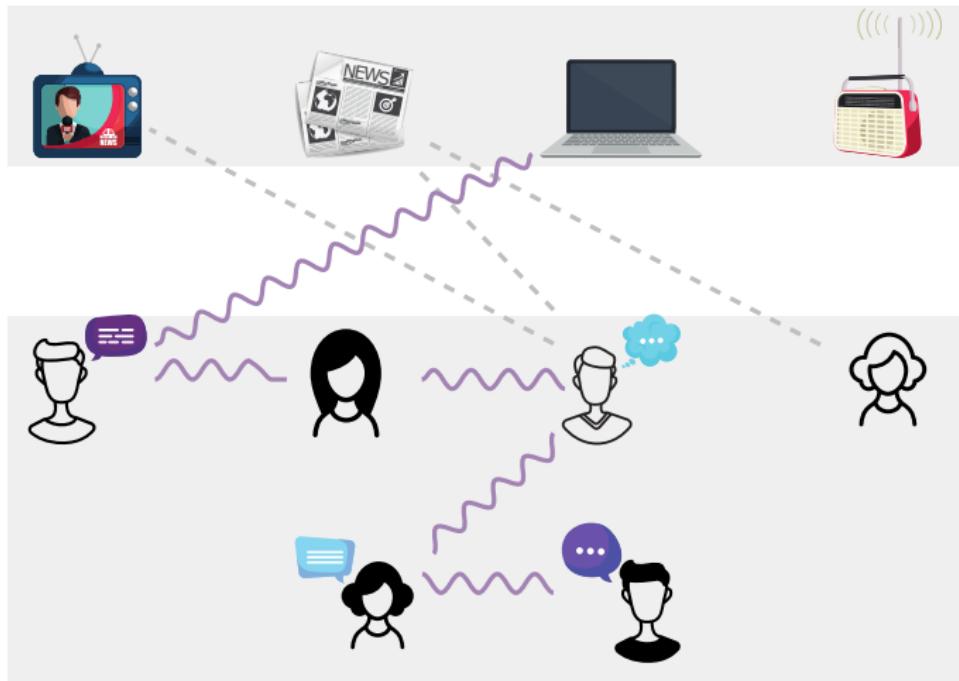
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$$\left. \begin{array}{c} t_D \\ t_G \end{array} \right\} := \text{time scale of the} \left\{ \begin{array}{c} \text{dynamics} \\ \text{graph} \end{array} \right\}$$




$$t_D \gg t_G$$



$$t_D \ll t_G$$



$$t_D \sim t_G$$

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$$t_D \gg t_G$$



$$t_D \ll t_G$$



$$t_D \sim t_G$$

# Outline

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## I Evolutionary Dynamics on Graphs ( $t_D \ll t_G$ )

# Outline

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## I Evolutionary Dynamics on Graphs ( $t_D \ll t_G$ )

- *The Modified Petford–Welsh Algorithm*

# Outline

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## I Evolutionary Dynamics on Graphs ( $t_D \ll t_G$ )

- *The Modified Petford–Welsh Algorithm*

## II Evolving Graphs ( $t_D \sim t_G$ )

# Outline

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## I Evolutionary Dynamics on Graphs ( $t_D \ll t_G$ )

- *The Modified Petford–Welsh Algorithm*

## II Evolving Graphs ( $t_D \sim t_G$ )

- *Co-evolution of the Multilayer News Flow*

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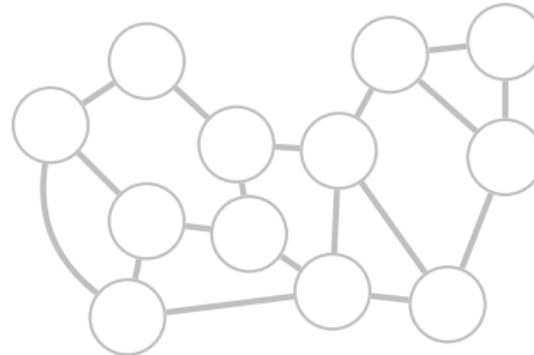
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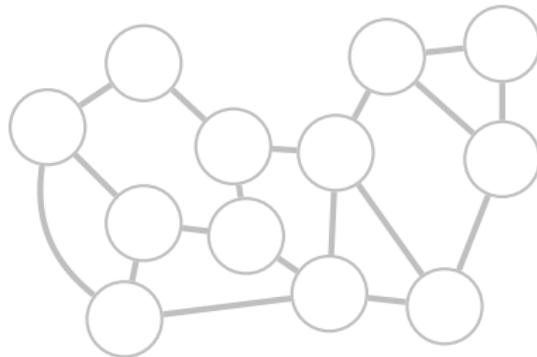
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# The Modified Petford–Welsh Algorithm

## Petford–Welsh algorithm

Petford, A. D. & Welsh, D. J. A. A Randomised 3-Colouring Algorithm, *Discrete Math.* **74** (1989), 253–261.

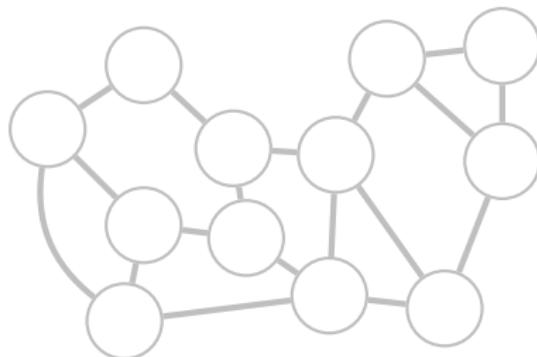


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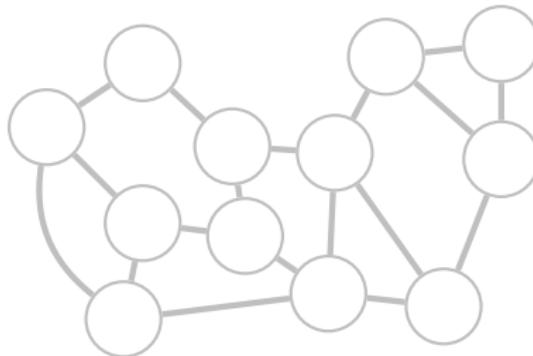


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## Petford–Welsh algorithm

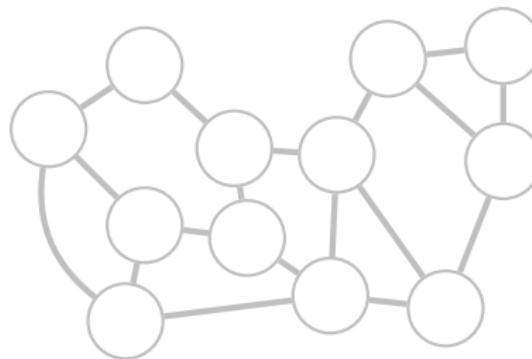
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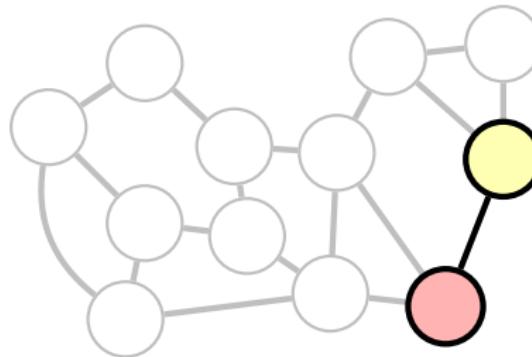
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A **proper colouring** is an assignment of colours to the vertices of a graph so that no two adjacent vertices have the same colour, i.e.,  $c : V \rightarrow \{1, 2, \dots, k\}$  s.t.  $c(i) \neq c(j)$  for all  $ij \in E$ .



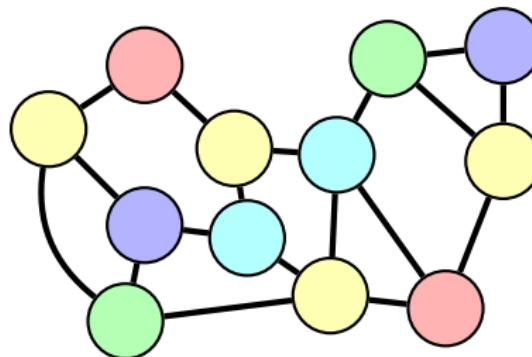
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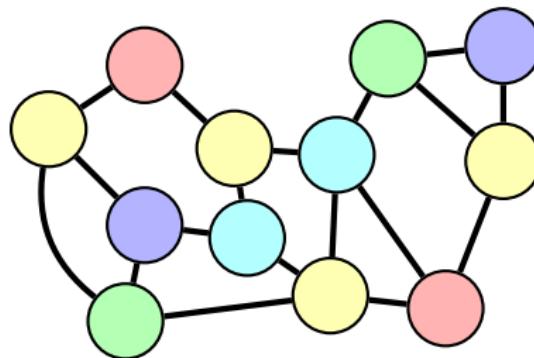
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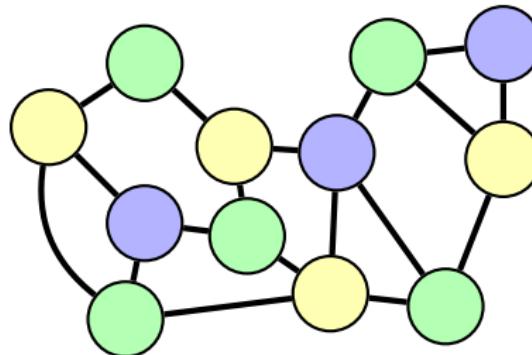
A graph that has a  $k$ -colouring is said to be  **$k$ -colourable**.



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A **proper colouring** is an assignment of colours to the vertices of a graph so that no two adjacent vertices have the same colour, i.e.,  $c : V \rightarrow \{1, 2, \dots, k\}$  s.t.  $c(i) \neq c(j)$  for all  $ij \in E$ .

A graph that has a  $k$ -colouring is said to be  **$k$ -colourable**.

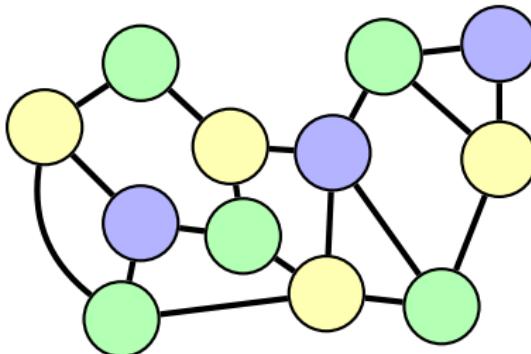


# The Modified Petford–Welsh Algorithm

## Petford–Welsh algorithm

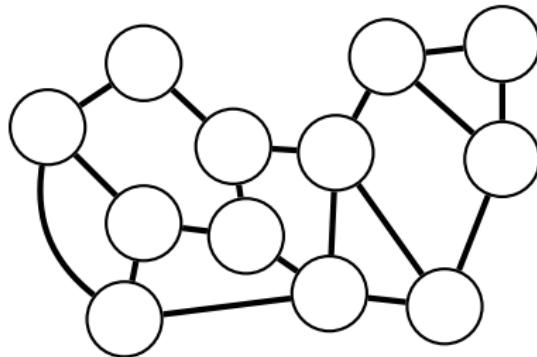
Petford, A. D. & Welsh, D. J. A. A Randomised 3-Colouring Algorithm, *Discrete Math.* **74** (1989), 253–261.

Žerovník, J. A Randomized Algorithm for  $k$ -Colorability, *Discrete Math.* **131** (1994), 379–393.



# The Modified Petford–Welsh Algorithm

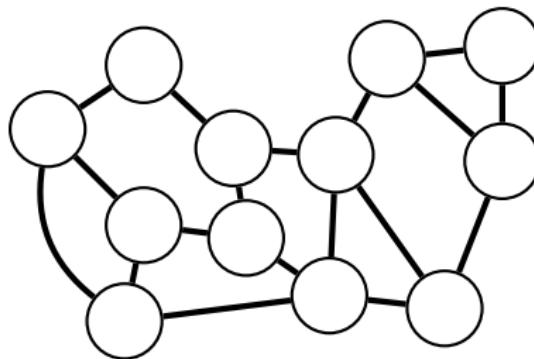
## A randomised $k$ -colouring algorithm



# The Modified Petford–Welsh Algorithm

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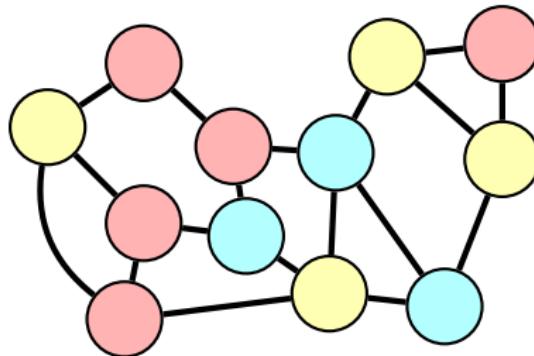
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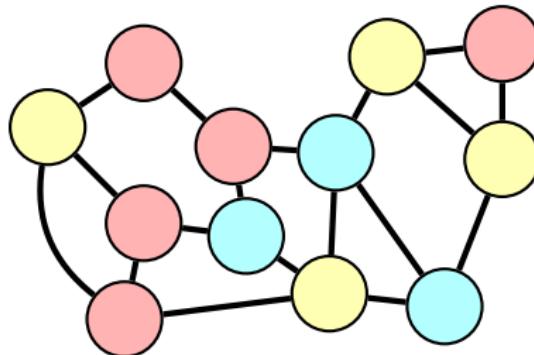
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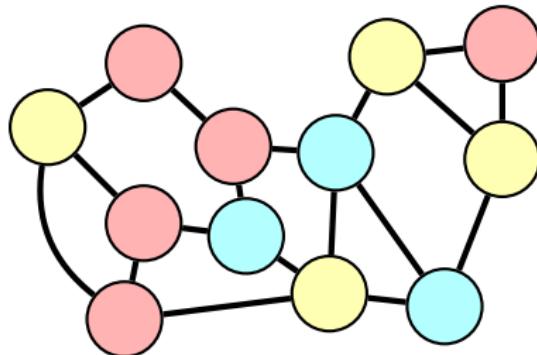
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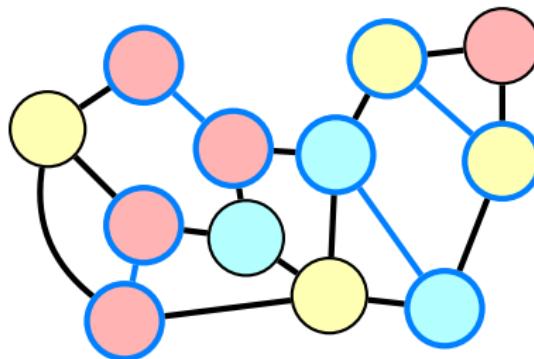
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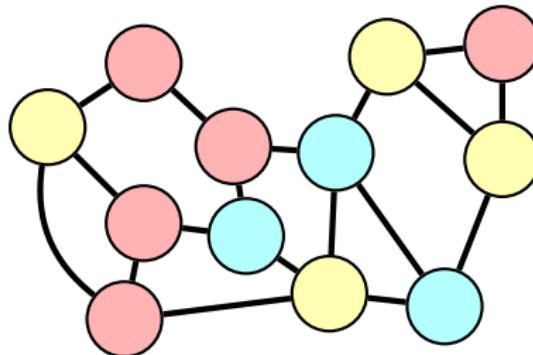
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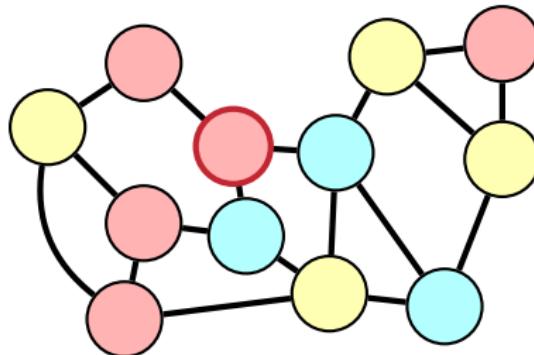
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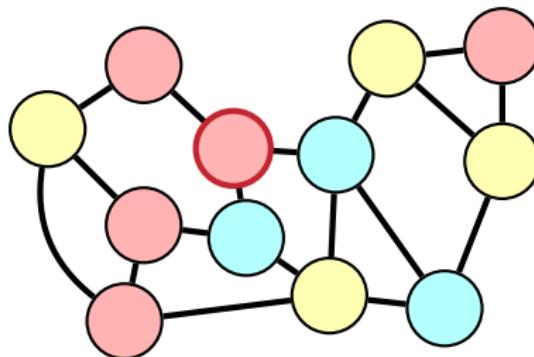
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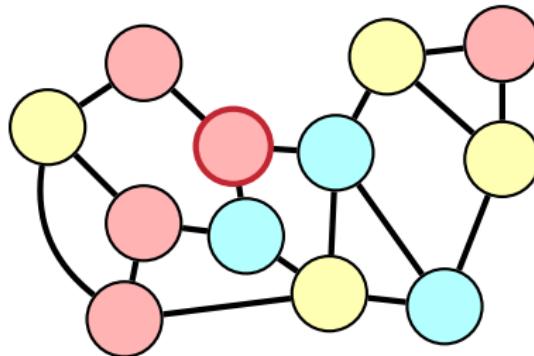
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$$\mathcal{N}(\text{red}, \text{red}) = 1$$

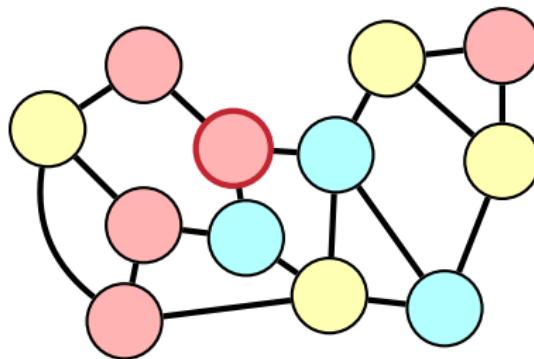
$$\mathcal{N}(\text{red}, \text{cyan}) = 2$$

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$$\mathcal{N}(\textcolor{red}{\bullet}, \textcolor{red}{r}) = 1$$

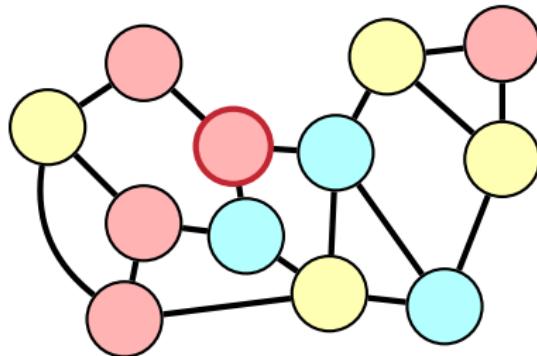
$$\mathcal{N}(\textcolor{red}{\bullet}, \textcolor{cyan}{b}) = 2$$

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# The Modified Petford–Welsh Algorithm

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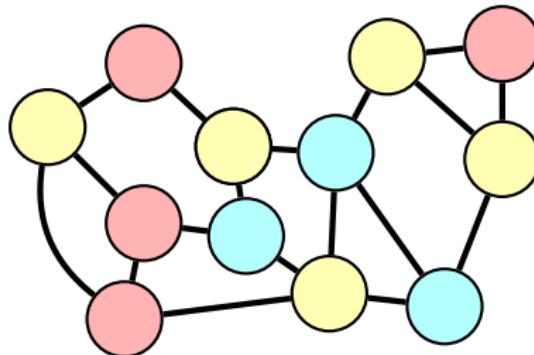
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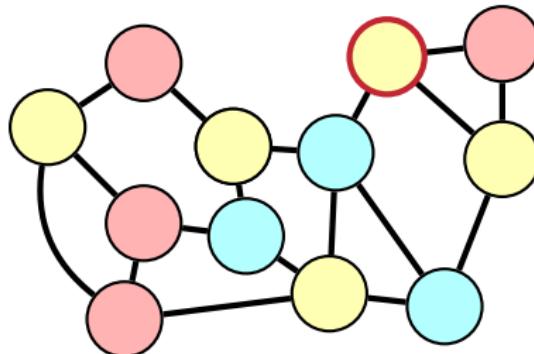
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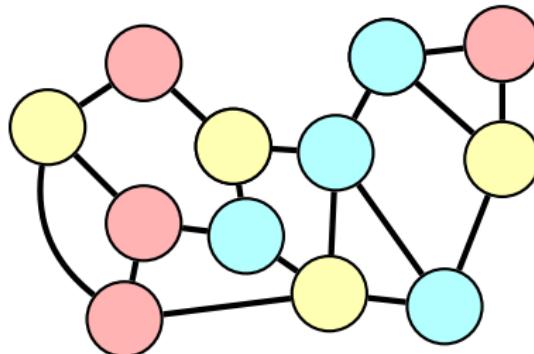
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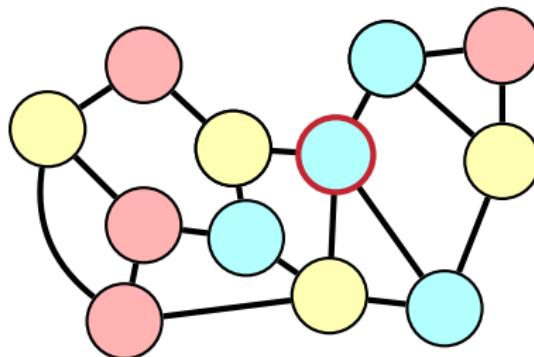
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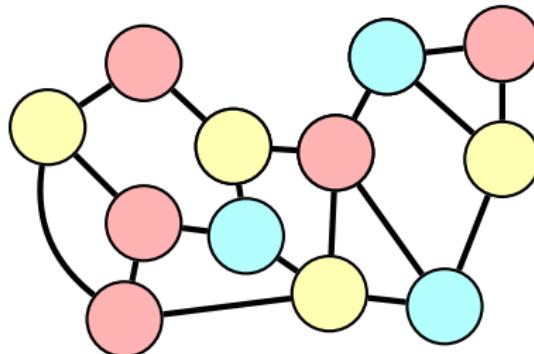
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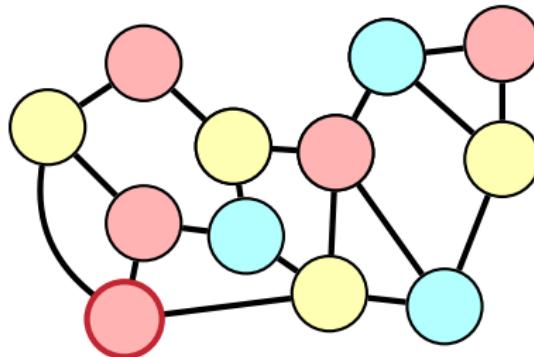
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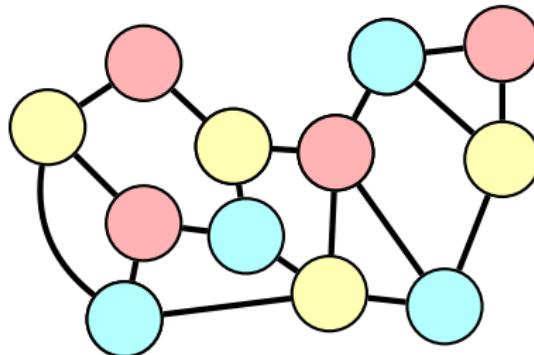
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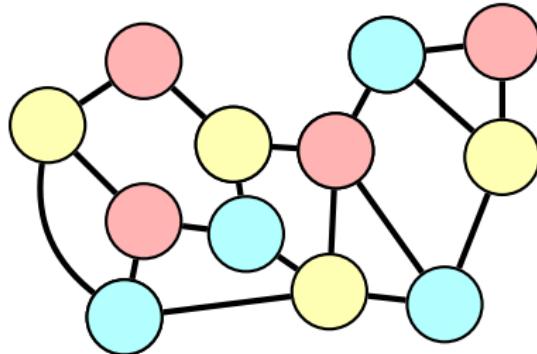
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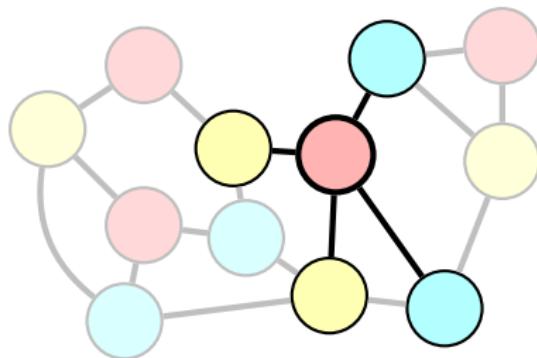
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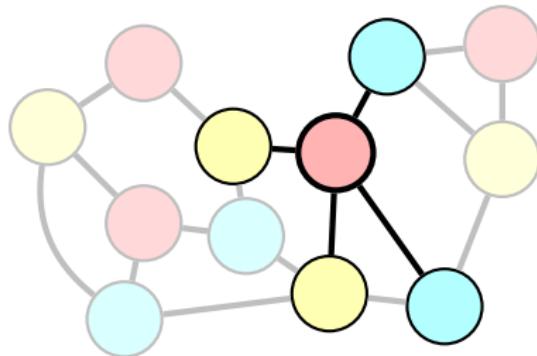


$$\omega^{-\mathcal{N}}(\bullet, \textcolor{red}{\textcolor{pink}{\textcolor{cyan}{\square}}})$$

