Expected genotype counts

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After dispersal

$$\mathrm{E}[\mathrm{n'}_{AA}(x,t=0)] = \frac{me^{\frac{-x^2}{4D}}}{\sqrt{4\pi D}}$$
$$\mathrm{E}[\mathrm{n'}_{Aa}(x,t=0)] = 0$$
$$\mathrm{E}[\mathrm{n'}_{aa}(x,t=0)] = \rho$$

After dispersal and reproduction (but before viability selection)

$$\mathrm{E}[\mathrm{n''}_{AA}(x,t=1)] = \frac{\left[me^{\frac{-x^2}{4D}} + \rho\sqrt{4\pi D}\right]\left[m^2e^{\frac{-x^2}{2D}}\right]}{\sqrt{4\pi D}\left[me^{\frac{-x^2}{4D}} + \rho\sqrt{4\pi D}\right]^2}$$
$$\mathrm{E}[\mathrm{n''}_{Aa}(x,t=1)] = \frac{\left[me^{\frac{-x^2}{4D}} + \rho\sqrt{4\pi D}\right]\left[2m\rho e^{\frac{-x^2}{4D}}\sqrt{4\pi D}\right]}{\sqrt{4\pi D}\left[me^{\frac{-x^2}{4D}} + \rho\sqrt{4\pi D}\right]^2}$$
$$\mathrm{E}[\mathrm{n''}_{aa}(x,t=1)] = \frac{\left[me^{\frac{-x^2}{4D}} + \rho\sqrt{4\pi D}\right]\left[4\pi D\rho^2\right]}{\sqrt{4\pi D}\left[me^{\frac{-x^2}{4D}} + \rho\sqrt{4\pi D}\right]^2}$$

After dispersal, reproduction, and viability selection

$$\mathrm{E}[\mathrm{n}_{AA}(x,t=1)] = \frac{[1+2\alpha k]\left[m^2 e^{\frac{-x^2}{2D}}\right]\left[me^{\frac{-x^2}{4D}}+\rho\sqrt{4\pi D}\right]}{\left[\sqrt{4\pi D}\right][1+2\alpha k]\left[m^2 e^{\frac{-x^2}{2D}}\right]+[1+(\alpha-1)k]\left[8\pi D m\rho e^{\frac{-x^2}{4D}}\right]+\left[4\pi D\rho^2\sqrt{4\pi D}\right]}$$

$$\mathrm{E}[\mathrm{n}_{Aa}(x,t=1)] = \frac{[1+(\alpha-1)k]\left[me^{\frac{-x^2}{4D}}+\rho\sqrt{4\pi D}\right]\left[2m\rho e^{\frac{-x^2}{4D}}\right]}{[1+2\alpha k]\left[m^2 e^{\frac{-x^2}{2D}}\right]+[1+(\alpha-1)k]\left[2m\rho e^{\frac{-x^2}{4D}}\right]\left[\sqrt{4\pi D}\right]+[4\pi D\rho^2]}$$

$$\mathrm{E}[\mathrm{n}_{aa}(x,t=1)] = \frac{[4\pi D\rho^2]\left[me^{\frac{-x^2}{4D}}+\rho\sqrt{4\pi D}\right]}{\left[\sqrt{4\pi D}\right][1+2\alpha k]\left[m^2 e^{\frac{-x^2}{2D}}\right]+[1+(\alpha-1)k]\left[8\pi D m\rho e^{\frac{-x^2}{4D}}\right]+\left[4\pi D\rho^2\sqrt{4\pi D}\right]}$$

Unsimplified: after dispersal, reproduction, and viability selection

$$\begin{aligned}\text{denominator} = &\left[\sqrt{4\pi D}\right][1+2\alpha k]\left[m^2 e^{\frac{-x^2}{2D}}\right]\left[me^{\frac{-x^2}{4D}}+\rho\sqrt{4\pi D}\right]^2\\ &+\left[8\pi D m\rho e^{\frac{-x^2}{4D}}\right][1+(\alpha-1)k]\left[me^{\frac{-x^2}{4D}}+\rho\sqrt{4\pi D}\right]^2\\ &+[4\pi D]^{\frac{3}{2}}\left[\rho^2\right]\left[me^{\frac{-x^2}{4D}}+\rho\sqrt{4\pi D}\right]^2\end{aligned}$$

