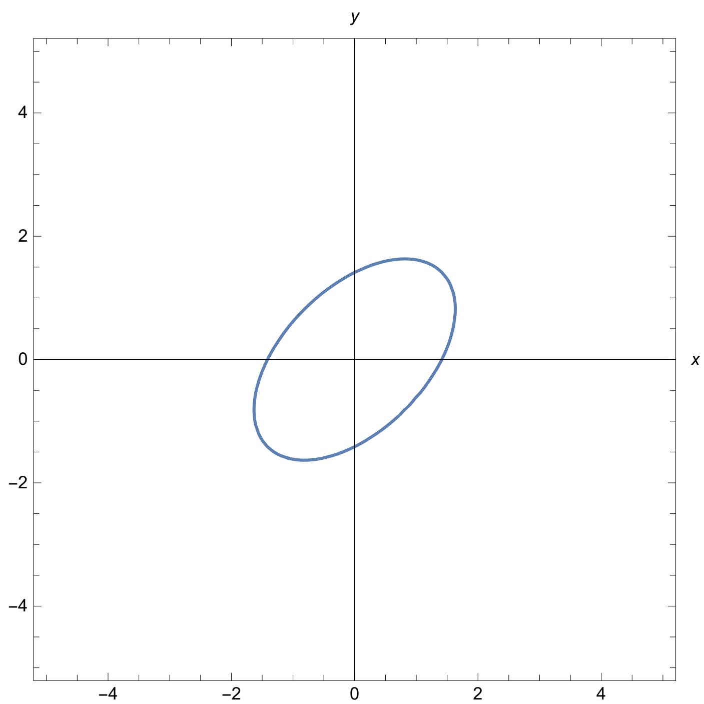
Do the following tasks using Mathematica.

$\iint_R (x^2 - xy + y^2)dA$, where R is the region bounded by the ellipse $x^2 - xy + y^2 = 2$. Use the transformation: $x = \sqrt{2}\, u - \sqrt{\left(\frac{2}{3}\right)}\, v$, $y = \sqrt{2}\, u + \sqrt{\left(\frac{2}{3}\right)}\, v$

(a) Plot R in both xy and uv planes.

In[1]:=
```
ContourPlot[x^2 - x y + y^2 == 2, {x, -5, 5}, {y, -5, 5}, Axes → True, AxesLabel → Automatic]
```

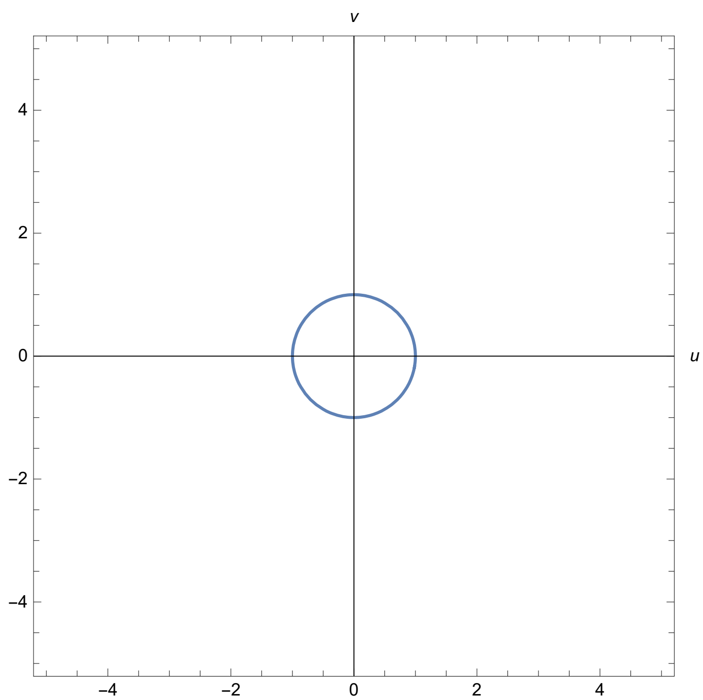
Out[1]=



In[2]:=
```
x = √2 u - √(2/3) v;
y = √2 u + √(2/3) v;
```

In[4]:= ContourPlot$\left[x^2 - x\,y + y^2 == 2, \{u, -5, 5\}, \{v, -5, 5\}, \text{Axes} \to \text{True}, \text{AxesLabel} \to \text{Automatic}\right]$

Out[4]=



(b) Find the Jacobian of the transformation

In[5]:= jac = Det[D[{x, y}, {{u, v}}]]

Out[5]= $\frac{4}{\sqrt{3}}$

(c) Evaluate the integral using the transformation

In[6]:= Solve$\left[\left\{x^2 - x\,y + y^2 == 2\right\}, \{v\}\right]$

Out[6]= $\left\{\left\{v \to -\sqrt{1-u^2}\right\}, \left\{v \to \sqrt{1-u^2}\right\}\right\}$

In[7]:= $\int_{-1}^{1}\int_{-\sqrt{1-u^2}}^{\sqrt{1-u^2}} \left(x^2 - x\,y + y^2\right)\,(\text{jac})\,dv\,du$

Out[7]= $\frac{4\,\pi}{\sqrt{3}}$