Workshop in experimental design and applied statistics: Day 2

Presentation and workshop materials developed by Maegen Simmonds

AIP App developed by Ian King Kyle, Jason Moore, and Maegen Simmonds Funded by USAID

Workshop Goals

- To understand experimental design and principles of statistical analysis to produce trustworthy results and interpretation.
- To learn how to use the free AIP computer app to analyze and visualize your data

what are the basic experimental designs that you use in your research?

Examples:

- 1. Complete Randomized Design
- 2. Randomized Complete Block Design
- 3. Split-plot Design

Note: "Factorial" is a treatment structure of one of the experimental designs, which we will also cover.

What are we doing today?

- Instruction 9:00 11:00 AM
 - Section 4: Testing assumptions of ANOVA and transforming data
- Tea break 11:00 11:30 AM
- Instruction 11:30 1:00 PM
 - Section 5: Presenting results, and RCBD with factorial treatment structure
- Lunch 1:00 2:00 PM
- Instruction 2:00 2:30 PM
 - Section 6: Split plot experimental design
- Tea break 2:30 3:00 PM
- Instruction 3:00 4:00 PM
 - Section 7: Multi-location studies, missing data

Have data. Want analysis.

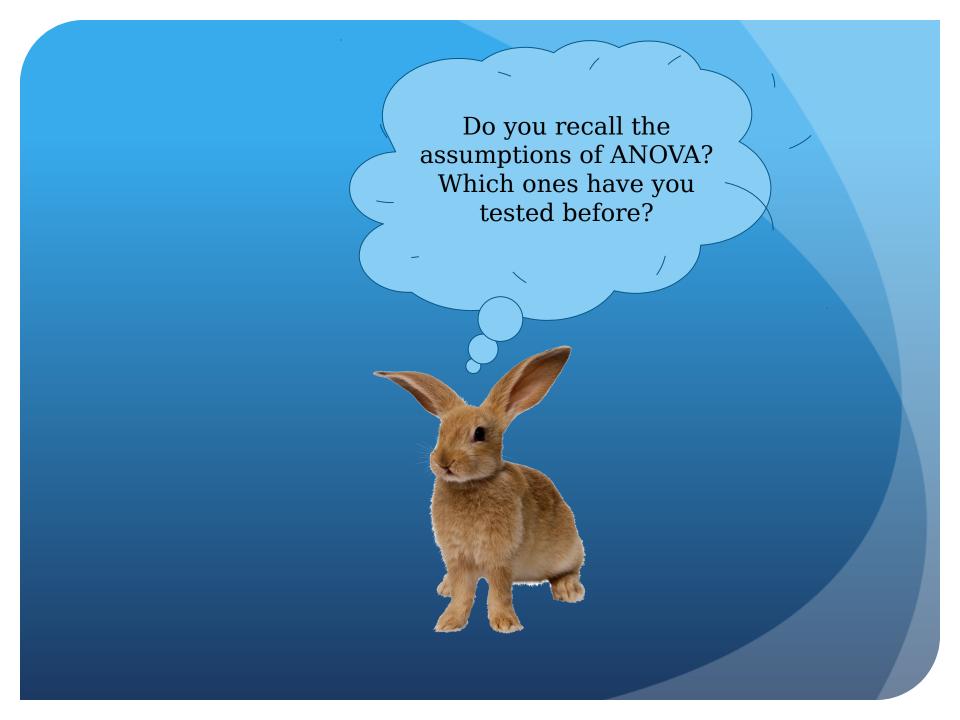


Load and run a warm-up dataset.

- What are the results?
- Is everything OK with it?

vali d Have data. Want analysis.





Section 4 Learning Objectives

Have basic understanding of:

- 1. The assumptions of ANOVA and transformations
 - a) Know the assumptions, and how to check your data
 - b) What to do when assumption tests fail, and what transformation to choose for your data
 - c) How to do all of this with the app!

Follow these four key assumptions of ANOVA to ensure that your analysis and conclusions are valid and correct:

ANOVA Assumptions	When might I have a problem?
1. Errors are statistically independent	Did I forget to <i>randomly</i> assign treatments to experimental units?(e.g. fertilizer rate treatments assigned to plots)
2. Additive effects: the variables are <i>added</i> , not multiplied	Does the <i>effect</i> of the treatments depend on what block they're in?
3. Normal distribution of errors	Do the data represent a rare event, or are there outliers?
4. Equal variance	Is there a very strong treatment effect, or is this a multi-site study?

Four key assumptions are used in ANOVA to ensure that your analysis and conclusions are valid at Ensure this by

ANUVA ASSUMPTIONS

1. Errors are statistically independent

Ensure this by randomizing treatment assignments to experimental units!

units?(e.g. fertilizer rate treatments assigned to plots)

2. Additive effects: the variables are *added*, not multiplied

Does the *effect* of the treatments *depend* on what block they're in?

3. Normal distribution of errors

Do the data represent a rare event, or are there outliers?

4. Equal variance

Is there a very strong treatment effect, or is this a multi-site study?

Four key assumptions are used in ANOVA to ensure that your analysis and conclusions are valid and correct.

When might I have a problem? **ANOVA Assumptions** Did I forget to randomly assign 1. Errors are statistically independent treatments to experimental units?(e.g. fertilizer rate It's standard practice to perform statistical tests 2. Additive effects: the variables for #3 and #4, and de are added, not multiplied include it in materials 3. Normal distribution of errors and methods section of research reports. 4. Equal variance cre a very strong zeatment effect, or is this a multi-site study?

Four key assumptions are used in

ANOVA conclus

The app will give you the test results, but YOU must interpret and address any problems.

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1. Erronindepen

(we will learn how)

de

mly assign

treatments to experimental units?(e.g. fertilizer rate

2. Additive effects: the variables are *added* not multiplied

It's standard practice to perform statistical tests

for #3 and #4, and include it in materials and methods section of

research_reports.

dere a very strong deatment effect, or is this a multi-site study?

3. Normal distribution of errors

4. Equal variance

Four key assumptions are used in ANOVA to ensure that your analysis and conclusions are valid and correct.

ANOVA Assumptions

1. Errors are statistically independent

The app provides test results for this too!

*Only for basic RCBD with 1 replication per block...

2. Additive effects: the variables are *added*, not multiplied

Desthe *effect* of the treatments depend on what block they're in?

3. Normal distribution of errors

Do the data represent a rare event, or are there outliers?

4. Equal variance

Is there a very strong treatment effect, or is this a multi-site study?

Inspect all assumption tests and corresponding plots from your app output before looking at ANOVA table

Let's use the app to check the ANOVA assumptions on the wheat variety dataset (RCBD) from yesterday (wheat yield data.csv)

- □ Normality
- Equal variance
- ☐ Additivity (only for RCBD with 1 replication per block)

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- □ Normality
- Equal variance
- ☐ Additivity (only for RCBD with 1 replication per block)

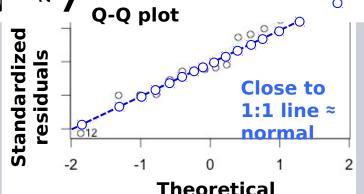
Shapiro-Wilk Normality Test

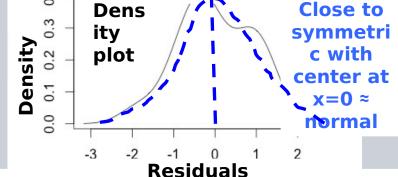
Good (Wheat example)

Bad

p-value > 0.05 CONCLUSION: Errors are

normally distributed.



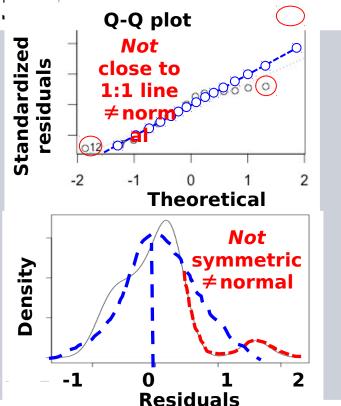


Ouantiles

p-value ≤ 0.05

CONCLUSION: Errors are *not* normally distributed.

W = 0.8784 , p-value = 0.0



Wheat variety dataset:

- **Normality**
- □ Equal variance
- □ Additivity

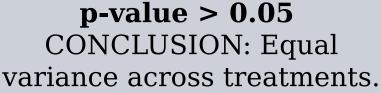
Wheat variety dataset:

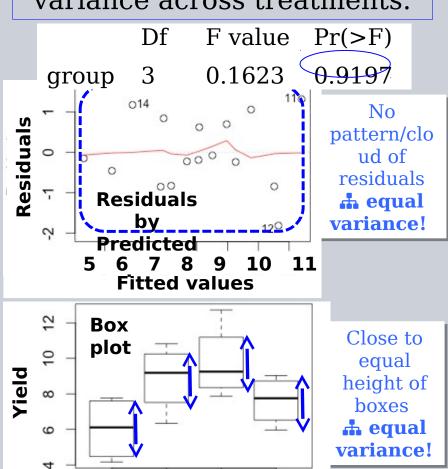
- **Normality**
- □ Equal variance
- □ Additivity

Levene's Test for Homogeneity of

Good (Wheat example) riance

Bad



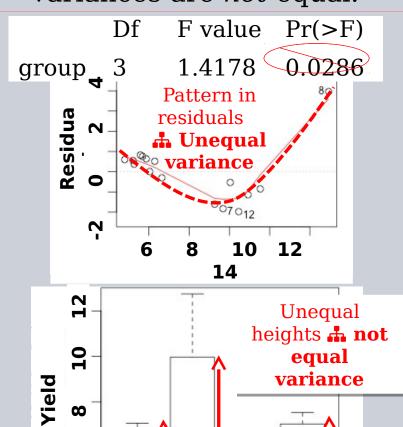


D

В

p-value ≤ 0.05

CONCLUSION: Treatments variances are *not* equal.



B

D

9

Wheat variety dataset:

- Normally distributed errors
- Tequal variance
- □ Additivity

Wheat variety dataset:

- Normally distributed errors
- Tequal variance
- □ Additivity

Tukey's Test for Nonadditivity

Good (Wheat example)

Bad

p-value > 0.05 CONCLUSION: Additive model. **p-value ≤ 0.05**CONCLUSION: Nonadditivity in model.

```
Analysis of Variance Table
```

```
Response: Yield
```

Df Sum Sq Mean Sq F value Pr(>F)

Treatment 3 31.5311 10.5104 7.2465 0.01141

Block 3 28.2354 9.4118 6.4891 0 01550

Restauals 8 11.6033 1.4504

Signif. codes: 0 '***' 0.001 '**' 0<mark>.</mark>01 '*' 0.05

Response: Yield

Df Sum Sq Mean Sq F value Pr(>F)

Treatment 3 4.9189e+19 1.6396e+19 21.895 0.0003267

Rlock 3 8 4309e+19 2 8103e+19 37 527 4 640e-05

Block 3 8.4309e+19 2.8103e+19 37.527 4.640e-05 **

sq_preds 1 1.1685e+20 1.1685e+20 156.030 1.578e-06

Residuals 8 5.9910e+18 7.4888e+17

Analysis of Variance Table

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1

Only look at p-value for 'sq preds'.

Wheat variety dataset:

- Normally distributed errors
- Equal variance
- Additivity

All our assumption tests were GOOD. Thus, the ANOVA of the field study is valid.

What if we failed 1 or more of the assumption tests?

All our as the Al VA of the field study is valid.

Transformations

- † Perform ANOVA on the dataset that *best* meets assumptions
- * May not be the original scale, such as lb/ac
- Thus, when data do *not* meet assumptions, one approach is to transform your data to see if the assumption tests improve.
 - Note: the transformation is applied to the dependent variable only (Y_{iik})

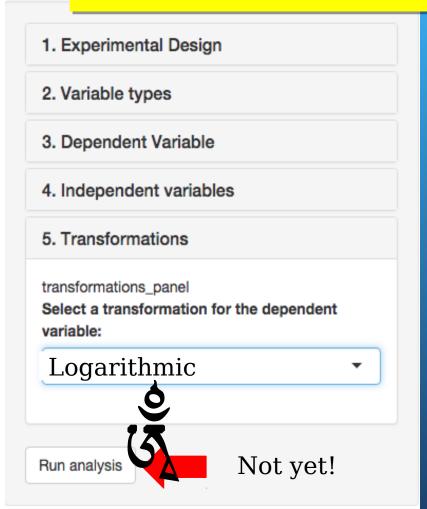
WARNING: We do <u>not</u> transform data to get the results we want. We transform data so that the analysis and results are *valid* and *correct*.

Transformations

If you decide to transform your data, you can choose from 2 transformations using the app:

- 1. Log transformation: $log_{10}(Y_{ij}) = \mu + \tau_i + \beta_j + \epsilon_{ij}$
- 2. Power transformation: $(Y_{ij})^a = \mu + \tau_i + \beta_j + \epsilon_{ii}$

Try both transformations. Use the one that most improves the assumption tests.



CSV file and reload it to the app:

1. Experimental Design 2. Variable types 3. Dependent Variable 4. Independent variables 5. Transformations transformations_panel Select a transformation for the dependent variable: Logarithmic Run analysis Not yet!

CSV file and reload it to the app:

- 1. Are all observations (Y_{ij}) greater than 0 and some (or all) between 0 and 1?
 - ✓ If so, multiply your entire Y_{ij} dataset by a constant 'X' so that all values are ≥ 1 .

Example dataset

	Α	В	С
1	Block	Independent	Dependent
2	1	Α	0.60230465
3	2	Α	0.16753883
4	3	Α	12.0412073
5	4	Α	26.4773239
6	1	В	29.8006572
7	2	В	0.54721707
8	3	В	0.2823309
9	4	В	168.656146
10	1	С	12.2879646
11	2	С	0.0048015
12	3	С	3.26729745
13	4	С	5.11869392
14	1	D	0.45601859
15	2	D	11.427267
16	3	D	13.288801
17	4	D	38.3352375
18			

CSV file and reload it to the app if:

- 1. Are all observations (Y_{ij}) greater than 0 and some (or all) between 0 and 1?
- If so, multiply your entire Y_{ij} dataset by a
 What X value 1.
 would you

choose for this dataset?

CSV file and reload it to the app if:

					u
	Α	В	С	D	
1	Block	Independent	Dependent		_
2	1	Α	0.60230465	=C2*1000	
3	2	Α	0.16753883		
4	3	Α	12.0412073		
5	4	Α	26.4773239		
6	1	В	29.8006572	At least	
7	2	В	0.54721707		\circ
8	3	В	0.2823309	X = 1000	
9	4	В	168.656146		
10	1	С	12.2879646		
11	2	С	0.0048015		
12	3	С	3.26729745		
13	4	С	5.11869392		
14	1	D	0.45601859		
15	2	D	11.427267		
16	3	D	13.288801		
17	4	D	38.3352375		
1.0					

- . Are all observations (Y_{ij}) greater than 0 and some (or all) between 0 and 1?
 - ✓ If so, multiply your entire Y_{ij} dataset by a constant 'X' so that all values are ≥ 1 .

CSV file and reload it to the app:

multiplie

d by

1000,

and are

greater than 1.

