Principles of Physics II (PHY112)

Lab

Experiment no: 5

Name of the experiment: Determination of the time constant of an RC series circuit

YOU HAVE TO BRING A GRAPH PAPER (cm scale) TO DO THIS EXPERIMENT.

Theory

Capacitor is a device to store energy in a form of electric charge. A capacitor (Figure 1) can be made by keeping a dielectric material between two isolated conducting plates which are close to each other. To store energy one of the plates gains positive charge and the other one gains equal amount of negative charge.

(a) A parallel plates capacitor (b) capacitor (c) Symbol of a non polarized capacitor (d) Symbol of a polarized capacitor

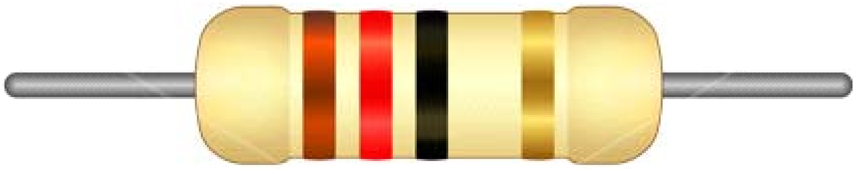
Figure 1: Capacitor (courtesy: wikimedia.org)

Capacitance is the ability of a capacitor to store energy as a form of charge. The amount of electric charge required to be stored in each plate of a capacitor to produce one unit potential difference (1V in SI system) between them is called the capacitance of that capacitor. If the charge of magnitude Q is needed to be stored in each of the plates of the capacitor to cause V_C amount of potential difference between them then the capacitance, C is given by,

$$C = \frac{Q}{V_C} \Rightarrow V_C = \frac{Q}{C} \tag{1}$$

According to Ohm's law we know that at a certain temperature, current, I passing through a conductor is directly proportional to the potential difference, V across it, i.e., $I \propto V \Rightarrow I = \frac{V}{R}$.

Here, R is the resistance of the conductor, which is a measure of difficulty to pass an electric current through the conductor.



(a) External view of a 12 Ω resistor (courtesy: shutterstock.com)

(b) Symbol of resistor

Figure 2: Resistor

Mnemonic	Color
B	Black
B	Brown
R	Red
O	Orange
Y	Yellow
Good	Green
Boy	Blue
Very	Violet
Good	Gray
Worker	White

