CSE-250

Assignment- 04

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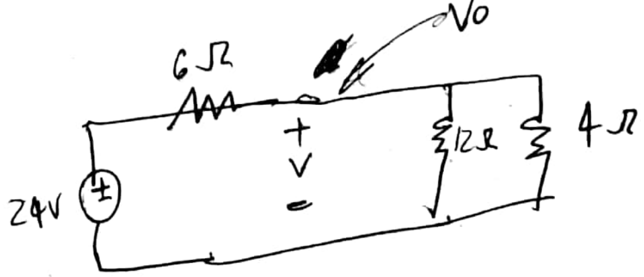
Sec : 07

Slide-22

Problem 2



⇐ for $t<0$

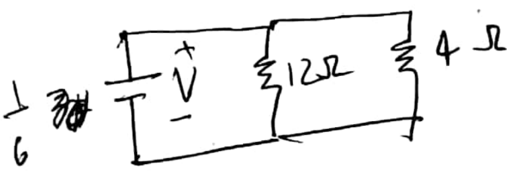
$-24+6i+3i=0$

$\therefore i = 2.67A$

$-24+6\times2.67+V_0$

$\therefore V_0 = 8$

for $t>0$,



$R_{eq} = 12 \| 4 = 3$

$\tau = RC = 3\times\frac{1}{6} = \frac{1}{2}$ sec

$\Rightarrow V_c = V_0 e^{-t/RC}$

$\therefore V_c = 8e^{-t/RC}$ V

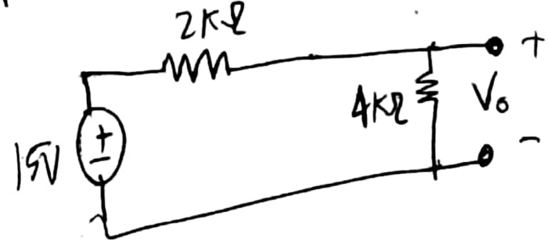
$\therefore W_c = \frac{1}{2} C V_0^2$

$= \frac{1}{2}\times\frac{1}{6}\times(8)^2$

$= 5.333$ J (Ans)

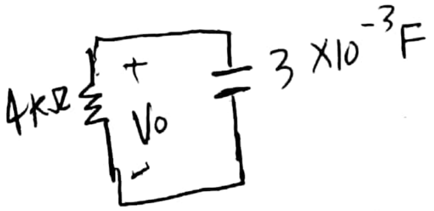
Problem-3

For $t < 0$,



$$V_0 = \frac{4}{2+4} \times 15 = 10\,V$$

For $t > 0$,



$$\tau = RC = 4 \times 10^{3} \times 3 \times 10^{-3} = 12\ \text{sec}$$

$$\Rightarrow V_C = V_0\, e^{-t/\tau}$$

$$= 10\, e^{-t/12}$$

$\therefore V_C$

Problem 4)

a) $i = \frac{V}{R}$

$\Rightarrow R = \frac{V}{i}$

$= \frac{10e^{-4t}}{0.2e^{-4t}} = 50$

$\tau = RC$

$\Rightarrow C = \frac{\tau}{R} = \frac{\frac{1}{4}}{50} = 5 \times 10^{-3} F$

b) $\tau = \frac{1}{4} = 0.25$ sec

c) $W_e(0) = \frac{1}{2} C V_0^2 = \frac{1}{2} \times 5 \times 10^{-3} \times (10)^2$

$= 0.25 J$

d) $W = \frac{1}{2} C V_0^2 (1 - e^{-2t/\tau})$

$\Rightarrow 0.25 \times \frac{50}{100} = \frac{1}{2} \times 5 \times 10^{-3} \times (10)^2 \times (1 - e^{-\frac{2t}{0.25}})$