	Α	В	С	D
1	Block	Independent	Dependent	
2	1	Α	0.60230465	602.304648
3	2	Α	0.16753883	167.538834
4	3	Α	12.0412073	12041.2073
5	4	Α	26.4773239	26477.3239
6	1	В	29.8006572	29800.6572
7	2	В	0.54721707	547.217069
8	3	В	0.2823309	282.330903
9	4	В	168.656146	168656.146
10	1	С	12.2879646	12287.9646
11	2	С	0.0048015	4.80149538
12	3	С	3.26729745	3267.29745
13	4	С	5.11869392	5118.69392
14	1	D	0.45601859	456.01859
15	2	D	11.427267	11427.267
16	3	D	13.288801	13288.801
17	4	D	38.3352375	38335.2375

- 1. Are all observations (Y_{ij}) greater than 0 and some (or all) between 0 and 1?
 - ✓ If so, multiply your entire Y_{ij} dataset by a All tant 'X' so that all values es are ≥ 1 .

CSV file and reload it to the app:

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17	4	D	38.3352375	38335.2375
9 10 11 12 13 14 15	3 4 1 2 3 4 1 2 3	B B C C C C D	0.2823309 168.656146 12.2879646 0.0048015 3.26729745 5.11869392 0.45601859 11.427267 13.288801	282.3309 168656.1 12287.96 4.801495 3267.297 5118.693 456.018 11427.2 13288.8

- 1. Are all observations (Y_{ij}) greater than 0 and some (or all) between 0 and 1?
 - ✓ If so, <u>multiply</u> your entire Y_{ij} dataset by a constant 'X' so that all

Copy values

Copy es are ≥ 1 .

	Α	В	С
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13	4	С	5118.69392
14	1	D	456.01859
15	2	D	11427.267
16	3	D	13288.801
17	4	D	38335.2375

CSV file and reload it to the app:

1. Are all observations (Y_{ij}) greater than 0 and some (or all) between 0 and 1?

Paste values

If so, <u>multiply</u> your entire Y_{ij} dataset by a constant 'X' so that all values are ≥ 1 .

	Α	В	С
1	Block	Independent	Dependent
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3	2	Α	167.538834
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12	3	С	3267.29745
13	4	С	5118.69392
14	1	D	456.01859
15	2	D	11427.267
16	3	D	13288.801
17	4	D	38335.2375

CSV file and reload it to the app:

- 1. Are all observations (Y_{ij}) greater than 0 and some (or all) between 0 and 1?
 - ✓ If so, <u>multiply</u> your entire Y_{ij} dataset by a constant 'X' so that all

'Save as' *new* file name (e.g. yield_multiplied_1000.csv) and load the new file to the app.

Before choosing log transformation

CSV file and reload it to the app:

- 1. Are all observations (Y_{ij}) greater than 0 *and* some (or all) between 0 and 1?
 - ✓ If so, <u>multiply</u> your entire Y_{ij} dataset by a constant 'X' so that all values are ≥ 1 .
- 2. Are any observations $(Y_{ij}) \le 0$?
 - ✓ If so, <u>add</u> a constant 'X' to entire Y_{ij} dataset so that all values are ≥ 1 .

Note: you only do one or the other, and if all your data are greater than 1, no pretransformation is needed

Before choosing log transformation

- 1. Experimental Design
- 2. Variable types
- 3. Dependent Variable
- 4. Independent variables
- 5. Transformations

transformations_panel

Select a transformation for the dependent variable:

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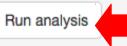
1. Are all observations (Y_{ii}) greater than 0 and some (or all) between 0 and 1?

the app if:

CSV file and reload it to

- \checkmark If so, <u>multiply</u> your entire Y_{ii} dataset by a constant 'X' so that all values are ≥ 1 .
- 2. Are any observations $(Y_{ii}) \leq 0$?
 - If so, <u>add</u> a constant 'X' so that all values are ≥ 1 .

Now your data are ready for the log transformation.



Section 4 Learning Objectives

You should now have a basic understanding of:

- ✓ The assumptions of ANOVA and transformations
 - a) What are the four assumptions of ANOVA?
 - b) How do you ensure your data meets the assumption of independent errors?
 - c) How do you use the app to check if your data meets the assumptions?
 - d) What can you do if your data fail an assumption test?
 - e) What types of transformations can you apply to your data using the app?
 - f) What range of values must your dataset be in before applying the log transformation?
 - What can you do if it is outside of that range?

Section 5 Learning Objectives

Have basic understanding of:

- 1. Factorial treatment structures
 - Linear model and ANOVA of factorial experiments
 - Interpreting ANOVA results and interactions
 - Post-hoc tests
- 2. Using the app to check assumptions *and* to transform data
- 3. Back-transformations and presenting results

Recall the CRD milking experiment. What was the independent variable(s)?

(kg)

Replicatio n	(1) Hand- milk 2x	(2) Hand- milk 3x	(3) Machine- milk 2x	(4) Machine- milk 3x
(1)				
(2)	9.3	10.8	9.3	12.0
(3)	9.1	11.7	9.0	11.8
Treatmen t means:	9.0	11.2	9.3	11.8

Independent variable = milking practice, which consists of **2** *factors*:

- (1) Factor A = 'milking method' with 2 *levels* (hand, machine)
- (2) Factor B = 'milking frequency' with 2 *levels* (2x per day, 3x per day)
- The image of the second secon

II			milk 2x	milk 3x
(1)				
(2)	9.3	10.8	9.3	12.0
(3)	9.1	11.7	9.0	11.8
Treatmen t means:	9.0	11.2	9.3	11.8

We used the following linear model to predict milk production based on milking practice:

$$Y_{ij} = \mu + \tau_i + \epsilon_{ij,}$$

where Y_{ij} = observation of ith treatment and jth replication, μ = grand mean, τ_i = effect of ith treatment (Hand-milk 2x, Hand-milk 3x, Machine-milk 2x, Machine-milk 3x), and ϵ_{ii} = random error.

We can get *more* information about the treatment effect by partitioning τ_i into: Factor A (milking method), Factor B (milking frequency), and AxB interaction. This linear model is given by:

$$Y_{ij} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk},$$

where $\alpha = \text{effect of } i^{\text{th}} \text{ level of Factor A (hand-milk, machine-$

This is a *factorial* treatment structure. Each level of Factor A must be combined with

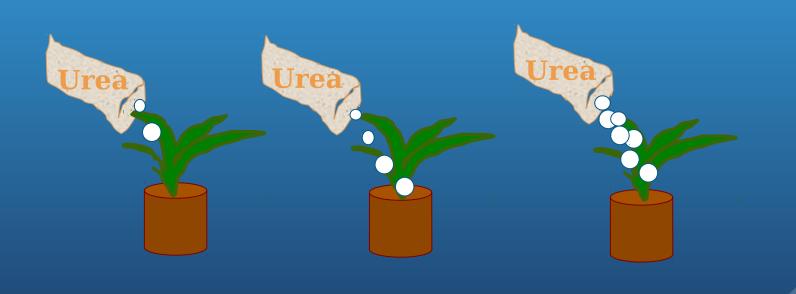
To otom D

Let's work on an example of an experiment with a factorial treatment structure...

• Does yield differ among clones (3 types)?



- Does yield differ among clones (3 types)?
- Does nitrogen fertilization affect yield (3 levels)?



- Does yield differ among clones (3 types)?
- Does nitrogen fertilization affect yield (3 levels)?
- Does the effect of nitrogen on yield change from clone?

- Does yield differ among clones (3 types)?
- Does nitrogen fertilization affect yield (3 levels)?
- Does the effect of nitrogen on yield change from clone?

In groups, outline a field experiment to answer the questions above. Include:

- Dependent variable?
- Independent variable(s)?
- Experimental design and treatment structure?
- Field layout of experimental plots? (Assume 4 reps)

- Dependent variable Yield
- Independent variable(s)Clone and
- Experimental design and treatment structRCBD experiment with 2x2 factorial treatment structure (Clone, Nitrogen), 3 levels each, 4 blocks.

3 levels for each factor (3 x 3) 9 unique combinations

	Clone	Nitrogen
1	c1	n1
2	c1	n2
3	c1	n3
4	c2	n1
5	c2	n2
6	c2	n3
7	c3	n1
8	c3	n2
9	c3	n3

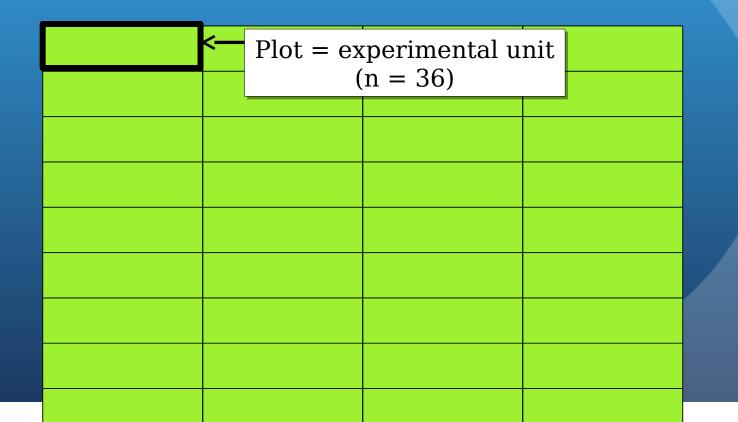
What does the layout of your field experiment look like?

What is the experimental unit?

How many are there?

 $9 \times 4 \text{ replications} = 36$

- Dependent variableYield
- Independent variable(s)Clone and
- Experimental design and treatment structRCBD experiment with 2x2 factorial treatment structure (Clone, Nitrogen), 3 levels each, 4 blocks.



- Dependent variableYield
- Independent variable(s)Clone and
- Experimental design and treatment structRCBD experiment with 2x2 factorial treatment structure (Clone, Nitrogen), 3 levels each, 4 blocks.

Block 1	Block 2	Block 3	Block 4	
c3 n2	Randomly assign the 9 treatment			nt
c1 n1	combinations to each plot within a block			olock
c3 n3				
c2 n1				
c1 n2				
c1 n3				
c3 n1				
c2 n3				
c2 n2				

- Dependent variable Yield
- Independent variable(s)Clone and
- Experimental design and treatment structRCBD experiment with

Repeat randomization of treatments for each block

2x2 factorial treatment structure (Clone, Nitrogen), 3 levels each, 4 blocks.

Block 1	Block 2	Block 3	Block 4
c3 n2	c1 n2	c3 n1	c2 n3
c1 n1	c3 n1	c2 n3	c1 n3
c3 n3	c3 n2	c1 n3	c2 n1
c2 n1	c1 n1	c3 n2	c3 n2
c1 n2	c2 n2	c2 n1	c3 n1
c1 n3	c2 n3	c1 n1	c1 n1
c3 n1	c1 n3	c1 n2	c2 n2
c2 n3	c1 n1	c2 n2	c3 n3
c2 n2	c2 n1	c1 n1	c1 n2

- Dependent variableYield
- Independent variable(s)Clone and
- Experimental design and treatment structRCBD experiment with 2x2 factorial treatment structure (Clone,

Layout of field experiment

Nitrogen), 3 levels each, 4 blocks.

Block 1	Block 2	Block 3	Block 4
c3 n2	c1 n2	c3 n1	c2 n3
c1 n1	c3 n1	c2 n3	c1 n3
Your	yield da	ata <u>ar</u> e r	eady
c1 n2	c2 n2	c2 n1	c3 n1
c1 n3	c2 n3	c1 n1	c1 n1
c3 n1	c1 n3	c1 n2	c2 n2
c2 n3	c1 n1	c2 n2	c3 n3
c2 n2	c2 n1	c1 n1	c1 n2

Let's use the app to analyze the clone x irrigation dataset!

- Load 'rcbd two var.csv' to the app
- This time, we will do all the analysis steps in the correct order:
 - 1) Check assumptions, and transform data if necessary
 - 2) Check ANOVA results, and perform Fisher's Protected LSD test

Does everything look okay?

```
yield ~ clone + nitrogen + clone:nitrogen + block
```

Model Fit Summary

ANOVA Table

```
Df Sum Sq Mean Sq F value Pr(>F)

clone 2 497.7 248.86 209.567 6.37e-16 ***

nitrogen 2 54.1 27.03 22.760 2.87e-06 ***

block 3 20.8 6.92 5.825 0.003875 **

clone:nitrogen 4 43.3 10.82 9.111 0.000126 ***

Residuals 24 28.5 1.19
```

Does everything look okay?

```
yield ~ clone + nitrogen + clone:nitrogen + block
```

Model Fit Summary ANOVA Table

The model looks good

```
Df Sum Sq Mean Sq F value Pr(>F)

clone 2 497.7 248.86 209.567 6.37e-16 ***

nitrogen 2 54.1 27.03 22.760 2.87e-06 ***

block 3 20.8 6.92 5.825 0.003875 **

clone:nitrogen 4 43.3 10.82 9.111 0.000126 ***

Residuals 24 28.5 1.19
```

Does everything look okay?

yield ~ clone + nitrogen + clone:

Model Fit Summary ANOVA Table

But we don't know yet if the ANOVA is valid! Look below this in the app output to check assumption tests and associated plots...

```
Df Sum Sq Mean Sq F value
                                         Pr(>F)
clone
                        248.86 209.567 6.37e-16 ***
nitrogen
                   54.1
                         27.03
                                22.760 2.87e-06 ***
block
                   20.8 6.92
                                 5.825 0.003875 **
clone:nitrogen 4
                 43.3
                         10.82 9.111 0.000126 ***
Residuals
              24
                   28.5
                          1.19
```

Kernel Density of the Residuals

density.default(x = residuals(model.fit))

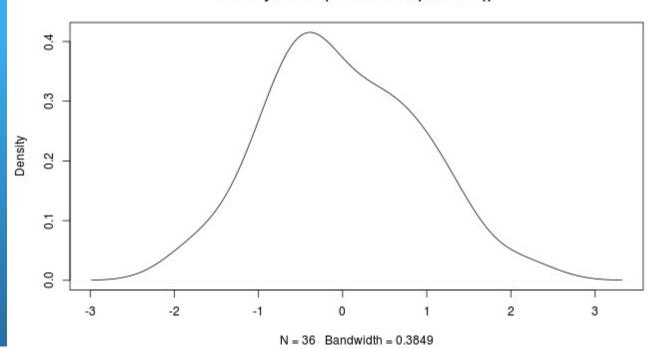
Normalit

y

Equal

variance

Additive



Shapiro-Wilk Normality Test Results

Shapiro-Wilk normality test

data: residuals(fit)
W = 0.98486, p-value = 0.893

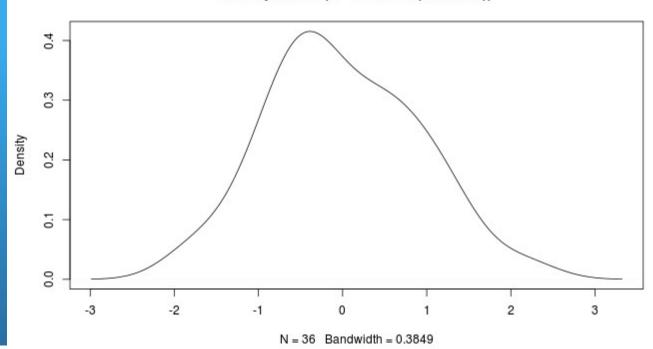
Kernel Density of the Residuals

density.default(x = residuals(model.fit))

Normalit
y
I Equal

variance

☐ Additive



Shapiro-Wilk Normality Test Results

Shapiro-Wilk normality test

data: residuals(fit)
W = 0.98486, p-value = 0.893

P-value > 0.05 ♣ GOOD!