$$\mathrm{E}[\mathrm{n}_{AA}(x,t=1)] = \frac{[1+2\alpha k]\left[m^2 e^{\frac{-x^2}{2D}}\right]\left[me^{\frac{-x^2}{4D}}+\rho\sqrt{4\pi D}\right]^3}{\text{denominator}}$$

$$\mathrm{E}[\mathrm{n}_{Aa}(x,t=1)] = \frac{\left[\sqrt{4\pi D}\right]\left[2m\rho e^{\frac{-x^2}{4D}}\right][1+(\alpha-1)k]\left[me^{\frac{-x^2}{4D}}+\rho\sqrt{4\pi D}\right]^3}{\text{denominator}}$$

$$\mathrm{E}[\mathrm{n}_{aa}(x,t=1)] = \frac{[4\pi D\rho^2]\left[me^{\frac{-x^2}{4D}}+\rho\sqrt{4\pi D}\right]^3}{\text{denominator}}$$

Expected rate of drive at this point

$$\mathrm{E}[\mathrm{u}_A(x,t=1)] = \frac{[m^2][1+2\alpha k]\left[e^{\frac{-x^2}{2D}}\right]+[m\rho]\left[\sqrt{4\pi D}\right][1+(\alpha-1)k]\left[e^{\frac{-x^2}{4D}}\right]}{[m^2][1+2\alpha k]\left[e^{\frac{-x^2}{2D}}\right]+[2m\rho]\left[\sqrt{4\pi D}\right][1+(\alpha-1)k]\left[e^{\frac{-x^2}{4D}}\right]+[4\pi D\rho^2]}$$

First derivative: $\frac{d}{dx}\mathbf{E}[\mathbf{u}_A(x,t=1)]$

$$\text{numerator} = f(x) = \left[\frac{-m\rho^3}{2D}\right][4\pi D]^{\frac{3}{2}}[1+(\alpha-1)k]\left[xe^{\frac{-x^2}{4D}}\right] + \left[\frac{-m^2\rho^2}{D}\right][4\pi D][1+2\alpha k]\left[xe^{\frac{-x^2}{2D}}\right]$$
$$+ \left[\frac{-m^3\rho}{2D}\right]\left[\sqrt{4\pi D}\right][1+2\alpha k][1+(\alpha-1)k]\left[xe^{\frac{-3x^2}{4D}}\right]$$

$$\text{denominator} = g(x) = \left[\rho^4\right][4\pi D]^2 + \left[m^4\right][1+2\alpha k]^2\left[e^{\frac{-x^2}{D}}\right] + \left[4m\rho^3\right][4\pi D]^{\frac{3}{2}}[1+(\alpha-1)k]\left[e^{\frac{-x^2}{4D}}\right]$$
$$+ \left[m^2\rho^2\right][4\pi D]\left[2(1+2\alpha k)+4(1+(\alpha-1)k)^2\right]\left[e^{\frac{-x^2}{2D}}\right]$$
$$+ \left[4m^3\rho\right]\left[\sqrt{4\pi D}\right][1+2\alpha k][1+(\alpha-1)k]\left[e^{\frac{-3x^2}{4D}}\right]$$

$$\frac{d}{dx}\mathrm{E}[\mathrm{u}_A(x,t=1)] = \frac{\text{numerator}}{\text{denominator}}$$

$$\frac{d}{dx}\text{numerator} = f'(x) = \left[\frac{-m\rho^3}{2D}\right][4\pi D]^{\frac{3}{2}}[1+(\alpha-1)k]\left[1-\frac{1}{2D}(x^2)\right]\left[e^{\frac{-x^2}{4D}}\right]$$
$$+ \left[\frac{-m^2\rho^2}{D}\right][4\pi D][1+2\alpha k]\left[1-\frac{1}{D}(x^2)\right]\left[e^{\frac{-x^2}{2D}}\right]$$
$$+ \left[\frac{-m^3\rho}{2D}\right]\left[\sqrt{4\pi D}\right][1+2\alpha k][1+(\alpha-1)k]\left[1-\frac{3}{2D}(x^2)\right]\left[e^{\frac{-3x^2}{4D}}\right]$$

$$\frac{d}{dx}\text{denominator} = g'(x) = \left[\frac{-m^4}{D}\right][1+2\alpha k]^2\left[2xe^{\frac{-x^2}{D}}\right]$$
$$+ \left[\frac{-2m\rho^3}{D}\right][4\pi D]^{\frac{3}{2}}[1+(\alpha-1)k]\left[xe^{\frac{-x^2}{4D}}\right]$$
$$+ \left[\frac{-m^2\rho^2}{D}\right][4\pi D]\left[2(1+2\alpha k)+4(1+(\alpha-1)k)^2\right]\left[xe^{\frac{-x^2}{2D}}\right]$$
$$+ \left[\frac{-6m^3\rho}{D}\right]\left[\sqrt{4\pi D}\right][1+2\alpha k][1+(\alpha-1)k]\left[xe^{\frac{-3x^3}{4D}}\right]$$