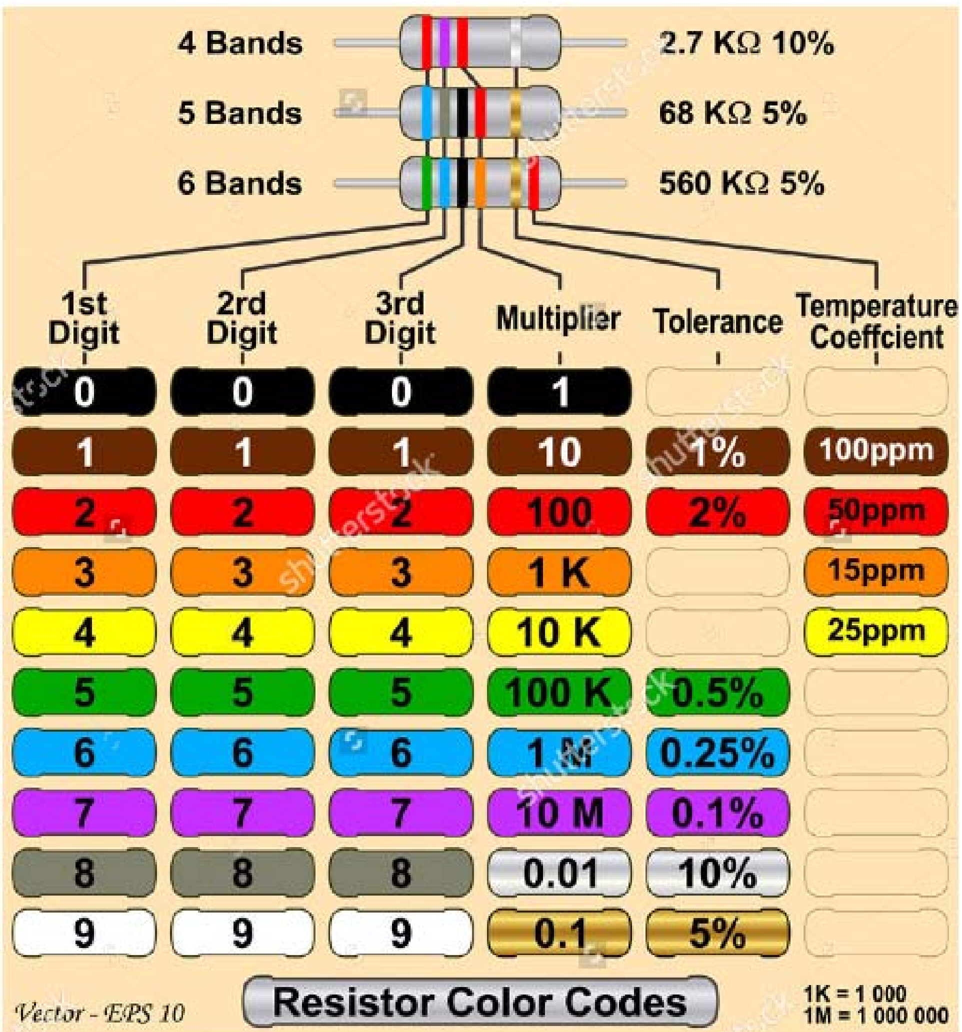


Figure 3: Color code to determine the resistance of a resistor (courtesy: shutterstock.com)

A resistor implements resistance in a circuit. Figure 2 shows the external view of a resistor and its symbol used in a circuit diagram. How to determine the resistance from its color code is shown in the Figure 3. We can determine the resistance of the resistor of Figure 1(a) by observing its color code. It is a resistor of 4 band color code. Sequence of the colors of the band is brown, red, black and gold. First two bands represent the number 12. The third band represents the multiplier $1\,\Omega$. The fourth band means the tolerance $\pm$ 5%. That means its resistance is (12.0 $\pm$ 0.6) Ω.

If V_R is the potential difference across a resistor, R is its resistance and I is the current passing through it, then

$$V_R = IR = R\frac{dQ}{dt} \tag{2}$$

Since, current I is the instantaneous rate at which charge, Q flows through any cross section of the conducting wire or the resistor with respect to time, t, $I = \frac{dQ}{dt}$. This is also equal to the instantaneous rate at which charge, Q is stored with respect to time, t, at each plate of the capacitor.

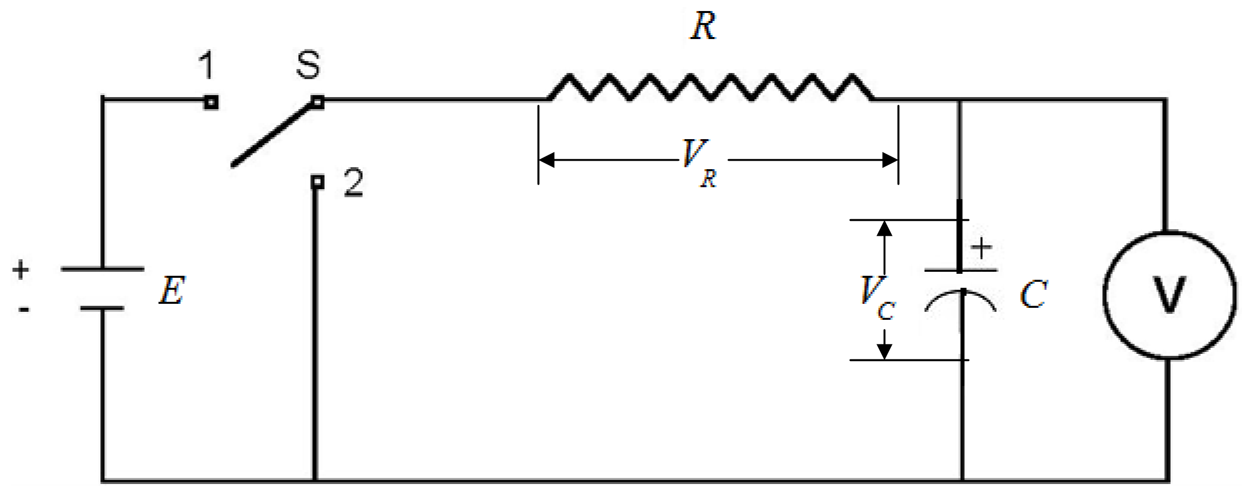
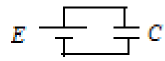


