TEXAS
The University of Texas at Austin

M408C EXAM 1-VERSION B OCTOBER 26

Print Name:	Signature:
UTEID #:	

Directions: You have **50 minutes** to answer the following questions. ***You must show all your work*** as neatly and clearly as possible and indicate the final answer clearly. Cell phones and other electronic devices may **NOT** be used during the exam.

Free Response

Show all work and justification.

1. (12 points) A longhorn starts at PMA and moves in a straight line so that at any time time t in $[0, 4]$ her position away PMA in feet is given by $s(t) = (4/3)t^3 - 7t^2 + 6t$ where t is in minutes.

   (a) (4 points) Determine if her velocity is increasing or decreasing at $t = \frac{1}{4}$ min.

   $v(t) = s'(t) = 4t^2 - 14t + 6$

   $a(t) = v'(t) = 8t - 14$

   $a(1/4) = \frac{8}{4} - 14 = -12 < 0$

   $v(t)$ is decreasing

   (b) (4 points) For what value(s) of t is the longhorn at rest?

   $v(t) = 0$

   $2(2t^2 - 7t + 3)$

   $2(2t - 1)(t - 3)$

   $t = 1/2, \quad t = 3$

   (c) (4 points) What are the ~~intervals of concavity for~~ $s(t)$? What (if any) are the points of inflection?

   no need for this exam

2. (12 points) Consider the equation $y^3 - xy = 2$

(a) (4 points) Find $\frac{dy}{dx}$

$$3y^2 \frac{dy}{dx} - \left[y + x\frac{dy}{dx}\right] = 0$$

$$(3y^2 - x)\frac{dy}{dx} = y$$

$$\frac{dy}{dx} = \frac{y}{3y^2 - x}$$

(b) (4 points) Find points on the curve where there are vertical tangents.

$3y^2 - x = 0 \quad (y \neq 0)$

$3y^2 = x \longrightarrow$ plug into original

$y^3 - 3y^3 = 2$

$-2y^3 = 2$

$y^3 = -1$

$$\boxed{\begin{array}{l} y = -1 \\ x = 3 \end{array}}$$

(c) (4 points) Find $\frac{d^2y}{dx^2}$ at $y = 1$

$$\frac{d^2y}{dx^2} = \frac{\frac{dy}{dx}(3y^2 - x) - y\left(6y\frac{dy}{dx} - 1\right)}{(3y^2 - x)^2}$$

When $y = 1$, $1 - x = 2$, $x = -1$

$$\left.\frac{dy}{dx}\right|_{y=1} = \frac{1}{3+1} = \frac{1}{4}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{1}{4}(3+1) - 1\left(6\cdot\frac{1}{4} - 1\right)}{} = \frac{1 - \frac{1}{2}}{} = \frac{1}{32}$$

$(3+1)^2$ $\quad$ 16

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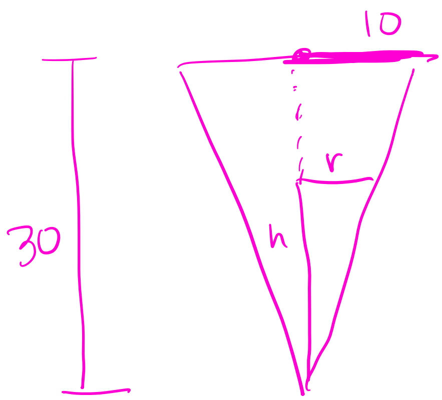
3. (8 points) A funnel in the shape of an inverted cone is 30 cm deep and has a diameter across the top of 20 cm. Liquid is flowing out of the funnel at the rate of $12cm^3$ per second. At what rate is the height of the liquid decreasing at the instant when the liquid in the funnel is 20 cm deep?



$$\frac{h}{r} = \frac{30}{10} = 3$$

$$\frac{dV}{dt} = -12$$

$$\frac{h}{3} = r$$

$$\boxed{h = 20}$$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 \cdot h$$

$$= \frac{\pi}{27} h^3$$

$$\frac{dV}{dt} = \frac{3\pi}{27} h^2 \cdot \frac{dh}{dt}$$

$$\frac{\pi}{9}(20)^2(-12)$$

4. (12 points)

(a) (8 points) Find the difference in the absolute maximum and the absolute minimum value of $f(x) = \sqrt{2}\cos(x) - \sin^2(x)$ on $[0, \pi]$

x	$f(x)$	
0	$\sqrt{2}$	max
π	$-\sqrt{2} - 0$	
$\frac{3\pi}{4}$	$-\frac{\sqrt{2}\sqrt{2}}{2} - \left(\frac{\sqrt{2}}{2}\right)^2 =$ $-1 - \frac{2}{4} = -\frac{3}{2}$	min