Fitted

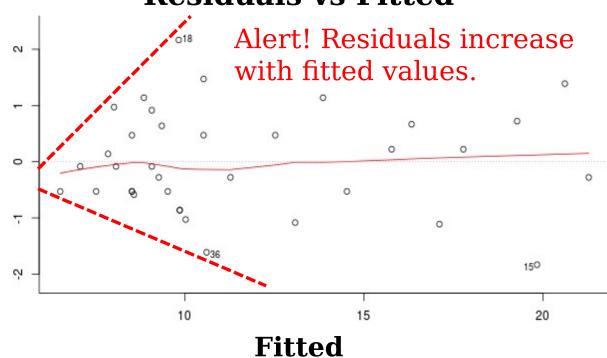
Residuals

Yield ~ Clone + Nitrogen + Clone:Nitrogen + Block

Normalit

- EqualvarianceAdditive
 - Does the plot display any trend that may be a problem?

Residuals vs Fitted



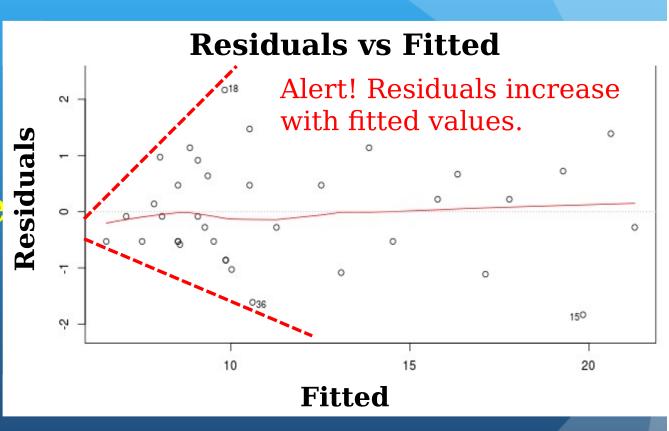
Fitted

Yield ~ Clone + Nitrogen + Clone:Nitrogen + Block



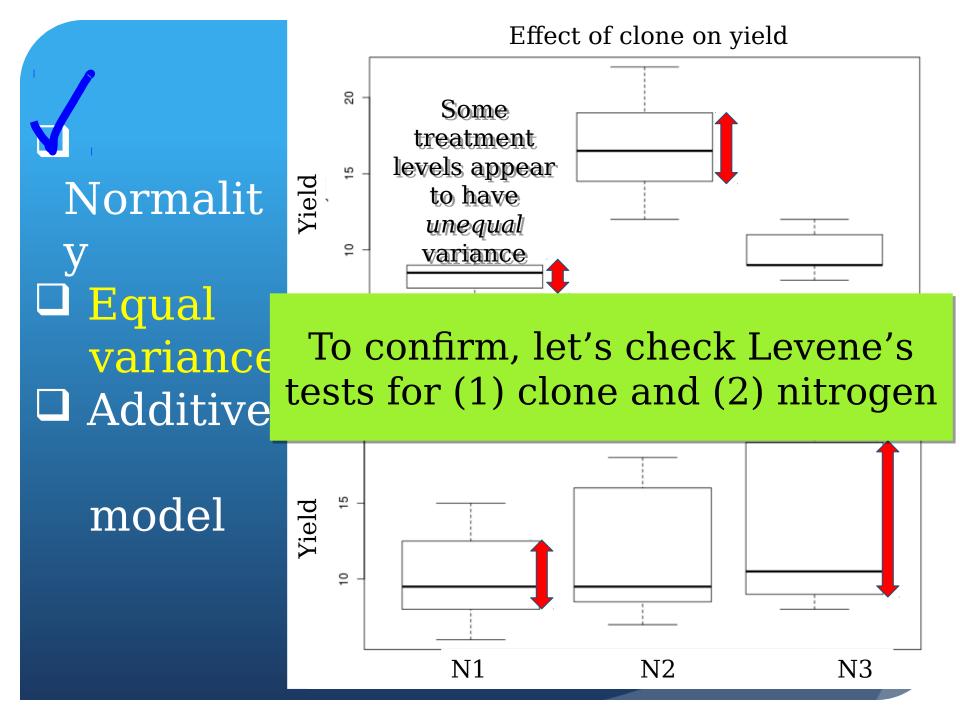
EqualvarianceAdditive

model



Let's check the box plots to see how variance compares among clone and nitrogen treatment levels

Effect of clone on yield Some treatment levels appear Yield Normalit to have unequal variance Equal variance c1c2c3Additive Effect of nitrogen on yield 20 What do these box plots tell you about variance across treatment 9 levels? / N1N2N3



Normalit y Lequal

variance

Does everythin g look OK?

Levene's Test for Homogeneity of Variance

Normalit v

□ Equal variance□ Additive

model

Levene's Test for Homogeneity of Variance

```
'yield ~ clone'
Levene's rest for Homogeneity of Variance (center = median)
     Df F value
                Pr(>F)
                              P-value < 0.05 \, \clubsuit
group 2 5.2195 0.01073
     33
                              Unequal variance
                     0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
Signif. codes:
$`yield ~ nitrogen`
Levene's rest for Homogeneity of Variance (center = median)
     Df F value Pr(>F)
                           group 2 1.0005 0.3786
                                   variance
```

Normalit y

□ Equalvariance□ Additive

Levene's Test for Homogeneity of Variance

```
'vield ~ clone'
Levene's rest for Homogeneity of Variance (center = median)
     Df F value
                Pr(>F)
                              P-value < 0.05 \, \clubsuit
group 2 5.2195 0.01073
     33
                              Unequal variance
                     0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
Signif. codes:
$`yield ~ nitrogen`
Levene's rest for Homogeneity of Variance (center = median)
     Df F value Pr(>F)
                           group 2 1.0005 0.3786
                                   variance
```

model

Let's check the last test, Tukey's Test for Nonadditivity.

Normalit y

- Equal variance
- Does
 everythin
 g look
 OK?

Tukey's Test for Nonadditivity

Attention: Refer only to the 'yield.pred.sq' row in this table, ignore all other rows.

```
Analysis of Variance Table
Response: yield
             Df Sum Sq Mean Sq F value
clone
              2 497.72 248.861 201.4245 2.651e-15 ***
nitrogen
              2 54.06 27.028 21.8759 4.770e-06 ***
block
              3 20.75 6.917
                                5.5982
                                       0.004935 **
yield.pred.sq 1 41.19 41.189 33.3376 6.993e-06 ***
clone:nitrogen 4 2.17 0.543
                                       0.778695
                                0.4396
Residuals
             23 28.42 1.236
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
```

Normalit

Equal variance

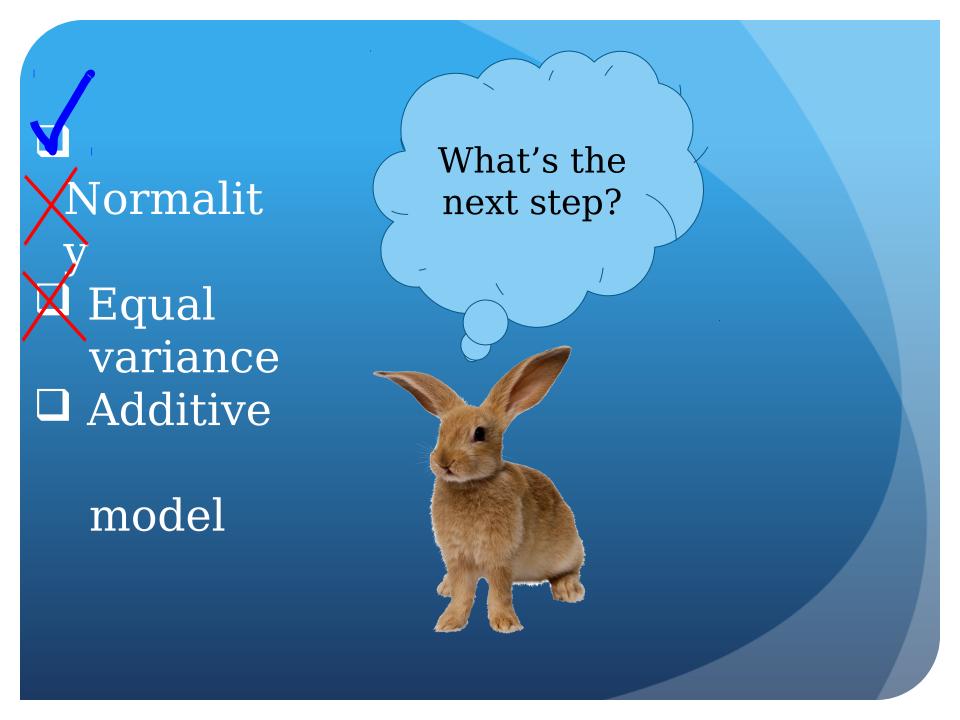
Does
everythin
g look
OK?

Tukey's Test for Nonadditivity

Attention: Refer only to the 'yield.pred.sq' row in this table, ignore all other rows.

```
Analysis of Variance Table
Response: yield
             Df Sum Sq Mean Sq F value
                                        Pr(>F)
clone
              2 497.72 248.861 201.4245 2.651e-15 ***
nitrogen
              2 54.06
                       27.028 21.8759 4.770e-06 ***
block
              3 20.75 6.917
                                5.5982
                                       0.004935 **
yield.pred.sq 1 41.19 41.189 33.3376 6.993e-06 ***
clone:nitrogen 4 2.17 0.543
                                0.4396
                                       0.738695
Residuals
             23 28,42 1,236
                               P-value < 0.05 🚠
Signif. codes: 0 '***' 0.001 '**
                               Nonadditivity in
```

model



Normalit

Equal variance

Additive

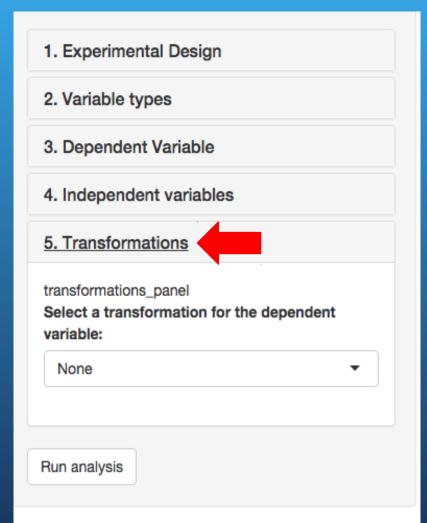
We need to transform the yield data to address (1)unequal variance and (2) nonadditivity.

Let's try both log and power transformations.

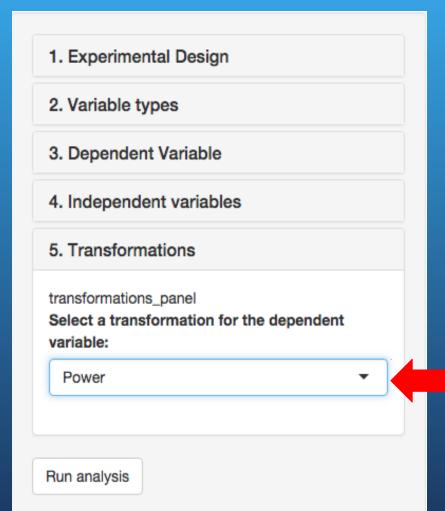
model

First the power transformation....

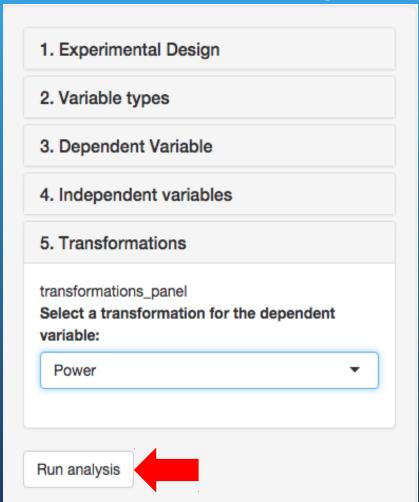
Click Transformations and select Power.



Click Transformations and select Power.



Click Run analysis



Note the new model formula with the dependent variable, yield.pow

Model Formula

```
yield.pow ~ clone + nitrogen + clone:nitrogen + block
```

Exponent from Power Transformation

0.906523802529747

And the exponent used in the power transformation

Assumption of normality

Original data

Shapiro-Wilk Test

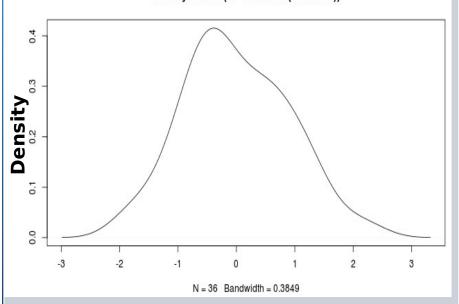
Data: residuals(fit)

W = 0.98486, p-value = 0.893



Recall, p>0.05 A normally distributed residuals

density.default(x = residuals(model.fit))



Power-transformed

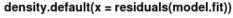
Shapiro-Wilk Test

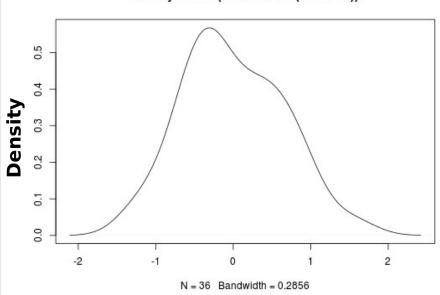
Data: residuals(fit)

W = 0.98225, p-value = 0.8185



Also OK

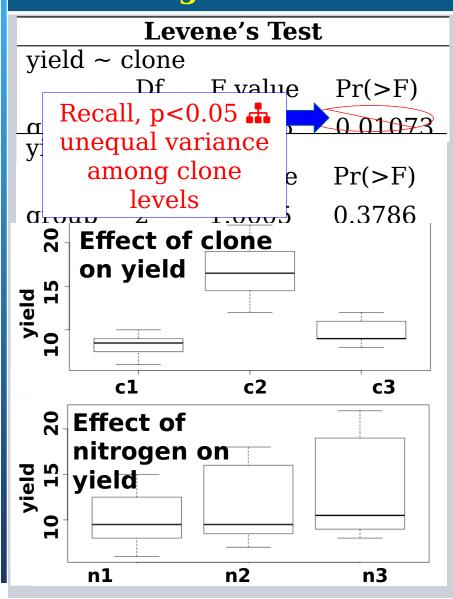


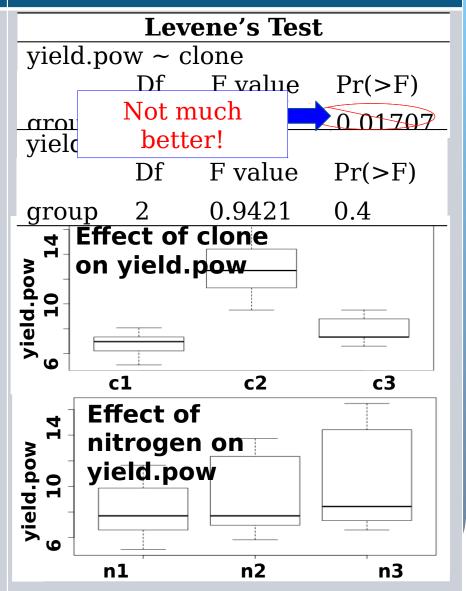


Assumption of equal variance



Power-transformed





Assumption of additivity

Original data

Tukey's Test for Nonadditivity

Attention: Refer only to the 'yield.pred.sq' row in this table, ignore all other rows.

```
Analysis of Variance Table

Response: yield

Df Sum Sq Mean Sq F value Pr(>F)

clone 2 497.72 248.861 201.4245 2.651e-15 ***
nitrogen 2 54.06 27.028 21.8759 4.770e-06 ***
block 3 20.75 6.917 5.5982 0.004935 **
yield.pred.sq 1 41.19 41.189 33.3376 6.993e-06 ***

ctone:nitrogen 4 2.17 0.543 0.4396 0 778695

Residuals 23 28.42 1.236

---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 05 '.' 0.
```

Recall, p-value < 0.05

♣ Nonadditivity in model

Power-transformed

Tukey's Test for Nonadditivity

Attention: Refer only to the 'yield.pred.sq' row in this table, ignore all other rows.

```
Analysis of Variance Table
Response: yield.pow
                Df Sum Sq Mean Sq F value
                                             Pr(>F)
clone
                 2 253.825 126.912 199.0748 3.012e-15 ***
nitrogen
                 2 27.070 13.535 21.2308 5.971e-06 ***
             3 10.949 3.650 5.7251 0.004447 **
block
yield.pow.pred.sq 1 19.411 19.411 30.4483 1.303e-05 ***
clone:nitrogen 4 1.255 0.314 0.4923 041386
Residuals
                23 14.663 0.638
              0 '***' 0.001 '**' 0.01 '*' 0.05 .' 0.1 '
Signif. codes:
```

Not much better! p<0.05 ♣ Nonadditivity in model

Now let's try the logarithmic (log) transformation...

