Calculus I

—— Instructions ——

- This homework should be submitted via Gradescope by 23:59 on the date listed above. You can find instructions on how to submit to Gradescope on our Campuswire channel.
- There are three main ways you might want to write up your work.
  - Write on this pdf using a tablet
  - Print this worksheet and write in the space provided
  - Write your answers on paper, clearly numbering each question and part.
    - If using either of the last two options, you can use an app such as OfficeLens to take pictures of your work with your phone and convert them into a single pdf file. Gradescope will only allow pdf files to be uploaded.
- **You must show all work**. You may receive zero or reduced marks for insufficient work. **Your work must be neatly organised and written**. You may receive zero or reduced marks for incoherent work.
- If you are writing your answers on anything other than this sheet, you should only have **one question per page**. You can have parts a), b) and c) on the page for example, but problems 1) and 2) should be on separate pages.
- When uploading to Gradescope, **you must match each question to the page that your answer appears on**. If you do not you will be docked a significant portion of your score.
- **Put a box or circle around your final answer** for each question.
- These problems are designed to be done without a calculator. Whilst there is nothing stopping you using a calculator when working through this assignment, be aware of the fact that you are not permitted to use calculators on exams so you might want to practice without one.

Problem 1: For the following functions, find all values of c on the given interval that satisfy the conclusion of the Mean Value Theorem.

(a) $f(x) = x^{2/3}$ on $[0, 1]$

(b) $g(x) = \sqrt{x-1}$ on $[1, 3]$

(c) $s(t) = t^3 - t^2$ on $[-1, 2]$

(d) $w(z) = \ln(z - 1)$ on $[2, 4]$

Problem 2: In each of the following, you are given the function, $f(x)$, its derivative, $f'(x)$ and its second derivative, $f''(x)$. For each of these, do the following.

i) Find the domain of $f(x)$.

ii) Find the zeros of $f(x)$.

iii) Find the vertical asymptotes of $f(x)$, if it has any.

iv) Find the horizontal asymptotes of $f(x)$, if it has any.

v) Find the zeros of $f'(x)$.

vi) Find the interval(s) where $f(x)$ is increasing.

vii) Find the interval(s) where $f(x)$ is decreasing.

viii) Find all local maximum and minimum values of $f(x)$

ix) Find the zeros of $f''(x)$.

x) Find the interval(s) where $f(x)$ is concave up.

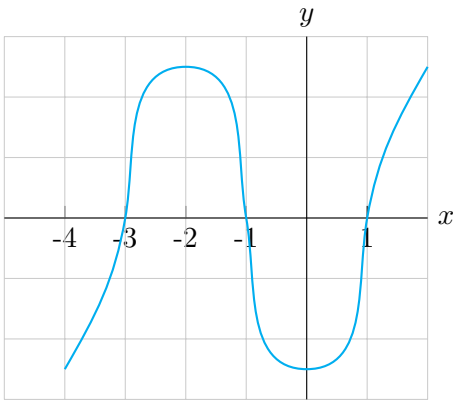
xi) Find the interval(s) where $f(x)$ is concave down.

xii) Find all points of inflection of $f(x)$.

(a) $f(x) = (x+4)^3(x-3), \quad f'(x) = (x+4)^2(4x-5), \quad f''(x) = 6(x+4)(2x+1)$

(b) $f(x) = \dfrac{e^x}{x^2}, \quad f'(x) = \dfrac{e^x\,(x-2)}{x^3}, \quad f''(x) = \dfrac{e^x\left(x^2-4x+6\right)}{x^4}$

Problem 3: The graph of $f'(x)$ is given below.



(a) Find the interval(s) where $f(x)$ is increasing.

(b) Find the interval(s) where $f(x)$ is decreasing.

(c) Find the x-coordinate(s) where $f(x)$ achieves a local maximum.

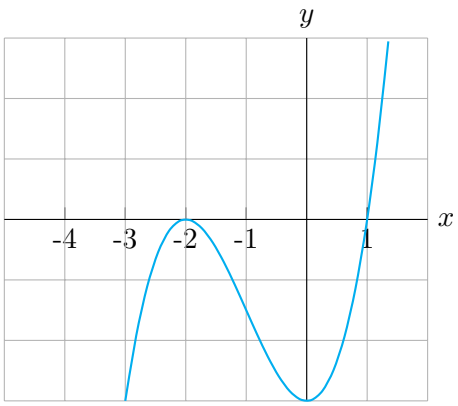
(d) Find the x-coordinate(s) where $f(x)$ achieves a local minimum.

(e) Find the interval(s) where $f(x)$ is concave up.

(f) Find the interval(s) where $f(x)$ is concave down.

(g) Find the x-coordinate(s) of any inflection points that $f(x)$ has.

Problem 4: The graph of $f(x)$ is given below.



(a) Find the interval(s) where $f(x)$ is increasing.

(b) Find the interval(s) where $f(x)$ is decreasing.

(c) Find the x-coordinate(s) where $f(x)$ achieves a local maximum.

(d) Find the x-coordinate(s) where $f(x)$ achieves a local minimum.