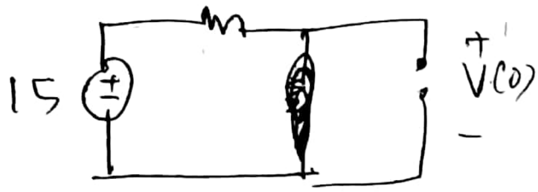
$\Rightarrow e^{-2t/RC} = 0.5$

$\Rightarrow t = 0.0866$

$= 86 ms$

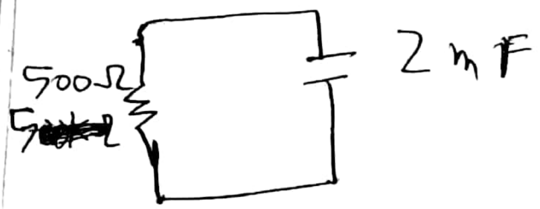
Problem 5

$t < 0$



Here; we can easily say $V(0) = 15$

∴ $1 < t < 0$,



Here,

$\tau = RC$

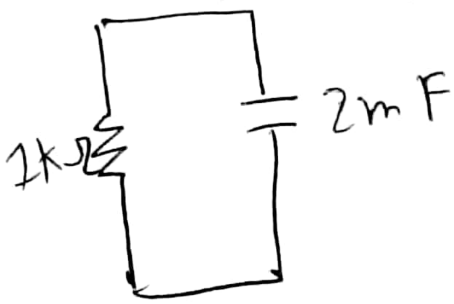
$= 500 \times 10^{3} \times 2 \times 10^{-3}$

$= \cancel{1000}\ 1$

$V(t) = 15 e^{-t/\tau}$

∴ $V(t) = 15 e^{-t/1}$

For $t > 1$,



we got previously,

$$V_o(t) = 15 e^{-t}$$

$$V_o(1) = 5.518$$

$$V = V_o e^{-\frac{(t-t_o)}{\tau}}$$

$$\therefore V(t) = 5.518 e^{-\frac{1}{2}(t-1)}$$

$$\tau = RC$$

$$= 1 \times 10^3 \times 2 \times 10^{-3}$$

$$= 2 \text{ sec}$$

Problem 6

← For $t<0$

Here,

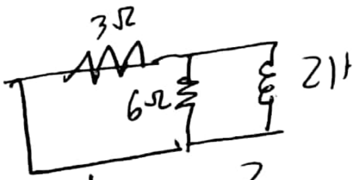
$5i = 10$

$\therefore i = 2A$

$i_0 = 0$

$\therefore V_0 = \frac{3}{3+2} \times 10 = 6V$

For $t>0$



$Req = 3 \| 6 = 2\Omega$

$\tau = \frac{L}{R_{eq}} = \frac{2}{2} = 1$ sec

$\therefore i(t) = i e^{-t/\tau} = 2e^{-t}$

$\therefore V_0(t) = iR$
$= 2 \times 2e^{-t}$
$= 4e^{-t}$

$i_0 = \frac{R_{eq}}{R_1} \times 2e^{-t}$
$= -\frac{2}{6} \times 2e^{-t}$
$= -\frac{2}{3} e^{-t}$

Problem 7

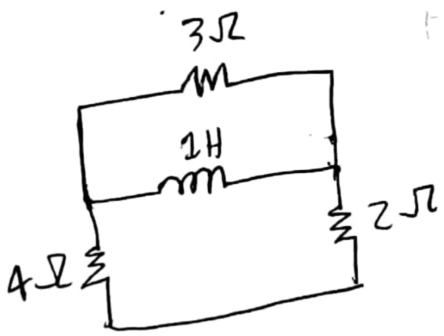
For $t<0$

$$i = \frac{4 \parallel 2}{2} \times 24 = 16\,A$$

$$i_0 = 24 - 16 = 8\,A$$

$$\therefore V_0 = iR = 16 \times 2 = 32\,V$$

For, $t>0$



$$\tau = \frac{L}{R} = \frac{1}{2}$$

$$i(t) = i\,e^{-\cancel{2}t/\tau}$$

$$= 16e^{-2t}$$

$$i_0 = \frac{-2}{6} \times 16e^{-2t}$$

$$= -5.333e^{-2t}$$

$$V_0 = i_{\cancel{x}} R$$

$$= 5.333e^{-2t} \times 2$$

$$= 10.667\,e^{-2t}$$

$\therefore v(t) = 5.518\, e^{-\frac{1}{2}(t-1)}$

Problem - 8]

