Sporadic Notes Taken While at LPS Probabilistic Computing

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§ *Entry 1*

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1 Reverse Ising Model Description

The Ising model is a model of ferromagnetism. We care more about the model itself and less about its physical properties, so here we provide an abstraction.

Definition 1.1. Suppose you are given an undirected graph G with vertices and edges decorated as follows.

- Vertex i receives a *spin* $s_i \in \{\pm 1\}$ and a *local bias* $h_i \in \mathbb{R}$. The former controls the direct of a vertex's intrinsic contribution to the energy of the system while the second controls the weight of the contribution.
- Edge e_{ij} is given a *coupling strength* $J_{ij} \in \mathbb{R}$. If edge e_{ij} does not exist then $J_{ij} = 0$ by convention.

Call such a graph an *Ising graph* and call the verticies *sites*. The total energy of the system is given by the Hamiltonian

$$H(G) = \sum_{i \in V} h_i s_i \; + \; \sum_{i,j \in V} J_{ij} s_j.$$

The **forward Ising problem** takes as input the values of the local biases $\{h_i\}$ and the coupling strengths $\{J_{ij}\}$ and then outputs the (or rather, "a") tuple $(s_1, ..., s_n)$ of spins which minimize the Hamiltonian.

The **reverse Ising problem** takes as input a tuple of spins and then outputs the set of h_i and J_{ij} which minimize the Hamiltonian, if possible. It is of interest as a slight modification of this problem would allow for the construction of logic gates from Ising graphs, in which a spin value of 1 and -1 represent true and false respectively.

Example 1.2.