Calculus I

The Fundamental Theorem of Calculus

1. Use the Fundamental Theorem of Calculus to compute the following, without integrating anything.

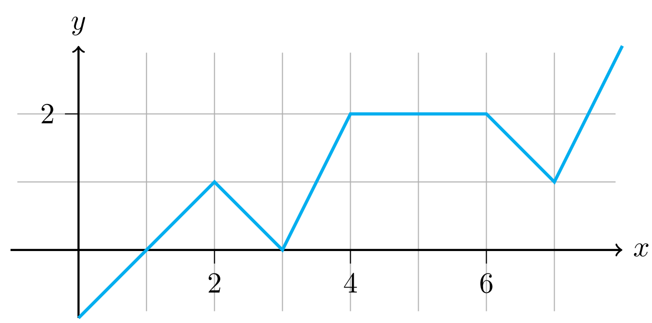
   a) $\dfrac{d}{dx}\displaystyle\int_0^x \sqrt{1+t^2}\,dt$ b) $\dfrac{d}{dy}\displaystyle\int_1^y 3x^2\,dx$ c) $\dfrac{d}{dz}\displaystyle\int_z^5 \sin\left(y^2\right)\,dy$

   d) $\dfrac{d}{dw}\displaystyle\int_w^{-2} \sec\left(z^3\right)\,dz$ e) $\dfrac{d}{dv}\displaystyle\int_7^{v^2} \ln\left(w^2+1\right)\,dw$ f) $\dfrac{d}{du}\displaystyle\int_3^{u^3+u} \tan\left(v\right)\,dv$

   g) $\dfrac{d}{ds}\displaystyle\int_{\sqrt{s}}^6 \frac{u^2}{u^2+4}\,du$ h) $\dfrac{d}{dr}\displaystyle\int_{\cos(r)}^{\sin(r)} e^{s^2}\,ds$ i) $\dfrac{d}{dt}\displaystyle\int_{\sqrt{t}}^{t^2} \sin^{-1}\left(r\right)\,dr$

   The next three problems are taken from the textbook section 5.4.

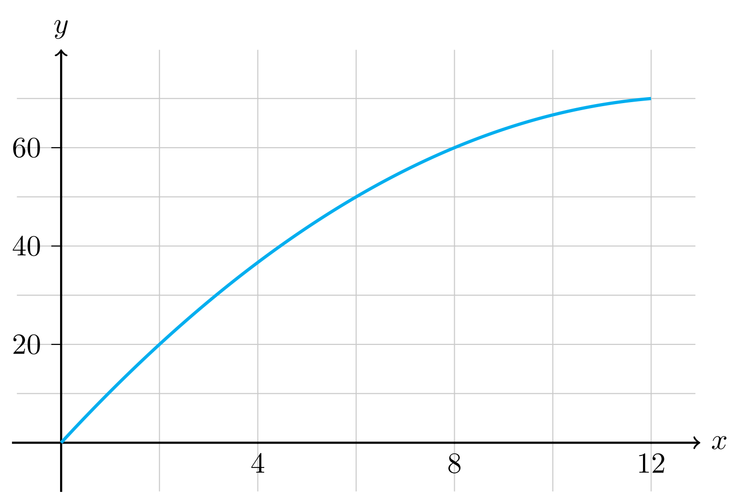
2. Find the average value of f, graphed on the right, on $[0, 8]$.



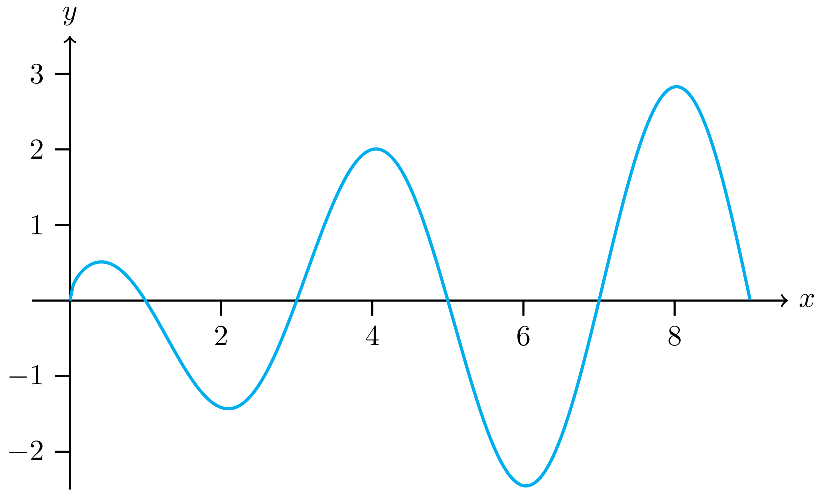
3. The velocity graph of an accelerating car is shown on the right

   a) Estimate the average velocity of the car during the first 12 seconds.

   b) Approximately at what time was the instantaneous velocity equal to the average velocity?



4. Let $g(x) = \int_0^x f(t)\, dt$, where f is the function whose graph is shown.



a) At what values of x do the local maximum and minimum values of g occur.

b) Where does g attain its absolute maximum value?

c) On what intervals is g concave downward?

d) Sketch the graph of g.

