Math Assignment 2

SET 13

Section 13

MD Ikramul Kayes
ID:21301576

1) $y = \frac{3}{4} x^{\frac{4}{3}} - \frac{3}{8} x^{\frac{2}{3}} + 5$

∴ Deferentiating y we get

$$\frac{\partial y}{\partial x} = x^{\frac{1}{3}} - \frac{1}{4} x^{-\frac{1}{3}}$$

we know,

$$L = \int_a^b \sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2} \, \partial x$$

∴ length of the curve is,

$$L = \int_1^8 \sqrt{1 + \left(x^{\frac{1}{3}} - \frac{1}{4} x^{-\frac{1}{3}}\right)^2} \, \partial x$$

$$= \int_1^8 \sqrt{1 + x^{\frac{2}{3}} - \frac{1}{2} + \frac{1}{16} x^{-\frac{2}{3}}} \, \partial x$$

$$= \int_1^8 \sqrt{\left(x^{\frac{1}{3}} + \frac{1}{4} x^{-\frac{1}{3}}\right)^2} \, \partial x$$

$$= \int_1^8 \left(x^{\frac{1}{3}} + \frac{1}{4} x^{-\frac{1}{3}}\right) \partial x$$

$$= \left[\frac{3}{4} x^{\frac{4}{3}} + \frac{3}{8} x^{\frac{2}{3}}\right]_1^8$$

$$= \frac{99}{8}$$

$= 12.375$ (Ans)

2) $y = \sqrt{x}$

Deferentiating y ~~to~~ we get,

$$\frac{\partial y}{\partial x} = \frac{1}{2\sqrt{x}}$$

We know,

$$S = \int_a^b 2\pi y \sqrt{1+\left(\frac{\partial y}{\partial x}\right)}\ \partial x$$

$$= \int_{3/4}^{15/4} 2\pi y \sqrt{1+\left(\frac{1}{2\sqrt{x}}\right)^2}\ \partial x$$

$$= \int_{3/4}^{15/4} 2\pi y \sqrt{1+\frac{1}{4x}}\ \partial x$$

$$= \int_{3/4}^{15/4} 2\pi \sqrt{x} \sqrt{\frac{4x+1}{4x}}\ \partial x$$

$$= \pi \int_{3/4}^{15/4} \sqrt{4x+1}$$

$$= \pi \left[\frac{1}{6}(4x+1)^{\frac{3}{2}}\right]_{3/4}^{15/4}$$

$$= \frac{\pi}{6}\left[\left(4\times\frac{15}{4}+1\right)^{\frac{3}{2}} - \left(4\times\frac{3}{4}+1\right)^{\frac{3}{2}}\right]$$

$$= \frac{\pi}{6}\times 56$$

$$\cancel{= \frac{28}{3}\pi}$$

$$= \frac{28}{3}\pi \quad \text{(Ans)}$$

3) $$\int_0^1\int_0^{y^2} (3y^3 e^{xy})\,\partial x\,\partial y$$

$$= \int_0^1 \left[3y^2 e^{xy}\right]_0^{y^2} \partial y$$

$$= \int_0^1 \left[3y^2 e^{y^3} - 3y^2\right] \partial y$$

$$= \int_0^1 3y^2 (e^{y^3} - 1)\,\partial y$$

$$= \int_0^1 (e^u - 1)\, \partial u$$

$$u = y^3$$

$$\partial u = 3y^2 \cdot \partial y$$

y	1	0
u	1	0

$$= \left[e^u - u \right]_0^1$$

$$= [e - 1 - 1]$$

$$= (e - 2)$$

~~= 0.1~~

~~= 0.7188~~

$= 0.718282$ (Ans)

4) Given,

$y = \sqrt{x}$

$y = 6 - x$

We can also write them as,

$x = y^2$

$x = 6 - y$

$\therefore$ The region enclosed by $x = y^2$ and $x = 6 - y$

is,

$$\int_0^2 \int_{y^2}^{6-y} (xy)\, \partial x\, \partial y$$

$$= \frac{1}{2} \int_0^2 [x^2 y]_{y^2}^{6-y}\, \partial y$$

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$$= \frac{1}{2} \int_0^2 y\, [36 - 12y + y^2 - y^4]\, dy$$

$$= \frac{1}{2} \int_0^2 (36y - 12y^2 + y^3 - y^5)\, dy$$

$$= \frac{1}{2} \left[18y^2 - 4y^3 + \frac{1}{4} y^4 - \frac{1}{6} y^6 \right]_0^2$$

$$= \frac{50}{3}$$

5) $\displaystyle \int_{-4}^{0} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} 3x \, dy \, dx$

This equation can be written as ~~g~~ after converting it into polar co-ordinates

$$\int_{\pi/2}^{3\pi/2} \int_0^{-4} 3r \cos\theta \; r \, dr \, d\theta$$

$$= \int_{\pi/2}^{3\pi/2} \int_{0}^{4} 3r^2 \cos\theta \, \partial r \, \partial\theta$$

$$= \int_{\pi/2}^{3\pi/2} \int_{0}^{4} 3r^2 \cos\theta \, \partial r \, \partial\theta$$

$$= \int_{\pi/2}^{3\pi/2} \left[r^3 \cos\theta \right]_0^4 \partial\theta$$

$$= 64 \int_{\pi/2}^{3\pi/2} \cos\theta \, \partial\theta$$

$$= 64 \left[\sin\theta \right]_{\pi/2}^{3\pi/2}$$

$$= -128$$

Do the following tasks using Mathematica.

y

2 = 2x + 6

y = x − 1

(a) Plot the curves in a single graph for $-5 \le x \le 10$ and shade the bounded

region only. Then find the area of the bounded region using Area[ ] function.