a] we know,

$$\tau = \frac{L}{R}$$

$$= \frac{L}{V/i} = \frac{L}{\frac{90e^{-50t}}{30/e^{-50t}}} = \frac{L}{3}$$

$\therefore L = 3\tau$

$$= 3 \times \frac{1}{50}$$

$$= 0.06$$

$\therefore L = 60\, mH$

Also, $R = \frac{V}{i} = \frac{90 e^{-50t}}{30 e^{-50t}} = 3\Omega$

b) we know,

$$V(t) = 90 e^{-\frac{t}{\tau}}$$

if we compare the equation with $V = 90e^{-\frac{t}{\tau} \times 50}$

$\therefore \tau = \frac{1}{50} = 0.02 \text{sec}$

c) we know, $W_0 = \frac{1}{2} L i_0^2$

$= \frac{1}{2} \times 60 \times 10^{-3} (30)^2$

$= 27 \text{ J}$

d) we know,

$W_R(t) = \frac{1}{2} L i^2 (1 - e^{-2t/\tau})$

$= 6.2786$

$\therefore$ fraction of energy is $(100 - W_R(t))$

$= 93.72$

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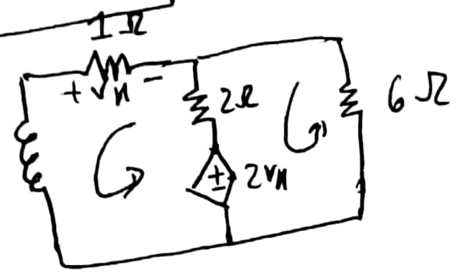
Problem 9



$V_x = -i$

$$\Rightarrow -V_x + L\frac{\partial i_1}{\partial t} - 2V_x + 2(i_1 - i_2) = 0$$

$$\Rightarrow i_1 + 2\frac{\partial i}{\partial t} + 2i_1 + 2i_1 - 2i_2 = 0$$

$$\Rightarrow 5i_1 + 2\frac{\partial i_1}{\partial t} - 2i_2 = 0 \quad \text{—— (1)}$$

also,

$$6i_2 + 2(i_2 - i_1) + 2V_x = 0$$

$$\Rightarrow 6i_2 + 2i_2 - 2i_1 - 2i_1 = 0$$

$$\Rightarrow 2i_2 = i_1$$

$$\therefore i_2 = 0.5 i_1$$

Putting i_2 in eq (1),

$$\Rightarrow 5i_1 + 2\frac{\partial i_1}{\partial t} - i_1 = 0$$

$$\Rightarrow \frac{\partial i_1}{\partial t} = -2i_1$$

$$\therefore \int \frac{\partial i_1}{i_1} = \int -2\,\partial t$$

$$\Rightarrow \ln(i_1) = -2t + c$$

$$\Rightarrow i_1 = e^{-2t + c}$$

$$= c_2 e^{-2t}$$

$$\therefore i(0) = \cancel{7}\, c_0 e^{-2\times 0} \qquad [\text{As } i(0) = 7]$$

$$\Rightarrow 7 = c_0$$

$$\therefore i(t) = 7e^{-2t}$$

$$\therefore v_n = -i_1$$

$$= -7e^{-2t}$$

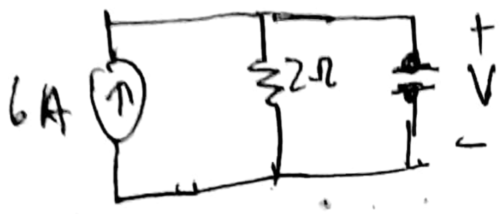
Problem 12

" '

