Math Assignment 4

SET 13

Section 13

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1) $(5xy+4y^2+1)\,\partial x+(x^2+2xy)\,\partial y=0$

$\Rightarrow (5xy+4y^2+1)+(x^2+2xy)\frac{\partial y}{\partial x}=0$

$\therefore M=5xy+4y^2+1$ | $N=x^2+2xy$

$\therefore \frac{\partial M}{\partial y}=5x+8y$ | $\frac{\partial N}{\partial x}=2x+2y$

As $\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$ [We need to find intregating factor As it is ~~inex~~ inexact equation]

$\therefore \frac{M_y-N_x}{N}$

$=\frac{5x+8y-2x-2y}{x^2+2xy}$

$=\frac{3x+6y}{x^2+2xy}$

$$= \frac{3(x+2y)}{x(x+2y)}$$

$$= \frac{3}{x}$$

we know,

$$\frac{\partial \mu}{\partial x} = \frac{3}{x} \mu$$

$$\Rightarrow \frac{\partial \mu}{\mu} = \frac{3\,\partial x}{x}$$

$$\Rightarrow \ln|\mu| = 3\ln|x|$$

$$\therefore \mu = x^3$$

Multiplying the equation with intregating factor.

we get,

$$(5x^4y + 4x^3y^2 + x^3) + (x^5 + 2x^4y)\,y' = 0$$

$M = 5x^4y + 4x^3y^2 + x^3$ | $N = x^5 + 2x^4y$

$\frac{\partial M}{\partial y} = 5x^4 + 8x^3y$ | $\frac{\partial N}{\partial x} = 5x^4 + 8x^3y$

As, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$$\psi_x = M(x,y) = 5x^4y + 4x^3y^2 + x^3 \text{ --- (I)}$$

$$\psi_y = N(x,y) = x^5 + 2x^4y \text{ --- (II)}$$

Now,

$$\int \psi_x = x^5y + x^4y^2 + \frac{1}{4}x^4 + h(y)$$

$$\therefore \psi = x^5y + x^4y^2 + \frac{1}{4}x^4 + h(y)$$

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Now,

$$\frac{\partial}{\partial y}(\psi) = \frac{\partial}{\partial y}\left(x^5 y + x^4 y^2 + \frac{1}{4}x^4 + h(y)\right)$$

$$\psi_y = x^5 + 2x^4 y + h'(y) \text{ --- (iii)}$$

From equation (ii) and (iii) we can write,

$$x^5 + 2x^4 y + h'(y) = x^5 + 2x^4 y$$

$$\therefore h'(y) = 0$$

$$\Rightarrow \int h'(y) = \int 0$$

$$\Rightarrow h(y) = c$$

$$\therefore \psi(x,y) = x^5 y + x^4 y^2 + \frac{1}{4}x^4 + c$$

$$\therefore x^5 y + x^4 y^2 + \frac{1}{4}x^4 = c$$

(Ans)

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2] $\left(\frac{3-y}{x^2}\right)\partial x + \left(\frac{y^2-2x}{xy^2}\right)\partial y = 0$

$$\Rightarrow \frac{3-y}{x^2} + \left(\frac{y^2-2x}{xy^2}\right) \cdot \frac{\partial y}{\partial x} = 0$$

$$M = \frac{3-y}{x^2} \qquad N = \frac{y^2-2x}{xy^2}$$

$$\frac{\partial M}{\partial y} = -\frac{1}{x^2} \qquad \frac{\partial N}{\partial x} = -\frac{1}{x^2}$$

As, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ so we can say it is an exact equation.

$$\psi_x = M(x,y) = \frac{3-y}{x^2} \quad \text{--- (i)}$$

$$\psi_y = N(x,y) = \frac{y^2-2x}{xy^2} \quad \text{--- (ii)}$$

$$\int \psi_x = \int \left(\frac{3}{x^2} - \frac{y}{x^2}\right) \partial x$$

$$\psi = -\frac{3}{x} + \frac{y}{x} + h(y)$$

$$\frac{\partial}{\partial y}(\psi) = \frac{\partial}{\partial y}\left(-\frac{3}{x} + \frac{y}{x} + h(y)\right)$$

