Inverse Functions, Inverse Trigonometric Functions, and the Exponential and Logarithm

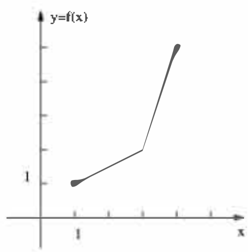
1. Let $f(x) = 2 + \frac{1}{x+3}$. Determine the inverse function of f, f^{-1}. Give the domain and range of f and the inverse function f^{-1}.

2. Solve $10^{2x+1} = 100$.

3. Suppose a and b are positive real numbers and $\ln(ab) = 3$ and $\ln(ab^2) = 5$. Find $\ln(a)$, $\ln(b)$, and $\ln(a^3/\sqrt{b})$.

4. Consider the function $f(x) = 1 + \ln(x)$. Determine the inverse function of f. Give the domain and range of f and of the inverse function f^{-1}.

5. Consider the function whose graph appears below.



   (a) Find $f(3)$, $f^{-1}(2)$ and $f^{-1}(f(2))$.
   (b) Give the domain and range of f and of f^{-1}.
   (c) Sketch the graph of f^{-1}.

6. Find the exact values of the following expressions. Do not use a calculator.

   (a) $\tan^{-1}(1)$
   (b) $\tan(\tan^{-1}(10))$
   (c) $\sin^{-1}(\sin(7\pi/3))$
   (d) $\tan(\sin^{-1}(0.8))$

7. Give a simple expression for $\sin(\cos^{-1}(x))$.

8. Let f be the function with domain $[\pi/2, 3\pi/2]$ with $f(x) = \sin(x)$ for x in $[\pi/2, 3\pi/2]$. Since f is one to one, we may let g be the inverse function of f. Give the domain and range of g. Find $g^{-1}(1/2)$.

9. True or False:

   (a) Every function has an inverse.
   (b) If $f \circ g(x) = x$ for all x in the domain of g, then f is the inverse of g.
   (c) If $f \circ g(x) = x$ for all x in the domain of g and $g \circ f(x) = x$ for all x in the domain of f, then f is the inverse of g.
   (d) If $f(x) = 1/(x+2)^3$ and g is the inverse function of f, then $g(x) = (x+2)^3$.
   (e) The function $f(x) = \sin(x)$ is one to one.
   (f) The function $f(x) = 1/(x+2)^3$ is one to one.

10. Let f be a linear function with slope m with $m \neq 0$. What is the slope of the inverse function f^{-1}.

Limits: A Numerical and Graphical Approach, the Limit Laws

1. Comprehension check:

   (a) In words, describe what "$\lim_{x\to a} f(x) = L$" means.

   (b) In words, what does "$\lim_{x\to a} f(x) = \infty$" mean?

   (c) Suppose $\lim_{x\to 1} f(x) = 2$. Does $f(1) = 2$?

   (d) Suppose $f(1) = 2$. Does $\lim_{x\to 1} f(x) = 2$?

2. Compute the value of the following functions near the given $x-$value. Use this information to guess the value of the limit of the function (if it exists) as x approaches the given value.

   (a) $f(x) = \frac{4x^2-9}{2x-3}$, $x = \frac{3}{2}$

   (b) $f(x) = \frac{x}{|x|}$, $x = 0$

   (c) $f(x) = \frac{\sin(2x)}{x}$, $x = 0$

   (d) $f(x) = \sin(\pi/x)$, $x = 0$

3. Let $f(x) = \begin{cases} x^2 & \text{if } x \le 0 \\ x-1 & \text{if } 0 < x \text{ and } x \ne 2 \\ -3 & \text{if } x = 2 \end{cases}$.

   (a) Sketch the graph of f.

   (b) Compute the following:

      i. $\lim_{x\to 0^-} f(x)$

      ii. $\lim_{x\to 0^+} f(x)$

      iii. $\lim_{x\to 0} f(x)$

      iv. $f(0)$

      v. $\lim_{x\to 2^-} f(x)$

      vi. $\lim_{x\to 2^+} f(x)$

      vii. $\lim_{x\to 2} f(x)$

      viii. $f(2)$

4. Compute the following limits or explain why they fail to exist:

   (a) $\displaystyle\lim_{x\to -3^+} \frac{x+2}{x+3}$

   (b) $\displaystyle\lim_{x\to -3^-} \frac{x+2}{x+3}$

   (c) $\displaystyle\lim_{x\to -3} \frac{x+2}{x+3}$

   (d) $\displaystyle\lim_{x\to 0^-} \frac{1}{x^3}$

5. Let $f(x) = \begin{cases} 2x+2 & \text{if } x > -2 \\ a & \text{if } x = -2 \\ kx & \text{if } x < -2 \end{cases}$. Find k and a so that $\lim_{x\to -2} f(x) = f(-2)$.

6. Given $\lim_{x\to 2} f(x) = 5$ and $\lim_{x\to 2} g(x) = 2$, use limit laws to compute the following limits or explain why we cannot find the limit. Note when working through a limit problem that your answers should be a chain of true equalities. Make sure to keep the $\lim_{x\to a}$ operator until the very last step.

   (a) $\lim_{x\to 2}(2f(x) - g(x))$

   (b) $\lim_{x\to 2}(f(x)g(2))$

   (c) $\displaystyle\lim_{x\to 2} \frac{f(x)g(x)}{x}$

   (d) $\lim_{x\to 2} f(x)^2 + x \cdot g(x)^2$

   (e) $\lim_{x\to 2}[f(x)]^{\frac{3}{2}}$

   (f) $\displaystyle\lim_{x\to 2} \frac{f(x)-5}{g(x)-2}$