(e) Find the interval(s) where $f(x)$ is concave up.

(f) Find the interval(s) where $f(x)$ is concave down.

(g) Find the x-coordinate(s) of any inflection points that $f(x)$ has.

Problem 5: Sketch the graph of a function $f(x)$ which has *all* of the following properties.

(1) $\lim_{x\to 2^-} f(x) = -\infty$

(2) $\lim_{x\to 2^+} f(x) = \infty$

(3) $\lim_{x\to -\infty} f(x) = 0$

(4) $f(-2) = 2$

(5) $f(5) = 1$

(6) $f(0) = 0$

(7) $f'(x) > 0$ if $x < -2$ or $x > 5$

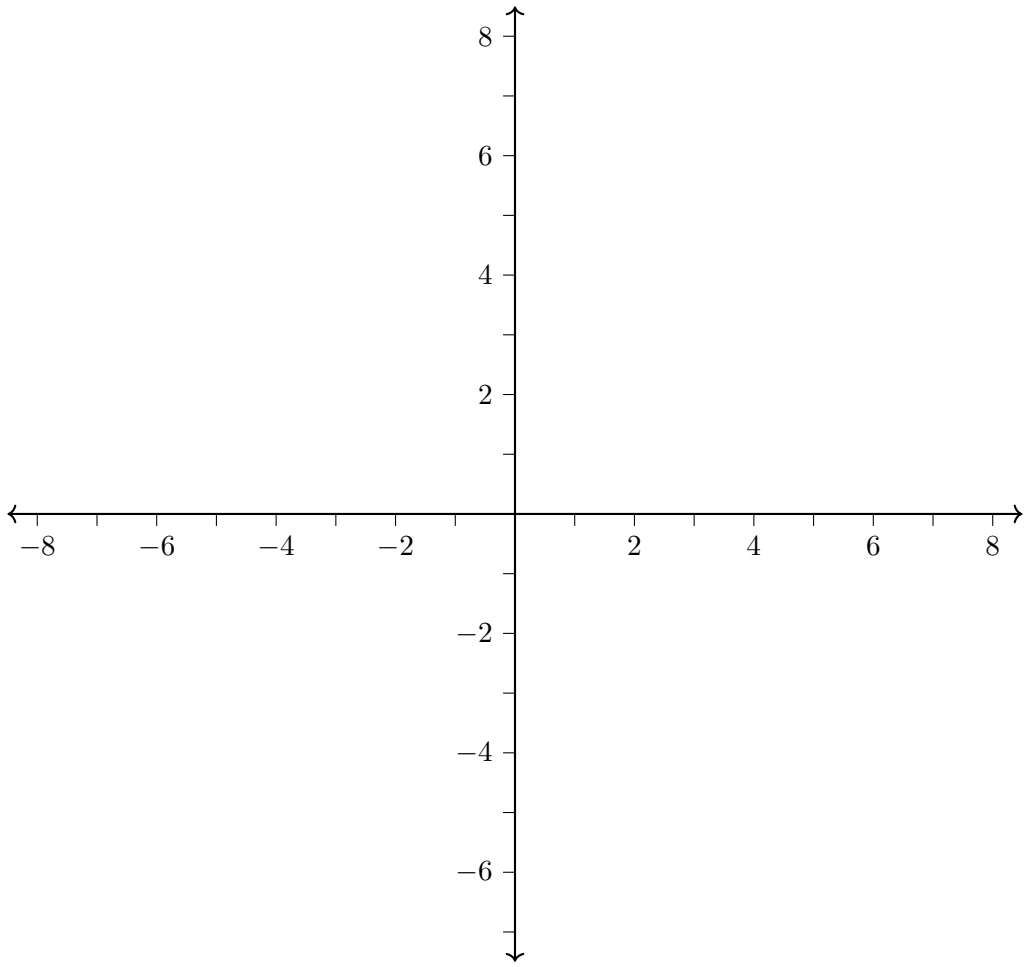
(8) $f'(x) < 0$ if $-2 < x < 2$ or $2 < x < 5$

(9) $f'(5) = 0$

(10) $f'(-2) = 0$

(11) $f''(x) > 0$ if $x < -3$ or $x > 2$

(12) $f''(x) < 0$ if $-3 < x < 2$



Problem 6: Sketch the graph of a function $f(x)$ which has *all* of the following properties.

(i) $f(x)$ has domain $(-\infty, -2) \cup (-2, \infty)$

(ii) $f(x)$ has range $(-5, \infty)$

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(iii) The graph of $f(x)$ has a vertical asymptote at $x = -2$

(iv) $\lim_{x \to -\infty} f(x) = 2$

(v) $\lim_{x \to \infty} f(x) = -5$

(vi) $\lim_{x \to 3} f(x) = 2$

(vii) $f(x)$ is discontinuous at $x = 3$

(viii) $f'(x) > 0$ on $(-\infty, -2)$

(ix) $f'(x) < 0$ on $(-2, 3)$

(x) $f'(4)$ is not defined, but $f(x)$ is continuous at 4