Figure 4: RC series circuit construction for this experiment

In this experiment we will examine the time dependence of the voltage across a capacitor when it is connected with a resistor in a series combination during charging process when the RC series combination is connected with a cell (of emf E) and, as well as, during discharging process when this RC combination makes a closed loop without any voltage source. Figure 4 depicts the circuit configuration of this experiment. S is a single pole double throw switch. When the throw 1 and the pole of S are connected then the circuit containing R, C and E is closed and the capacitor gets charged gradually, causing a gradual increase of the potential difference, V_C across it. When the throw 2 and the pole of S are connected then the circuit containing only R and C is closed and the capacitor gets discharged gradually, causing a gradual decrease of the potential difference, V_C across it.

Charging process of the capacitor

If a capacitor is connected with a cell (

) it gets charged almost instantaneously and achieves the voltage across it equal to the emf of the cell. Inclusion of a resistor, in series combination with the capacitor, slows down this charging process. Hence, more the value of resistance R is, slower is the capacitor to get charged and attain the voltage across it close to E. On the other hand, if the capacitance, C is high then large amount of charges are required to be stored to attain voltage near E. Hence, bigger the capacitance, C is, more is the required time to attain a voltage close to E. Let us define a parameter called 'time constant', $\tau = RC$. Time constant of an RC series circuit is a measure of how slowly the capacitor gets charged to attain a voltage across it close to the applied voltage of the cell, during the charging process of the capacitor.

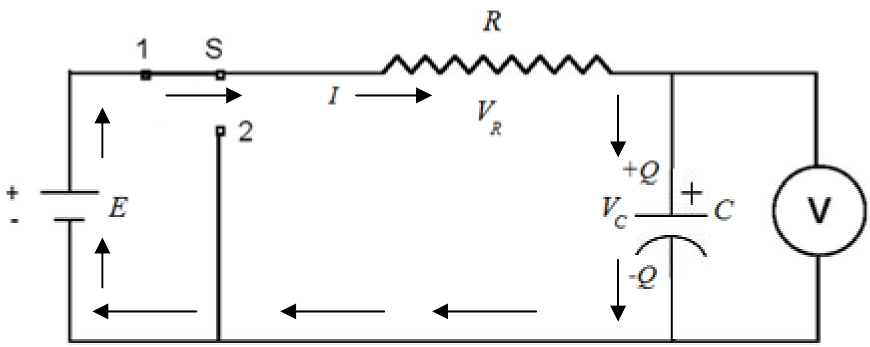


Figure 5: Charging of the capacitor

Figure 5 shows the charging process of the capacitor where the RC series combination is connected with the cell of emf E to form a closed loop. According to Kirchhoff's loop voltage rule,

$$E = V_C + V_R$$

$$\Rightarrow E = \frac{Q}{C} + R\frac{dQ}{dt}$$

$$\Rightarrow \frac{dQ}{dt} + \frac{Q}{RC} = \frac{E}{R}$$

$$\Rightarrow \frac{dQ}{dt} + \frac{Q}{\tau} = \frac{E}{R} \qquad (3)$$

To solve this differential equation for $Q(t)$ we need to multiply it by an integrating factor $e^{\int \frac{1}{\tau} dt} = e^{\frac{t}{\tau}}$

By multiplying equation (3) by this integrating factor we get,

$$e^{\frac{t}{\tau}}\frac{dQ}{dt}+\frac{Q}{\tau}e^{\frac{t}{\tau}}=\frac{E}{R}e^{\frac{t}{\tau}} \tag{4}$$