$$= -7e^{-2t}$$

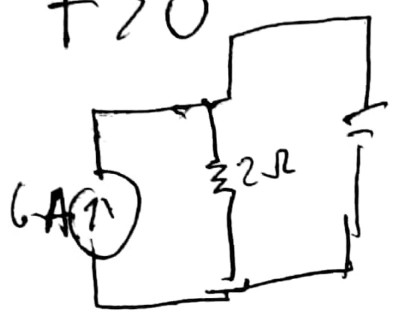
Problem 12

$t < 0$



$$V_0 = 6 \times 2 = 12V$$

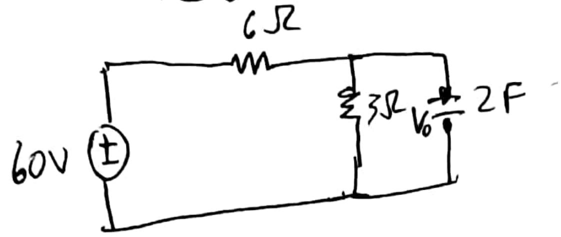
$t > 0$



$$V(t) = 6 \times 2 = 12V$$

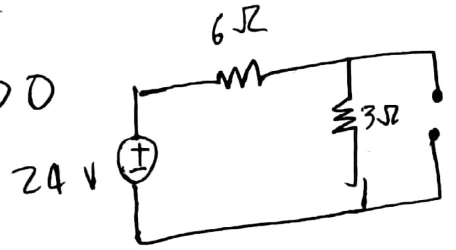
Problem-13

$t < 0$



$$V_0 = \frac{3}{3+6} \times 60 = 20\,V$$

$t > 0$



$$\tau = 3 \| 6 \times 2 = 4$$

$$V(\infty) = \frac{3}{6+3} \times 24 = 8\,V$$

$$V(t) = V(\infty) + [V(0) - V(\infty)]\, e^{-t/RC}$$

$$= 8 + (20 - 8)\, e^{-t/4}$$

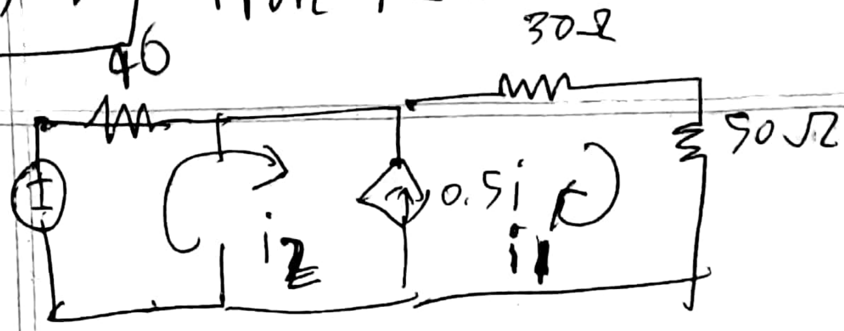
$$= 8 + 12 e^{-t/4}$$

$$i(t) = C \frac{\partial V}{\partial t}$$

$$= 2 \times \left(0 - \frac{12}{4} e^{-t/4}\right)$$

$$= -6 e^{-t/4}$$

Prob-19) For $t<0$



$-80 + 40i_2 + 30i_1 + 50i_1 = 0$

$\Rightarrow 80i_1 + 40i_2 = 8$ —— (i)

$i_1 - i_2 = 0.5i_1$

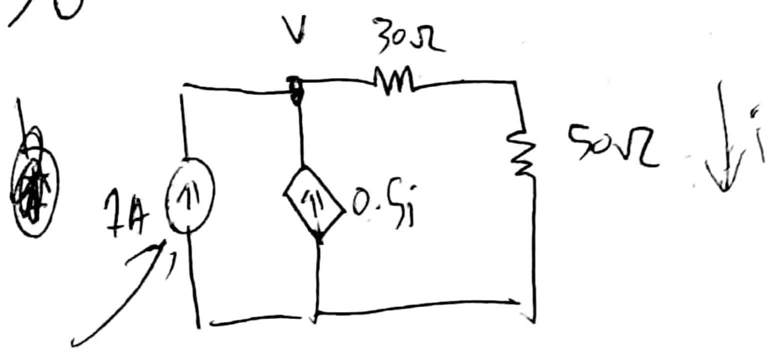
$\Rightarrow 0.5i_1 - i_2 = 0$ —— (ii)

solving eq (i) (ii) we get

$i_1 = 0.8$, $i_2 = 0.4$

~~Let~~ $i(0) = 0.8$

For $t > 0$



dummy source

$i = \frac{V}{80}$

$V\left(\frac{1}{80}\right) - 1 - \frac{V}{80} \times 0.5 = 0$

$V = 160\,V$

$R = \frac{V}{i} = \frac{160}{1} = 160\,\Omega$

$\tau = RC = 160 \times 3 = 480$ sec

~~$i(t) = i(\infty$~~

Also, $i(\infty) = 0$

$\Rightarrow i(t) = i(\infty) + [i(0) - i(\infty)]\, e^{-t/\tau}$

$\therefore i(t) = +0.8 e^{-t/480}$

Problem 15

Prob 15)

(i) Ans: The time constant of a circuit is the time required the response to decay to a factor of 1/e or 36.8% of it's initial value.