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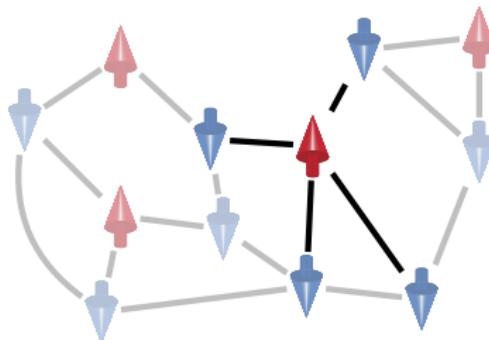


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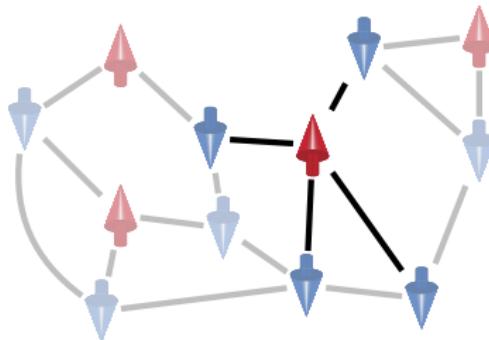


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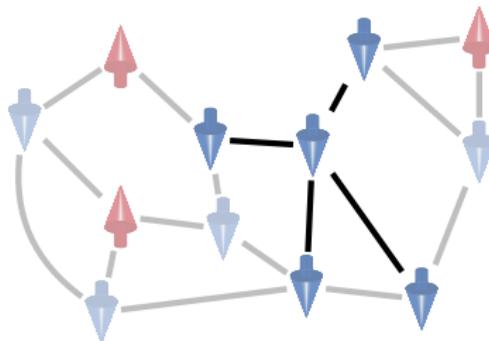
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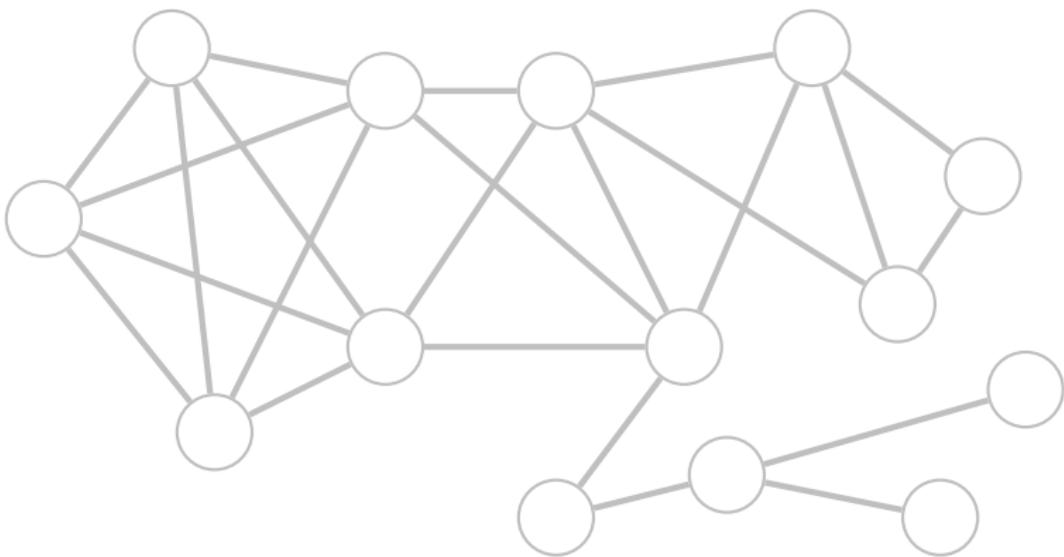


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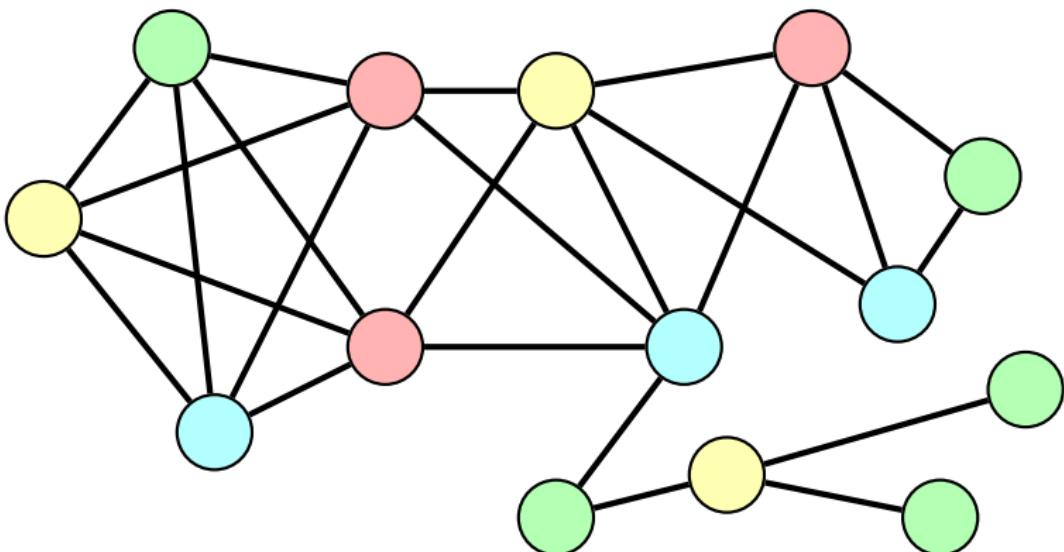
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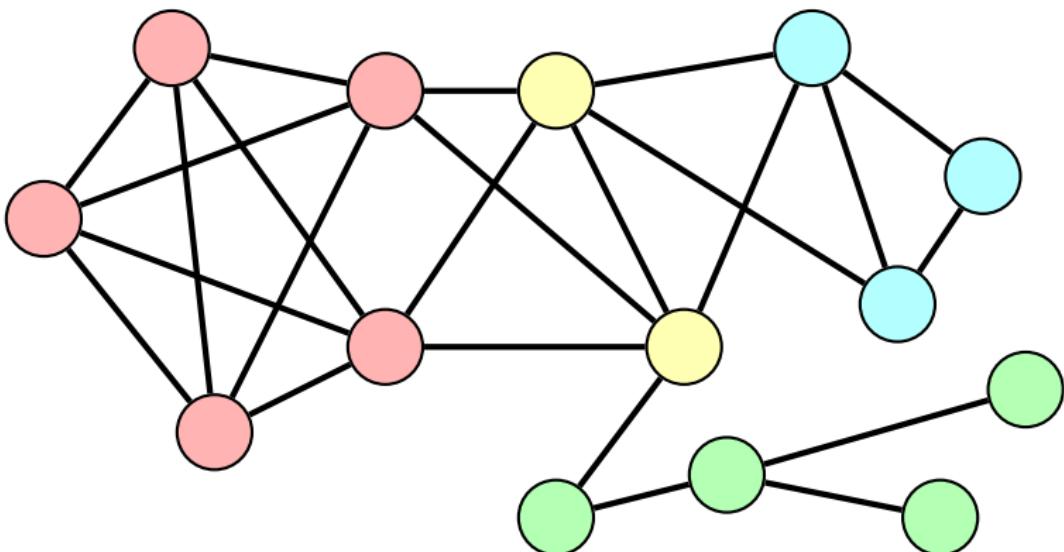


# The Modified Petford–Welsh Algorithm

## Graph colouring

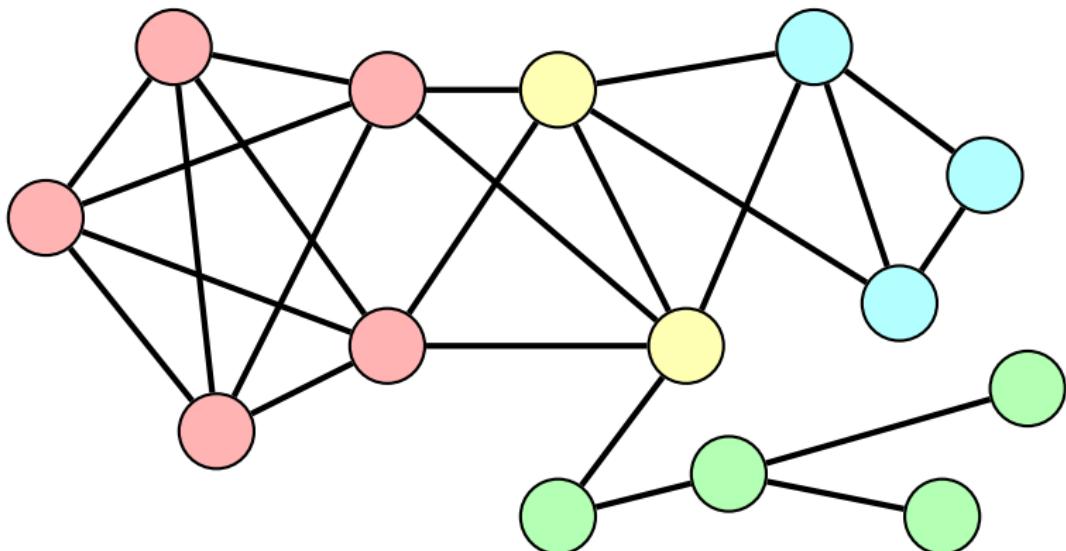


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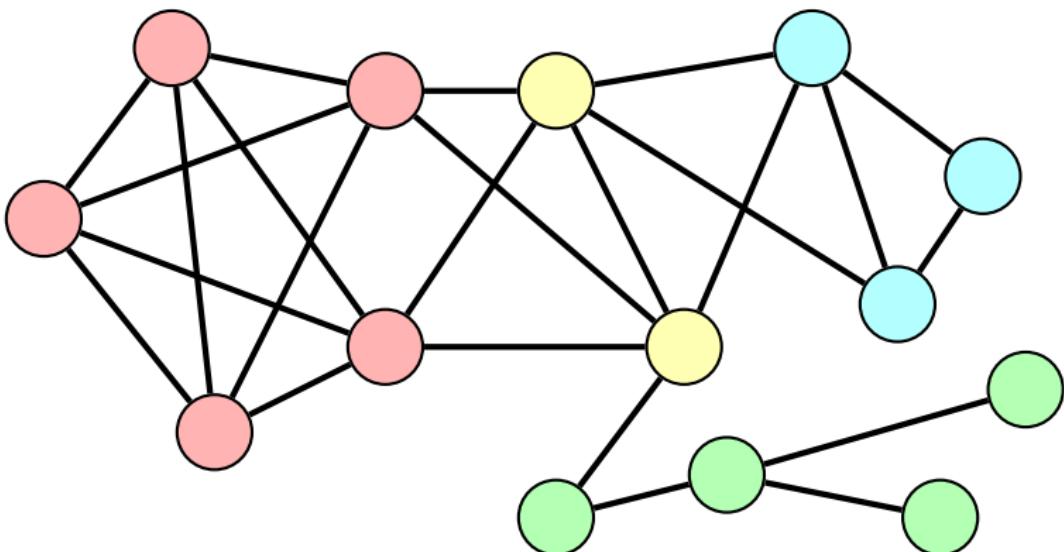
# The Modified Petford–Welsh Algorithm

## Graph clustering



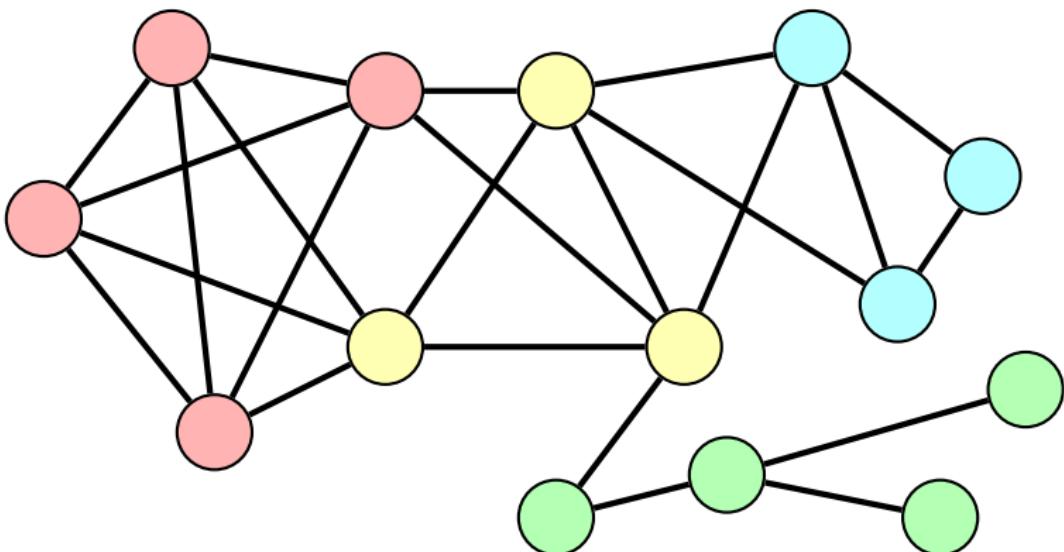
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**clustering** [partitioning or grouping data into *similar* subsets]



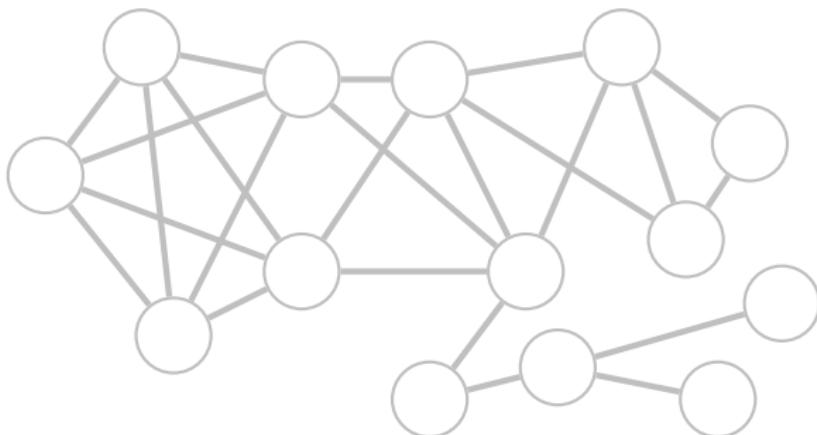
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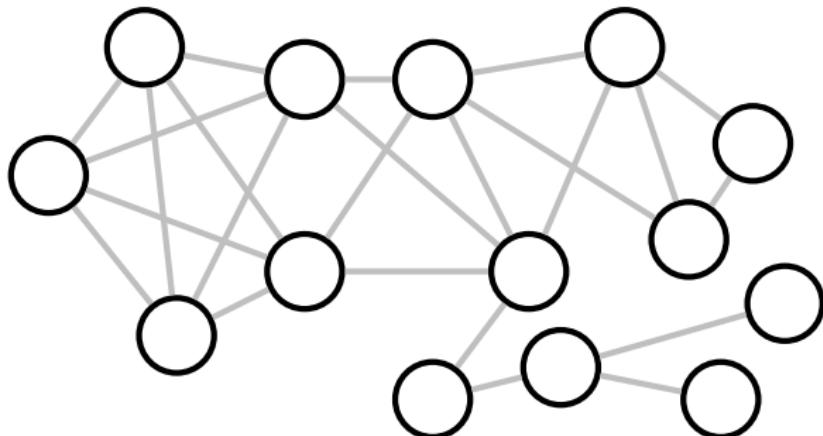
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The goal of **clustering** is to separate a given set of objects  $X = \{x_1, x_2, \dots, x_n\}$  into non-overlapping groups/*clusters*  $\mathcal{C} = \{C_1, C_2, \dots, C_m\}$  that satisfy



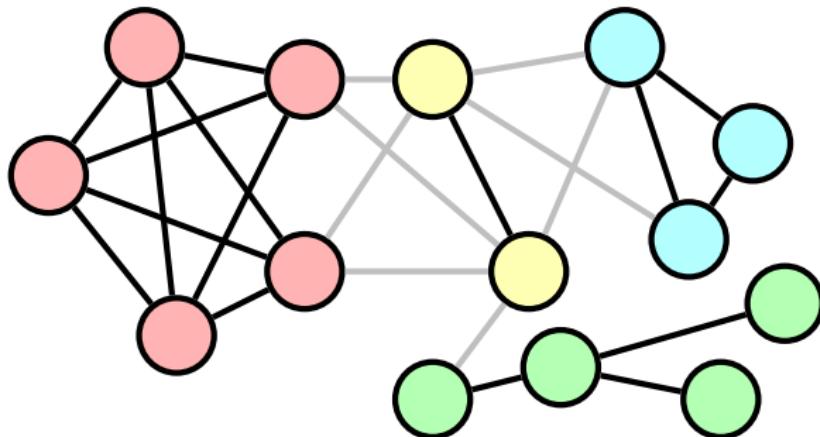
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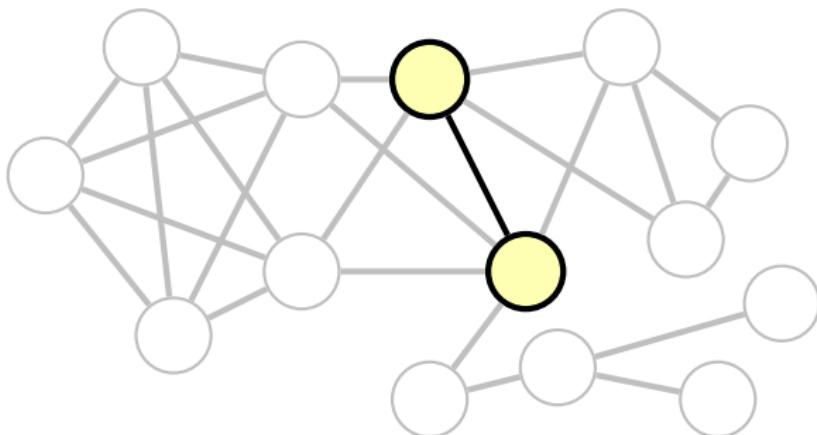
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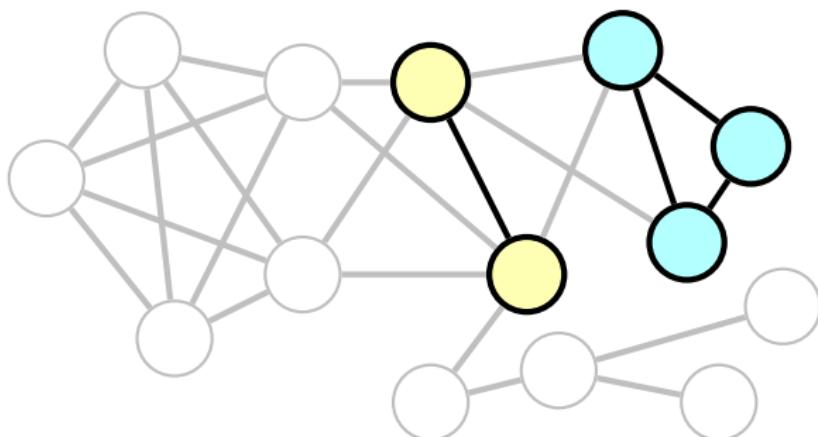


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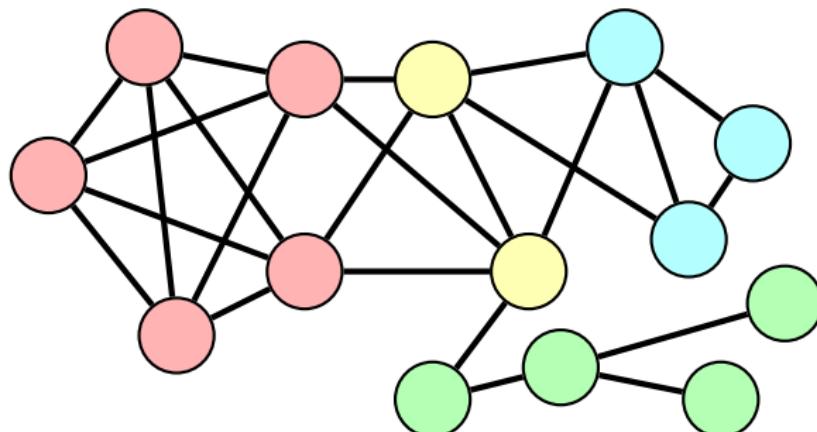
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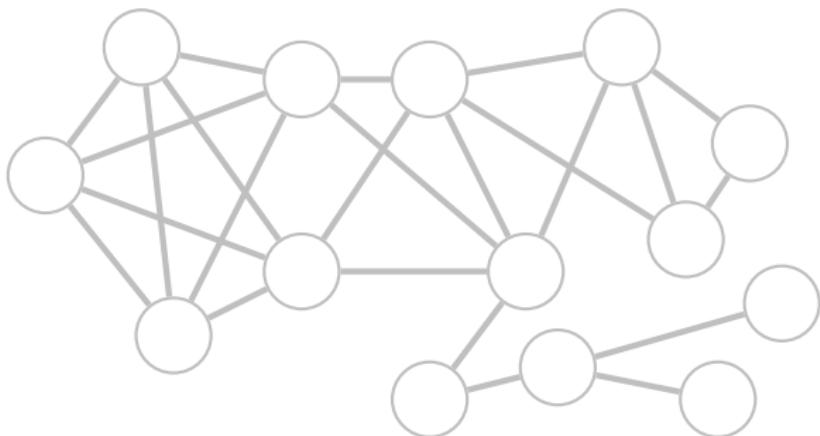
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$$\bigcup_{i=1}^m C_i = X.$$