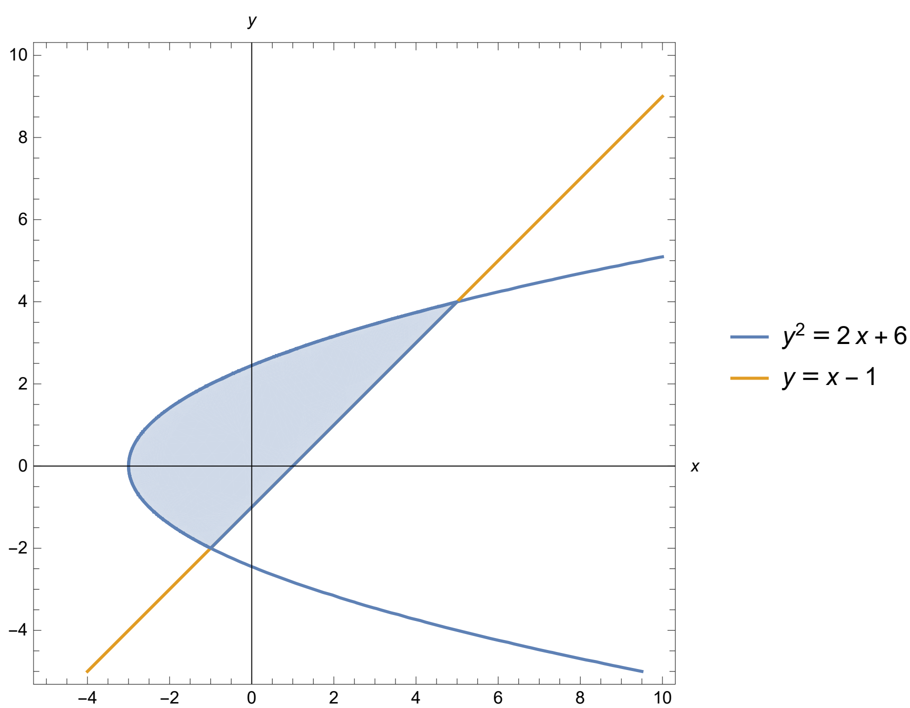
Ans:

In[1]:=
```
plot1 = ContourPlot[{y^2 == 2 x + 6, y == x - 1}, {x, -5, 10}, {y, -5, 10},
   Axes → True, AxesLabel → Automatic, PlotLegends → "Expressions"];
```

In[2]:=
```
region1 = ImplicitRegion[x < y + 1 && x > (y^2 - 6)/2, {{x, -5, 10}, {y, -5, 10}}];
```

In[3]:=
```
Show[plot1, RegionPlot[region1]]
```

Out[3]=



In[4]:=
```
Area[region1]
```

Out[4]= 18

(b) Find the length of the curve $y = \frac{1}{x}$ for $1 \le x \le 2$. Do not use the built-in ArcLength[ ] function.

Ans:

In[5]:=
```
ClearAll["Global`*"]
```

In[6]:= $D\left[\frac{1}{x}, x\right]$

Out[6]= $-\frac{1}{x^2}$

In[7]:= $L = \int_1^2 \sqrt{1 + \left(-\frac{1}{x^2}\right)^2}\, dx$ // N

Out[7]= $1.13209 - 4.44089 \times 10^{-16}\, i$

As $4.44089 \times 10^{-16}\, i$ is close to 0 we can say the length of the curve $y = \frac{1}{x}$ is 1.13209

So the answer is 1.13209

(c) Find the surface area obtained by rotating the curve

i. $y = x^3$, $0 \le x \le 1$ about y axis.

ii. $y = \cos\left(\frac{x}{2}\right)$, $0 \le x \le \pi$ about x axis.

Ans:

i)

In[8]:= $D\left[y^{\frac{1}{3}}, y\right]$

Out[8]= $\frac{1}{3\, y^{2/3}}$

In[9]:= $S1 = \int_0^1 2\pi y^{\frac{1}{3}} \sqrt{1 + \left(\frac{1}{3\, y^{2/3}}\right)^2}\, dy$ // N

Out[9]= 5.91943

ii)

In[10]:= $D\left[\mathrm{Cos}\left[\frac{x}{2}\right], x\right]$

Out[10]= $-\frac{1}{2} \mathrm{Sin}\left[\frac{x}{2}\right]$

In[11]:= $S2 = \int_0^\pi 2\pi\, \mathrm{Cos}\left[\frac{x}{2}\right] \sqrt{1 + \left(-\frac{1}{2} \mathrm{Sin}\left[\frac{x}{2}\right]\right)^2}\, dx$ // N

Out[11]= 13.0719