$$\Rightarrow \psi_y = \frac{1}{x} + h'(y) \quad \text{— (iii)}$$

From equation (ii) and (iii) we can write,

$$\frac{1}{x} + h'(y) = \frac{1}{x} - \frac{2}{y^2}$$

$$\therefore h'(y) = -\frac{2}{y^2}$$

$$\Rightarrow \int h'(y) = \int \left(-\frac{2}{y^2}\right) \partial y$$

$$= h(y) = \frac{2}{y} + c$$

$\therefore \cancel{\psi = \frac{3}{x} + \frac{y}{x}}$

Putting the value of $h(y)$ in the equation,

$$\therefore \psi = -\frac{3}{x} + \frac{y}{x} + \frac{2}{y} + c$$

$$\therefore -\frac{3}{x} + \frac{y}{x} + \frac{2}{y} = c$$

As, $y(-1) = 2$ we can get

$$\Rightarrow \frac{-3}{-1} + \frac{2}{-1} + \frac{2}{2} = c$$

$$\therefore c = 2$$

Putting the value of c in the equation we ge

$$-\frac{3}{x} + \frac{y}{x} + \frac{2}{y} = 2$$

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The equation we get is,

$$\Rightarrow -\frac{3}{x} + \frac{y}{x} + \frac{z}{y} = 2$$

(Ans)

3] $y'' + y' + 2y = 0$

The corresponding auxilary equation is,

$$r^2 + r + 2 = 0$$

$$\therefore r = -\frac{1}{2} + \frac{\sqrt{7}}{2}i \; , \; -\frac{1}{2} - \frac{\sqrt{7}}{2}i$$

We know,

$$y_c = e^{\lambda x}(c_1 \cos(\alpha x) + c_2 \sin(\alpha x))$$

Here,

$$\lambda \pm \alpha i = -\frac{1}{2} \pm \frac{\sqrt{7}}{2}i$$

$$\therefore \lambda = -\frac{1}{2}$$

$$\alpha = \frac{\sqrt{7}}{2}$$

$$\therefore y(x) = e^{-\frac{1}{2}x}\left(c_1 \cos\left(\frac{\sqrt{7}}{2}x\right) + c_2 \sin\left(\frac{\sqrt{7}}{2}x\right)\right)$$

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$$y'(x) = -\frac{1}{2}e^{-\frac{1}{2}x}\left(c_1\cos\left(\frac{\sqrt{7}}{2}x\right) + c_2\sin\left(\frac{\sqrt{7}}{2}x\right)\right)$$

$$+\frac{\sqrt{7}}{2}e^{-\frac{1}{2}x}\left(c_2\cos\left(\frac{\sqrt{7}}{2}x\right) - c_1\sin\left(\frac{\sqrt{7}}{2}x\right)\right)$$

Now,

$\cancel{y(0) = 0 - \frac{1}{2}e}$

$\therefore$ For, $y(0)=0$ we get,

$$\Rightarrow 0 = e^{-\frac{1}{2}0}\left(c_1\cos\left(\frac{\sqrt{7}}{2}0\right) + c_2\sin\left(\frac{\sqrt{7}}{2}\cdot 0\right)\right)$$

$$\Rightarrow 0 = c_1$$

$$\therefore c_1 = 0$$

~~For, $y'(0)$~~ we

For, $y'(0)=0$ we get

$$\Rightarrow 0 = -\frac{1}{2} e^{-\frac{1}{2}0}(c_1 \cos(\frac{\sqrt{7}}{2}0) + c_2 \sin(\frac{\sqrt{7}}{2}0))$$

$$+ \frac{\sqrt{7}}{2} e^{-\frac{1}{2}0}(c_2 \cos(\frac{\sqrt{7}}{2}.0) - c_1 \sin(\frac{\sqrt{7}}{2}0))$$

$$\Rightarrow 0 = -\frac{1}{2} c_1 + \frac{\sqrt{7}}{2} c_2$$

Lets put the value of c_1 in the equation,

$$\Rightarrow -\frac{1}{2}(0) + \frac{\sqrt{7}}{2} c_2 = 0$$

$$\therefore c_2 = 0$$

$$\therefore y(x) = e^{-\frac{1}{2}x}(0.\cos(\frac{\sqrt{7}}{2}x) + 0 \sin(\frac{\sqrt{7}}{2}x))$$

$$= 0.$$

$$\therefore y(x) = 0$$

(Ans)

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4) $y'' + 4y = 3\sin 2x$

As it is inhomogeneous equation, defferent equation will be, $y = y_c + y_p$

For y_c,

~~$y''+4y$~~

The corresponding auxiliary equation.