(ii) Given,

$$5\tau = 45 \times 10^{-3} \text{ sec} \leftarrow \text{Total time}$$

$$\Rightarrow \tau = 9 \times 10^{-3} \text{ sec}$$

(iii) Given, $V(\infty) = 6V$

$$V(0) = -2V$$

$$V(t) = V(\infty) + [V(0) - V(\infty)]\, e^{-t/\tau}$$

$$= 6 + [-2 - 6]\, e^{-t/9\times10^{-3}}$$

$$= 6 - 8e^{-\frac{1000t}{9}}$$

IV) We know, $W_0 = \frac{1}{2} C V_0^2$

Here,

$$\tau = RC$$

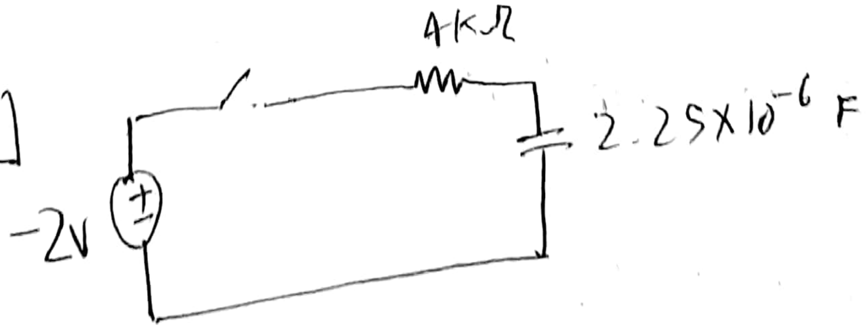
$$\Rightarrow C = \frac{9 \times 10^{-3}}{4 \times 10^{3}} = 2.25 \times 10^{-6}$$

$$\therefore W_0 = \frac{1}{2} C V_0^2$$

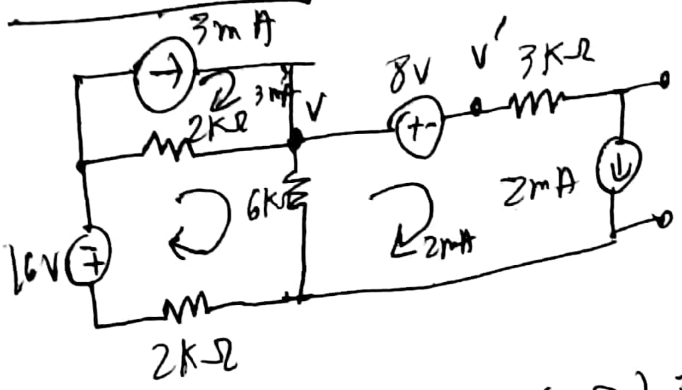
$$= \frac{1}{2} \times 2.25 \times 10^{-6} \times (-2)^2$$

$$= 4.5 \times 10^{-6} \text{ J}$$

V)



Problem 16



$$2i + 16 + 2(i-3) + 6(i-2) = 0$$

$$\Rightarrow 10i = 2$$

$$\therefore i = 0.2$$

$$V - 0 = 6(i-2)$$

$$\Rightarrow V = 6(0.2-2)$$

$$= -10.8$$

Also,

$$V - V' = 8$$

$$\Rightarrow V' = V - 8$$

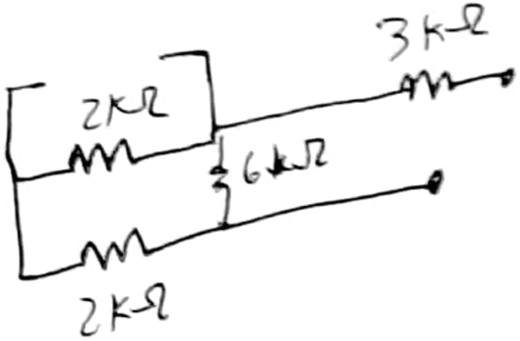
$$= -10.8 - 8$$

$$= -18.8$$

$$V' - V_{th} = 3 \times 2$$

$$\Rightarrow V_{th} = -6 - 18.8 = -24.8$$

For R_{th},



$$R_{th} = (4 \| 6 + 3) = 5.4\,k\Omega$$

$$\tau = RC = 5.4 \times 10^{3} \times 1 \times 10^{-6}$$

$$= 5.4\,ms$$

$$V(t) = -24.5 + 24.5\, e^{-t/(5.4 \times 10^{-3})}$$

$$i(t) = C \frac{\partial v}{\partial t} = 1 \times 10^{-6} \times \left(- \frac{24.5}{5.4 \times 10^{-3}} e^{-\frac{t}{(5.4 \times 10^{-3})}} \right) \times 10^{3}$$

$$= 4.59\, e^{-\frac{t}{5.4 \times 10^{-3}}}$$

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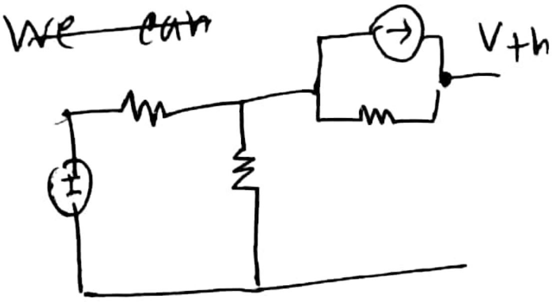
(11) ~~T = 9ms from~~