Note the new model formula with the dependent variable: yield.log10

Model Formula

```
yield.log10 ~ clone + nitrogen + clone:nitrogen + block
```

Assumption of normality

Original data

log-transformed

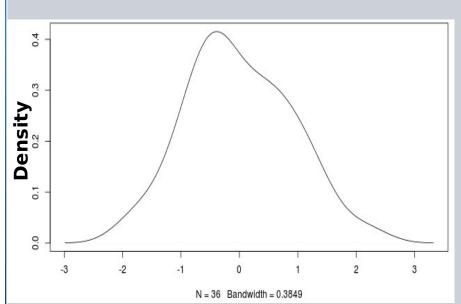
Shapiro-Wilk Test

Data: residuals(fit)

W = 0.98486, p-value = 0.893



Recall, p>0.05 ♣ OK



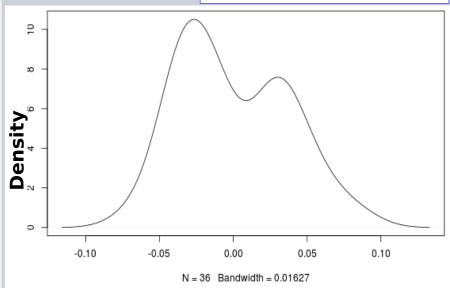
Shapiro-Wilk Test

Data: residuals(fit)

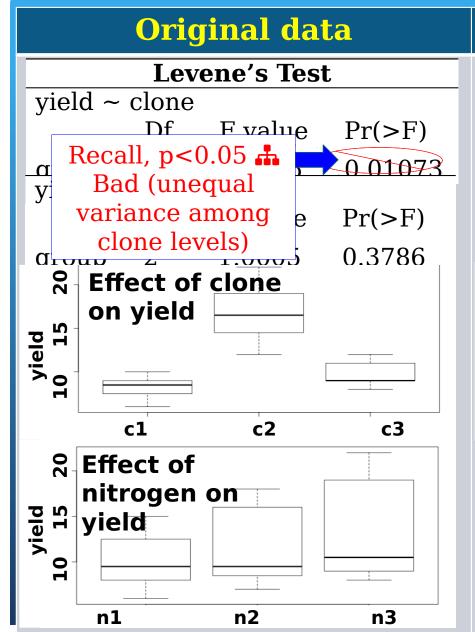
$$W = 0.94738$$
, p-value =

0.08646

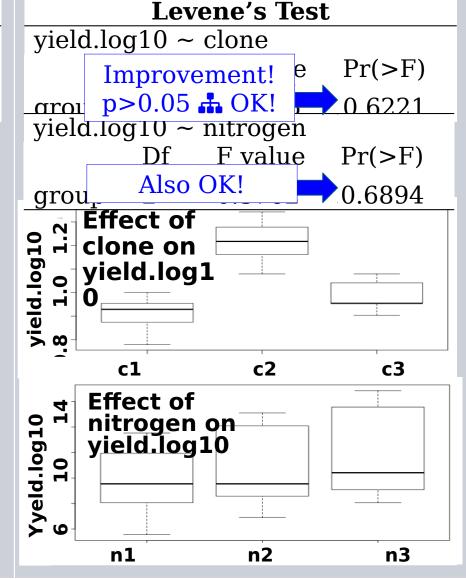
Also OK, but worse than original



Assumption of equal variance



log-transformed



Assumption of additivity

Original data

Tukey's Test for Nonadditivity

Attention: Refer only to the 'yield.pred.sq' row in this table, ignore all other rows.

```
Analysis of Variance Table

Response: yield

Df Sum Sq Mean Sq F value Pr(>F)

clone 2 497.72 248.861 201.4245 2.651e-15 ***
nitrogen 2 54.06 27.028 21.8759 4.770e-06 ***
block 3 20.75 6.917 5.5982 0.004935 **

yield.pred.sq 1 41.19 41.189 33.3376 6.993e-06 ***

ctone:nitrogen 4 2.17 0.543 0.4396 0 778695

Residuals 23 28.42 1.236

---

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 05 '.' 0.5
```

Recall, p-value<0.05 Bad (nonadditivity in model)

log-transformed

Tukey's Test for Nonadditivity

Attention: Refer only to the 'yield.log10.pred.sq' row in this table, ignore all other rows.

```
Analysis of Variance Table
Response: yield.log10
                   Df Sum Sq Mean Sq F value
clone
                    2 0.60563 0.302815 165.9591 2.155e-14 ***
nitrogen
                    2 0.05674 0.028371 15.5491 5.349e-05 ***
                    3 0.03799 0.012662
                                        6.9395 0.001713 **
yield.log10.pred.sq 1 0.01893 0.018925 10.3722 0.003787 **
                                        1.3605
ctone:nitrogen
                 4 0.00993 0.002482
                                                 278312
Residuals
                   23 0.04197 0.001825
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '
```

Better, but p<0.05 ABB Bad (Nonadditivity in model)

Overall, the log transformation improved the assumption tests the most.

Thus, we will use the logtransformed data for the final analysis.



If your data transformation does not improve your Levene's test, you may want to use a Weighted Least Squares (WLS) approach, but this is not covered in the course or available (yet) with the app.

Model Formula

```
yield.log10 ~ clone + nitrogen + clone:nitrogen + block
```

Model Fit Summary

ANOVA Table

Does clone have an effect on yield?

Model Formula

```
yield.log10 ~ clone + nitrogen + clone:nitrogen + block
```

Model Fit Summary

ANOVA Table

The effect of clone on yield depends on the nitrogen level.

Model Formula

```
yield.log10 ~ clone + nitrogen + clone:nitrogen + block
```

Model Fit Summary

ANOVA Table

We know this
because there is
a significant
interaction.
Interaction = the
effect of one
factor depends
on the level of
the other factor.

Model Formula

```
yield.log10 ~ clone + nitrogen + cloopen - block
```

Model Fit Summary

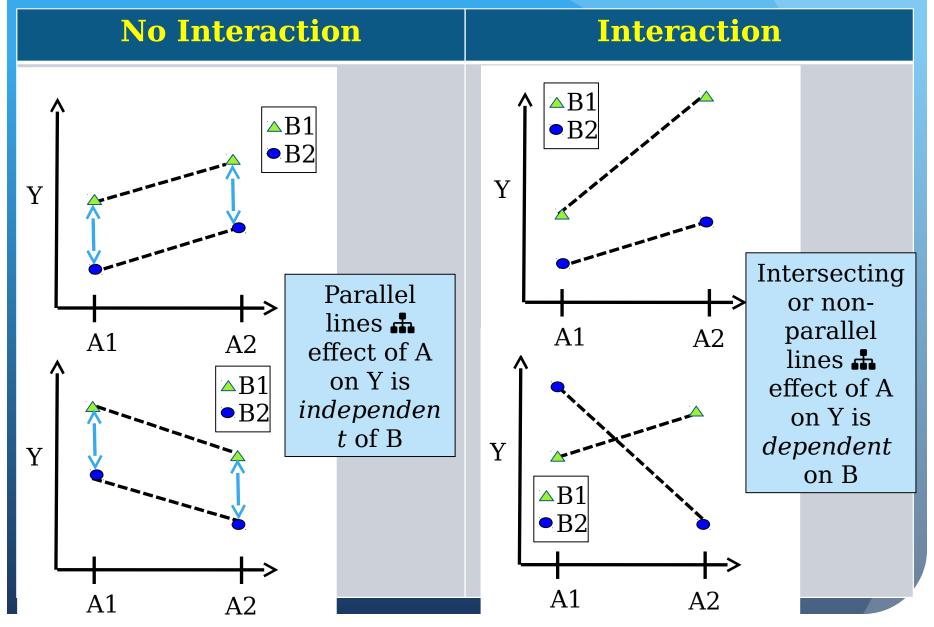
ANOVA Table

In a 2-factor factorial, there are 3 terms to look at in ANOVA table:

- 1. Factor A (clone)
- 2. Factor B (nitrogen)
- 3. AxB (clone:nitrogen)

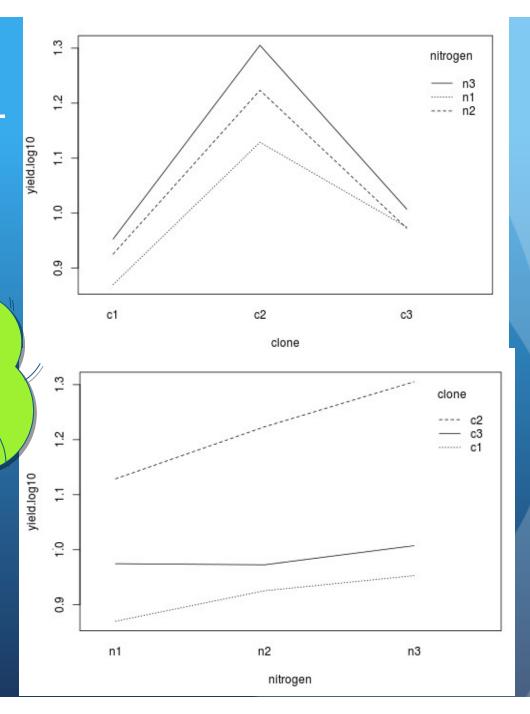
If interaction is significant (p<0.05), LSD test is performed on all treatment level combinations (simple effects). For example, we can't say what N level produced the highest yield (main

Interaction plots

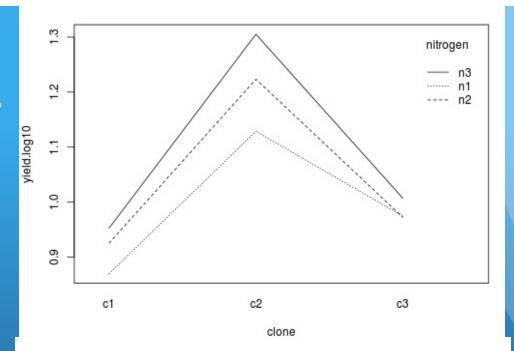


Interaction plots from logtransformed yield variable

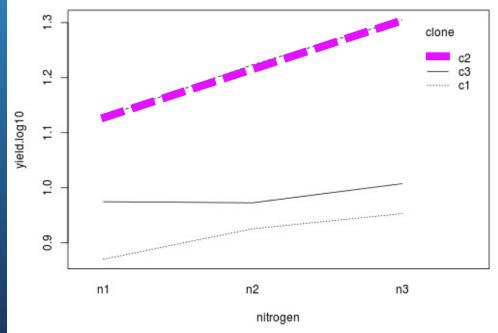
What do these plots say about the nitrogen by clone interaction on yield?



Interaction plots from logtransformed yield variable



The effect of N rate differs for each clone, with the strongest N effect exhibited in clone 2.



Click: Post-hoc tests



Agricultural Field Trial Statistics Package

1. Load data

2. Analysis

Post-hoc tests

4. Report

About

1. Experimental Design 2. Variable types 3. Dependent Variable 4. Independent variables 5. Transformations transformations_panel Select a transformation for the dependent variable: Logarithmic Run analysis

```
# convert categorical variables to factors
my.data$block <- as.factor(my.data$block)
my.data$clone <- as.factor(my.data$clone)
my.data$nitrogen <- as.factor(my.data$nitrogen)

# transform the dependent variable
my.data$yield.log10 <- log10(my.data$yield)

# fit the model</pre>
```

Model Formula

yield.log10 ~ clone + nitrogen + clone:nitrogen + block

Model Fit Summary

ANOVA Table

Click: Run post-hoc analysis

Agricultural Field Trial Statistics Package

1. Load data

2. Analysis

3. Post-hoc tests

4. Report

About

Post hoc analysis Run post hoc analysis

Post hoc tests and figures

The interaction, clone:nitrogen, is significant (alpha = 0.05). Both factors are significant.

Study: fit ~ lsd.vars

LSD t Test for yield.log10

The app chooses the correct post-hoc tests and figures based on the ANOVA.

Mean Square Error: 0.001999199

clone:nitrogen, means and individual (95 %) CI

```
yield.log10 std r LCL
                                         UCL
                                                     Min
                                                               Max
c1:n1 0.8701454 0.07582428 4 0.8240045 0.9162864 0.7781513 0.9542425
c1:n2
       0.9256076 0.06669088 4 0.8794667 0.9717486 0.8450980 1.0000000
c1:n3
       0.9528938 0.03959399 4 0.9067528 0.9990347 0.9030900 1.0000000
c2:n1
       1.1288360 0.04171115 4 1.0826950 1.1749769 1.0791812 1.1760913
c2:n2
       1.2234903 0.02455571 4 1.1773494 1.2696313 1.2041200 1.2552725
c2:n3
       1.3052361 0.03735114 4 1.2590952 1.3513771 1.2552725 1.3424227
c3:n1
       0.9746813 0.05953852 4 0.9285404 1.0208222 0.9030900 1.0413927
c3:n2
       0.9726891 0.07497815 4 0.9265481 1.0188300 0.9030900 1.0791812
c3:n3
       1.0072647 0.06313851 4 0.9611238 1.0534057 0.9542425 1.0791812
```

alpha: 0.05; Df Error: 24 Critical Value of t: 2.063899

Least Significant Difference 0.06525313

Means with the same letter are not significantly different.

The interaction, clone:nitrogen, is significant (alpha = 0.05). Both factors are significant.

Study: fit ~ lsd.vars LSD t Test for yield.log10

NOTE: Output is in log-scale due to the transformation. We will learn how to present results in the original units in the next Mean Square Error: 0.001999 Section.

clone:nitrogen, means and individual (95 %) CI

```
yield.log10
                  std r
                                    LCL
                                              UCL
                                                        Min
                                                                  Max
c1:n
       0.8701454 0.07582428 4 0.8240045 0.9162864 0.7781513 0.9542425
c1:n
       0.9256076 0.06669088 4 0.8794667 0.9717486 0.8450980 1.0000000
c1:n
       0.9528938 0.03959399 4 0.9067528 0.9990347 0.9030900 1.0000000
c2:n
       1.1288360 0.04171115 4 1.0826950 1.1749769 1.0791812 1.1760913
c2:n2
       1.2234903 0.02455571 4 1.1773494 1.2696313 1.2041200 1.2552725
c2:n
       1.3052361 0.03735114 4 1.2590952 1.3513771 1.2552725 1.3424227
c3:n
       0.9746813 0.05953852 4 0.9285404 1.0208222 0.9030900 1.0413927
c3:n
       0.9726891 0.07497815 4 0.9265481 1.0188300 0.9030900 1.0791812
c3:n3
       1.0072647 0.06313851 4 0.9611238 1.0534057 0.9542425 1.0791812
```

alpha: 0.05; Df Error: 24 Critical Value of t: 2,063899

Least Significant Difference 0.06525313 Means with the same letter are not significantly different.

