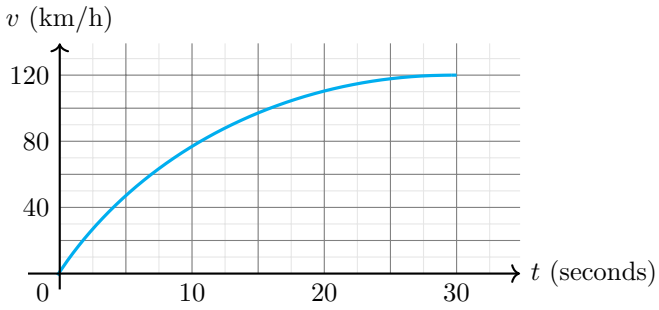
Calculus I

—— Instructions ——

- This homework should be submitted via Gradescope by 23:59 on the date listed above. You can find instructions on how to submit to Gradescope on our Campuswire channel.
- There are three main ways you might want to write up your work.
  - Write on this pdf using a tablet
  - Print this worksheet and write in the space provided
  - Write your answers on paper, clearly numbering each question and part.
    - If using either of the last two options, you can use an app such as OfficeLens to take pictures of your work with your phone and convert them into a single pdf file. Gradescope will only allow pdf files to be uploaded.
- **You must show all work**. You may receive zero or reduced marks for insufficient work. **Your work must be neatly organised and written**. You may receive zero or reduced marks for incoherent work.
- If you are writing your answers on anything other than this sheet, you should only have **one question per page**. You can have parts a), b) and c) on the page for example, but problems 1) and 2) should be on separate pages.
- When uploading to Gradescope, **you must match each question to the page that your answer appears on**. If you do not you will be docked a significant portion of your score.
- **Put a box or circle around your final answer** for each question.
- These problems are designed to be done without a calculator. Whilst there is nothing stopping you using a calculator when working through this assignment, be aware of the fact that you are not permitted to use calculators on exams so you might want to practice without one.
-

Problem 1: The velocity graph of a car accelerating from rest to a speed of 120 km/h over a period of 30 seconds is given below.



(a) Find a lower estimate for the distance travelled by the car during these thirty seconds.

(b) Find an upper estimate for the distance travelled by the car during these thirty seconds.

Problem 2: The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table below.

t (s)	0	0.5	1.0	1.5	2.0	2.5	3.0
v (ft/s)	0	6.2	10.8	14.9	18.1	19.4	20.2

(a) Approximate the distance she travelled during the first three seconds using a left endpoint estimate.

(b) Approximate the distance she travelled during the first three seconds using a right endpoint estimate.

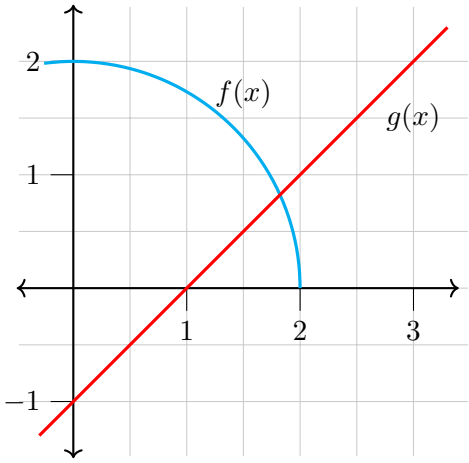
Problem 3: Use a Riemann sum with five equal width subintervals to estimate the area under each of the following curves over the indicated interval, using the indicated endpoints and state whether the estimate is an underestimate or overestimate.

(a) $y = \dfrac{1}{x}$ over $\left[\dfrac{1}{2}, 3\right]$ using left endpoints.

(b) $y = x^3$ over $[0, 1]$ using right endpoints.

(c) $y = x^2$ over $[1, 3]$ using midpoints.

Problem 4: Consider the two functions graphed below.



(a) Find $\int_0^2 f(x)\,dx$

(b) Find $\int_0^3 g(x)\,dx$

Problem 5: Suppose that $f(x)$ and $g(x)$ are integrable and that

$$\int_1^2 f(x)\,dx = -4, \qquad \int_1^5 f(x)\,dx = 6, \qquad \int_1^5 g(x) = 8.$$

Use the properties of definite integrals to calculate the following.

(a) $\displaystyle\int_1^2 3f(x)\,dx$

(b) $\displaystyle\int_1^5 f(x) - g(x)\,dx$

(c) $\displaystyle\int_1^5 4f(x) - g(x)\,dx$

(d) $\displaystyle\int_5^1 g(x)\,dx$

(e) $\displaystyle\int_2^5 f(x)\,dx$

(f) $\displaystyle\int_2^2 g(x)\,dx$