Quotient rule to get the second derivative: $\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$

$$g(x)f'(x) = \left[\frac{-m\rho^7}{2D}\right][4\pi D]^{\frac{7}{2}}$$
$$+\left[\frac{-m^7\rho}{2D}\right]\left[\sqrt{4\pi D}\right][1+2\alpha k]^3\,[1+(\alpha-1)k]\left[1-\frac{3x^2}{2D}\right]\left[e^{\frac{-7x^2}{4D}}\right]$$
$$+\left[\frac{-m^5\rho^3}{2D}\right][4\pi D]^{\frac{3}{2}}\,[1+2\alpha k]\,[1+(\alpha-1)k]\ \text{X}$$
$$\left[(1+2\alpha k)(9-\frac{17x^2}{2D})+\left[2(1+2\alpha k)+4(1+(\alpha-1)k)^2\right](1-\frac{3x^2}{2D})\right]\left[e^{\frac{-5x^2}{4D}}\right]$$
$$+\left[\frac{-m^2\rho^6}{D}\right][4\pi D]^3\left[(1+2\alpha k)(1-\frac{x^2}{D})+2(1+(\alpha-1)k)^2(1-\frac{x^2}{2D})\right]\left[e^{\frac{-x^2}{2D}}\right]$$
$$+\left[\frac{-m^3\rho^5}{2D}\right][4\pi D]^{\frac{5}{2}}\,[1+(\alpha-1)k]\ \text{X}$$
$$\left[(1+2\alpha k)(9-\frac{19x^2}{2D})+\left[2(1+2\alpha k)+4(1+(\alpha-1)k)^2\right](1-\frac{x^2}{2D})\right]\left[e^{\frac{-3x^2}{4D}}\right]$$
$$+\left[\frac{-4m^4\rho^4}{D}\right][4\pi D]^2\,[1+2\alpha k]\ \text{X}$$
$$\left[(1+(\alpha-1)k)^2(1-x^2)+\left[\frac{1}{2}(1+2\alpha k)+(1+(\alpha-1)k)^2\right](1-\frac{x^2}{D})\right]\left[e^{\frac{-x^2}{D}}\right]$$
$$+\left[\frac{-m^6\rho^2}{D}\right][4\pi D]\,[1+2\alpha k]^2\left[(1+2\alpha k)(1-\frac{x^2}{D})+2(1+(\alpha-1)k)^2(1-\frac{3x^2}{2D})\right]\left[e^{\frac{-3x^2}{2D}}\right]$$

$$f(x)g'(x) = \left[\frac{m^2\rho^6}{D^2}\right][4\pi D]^3\,[1+(\alpha-1)k]^2\left[x^2e^{\frac{-x^2}{2D}}\right]$$
$$+\left[\frac{m^7\rho}{D^2}\right]\left[\sqrt{4\pi D}\right][1+2\alpha k]^3\,[1+(\alpha-1)k]\left[x^2e^{\frac{-7x^2}{4D}}\right]$$
$$+\left[\frac{2m^5\rho^3}{D^2}\right][4\pi D]^{\frac{3}{2}}\,[1+2\alpha k]\,[1+(\alpha-1)k]\left[4+8\alpha k+(1+(\alpha-1)k)^2\right]\left[x^2e^{\frac{-5x^2}{4D}}\right]$$
$$+\left[\frac{m^3\rho^5}{D^2}\right][4\pi D]^{\frac{5}{2}}\,[1+(\alpha-1)k]\left[3+6\alpha k+2(1+(\alpha-1)k)^2\right]\left[x^2e^{\frac{-3x^2}{4D}}\right]$$
$$+\left[\frac{2m^4\rho^4}{D^2}\right][4\pi D]^2\,[1+2\alpha k]\left[1+2\alpha k+4(1+(\alpha-1)k)^2\right]\left[x^2e^{\frac{-x^2}{D}}\right]$$
$$+\left[\frac{m^6\rho^2}{D^2}\right][4\pi D]\,[1+2\alpha k]^2\left[2(1+2\alpha k)+3(1+(\alpha-1)k)^2\right]\left[x^2e^{\frac{-3x^2}{2D}}\right]$$

