Calculus I

—— Instructions ——

- This homework should be submitted via Gradescope by 23:59 on the date listed above. You can find instructions on how to submit to Gradescope on our Campuswire channel.
- There are three main ways you might want to write up your work.
  - Write on this pdf using a tablet
  - Print this worksheet and write in the space provided
  - Write your answers on paper, clearly numbering each question and part.
    - If using either of the last two options, you can use an app such as OfficeLens to take pictures of your work with your phone and convert them into a single pdf file. Gradescope will only allow pdf files to be uploaded.
- **You must show all work**. You may receive zero or reduced marks for insufficient work. **Your work must be neatly organised and written**. You may receive zero or reduced marks for incoherent work.
- If you are writing your answers on anything other than this sheet, you should only have **one question per page**. You can have parts a), b) and c) on the page for example, but problems 1) and 2) should be on separate pages.
- When uploading to Gradescope, **you must match each question to the page that your answer appears on**. If you do not you will be docked a significant portion of your score.
- **Put a box or circle around your inal answer** for each question.
- These problems are designed to be done without a calculator. Whilst there is nothing stopping you using a calculator when working through this assignment, be aware of the fact that you are not permitted to use calculators on exams so you might want to practice without one.
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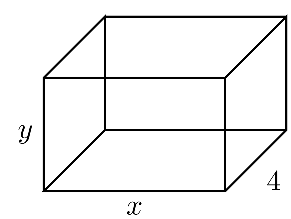
Problem 1: A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with maximum volume?

Problem 2: A tank with a rectangular base and rectangular sides is open at the top. It is to be constructed so that its width is 4 metres and its volume is 36 cubic metres. If building the tank costs \$10/m^2 for the base and \$5/m^2 for the sides, what is the cost of the least expensive tank and what are its dimensions?
Hints:

- First, convince yourself that the surface area of the box, S, is given by $S = 4x + 2xy + 8y$.
- Then, convince yourself that the cost of making the box, C, is given by $C = 40x + 10xy + 40y$.
- Use C and a constraint equation to solve the problem.



Problem 3: A cylindrical metal container, open at the top, is to have a capacity of 24π in^3. The cost of material for the bottom of the container is \$0.15/in^2 and the cost of the material used for the curved part is \$0.05/in^2. Find the dimensions that will minimise the cost of the material and find the minimum cost.

Hint: Like the previous problem, first find an expression for the surface area and then use it to find the expression for the cost of materials.

Problem 4: Find an antiderivative for each of the following functions. Remember, you can check your answer by differentiating it.

(a) $f(x) = x^{-4} + 2x + 3$

(b) $g(t) = \sqrt{t} - \dfrac{1}{\sqrt[3]{t}}$

(c) $h(z) = \dfrac{1}{z} + 5\sin(z)$

(d) $w(x) = \dfrac{2}{3}\sec^2\left(\dfrac{x}{3}\right)$

Problem 5: Find a function f that satisfies the given criteria.

(a) $f''(x) = 20x^3 - 12x^2 + 6x$

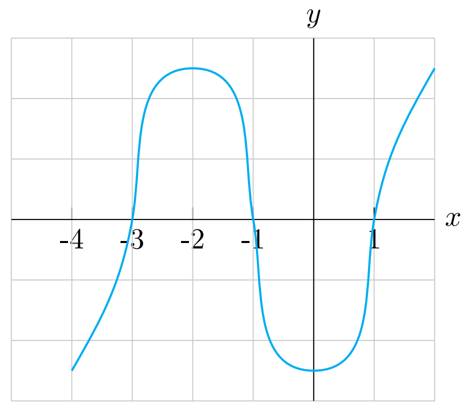
(b) $f'(t) = \dfrac{4}{1+t^2}, \quad f(1) = 0$

(c) $f''(y) = 8y^3 + 5, \quad f(1) = 0, \quad f'(1) = 8$

Problem 6: A particle moves in a straight line and has acceleration given by $a(t) = 4t - 1$. Its initial velocity is $v(0) = -3\,\text{cm/s}$ and it's initial displacement is $s(0) = 4\,\text{cm}$. Find its position function $s(t)$.

Problem 7: A stone was dropped off a cliff and hit the ground with a speed of $120\,\text{ft/s}$. What is the height of the cliff? Assume that there is no air resistance and that the acceleration of the ball is $a(t) = -32\,\text{ft/s}^2$.

Problem 8: The graph of $f'(x)$ is given below.



(a) Find the interval(s) where $f(x)$ is increasing.

(b) Find the interval(s) where $f(x)$ is decreasing.

(c) Find the x-coordinate(s) where $f(x)$ achieves a local maximum.

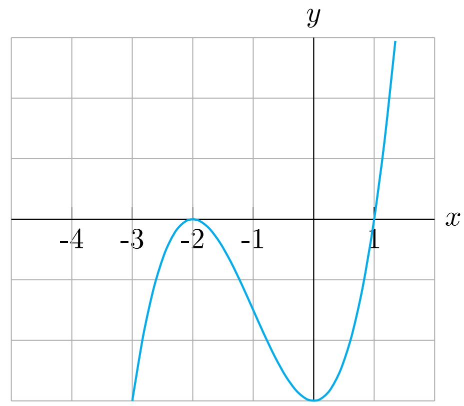
(d) Find the x-coordinate(s) where $f(x)$ achieves a local minimum.

(e) Find the interval(s) where $f(x)$ is concave up.

(f) Find the interval(s) where $f(x)$ is concave down.

(g) Find the x-coordinate(s) of any inflection points that $f(x)$ has.

Problem 9: The graph of $f(x)$ is given below.



(a) Find the interval(s) where $f(x)$ is increasing.

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(d) Find the x-coordinate(s) where $f(x)$ achieves a local minimum.

(e) Find the interval(s) where $f(x)$ is concave up.

(f) Find the interval(s) where $f(x)$ is concave down.

(g) Find the x-coordinate(s) of any inflection points that $f(x)$ has.