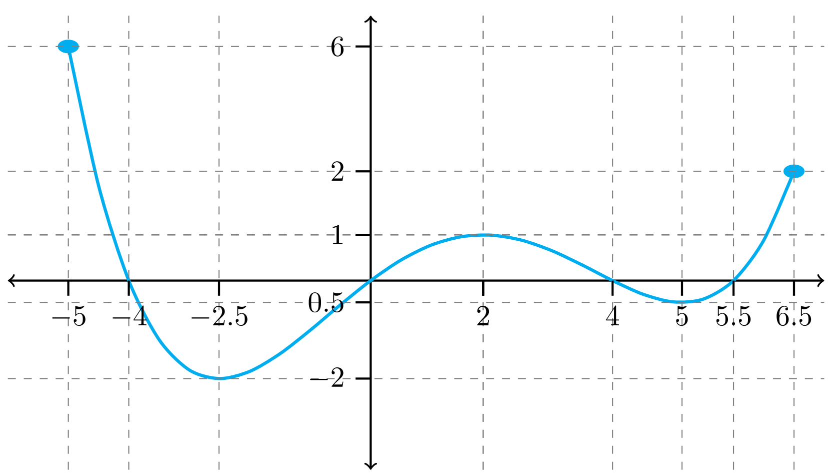
5. The graph of $f(t)$, defined on the interval $[-5, 6.5]$, is given. Define a function by $h(x) = \int_{-5}^x f(t)\, dt$ for $-5 \le x \le 6.5$.

a) Determine the interval(s) where $h(x)$ is increasing.

b) Determine the critical points of $h(x)$.

c) Find all local maximum points.

d) Determine the interval(s) where $h(x)$ is concave down.



Calculus I

Fall 2023

Evaluating Definite Integrals

1. Compute the following integrals.

   a) $\displaystyle\int_{-1}^{2} 3x^2 - 2x + 1\,dx$ b) $\displaystyle\int_{-1}^{0} x - x^2\,dx$ c) $\displaystyle\int 2\sec^2(\theta)\,d\theta$

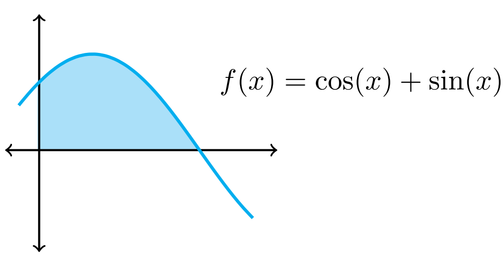
   d) $\displaystyle\int_{0}^{1} 3x^2 + x - 5\,dx$ e) $\displaystyle\int (y+1)^2\;dy$ f) $\displaystyle\int_{0}^{1/2} \frac{4}{\sqrt{1-x^2}}\,dx$

   g) $\displaystyle\int_{-\pi/3}^{-\pi/4} 4\sec^2(\theta) + \frac{\pi}{\theta^2}\,d\theta$ h) $\displaystyle\int \frac{z^5 - 2z}{z^3}$ i) $\displaystyle\int \frac{3}{1+v^2} - \csc^2(v)\;dv$

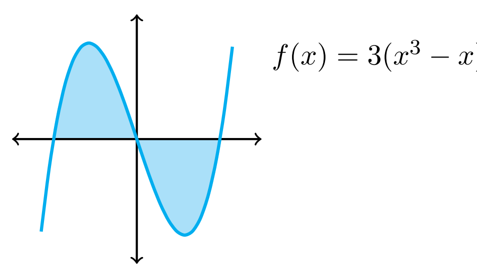
   j) $\displaystyle\int_{1}^{\sqrt{2}} \frac{t^2 + \sqrt{t}}{t^2}\,dt$ k) $\displaystyle\int \frac{\left(x^{1/3}+1\right)\left(2 - x^{2/3}\right)}{x^{1/3}}\,dx$ l) $\displaystyle\int_{0}^{\pi/3} \left(\sec(\theta) + \tan(\theta)\right)^2\;d\theta$

2. Find the area of the indicated shaded regions.

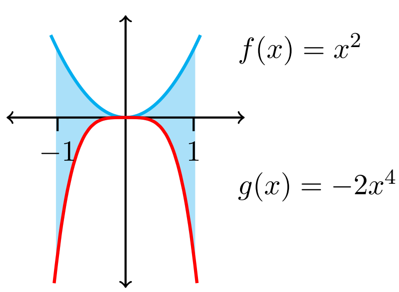
   a)



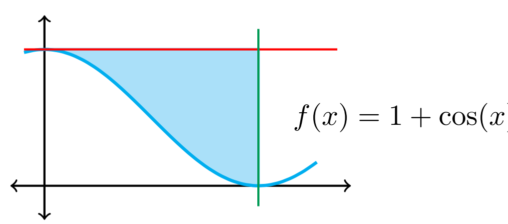
   b)



   c)



   d)



3. A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week. What does $100 + \int_0^{15} n'(t)\,dt$ represent?

4. If $f(x)$ is the slope of a trail a distance of x miles from the start of the trail, what does $\int_3^5 f(x)\,dx$ represent?

5. The acceleration function (in $\mathrm{m/s^2}$) and the initial velocity are given for a particle moving along a line. Find the

(i) velocity at time t

(ii) distance travelled during the given time interval.

a) $a(t) = t + 4$, $v(0) = 5$, $0 \le t \le 10$

b) $a(t) = 2t + 3$, $v(0) = -4$, $0 \le t \le 3$

6. Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ litres per minute, where $0 \le t \le 50$. Find the amount of water that flows from the tank during the first 10 minutes.