$$
\begin{aligned}
g(x)f'(x) - f(x)g'(x) = &\left[\frac{-m\rho^7}{2D}\right][4\pi D]^{\frac{7}{2}} \\
&+ \left[\frac{-m^7\rho}{2D}\right]\left[\sqrt{4\pi D}\right][1+2\alpha k]^3\,[1+(\alpha-1)k]\left[1+\frac{x^2}{2D}\right]\left[e^{\frac{-7x^2}{4D}}\right] \\
&+ \left[\frac{-m^5\rho^3}{D}\right][4\pi D]^{\frac{3}{2}}\,[1+2\alpha k]\,[1+(\alpha-1)k]\ \text{ X} \\
&\quad \left[(\frac{11}{2}+\frac{9x^2}{4D})(1+2\alpha k)+(2-\frac{x^2}{D})(1+(\alpha-1)k)^2\right]\left[e^{\frac{-5x^2}{4D}}\right] \\
&+ \left[\frac{-m^2\rho^6}{D}\right][4\pi D]^3\left[(1+2\alpha k)(1-\frac{x^2}{D})+2(1+(\alpha-1)k)^2\right]\left[e^{\frac{-x^2}{2D}}\right] \\
&+ \left[\frac{-m^3\rho^5}{D}\right][4\pi D]^{\frac{5}{2}}\,[1+(\alpha-1)k]\ \text{ X} \\
&\quad \left[(1+2\alpha k)(\frac{11}{2}-\frac{9x^2}{4D})+(2+\frac{x^2}{D})(1+(\alpha-1)k)^2\right]\left[e^{\frac{-3x^2}{4D}}\right] \\
&+ \left[\frac{-2m^4\rho^4}{D}\right][4\pi D]^2\,[1+2\alpha k]\ \text{ X} \\
&\quad \left[(1-\frac{x^2}{2D})(1+2\alpha k)+(4-2x^2)(1+(\alpha-1)k)^2\right]\left[e^{\frac{-x^2}{D}}\right] \\
&+ \left[\frac{-m^6\rho^2}{D}\right][4\pi D]\,[1+2\alpha k]^2\left[(1+\frac{x^2}{D})(1+2\alpha k)+2(1+(\alpha-1)k)^2\right]\left[e^{\frac{-3x^2}{2D}}\right]
\end{aligned}
$$

$$
\begin{aligned}
[g(x)]^2 = &\left[\rho^8\right][4\pi D]^4 \\
&+ \left[m^8\right][1+2\alpha k]^4\left[e^{\frac{-2x^2}{D}}\right] \\
&+ \left[2m^4\rho^4\right][4\pi D]^2\left[3(1+2\alpha k)^2+8(1+(\alpha-1)k)^4+24(1+2\alpha k)(1+(\alpha-1)k)^2\right]\left[e^{\frac{-x^2}{D}}\right] \\
&+ \left[4m^2\rho^6\right][4\pi D]^3\left[1+2\alpha k+6(1+(\alpha-1)k)^2\right]\left[e^{\frac{-x^2}{2D}}\right] \\
&+ \left[8m^3\rho^5\right][4\pi D]^{\frac{5}{2}}\,[1+(\alpha-1)k]\left[3(1+2\alpha k)+4(1+(\alpha-1)k)^2\right]\left[e^{\frac{-3x^2}{4D}}\right] \\
&+ \left[8m^5\rho^3\right][4\pi D]^{\frac{3}{2}}\,[1+2\alpha k]\,[1+(\alpha-1)k]\left[3(1+2\alpha k)+4(1+(\alpha-1)k)^2\right]\left[e^{\frac{-5x^2}{4D}}\right] \\
&+ \left[8m\rho^7\right][4\pi D]^{\frac{7}{2}}\,[1+(\alpha-1)k]\left[e^{\frac{-x^2}{4D}}\right] \\
&+ \left[4m^6\rho^2\right][4\pi D]\,[1+2\alpha k]^2\left[1+2\alpha k+6(1+(\alpha-1)k)^2\right]\left[e^{\frac{-3x^2}{2D}}\right] \\
&+ \left[8m^7\rho\right]\left[\sqrt{4\pi D}\right][1+2\alpha k]^3\,[1+(\alpha-1)k]\left[e^{\frac{-7x^2}{4D}}\right]
\end{aligned}
$$