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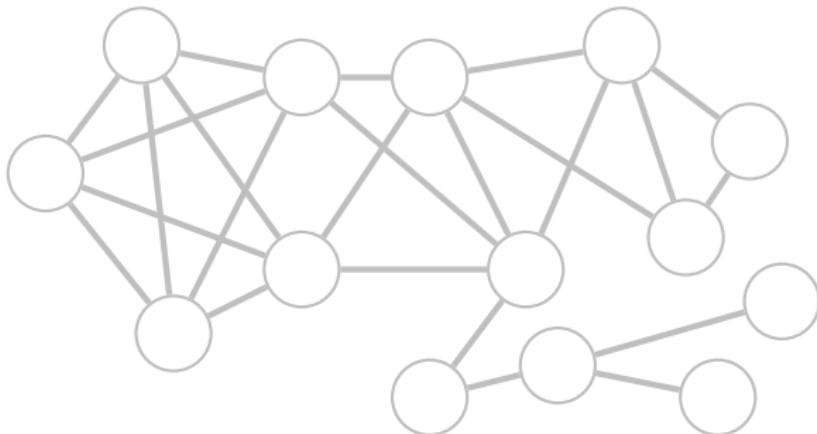
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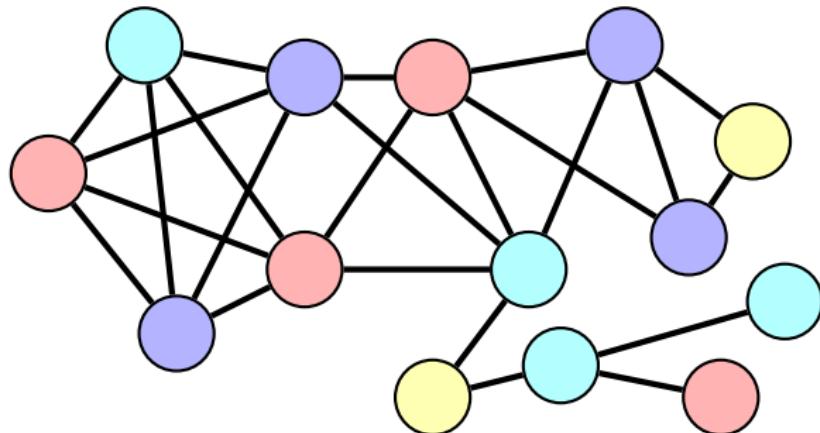
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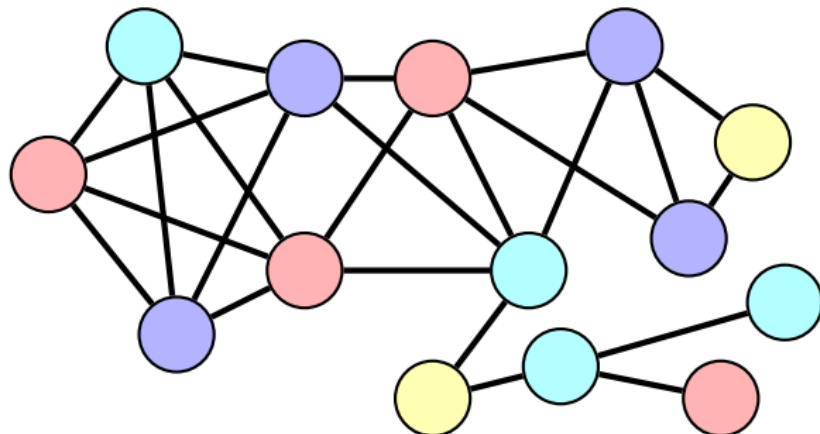
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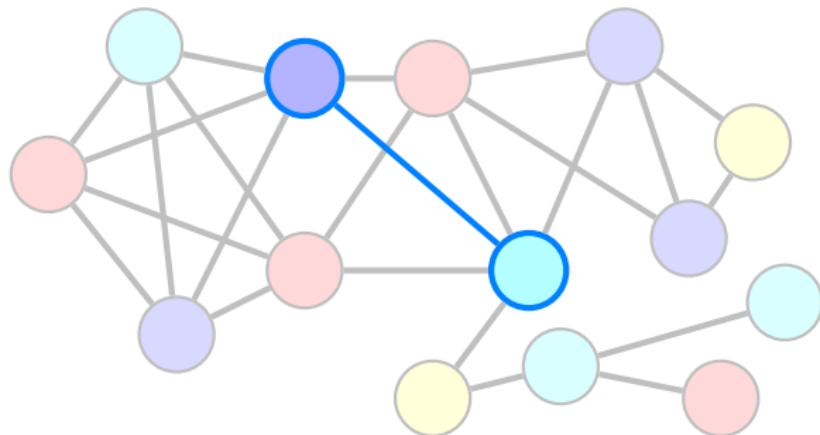
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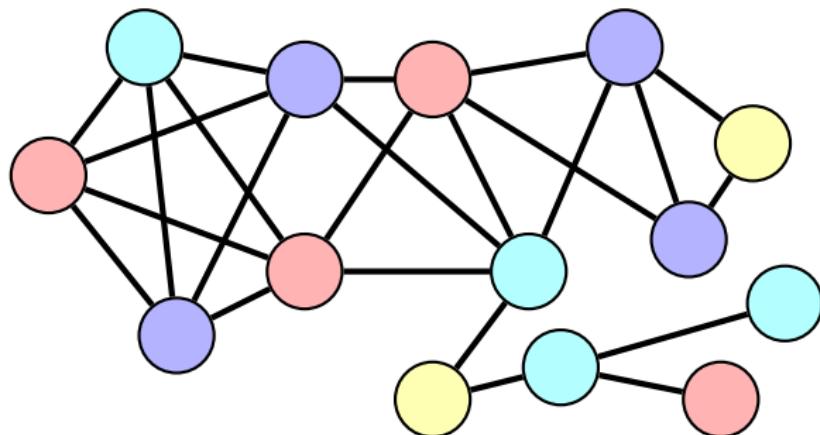
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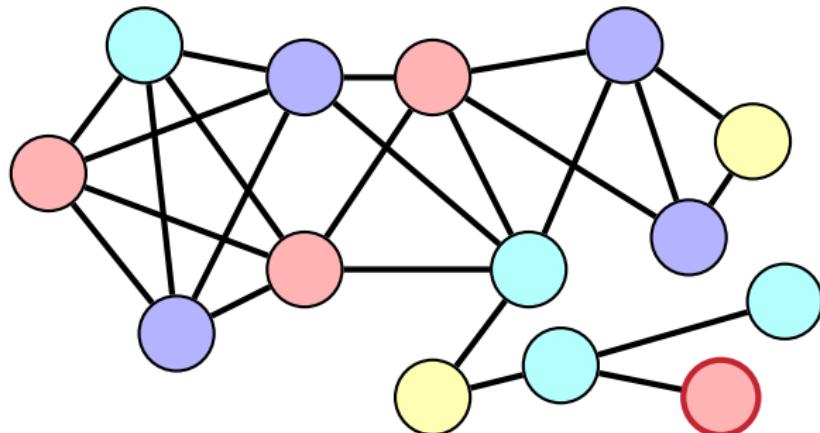
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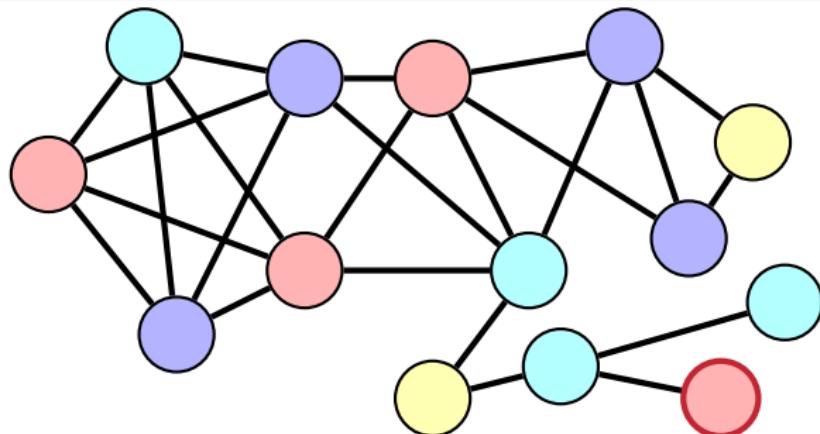
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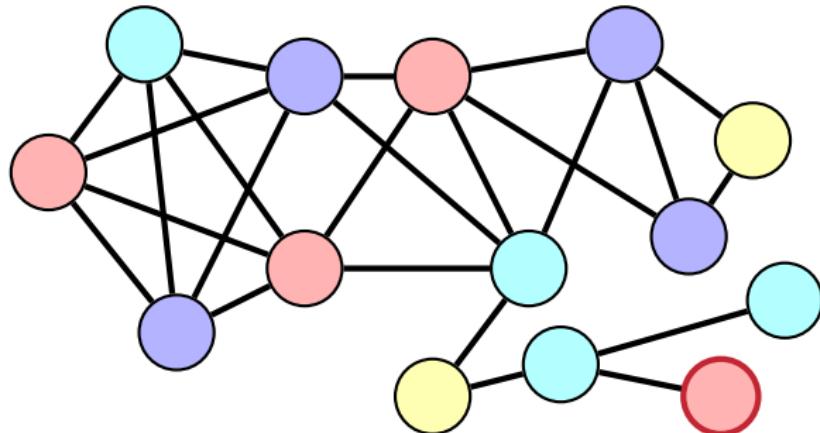
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  - 2.2 choose a new colour  $i$  for  $v$  proportionally to  $\omega + \mathcal{N}(v, i)$ ,  $\omega > 1$



# The Modified Petford–Welsh Algorithm

A randomised *clustering* algorithm [<https://github.com/ikicab/mPW>]

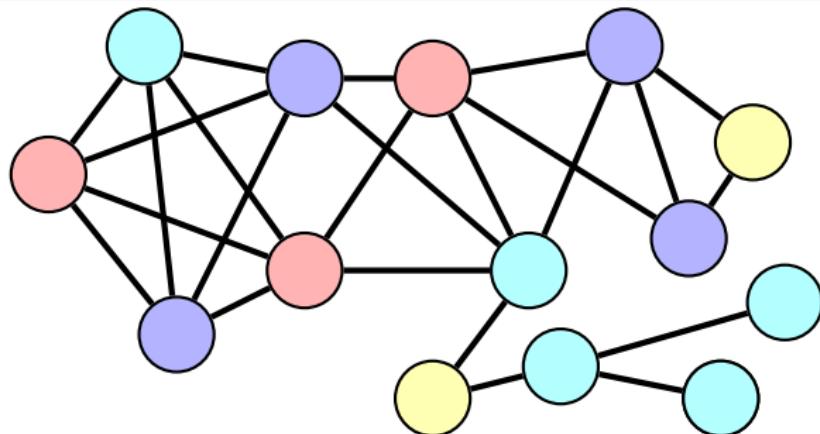
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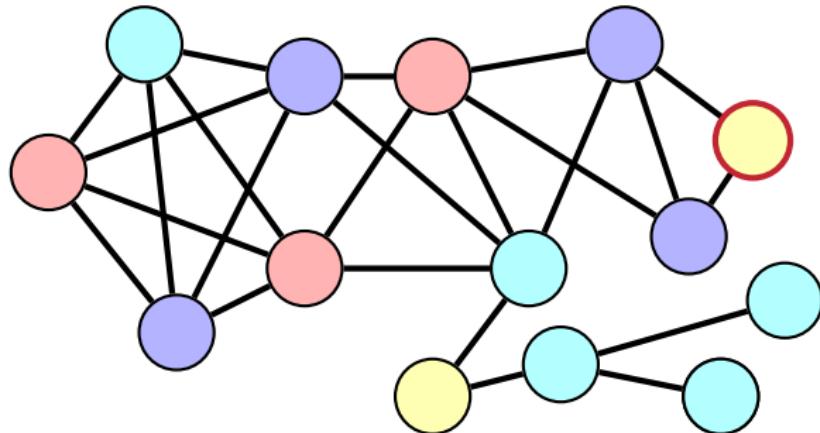
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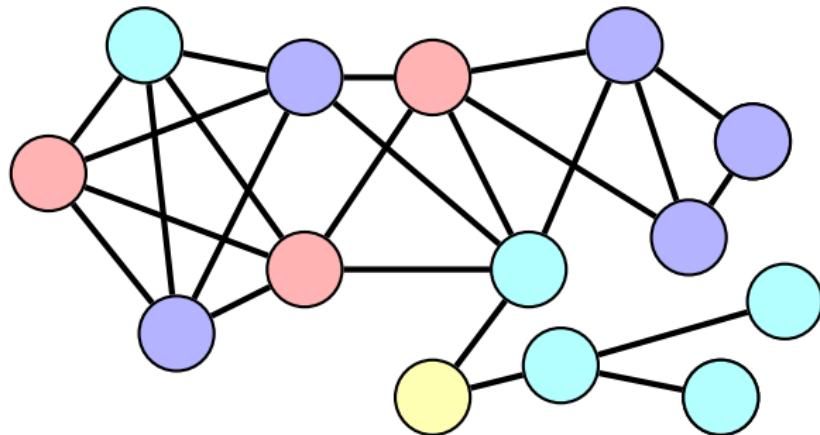
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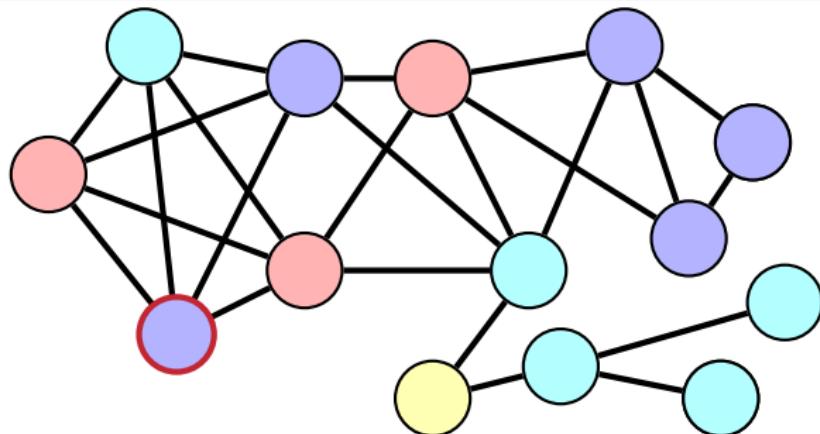
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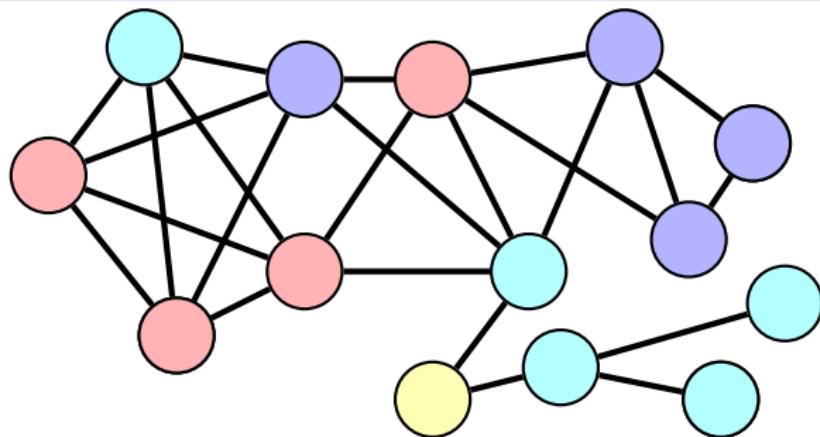
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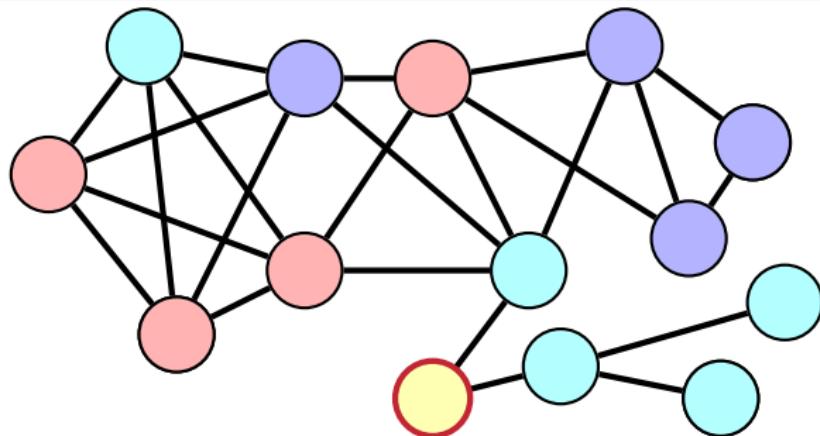
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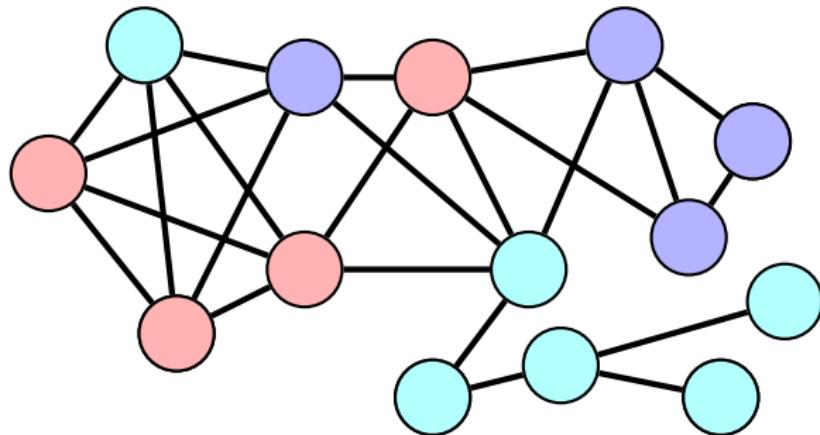
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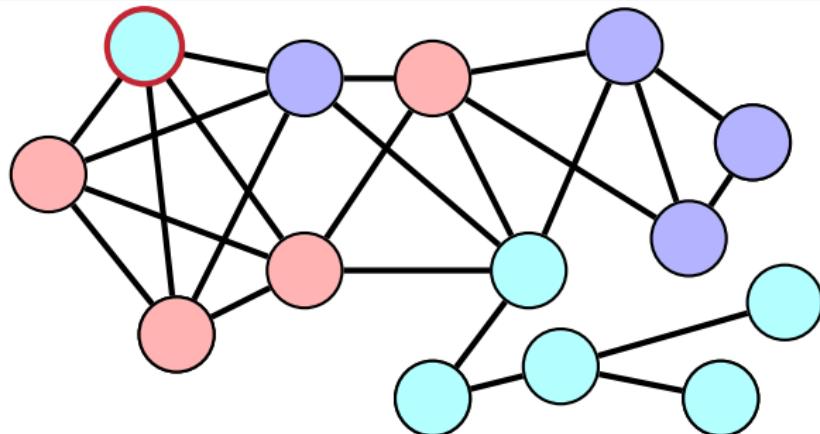
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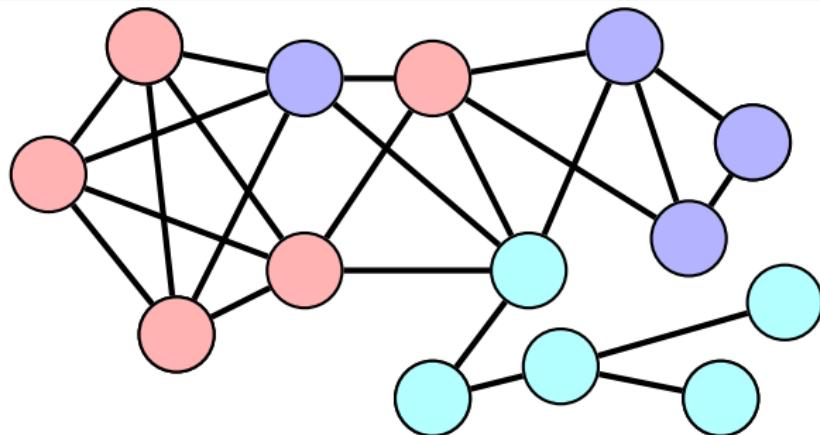
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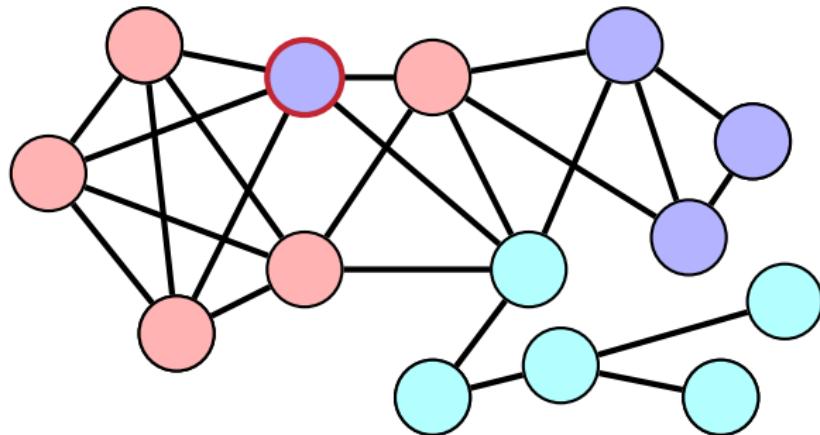
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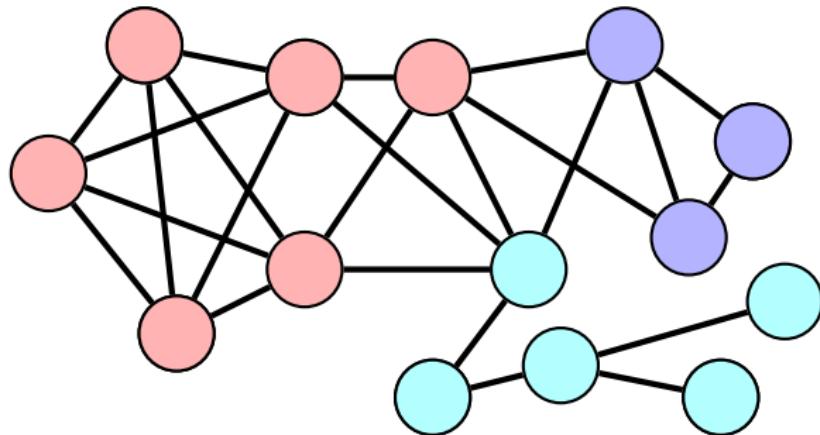
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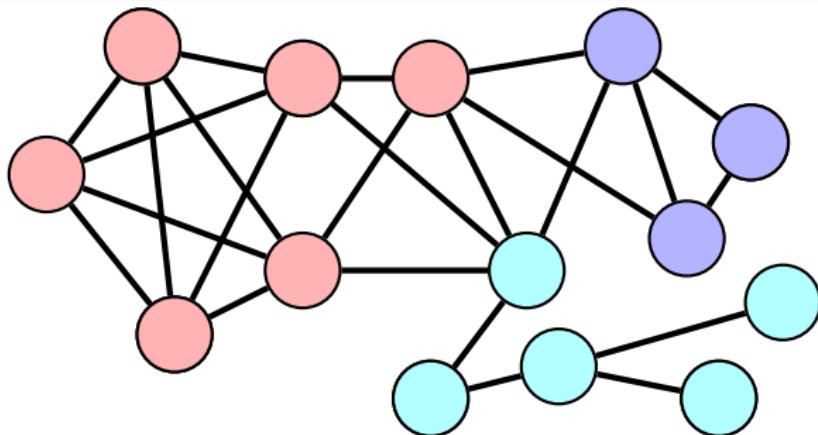
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2. **while** (there is a *bad vertex*) **and** ( $\text{Var}[\text{bad edges}] \geq \text{tol}$ ) **repeat**
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  - 2.2 choose a new colour  $i$  for  $v$  proportionally to  $\omega + \mathcal{N}(v, i)$ ,  $\omega > 1$



# The Modified Petford–Welsh Algorithm

## Stopping Condition

---

$$\text{Var}_{\text{step}} = \text{Var}(\text{bad\_edges}[1 : \text{step}]) < \text{tol}$$

# The Modified Petford–Welsh Algorithm

## Stopping Condition

---

$$\text{Var}_{\text{step}} = \text{Var}(\text{bad\_edges}[\text{step} - L + 1 : \text{step}]) < \text{tol}$$

# The Modified Petford–Welsh Algorithm

## Stopping Condition

---

**bad edges = []**

$$\mu_L = \frac{1}{L}(b_1 + b_2 + \dots + b_L)$$

$$\text{Var}_L = \frac{1}{L-1} (b_1^2 + b_2^2 + \dots + b_L^2) - \frac{L}{L-1} \mu_L^2$$

# The Modified Petford–Welsh Algorithm

## Stopping Condition

---

**step = 1 :**      **bad edges = [b<sub>1</sub>]**

$$\mu_L = \frac{1}{L}(b_1 + b_2 + \dots + b_L)$$

$$\text{Var}_L = \frac{1}{L-1} (b_1^2 + b_2^2 + \dots + b_L^2) - \frac{L}{L-1} \mu_L^2$$

# The Modified Petford–Welsh Algorithm

## Stopping Condition

---

**step** = 1 :           **bad edges** = [ $b_1$ ]

$$\mu_L = \frac{1}{L} (b_1 + b_2 + \dots + b_L)$$

$$\text{Var}_L = \frac{1}{L-1} (b_1^2 + b_2^2 + \dots + b_L^2) - \frac{L}{L-1} \mu_L^2$$

# The Modified Petford–Welsh Algorithm

## Stopping Condition

---

**step = 2 :**      **bad edges = [ $b_1, b_2$ ]**

$$\mu_L = \frac{1}{L}(b_1 + b_2 + \dots + b_L)$$

$$\text{Var}_L = \frac{1}{L-1} (b_1^2 + b_2^2 + \dots + b_L^2) - \frac{L}{L-1} \mu_L^2$$

# The Modified Petford–Welsh Algorithm

## Stopping Condition

---

**step** =  $L$  :      **bad edges** =  $[b_1, b_2, \dots, b_L]$

$$\mu_L = \frac{1}{L} (b_1 + b_2 + \dots + b_L)$$

$$\text{Var}_L = \frac{1}{L-1} (b_1^2 + b_2^2 + \dots + b_L^2) - \frac{L}{L-1} \mu_L^2$$

# The Modified Petford–Welsh Algorithm

## Stopping Condition

---

**step** =  $L$  :      **bad edges** =  $[b_1, b_2, \dots, b_L]$

$$\mu_L = \frac{1}{L} (b_1 + b_2 + \dots + b_L)$$

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# The Modified Petford–Welsh Algorithm

## Stopping Condition

---

**step** =  $L$  :      **bad edges** =  $[b_1, b_2, \dots, b_L]$

$$\mu_L = \frac{1}{L} (b_1 + b_2 + \dots + b_L)$$

$$\text{Var}_L = \frac{1}{L-1} (b_1^2 + b_2^2 + \dots + b_L^2) - \frac{L}{L-1} \mu_L^2$$

# The Modified Petford–Welsh Algorithm

## Stopping Condition

---

**step** =  $L + 1$  :     **bad edges** =  $[b_1, b_2, \dots, b_L, b_{L+1}]$

$$\mu_L = \frac{1}{L} (b_1 + b_2 + \dots + b_L)$$

$$\text{Var}_L = \frac{1}{L-1} (b_1^2 + b_2^2 + \dots + b_L^2) - \frac{L}{L-1} \mu_L^2$$