The interaction, clone:nitrogen, is significant (alpha = 0.05). Both factors are significant.

Key output includes:

```
1. Least square means
Study: fit ~ lsd.vars
LSD t Test for yield.log10
Mean Square Error: 0.001999199
clone:nitrogen, means and individual ( 95 %) CI
     yield.log10 std r LCL UCL Min
                                                                Max
       0.8701454 0.07582428 4 0.8240045 0.9162864 0.7781513 0.9542425
c1:n
       0.9256076 0.06669088 4 0.8794667 0.9717486 0.8450980 1.0000000
c1:n
c1:n
       0.9528938 0.03959399 4 0.9067528 0.9990347 0.9030900 1.0000000
       1.1288360 0.04171115 4 1.0826950 1.1749769 1.0791812 1.1760913
c2:n
c2:n
       1.2234903 0.02455571 4 1.1773494 1.2696313 1.2041200 1.2552725
c2:n
       1.3052361 0.03735114 4 1.2590952 1.3513771 1.2552725 1.3424227
c3:n
       0.9746813 0.05953852 4 0.9285404 1.0208222 0.9030900 1.0413927
       0.9726891 0.07497815 4 0.9265481 1.0188300 0.9030900 1.0791812
c3:n
       1.0072647 0.06313851 4 0.9611238 1.0534057 0.9542425 1.0791812
c3:n
alpha: 0.05; Df Error: 24
Critical Value of t: 2,063899
Least Significant Difference 0.06525313
```

Means with the same letter are not significantly different.

The interaction, clone:nitrogen, is significant (alpha = 0.05). Both factors are significant.

```
Key output includes:
                                             Least square means
Study: fit ~ lsd.vars
                                         2. Standard deviation
LSD t Test for yield.log10
Mean Square Error: 0.001999199
clone:nitrogen, means and individual ( 95 %) CI
                      std r
     yield.log1)
                               LCL UCL
                                                 Min
                                                           Max
c1:n1 0.870145 0.07582428 4 0.8240045 0.9162864 0.7781513 0.9542425
       c1:n2
       0.952893 0.03959399 4 0.9067528 0.9990347 0.9030900 1.0000000
c1:n3
       1.128836 0.04171115 4 1.0826950 1.1749769 1.0791812 1.1760913
c2:n1
       1.2234903 0.02455571 4 1.1773494 1.2696313 1.2041200 1.2552725
c2:n2
c2:n3
       1.305236 0.03735114 4 1.2590952 1.3513771 1.2552725 1.3424227
c3:n1
       0.9746813 0.05953852 4 0.9285404 1.0208222 0.9030900 1.0413927
       0.972689 0.07497815 4 0.9265481 1.0188300 0.9030900 1.0791812
c3:n2
       1.0072647 0.06313851 4 0.9611238 1.0534057 0.9542425 1.0791812
c3:n3
alpha: 0.05; Df Error: 24
Critical Value of t: 2,063899
Least Significant Difference 0.06525313
Means with the same letter are not significantly different.
```

Least Significant Difference 0.06525313

Means with the same letter are not significantly different.

Study: fit ~ lsd.vars

The interaction, clone:nitrogen, is significant (alpha = 0.05). Both factors are significant.

Key output includes:

Least square means

```
2. Standard deviation
LSD t Test for yield.log10
                                            3. 95% confidence interval
Mean Square Error: 0.001999199
clone:nitrogen, means and individual ( 95 %) CI
     yield.log10
                       std r
                                  LCL
                                            UCL
                                                     Min
                                                               Max
c1:n1 0.8701454 0.07582428 4 0.8240045 0.9162864
                                                 .7781513 0.9542425
c1:n2 0.9256076 0.06669088 4 0.8794667 0.9717486
                                                 .8450980 1.0000000
       0.9528938 0.03959399 4 0.9067528 0.9990347 .9030900 1.0000000
c1:n3
c2:n1 1.1288360 0.04171115 4 1.0826950 1.1749769
                                                 .0791812 1.1760913
c2:n2 1.2234903 0.02455571 4 1.1773494 1.2696313
                                                 .2041200 1.2552725
c2:n3 1.3052361 0.03735114 4 1.2590952 1.3513771 .2552725 1.3424227
c3:n1
       0.9746813 0.05953852 4 0.9285404 1.0208222
                                                 .9030900 1.0413927
       0.9726891 0.07497815 4 0.9265481 1.0188300 .9030900 1.0791812
c3:n2
c3:n3 1.0072647 0.06313851 4 0.9611238 1.0534057 .9542425 1.0791812
alpha: 0.05; Df Error: 24
Critical Value of t: 2,063899
```

The interaction, clone:nitrogen, is significant (alpha = 0.05). Both factors are significant.

```
Study: fit ~ lsd.vars

LSD t Test for yield.log10

Mean Square Error: 0.001999199

clone:nitrogen, means and individual ( 95 %)
```

Key output includes:

- 1. Least square means
- 2. Standard deviation
- 3. 95% confidence interval
- 4. Minimum and Maximum values

```
yield.log10 std r LCL
                                            UCL
                                                      Min
                                                                Max
c1:n1 0.8701454 0.07582428 4 0.8240045 0.9162864 0.7781513 0.9542425
c1:n2 0.9256076 0.06669088 4 0.8794667 0.9717486 0.8450980 1.0000000
c1:n3
       0.9528938 0.03959399 4 0.9067528 0.9990347 0.9030900 1.0000000
c2:n1 1.1288360 0.04171115 4 1.0826950 1.1749769 1.0791812 1.1760913
c2:n2 1.2234903 0.02455571 4 1.1773494 1.2696313 1.2041200 1.2552725
c2:n3 1.3052361 0.03735114 4 1.2590952 1.3513771 1.2552725 1.3424227
c3:n1
       0.9746813 0.05953852 4 0.9285404 1.0208222 0.9030900 1.0413927
c3:n2
       0.9726891 0.07497815 4 0.9265481 1.0188300 0.9030900 1.0791812
c3:n3 1.0072647 0.06313851 4 0.9611238 1.0534057 0.9542425 1.0791812
```

```
alpha: 0.05 ; Df Error: 24
Critical Value of t: 2.063899
```

Least Significant Difference 0.06525313

Means with the same letter are not significantly different.

The interaction, clone:nitrogen, is significant (alpha = 0.05). Both factors are significant.

alpha: 0.05 ; Df Error: 24

Critical Value of t: 2.063899

Least Significant Difference 0.06525313

Means with the same letter are not significantly different.

Groups, Treatments and means

b.

de.

de

de.

ef

-2--2 1 205

c2:n3 1.305

c2:n2 1.223

c2:n1 1.129

c3:n3 1.007

c3:n1 0.9747

c3:n2 0.9727

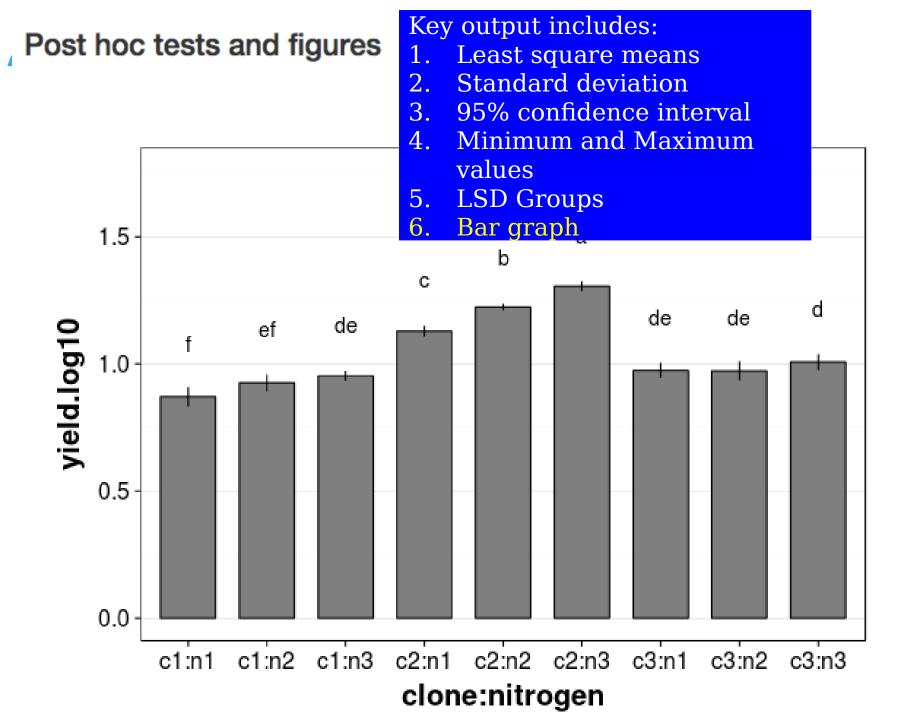
c1:n3 0.9529

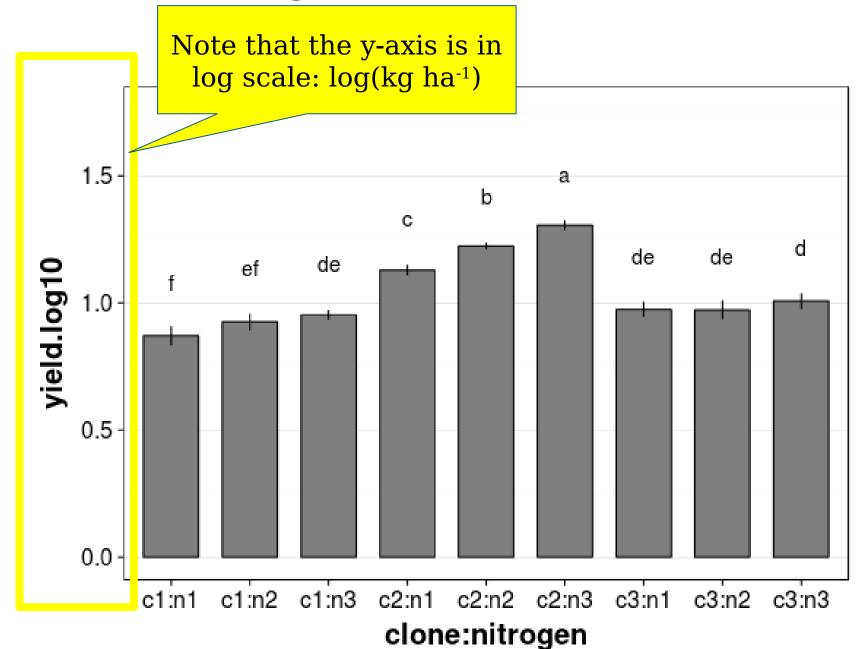
c1:n2 0.9256

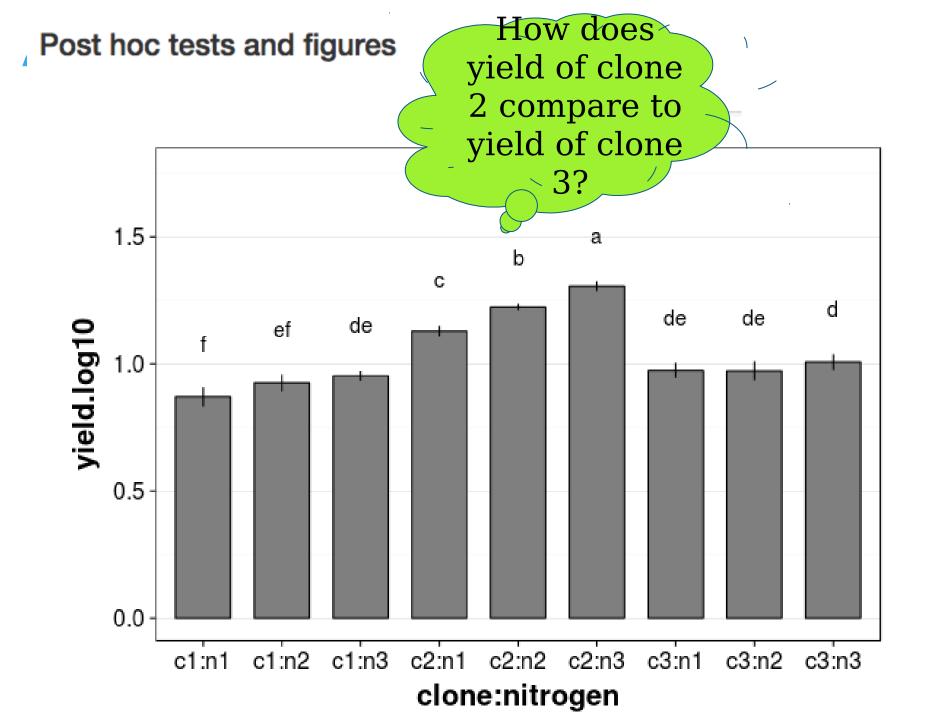
c1:n1 0.8701

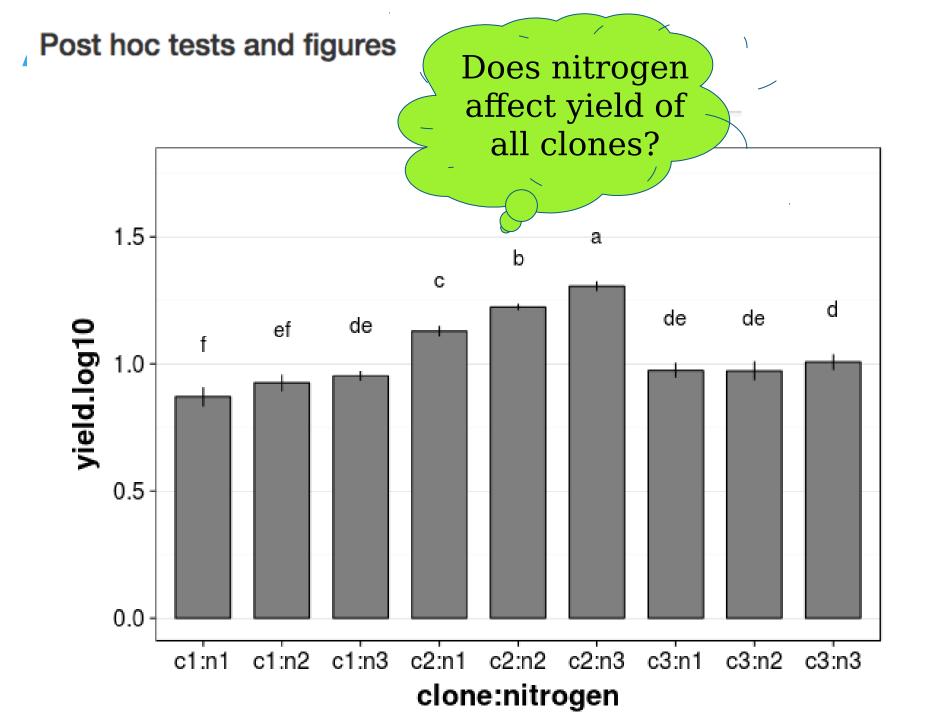
Key output includes:

- 1. Least square means
- 2. Standard deviation
- 3. 95% confidence interval
- 4. Minimum and Maximum values
- 5. LSD Groups

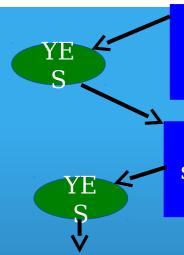








Next, a quick review...