~~(111)~~

Problem 17

~~We can~~

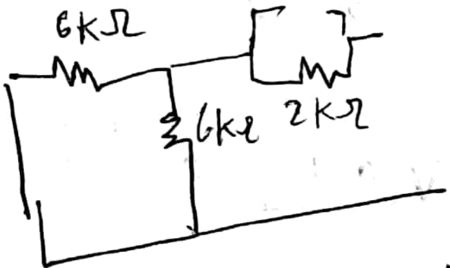


↓

From the circuit we can say the V_{th} is 20 V

$\therefore V_{th} = 20V$

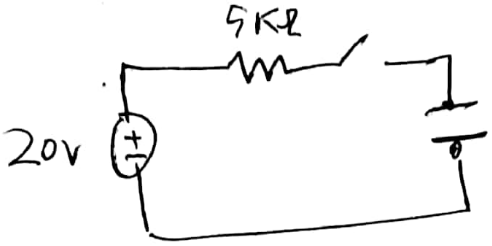
For R_{th}



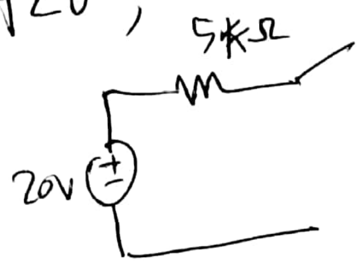
$R_{th} = 2 + (6 \| 6)$

$= 5K\Omega$

Thevenin circuit

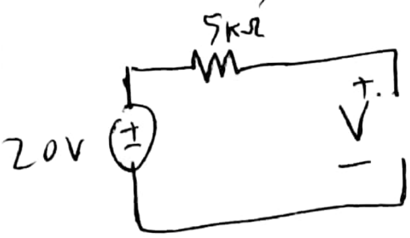


For $t<0$,



$\therefore v(0) = 0$

for $t>0$,



$V(\infty) = 20V$

$\tau = RC = 5\times10^{3} \times 100\times10^{-6} = 0.5 \text{ sec}$

$V_c(t) = \cancel{8}\, V(\infty) + [V(0) - V(\infty)]\, e^{-t/\tau}$

$\cancel{= 20 + 20e^{-t/0}}$

$= 20 - 20e^{-t/0.5}$

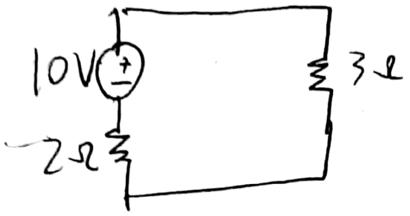
$i(t) = C \frac{\partial}{\partial t}$ ... $- \frac{t}{5.4 \times 10^{-3}}$

$= 4.59\, e^{-\frac{t}{5.4 \times 10^{-3}}}$

~~Problem 18~~

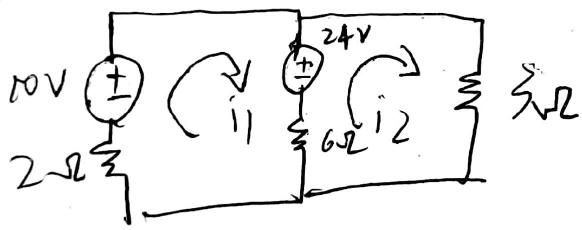
Problem 18

$t < 0$



$i_0 = 2A$

$t > 0$



$2i_1 - 10 + 24 + 6(i_1 - i_2)$

$\Rightarrow 8i_1 - 6i_2 = -14$ — (i)

$3i_2 + 6(i_2 - i_1) - 24 = 0$

$\Rightarrow -6i_1 + 9i_2 = 24$ — (ii)