# The Modified Petford–Welsh Algorithm

## Stopping Condition

---

**step** =  $L + 1$  :     **bad edges** = [ $b_1, b_2, \dots, b_L, b_{L+1}$ ]

$$\mu_L = \frac{1}{L}(b_1 + b_2 + \dots + b_L)$$

$$\text{Var}_L = \frac{1}{L-1} (b_1^2 + b_2^2 + \dots + b_L^2) - \frac{L}{L-1} \mu_L^2$$

$$\mu_{L+1} = \mu_L + \frac{1}{L} (b_{L+1} - b_1)$$

# The Modified Petford–Welsh Algorithm

## Stopping Condition

---

**step** =  $L + 1$  :     **bad edges** = [ $b_1, b_2, \dots, b_L, b_{L+1}$ ]

$$\mu_L = \frac{1}{L}(b_1 + b_2 + \dots + b_L)$$

$$\text{Var}_L = \frac{1}{L-1} (b_1^2 + b_2^2 + \dots + b_L^2) - \frac{L}{L-1} \mu_L^2$$

$$\mu_{L+1} = \mu_L + \frac{1}{L} (b_{L+1} - b_1)$$

$$\text{Var}_{L+1} = \text{Var}_L + \frac{1}{L-1} (b_{L+1} - b_1)(b_{L+1} + b_1 - \mu_{L+1} - \mu_L)$$

# The Modified Petford–Welsh Algorithm

## Stopping Condition

---

**step** =  $L + 2$  :     **bad edges** =  $[b_1, b_2, \dots, b_L, b_{L+1}, b_{L+2}]$

$$\mu_L = \frac{1}{L}(b_1 + b_2 + \dots + b_L)$$

$$\text{Var}_L = \frac{1}{L-1} (b_1^2 + b_2^2 + \dots + b_L^2) - \frac{L}{L-1} \mu_L^2$$

$$\mu_{L+2} = \mu_{L+1} + \frac{1}{L} (b_{L+2} - b_2)$$

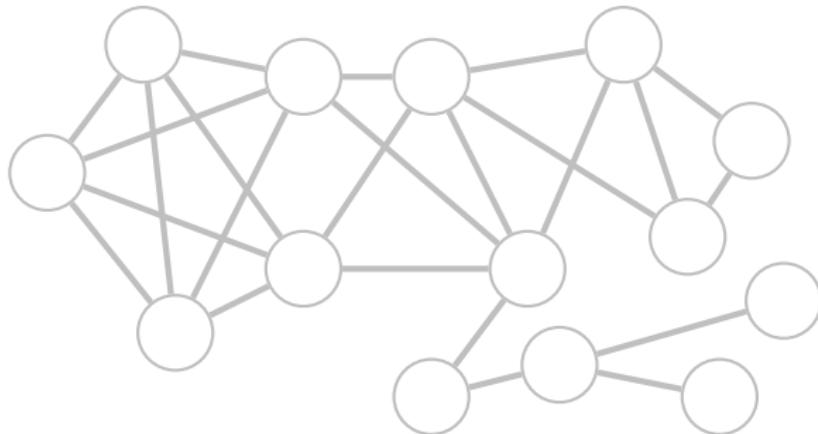
$$\text{Var}_{L+2} = \text{Var}_{L+1} + \frac{1}{L-1} (b_{L+2} - b_2)(b_{L+2} + b_2 - \mu_{L+2} - \mu_{L+1})$$

# The Modified Petford–Welsh Algorithm

## Fine-tuning

---

### Problems



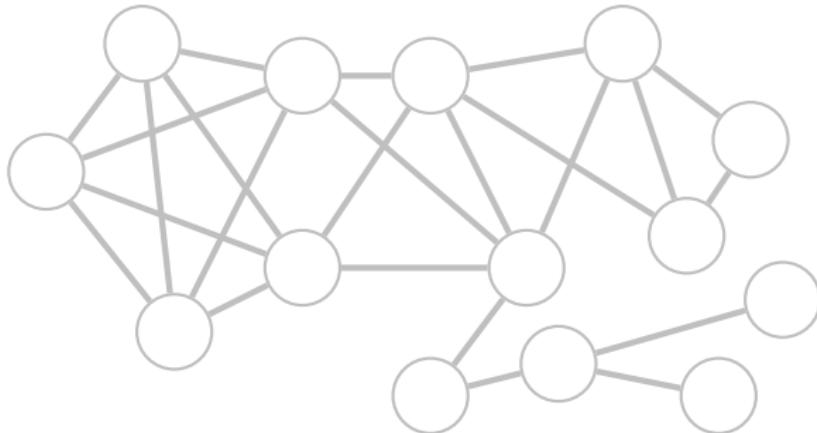
# The Modified Petford–Welsh Algorithm

## Fine-tuning

---

### Problems

- different clusters get assigned the same colour due to random seeds



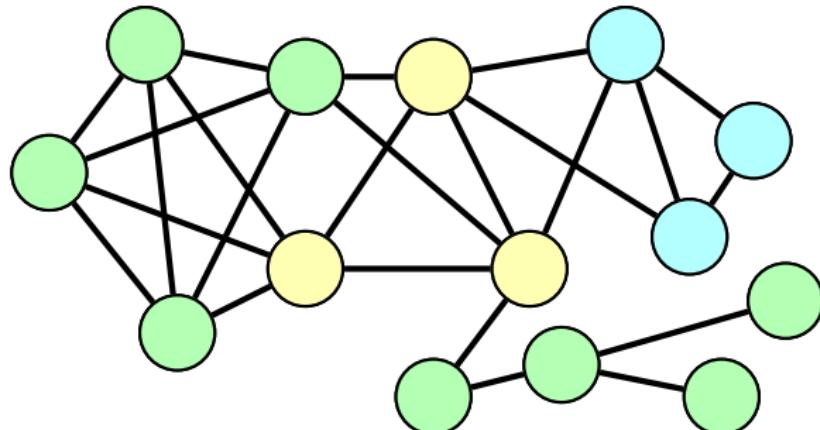
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## Fine-tuning

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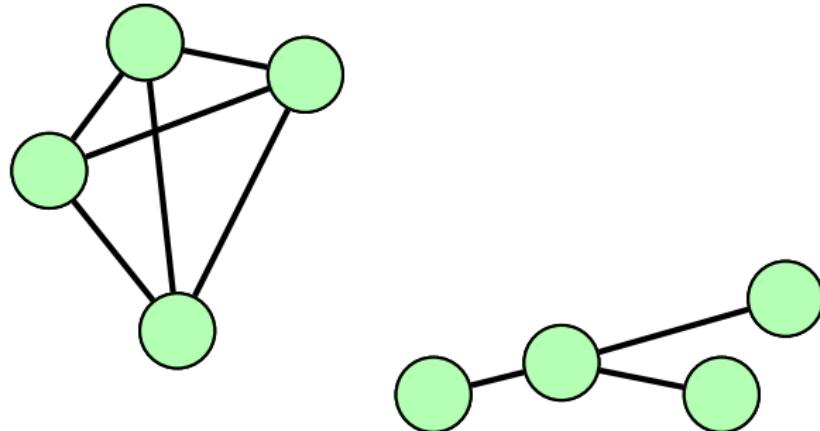
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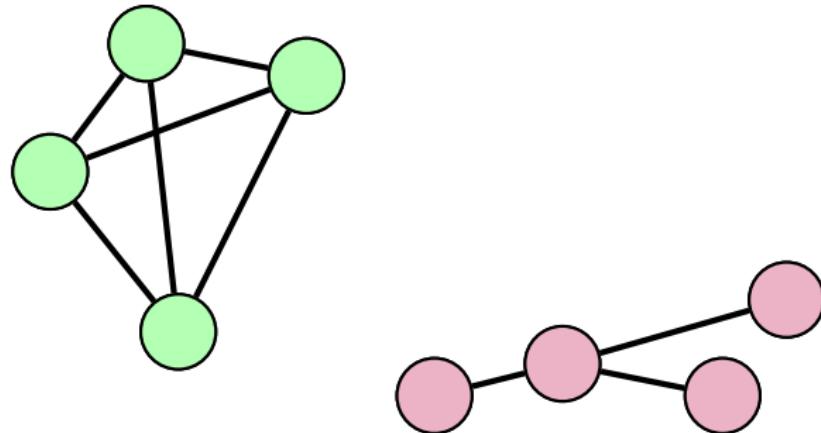
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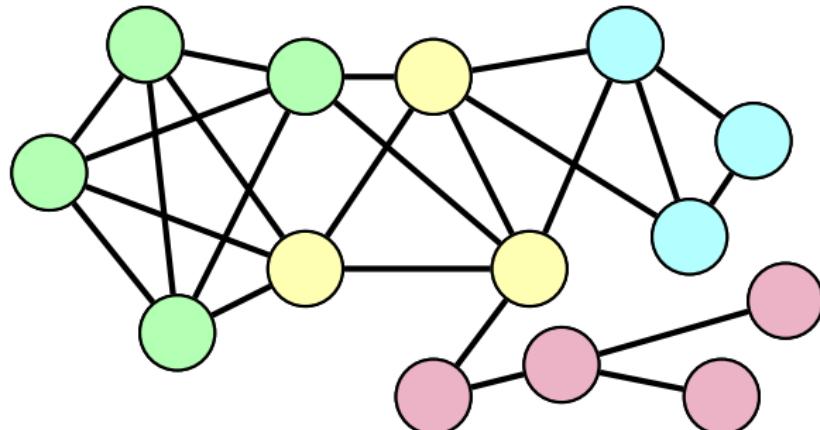


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## Fine-tuning

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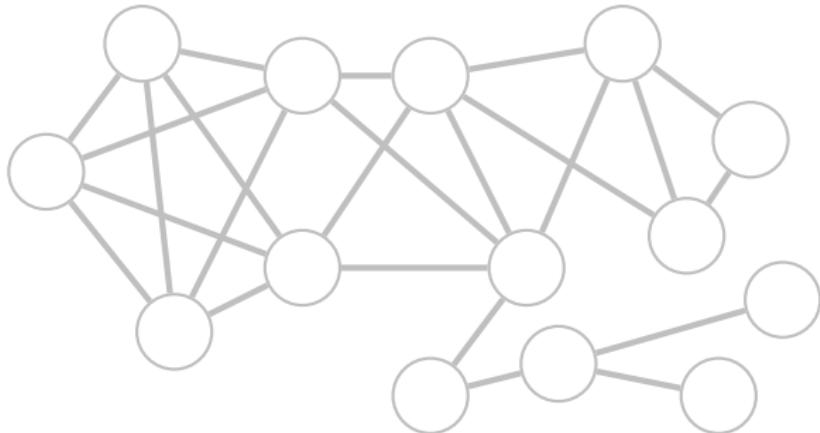
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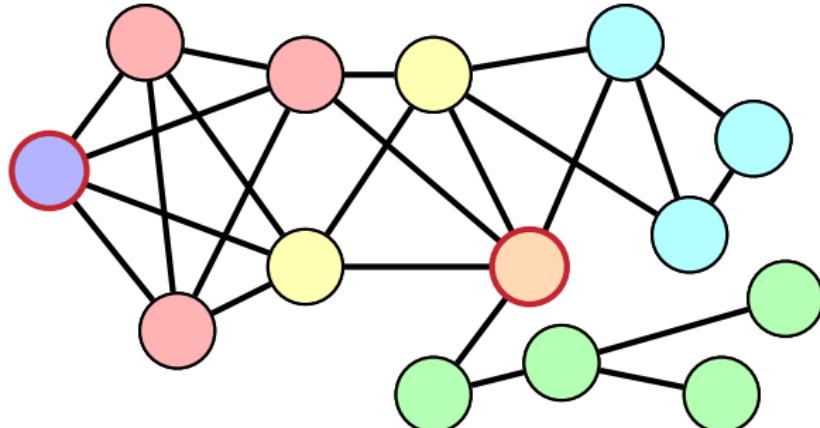


# The Modified Petford–Welsh Algorithm

## Fine-tuning

### Problems

- different clusters get assigned the same colour due to random seeds
- **outliers** (singleton clusters)

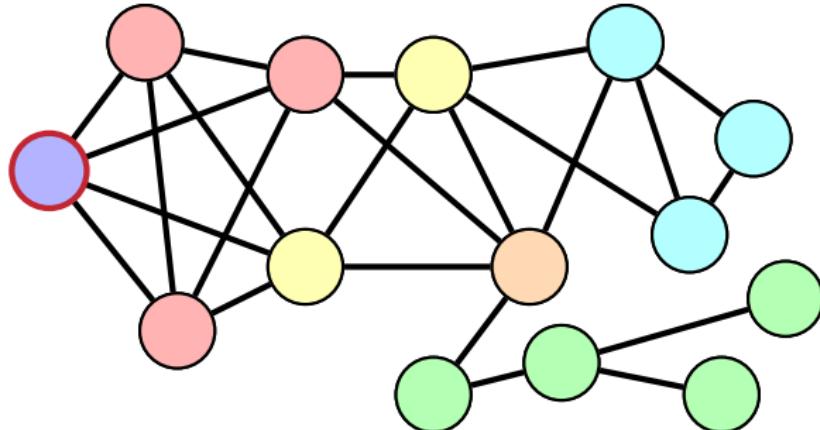


# The Modified Petford–Welsh Algorithm

## Fine-tuning

### Problems

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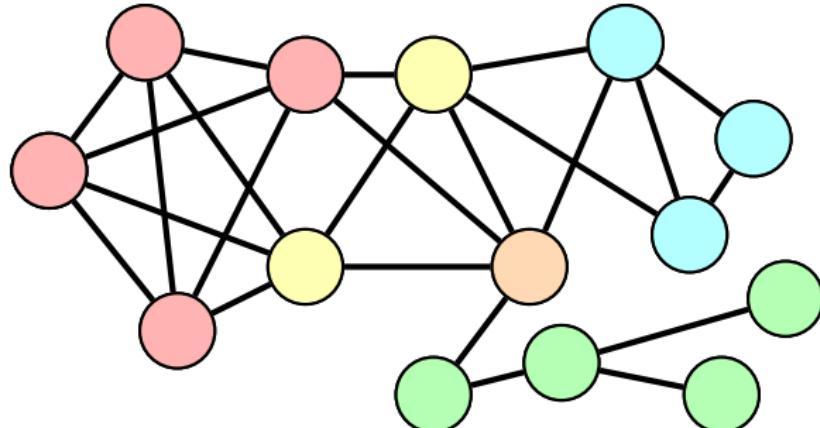
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## Fine-tuning

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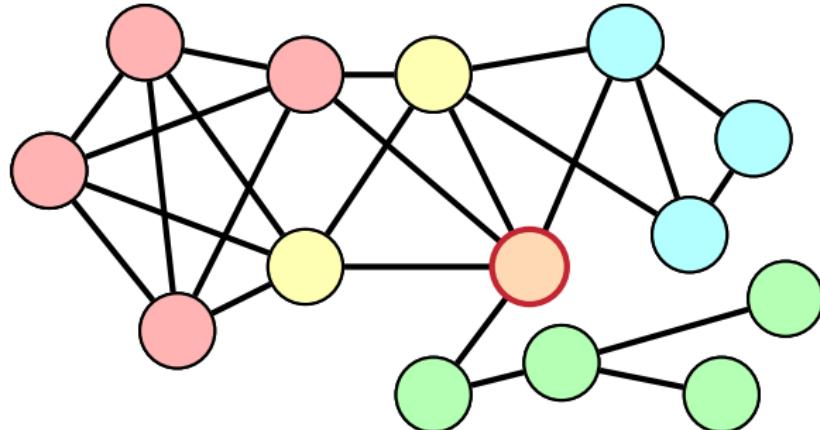


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## Fine-tuning

### Problems

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- outliers (singleton clusters)



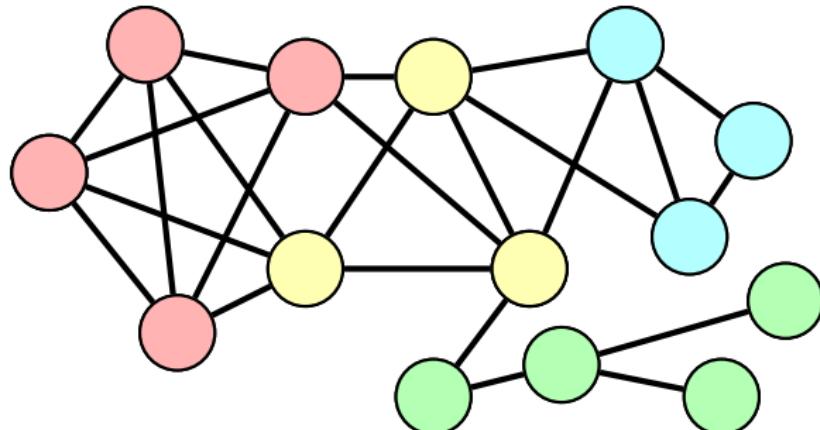
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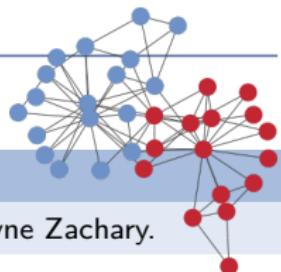


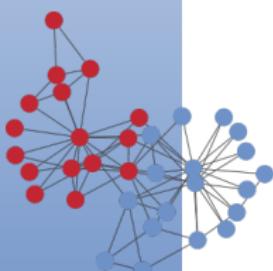
# The Modified Petford–Welsh Algorithm

## Experiments

Zachary ( $|V| = 34, |E| = 78$ )

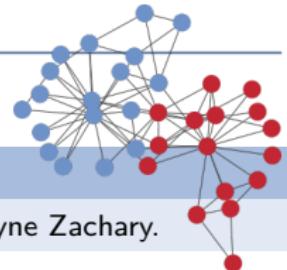
Ties among the members of a university karate club by Wayne Zachary.





# The Modified Petford–Welsh Algorithm

## Experiments



Zachary ( $|V| = 34$ ,  $|E| = 78$ )

Ties among the members of a university karate club by Wayne Zachary.

Method	NMI	ARI	$\phi$	$\gamma$	$Q$	$ \mathcal{C} $
Edge betweenness	0.517	0.392	0.424	0.692	0.401	5
Fastgreedy	0.576	0.568	0.574	0.756	0.381	3
Infomap	0.578	0.591	0.668	0.821	0.402	3
Label propagation	0.865	0.882	<b>0.773</b>	<b>0.949</b>	0.415	3
Leading eigenvector	0.612	0.435	0.487	0.667	0.393	4
Multilevel	0.516	0.392	0.558	0.731	0.419	4
Spinglass	0.627	0.509	0.563	0.756	<b>0.420</b>	4
Walktrap	0.531	0.321	0.434	0.590	0.353	5
<b>mPW</b>	<b>1.000</b>	<b>1.000</b>	<b>0.773</b>	<b>0.949</b>	0.403	2



# The Modified Petford–Welsh Algorithm

## Experiments

International E-road network

( $|V| = 1040$ ,  $|E| = 1305$ )  $\text{tol} = 0.0007$

An international system for numbering  
and designating roads stretching  
throughout Europe and some parts of  
Central Asia.



# The Modified Petford–Welsh Algorithm

## Experiments

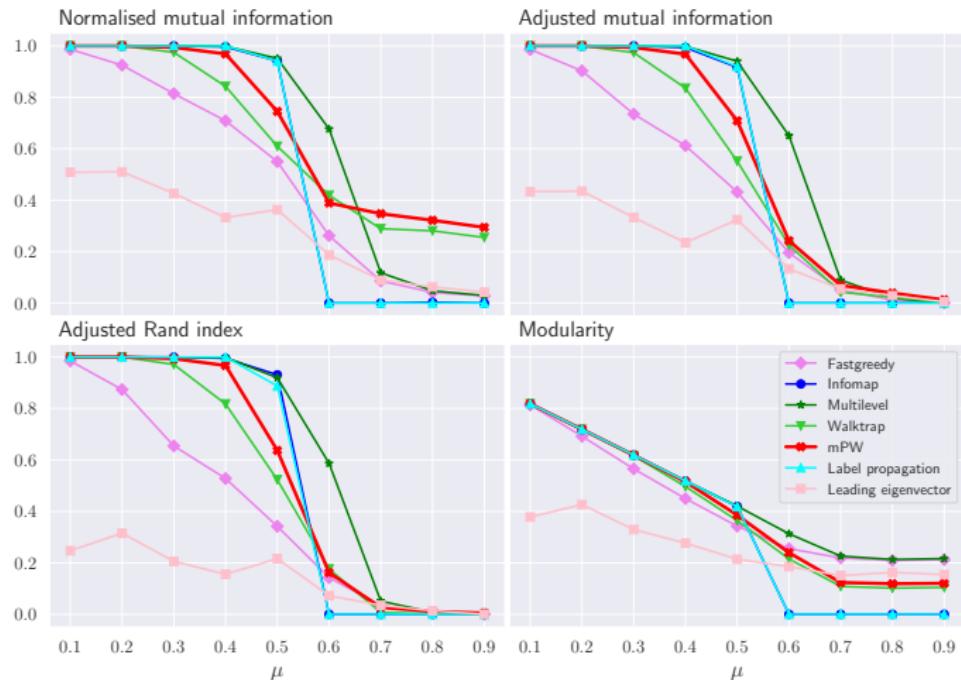
International E-road network  
 $(|V| = 1040, |E| = 1305)$

An international system for numbering  
and designating roads stretching  
throughout Europe and some parts of  
Central Asia.

Method	$\phi$	$\gamma$	$Q$	$ \mathcal{C} $
Edge betweenness	—	—	—	—
Fastgreedy	0.860	0.917	0.861	24
Infomap	0.663	0.787	0.777	126
Label propagation	0.731	0.856	0.828	82
Leading eigenvector	0.794	0.887	0.835	26
Multilevel	0.873	0.921	0.867	24
Spinglass	0.866	0.924	<b>0.872</b>	25
Walktrap	0.757	0.886	0.828	67
<b>mPW</b>	<b>0.945</b>	<b>0.979</b>	0.845	17

# The Modified Petford–Welsh Algorithm

## Experiments



LFR( $|V| = 1000, \gamma = 2, \beta = 1, k_{avg} = 15, k_{max} = 100, c_{min} = 50, c_{max} = 100$ )

# Co-evolution of the Multilayer News Flow

Science

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**SHARE** REPORT

## The spread of true and false news online

Soroush Vosoughi<sup>1</sup>, Deb Roy<sup>1</sup>, Sinan Aral<sup>2,\*</sup>

<sup>1</sup>Massachusetts Institute of Technology (MIT), the Media Lab, E14-526, 75 Amherst Street, Cambridge, MA 02142, USA.  
<sup>2</sup>MIT, E62-364, 100 Main Street, Cambridge, MA 02142, USA.

\*Corresponding author. Email: sinan@mit.edu  
—Hide authors and affiliations

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Vol. 359, Issue 6380, pp. 1146-1151  
DOI: 10.1126/science.aap9559

Article Figures & Data Info & Metrics eLetters PDF

**Lies spread faster than the truth**

There is worldwide concern over false news and the possibility that it can influence political, economic, and social well-being. To understand how false news spreads, Vosoughi *et al.* used a data set of rumor cascades on Twitter from 2006 to 2017. About 126,000 rumors were spread by ~3 million people. False news reached more people than the truth; the top 1% of false news cascades diffused to between 1000 and 100,000 people, whereas the truth rarely diffused to more than 1000 people. Falsehood also diffused faster than the truth. The degree of novelty and the emotional reactions of recipients may be responsible for the differences observed.

Vosoughi, S., Roy, D. & Aral, S. The spread of true and false news online, *Science* 359(6380) (2018), 1146–1151.



Science  
Vol 359, Issue 6380  
09 March 2018  
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# Co-evolution of the Multilayer News Flow

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**news**

any asserted claim

Vosoughi, S., Roy, D. & Aral, S. The spread of true and false news online, *Science* **359**(6380) (2018), 1146–1151.