Check assumptions: Do the data meet all ANOVA assumptions?

Run ANOVA: Is there a significant FactorA x FactorB interaction?

Transform data

NO

NO

Look at *simple* effects (all factor *combinations*) using LSD test: Which levels don't share the same letter?

Conclusion: These levels differ significantly at p<0.05

YE Z

Conclusion: there is a significant effect of factor(A or B) at p<0.05.

Is there a significant effect of FactorA or FactorB?



Look at *main* effects of *significant* factors using LSD test: Which levels don't share the same letter?

Conclusion: These levels differ significantly at p<0.05

Conclusion: no sufficient evidence to support that there is an effect of factor(A or B) on Y at p<0.05.

(1) If the p-value = 0.03 for the Shapiro-Wilk test, can we proceed with the ANOVA? Why or why not?

Example dataset

Independent variable: N fertilizer rate (kg nitrogen ha-1)	Bloc k	Dependent variable: Yield (kg grain ha ⁻¹)
0	1	7.1
50	1	9
100	1	9.5
0	2	6.2
50	2	8.8
100	2	9.2

(2) If the app determines the exponent *a* for power transformation of the dataset below:

a = 0.2. What are the power-transformed values?

N fertilizer rate (kg nitrogen ha ⁻¹)	Bloc k	Yield (kg grain ha ⁻¹)	Power- transformed Yield
0	1	7.1	
50	1	9	
100	1	9.5	
0	2	6.2	
50	2	8.8	
100	2	9.2	

(2) If the app determines the exponent *a* for power transformation of the dataset below:

a = 0.2. What are the power-transformed values?

N fertilizer rate (kg nitrogen ha ⁻¹)	Bloc k	Yield (kg grain ha ⁻¹)	Power- transformed Yield: (Y _{ij}) ^{0.2}
0	1	7.1	$(7.1)^{0.2} = 1.48$
50	1	9	1.55
100	1	9.5	1.57
0	2	6.2	1.44
50	2	8.8	1.54
100	2	9.2	1.56

(3) What are the units of the power-transformed data?

N fertilizer rate (kg nitrogen ha ⁻¹)	Bloc k	Yield (kg grain ha ⁻¹)	Power- transformed Yield
0	1	7.1	1.48
50	1	9	1.55
100	1	9.5	1.57
0	2	6.2	1.44
50	2	8.8	1.54
100	2	9.2	1.56

(3) What are the units of the power-transformed data?

N fertilizer rate (kg nitrogen ha ⁻¹)	Bloc k	Yield (kg grain ha ⁻¹)	Power- transformed Yield (kg grain ha ⁻
0	1	7.1	1.48
50	1	9	1.55
100	1	9.5	1.57
0	2	6.2	1.44
50	2	8.8	1.54
100	2	9.2	1.56

Presenting results from analyses of transformed data

When your data has been transformed, the units change too!

Log Log-transform

Original units Ex. Yield (ton/ha)

Log transformation Log-transformed units

Tests of significance (e.g. F tests in ANOVA table) and multiple mean comparisons (e.g. LSD) are performed on the *transformed* data.

However, it's common practice to present results using the *original* units. This can be done in two ways: (1) present original means, or (2) back-transform means IMPORTANT: When reporting results using original units, make sure to state that *the statistical analyses were performed on (power- or loa-) transformed data*.

Example: If this was the app output for the LSD test, how would you formally present the results?

Fertilizer N Treatment (kg N ha ⁻¹)	Power- Transformed Mean Yield (kg grain ha ⁻ ¹) ^{0.2}	LSD Groupings of Power- Transformed Means	
0	1.46	a	
50	1.55	b	
100	1.57	b	

Two ways to present results in original units:

1. Back-transformation

- a. Convert power-transformed mean yield (Y_{trans}) back to the original units $(kg ha^{\prime})_{trans}^{(1/a)} = Y$
- b. Convert log-transformed mean yield (Y_{trans}) back to the original units $(kg ha_1^{-1})^{1/2} Y_{trans} = Y$

- If data were multiplied by constant 'x' before doing the log-transformatiqn (first) divide each transformed mean by x:
- If data were added to $x \in \mathbb{R}^{r}$ is the log-transformation, first subtract each transformed mean by x:

Example: If this were your post-hoc output for the LSD test, how would you formally present the results?

Fertilizer N Treatment (kg N ha ⁻¹)	Power- Transformed Mean Yield (kg grain ha ⁻ ¹) ^{0.2}	LSD Groupings of Power- Transformed Means	
0	1.46	a	
50	1.55	b	
100	1.57	b	

Convert power-transformed mean yield (kg ha⁻¹)^a back to the original units (kg ha⁻¹) by: $(Y_{i,trans})^{(1/a)} = Y_i$

Note: these are weighted means ≠ original means means

Fertilizer N Treatment (kg N ha ⁻¹)	Power- transformed Mean Yield (kg grain ha ⁻¹) ^{0.2}	Back- Transforme d Mean Yield (kg ha ⁻¹)	LSD Groupings of Power- Transforme d Means
0	1.46	(1.46) ^{1/a}	a
50	1.55	(1.55) ^{1/a}	b
100	1.57	(1.57) ^{1/a}	b

Convert power-transformed mean yield (kg ha⁻¹)^a back to the original units (kg ha⁻¹) by: $(Y_{i,trans})^{(1/a)} = Y_i$

Fertilizer N Treatment (kg N ha ⁻¹)	Power- transformed Mean Yield (kg grain ha ⁻¹) ^{0.2}	Back- Transforme d Mean Yield (kg ha ⁻¹)	LSD Groupings of Power- Transforme d Means
0	1.46	$(1.46)^{1/0.2}$	a
50	1.55	(1.55)1/0.2	b
100	1.57	$(1.57)^{1/0.2}$	b

Convert power-transformed mean yield (kg ha⁻¹)^a back to the original units (kg ha⁻¹) by: $(Y_{i,trans})^{(1/a)} = Y_i$

Fertilizer N Treatment (kg N ha ⁻¹)	Power- transformed Mean Yield (kg grain ha ⁻¹) ^{0.2}	Back- Transforme d Mean Yield (kg ha ⁻¹)	LSD Groupings of Power- Transforme d Means
0	1.46	6.63	a
50	1.55	8.80	b
100	1.57	9.39	b

A formatted table may look something like this:

Table 1. Yield response to N fertilizer additions.

Fertilizer N Rate	Mean Yield ¹
(kg N ha ⁻¹)	(kg ha ⁻¹)
0	6.63a ²
50	8.80b
100	9.39b

¹Least square means after transformation back to the original measurement scale.

²Mean yields not sharing the same letter are significantly different at p<0.05 based on Fisher's protected least significant difference test applied to power-transformed data.

Convert power-transformed mean yield (Y_{trans}) back to the original units $(kq\gamma_{trans})^{(1/2)} = \gamma$ by:

Fertilize r N Treatme nt (kg N ha ⁻¹)	Power- transform ed Mean Yield (kg ha ⁻¹) ^{0.2}	Back- Transform ed Mean Yield (kg ha ⁻¹)	LSD Groupings Of Power- Transform ed Means
0	1.46	6.63	a
50	1.55	8.80	b
100	1.57	9.39	b

Convert power-transformed mean yield (Y_{trans}) back to the original units $(kg\gamma_{trans}^{ha})^{(1/p)} = \gamma$

Option 2:

Fertilize r N Treatme nt (kg N ha ⁻¹)	Power- transform ed Mean Yield (kg ha ⁻¹) ^{0.2}	Back- Transform ed Mean Yield (kg ha ⁻¹)	LSD Groupings Of Power- Transform ed Means
0	1.46	6.63	a
50	1.55	8.80	b
100	1.57	9.39	b

Convert power-transformed mean yield (Y_{trans}) back to the original units $(kg\gamma_{trans}^{ha})^{(1/p)} = \gamma$

bytion 2:

Fertilize r N rate (kg N	Bloc k	Yield (kg ha ⁻¹)	Origin al datase			
ha ⁻¹)		11th /	I t	Back-	Mean	LSD
0	1	7.1	Treatme	Transform ed Mean	Yield (kg ha [.]	Groupings of Power-
50	1	9	nt (kg N	Yield (kg ha ⁻¹)	1)	Transform ed Means
100	1	9.5	ha ⁻¹)	(Ky IIa)		eu Means
0	2	6.2	0	6.63		a
50	2	8.8	50	8.80		b
100	2	9.2	100	9.39		b

Convert power-transformed mean yield (Y_{trans}) back to the original units $(kg\gamma_{trans}^{ha})^{(1/p)} = \gamma$

Dytion 2:

Fertilize r N rate (kg N ha ⁻¹)	Bloc k	Yield (kg ha ⁻¹)
0	1	7.1
50	1	9
100	1	9.5
0	2	6.2
50	2	8.8
100	2	9.2

Fertilize r N Treatme nt (kg N ha ⁻¹)	Back- Transform ed Mean Yield (kg ha ⁻¹)	Mean Yield (kg ha ⁻	LSD Groupings Of Power- Transform ed Means
0	6.63	(7.1+6.2)/ 2	a
50	8.80		b
100	9.39		b

Convert power-transformed mean yield (Y_{trans}) back to the original units $(kg\gamma_{trans}^{(1)})^{(1)} = \gamma$

Dytion 2:

Fertilize r N rate (kg N ha ⁻¹)	Bloc k	Yield (kg ha ⁻¹)
0	1	7.1
50	1	9
100	1	9.5
0	2	6.2
50	2	8.8
100	2	9.2

Fertilize r N Treatme nt (kg N ha ⁻¹)	Back- Transform ed Mean Yield (kg ha ⁻¹)	Mean Yield (kg ha ⁻	LSD Groupings of Power- Transform ed Means
0	6.63	6.65	a
50	8.80	8.90	b
100	9.39	9.35	b

A formatted table may look something like this:

Table 1. Yield response to N fertilizer additions.

Fertilizer N Rate	Mean Yield
(kg N ha ⁻¹)	(kg ha ⁻¹)
0	$6.7a^{1}$
50	8.9b
100	9.4b

¹Mean yields not sharing the same letter are significantly different at p<0.05 based on Fisher's protected least significant difference test applied to power transformed data.

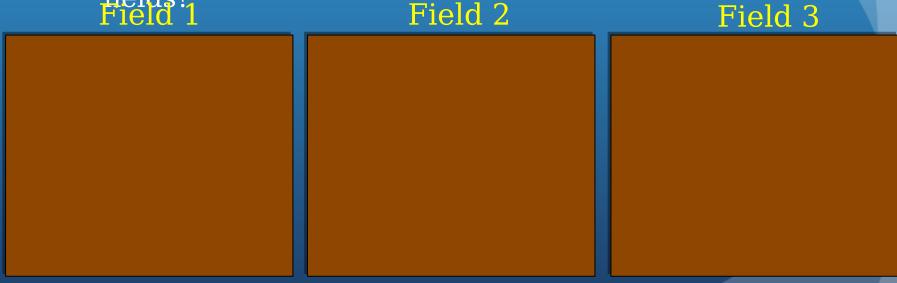
Section 6 Learning Objectives

Have basic understanding of:

- 1. Split-plot experimental designs
 - What are they, and why are they used?
 - What are the linear models for a split-plot CRD and splitplot RCBD?
 - How to use the app to analyze both of these types of split-plot designs?

Research question: In a rice system, does the type of irrigation method (mid-season drain, no drain) and rate of nitrogen fertilizer application (0, 80, 140 kg N ha⁻¹) affect methane emissions?

Field separation methods could be established per field due to practical constraints (e.g. risk of seepage), how could you design an experiment to investigate these research questions if you had 3 fields?



Ι.	Factoria	l experi	ment in	CRD =	ONE s	stage ra	ndomiza	ation.	

A1-	A2-	A2-	A3-	A1-	A3-	A2-	A2-	A2-
B1	B1	B2	B2	B2	B1	B1	B2	B2
A1-	A3-	A1-	A1-	A3-	A1-	A3-	A2-	A3-
B2	B1	B1	B2	B1	B1	B2	B1	B2

	A1- B1	A2- B1	A2- B2	A3- B2	A1- B2		A2- B1	A2- B2	A2- B2
II.	A1- B2	A3- B1	A1- B1	A1- B2	A3- B1	A1- B1	A3- B2	A2- B1	A3- B2
b									

	A1- B1	A2- B1	A2- B2	A3- B2	A1- B2	A3- B1	A2- B1	A2- B2	A2- B2	
II	A1- B2	A3- B1	A1- B1	A1- B2	A3- B1	A1- B1	A3- B2	A2- B1	A3- B2	
b	A1- B2	A3- B2	A3- B1							
	A2- B2	A2- B1	A1- B1							

	A1- B1	A2- B1	A2- B2	A3- B2	A1- B2	A3- B1	A2- B1	A2- B2	A2- B2	
II	A1- B2	A3- B1	A1- B1	A1- B2	A3- B1	A1- B1	A3- B2	A2- B1	A3- B2	
b	A1- B2	A3- B2	A3- B1	A2- B2	A2- B1	A3- B2				
	A2- B2	A2- B1	A1- B1	A3- B1	A1- B2	A1- B1				

	A1- B1	A2- B1	A2- B2	A3- B2	A1- B2	A3- B1	A2- B1	A2- B2	A2- B2
II.	A1- B2	A3- B1	A1- B1	A1- B2	A3- B1		A3- B2	A2- B1	A3- B2
b	A1- B2	A3- B2	A3- B1	A2- B2	A2- B1	A3- B2	A3- B2		
	A2- B2	A2- B1	A1- B1	A3- B1	A1- B2	A1- B1	A2- B1		A1- B2

	A1- B1	A2- B1	A2- B2	A3- B2	A1- B2	A3- B1	A2- B1	A2- B2	A2- B2	
II.	A1- B2	A3- B1	A1- B1	A1- B2	A3- B1	A1- B1	A3- B2	A2- B1	A3- B2	
b	A1- B2	A3- B2	A3- B1	A2- B2	A2- B1	A3- B2	A3- B2	I	A3- B1	
I	A2- B2	A2- B1	A1- B1 ni	A3- B <mark>plit</mark>	A1- -pl <u>B2</u> Cl	A1- RDB1	A2- V(B1		A1- B2	

1. Factor A randomly assigned to Main-Plots

	A1- B1	A2- B1	A2- B2		A3- B2	A1- B2	A3- B1		A2- B1	A2- B2	A2- B2	
II.	A1- B2	A3- B1	A1- B1		A1- B2	A3- B1	A1- B1		A3- B2	A2- B1	A3- B2	
b	A1- B2	A3- B2	A3- B1		A2- B2	A2- B1	A3- B2		A3- B2			
I	A2- B2	A2- B1	A1- B1	nt	A3- Bplit	A1-	A1- RDB1	VC	A2- B1	A2- B2	A1- B2	

- 1. Factor A randomly assigned to Main-Plots
- 2. Factor B randomly assigned to Sub-Plots within Main Plots

A2	A3	A3	A2	A1	A1	A3	A2	A1
----	----	----	----	----	----	----	----	----

	A1- B1	A2- B1	A2- B2	A3- B2	A1- B2	A3- B1	A2- B1	A2- B2	A2- B2	
II.	A1- B2	A3- B1	A1- B1	A1- B2	A3- B1	A1- B1	A3- B2	A2- B1	A3- B2	
b	A1- B2	A3- B2	A3- B1	A2- B2	A2- B1	A3- B2	A3- B2	I	A3- B1	
I	A2- B2	A2- B1	A1- B1 at	A3- B <mark>plit</mark>	A1- plB2Cl	A1- RDB1	A2- V(B1		A1- B2	

- 1. Factor A randomly assigned to Main-Plots
- 2. Factor B randomly assigned to Sub-Plots within Main Plots

A2-	A3-	A3-	A2-	A1-	A1-	A3-	A2-	A1-
B1	B2	B2	B2	B1	B2	B2	B1	B1
A2-	A3-	A3-	A2-	A1-	A1-	A3-	A2-	A1-
B2	B1	B1	B1	B2	B1	B1	B2	B2

I. Factoria	l experiment in	CRD = ONE	stage rand	lomization.

	A1- B1	A2- B1	A2- B2		A3- B2	A1- B2	A3- B1		A2- B1	A2- B2	A2- B2	
II.	A1- B2	A3- B1	A1- B1		A1- B2	A3- B1	A1- B1		A3- B2	A2- B1	A3- B2	
b	A1- B2	A3- B2	A3- B1		A2- B2	A2- B1	A3- B2		A3- B2			
I	A2- B2	A2- B1	A1- B1	nt	A3- Bplit	A1-	A1- RDB1	VC	A2- B1	A2- B2	A1- B2	

- 1. Factor A randomly assigned to Main-Plots
- 2. Factor B randomly assigned to Sub-Plots within Main Plots

	A2- B1	A3- B2	A3- B2	A2- B2	A1- B1	A1- B2	A3- B2	A2- B1	A1- B1	
IV.	A2- B2	A3- B1	A3- B1	A2-	A1- old ² RC	A1- BB1	A3- B1	A2- B2	A1- B2	on

per block.