$f'(x) = -\sqrt{2}\sin x - 2\sin x\cos x = 0$

$\sin x(-\sqrt{2} - 2\cos x) = 0$

$\sin x = 0$ $\quad$ $-\sqrt{2} = 2\cos x$

$x = 0, \pi$ $\quad$ $-\frac{\sqrt{2}}{2} = \cos x$

$x = 3\pi/4$

Diff $\quad \sqrt{2} + \frac{3}{2}$

(b) (4 points) Find $\dfrac{dy}{dx}$ for $y = (\sin x)^x$

$\ln y = x\ln(\sin x)$

$\frac{1}{y}\frac{dy}{dx} = \ln(\sin x) + x \cdot \frac{\cos x}{\sin x}$

$\frac{dy}{dx} = y\left[\ln(\sin x) + x\cot x\right]$

$(\sin x)^x\left[\ln(\sin x) + x\cot x\right]$

Multiple Choice

(2 points each) Select the correct option. You need not explain your answer. No partial credit will be given.

Use this chart for both 1 and 2. Consider two functions $f(x)$ and $g(x)$. **We do not know the formulas for f and g, but we know that f and g are continuous and differentiable everywhere.** Furthermore, we have the following table of values:

x	0	4	5	6
$f(x)$	6	2	3	3
$g(x)$	4	6	3	9
$f'(x)$	2	1	4	5
$g'(x)$	5	3	2	4

1. If $h(x) = \sqrt{7 + [g(x)]^2}$, find $h'(5)$

   (A) $\frac{6}{7}$

   $h'(x) = \cancel{\tfrac{1}{2}}(7+(g(x))^2)^{-1/2}\,\cancel{2}\,g(x)\cdot g'(x)$

   (B) $\frac{3}{2}$

   $\frac{g(5)g'(5)}{\sqrt{7+g(5)^2}} = \frac{3\cdot 2}{\sqrt{7+9}} = \frac{6}{4} = 3/2$

   (C) $\frac{18}{7}$

   (D) 126

   (E) There is not enough information.

2. If $h(x) = \dfrac{x + g(x)}{f(x) - g(x)}$, find $h'(0)$

   $h'(x) = \dfrac{(1+g'(x))[f(x)-g(x)] - (x+g(x))(f'-g')}{(f-g)^2}$

   $h'(0) = \dfrac{(1+g'(0))[f(0)-g(0)] - (0+g(0))(f'(0)-g'(0))}{(f(0)-g(0))^2}$

   $= \dfrac{(1+5)(6-4) - (4)(2-5)}{(6-4)^2}$

   (A) $\frac{8}{19}$

   (B) $\frac{9}{26}$

   (C) $\frac{27}{4}$

   (D) $\frac{1}{8}$

   (E) none of the above.

   $\frac{6(2)+12}{4} = 6$

3. Calculate $f'(\ln(6))$ when

$$f(x) = \ln(\sqrt{3 + 2e^x})$$

(A) $\frac{8}{19}$

(B) $\frac{9}{26}$

(C) $\frac{2}{5}$

(D) none of the above.

$$\frac{1}{\sqrt{3+2e^x}} \cdot \cancel{\tfrac{1}{2}}(3+2e^x)^{-1/2}(\cancel{2}e^x)$$

$$\frac{e^x}{3+2e^x} \qquad \frac{6}{3+2\cdot 6} = \frac{6}{15}$$

4. For the function f, if $f(5) = -3$, $f'(x) = 2x - 4$, what is the approximation of $f(5.3)$ using linearization?

(A) -1.8

(B) -1.2

(C) -1

(D) 4.2

(E) 6.4

$$L = f(5) + f'(5)(x-5)$$

$$L = -3 + (6)(x-5)$$

$$L(5.3) = -3 + 6(.3)$$

$$= -3 + 1.8 = -1.2$$

5. If $x = 5$ is in the domain of f and $f'(x) > 0$ on $(0, 5)$, $f'(x) < 0$ on $(5, 6)$ and $f''(x) > 0$ on $(0, 6)$ then which of the following is true:

(I) f has a local max at $x = 5$

(II) f has a local min at $x = 5$

(III) f has an inflection point at $x = 5$

(IV) f does not have an inflection point at $x = 5$

(A) II only

(B) I and III

(C) I and IV

(D) I only

(E) II and IV