At Montessori kindergarten in Germany, my classroom had small sheets of paper with 10x4 empty grids. We penciled in the integers one at a time, glued the sheets end to end, and wrapped them around a pencil for storage. Something about these number scrolls captured my attention. I had been told that "numbers go on forever," but I wasn't entirely convinced – how could someone be certain? For days I intently penciled increasing integers on my scroll in batches of ten. I eventually realized the process of "adding one" was exactly the same for 1 to 2 as it was from 1999 to 2000, and that if I did find "the biggest number," I could simply add one to it. That realization impacted me deeply. I had unlocked a secret about the nature of numbers and had felt the bounds of my understanding expand – it was then I first glimpsed the intense satisfaction gained when curiosity becomes knowledge. It felt incredible.

Throughout my childhood, much was uncertain. My parents enjoy a nomadic lifestyle, and by the time I turned 18, I had moved internationally six times across three continents. To facilitate this constant change, from first grade onwards I attended online school, which consisted exclusively of independent self-taught courses beginning in seventh grade. Neither the adaptability I learned from constantly moving nor the independence of thought I developed through self-taught coursework prepared me for a crisis of faith, however, when at age 15, I was forced to confront the validity of my own – incredibly orthodox – religious beliefs. Yet, through all of this constant change, since the days I spent penciling numbers onto small pieces of paper, one goal has never wavered: I want to become a scientist and spend my days furthering humanity's collective knowledge. Specifically, I am interested in using tools from commutative algebra and algebraic geometry to study problems in mathematical physics. I intend to work primarily as a pure mathematician throughout my PhD but intend to use my early experience with physics research to both effectively collaborate with physicists and motivate the topics I study.

Intellectual Merit

Space Debris in Low Earth Orbit: I moved back to the U.S. the summer before my senior year of high school and opted to enroll in a community college for a year rather than continue self-paced coursework. It was in there, in my modern physics course, that I first experienced a taste of independent research. I developed an original physics research project under the supervision of my mentor in the spring of 2017. I created two models for novel techniques of space debris detection in Low Earth Orbit, and though I ended up demonstrating that my ideas were largely infeasible, the process of identifying a question, developing a model, and presenting my results at my school's research symposium proved intensely rewarding.

Fabrication of Topological Insulators: I was initially unsure whether I would pursue a PhD in math or theoretical physics, and because experimental research was more accessible to undergraduates, I immediately began seeking out a position in a physics lab. Beginning in the spring of 2018, I worked under Su Kong Chong, one of Dr. Vikram Deshpande's graduate students, to study the relationship between the thickness of a bulk-insulating Topological Insulator (TI) and the integer quantum Hall conductance on the TI's surface. Using polymer film pick-up techniques and a micro manipulator, I designed and built heterostructures (thin, layered materials held together by van der Waals forces) which enabled us to tune the chemical potential of the TI while it was inside an optical cryostat. Although I created several successful devices, the lack of theory in my research frustrated me, and I left the lab in the fall of my sophomore year to join a theoretical research group. Nevertheless, I hope my experience in the lab will prove useful for future collaborations.

Dark Photon Dark Matter: I knew from the time I started my undergraduate degree that I wanted to work in theoretical research, so when Dr. Yue Zhao offered me a position in his theoretical physics group, I leapt at the opportunity. At the time, Dr. Zhao studied dark photon dark matter (DPDM): a theorized extension to the standard model characterized by the dark photon, a force carrier to a dark-matter analog of the electromagnetic force. Our group was trying to demonstrate that the Laser Interferometer Gravitational-Wave Observatory (LIGO) could confirm the existence of DPDM. I helped write both the Mathematica code used to generate a mock DPDM signal and the analysis code used to convert the data to a frequency series and identify the DPDM signal. I learned to use the Einstein Toolkit, a numerical general relativity simulation package, and helped the cosmology group at Utah install and run it on our school's cluster computer. However, though I found the theory of DPDM fascinating, my limited knowledge only enabled me to implement it in simulations. I still hadn't contributed to the actual development of the theory itself.

During the semester I worked with Dr. Zhao, I took two math and physics classes each: quantum mechanics, solid state physics, pointset topology, and modern algebra. In contrast to my physics research and coursework, I found my math classes to be utterly fascinating and immensely satisfying, and I began to suspect my true passion lay in mathematics. The following semester I deliberately filled my schedule with exclusively math, including a graduate course in algebraic topology, and haven't looked back since.

Real Almost Abelian Lie Theory: At this point I hadn't experienced math research, so I resolved to change that during the summer of 2019 following my sophomore year. I giddily accepted an offer from Zhirayr Avetisyan, a mathematical physicist, to work on Lie theory at the UCSB REU. There, I studied the subgroups, representations, and automorphisms of real finite dimensional almost Abelian Lie groups. These are a special class of Lie groups which serve as generalizations to the Heisenberg group. I found the work immensely satisfying, for though it was securely rooted in pure math, my work had a plethora of applications to cosmology and crystallography. I proved a theorem regarding the faithful matrix representations of almost Abelian Lie groups and explicitly found a faithful finite-dimensional representation when it exists. I also derived an implicit description of the automorphism group of an arbitrary real connected almost Abelian Lie group. I presented my findings at three different conferences, including the Young Mathematician's Conference at OSU. Once I returned to Utah, I continued working with the UCSB group remotely for several months, during which we arrived at a full description of the automorphism group of an arbitrary real finite dimensional almost Abelian Lie group and provided sufficient and necessary conditions for a faithful matrix representation to exist. Our paper is currently under review, and the preprint is available on arXiv. In contrast to my prior experiences with physics, this research was a perfect fit for me and strengthened my resolve to pursue a future in mathematics.