2nd derivative at 0 – the numerator $g(0)f'(0) - f(0)g'(0)$

$$
\begin{aligned}
g(0)f'(0) - f(0)g'(0) =&\{\frac{-m\rho}{D}\sqrt{4\pi D}\}\ \mathrm{X} \\
&\{\left[\frac{\rho^6}{2}\right][4\pi D]^3 \\
&+\left[\frac{m^6}{2}\right][1+2\alpha k]^3[1+(\alpha-1)k] \\
&+\left[m^4\rho^2\right][4\pi D][1+2\alpha k][1+(\alpha-1)k]\left[\frac{11}{2}(1+2\alpha k)+2(1+(\alpha-1)k)^2\right] \\
&+\left[m\rho^5\right][4\pi D]^{\frac{5}{2}}\left[1+2\alpha k+2(1+(\alpha-1)k)^2\right] \\
&+\left[m^2\rho^4\right][4\pi D]^2[1+(\alpha-1)k]\left[\frac{11}{2}(1+2\alpha k)+2(1+(\alpha-1)k)^2\right] \\
&+\left[2m^3\rho^3\right][4\pi D]^{\frac{3}{2}}[1+2\alpha k]\left[1+2\alpha k+4(1+(\alpha-1)k)^2\right] \\
&+\left[m^5\rho\right]\left[\sqrt{4\pi D}\right][1+2\alpha k]^2\left[1+2\alpha k+2(1+(\alpha-1)k)^2\right]\}
\end{aligned}
$$

2nd derivative at 0 – the numerator $g(0)f'(0) - f(0)g'(0)$ TIMES negative D

$$
\begin{aligned}
g(0)f'(0) - f(0)g'(0) =&\{m\rho\sqrt{4\pi D}\}\ \mathrm{X} \\
&\{\left[\frac{\rho^6}{2}\right][4\pi D]^3 \\
&+\left[\frac{m^6}{2}\right][1+2\alpha k]^3[1+(\alpha-1)k] \\
&+\left[m^4\rho^2\right][4\pi D][1+2\alpha k][1+(\alpha-1)k]\left[\frac{11}{2}(1+2\alpha k)+2(1+(\alpha-1)k)^2\right] \\
&+\left[m\rho^5\right][4\pi D]^{\frac{5}{2}}\left[1+2\alpha k+2(1+(\alpha-1)k)^2\right] \\
&+\left[m^2\rho^4\right][4\pi D]^2[1+(\alpha-1)k]\left[\frac{11}{2}(1+2\alpha k)+2(1+(\alpha-1)k)^2\right] \\
&+\left[2m^3\rho^3\right][4\pi D]^{\frac{3}{2}}[1+2\alpha k]\left[1+2\alpha k+4(1+(\alpha-1)k)^2\right] \\
&+\left[m^5\rho\right]\left[\sqrt{4\pi D}\right][1+2\alpha k]^2\left[1+2\alpha k+2(1+(\alpha-1)k)^2\right]\}
\end{aligned}
$$

2nd derivative at 0 – the denominator $[g(0)]^2$

$$
\begin{aligned}
[g(0)]^2 = {} & \left[\rho^8\right][4\pi D]^4 \\
& + \left[m^8\right][1+2\alpha k]^4 \\
& + \left[2m^4\rho^4\right][4\pi D]^2\left[3(1+2\alpha k)^2 + 8(1+(\alpha-1)k)^4 + 24(1+2\alpha k)(1+(\alpha-1)k)^2\right] \\
& + \left[4m^2\rho^6\right][4\pi D]^3\left[1+2\alpha k + 6(1+(\alpha-1)k)^2\right] \\
& + \left[8m^3\rho^5\right][4\pi D]^{\frac{5}{2}}[1+(\alpha-1)k]\left[3(1+2\alpha k)+4(1+(\alpha-1)k)^2\right] \\
& + \left[8m^5\rho^3\right][4\pi D]^{\frac{3}{2}}[1+2\alpha k][1+(\alpha-1)k]\left[3(1+2\alpha k)+4(1+(\alpha-1)k)^2\right] \\
& + \left[8m\rho^7\right][4\pi D]^{\frac{7}{2}}[1+(\alpha-1)k] \\
& + \left[4m^6\rho^2\right][4\pi D][1+2\alpha k]^2\left[1+2\alpha k+6(1+(\alpha-1)k)^2\right] \\
& + \left[8m^7\rho\right]\left[\sqrt{4\pi D}\right][1+2\alpha k]^3[1+(\alpha-1)k]
\end{aligned}
$$