$r^2 + 4 = 0$

$\Rightarrow r^2 = -4$

$\therefore r = \pm 2i$

We know,

$y_c = e^{\lambda x}(c_1 \cos(\alpha x) + c_2 \sin(\alpha x))$

Here,

$\lambda \pm \alpha i = \pm 2i$

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$\therefore \lambda = 0,$

$\alpha = 2$

$\therefore y_c = e^0 (c_1 \cos 2x + c_2 \sin 2x)$

$= c_1 \cos 2x + c_2 \sin 2x$

For y_p part,

$y_p = Ax \cos 2x + Bx \sin 2x$

$y'_p = A \cos 2x - 2Ax \sin 2x + B \sin 2x + 2Bx \cos 2x$

$y''_p = -2A \sin 2x - 2A \sin 2x - 4Ax \cos 2x + 2B \cos 2x$
$+ 2B \cos 2x - 4Bx \cos 2x$

$= -4A \sin 2x + 4B \cos 2x - 4x \cos 2x - 4B \sin 2x$

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Given equation,

$y'' + 4y = 3\sin 2x$

$\Rightarrow -4A\sin 2x + 4B\cos 2x - 4Ax\cos 2x - 4Bx\sin 2x$

$+ 4Ax\cos 2x + 4Bx\cos 2x = 3\sin(2x)$

$\Rightarrow -4A\sin(2x) + 4B\cos(2x) = 3\sin 2x$

$\therefore$ We can say,

$-4A = 3$

$\therefore A = -\frac{3}{4}$

$4B = 0$

$\therefore B = 0$

$y_p = -\frac{3}{4}x\cos(2x) + 0 \cdot x\sin(2x)$

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$\therefore y_p = -\frac{3}{4} x \cos(2x)$

As,

$y = y_c + y_p$

$= c_1 \cos(2x) + c_2 \sin(2x) - \frac{3}{4} x \cos(2x)$

$\therefore y(x) = c_1 \cos(2x) + c_2 \sin(2x) - \frac{3}{4} x \cos(2x)$

(Ans:)

5] $y''' + y'' - 2y = 0$

The corresponding auxilary equation is,

$r^3 + r^2 - 2 = 0$

$\Rightarrow (r-1)(r^2 + 2r + 2) = 0$

$\therefore r = 1, -1+i, -1-i$

~~we~~ For $r = 1$ we get,

$c_1 e^x$ — ①

For $r = -1 \pm i$ we get,

we know,

$e^{\lambda x}(c_2 \cos \alpha x + c_3 \sin \alpha x)$

Here,

$\lambda \pm \alpha i = -1 \pm i$

$\therefore \lambda = -1$

$\alpha = 1$

We get,

$$e^{-x}(c_2\cos x + c_3\sin x) \quad \text{--- (ii)}$$

From, equation (i) and (ii)

$$\therefore y(x) = c_1e^{x} + e^{-x}(c_2\cos x + c_3\sin x)$$

(Ans)

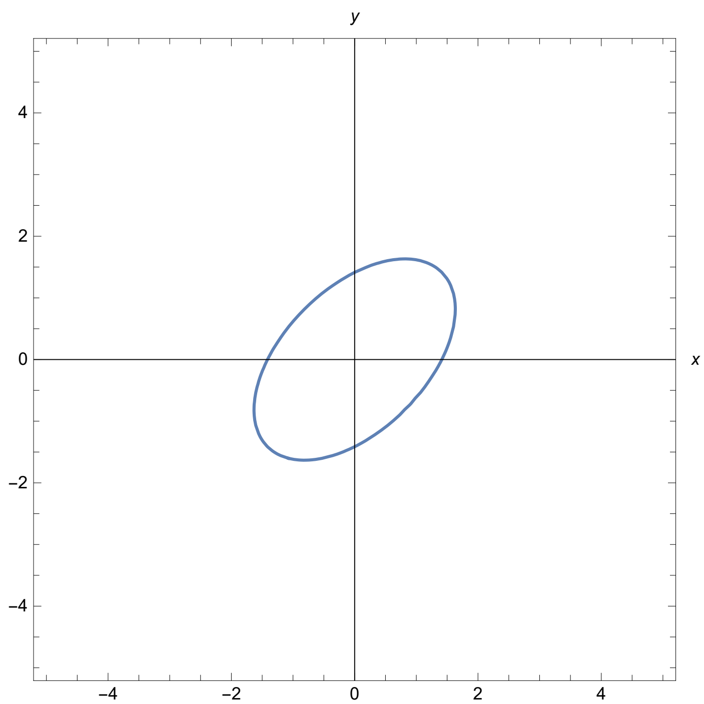
Do the following tasks using Mathematica.