# Co-evolution of the Multilayer News Flow



**news**

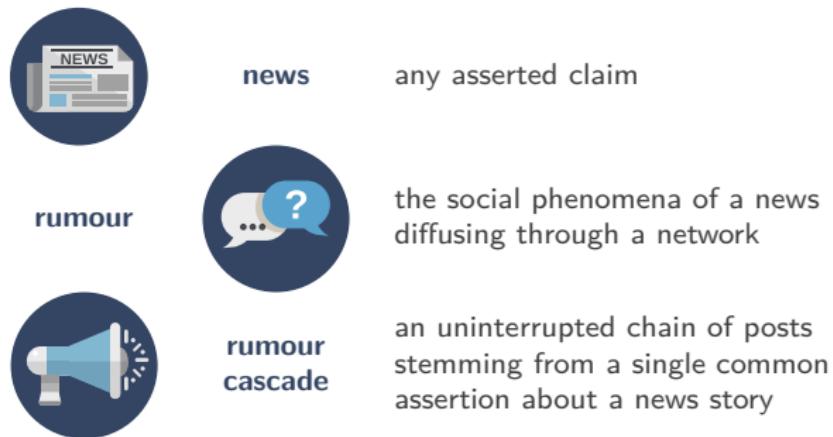
any asserted claim



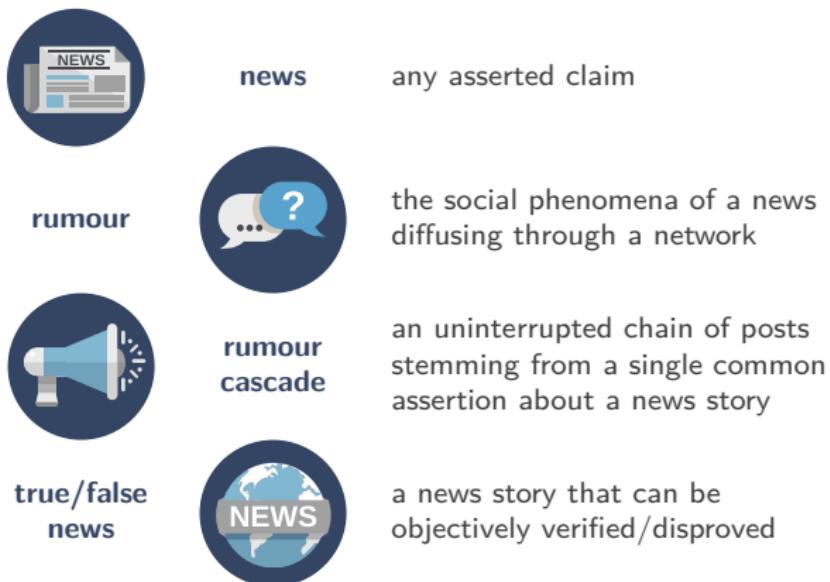
**rumour**

the social phenomena of a news  
diffusing through a network

# Co-evolution of the Multilayer News Flow



# Co-evolution of the Multilayer News Flow



# Co-evolution of the Multilayer News Flow

## RESEARCH

### SOCIAL SCIENCE

## The spread of true and false news online

Soroush Vosoughi,<sup>1</sup> Deb Roy,<sup>1</sup> Sinan Aral<sup>2\*</sup>

We investigated the differential diffusion of all of the verified true and false news stories distributed on Twitter from 2006 to 2017. The data comprise ~126,000 stories tweeted by ~3 million people more than 4.5 million times. We classified news as true or false using information from six independent fact-checking organizations that exhibited 95 to 98% agreement on the classifications. Falsehood diffused significantly farther, faster, deeper, and more broadly than the truth in all categories of information, and the effects were more pronounced for false political news than for false news about terrorism, natural disasters, science, urban legends, or financial information. We found that false news was more novel than true news, which suggests that people were more likely to share novel information. Whereas false stories inspired fear, disgust, and surprise in replies, true stories inspired anticipation, sadness, joy, and trust. Contrary to conventional wisdom, robots accelerated the spread of true and false news at the same rate, implying that false news spreads more than the truth because humans, not robots, are more likely to spread it.

**F**oundational theories of decision-making (*1–3*), cooperation (*4*), communication (*5*), and markets (*6*) all view some conceptualization of truth or accuracy as central to the functioning of nearly every human endeavor. Yet, both true and false information

Current work analyzes the spread of single rumors, like the discovery of the Higgs boson (*12*) or the Haitian earthquake of 2010 (*14*), and multiple rumors from a single disaster event, like the Boston Marathon bombing of 2013 (*10*), or it develops theoretical models of rumor diffusion

support their positions as unreliable or fake news, whereas sources that support their positions are labeled reliable or not fake, the term has lost all connection to the actual veracity of the information presented, rendering it meaningless for use in academic classification. We have therefore explicitly avoided the term fake news throughout this paper and instead use the more objectively verifiable terms “true” or “false” news. Although the terms fake news and misinformation also imply a willful distortion of the truth, we do not make any claims about the intent of the purveyors of the information in our analyses. We instead focus our attention on veracity and stories that have been verified as true or false.

We also purposefully adopt a broad definition of the term news. Rather than defining what constitutes news on the basis of the institutional source of the assertions in a story, we refer to any asserted claim made on Twitter as news (we defend this decision in the supplementary materials section on “reliable sources,” section S1.2). We define news as any story or claim with an assertion in it and a rumor as the social phenomena of a news story or claim spreading or diffusing through the Twitter network. That is, rumors are inherently social and involve the sharing of claims between people. News, on the other hand, is an assertion with claims, whether it is shared or not.

A rumor cascade begins on Twitter when a

# Co-evolution of the Multilayer News Flow

Falsehood diffused significantly **farther, faster, deeper, and more broadly** than the truth in **all** categories of information.



# Co-evolution of the Multilayer News Flow

Falsehood diffused significantly farther, faster, deeper, and more broadly than the truth in all categories of information.

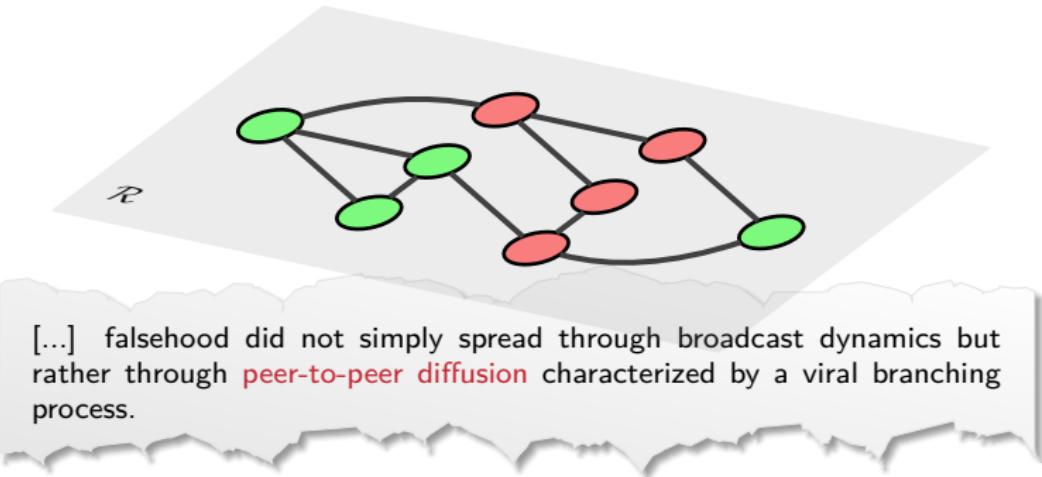
Whereas the truth rarely diffused to more than 1000 people, the top 1% of false-news cascades routinely diffused to between 1000 and **100,000** people. [...] It took the truth about **six times** as long as falsehood to reach 1500 people and **20 times** as long as falsehood to reach a cascade depth of 10. [...] falsehoods were **70% more likely** to be retweeted than the truth, even when controlling for [user characteristics]

# Co-evolution of the Multilayer News Flow

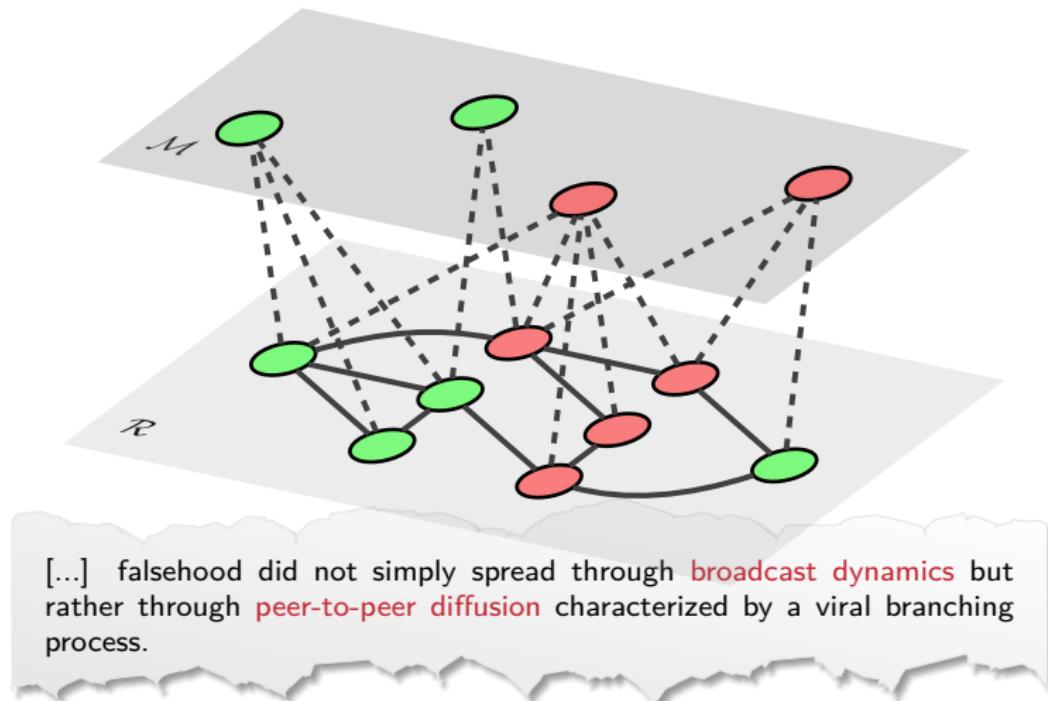
Falsehood diffused significantly farther, faster, deeper, and more broadly than the truth in all categories of information.

Whereas the truth rarely diffused to more than 1000 people, the top 1% of false-news cascades routinely diffused to between 1000 and 100,000 people. [...] It took the truth about six times as long as falsehood to reach 1500 people and 20 times as long as falsehood to reach a cascade depth of 10. [...] falsehoods were 70% more likely to be retweeted than the truth, even when controlling for [user characteristics]

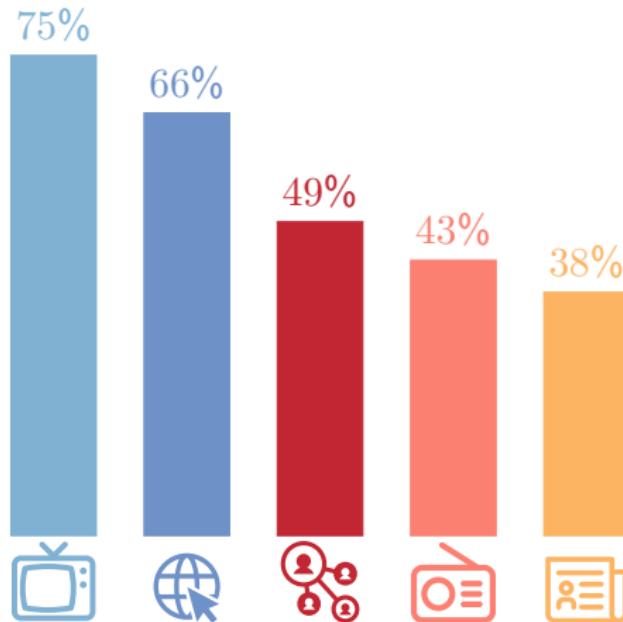
[...] falsehood did not simply spread through **broadcast dynamics** but rather through **peer-to-peer diffusion** characterized by a viral branching process.



# Co-evolution of the Multilayer News Flow



# Co-evolution of the Multilayer News Flow



Motivation

Outline

Evolutionary  
Dynamics on  
Graphs

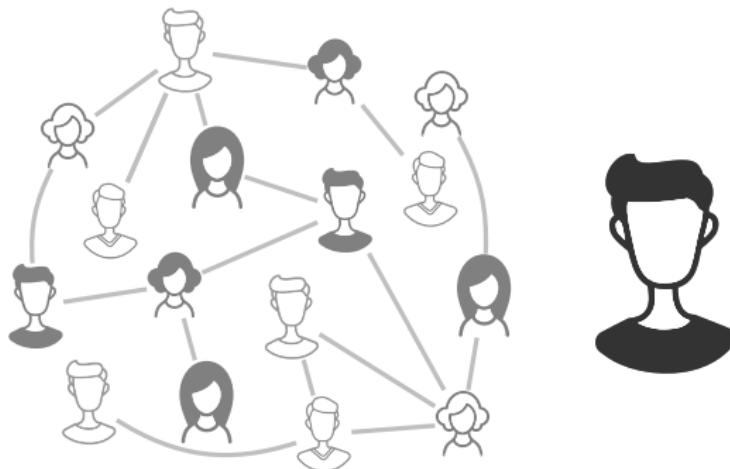
The Modified  
Petford-Welsh  
Algorithm

Evolving  
Graphs

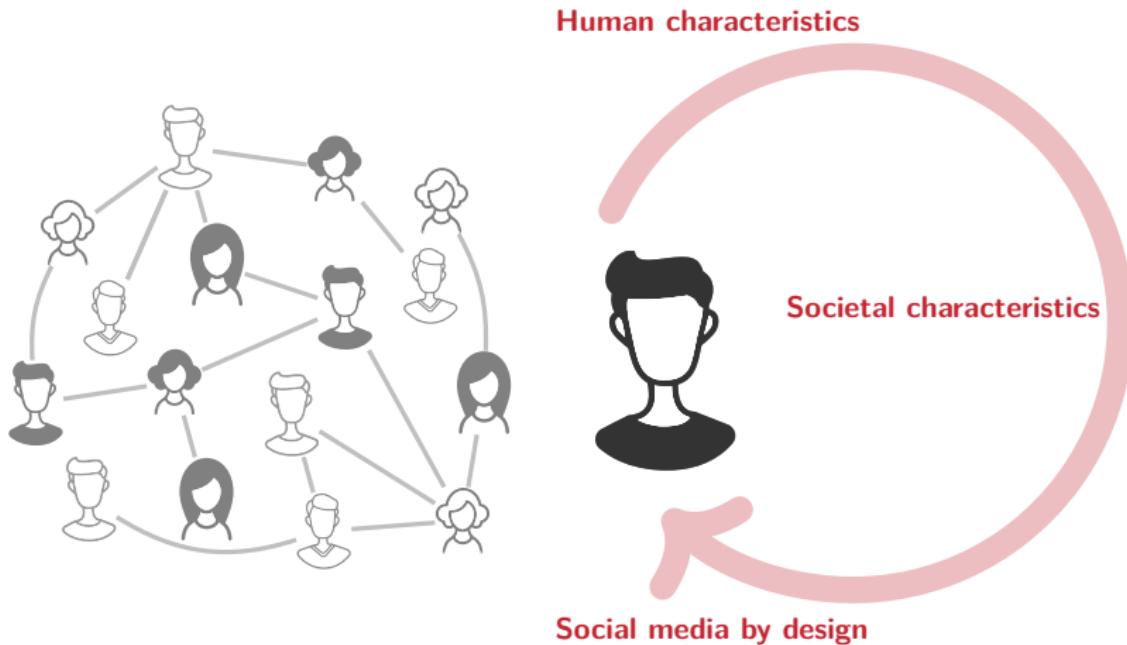
Co-evolution of the  
Multilayer News Flow

# Co-evolution of the Multilayer News Flow

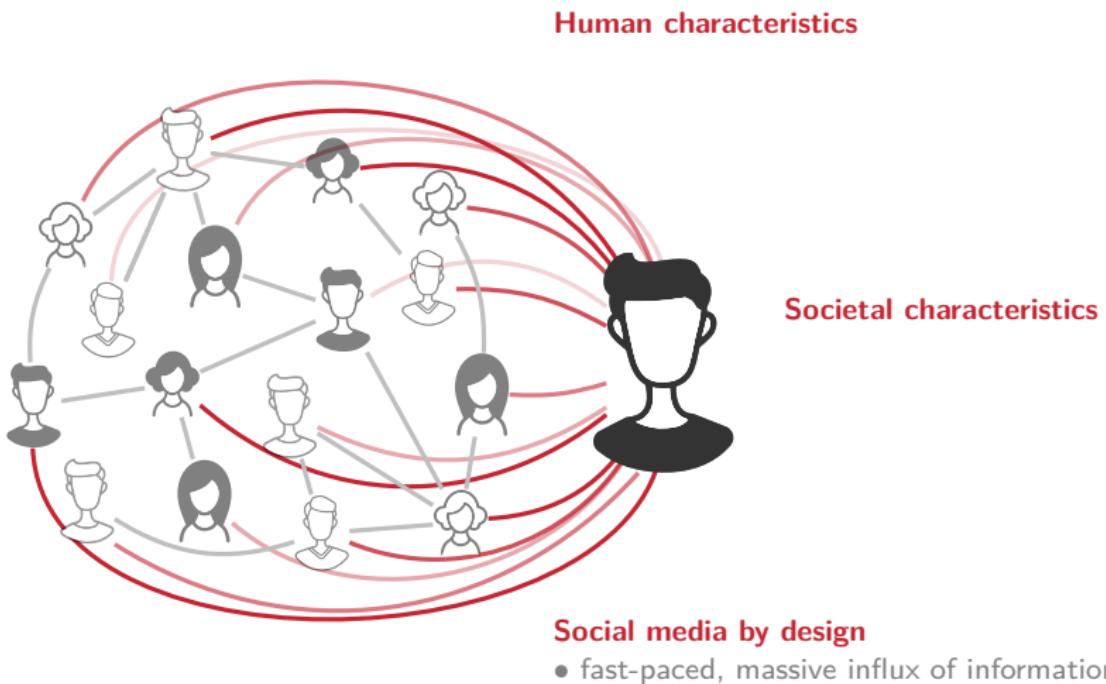
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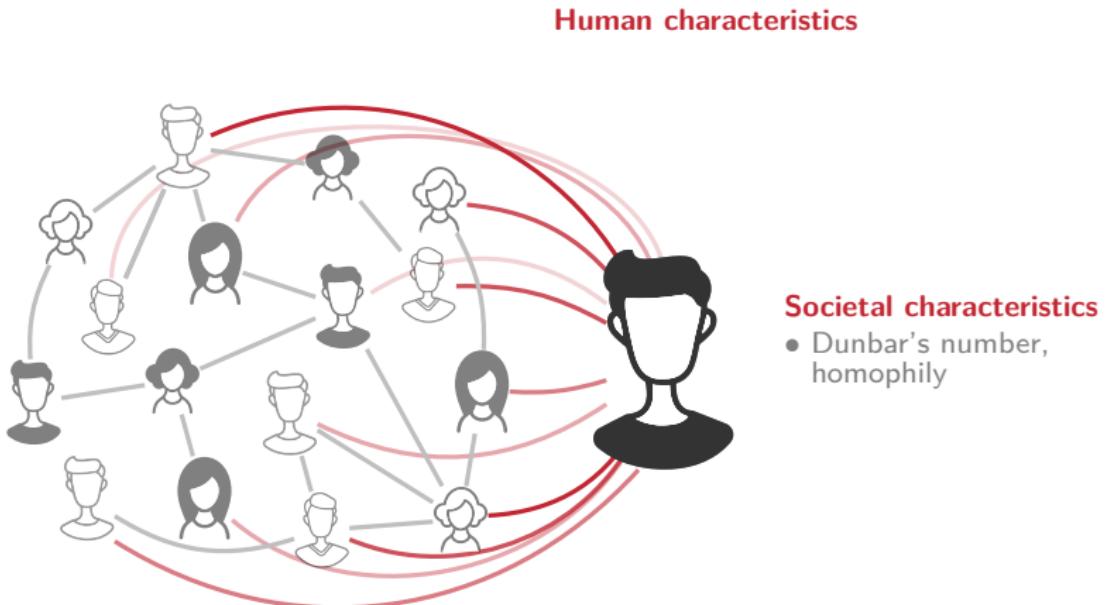
# Co-evolution of the Multilayer News Flow



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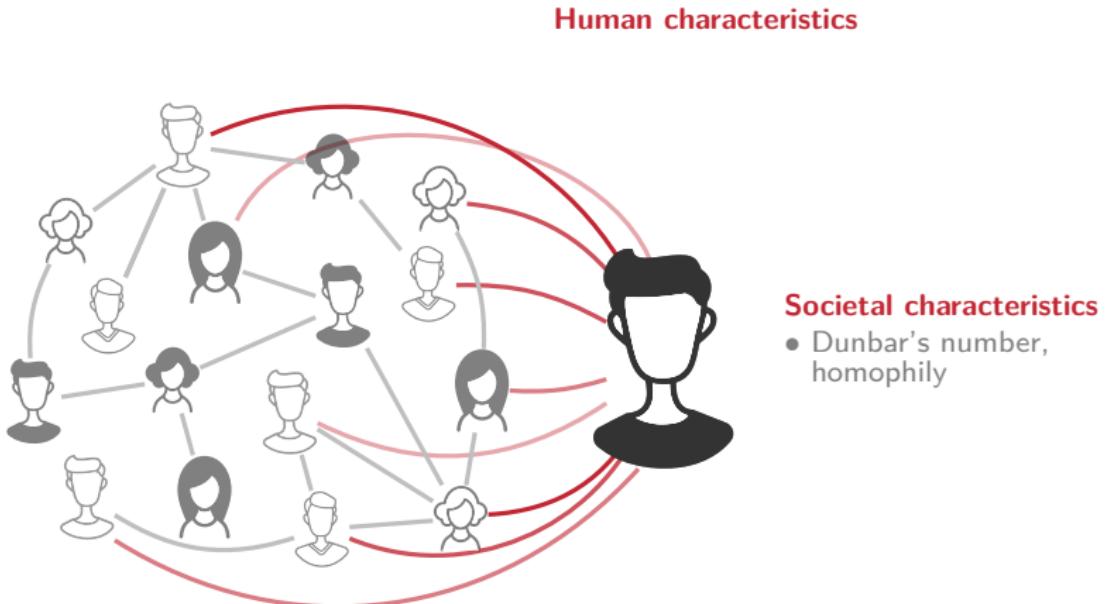
# Co-evolution of the Multilayer News Flow



## Social media by design

- fast-paced, massive influx of information

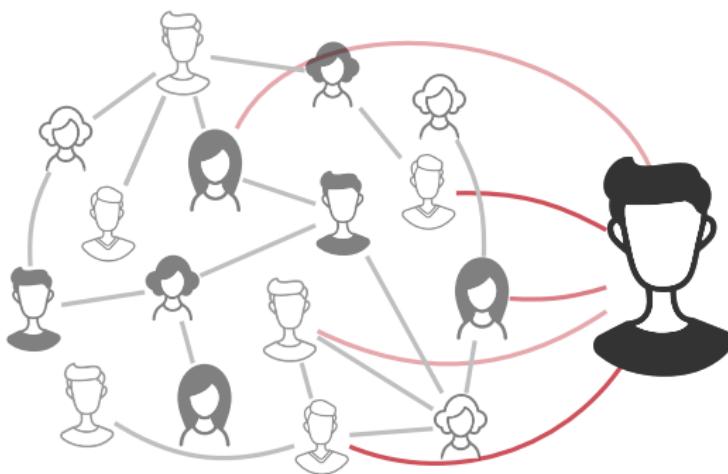
# Co-evolution of the Multilayer News Flow



## Social media by design

- fast-paced, massive influx of information
- engagement-boosting business model

# Co-evolution of the Multilayer News Flow



## Human characteristics

- limited time, short attention span

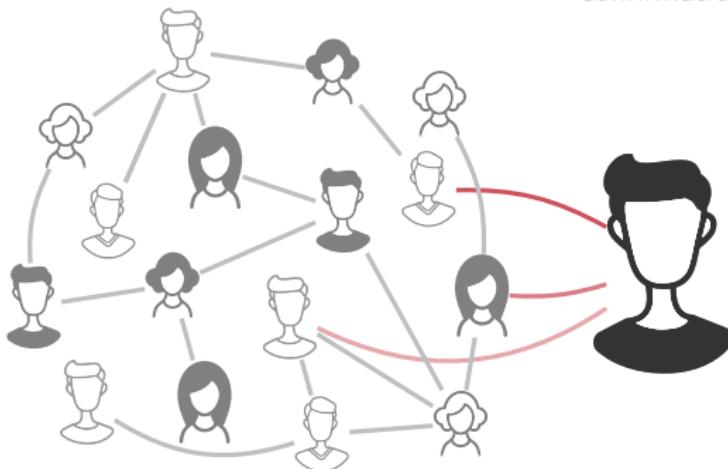
## Societal characteristics

- Dunbar's number, homophily

## Social media by design

- fast-paced, massive influx of information
- engagement-boosting business model

# Co-evolution of the Multilayer News Flow



## Human characteristics

- limited time, short attention span
- confirmation bias, familiarity heuristics

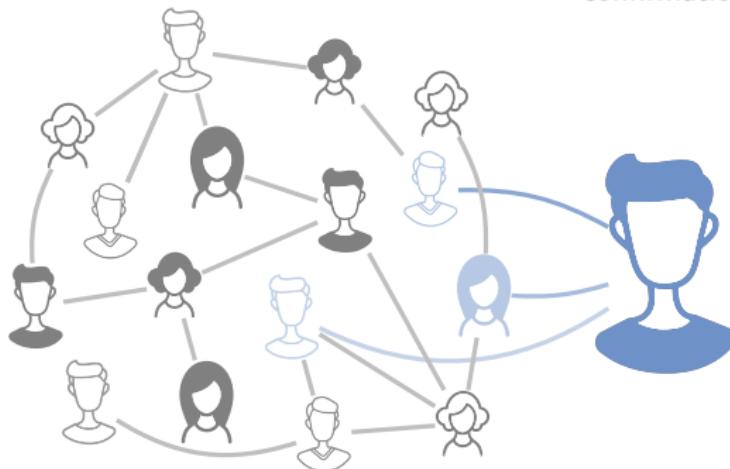
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- engagement-boosting business model