[Negative] diffusion term $-D\frac{d^2}{dx^2}E\left[u_A(x,t=1)\right]$:

$$
\begin{aligned}
\text{numerator} = &\{m\rho\sqrt{4\pi D}\}\ \text{X}\\
&\{\left[\frac{\rho^6}{2}\right][4\pi D]^3\\
&+\left[\frac{m^6}{2}\right][1+2\alpha k]^3\,[1+(\alpha-1)k]\\
&+\left[m^4\rho^2\right][4\pi D]\,[1+2\alpha k]\,[1+(\alpha-1)k]\left[\frac{11}{2}(1+2\alpha k)+2(1+(\alpha-1)k)^2\right]\\
&+\left[m\rho^5\right][4\pi D]^{\frac{5}{2}}\left[1+2\alpha k+2(1+(\alpha-1)k)^2\right]\\
&+\left[m^2\rho^4\right][4\pi D]^2\,[1+(\alpha-1)k]\left[\frac{11}{2}(1+2\alpha k)+2(1+(\alpha-1)k)^2\right]\\
&+\left[2m^3\rho^3\right][4\pi D]^{\frac{3}{2}}\,[1+2\alpha k]\left[1+2\alpha k+4(1+(\alpha-1)k)^2\right]\\
&+\left[m^5\rho\right]\left[\sqrt{4\pi D}\right][1+2\alpha k]^2\left[1+2\alpha k+2(1+(\alpha-1)k)^2\right]\}
\end{aligned}
$$

$$
\begin{aligned}
\text{denominator} = &\left[\rho^8\right][4\pi D]^4\\
&+\left[m^8\right][1+2\alpha k]^4\\
&+\left[2m^4\rho^4\right][4\pi D]^2\left[3(1+2\alpha k)^2+8(1+(\alpha-1)k)^4+24(1+2\alpha k)(1+(\alpha-1)k\right.\\
&+\left[4m^2\rho^6\right][4\pi D]^3\left[1+2\alpha k+6(1+(\alpha-1)k)^2\right]\\
&+\left[8m^3\rho^5\right][4\pi D]^{\frac{5}{2}}\,[1+(\alpha-1)k]\left[3(1+2\alpha k)+4(1+(\alpha-1)k)^2\right]\\
&+\left[8m^5\rho^3\right][4\pi D]^{\frac{3}{2}}\,[1+2\alpha k]\,[1+(\alpha-1)k]\left[3(1+2\alpha k)+4(1+(\alpha-1)k)^2\right]\\
&+\left[8m\rho^7\right][4\pi D]^{\frac{7}{2}}\,[1+(\alpha-1)k]\\
&+\left[4m^6\rho^2\right][4\pi D]\,[1+2\alpha k]^2\left[1+2\alpha k+6(1+(\alpha-1)k)^2\right]\\
&+\left[8m^7\rho\right]\left[\sqrt{4\pi D}\right][1+2\alpha k]^3\,[1+(\alpha-1)k]
\end{aligned}
$$

$$-D\frac{d^2}{dx^2}\left[E(u_A(x,t=1))\right]=\frac{\text{numerator}}{\text{denominator}}$$

Reaction term $[2ku_A][1-u_A][u_A-\hat{u}]$

Subsections

$$u_A(x=0,t=1) = \frac{\left[m^2\right][1+2\alpha k]+[m\rho]\left[\sqrt{4\pi D}\right][1+(\alpha-1)k]}{[m^2][1+2\alpha k]+[2m\rho]\left[\sqrt{4\pi D}\right][1+(\alpha-1)k]+[4\pi D\rho^2]}$$

$$2ku_A(x=0,t=1) = \frac{\left[2km^2\right][1+2\alpha k]+[2km\rho]\left[\sqrt{4\pi D}\right][1+(\alpha-1)k]}{[m^2][1+2\alpha k]+[2m\rho]\left[\sqrt{4\pi D}\right][1+(\alpha-1)k]+[4\pi D\rho^2]}$$