F-signature and Torsion Divisors of Local Strongly F-regular Rings: My longest project to date began August 2019 when I enrolled in a polynomials research class taught by Dr. Thomas Polstra, who was, at the time, a post doc at the University of Utah working in algebraic geometry and commutative algebra. This course provided undergraduate students with a partial research stipend and was a mix between independent research and supervised reading. Along with five other students, I worked primarily out of the books *Polynomials* by Prasolov and *Algebraic Curves* by Fulton, and I delivered lectures on Grace's Theorem and on a surprising result regarding the vanishing sets of polynomials over non-algebraically closed fields. I took this class in conjunction with a second-year graduate commutative algebra class and the graduate algebra sequence. By the end of the semester, I knew I wanted to study commutative algebra and algebraic geometry.

The following spring, Dr. Polstra invited me to take a supervised reading course with him on *F*-singularities, detailed in my research proposal. We primarily studied Weil divisors, *F*-regular rings, and invariants associated to local prime characteristic rings. Two such invariants are *F*-signature and the number of torsion divisors present in the divisor class group. Javier Carvajal-Rojas had previously shown that the order of an individual torsion divisor was bounded by the reciprocal of *F*-signature in a strongly *F*-regular ring, while Polstra proved that the torsion subgroup of a strongly *F*-regular ring was guaranteed to be finite. Together, these results lent plausibility to the following conjecture: "the reciprocal of the *F*-signature of a strongly *F*-regular ring is an upper bound on the number of torsion divisors". I began work on this conjecture in earnest during the spring of 2020, and part-time research funding from my university helped me to continue my research through the summer. During this time, I also began meeting remotely with Dr. Karl Schwede, who played a pivotal role in helping me understand the geometric side of *F*-singularities. After studying the techniques used to extend the notion of *F*-signature to finitely generated modules, I had a breakthrough, and in an exhilarating moment of inspiration, proved my conjecture. These results were the topic of my talk at the 2020 YMC and became my first single-author paper. It is available as a preprint on arXiv and is under review for publication in Communications in Algebra.

Because Dr. Polstra is now at UV and Dr. Schwede is on sabbatical, Dr. Anurag Singh graciously agreed to advise my undergraduate honors thesis, for which I am also studying *F*-singularities. We are currently investigating various examples of strongly *F*-regular rings whose divisor class groups exhibit desirable properties. These inquiries will likely illuminate future directions of research. I am particularly interested in better understanding exactly when the number of torsion divisors is *equal* to the reciprocal of *F*-signature. I still meet regularly with Dr. Polstra to discuss other conjectured features of the divisor class groups of strongly *F*-regular rings, as well as loftier goals such as the weak implies strong conjecture.

Algebraic de Rham Cohomology and the Degeneration of the Hodge Spectral Sequence: During the summer of 2020, I was a full participant of the University of Chicago's remote REU program. I worked under the guidance of Ignacio Darago, a fourth-year graduate student at UChicago. Using prime characteristic techniques, I proved the equivalence of Hodge and algebraic de Rham cohomology for varieties. Serre's GAGA principle is used to prove this equivalence in the characteristic zero case but fails in prime characteristic. As it turns out, the techniques used in the prime characteristic proof can be cleverly extended to the characteristic zero case to bypass the GAGA principle entirely. This project exposed me to spectral sequences as well as the analytic side of algebraic geometry. In addition, because COVID-19 limited the availability of my mentor, this project promoted academic independence and forced me to become a self-sufficient learner. I presented my work at the REU's participant conference.

Broader Impacts

I look up to Carl Sagan as one of the great teachers of science. He once lamented that the natural curiosity we are all born with is too often lost in the transition from childhood to adulthood. Humans are matter endowed with thought, so what purpose could be grander than the quest to understand the nature of reality? In this sense, the natural drive to understand is divine, and it all the more tragic when a person loses the curiosity within them. A large part of why I seek to become a professor of mathematics is to help combat this apathy and inspire others to remain curious about science and math.

Since January of 2018, I have worked 10 hours a week as a tutor at the U's mathematics center, where I have helped students with math ranging from pre-calculus to PDEs and modern algebra. I have also been a TA for Dr. Henryk Hecht's introductory analysis courses for a year and a half. This is the first rigorous proof-based math class many students encounter, and it has been incredibly rewarding to see my students navigate their first experiences in pure math. This semester I have assumed an elevated teaching role and have helped Professor Hecht by preparing short videos and lectures. The consistent teaching experience I've received through my time as an undergraduate has given me a passion and respect for pedagogy, which I believe will help me become an effective teacher one day.

I have also been an active member of my school's AWM chapter for the past two years, where I help plan and run outreach events. My favorite experience was visiting a local 6th grade classroom to talk with the students about "different sizes of infinity". Nearly all the students seemed genuinely thrilled by the discussion, as evidenced by their wonderful questions and surprising insights. Too often math is alienating, but programs such as this help make it accessible and exciting. I also help perform services for disadvantaged students through AWM. COVID-19 has disproportionately impacted these students, and to provide help, I along with several graduate students hold weekly conference meetings to help local students with math homework questions. Once it is deemed safe, we have also arranged to visit several high schools to talk with students about accessible, novel mathematics.

Future Plans

My desire to become a scientist has only grown stronger since my youth. While I feel that I have found my passion in commutative algebra and algebraic geometry, my interest in physics remains, and I hope to contribute to the field through collaborations with theoretical physicists. As the beneficiary of wonderful mentors who have done nothing but encourage my passion for science and math, as a graduate student and eventually as a professor, I want to return the favor by mentoring undergraduates. Inspired by the success I've seen through my involvement in AWM, I also hope to implement more permanent programs at the K-12 level to better enable students to pursue careers in the sciences, and above all, to remain curious.