# Co-evolution of the Multilayer News Flow



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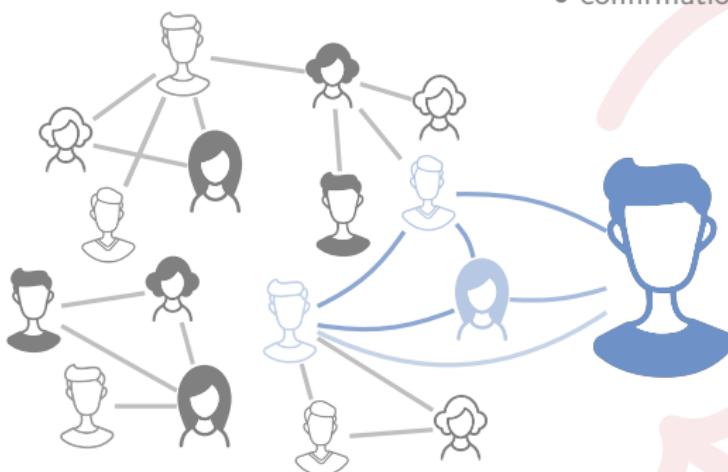
## Societal characteristics

- Dunbar's number,
- homophily
- conformity bias

## Social media by design

- fast-paced, massive influx of information
- engagement-boosting business model

# Co-evolution of the Multilayer News Flow



## Human characteristics

- limited time, short attention span
- confirmation bias, familiarity heuristics

## Societal characteristics

- Dunbar's number, homophily
- conformity bias

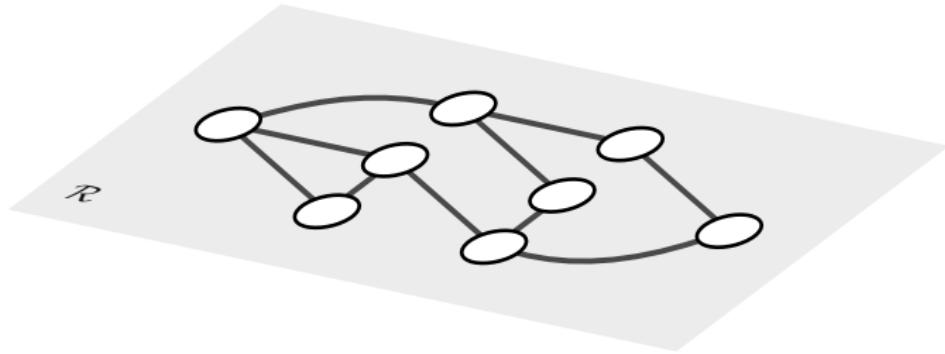
Filter bubbles &  
echo chambers

## Social media by design

- fast-paced, massive influx of information
- engagement-boosting business model

# Co-evolution of the Multilayer News Flow

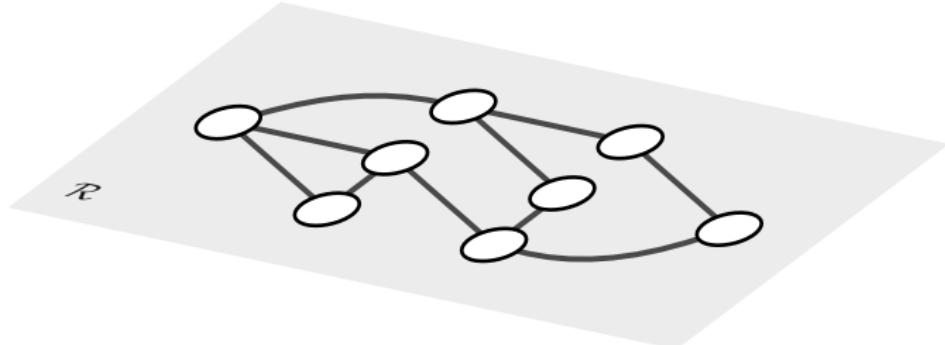
$$G_R = (\mathcal{R}, E_R)$$



# Co-evolution of the Multilayer News Flow

$$G_R = (\mathcal{R}, E_R)$$

Barabási-Albert ( $P(k) \sim k^{-\gamma}$ ,  $\gamma = 2.5$ )  
 $\Delta(G_R) \lesssim$  Dunbar's number

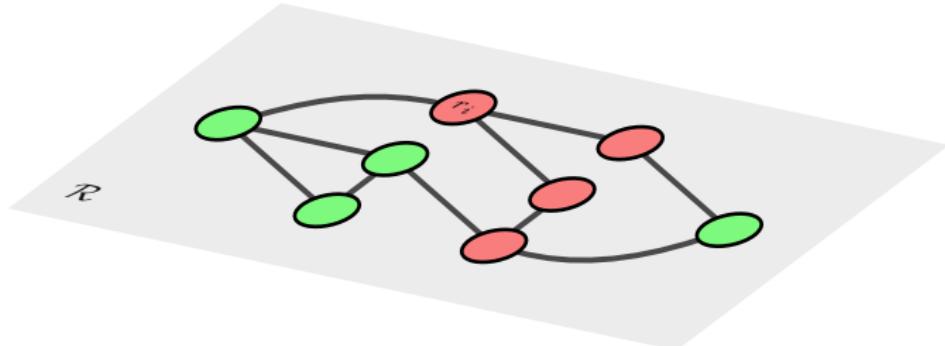


# Co-evolution of the Multilayer News Flow

$$G_R = (\mathcal{R}, E_R)$$

$$r_i(t) \in \{0, 1\}$$

Barabási-Albert ( $P(k) \sim k^{-\gamma}$ ,  $\gamma = 2.5$ )  
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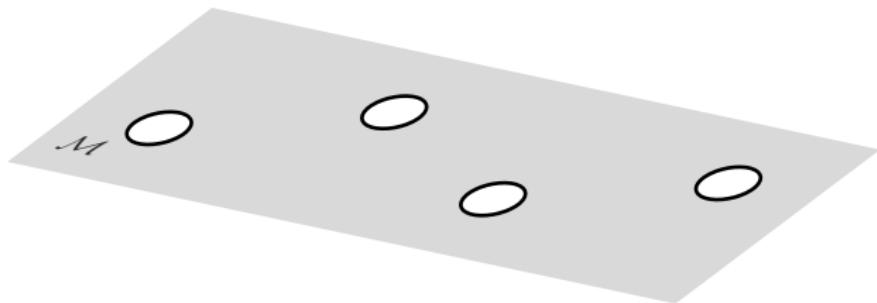
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Barabási-Albert ( $P(k) \sim k^{-\gamma}$ ,  $\gamma = 2.5$ )  
 $\Delta(G_R) \lesssim$  Dunbar's number

$$G_M = (\mathcal{M}, E_M)$$



# Co-evolution of the Multilayer News Flow

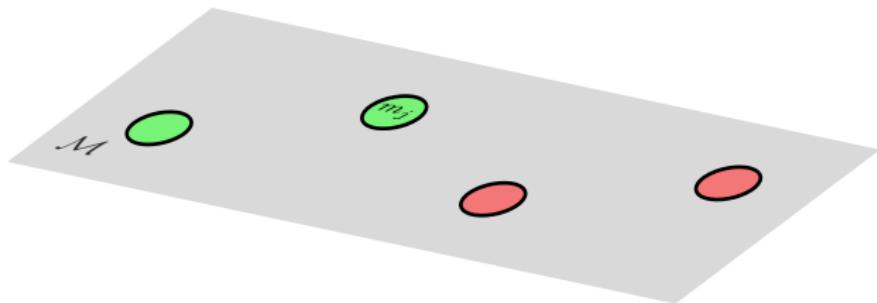
$$G_R = (\mathcal{R}, E_R)$$

$$r_i(t) \in \{0, 1\}$$

$$G_M = (\mathcal{M}, E_M)$$

$$m_j \in \{0, 1\}$$

Barabási-Albert ( $P(k) \sim k^{-\gamma}$ ,  $\gamma = 2.5$ )  
 $\Delta(G_R) \lesssim$  Dunbar's number



# Co-evolution of the Multilayer News Flow

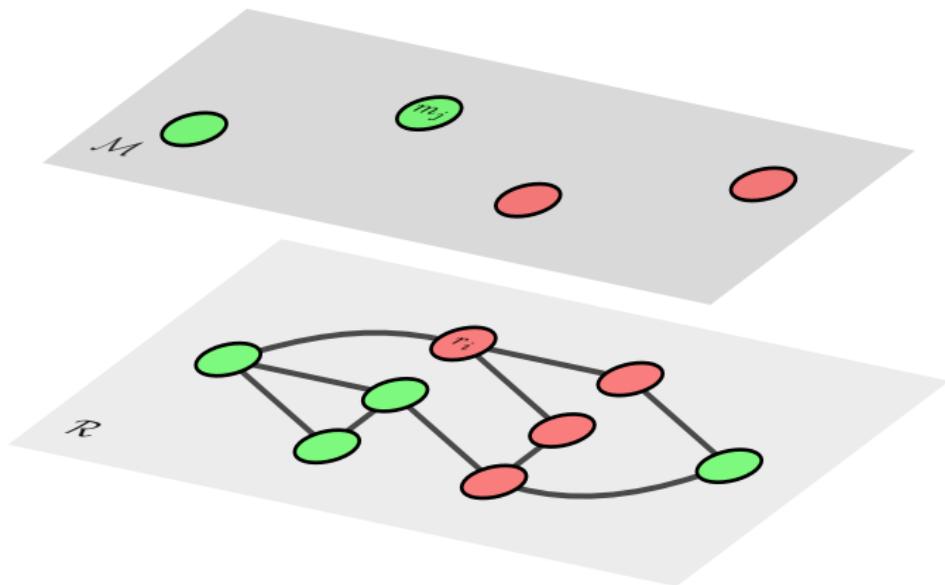
$$G_R = (\mathcal{R}, E_R)$$

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# Co-evolution of the Multilayer News Flow

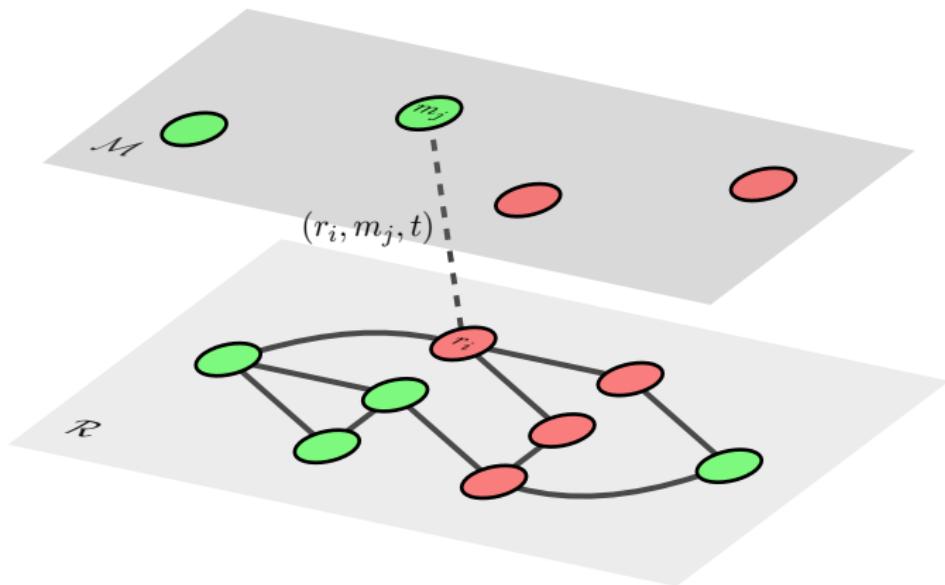
$$G_R = (\mathcal{R}, E_R)$$

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# Co-evolution of the Multilayer News Flow

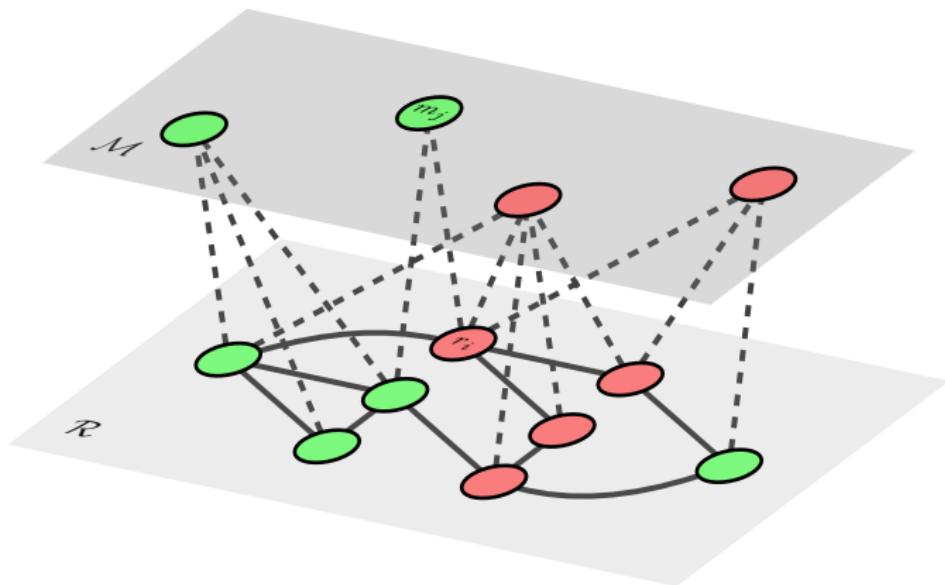
$$G_R = (\mathcal{R}, E_R)$$

$$r_i(t) \in \{0, 1\}$$

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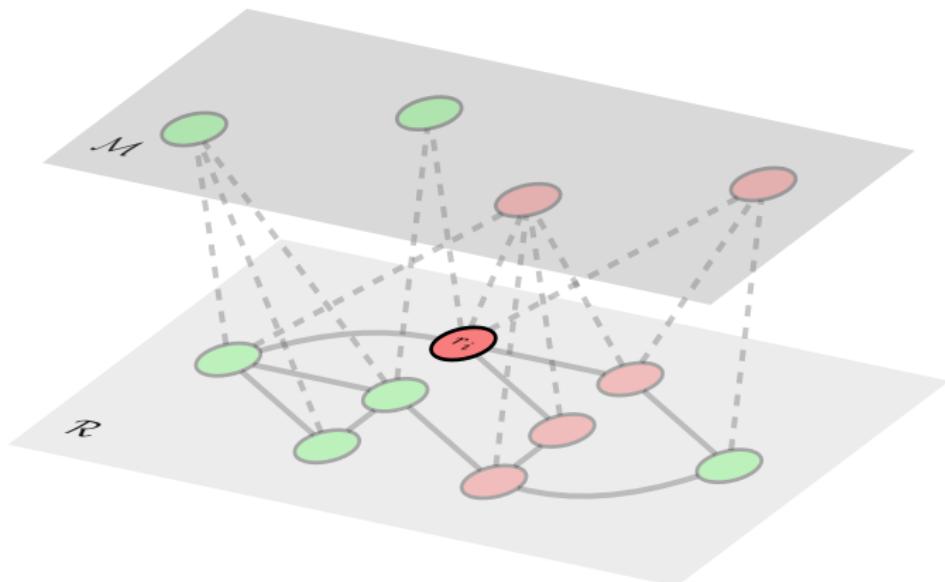
$$m_j \in \{0, 1\}$$

Barabási-Albert ( $P(k) \sim k^{-\gamma}$ ,  $\gamma = 2.5$ )  
 $\Delta(G_R) \lesssim$  Dunbar's number



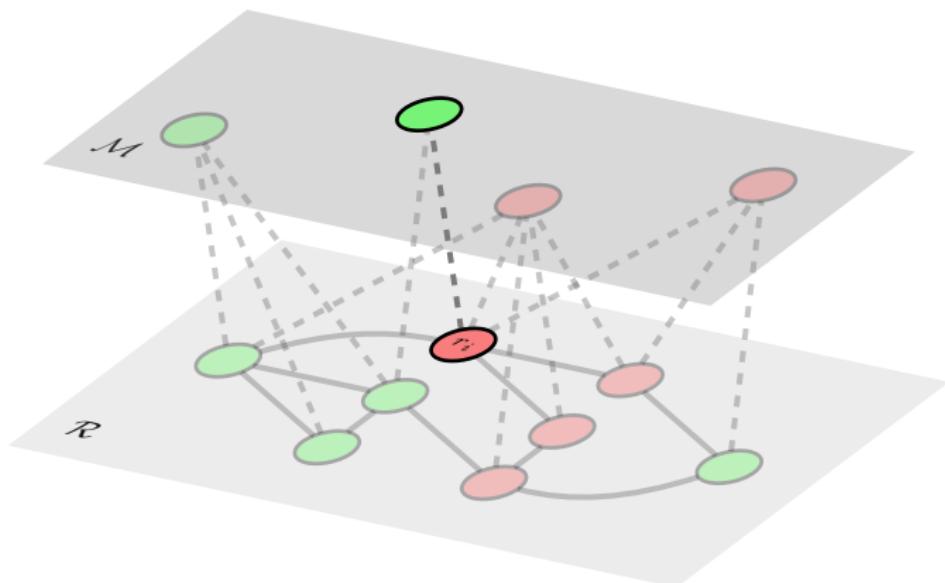
# Co-evolution of the Multilayer News Flow

Media repertoire  $(r_i^T(t), r_i^F(t))$        $r_i^T(t) + r_i^F(t) \leq M \ll R$



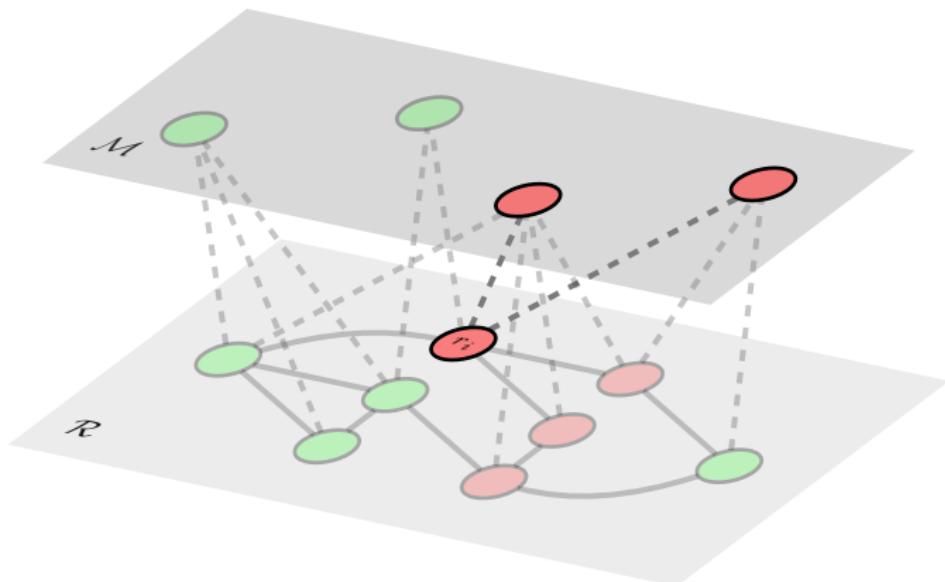
# Co-evolution of the Multilayer News Flow

Media repertoire  $(r_i^T(t), r_i^F(t))$   $r_i^T(t) + r_i^F(t) \leq M \ll R$



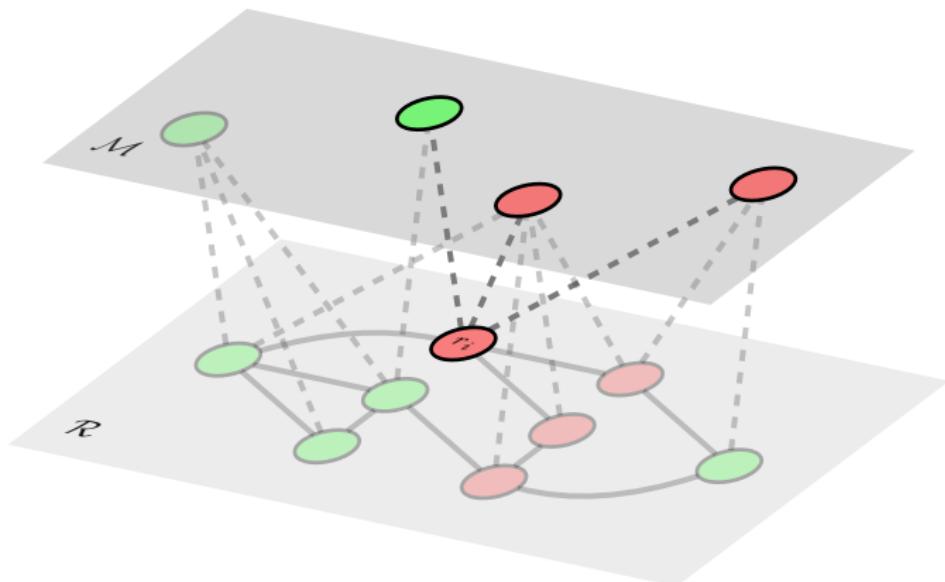
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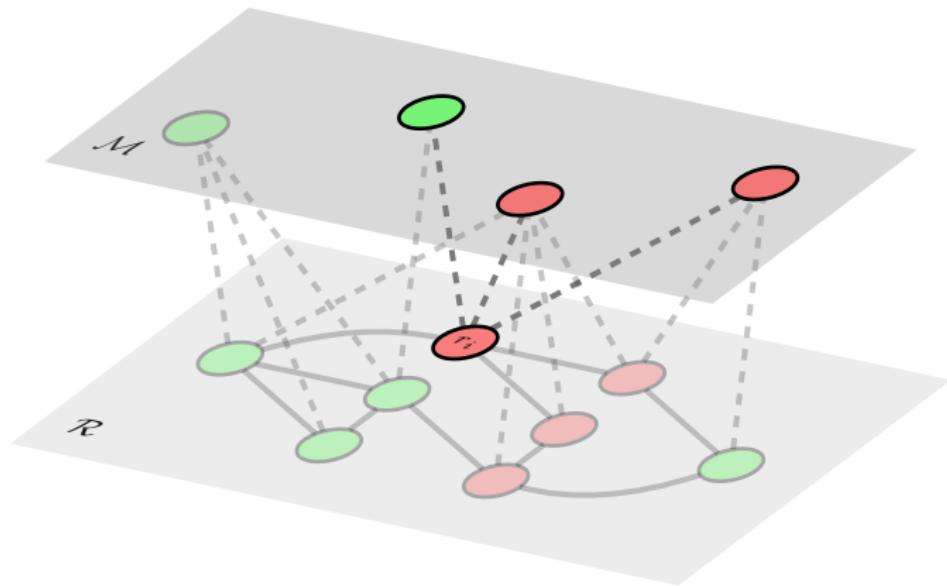
# Co-evolution of the Multilayer News Flow

Media repertoire  $(r_i^T(t), r_i^F(t))$   $r_i^T(t) + r_i^F(t) \leq M \ll R$



# Co-evolution of the Multilayer News Flow

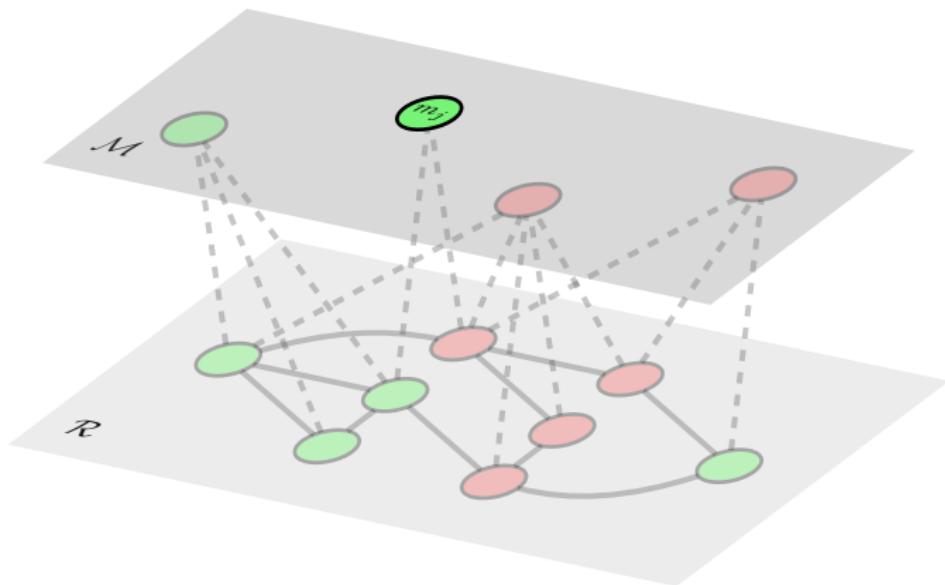
**Media repertoire**  $(\rho_i^T(t), \rho_i^F(t))$   $r_i^T(t) + r_i^F(t) \leq M \ll R$



# Co-evolution of the Multilayer News Flow

Media repertoire     $(r_i^T(t), r_i^F(t)) \quad r_i^T(t) + r_i^F(t) \leq M \ll R$

