Fundamental Theorem of Arithmetic in the Tropics

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1 An attempted formulation of the FTA in the tropics

This formulation only works for what I'm calling **irredundant** tropical polynomials. By this, I mean a tropical polynomial

$$f = a_{i_n} \odot x^{\odot i_n} \oplus ... \oplus a_{i_0}$$

such that the tropical polynomial $\hat{f}_j$ obtained by removing the monomial $a_{i_j} \oplus x^{\odot i_j}$ from f is distinct from f for each $j \in \{0..n\}$. If f is not irredundant, then it is **redundant**.

I'm not calling this a "minimal" tropical polynomial yet because I don't yet know if the irredundant presentaiton of f minimizes the number of monomials needed or not. I believe it does, I just haven't shown that yet. Note also that this formulation mandates assigning subscripts to the powers of the monomials, as setting a coefficient to ∞ would make f redundant.

Redundant is nice because of the following fact:

Anyways, here's an attempt at the Fundamental Theorem of Algebra.

Theorem 1.1. *Let f be an irredundant tropical polynomial of degree d so that*

$$f = a_{i_n} \odot x^{\odot i_n} \oplus ... \oplus a_{i_0}$$

where $0 = i_0 < i_1 < ... < i_n = d$. Then for all $x \in \mathbb{R} \cup \{\infty\}$,

$$f(x) = a_{i_n} \bigodot_{j=1}^{n} (x \oplus b_j)^{\odot m_j}$$

where $m_j = i_j - i_{j-1}$ is the gap between subsequent integer powers and $b_j = \frac{a_{j-1} - a_j}{m_j}$.

Proof: Set $g(x) = a_{i_n} \bigodot_{j=1}^{n} (x \oplus b_j)^{\odot m_j}$. Then we get

$$g(x) = a$$

$\square$