By using Leibnitz rule ($\frac{d(uv)}{dt}=u\frac{dv}{dt}+v\frac{du}{dt}$) you can check that $\frac{d}{dt}\left(e^{\frac{t}{\tau}}Q\right)=e^{\frac{t}{\tau}}\frac{dQ}{dt}+\frac{Q}{\tau}e^{\frac{t}{\tau}}$

Therefore, the equation (4) can be written as $\frac{d}{dt}\left(e^{\frac{t}{\tau}}Q\right)=\frac{E}{R}e^{\frac{t}{\tau}}$

Now, we can integrate both sides of this equation:

$\int\frac{d}{dt}\left(e^{\frac{t}{\tau}}Q\right)dt=\frac{E}{R}\int e^{\frac{t}{\tau}}dt$ [$\frac{E}{R}$ does not depend on time, it is a constant. So we can keep it outside the integration operator]

$\Rightarrow e^{\frac{t}{\tau}}Q=\frac{E}{R}\int e^{\frac{t}{\tau}}dt=\frac{E}{R}\frac{e^{\frac{t}{\tau}}}{\left(\frac{1}{\tau}\right)}+A=\frac{E}{R}\tau e^{\frac{t}{\tau}}+A$ [Here, we use the rule $\int e^{ax}dx=\frac{e^{ax}}{a}+A$, A is an arbitrary constant]

$$\Rightarrow Q=\frac{E}{R}\tau+Ae^{\frac{-t}{\tau}}=\frac{E}{\cancel{R}}\cancel{R}C+Ae^{\frac{-t}{\tau}}$$ [Since, $\tau = RC$]

$$\Rightarrow \frac{Q}{C}=E+\frac{A}{C}e^{\frac{-t}{\tau}}$$

$$\Rightarrow \frac{Q}{C}=E+A'e^{\frac{-t}{\tau}}$$ [Say, $A'=\frac{A}{C}$]

$$\Rightarrow V_C(t)=E+A'e^{\frac{-t}{\tau}} \tag{5}$$ [Using equation (1)]

We can fix the value of the arbitrary constant A' by imposing initial condition. At time, *t=0*, i.e., when we turn on the switch, the capacitor does not have any charge and voltage across it is zero. That means $V_C(0)=0$. If we use it in equation (5), we get $A'=-E$. Finally, we get the equation showing how voltage across the capacitor, V_C will depend on time, *t* during its charging process.

$$\boxed{V_C(t) = E\left(1 - e^{\frac{-t}{\tau}}\right)} \tag{6}$$

When time $t \to \infty$, then $e^{-t/\tau} \to 0$. Hence, from equation (6) we can write $V_C(t \to \infty) = E(1-0) = E$. As time goes on, the voltage across the capacitor approaches the value of the electromotive force of the cell.

The argument of an exponential function must be a dimensionless quantity. In equation (6), we see the argument of the exponential function is $-\frac{t}{\tau}$. It should be dimensionless. Therefore, the time constant τ which is defined as the product of *R* and *C*, must be an amount of time. If we put $t = \tau$ in equation (6) then $V_C(\tau) = E(1-e^{-1}) \approx 0.63E$. τ amount of time later the voltage across the capacitor reaches up to *0.63E*. Therefore,

"***During the charging process of the capacitor of an RC series circuit connected with a cell of electromotive force E, the time required by the capacitor, which is initially discharged, to attain voltage across it up to 0.63E is the time constant of that circuit.*** "

Figure 6 shows the V_C vs. *t* curve during the charging process of the capacitor of an RC series circuit. The point having Y coordinate 0.63*E* is located on the Y axis. A line parallel to the X axis is drawn up to the curve. From the point on the curve at which this line intersects the curve, another line parallel to Y axis is drawn down to the X axis. The X coordinate of the point at which this line intersects the X axis is the time constant of the circuit.

