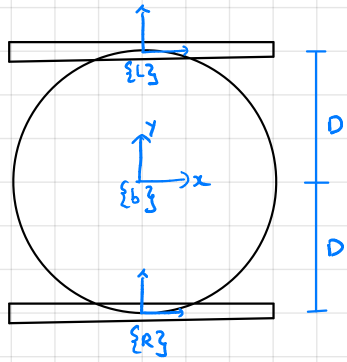
ME 495 – HW#2
Kinematics of Differential Drive Robot



Transform matrices:

$T_{bL} = (0, 0, -D)$

$T_{bR} = (0, 0, D)$

Adjoints:

$$A_{bL} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad A_{bR} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Inverse Adjoints:

$$A_{Lb} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad A_{Rb} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Body Twists

Let $$V_b = \begin{bmatrix} \dot\theta \\ V_x \\ V_y \end{bmatrix} \qquad V_L = \begin{bmatrix} \dot\theta \\ V_{xL} \\ V_{yL} \end{bmatrix} \qquad V_R = \begin{bmatrix} \dot\theta \\ V_{xR} \\ V_{yR} \end{bmatrix}$$

Then $V_i = A_{ib} V_b$

$$V_{Left} = \begin{bmatrix} \dot\theta \\ V_{xL} \\ V_{yL} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot\theta \\ V_x \\ V_y \end{bmatrix} = \begin{bmatrix} \dot\theta \\ -D\dot\theta + V_x \\ V_y \end{bmatrix}$$

$$V_{Right} = \begin{bmatrix} \dot\theta \\ V_{xL} \\ V_{yL} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot\theta \\ V_x \\ V_y \end{bmatrix} = \begin{bmatrix} \dot\theta \\ D\dot\theta + V_x \\ V_y \end{bmatrix}$$

Conventional wheel:

Let $$\begin{bmatrix} V_{xL} \\ V_{yL} \end{bmatrix} = \begin{bmatrix} r\dot\phi_L \\ 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} V_{xR} \\ V_{yR} \end{bmatrix} = \begin{bmatrix} r\dot\phi_R \\ 0 \end{bmatrix}$$

Then:

Left wheel:

$$\begin{bmatrix} \dot\theta \\ r\dot\phi_L \\ 0 \end{bmatrix} = \begin{bmatrix} \dot\theta \\ -D\dot\theta + V_x \\ V_y \end{bmatrix} \Rightarrow \dot\phi_L = -\frac{D}{r}\dot\theta + \frac{1}{r}V_x$$

$$\Rightarrow \begin{bmatrix} \dot\phi_L \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{D}{r} & \frac{1}{r} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot\theta \\ V_x \\ V_y \end{bmatrix}$$

EQN 1

Right wheel:

$$\begin{bmatrix} \dot\theta \\ r\dot\phi_R \\ 0 \end{bmatrix} = \begin{bmatrix} \dot\theta \\ D\dot\theta + V_x \\ V_y \end{bmatrix} \Rightarrow \dot\phi_R = \frac{D}{r}\dot\theta + \frac{1}{r}V_x$$

$$\Rightarrow \begin{bmatrix} \dot\phi_R \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{D}{r} & \frac{1}{r} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot\theta \\ V_x \\ V_y \end{bmatrix}$$

EQN 2

Forward Kinematics

$$\begin{bmatrix} \dot{\phi}_L \\ \phi_R \end{bmatrix} = \begin{bmatrix} -\frac{D}{r} & \frac{1}{r} & 0 \\ D/r & 1/r & 0 \end{bmatrix} \xrightarrow{\text{Solving simultaneously}}$$

$$\dot{\theta} = \frac{r}{2D}\left(\dot{\phi}_R - \dot{\phi}_L\right) \qquad \text{EQN 3}$$

$$v_x = \frac{r}{2}\left(\dot{\phi}_R + \dot{\phi}_L\right) \qquad \text{EQN 4}$$