Amat 491	**Assignment 3**	**F2017**

Integration, Differentiation and Linear Systems

DUE: NOVEMBER 27^{TH}

01. If the interval $[a, b]$ is subdivided into n subinterval $[x_i, x_{i+1}]$, $1 \le i \le n$ of equal length $h = \dfrac{b-a}{n}$, and if Simpson's rule is successively applied to each of those intervals, one gets the n-interval Composite Simpson's Rule, denoted by $\mathcal{Q}_S^{c,n}(f)$

a. Show that

$$Q_S^{c,n}(f)) = \frac{h}{6}\,\big(\,f(a) + f(b)\,\big) + \frac{h}{3}\,\Big(f(a+h) + f(a+2\,h) + \cdot + f\big(a + (n-1)\,h\big)\Big)$$
$$+ \frac{2\,h}{3}\,\Big(f\big(a + \frac{h}{2}\big) + f\big(a + 3\,\frac{h}{2}\big) + \cdots + f\big(a + (2\,n-1)\,\frac{h}{2}\big)\Big)$$

b. Show that if $f(t)$ has a continuous fourth derivative, then

$$\int_a^b f(t)\,\mathrm{d}t = Q_S^{c,n}(f)) - \frac{(b-a)^5}{2880\,n^4}\,f^{(4)}(\theta), \qquad \theta \in [a\,,\,b]$$

c. Suppose $-9 \le f^{(4)}(t) \le 2$, in the interval $[0\,,\,2]$. Find the smallest number of intervals n, to guarantee that

$$\left|\int_0^2 f(t)\,\mathrm{d}t - \mathcal{Q}_S^{c,n}(f)\right| \le 10^{-5}$$

02. Write a MATLAB function Composite_Simpson() that implements the composite Simpson's rule. The function should take the following arguments

- fname the handle of the function $f(t)$ whose definite integral is being computed.
- a and b, the end points of the interval of integration.
- n the number of intervals into which the $[a, b]$ gets subdivided.

Composite_Simpson() should return the value of the integral.

Test your function for $n = 1, 2, 3$ on the integrals

- $\displaystyle\int_0^1 x^k\,\mathrm{d}x$, $k = 0, 1\,2, 3$, to make sure the degree of precision is 3
- $\displaystyle\int_0^1 \mathrm{e}^{3\,x}\,\mathrm{d}x$, to make sure that $\mathcal{Q}_S^{c,n}(f)$ is close to the exact value of the integral.

03. Derive a difference formula that approximates $f'(x_0)$, and uses the points

$$x_0 - 2\,h,\ x_0 - h,\ x_0 + h,\ x_0 + 2\,h$$

What is the order of convergence of the formula?

04. Let $f(x)$ be a function with as many derivatives as needed at and around the point x_0. Find the error term for the difference formula

$$f''(x_0) \approx \frac{-f(x_0+3h)+4f(x_0+2h)-5f(x_0+h)+2f(x_0)}{h^2}$$

05. Use hand calculations to find an $\boldsymbol{LU}$ factorization with partial pivoting of the matrix $\boldsymbol{A} = \begin{bmatrix} 1 & 1 & -1 & 2 \\ 2 & 2 & 4 & 5 \\ 1 & -1 & 1 & 7 \\ 2 & 3 & 4 & 6 \end{bmatrix}$, then use it to solve the system $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, where $\boldsymbol{b} = \begin{bmatrix} 6 \\ 21 \\ 25 \\ 23 \end{bmatrix}$

06. Consider the matrix $\boldsymbol{A} = \begin{bmatrix} 4 & 1 & 1 & 1 \\ 1 & 3 & 0 & -1 \\ 1 & 0 & 2 & 1 \\ 1 & -1 & 1 & 4 \end{bmatrix}$. Use hand calculations to find

a. an $\boldsymbol{L}\boldsymbol{D}\boldsymbol{L}^t$ factorization of $\boldsymbol{A}$ **b.** a Cholesky factorization of $\boldsymbol{A}$