$$1-u_A(x=0,t=1) = \frac{[m\rho]\left[\sqrt{4\pi D}\right][1+(\alpha-1)k]+\left[4\pi D\rho^2\right]}{[m^2][1+2\alpha k]+[2m\rho]\left[\sqrt{4\pi D}\right][1+(\alpha-1)k]+[4\pi D\rho^2]}$$

$$u_A(x=0,t=1)-\hat{u} = \frac{\left[m^2\right][1-\hat{u}][1+2\alpha k]+[m\rho][1-2\hat{u}]\left[\sqrt{4\pi D}\right][1+(\alpha-1)k]-\left[\hat{u}\rho^2\right][4\pi D]}{[m^2][1+2\alpha k]+[2m\rho]\left[\sqrt{4\pi D}\right][1+(\alpha-1)k]+[4\pi D\rho^2]}$$

Full reaction term $[2ku_A]\,[1-u_A]\,[u_A-\hat{u}]$

$$
\begin{aligned}
\text{numerator} = &\left[-2km^2\rho^4\hat{u}\right][4\pi D]^2\,[1+2\alpha k] \\
&+ \left[-2km\rho^5\hat{u}\right][4\pi D]^{\frac{5}{2}}\,[1+(\alpha-1)k] \\
&+ \left[4km^3\rho^3\right][4\pi D]^{\frac{3}{2}}\,[1-2\hat{u}]\,[1+2\alpha k]\,[1+(\alpha-1)k] \\
&+ \left[2km^4\rho^2\right][4\pi D]\,[1-\hat{u}]\,[1+2\alpha k]^2 \\
&+ \left[2km^2\rho^4\right][4\pi D]^2\,[1-3\hat{u}]\,[1+(\alpha-1)k]^2 \\
&+ \left[2km^5\rho\right]\left[\sqrt{4\pi D}\right][1-\hat{u}]\,[1+2\alpha k]^2\,[1+(\alpha-1)k] \\
&+ \left[2km^4\rho^2\right][4\pi D]\,[2-3\hat{u}]\,[1+2\alpha k]\,[1+(\alpha-1)k]^2 \\
&+ \left[2km^3\rho^3\right][4\pi D]^{\frac{3}{2}}\,[1-2\hat{u}]\,[1+(\alpha-1)k]^3
\end{aligned}
$$

$$
\begin{aligned}
\text{denominator} = &\left[\rho^6\right][4\pi D]^3 \\
&+ \left[3m^2\rho^4\right][4\pi D]^2\,[1+2\alpha k] \\
&+ \left[6m\rho^5\right][4\pi D]^{\frac{5}{2}}\,[1+(\alpha-1)k] \\
&+ \left[12m^3\rho^3\right][4\pi D]^{\frac{3}{2}}\,[1+2\alpha k]\,[1+(\alpha-1)k] \\
&+ \left[3m^4\rho^2\right][4\pi D]\,[1+2\alpha k]^2 \\
&+ \left[12m^2\rho^4\right][4\pi D]^2\,[1+(\alpha-1)k]^2 \\
&+ \left[6m^5\rho\right]\left[\sqrt{4\pi D}\right][1+2\alpha k]^2\,[1+(\alpha-1)k] \\
&+ \left[12m^4\rho^2\right][4\pi D]\,[1+2\alpha k]\,[1+(\alpha-1)k]^2 \\
&+ \left[m^6\right][1+2\alpha k]^3 \\
&+ \left[8m^3\rho^3\right][4\pi D]^{\frac{3}{2}}\,[1+(\alpha-1)k]^3
\end{aligned}
$$

$$
\text{reaction term} = [2ku_A]\,[1-u_A]\,[u_A-\hat{u}] = \frac{\text{numerator}}{\text{denominator}}
$$

Setting the (negative) diffusion term = the reaction term

Dividing out $m\rho\sqrt{4\pi D}$ leads to

[Negative] diffusion term:

$$
\begin{aligned}
\text{numerator} = & \left[\frac{\rho^6}{2}\right][4\pi D]^3 \\
& + \left[\frac{m^6}{2}\right][1+2\alpha k]^3 [1+(\alpha-1)k] \\
& + \left[m^4\rho^2\right][4\pi D][1+2\alpha k][1+(\alpha-1)k]\left[\frac{11}{2}(1+2\alpha k)+2(1+(\alpha-1)k)^2\right] \\
& + \left[m\rho^5\right][4\pi D]^{\frac{5}{2}}\left[1+2\alpha k+2(1+(\alpha-1)k)^2\right] \\
& + \left[m^2\rho^4\right][4\pi D]^2[1+(\alpha-1)k]\left[\frac{11}{2}(1+2\alpha k)+2(1+(\alpha-1)k)^2\right] \\
& + \left[2m^3\rho^3\right][4\pi D]^{\frac{3}{2}}[1+2\alpha k]\left[1+2\alpha k+4(1+(\alpha-1)k)^2\right] \\
& + \left[m^5\rho\right]\left[\sqrt{4\pi D}\right][1+2\alpha k]^2\left[1+2\alpha k+2(1+(\alpha-1)k)^2\right]
\end{aligned}
$$

