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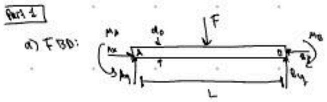
ME20: Project 2

Part 1: Determine the normal stress and deflection developed in the pull-up bar

Part 1

a) FBD:



b) 6 unknowns: $M_B, M_A, A_x, A_y, B_x, B_y$

$+\circlearrowleft \Sigma M_A = 0 = M_A - F(L/2) - M_B(L) + B_y(L)$

$+\circlearrowleft \Sigma M_B = 0 = M_A(L) - A_y(L) + F(L/2) - M_B$

$+\uparrow \Sigma F_y = 0 = A_y - F + B_y$

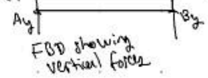
$\xrightarrow{+} \Sigma F_x = 0 = -A_x + B_x \therefore A_x = B_x = 0\ N$

$\therefore$ 4 Equations & 6 unknowns so not enough EQs to solve for unknowns $\therefore$ statistically indeterminate

c) Because this problem set-up is statistically indeterminate, we can cut the bar in half & form the deflections caused by loadings (F, M_B, B_y) independently given that $\delta_B = \delta_A$: deflection$_B$ = deflection$_A$ $\therefore V_1(L) = V_2(L) = 0$ & $V_1 + V_2 + V_3 = 0$.

d)

① $A_y = B_y$ b/c pull-up bar



FBD showing vertical forces

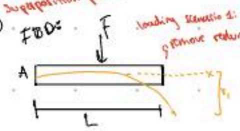
$+\uparrow \Sigma F_y = 0 = -F + B_y + A_y$ when bar is symmetric $\therefore$

$F = B_y + A_y$ when $A_y = B_y \therefore F = 2A_y$

$\therefore A_y = \frac{F}{2} = B_y$

Application of superposition principle

② FBD:

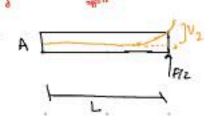


loading scenario 1: remove redundants

$V_1 = -\frac{5PL^3}{48EI} = -\frac{5FL^3}{48EI}$

loading scenario 2: only redundant By applied

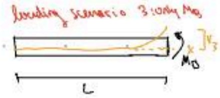
FBD:



$V_2 = -\frac{PL^3}{3EI} = \frac{B_y L^3}{3EI} = \frac{\frac{F}{2}L^3}{3EI}$

loading scenario 3: only M_B

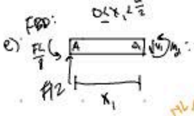
FBD:



$V_3 = \frac{M L^2}{2EI} = \frac{M_B L^2}{2EI}$

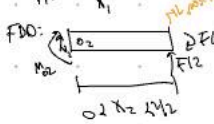
$\underbrace{0 = V_1 + V_2 + V_3}_{\text{superposition principle}} = -\frac{5FL^3}{48EI} + \frac{\frac{F}{2}L^3}{3EI} + \frac{M_B L^2}{2EI} \rightsquigarrow \frac{2\cancel{EI}}{\cancel{L^2}}\left(\frac{5FL^{\cancel{3}}}{48\cancel{EI}} - \frac{\frac{F}{2}L^{\cancel{3}}}{3\cancel{EI}}\right) = M_B = -\frac{FL}{8}$

e) FBD: $0 \le x_1 \le \frac{L}{2}$



$\circlearrowleft \Sigma M_{O_1} = 0 = \left(\frac{FL}{8}\right) + \frac{F}{2}x_1 + M_1 \therefore M_1 = -\frac{FL}{8} + \frac{Fx_1}{2}$ (units)

FBD:

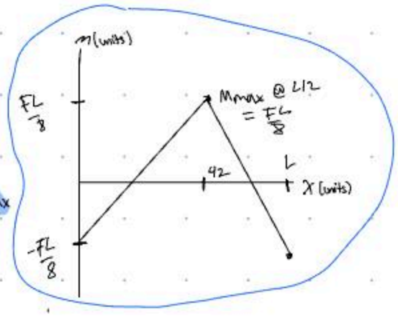


$0 \le x_2 \le L/2$

L/2 position

$\circlearrowleft FL/8$ $\circlearrowleft \Sigma M_{O_2} = 0 = \frac{FL}{8} - \frac{F}{2}x_2 + M_2 \therefore M_2 = \frac{F}{2}x_2 - \frac{FL}{8}$

