Equations

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Initial equation, $t = 0$

$$u(x, t = 0) = \begin{cases} b & \text{if } \frac{-a}{2} \leq x \leq \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

Dispersal kernel

$$P(x, y) = \frac{1}{2\sigma} e^{\frac{-1}{\sigma}|y-x|}$$

Migration, $t = 1$

$x < \frac{-a}{2}$:

$$u_1'(x, t = 1) = \frac{-b}{2} \left[e^{\frac{-1}{\sigma}(\frac{a}{2}-x)} - e^{\frac{-1}{\sigma}(\frac{-a}{2}-x)} \right]$$

$\frac{-a}{2} \leq x \leq \frac{a}{2}$:

$$u_2'(x, t = 1) = \frac{b}{2} \left[2 - e^{\frac{-1}{\sigma}(x+\frac{a}{2})} - e^{\frac{-1}{\sigma}(\frac{a}{2}-x)} \right]$$

$x > \frac{a}{2}$:

$$u_3'(x, t = 1) = \frac{b}{2} \left[e^{\frac{-1}{\sigma}(x-\frac{a}{2})} - e^{\frac{-1}{\sigma}(x+\frac{a}{2})} \right]$$

Adding the reaction, t=1

$x < \frac{-a}{2}$**:**

$$u_1(x,t=1) = b(k\hat{u} - \frac{1}{2})\left[e^{\frac{-1}{\sigma}(\frac{a}{2}-x)} - e^{\frac{-1}{\sigma}(\frac{-a}{2}-x)}\right] + \frac{kb^2}{2}(1+\hat{u})\left[e^{\frac{-1}{\sigma}(\frac{a}{2}-x)} - e^{\frac{-1}{\sigma}(\frac{-a}{2}-x)}\right]^2 + \frac{kb^3}{4}\left[e^{\frac{-1}{\sigma}(\frac{a}{2}-x)} - e^{\frac{-1}{\sigma}(\frac{-a}{2}-x)}\right]^3$$

$\frac{-a}{2} \leq x \leq \frac{a}{2}$**:**

$$u_2(x,t=1) = b\left[2k(b-\hat{u})(1-b)+1\right] + b\left[k\left[b(3b-2)+\hat{u}(1-2b)\right] - \frac{1}{2}\right]\left[e^{\frac{-1}{\sigma}(x+\frac{a}{2})} + e^{\frac{-1}{\sigma}(\frac{a}{2}-x)}\right] - \left[\frac{kb^2}{2}(3b-\hat{u}-1)\right]\left[e^{\frac{-1}{\sigma}(x+\frac{a}{2})} + e^{\frac{-1}{\sigma}(\frac{a}{2}-x)}\right]^2 + \frac{kb^3}{4}\left[e^{\frac{-1}{\sigma}(x+\frac{a}{2})} + e^{\frac{-1}{\sigma}(\frac{a}{2}-x)}\right]^3$$

$x > \frac{a}{2}$**:**

$$u_3(x,t=1) = b(\frac{1}{2} - k\hat{u})\left[e^{\frac{-1}{\sigma}(x-\frac{a}{2})} - e^{\frac{-1}{\sigma}(x+\frac{a}{2})}\right] + \frac{kb^2}{2}(1+\hat{u})\left[e^{\frac{-1}{\sigma}(x-\frac{a}{2})} - e^{\frac{-1}{\sigma}(x+\frac{a}{2})}\right]^2 - \frac{kb^3}{4}\left[e^{\frac{-1}{\sigma}(x-\frac{a}{2})} - e^{\frac{-1}{\sigma}(x+\frac{a}{2})}\right]^3$$

Solving for θ_1 (in Mathematica)

$$\theta_1 = \left[\frac{3\sigma b^2 k}{2}(3b-2\hat{u}-2) - 2abk(b-1)(b-\hat{u}) + ab\right] + [bk]\left[\sigma b(3\hat{u}-4b+3) - a(3b-\hat{u}-1)\right]\left[e^{\frac{-a}{\sigma}}\right] - \left[\frac{\sigma b^3 k}{2}\right]\left[e^{\frac{-2a}{\sigma}}\right]$$

when $b=1$**:**

$$\theta_1 = \left[\frac{3\sigma k}{2}(1-2\hat{u}) + a\right] + [k]\left[\sigma(3\hat{u}-1) - a(2-\hat{u})\right]\left[e^{\frac{-a}{\sigma}}\right] - \left[\frac{\sigma k}{2}\right]\left[e^{\frac{-2a}{\sigma}}\right]$$

Solving for $\Delta = \theta_1 - ab$ (in Mathematica)

$$\begin{aligned}\Delta = &\left[\frac{3\sigma b^2 k}{2}(3b - 2\hat{u} - 2) - 2abk(b-1)(b-\hat{u})\right] \\ &+ [bk]\left[\sigma b(3\hat{u} - 4b + 3) - a(3b - \hat{u} - 1)\right]\left[e^{\frac{-a}{\sigma}}\right] \\ &- \left[\frac{\sigma b^3 k}{2}\right]\left[e^{\frac{-2a}{\sigma}}\right]\end{aligned}$$

when $b = 1$:

$$\begin{aligned}\Delta = &\left[\frac{3\sigma k}{2}(1 - 2\hat{u})\right] \\ &+ [k]\left[\sigma(3\hat{u} - 1) - a(2 - \hat{u})\right]\left[e^{\frac{-a}{\sigma}}\right] \\ &- \left[\frac{\sigma k}{2}\right]\left[e^{\frac{-2a}{\sigma}}\right]\end{aligned}$$