1. Factor A randomly assigned to Main-Plots within blocks

I.	Factorial	experiment in	CRD =	ONE stage i	randomization.
		and the second of the second o			

	A1- B1	A2- B1	A2- B2	A3- B2	A1- B2	A3- B1	A2- B1	A2- B2	A2- B2	
II.	A1- B2	A3- B1	A1- B1	A1- B2	A3- B1	A1- B1	A3- B2	A2- B1	A3- B2	
b	A1- B2	A3- B2	A3- B1	A2- B2	A2- B1	A3- B2	A3- B2	I	A3- B1	
I.	A2- B2	A2- B1	A1- B1 at	A3- B <mark>plit</mark>	A1- -pl <u>B2</u> C1	A1- RDB1	A2- V(B1		A1- B2	

- 1. Factor A randomly assigned to Main-Plots
- 2. Factor B randomly assigned to Sub-Plots within Main Plots

	A2- B1	A3- B2				
IV.		A3- B1			A1- B2	D1

per block.

1. Factor A randomly assigned to Main-Plots within blocks

A1	A3	A2			

I. Factoria	l experiment in	CRD = ONE	E stage ran	domization.
	and the second s			

	A1- B1	A2- B1	A2- B2	A3- B2	A1- B2	A3- B1	A2- B1	A2- B2	A2- B2	
II.	A1- B2	A3- B1	A1- B1	A1- B2	A3- B1	A1- B1	A3- B2	A2- B1	A3- B2	
b	A1- B2	A3- B2	A3- B1	A2- B2	A2- B1	A3- B2	A3- B2	I	A3- B1	
I	A2- B2	A2- B1	A1- B1 at	A3- B <mark>plit</mark>	A1- plB2Cl	A1- RDB1	A2- V(B1		A1- B2	

- 1. Factor A randomly assigned to Main-Plots
- 2. Factor B randomly assigned to Sub-Plots within Main Plots

						A1- B2				
IV.	A2- B2	A3- B1	A3- B1	A2- <mark>- <mark>B</mark>1it-1</mark>	A1- olo ⁸ 2RC	A1- BB1	A3- B1	A2- B2	A1- B2	on

per block.

1. Factor A randomly assigned to Main-Plots within blocks.

A1	A3	A2	A2	A1	А3			
----	----	----	----	----	----	--	--	--

	A1- B1	A2- B1	A2- B2	A3- B2	A1- B2	A3- B1		A2- B1	A2- B2	A2- B2	
II.	A1- B2	A3- B1	A1- B1	A1- B2	A3- B1	A1- B1		A3- B2	A2- B1	A3- B2	
b	A1- B2	A3- B2	A3- B1	A2- B2	A2- B1	A3- B2		A3- B2	A1- B1	A3- B1	
I	A2- B2	A2- B1	A1- B1 at	A3- Bplit	A1- -plB2Cl	A1- RDB1	VC	A2- B1	A2- B2	A1- B2	

- 1. Factor A randomly assigned to Main-Plots
- 2. Factor B randomly assigned to Sub-Plots within Main Plots

	A2- B1		A3- B2							
IV.	A2- B2	A3- B1	A3- B1	A2-	A1-	A1- BB1	A3- B1	A2- B2	A1- B2	on

per block.

1. Factor A randomly assigned to Main-Plots within blocks.

			nη				S 1			
A1	A3	A2		A2	A1	A3		A2	A3	A1

	A1- B1	A2- B1	A2- B2	A3- B2	A1- B2	A3- B1	A2- B1	A2- B2	A2- B2	
II.	A1- B2	A3- B1	A1- B1	A1- B2	A3- B1	A1- B1	A3- B2	A2- B1	A3- B2	
b	A1- B2	A3- B2	A3- B1	A2- B2	A2- B1	A3- B2	A3- B2		A3- B1	
I	A2- B2	A2- B1	A1- B1	A3- B1	A1- B2	A1- B1	A2- V(B1		A1- B2	

- 1. Factor A randomly assigned to Main-Plots
- 2. Factor B randomly assigned to Sub-Plots within Main Plots

	A2- B1	A3- B2				
IV.		A3- B1			A1- B2	D1

per block.

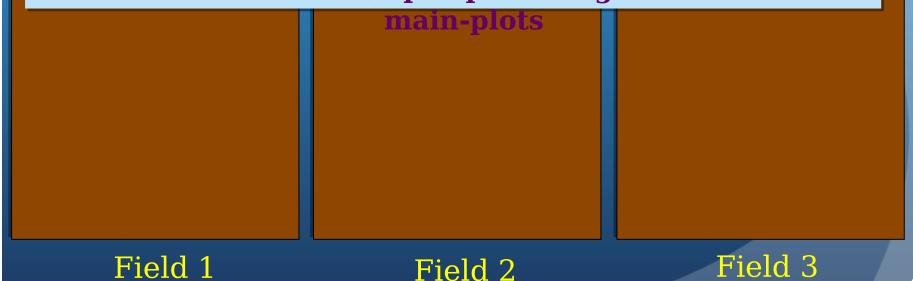
1. Factor A randomly assigned to Main-Plots within blocks

A1-	A3-	A2-	Ш	A2-	A1-	A3-	5	A2-	A3-	A1-
B2	B2	В1		B2	B1	B2		B2	B1	B1
A1-	A3-					A3-			A3-	A1-
D 1	D 1	מ		Q1	PΩ	D1		Q1	l D')	l D')

Research question: In a rice system, does the type of irrigation method (mid-season drain, no drain) and rate of nitrogen fertilizer application (0, 80, 140 kg N ha⁻¹) affect methane emissions?

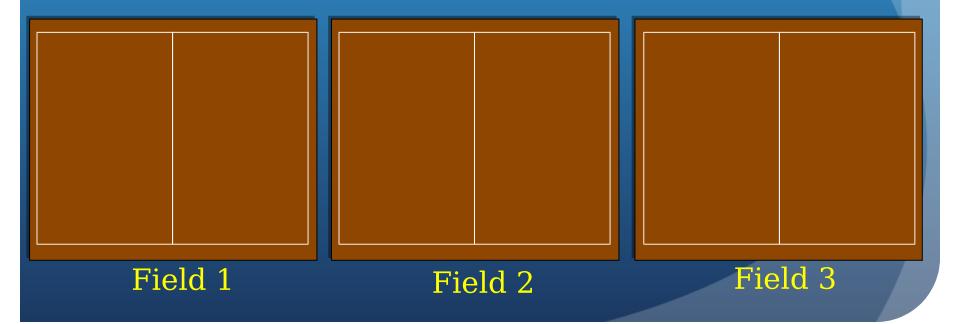
➤ If only 2 irrigation methods could be established per field due to practical constraints (e.g. risk of seepage), how could you design an experiment to investigate these research questions if you had 3 fields?

1. Experimental designation of the signature of the signa

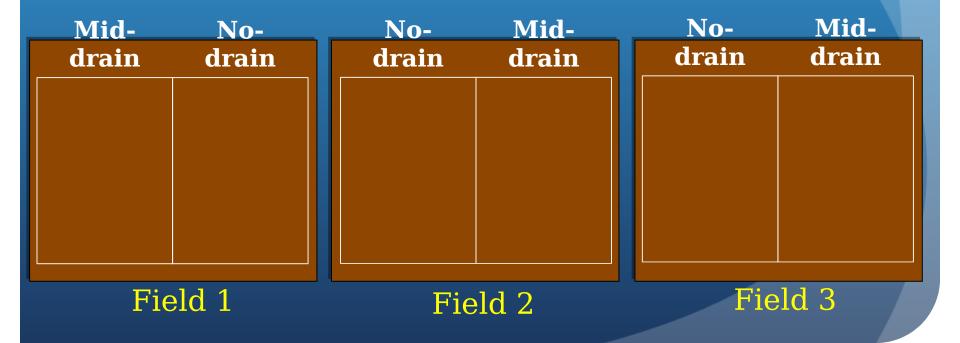


- If only 2 irrigation methods could be established per field due to practical constraints (e.g. risk of seepage), how could you design an experiment to investigate these research questions if you had 3
- 1. Experimental designation of the signature of the second section of the section o
- 2. Dependent variable (SMethane emissions
- 3. Independent variable(s)?rigation method and N rate
- 4. Main-plotarrigation method
- 5. Sub-plot?_{N rate}
- 6. Blocking variable ield

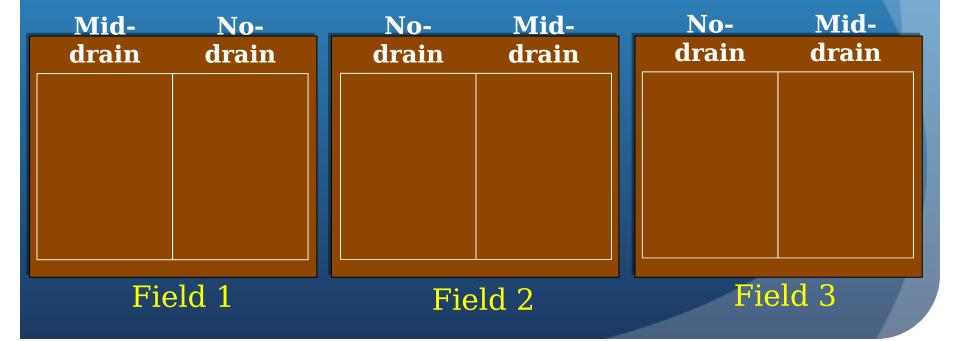
Step 1: Randomize main-plots within blocks



Step 1: Randomize main-plots within blocks



Step 2: Randomize sub-plots *within* mainplots



Step 2: Randomize sub-plots within mainplots

Mid-	No-		No-	Mid-		No-	Mid-
drain	drain		drain	drain	н	drain	drain
		ш			Ш		
					П		
		11-			П		
					Н		
		ш					
					Н		
		II-					
Field 1			Field 2			Fie	ld 3

Step 2: Randomize sub-plots within mainplots

Mid-	No-	No-	Mid-	No-	Mid-
drain	drain	drain	drain	drain	drain
80N					
0N					
140N					
Field 1		Fie	eld 2	Fi	eld 3

Step 2: Randomize sub-plots *within* mainplots

Mid-	No-
drain	drain
80N	0N
0N	80N
140N	140N

No-	Mid-
drain	drain
0N	80N
140N	140N
80N	0N

No-	Mid-
drain	drain
140N	80N
80N	140N
0N	0N

Field 1 Field 2

Field 3

Split-Plot experimental designs

use them?

- Good for experiments where it's difficult to change one factor at a small scale
- Less precision in estimating main-plot effect, but more precision for estimating sub-plot effect and sub-plot x main-plot interaction
- III. Factorial experiment in split-plot CRD = TWO stage randomization.
 - 1. Factor A randomly assigned to Main-Plots
 - 2. Factor B randomly assigned to Sub-Plots within Main Plots

	A2- B1	A3- B2	A3- B2	A2- B2	A1- B1	A1- B2	A3- B2	A2- B1	A1- B1	
IV.	A2- B2	A3- B1	A3- B1	A2- <mark></mark>	A1- pld[2RC	A1- BB1	A3- B1	A2- B2	A1- B2	on

per block.

1. Factor A randomly assigned to Main-Plots within blocks

B2 B1 B2 B1 B2 B1 B2 B1	A1-
	B1
A1- A3- A2- A2- A1- A3- A2- A3- A2- A3- A2- A3- A2- A3- A3- A3- A3- A3- A3- A3- A3- A3- A3	A1-

Split-Plot experimental

- > 2 stages of randomizates igns terms
- ➤ Make sure to choose "split-plot.." from experimental design in app!
- **Important note for split-plot CRD** Your .csv data file must include a column for 'replication' identification (e.g.
- III. Façt@rial) experiment in split-plot CRD = TWO stage randomization.
 - 1. Factor A randomly assigned to Main-Plots
 - 2. Factor B randomly assigned to Sub-Plots within Main Plots

	A2- B1	A3- B2	A3- B2	A2- B2	A1- B1	A1- B2	A3- B2	A2- B1	A1- B1	
IV.	A2- B2	A3- B1	A3- B1	A2- <mark>- 1</mark> 1t-1	A1-	A1- BB1	A3- B1	A2- B2	A1- B2	on

per block.

1. Factor A randomly assigned to Main-Plots within blocks

A1-	A3-	A2-	111	A2-	A1-	A3-	S	A2-	A3-	A1-
B2	B2	B1		B2	B1	B2		B2	B1	B1
		A2-							A3-	A1-
D 1	D 1	מ		D 1	D)	D1		D 1	D)	D)

Split-Plot experimental

Linear model for split-pesigns for main-plots:

$$Y_{ijk} = \mu + \alpha_i + (\alpha^* \gamma)_{ik} + \beta_j + (\alpha^* \beta)_{ij} + \varepsilon_{ijk}$$

Mainplot effect (A) Error A = random A

X
Replicatio
n
interactio
n

Subplot effect (B)

AxB interactio n

Error B, Error A*B

Split-Plot experimental

Linear model for split- Linear

$$Y_{ijk} = \mu + \nu_k + \alpha_i + (\alpha^* \nu)_{ik} + \beta_j + (\alpha^* \beta)_{ij}$$

rando m Block effect Mainplot effect (A)

Error A=
random
AxBlock
interactio
n

Subplot effect (B)

AxB interactio n

Error B, Error A*B

Let's practice using the app to analyze a new dataset¹!