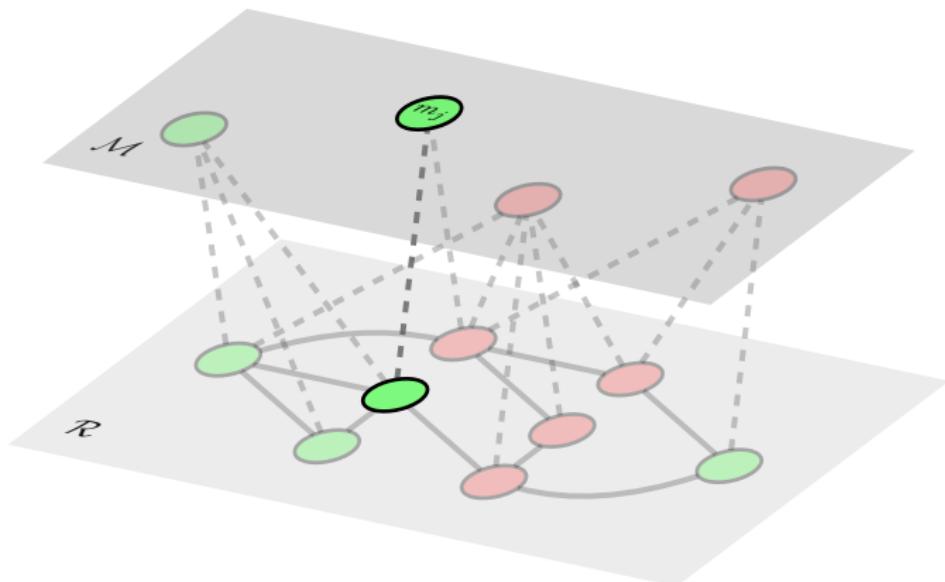
Media clientele     $(m_j^T(t), m_j^F(t))$



# Co-evolution of the Multilayer News Flow

Media repertoire     $(r_i^T(t), r_i^F(t)) \quad r_i^T(t) + r_i^F(t) \leq M \ll R$

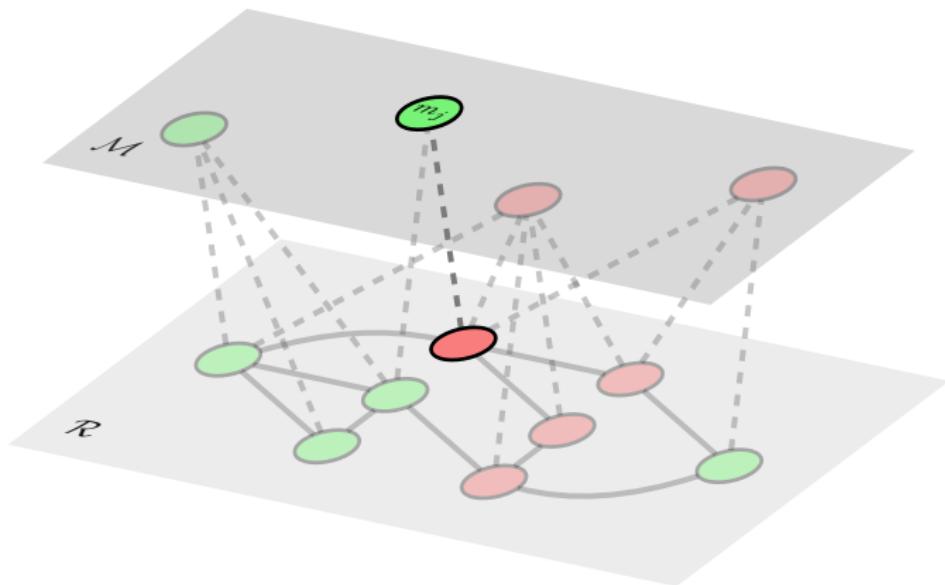
Media clientele     $(m_j^T(t), m_j^F(t))$



# Co-evolution of the Multilayer News Flow

Media repertoire     $(r_i^T(t), r_i^F(t))$      $r_i^T(t) + r_i^F(t) \leq M \ll R$

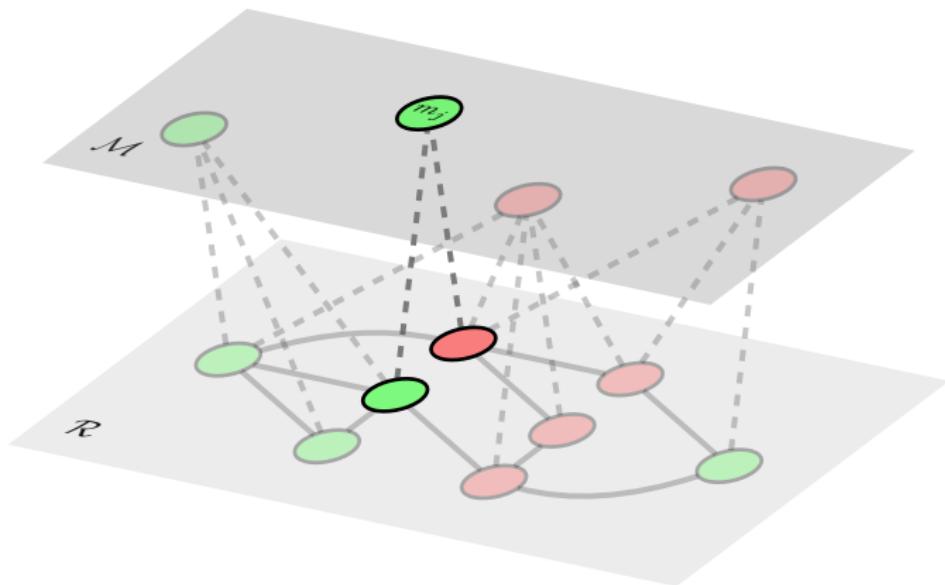
Media clientele     $(m_j^T(t), m_j^F(t))$



# Co-evolution of the Multilayer News Flow

Media repertoire     $(r_i^T(t), r_i^F(t)) \quad r_i^T(t) + r_i^F(t) \leq M \ll R$

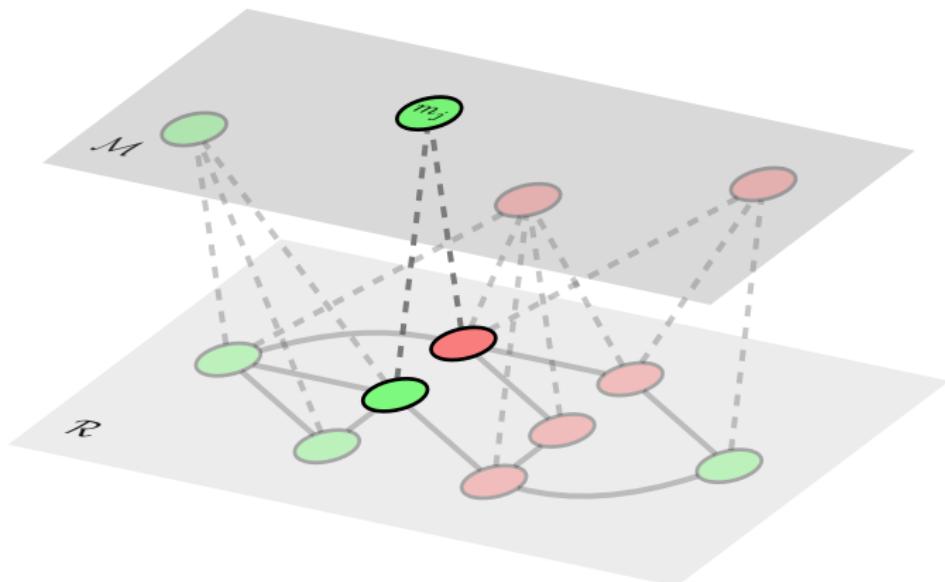
Media clientele     $(m_j^T(t), m_j^F(t))$



# Co-evolution of the Multilayer News Flow

Media repertoire     $(r_i^T(t), r_i^F(t)) \quad r_i^T(t) + r_i^F(t) \leq M \ll R$

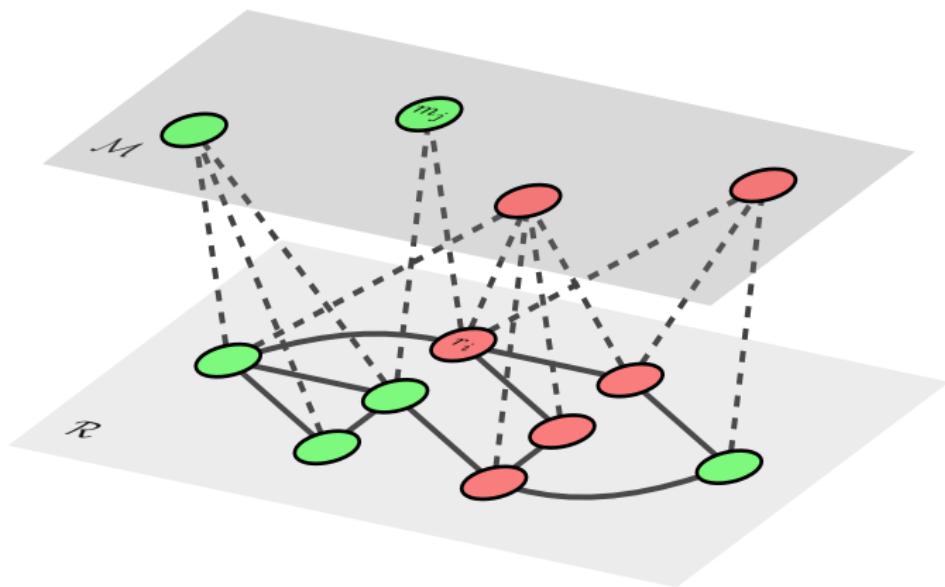
Media clientele     $(\mu_j^T(t), \mu_j^F(t))$



# Co-evolution of the Multilayer News Flow

**Media repertoire**  $(r_i^T(t), r_i^F(t)) \quad r_i^T(t) + r_i^F(t) \leq M \ll R$

**Media clientele**  $(m_j^T(t), m_j^F(t))$

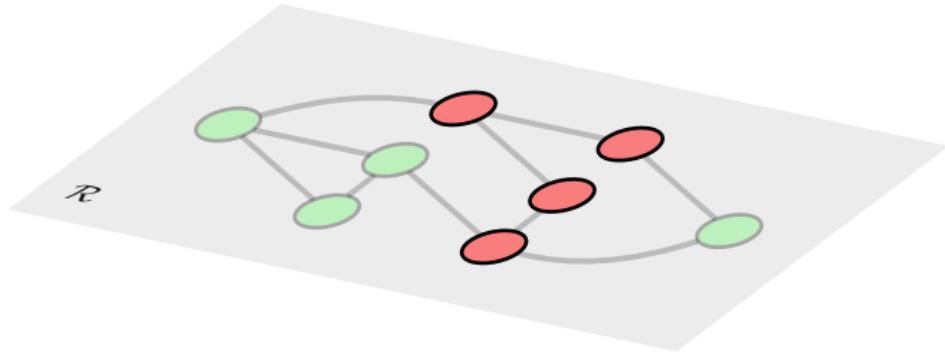


# Co-evolution of the Multilayer News Flow

False-news followers

$$\mathcal{R}^F(t) = \{r_i \mid r_i(t) = 0\}$$

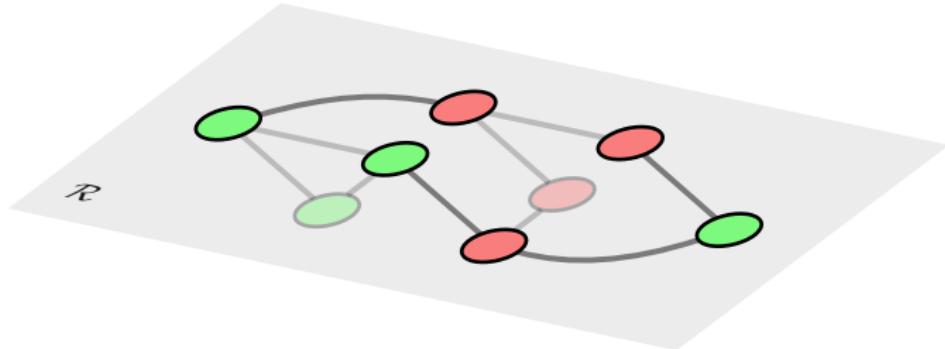
$$\rho^F(t)$$



# Co-evolution of the Multilayer News Flow

**False-news followers**       $\mathcal{R}^F(t) = \{r_i \mid r_i(t) = 0\}$        $\rho^F(t)$

**Unbalanced edges**       $E_R^{FT}(t) = \{r_i r_j \mid r_i(t) \neq r_j(t)\}$        $\rho^{FT}(t)$



# Co-evolution of the Multilayer News Flow

False-news followers

$$\mathcal{R}^F(t) = \{r_i \mid r_i(t) = 0\}$$

$$\rho^F(t)$$

Unbalanced edges

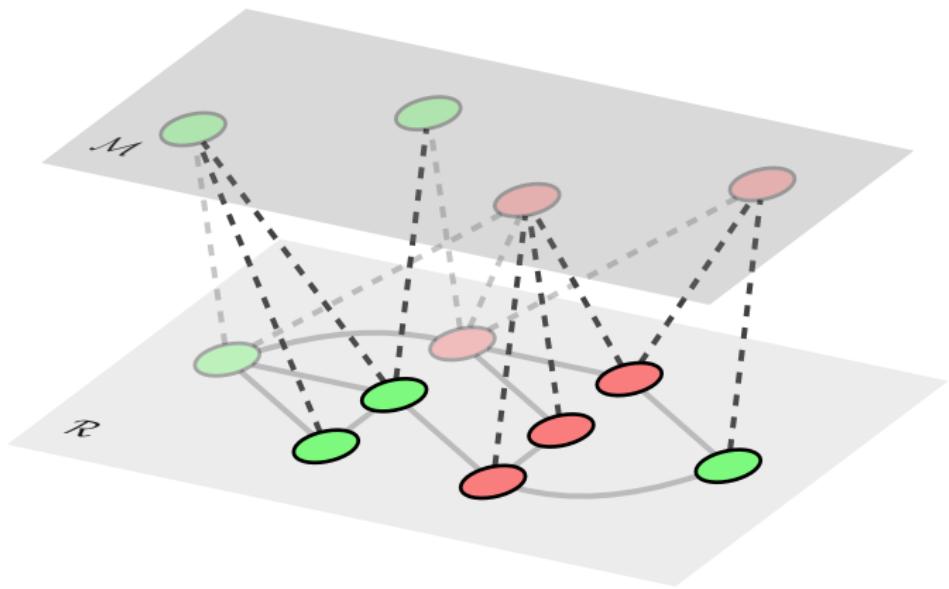
$$E_R^{FT}(t) = \{r_i r_j \mid r_i(t) \neq r_j(t)\}$$

$$\rho^{FT}(t)$$

Polarised individuals

$$\mathcal{P}(t) = \{r_i \mid r_i^T(t) \cdot r_i^F(t) = 0\}$$

$$\rho^P(t)$$



# Co-evolution of the Multilayer News Flow

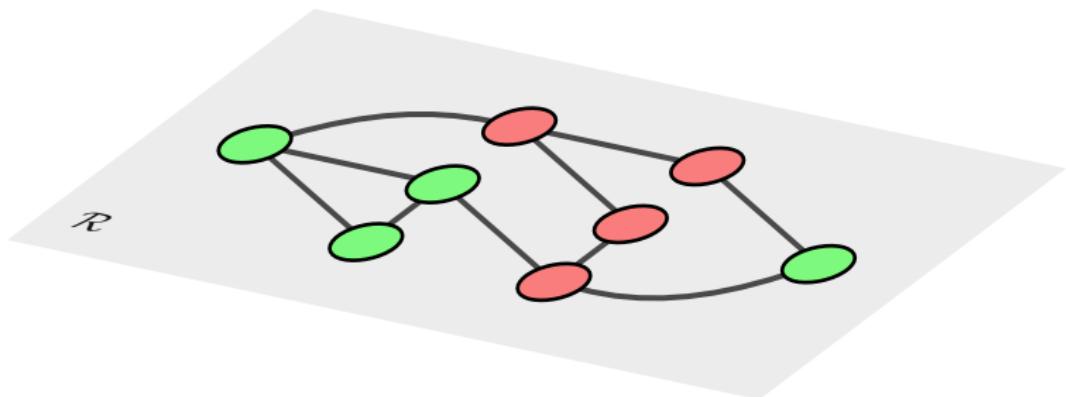
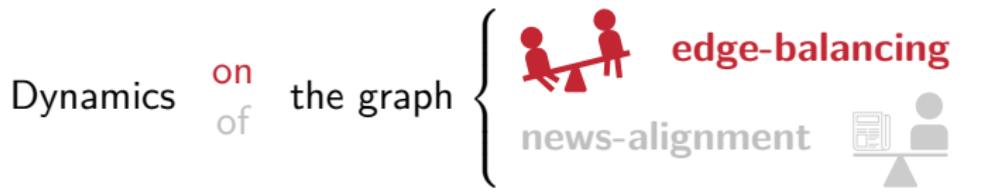
Dynamics **on** the graph {  **edge-balancing** }

# Co-evolution of the Multilayer News Flow

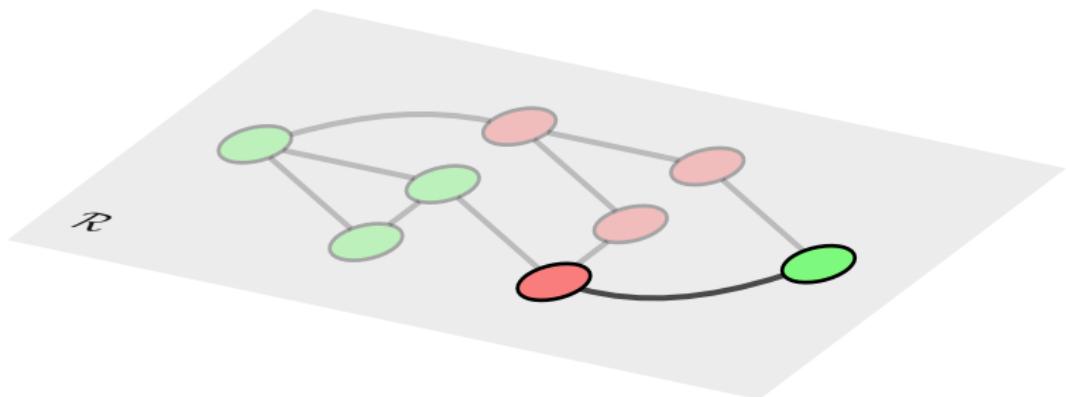
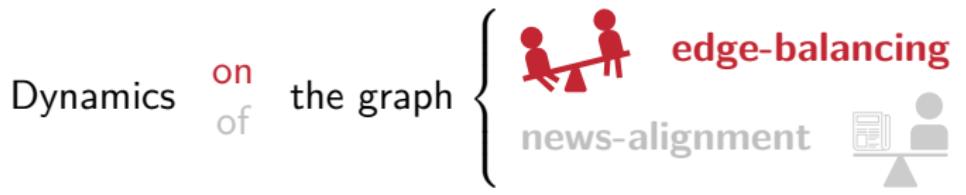
Dynamics on  
of the graph

{  **edge-balancing**  
 **news-alignment**

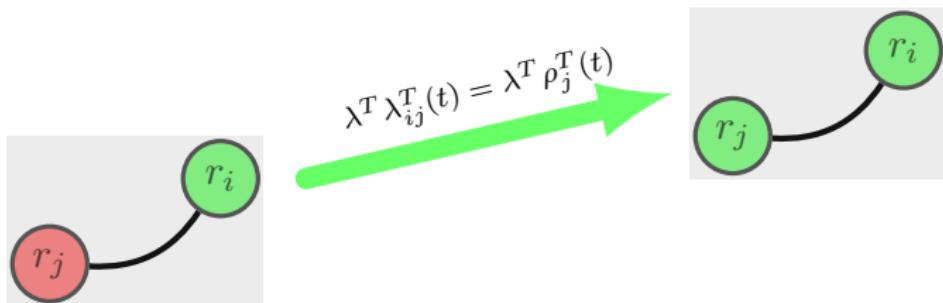
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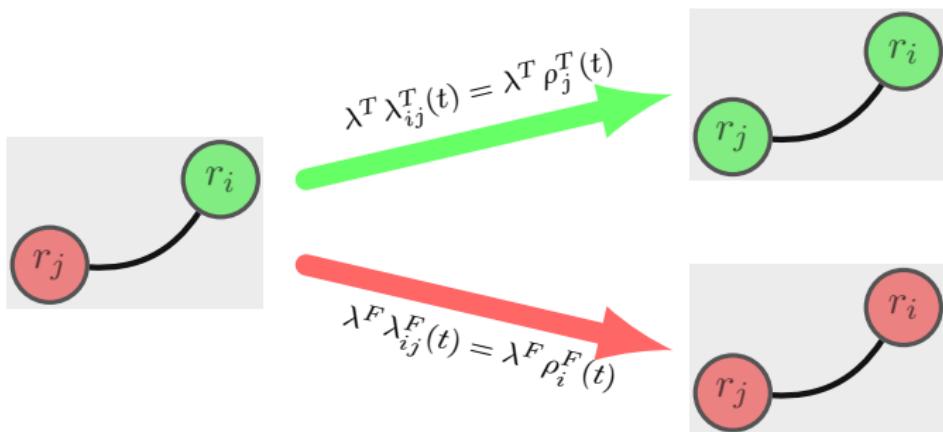
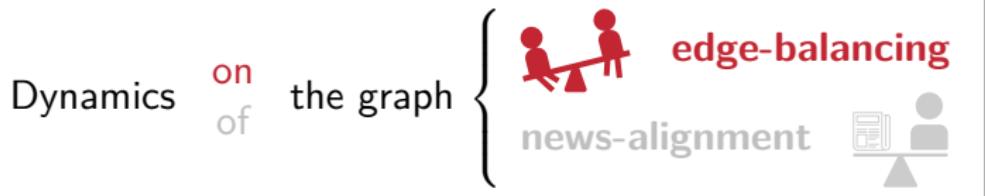
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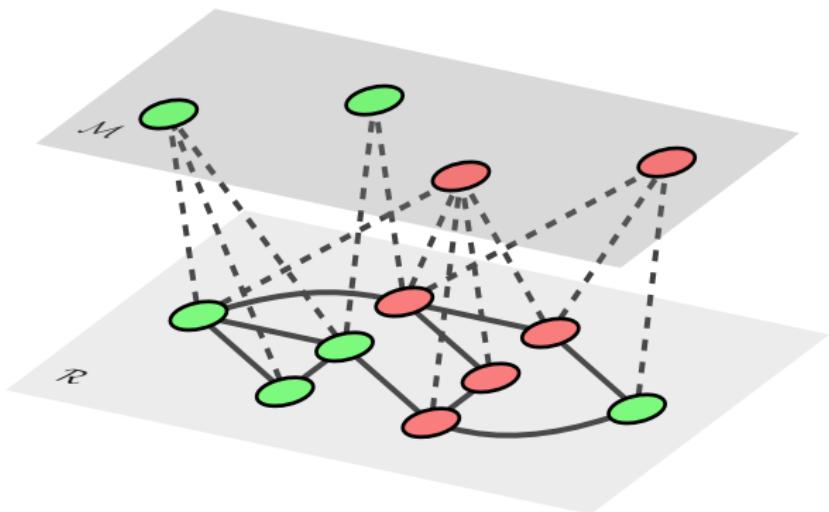
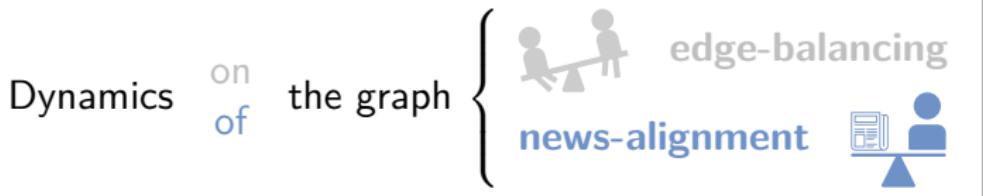
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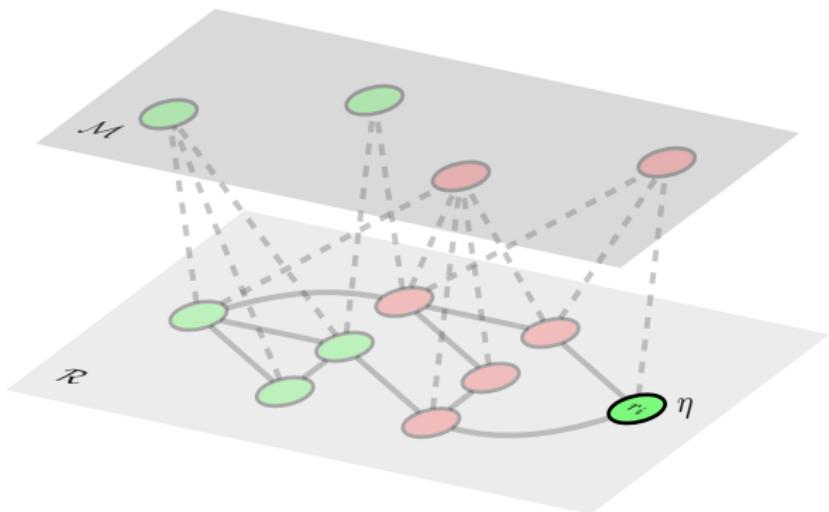
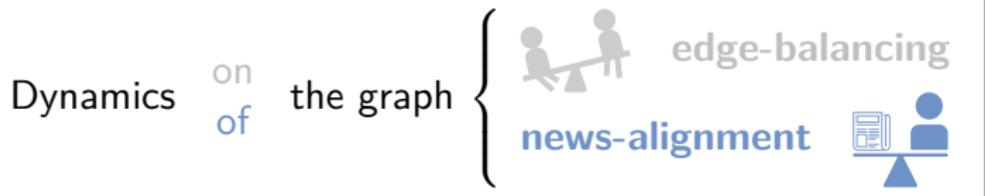
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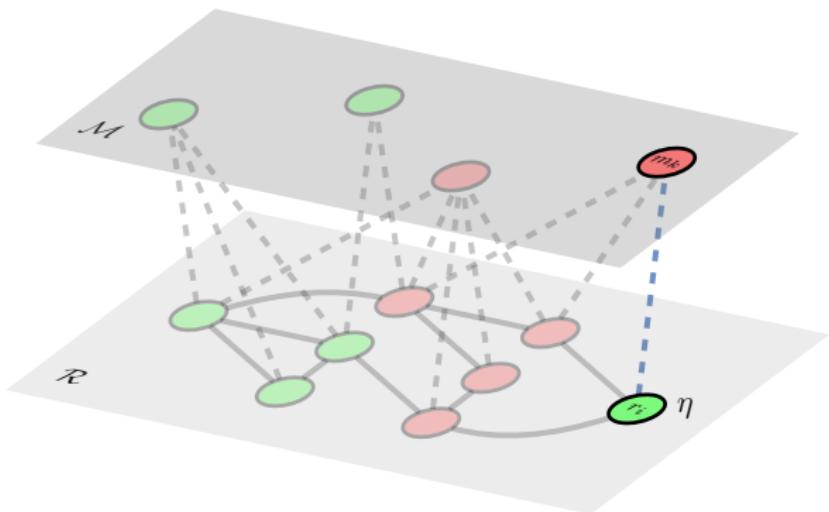
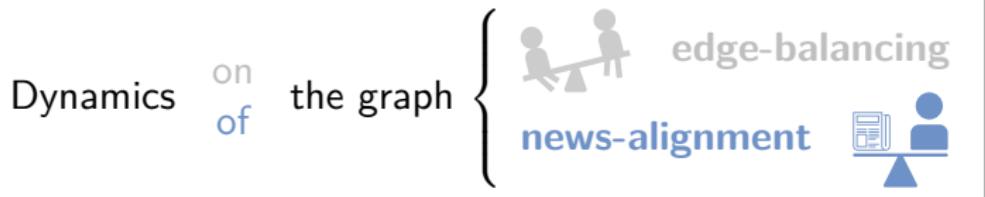
# Co-evolution of the Multilayer News Flow