$\iint_R (x^2 - xy + y^2)dA$, where R is the region bounded by the ellipse $x^2 - xy + y^2 = 2$. Use the transformation: $x = \sqrt{2}\,u - \sqrt{\left(\frac{2}{3}\right)}\,v$, $y = \sqrt{2}\,u + \sqrt{\left(\frac{2}{3}\right)}\,v$

(a) Plot R in both xy and uv planes.

In[1]:=
```
ContourPlot[x^2 - x y + y^2 == 2, {x, -5, 5}, {y, -5, 5}, Axes → True, AxesLabel → Automatic]
```

Out[1]=



In[2]:=
```
x = √2 u - √(2/3) v;
y = √2 u + √(2/3) v;
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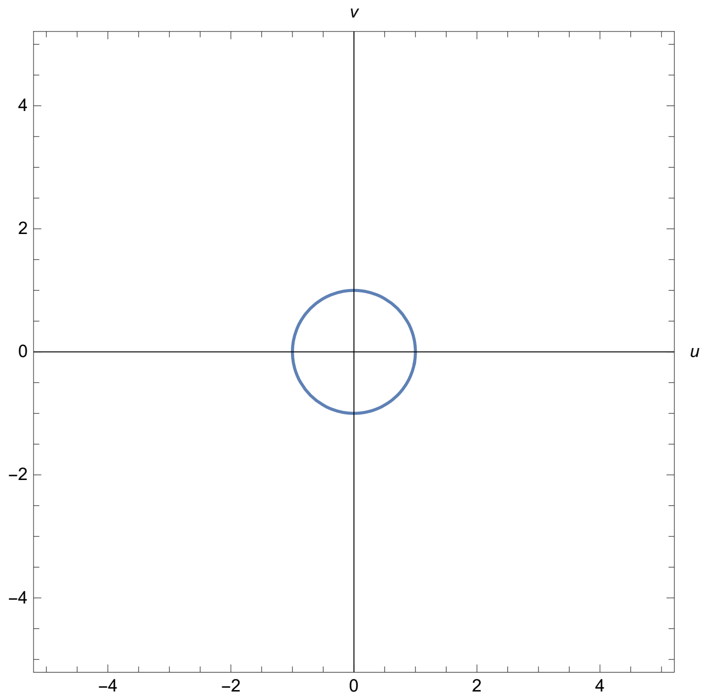
In[4]:= ContourPlot[$x^2 - x\,y + y^2$ == 2, {u, -5, 5}, {v, -5, 5}, Axes → True, AxesLabel → Automatic]

Out[4]=



(b) Find the Jacobian of the transformation

In[5]:= jac = Det[D[{x, y}, {{u, v}}]]

Out[5]= $\frac{4}{\sqrt{3}}$

(c) Evaluate the integral using the transformation

In[6]:= Solve[{$x^2 - x\,y + y^2$ == 2}, {v}]

Out[6]= $\left\{\left\{v \to -\sqrt{1 - u^2}\right\}, \left\{v \to \sqrt{1 - u^2}\right\}\right\}$

In[7]:= $\int_{-1}^{1}\int_{-\sqrt{1-u^2}}^{\sqrt{1-u^2}} \left(x^2 - x\,y + y^2\right)\,(\text{jac})\,dv\,du$

Out[7]= $\frac{4\,\pi}{\sqrt{3}}$