$M_{max}: -\frac{FL}{8} + \frac{F}{2}\left(\frac{L}{2}\right) = -\frac{FL}{8} + \frac{FL}{4} = -\frac{FL}{8} + \frac{2FL}{8} = \frac{FL}{8} = M_{max}$



f) $\sigma_{max} = -\frac{M_{max} y}{I}$ where $M_{max} = \frac{FL}{8}$ & $y = c = \frac{d_o}{2} \therefore \sigma_{max} = \frac{\frac{FL}{8}\left(\frac{d_o}{2}\right)}{I} = \frac{FLd_o}{16I} = \sigma_{max}$

g) $EIv_1'' = M_1 \Rightarrow v_1 = \frac{1}{EI}\iint M_1\,dx = \iint -\frac{FL}{8} + \frac{Fx_1}{2}\,dx \Rightarrow EIv_1' = \frac{1}{4}Fx_1^2 - \frac{1}{8}FLx_1 + C_1 \Rightarrow EIv = \frac{1}{12}Fx_1^3 - \frac{1}{16}FLx_1^2 + C_1x_1 + C_2$

$0 \le x < \frac{L}{2}$

B.C. {fixed end: @ $x = 0$, $v = 0$, $\theta_1 = 0$; $v_1 = 0: \frac{F}{12}(0)^3 - \frac{FL}{16}(0)^2 + C_1(0) + C_2 = 0 \therefore C_2 = 0$

$\theta_1 = 0: \frac{F}{4}(0)^2 - \frac{FL}{8}(0) + C_1 = 0 \therefore C_1 = 0$

$\rightarrow v_1(x) = \frac{\frac{1}{12}Fx_1^3 - \frac{1}{16}FLx_1^2}{EI}$ $\quad 0 \le x \le L/2$

For $0 \le x_2 \le L/2$: $EIv_2'' = -\frac{FL}{8} + \frac{Fx_2}{2} \therefore EIv_2' = \frac{1}{4}Fx_2^2 - \frac{1}{8}FLx_2 + C_1 \therefore EIv_2 = \frac{1}{12}Fx_2^3 - \frac{1}{16}FLx_2^2 + C_3x_2 + C_4$

x_2 @ location of x=L

B.C. {fixed end @ $x_2 = 0$: $v_2 = 0$, $\theta_2 = 0$: $v_2 = 0: \frac{F}{12}(0)^3 - \frac{FL}{16}(0)^2 + C_3(0) + C_4 = 0 \therefore C_4 = 0$

x=L

$\theta_2 = 0: \frac{F}{4}(0)^2 - \frac{FL}{8}(0) + C_3 = 0 \therefore C_3 = 0$

$\rightarrow v_2(x) = \frac{\frac{1}{12}Fx_2^3 - \frac{1}{16}FLx_2^2}{EI}$

v_{max}: occurs at $EIv' = 0$

For $v_1' = 0$; $\frac{F}{4}x^2 - \frac{FL}{8}x \therefore$ @ $x = L/2$ $\rightarrow$ plugging into: $v_1(x) = \frac{\frac{1}{12}Fx_1^3 - \frac{1}{16}FLx_1^2}{EI} = \frac{1}{12}F\left(\frac{L}{2}\right)^3 - \frac{1}{16}FL\left(\frac{L}{2}\right)^2 = \frac{\frac{FL^3}{96} - \frac{FL^3}{64}}{EI} = v_{max}$