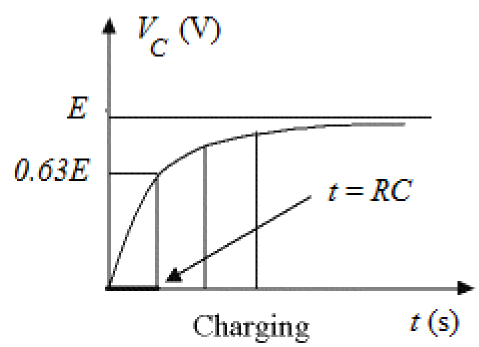
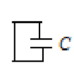


Figure 6: V_C vs. *t* curve during the charging process of the capacitor of an RC series circuit

Discharging process of the capacitor

If the two electrodes of a charged capacitor is shorted (

), then it gets discharged almost instantaneously and achieves the voltage across it equal to zero. Inclusion of a resistor, in series combination with the capacitor, slows down this discharging process. Hence, more the value of resistance *R* is, slower is the capacitor to get discharged and have the voltage across it close to *0*. On the other hand, if the capacitance, *C* is high then large amount of charges are required to move from one electrode to other to make the voltage difference between them to be zero.

Hence, bigger the capacitance, C is, more is the required time to decrease the voltage down to *0*. Hence, the time constant, $\tau = RC$ of an RC series circuit is a measure of how slowly the capacitor gets discharged to have the voltage across it close to zero during the discharging process of the capacitor.

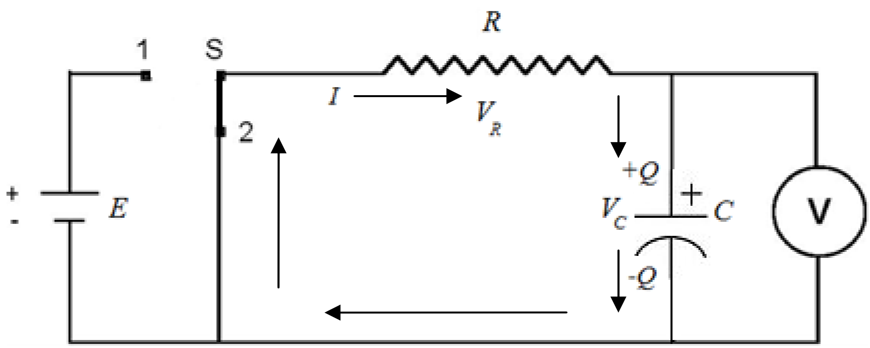


Figure 7: Discharging of the capacitor

Figure 7 shows the discharging process of the capacitor where the RC series combination forms a closed loop without any voltage source. According to Kirchhoff's loop voltage rule,

$$V_C + V_R = 0$$
$$\Rightarrow \frac{Q}{C} + R\frac{dQ}{dt} = 0$$
$$\Rightarrow \frac{dQ}{dt} = -\frac{1}{RC}Q = -\frac{1}{\tau}Q$$
$$\Rightarrow \frac{dQ}{Q} = -\frac{1}{\tau}dt$$

We can now integrate both sides of the above-shown differential equation to find

$$\int \frac{dQ}{Q} = -\frac{1}{\tau}\int dt$$

$$\Rightarrow \ln Q = -\frac{1}{\tau}t + A$$

Here, A is an arbitrary constant. The equation shown above can be written as,

$$Q(t) = e^{A}e^{-\frac{t}{\tau}} \tag{7}$$

Now, voltage across the capacitor is,

$$V_C(t) = \frac{Q(t)}{C} = \frac{e^A}{C} e^{\frac{-t}{\tau}} = A' e^{\frac{-t}{\tau}} \text{ [Say, } \frac{e^A}{C} = A' \text{]} \tag{8}$$

We can fix the value of the constant A' by imposing initial condition. At time $t = 0$, the capacitor is fully charged having voltage across it equals to E. Therefore, V_C ($t = 0$) = E. Using this condition we find that $A' = E$. Finally, we find out the equation showing how voltage across the capacitor, V_C will depend on time, t during its discharging process:

$$\boxed{V_C(t) = Ee^{\frac{-t}{\tau}}} \tag{9}$$

When time $t \to \infty$, then $e^{-t/\tau} \to 0$. Hence, from equation (9) we can write $V_C(t \to \infty) = 0$. As time goes on, the voltage across the capacitor approaches zero.

