M408C Worksheet 1: Functions

Name: _Solutions_

Exercise 1. *Functions and equations.*

(a) What is a function? (Write down the definition of a function)

A function is a machine which maps inputs to outputs. Each input prod exactly one output.

(b) What is an equation? (Write down the definition of an equation)

An equation is a statement equating two mathematical expressions w an =: sign. It can be either true or false.

(c) Give an example of an equation whose graph is NOT the graph of a function. Write an equation AND sketch its graph.

$$y^2 = x^3 + 3x^2 + 2$$
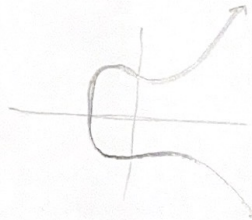
(d) If $f : \mathbb{R} \to \mathbb{R}$ is a function, can you always write an equation involving x and y whose graph is the exactly the graph of f? If yes, demonstrate how – if no, explain why not.

Yes, the "graph" of the function f is exactly the solution set of the equation $y = f(x)$.

Exercise 2. *Even and odd functions.* Let $f : \mathbb{R} \to \mathbb{R}$ be a function.

NOTE: *If you don't know the definition of an even or odd function in the following problem, copy it from the Wikipedia page titled, "Even and odd functions".*

(a) $f(x)$ is **even** if $\underline{f(x) = f(-x)}$. An example of an even function is $f(x) = \underline{0}$ (or x^2, or $|x|$, et

(b) $g(x)$ is **odd** if $\underline{f(x) = -f(-x)}$. An example of an odd function is $g(x) = \underline{0}$ (or x, or $-x^3$, et

(c) Define the **absolute value function** as a **piecewise function**:

$$|x| = \begin{cases} \underline{x} & \text{if } \underline{x \geq 0} \\ \underline{-x} & \text{if } \underline{x < 0} \end{cases}$$

Show that $|x|$ is an even function using the definition of an even function.

Case 1: $x > 0$. Then $-x < 0$ so $|-x| = -(-x) = x = |x|$.

Case 2: $x < 0$. Then $-x > 0$, so $|-x| = -x = |x|$.

Case 3: $x = 0$. Then $-x = 0$ so $|-x| = 0 = |x|$.

In all cases, $|x| = |-x|$. Thus $x \longmapsto |x|$ is even.