Oat field trial measuring yield of 3 varieties at 4 nitrogen fertilizer application rates.

- Varieties are randomized within blocks, and
 4 rates of fertilizer are randomized within
 varieties
 Split-plot RCBD
- Experimental design?
- Main-plot?rate
- Sub-pletck
- Block?

¹Source: Pinheiro, J. C. and Bates, D. M. (2000), Mixed-Effects Models in S and S-PLUS, Springer, New York. (Appendix A.15) Venables, W. N. and Ripley, B. D. (2002) Modern Applied Statistics with S. (4th ed), Springer, New York.

First let's create the experimental layout

Block 1

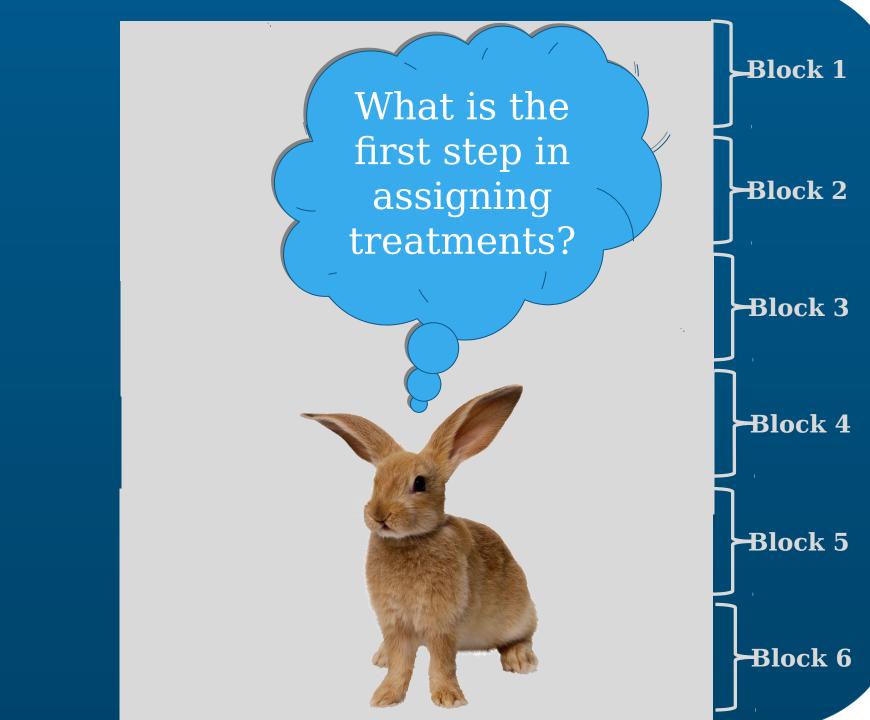
Block 2

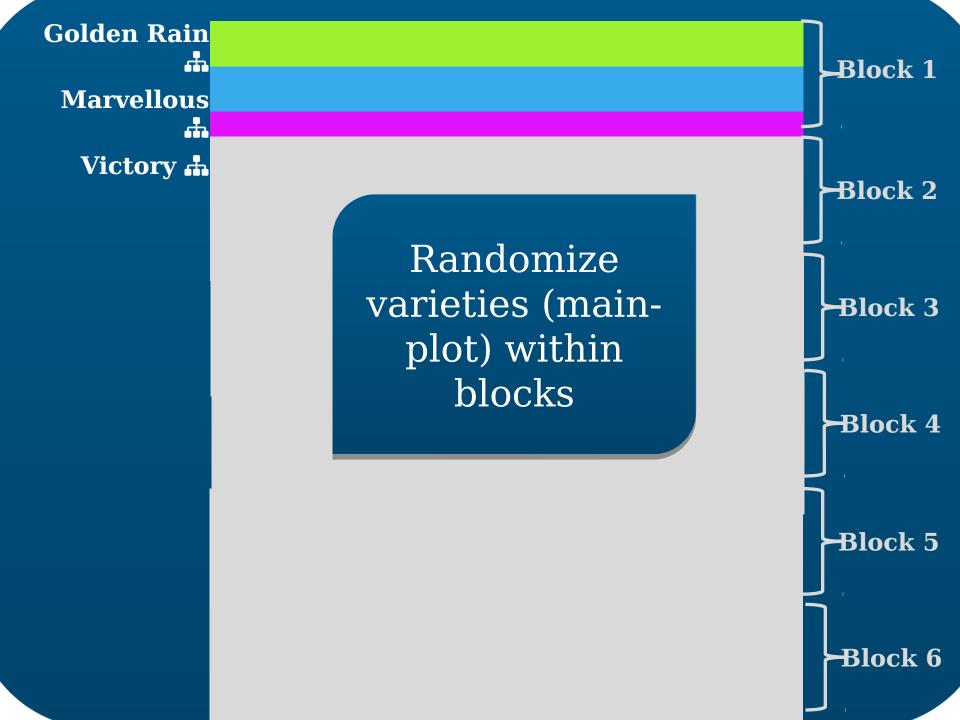
Block 3

Block 4

Block 5

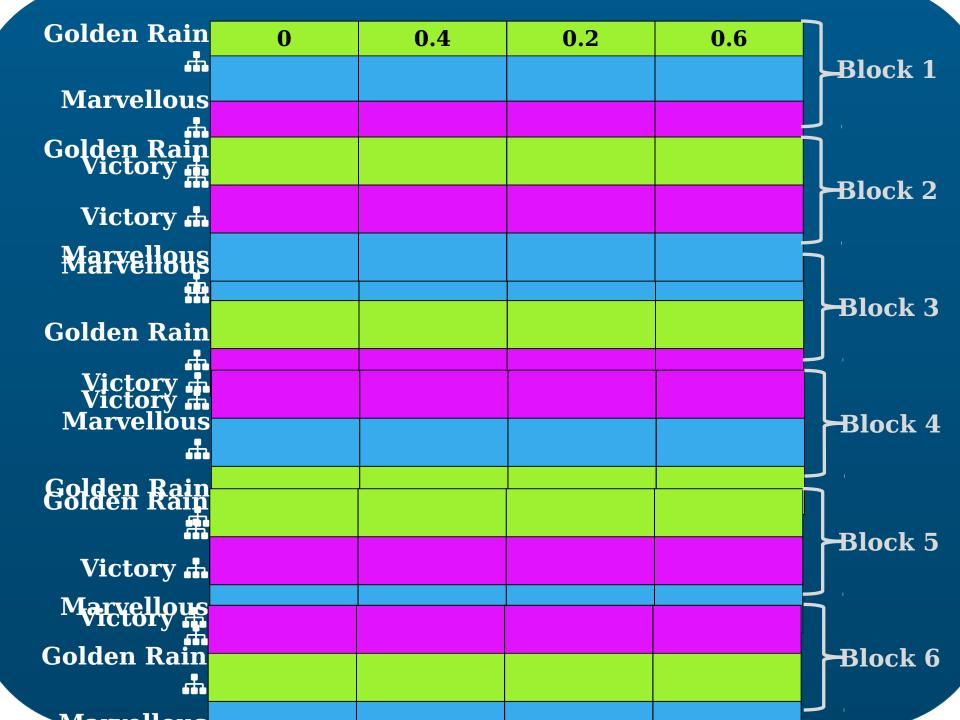
Block 6

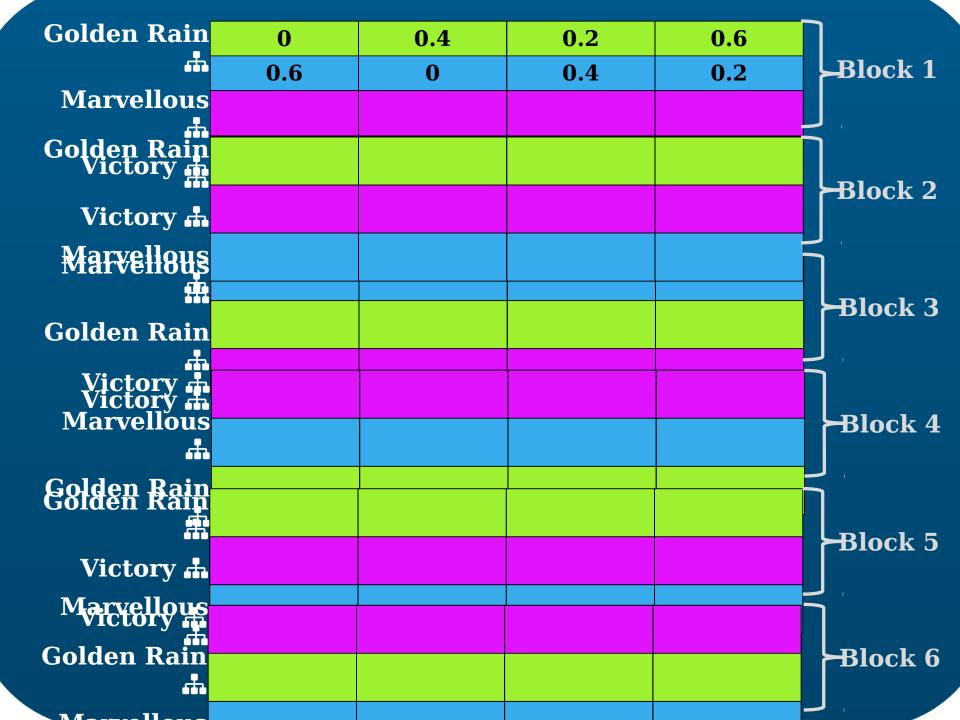


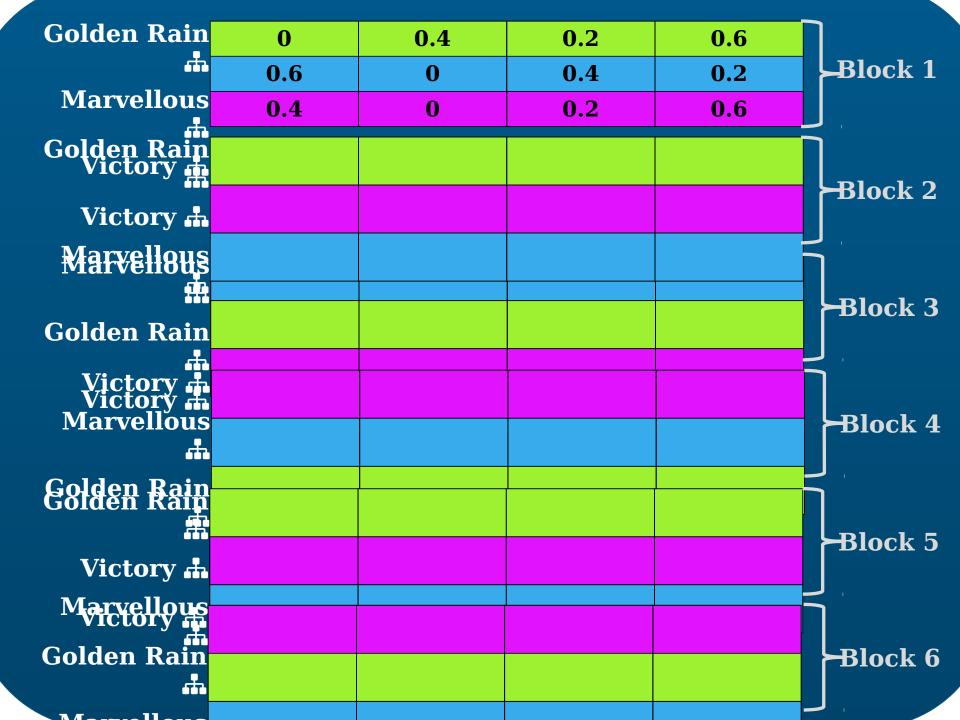












Golden Rain	0	0.4	0.2	0.6	
	0.6	0	0.4	0.2	Block 1
Marvellous #	0.4	0	0.2	0.6	
Golden Rain Victory 🛖	0	0.4	0.6	0.2	
	0	0.2	0.6	0.4	Block 2
Victory 🚓	0.4	0.2	0	0.6	
Marvellaus	0.4	0.6	0.2	0	
· · · · · · · · · · · · · · · · · · ·	0	0.6	0.2	0.4	Block 3
Golden Rain	0.6	0	0.4	0.2	
Victory A Victory A	0	0.4	0.6	0.2	
Marvellous	0.2	0.4	0	0.6	Block 4
4	0.4	0	0.6	0.2	
Golden Rain Golden Rain	0.6	0.4	0.2	0	֓֞֞֞֓֓֓֓֓֓֓֓֓֓֓֓֓֓֓֓֓֓֓֓֓֓֓֓֓֓֓֓֓֓֓֓֓
in the second second	0	0.2	0.4	0.6	Block 5
Victory 🚠	0.4	0.2	0.6	0	
Maryellous Victory	0.6	0.2	0.4	0	٦̈́
Golden Rain	0.6	0	0.4	0.2	Block 6
ф Мо	0.6	0.4	0.2	0	

Load oat data file to app

'oats-split-plot-rcbd.csv'

Section 6 Learning Objectives

You should now have a basic understanding of:

- 1. Split-plot designs
 - How to randomize treatments in 2 stages for CRD and RCBD main-plots
 - Benefit and cost of this experimental design
 - Statistical models for split-plot design CRD and split-plot design RCBD
 - Using the app to analyze both of these types of split-plot designs

Section 7 Learning Objectives

Have basic understanding of:

- 1. Multi-location studies
- 2. What to do when you have missing data

Experimental designs that we've discussed:

- 1. Complete Randomized Design (CRD) (1 or 2 factors)
- 2. Randomized Complete Block Design (RCBD) (1 or 2 factors)
- 3 Split-plot Design (CRD or RCRD main-plots with 2 Multi-site experiments

Experiments are often repeated at multiple locations, which may be analyzed as a single dataset using the app (mixed effects model). In the app, the variable you select as 'site' is designated as having a random effect, allowing for conclusions of the treatment effect to be extended to other representative sites (e.g. particular growing region for rice).

Be aware that this approach makes the following assumptions:

- 1. You are *not* particularly interested in differences among sites, or in these sites alone.
- 2. Sites were *randomly* selected from many potential sites of interest.

Linear model for RCBD experiment repeated at multiple locations:

$$Y_{ijk} = \mu + \tau_i + \nu(l)_{jk} + l_k + \varepsilon_{ijk}$$

- Y_{ijk} = observation of the ith treatment level in the jth block at the kth location
- $\succ \tau_i = \text{effect of the i}^{th} \text{ treatment}$
- $\triangleright \nu(l)_{jk} = random$ effect of the jth block at the kth location
- $> l_k = (random)$ effect of the kth location
- $\triangleright \varepsilon_{ijk} = \text{random error}$

Linear model for RCBD experiment repeated at multiple locations:

$$Y_{ijk} = \mu + \tau_i + \nu(l)_{jk} + l_k + (\tau l)_{ij} + \varepsilon_{ijk}$$

- Y_{ijk} = observation of the ith treatment level in the jth block at the kth location
- $\succ \tau_i = \text{effect of the i}^{\text{th}} \text{ treatment}$
- $\triangleright \nu(l)_{jk} = random$ effect of the jth block at the kth location
- $> l_k = random$ effect of the kth location
- $(\tau l)_{ik} = random$ effect of ijth location x treatment interaction (excluding it assumes that treatment effects are similar across sites)
- $\triangleright \varepsilon_{iik} = \text{random error}$

Missing data