If we put $t = \tau$ in equation (9) then, $V_C(t = \tau) = Ee^{-1} \approx 0.37E$. After the moment we close the circuit, τ amount of time later the voltage across the capacitor decreases down to $0.37E$. Therefore,

"***During the discharging process of the capacitor of an RC series circuit, the time required by the capacitor, which is initially charged, to reduce voltage across it down to 0.37 times of its initial voltage, is the time constant of that circuit.*** "

Figure 8 shows the V_C vs. t curve during the discharging process of the capacitor of an RC series circuit. The point having Y coordinate 0.37E is located on the Y axis. A line parallel to the X axis is drawn up to the curve. From the point on the curve at which this line intersects the curve, another line parallel to Y axis is drawn down to the X axis. The X coordinate of the point at which this line intersects the X axis is the time constant.

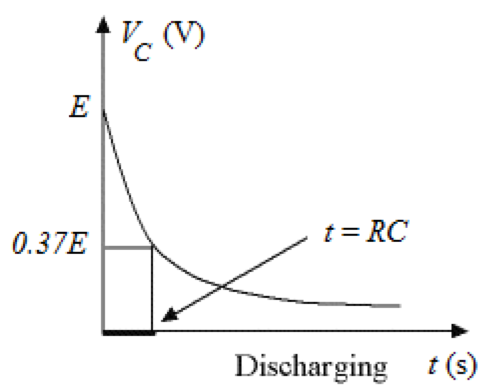


Figure 8: V_C vs. t curve during the discharging process of the capacitor

Apparatus

A bread board, a capacitor ($100\mu F$), a resistor ($0.220 M\Omega$), wires, a single pole double through switch, a cell, a stopwatch, and a multi-meter.

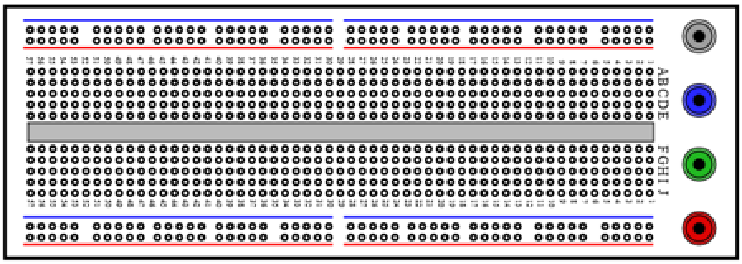


Figure 9: A breadboard (courtesy: wikimedia.org)

Figure 10: Polarized and non polarized Capacitor. Positive electrode of a polarized capacitor is longer than its negative electrode (courtesy: shutterstock.com)

Procedure

1) Construct the circuit according to the Figure 4. If you are using polarized capacitor, make sure that its negative or positive electrode will be connected to the negative or positive terminal of the cell respectively when you will turn on the switch.
2) Make sure that the initial voltage across the capacitor is zero. Use the multi-meter, by keeping its knob pointed towards voltage measurement region, to measure the initial voltage of the capacitor. If it is not zero short circuit the electrodes of the capacitor to discharge it instantaneously.
3) Connect the pole of the switch S with the throw 1, as shown in the Figure 5 and at the same moment turn on the stopwatch.
4) Record the voltage across the resistor in every 10 s interval. Continue it until the voltage shown on the display box of the multi-meter (which is now being used to measure voltage) becomes almost stable. These are the data for the charging process of capacitor.
5) Turn off the switch. Reset the stop watch's time to zero. Connect the pole of the switch S with the throw 2, as shown in Figure 7 and at the same moment turn on the stopwatch.
6) Record the voltage across the resistor in every 10 s interval. Continue it until the voltage shown on the display box of the multi-meter becomes almost zero.
7) Draw the voltage versus time curves for both charging and discharging process.
8) Find out the value of the time constant for both charging and discharging curves from the graph as discussed in the theory. Take their average which is the experimental value of the time constant.

