M408C

Stepp

WS

—— Instructions ——

- This homework should be submitted via Gradescope.
- There are three main ways you might want to write up your work.
  - Write on this pdf using a tablet
  - Print this worksheet and write in the space provided
  - Write your answers on paper, clearly numbering each question and part.
    - If using either of the last two options, you can use an app such as OfficeLens to take pictures of your work with your phone and convert them into a single pdf file. Gradescope will only allow pdf files to be uploaded.
- **You must show all work**. You may receive zero or reduced points for insufficient work. **Your work must be neatly organised and written**. You may receive zero or reduced points for incoherent work.
- If you are writing your answers on anything other than this sheet, you should only have **one question per page**. You can have parts a), b) and c) on the page for example, but problems 1) and 2) should be on separate pages.
- The problems on this assignment will be **graded on completeness**.
- These problems are designed to be done without a calculator. Whilst there is nothing stopping you using a calculator when working through this assignment, be aware of the fact that you are not permitted to use calculators on exams so you might want to practice without one.

1. Show, using the limit definition of the derivative, that $\frac{d}{dx}(\cos(x)) = -\sin(x)$.

2. Using the limit definition of the derivative, find $f'(x)$ if $f(x) = \sqrt{1-2x}$.

3. The equation of motion of a particle is $s = t^3 - 10t^2 + 24t$, for $t \geq 0$, where s is measured in metres and t is measured in seconds.

   (a) Find the velocity of the particle at time t.

   (b) What is the velocity of the particle after 3 seconds?

   (c) When is the particle at rest?

   (d) When is the particle moving in the positive direction?

   (e) Find the total distance travelled by the particle during the first 8 seconds?

   (f) Find the acceleration of the particle after 3 seconds.

4. Let $f(x) = x^2 + x$. In this question we are going to find the equations of both lines through the point $(2, -3)$ that are tangent to the graph of $f(x)$.

   (a) Find the slope of the line that goes through the points $(a, f(a))$ and $(2, -3)$. (Your answer will be an expression in terms of a).

   (b) Find the slope of the line tangent to $f(x)$ at the point $(a, f(a))$. (Your answer will be an expression in terms of a).

   (c) Use your answers in parts a) and b) to find the values of a for which the tangent lines to $f(x)$ at $(a, a^2 + a)$ go through the point $(2, -3)$.

   (d) For each of the points $(a, f(a))$ found in part c), find the equation of the tangent line to $f(x)$ at $(a, f(a))$.

5. Use the product rule (and maybe the chain rule) to find the derivatives of the following functions.

   (a) $y = x \sin(x)$

   (b) $g = te^t$

   (c) $h(y) = e^y \cos(y)$

   (d) $s(t) = 3t \ln(t)$

   (e) $f(x) = \sin(x) \ln(x)$

   (f) $u(z) = e^z (z^2 + 1)$

   (g) $y = (3 - 4x^2) x^{-1/2}$

   (h) $y = (1 + \sin(\theta))(1 - \sin(\theta))$

   (i) $g(z) = \sqrt{z} \ln(z)$

   (j) $h(x) = (x^2 + 3x)(e^x - 1)(\sqrt{x} - \sqrt[3]{x})$

   (k) $f(\theta) = \csc(\theta) \tan(\theta) \cot(\theta) \sin(\theta)$

6. Use the quotient rule to find the derivatives of the following functions.

(a) $y = \dfrac{x+1}{x-1}$

(b) $g = \dfrac{\sin(t)}{t}$

(c) $h(y) = \dfrac{\sin(y)}{e^y}$

(d) $s(t) = \dfrac{t^2+1}{t+1}$

(e) $f(x) = \dfrac{\cos(x)}{\sqrt{x}}$

(f) $u(z) = \dfrac{\ln(z)}{z}$

(g) $y = \dfrac{x}{\cos(x)}$

(h) $y = \dfrac{\theta + \sin(\theta)}{\cos(\theta)}$

(i) $g(z) = \dfrac{z^2}{1-\sqrt{z}}$

(j) $f(x) = \dfrac{x^2 - 3x + 1}{\sqrt{x}}$