January 8, 2022

Department of Mathematics and Natural Sciences

PHY111 - Principles of Physics-I

Final Assessment, Fall 2021

Time: 1 Hour (6:00 pm to 7:00 pm)

Total Marks: 20

Answer any two questions.

1. As shown in Fig. 1, a block of mass *m* is falling from some height h along an inclined plane. It then slides across a frictionless surface and enters a circular loop of radius R = 5 meters. The magnitude of gravitational acceleration is given by 9.81 m/s^2.

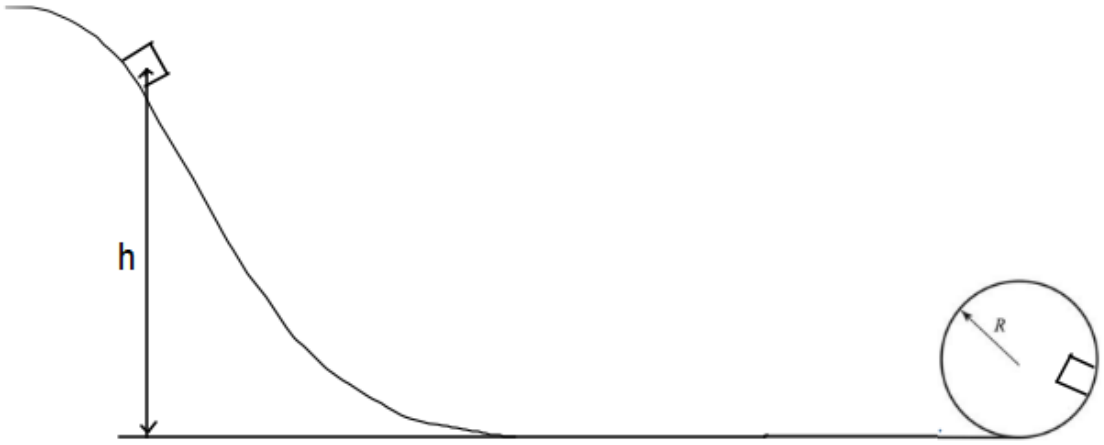

Fig. 1

(a) (*3 marks*) If h = 20 meters, what is the magnitude of velocity of the block when it is travelling along the horizontal surface?

(b) (*3 marks*) Calculate the magnitude of minimum tangential velocity with which the block must travel when it is at the top position of the circular loop such that it does not fall off. (For the minimum speed, the block will almost lost contact with the surface of the loop, making the normal reaction force zero)

(c) (*4 marks*) Find the minimum height h from where the block needs to be released on the inclined plane such that it can make a complete turn along the circular loop.

2. A satellite of mass 190 kg is placed into an orbit around Earth at a height of 700 km above the surface. (Earth's radius is 6.37×10^6 m and mass of the earth is 5.972×10^{24} kg)

(a) (*3 marks*) Assuming a circular orbit, how long does the satellite take to complete one orbit?

(b) (*3 marks*) What is the satellite's speed?

(c) (*4 marks*) Starting from the satellite on the Earth's surface, what is the minimum energy input necessary to place this satellite in orbit? Ignore air resistance but include the effect of the planet's daily rotation.

3. A particle of mass 20 g is connected to one end of a spring on a frictionless horizontal surface and moving in a simple harmonic motion about the origin. The displacement given by $x = 2 \sin 3t$, where *x* is in meters and time, *t* is in seconds. The motion starts when *t* = 0.

(a) (*3 marks*) What is the total distance travelled by the particle at time *t* = 3T/4, where T is the time period?

(b) (*3 marks*) Find the kinetic and potential energy of the particle at time *t* = 3T/4.

(c) (*4 marks*) What is the acceleration of the particle when it first comes to a rest?