9) Read the color code of the resistor to find its resistance. Measure its resistance by using the multi-meter as well.
10) See what the value of capacitance of the capacitor is.
11) Work out the theoretically predicted value of the time constant of an RC series circuit.
12) Find out how much your experimental value of the time constant is deviated from its theoretically predicted value.

Read carefully and follow the following instructions:

- **Please READ the theory carefully, TAKE printout of the 'Questions on Theory' and ANSWER the questions in the specified space BEFORE you go to the lab class.**

- **To get full marks for the 'Questions on Theory' portion, you must answer ALL of these questions CORRECTLY and with PROPER UNDERSTANDING, BEFORE you go to the lab class. However, to ATTEND the lab class you are REQUIRED to answer AT LEAST the questions with asterisk mark.**

- **Write down your NAME, ID, THEORY SECTION, GROUP, DATE, EXPERIMENT NO AND NAME OF THE EXPERIMENT on the top of the first paper.**

- **If you face difficulties to understand the theory, please meet us BEFORE the lab class. However, you must read the theory first.**

- **DO NOT PLAGIARIZE. Plagiarism will bring ZERO marks in this WHOLE EXPERIMENT. Be sure that you have understood the questions and the answers what you have written, and all of these are your own works. You WILL BE asked questions on these tasks in the class. If you plagiarize for more than once, WHOLE lab marks will be ZERO.**

- **After entering the class, please submit this portion before you start the experiment.**

Name: ______________________ ID: _______________ Sec: ___ Group: __ Date: __________

Experiment no: ___

Name of the Experiment: ____

____

Questions on theory (all diagrams should be drawn by using a pencil and a scale)

*1) What is a capacitor? Define capacitance. [0.25]

Ans:

*2) State Ohm's law. [0.25]

Ans:

3) Read the color code of the following resistors and find out their resistances. [0.5]

(a)

(b)

(c)

Ans:

*4) Draw the circuit construction for the experiment [0.25]

Ans:

*5) Apply Kirchhoff’s loop voltage rule around the circuit when the capacitor gets charged and find the associated differential equation. [0.25]

Ans:

*6) Solve this equation of the answer of question 5 to find charge stored in the capacitor as a function of time t, *i.e.*, $Q(t)$ during charging process. Next, show that $V_C(t) = E\left(1 - e^{-\frac{t}{\tau}}\right)$ [1]

Ans:

7) Apply Kirchhoff's loop voltage rule around the circuit when the capacitor gets discharged and find the associated differential equation. [0.25]

Ans:

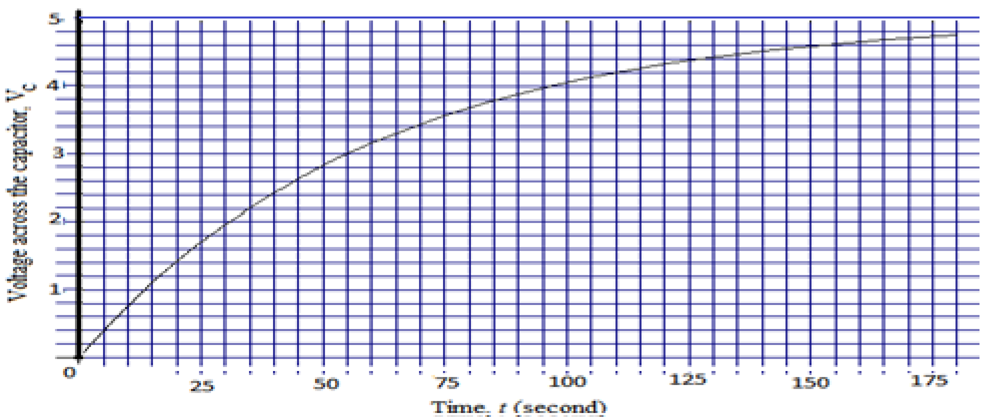
*8) Solve this equation of the answer of question 7 to find charge stored in the capacitor as a function of time *t*, i.e., *Q*(*t*) during discharging process. Next, show that

$V_C(t) = Ee^{\frac{-t}{\tau}}$ [1]