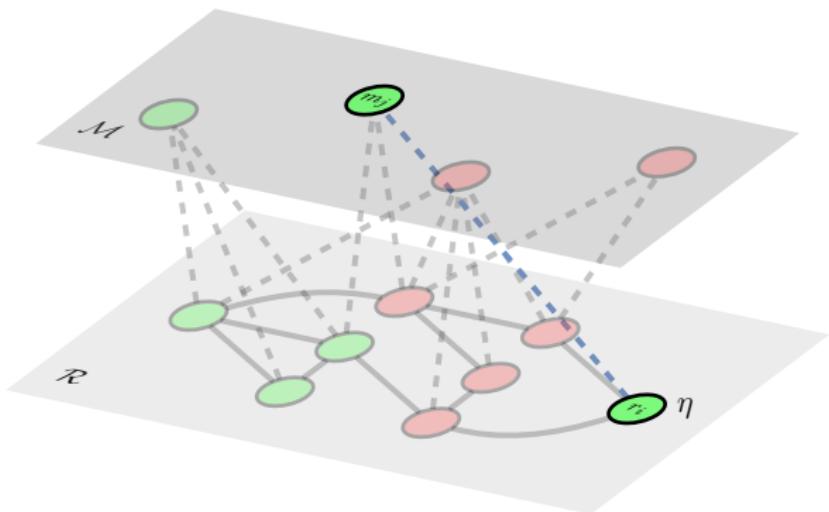
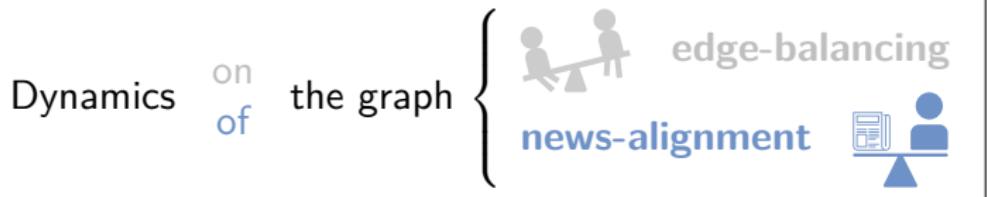
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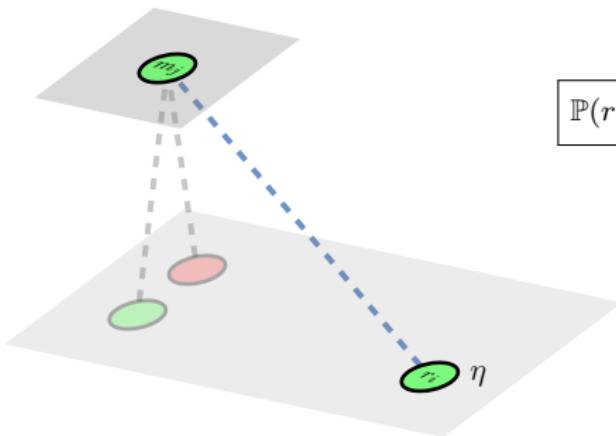
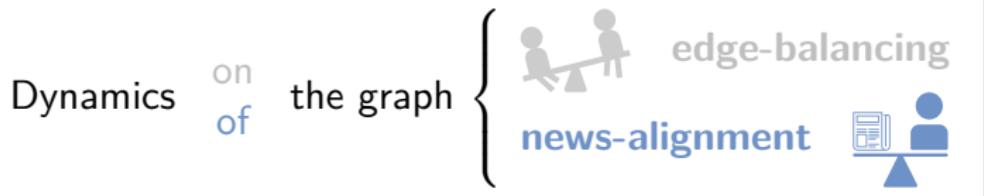
# Co-evolution of the Multilayer News Flow



# Co-evolution of the Multilayer News Flow

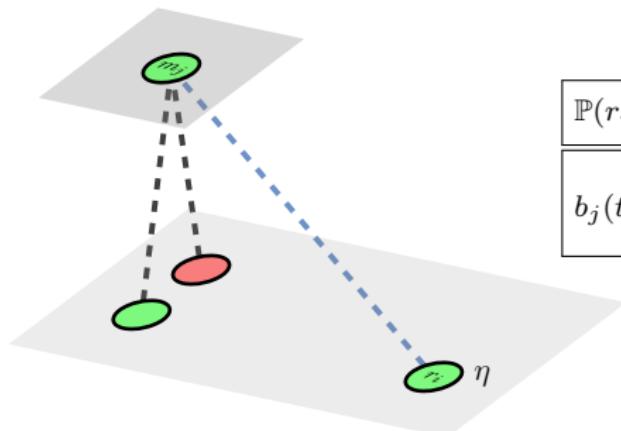
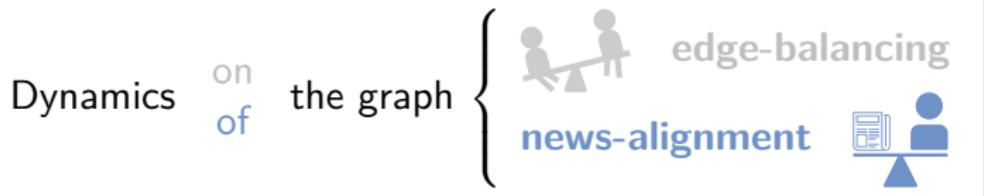


# Co-evolution of the Multilayer News Flow



$$\mathbb{P}(r_i \rightarrow m_j, t) \propto b_j(t) p_j^T(t)$$

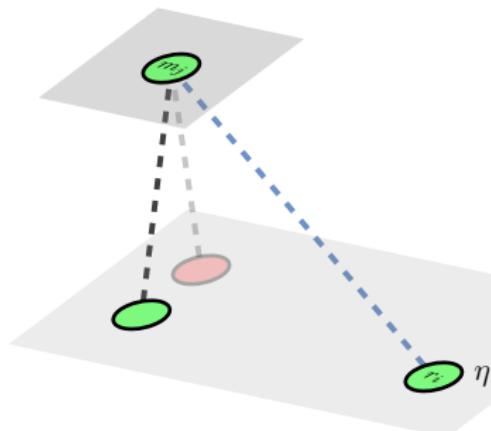
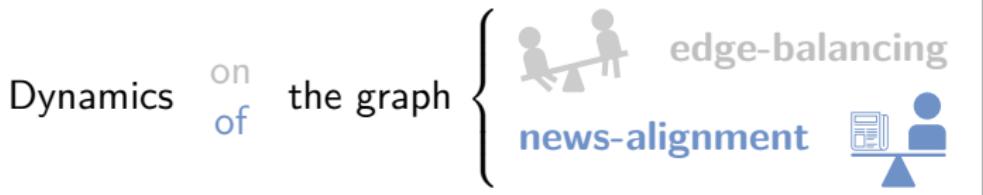
# Co-evolution of the Multilayer News Flow



$$\mathbb{P}(r_i \rightarrow m_j, t) \propto b_j(t) p_j^T(t)$$

$$b_j(t) := \frac{-\sum_S \mu_j^S(t) \log \mu_j^S(t)}{\log 2}$$

# Co-evolution of the Multilayer News Flow



$$\mathbb{P}(r_i \rightarrow m_j, t) \propto b_j(t) p_j^T(t)$$

$$b_j(t) := \frac{-\sum_S \mu_j^S(t) \log \mu_j^S(t)}{\log 2}$$

$$p_j^T(t) := \frac{1}{1 + e^{-m_j^T(t)}}$$

# Co-evolution of the Multilayer News Flow

[<https://github.com/ikicab/NewsFlow>]

## Gillespie algorithm

**while**  $(\text{Var}_n(P(t_n)) \geq \text{tol})$  **and**  $(\text{Var}_n(|E_R^{FT}(t_n)|) \geq \text{tol})$  **repeat**

1.  $\lambda(t_n) = R\eta + \lambda^T \Lambda^T(t_n) + \lambda^F \Lambda^F(t_n)$

2.  $\tau(t_n) = -\ln r / \lambda(t_n) = 1 / \lambda(t_n)$

3. draw  $\mathcal{A} \in \{\mathcal{A}_N, \mathcal{A}_B^T, \mathcal{A}_B^F\}$  at random:

$$\mathbb{P}(\mathcal{A}_N) = \frac{R\eta}{\lambda(t_n)}, \quad \mathbb{P}(\mathcal{A}_B^T) = \frac{\lambda^T \Lambda^T(t_n)}{\lambda(t_n)}, \quad \mathbb{P}(\mathcal{A}_B^F) = \frac{\lambda^F \Lambda^F(t_n)}{\lambda(t_n)}$$

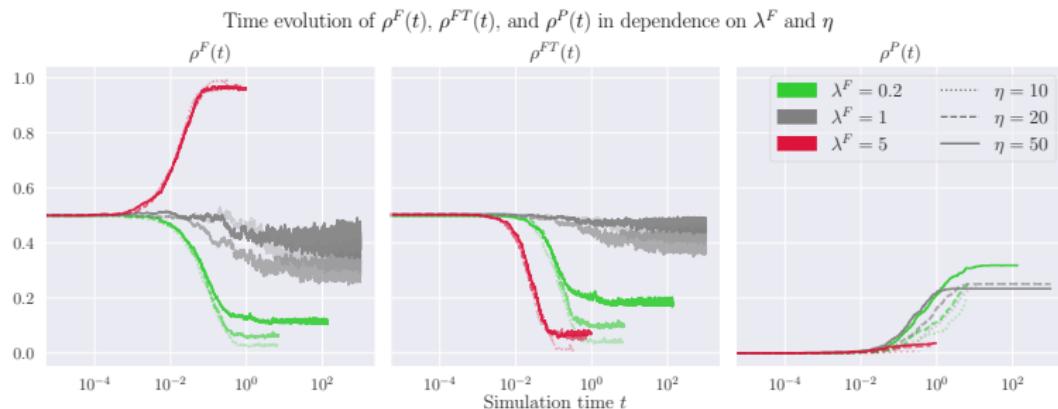
4. draw either  $r_i \in \mathcal{R}$  or  $r_i r_j \in E_R^{FT}(t_n)$  at random:

$$\mathbb{P}(r_i | \mathcal{A}_N) = \frac{1}{R}, \quad \mathbb{P}(r_i r_j | \mathcal{A}_B^T) = \frac{\lambda_{ij}^T(t_n)}{\Lambda^T(t_n)}, \quad \mathbb{P}(r_i r_j | \mathcal{A}_B^F) = \frac{\lambda_{ij}^F(t_n)}{\Lambda^F(t_n)}$$

5. update the system;  $t_n \leftarrow t_n + \tau(t_n)$

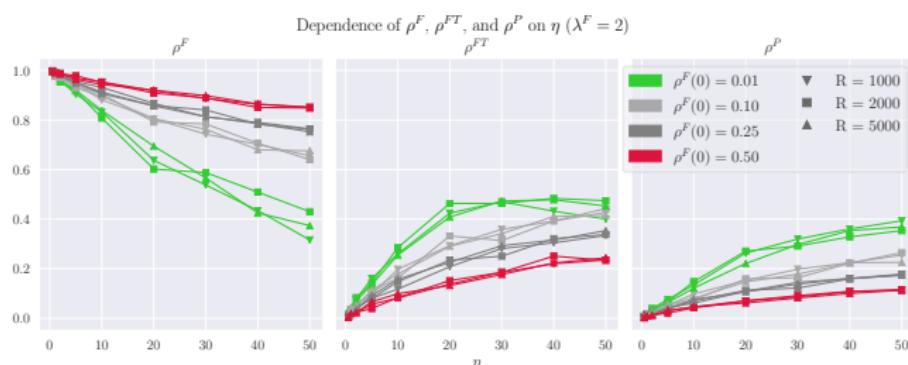
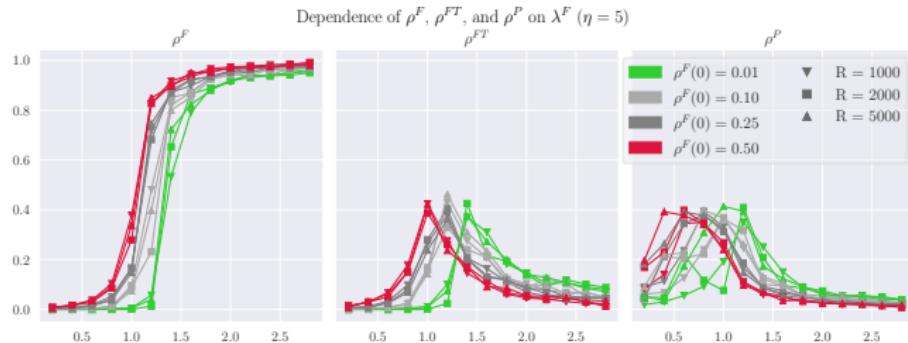
## Co-evolution of the Multilayer News Flow

## Experiments [macro-scale dynamics]



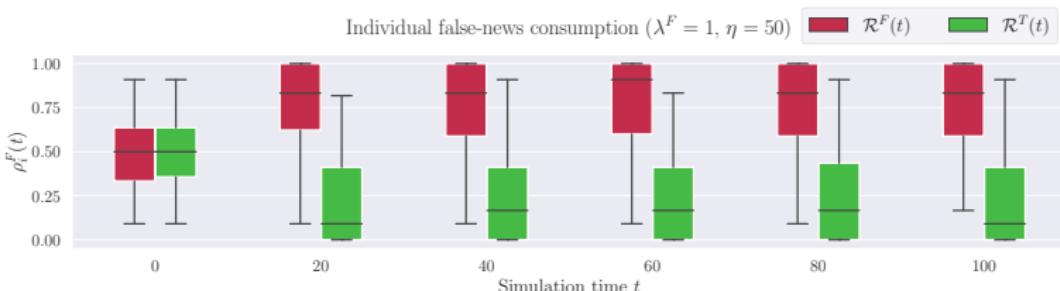
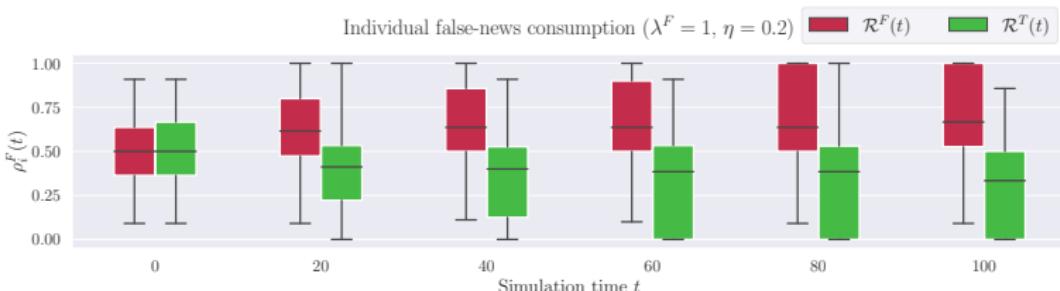
# Co-evolution of the Multilayer News Flow

## Experiments [macro-scale dynamics]



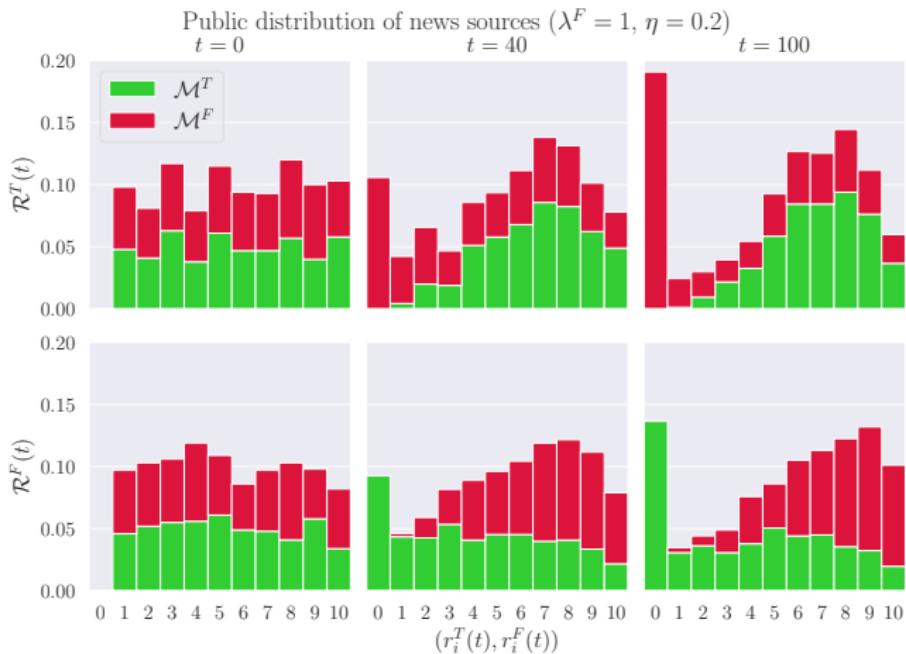
# Co-evolution of the Multilayer News Flow

Experiments [micro-scale dynamics; news consumers]



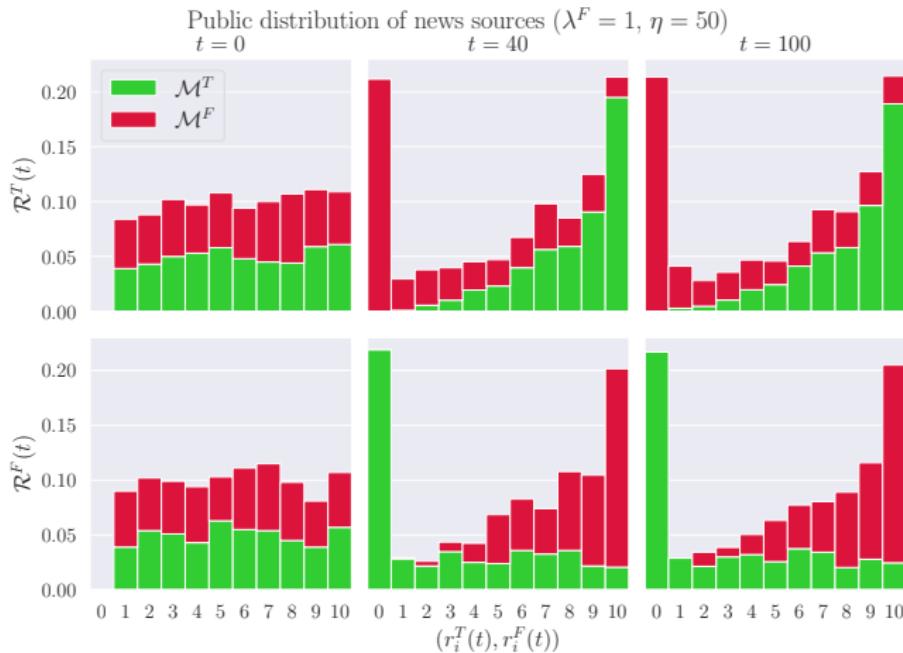
# Co-evolution of the Multilayer News Flow

Experiments [micro-scale dynamics; news consumers]



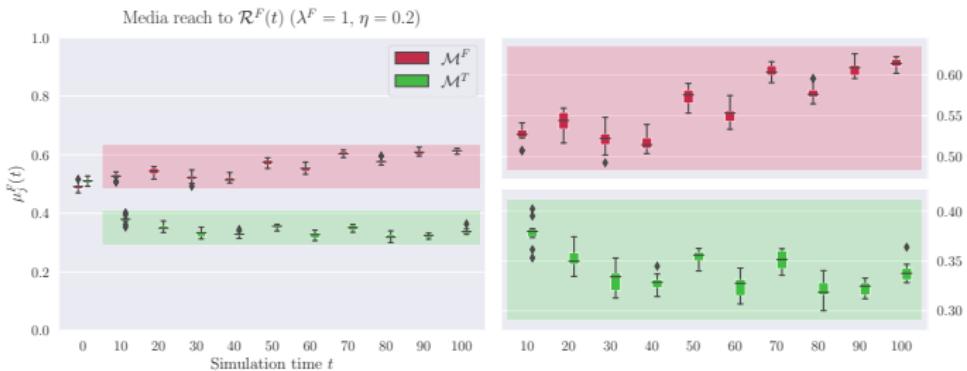
# Co-evolution of the Multilayer News Flow

Experiments [micro-scale dynamics; news consumers]



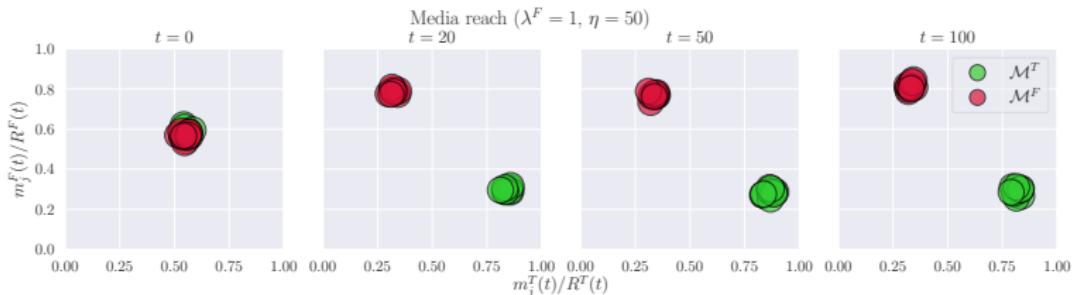
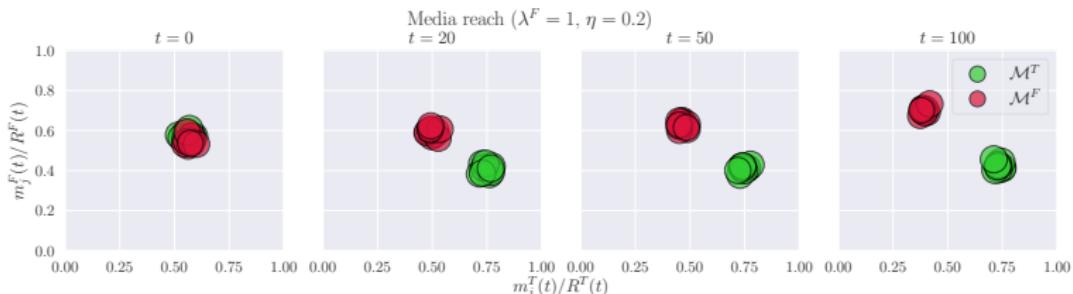
# Co-evolution of the Multilayer News Flow

Experiments [micro-scale dynamics; news providers]



# Co-evolution of the Multilayer News Flow

Experiments [micro-scale dynamics; news providers]



Thank  
you