2] a] Given that,

$$h = 700\ km$$

$$= 700 \times 10^3$$

we know,

$$T = \sqrt{\left(\frac{4\pi^2}{GM}\right)(R+h)^3}$$

$$= \sqrt{\left(\frac{4\pi^2}{6.67\times10^{-11} \times 5.972\times10^{24}}\right)(6.37\times10^6 + 700\times10^3)^3}$$

$$= 5918.15\ sec$$

$$= 1.644\ h$$

(Ans)

b)

we know,

Satelite's speed

$$v = \sqrt{\frac{GM}{(R+h)}} = \sqrt{\frac{6.67\times10^{-11}\times5.972\times10^{24}}{(700\times10^{3}+6.37\times10^{6})}}$$

$$= 7506.078\ ms^{-1}$$

c] we know,

satalites speed on earth surface is,

$$v_e = \frac{2\pi R}{24\times60\times60} = \frac{2\pi(6.37\times10^{6})}{86400} = 465.239\ ms^{-1}$$

On surface,

$$\text{Energy} = \frac{1}{2}mv_e^2 - G\frac{Mm}{R}$$

On orbit,

$$\text{Energy} = \frac{1}{2}mv^2 - G\frac{Mm}{R+h}$$

~~On surface~~ $E_E = \frac{1}{2}mv^2 - G\frac{Mm}{R}$

$\equiv$

~~Energy~~

The Energy required to put the satellite on orbit,

$$\Delta E = \frac{1}{2}mV_0^2 - G\frac{Mm}{R+h} - \frac{1}{2}\cdot mv_e^2 + G\frac{Mm}{R}$$

$$= \frac{1}{2}190 \times (7500.078)^2 - 6.67\times10^{-11}\times\frac{5.972\times10^{24}\times\cdots}{6.37\times10^6+700\times10^3}$$

$$- \frac{1}{2}190\times(463.239)^2 + \frac{6.67\times10^{-11}\times5.972\times10^{24}\times190}{6.37\times10^6}$$

$$= 6.52\times10^9\ \text{J} \quad (\text{Ans})$$

23] a) equation $x = 2\sin 3t$

we know: $x = A\sin \omega t$

$\omega = 2$

we know,

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

$$= \frac{2\pi}{3}$$

Given, $t = \frac{3T}{4}$

$x = 2\sin 3t$

$= 2\sin 3\frac{3T}{4}$

$= 2\sin \frac{9}{4}\frac{2\pi}{3}$

$= \sqrt{2} - 2$

∴ Total distance travelled $\sqrt{2} - 2$

(Ans)

b) From (a) we get at $\frac{3T}{4}$ the distance traveled $\sqrt{2} - 2$

∴ potential energy

$E_p = \frac{1}{2} kx^2$

$= \frac{1}{2} 0.08 \times (\sqrt{2})^2$

$= 0.08 \times 0.16$

$= 0.36 J$

$w = \sqrt{\frac{k}{m}}$

$\Rightarrow w^2 = \frac{k}{m}$

$\Rightarrow k = w^2 m$

$= (2)^2 \times 20 \times 10^{-3}$

$= 0.08 m$

kenetic energy,

d) $E_K = \frac{1}{2} k(A^2 - x^2)$

$= \frac{1}{2} \times 0.28 \times (4 - 4)$

$= 0$

Potential energy $E_p = 0.36\ J$

Kenetic energy $E_K = 0\ J$ (Ans)

e) we get, $w = 3\ rads^{-1}$

$A = 2$

we know,

$a = -w^2 A$

$= -3^2 \times 2$

$= -18\ rads^{-2}$

We know,

$$h = 2\sin 3t$$

$$v = 6\sin 6(90) 3t \quad @ 6\cos 3t$$

$$\Rightarrow 0 = 6s$$

$v = 0$, when it comes rest,

$$\Rightarrow 0 = 6\cos 3t$$

$$\Rightarrow 3t = \frac{\pi}{2}$$

$$t = \frac{\pi}{6}$$

$$\frac{dv}{dt} = -18\sin 3t$$

$$\Rightarrow a = -18\sin\left(3\,\frac{\pi}{6}\right) = -18 \text{ rads}^{-2}$$

(Ans)