For $v_2' = 0$; $\frac{F}{4}x^2 - \frac{FL}{8}x \therefore$ @ $x = L/2$

Part 2: Comparing the peak normal stress and deflection with the ones from FEA

Part 2:

h) $\sigma_{max} = \frac{FLd_o}{16I} = \frac{(1300N)(.77m)(.025m)}{16\pi\left(\frac{.025m^4 - .017m^4}{64}\right)} \approx \frac{25.025\ N \cdot m^2}{2.41 \cdot 10^{-7}\ m^4} = 103752539\ Pa = 103.75\ MPa = \sigma_{max}$

i) $\frac{\frac{FL^3}{96} - \frac{FL^3}{64}}{EI} = v_{max}: \frac{\frac{(1300N)(.77m)^3}{96} - \frac{(1300N)(.77m)^3}{64}}{\frac{\pi}{64}(.025m^4 - .017m^4)(210 \cdot 10^9 Pa)} = \frac{6.18 - 9.27\ N \cdot m^3}{3166\ N \cdot m^2} = \frac{-3.09\ N \cdot m^3}{3166\ N \cdot m^2} = 9.76 \cdot 10^{-4}\ m$

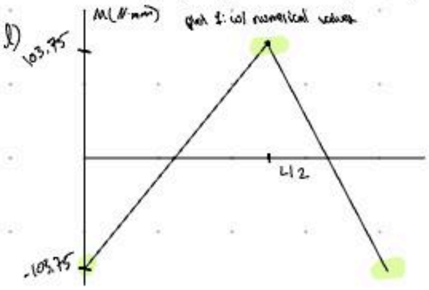
j)

	Steps h/i	Deliverables 1/2	Deliverables 3/4	Deliverables 5/6
Max normal stress (MPa)	103.75	89 MPa	89 MPa	91.3 MPa
Max deflection (mm)	.976 (↓ direction)	.889 (↓ direction)	.889 (↓ direction)	.907 (↓ direction)

k) The computations in steps i & h for the maximum normal stress & vertical deflection were an overestimation compared to the FEA results. The FEA model is more accurate b/c the by-hand computations contain assumptions of ideal conditions (homogenous material, point loads), whereas the FEA model has settings such as the 'meshing' function which breaks down the pulley 'bar' into smaller parts: the deflection is analyzed locally. The force in FEA is distributed over an area on the top surface of the bar whereas in the hand computations the force is applied exactly at the midpoint. These are some examples providing context as to why FEA provides more accurate results in its representation of more 'real-world' features.

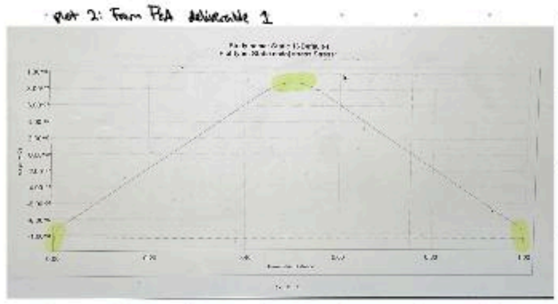
l)

plot 1: w/ numerical values



VS

plot 2: From FEA deliverable 1

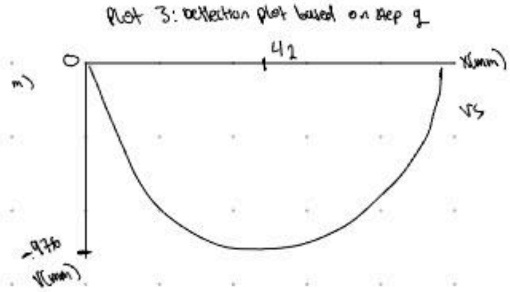


Plot 1 & 2 differ in their maximum moment values, which is commented on in reference to the max normal stress values. Plot 2 has a smoothed peak whereas plot 1 has a pointed peak. Plot 2 has steep slopes near x=0 & x=L which is not reflected in plot 1, which makes sense b/c the FEA can analyze moment locally.

Both plots have values for moment above & below the x-axis of similar magnitudes.

m)

Plot 3: Deflection plot based on step g



VS

Plot 4: Deliverable 2

Plot 3 is less specific than plot 4 in capturing change in slope, which makes sense as FEA employs computations specific to smaller components (mesh) & ∴ can identify small shifts.

Both plots are reasonably parabolic w/ their most negative values at L/2.