M408C Worksheet 1: Functions

Name: _____

Exercise 1. *Functions and equations.*

(a) What is a function? (Write down the definition of a function)

(b) What is an equation? (Write down the definition of an equation)

(c) Give an example of an equation whose graph is NOT the graph of a function. Write an equation AND sketch its graph.

(d) If $f : \mathbb{R} \to \mathbb{R}$ is a function, can you always write an equation involving x and y whose graph is the exactly the graph of f? If yes, demonstrate how – if no, explain why not.

Exercise 2. *Even and odd functions.* Let $f : \mathbb{R} \to \mathbb{R}$ be a function.

NOTE: If you don't know the definition of an even or odd function in the following problem, copy it from the Wikipedia page titled, "Even and odd functions".

(a) $f(x)$ is **even** if _____. An example of an even function is $f(x) = $ _____.

(b) $g(x)$ is **odd** if _____. An example of an odd function is $g(x) = $ _____.

(c) Define the **absolute value function** as a **piecewise function:**

$$|x| = \begin{cases} \underline{\hspace{2cm}} & \text{if} \quad \underline{\hspace{1.5cm}} \\ \underline{\hspace{2cm}} & \text{if} \quad \underline{\hspace{1.5cm}} \end{cases}$$

Show that $|x|$ is an even function using the definition of an even function.

(d) Can a function be neither even nor odd? If yes, give an example. If no, explain why.

Choose ONE of the following questions and respond to it in $\sim$ 3 sentences.

Question 1. What is a number?

Question 2. What is area? Does every closed shape have an area?

Question 3. Let S be some 2-dimensional shape. Is it possible for both of the following statements to be true simultaneously? Why/why not?

- S has an area

- It is impossible for us to calculate the area of S

Question 4. Consider a polynomial p, or for example, $p(x) = x^5 + 20x + 16$. Is it possible for both of the following statements to be true simultaneously? Why/why not?

- There exists a real number $a \in \mathbb{R}$ such that $p(a) = 0$. (We say that a is a **zero** of p.)

- We cannot calculate any of the zeros of p.