Algebraic Topology Homework 0

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EXERCISE 1. Construct an explicit deformation retraction of the torus with one point delted onto a graph consisting of two circles intersecting in a point, namely, longitude and meridian circles of the torus.

EXERCISE 2.

(a) Show that the composition of homotopy equivalences $X \longrightarrow Y$ and $Y \longrightarrow Z$ is a homotopy equivalence $X \longrightarrow Z$. Deduce that homotopy equivalence is an equivalence relation.

(b) Show that the relation of homotopy among maps $X \longrightarrow Y$ is an equivalence relation.

EXERCISE 3. Show that a retract of a contractible space is contractible.

EXERCISE 4. Show that S^∞ is contractible.

EXERCISE 5.

(a) Show that the mapping cylinder of every map $f : S^1 \longrightarrow S^1$ is a CW complex.

(b) Construct a 2-dimensional CW complex that contains both an annulus $S^1 \times I$ and a Möbius band as deformation retracts.

EXERCISE 6. Show that a CW complex is contractible if is the union of two contractible subcomplexes whose intersection is also contractible.

EXERCISE 7. Use Corollary 0.20 to show that if (X, A) has the homotopy extension property, then $X \times I$ deformation retracts to $X \times \{0\} \cup A \times I$. Deduce from this that Proposition 0.18 holds more generally for any pair (X_1, A) satisfying the homotopy extension property.