# Foundations of Data Science and Machine Learning – *Homework 3*

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EXERCISE 3. Find the mapping $\varphi(\mathbf{x})$ that gives rise to the polynomial kernel

$$K(\mathbf{x}, \mathbf{y}) = (x_1x_2 + y_1y_2)^2.$$

*Proof:* Consider the map $\varphi : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ defined $\varphi(x_1, x_2) = (x_1^2, x_2^2, \sqrt{2}x_1x_2)$. Interestingly, this is similar to the map one considers from a polynomial ring $R[x_1, x_2]$ to its $2^{\text{nd}}$ Veronese subring $R[x_1^2, x_1x_2, x_2^2]$. We then have that

$$\begin{aligned}\varphi(\mathbf{x}) \cdot \varphi(\mathbf{y})^T &= (x_1^2, x_2^2, \sqrt{2}x_1x_2) \cdot (y_1^2, y_2^2, \sqrt{2}y_1y_2) \\ &= x_1^2y_1^2 + x_2^2y_2^2 + 2x_1y_1x_2y_2 \\ &= (x_1y_1 + x_2y_2)^2,\end{aligned}$$

hence $\varphi$ gives rise to the desired kernel. $\square$