$$
\begin{aligned}
\text{denominator} = & \left[\rho^8\right][4\pi D]^4 \\
& + \left[m^8\right][1+2\alpha k]^4 \\
& + \left[2m^4\rho^4\right][4\pi D]^2\left[3(1+2\alpha k)^2+8(1+(\alpha-1)k)^4+24(1+2\alpha k)(1+(\alpha-1)k)^2\right] \\
& + \left[4m^2\rho^6\right][4\pi D]^3\left[1+2\alpha k+6(1+(\alpha-1)k)^2\right] \\
& + \left[8m^3\rho^5\right][4\pi D]^{\frac{5}{2}}[1+(\alpha-1)k]\left[3(1+2\alpha k)+4(1+(\alpha-1)k)^2\right] \\
& + \left[8m^5\rho^3\right][4\pi D]^{\frac{3}{2}}[1+2\alpha k][1+(\alpha-1)k]\left[3(1+2\alpha k)+4(1+(\alpha-1)k)^2\right] \\
& + \left[8m\rho^7\right][4\pi D]^{\frac{7}{2}}[1+(\alpha-1)k] \\
& + \left[4m^6\rho^2\right][4\pi D][1+2\alpha k]^2\left[1+2\alpha k+6(1+(\alpha-1)k)^2\right] \\
& + \left[8m^7\rho\right]\left[\sqrt{4\pi D}\right][1+2\alpha k]^3[1+(\alpha-1)k]
\end{aligned}
$$

Reaction term:

$$
\begin{aligned}
\text{numerator} = & \left[-2km\rho^3\hat{u}\right][4\pi D]^{\frac{3}{2}}[1+2\alpha k] \\
& + \left[-2k\rho^4\hat{u}\right][4\pi D]^2[1+(\alpha-1)k] \\
& + \left[4km^2\rho^2\right][4\pi D][1-2\hat{u}][1+2\alpha k][1+(\alpha-1)k] \\
& + \left[2km^3\rho\right]\left[\sqrt{4\pi D}\right][1-\hat{u}][1+2\alpha k]^2 \\
& + \left[2km\rho^3\right][4\pi D]^{\frac{3}{2}}[1-3\hat{u}][1+(\alpha-1)k]^2 \\
& + \left[2km^4\right][1-\hat{u}][1+2\alpha k]^2[1+(\alpha-1)k] \\
& + \left[2km^3\rho\right]\left[\sqrt{4\pi D}\right][2-3\hat{u}][1+2\alpha k][1+(\alpha-1)k]^2 \\
& + \left[2km^2\rho^2\right][4\pi D][1-2\hat{u}][1+(\alpha-1)k]^3
\end{aligned}
$$

$$
\begin{aligned}
\text{denominator} = & \left[\rho^6\right][4\pi D]^3 \\
& + \left[3m^2\rho^4\right][4\pi D]^2[1+2\alpha k] \\
& + \left[6m\rho^5\right][4\pi D]^{\frac{5}{2}}[1+(\alpha-1)k] \\
& + \left[12m^3\rho^3\right][4\pi D]^{\frac{3}{2}}[1+2\alpha k][1+(\alpha-1)k] \\
& + \left[3m^4\rho^2\right][4\pi D][1+2\alpha k]^2 \\
& + \left[12m^2\rho^4\right][4\pi D]^2[1+(\alpha-1)k]^2 \\
& + \left[6m^5\rho\right]\left[\sqrt{4\pi D}\right][1+2\alpha k]^2[1+(\alpha-1)k] \\
& + \left[12m^4\rho^2\right][4\pi D][1+2\alpha k][1+(\alpha-1)k]^2 \\
& + \left[m^6\right][1+2\alpha k]^3 \\
& + \left[8m^3\rho^3\right][4\pi D]^{\frac{3}{2}}[1+(\alpha-1)k]^3
\end{aligned}
$$