Toric Geometry: Example Sheet 1

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§ *Theory Problems*

EXERCISE 1.

EXERCISE 2.

EXERCISE 3.

(a) Show that every $\tau \in \mathfrak{h}$ is conjugate by an element $\gamma \in \mathrm{SL}_2(\mathbb{Z})$ to an element of

$$\mathcal{F}_1 = \{\tau \in \mathfrak{h} \mid -1 \leq \mathrm{Re}(\tau) \leq 0,\ |\tau| \geq 1,\ |\tau+1| \geq 1\}.$$

(b) Define $\Gamma(2)$ to be the kernel of the "reduction modulo 2" homomorphism $\mathrm{SL}_2(\mathbb{Z}) \to \mathrm{SL}_2(\mathbb{Z}/2\mathbb{Z})$, and let $\Gamma \leq \mathrm{SL}_2(\mathbb{Z})$ be the subgroup generated by $\Gamma(2)$ and S.
Show that every $\tau \in \mathfrak{h}$ is conjugate by an element $\gamma \in \Gamma$ to an element of

$$\mathcal{F}_2 = \{\tau \in \mathfrak{h} \mid -1 \leq \mathrm{Re}(\tau) \leq 0,\ |\tau+1/2| \geq 1/2\}.$$

(c) Deduce that Γ is generated by S and T^2.

Proof: $\square$