*9) What is the physical significance of the time constant of an RC series circuit?[0.25]

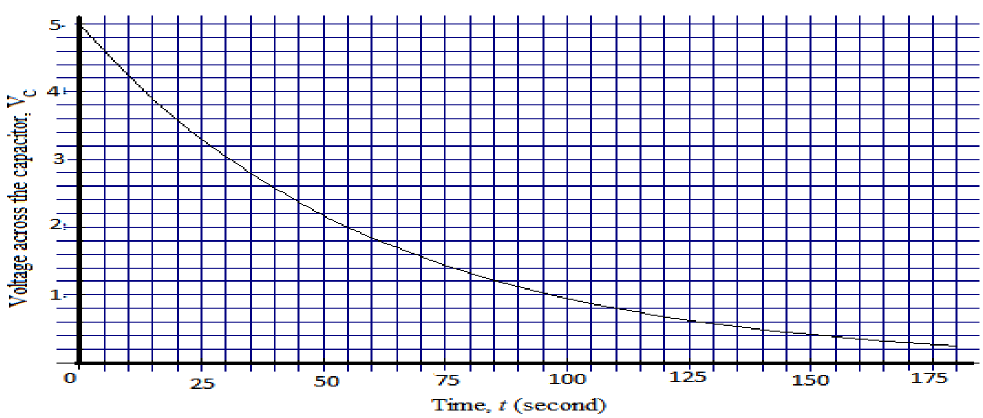
Ans:

10) A voltage across the capacitor (in Volts) vs. time (in seconds) curve for a series RC circuit while the capacitor is charging is shown below: Find out the time constant τ . [0.5]



Ans:

11) A voltage across the capacitor (in Volts) vs. time (in seconds) curve for a series RC circuit while the capacitor is discharging is shown below: Find out the time constant τ . [0.5]



Ans:

- **Draw the data table(s) and write down the variables to be measured shown below (in the 'Data' section), using pencil and ruler BEFORE you go to the lab class.**

- **Write down your NAME and ID on the top of the page.**
- **This part should be separated from your Answers of "Questions on Theory" part.**
- **Keep it with yourself after coming to the lab.**
- **Do not forget to bring a graph paper.**

Data

Table: Data showing the time dependence of voltage across the capacitor

Observation	Time, t (S)	Voltage across the capacitor V_C During charging process,(V)	Voltage across the capacitor V_C during discharging process (V)

Capacitance:

Capacitance, *C* =

Resistance:

Color				
Digit/Multiplier/Tolerance				

Value of resistance from the multi-meter, *R* =

Theoretically predicted value of the time constant:

- **READ the PROCEDURE carefully and perform the experiment by YOURSELVES. If you need help to understand any specific point draw attention of the instructors.**
- **DO NOT PLAGIARIZE data from other group and/or DO NOT hand in your data to other group. It will bring ZERO mark in this experiment. Repetition of such activities will bring zero mark for the whole lab.**
- **Perform calculations by following the PROCEDURE . Show every step in the Calculations section.**
- **Write down the final result(s)**

Calculations

Results:

- **TAKE printout of the ‘Questions for Discussions’ BEFORE you go to the lab class. Keep this printout with you during the experiment. ANSWER the questions in the specified space AFTER you have performed the experiment.**
- **Attach Data, Graph, Calculations, Results and the Answers of ‘Questions for Discussions’ parts to your previously submitted Answers of ‘Questions on Theory’ part to make the whole lab report.**
- **Finally, submit the lab report before you leave the lab.**

Name: ___________________________ **ID:** __________________

Questions for Discussions

1) In this experiment what type of capacitor have you used? Is it polarized or non-polarized capacitor? What type of precaution is required while connecting polarized capacitor in the circuit? [1]

Ans:

2) Sometimes it is observed that the capacitor is getting discharged, although the two electrodes of the capacitor are not anyhow connected. What might be the possible reason for it? [0.5]

Ans:

3) Sketch voltage across the resistor, V_R vs. time, t curve during charging and discharging process of the capacitor. [0.5]

Ans:

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