Solving eq (i) and (ii) we get,

$i_1 = 0.5$, $i_2 = 3$

Here, $i(\infty) = i_2 = 3$

For Req

$R_{eq} = 6 \| 2 + 3$
$= 4.5$

$$\tau = \frac{L}{R} = \frac{2}{4.5} = \frac{4}{9}$$

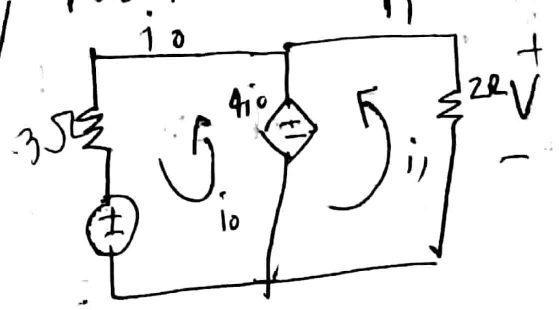
$$i(t) = i(\infty) + (i(0) - i(\infty))e^{-\frac{t}{\tau}}$$

$$= 3 + (2-3)e^{-\frac{9}{4}t}$$

$$= 3 - e^{-\frac{9}{4}t}$$

Problem 19

For $t < 0$



$3i_0 + 24 - 4i_0 = 0$ — (i)

$4i_0 + 2i_1 = 0$ — (ii)

solving equation (i), (ii) we get,

$i_0 = 24$

$i_1 = -2i_0 = -48$

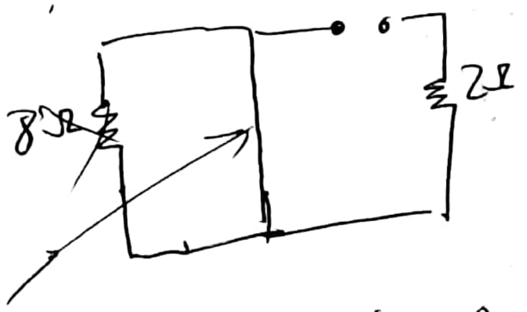
$V(t) = -2 \times -48 = 96 \cdot V$

$t > 0$,

Here, $i_0 = 0$, so $4i_0 = 0$,

The circuit can be drawn

For ~~R~~ Req,



As $i_0 = 0$, so $4i_0 = 0$

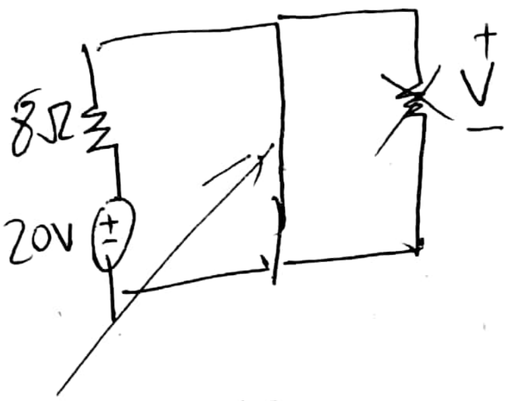
~~R~~ $R_{eq} = 2\Omega$

$\therefore \tau = \frac{L}{R}$

$= \frac{0.5}{2}$

$= 0.25$ sec

For $V(\infty)$



As $i_0 = 0$, so $4i_0 = 0$

~~∴ As~~

$\therefore V(\infty) = 0$

$\therefore V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$

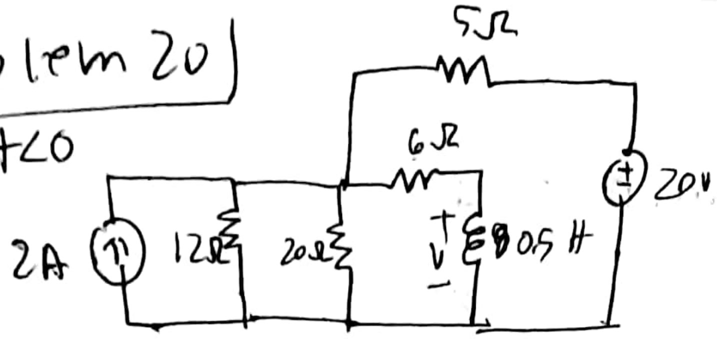
~~$= 0 + [96-0]e^{-4t}$~~

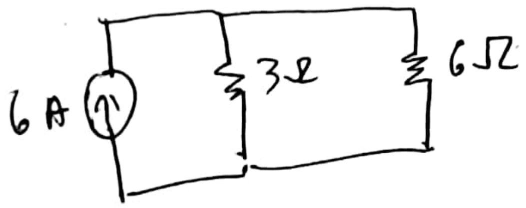
$= 0 + [96 - 0]e^{-\frac{t}{0.25}}$

$= 96e^{-4t}$ (Ans)

Problem 20

when $t<0$

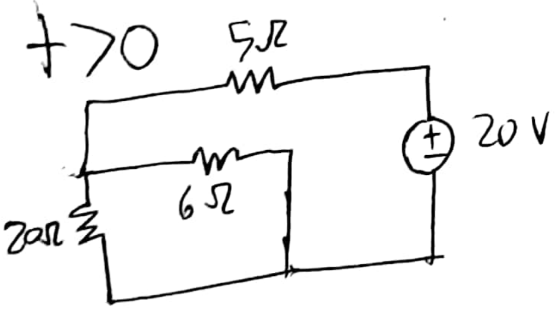




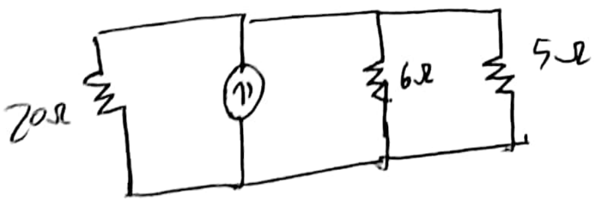
$$i(0) = \frac{3}{3+6} \times 6$$

$$i(0) = 2A$$

$t>0$



can be drawn as,



$$i(\infty) = \frac{20\|6\|5}{6} \times 4$$

$$\therefore i(\infty) = 1.6A$$

$$R_{eq} = 6 + (20\|5)$$

$$= 10\Omega$$

$$\therefore \tau = \frac{L}{R} = \frac{0.5}{10} = 0.05 \text{ sec}$$

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

$$= 1.6 + (2-1.6)e^{-\frac{t}{0.05}}$$

$$\therefore i(t) = 1.6 + 0.4\, e^{-\frac{t}{0.05}}$$

$$\therefore v(t) = L\frac{\partial i}{\partial t}$$

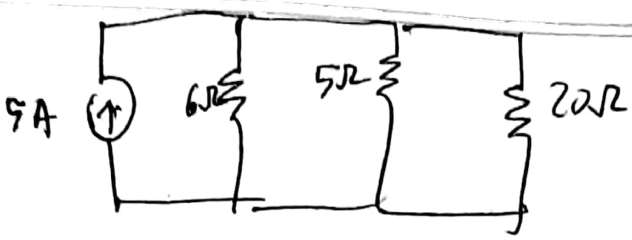
$$= 0.5\left(0 + \frac{-0.4}{0.05} e^{-\frac{t}{0.05}}\right)$$

$$\therefore v(t) = -4e^{-20t}$$

Prob-21

$R_{eq} = 2.4$

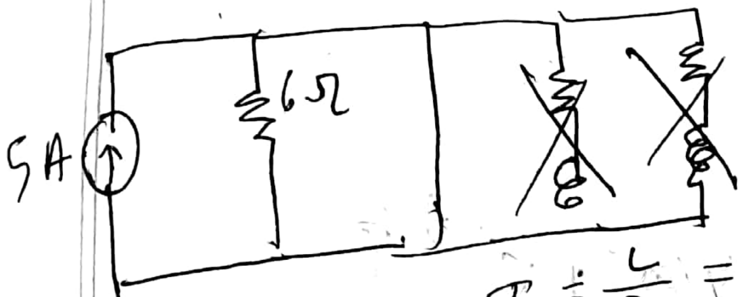
$t < 0$



$i_1(0) = \frac{2.4}{5} \times 5 = 2.4$

$i_2(0) = \frac{2.4}{20} \times 5 = 0.6$

$t > 0$



$i_1(\infty) = 0 \rightarrow \tau_1 = \frac{L}{R} = \frac{2.5}{5} = 0.5$

$i_2(\infty) = 0 \rightarrow \tau_2 = \frac{L}{R} = \frac{4}{20} = 0.2$

$i_1(t) = i_1(\infty) + [i_1(0) - i_1(\infty)]e^{-2t}$

$= 2.4e^{-2t}$

$i_2(t) = i_2(\infty) + [i_2(0) - i_2(\infty)]e^{-5t}$

$= 0.6e^{-5t}$