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Sec : 08

1] a] From the rule of ceter of mass,

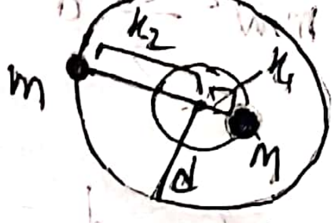
$M x_1 = m x_2$ [$M = 2m$ ← From the question]

$\Rightarrow 2\cancel{m} x_1 = \cancel{m} x_2$

$\Rightarrow \frac{x_2}{x_1} = 2$

$\Rightarrow x_2 = 2x_1$ [Here $x_1 + x_2 = d$]

For the small star,



The mass of small star = m

The distance from the center of mass = r_2

So, the centripetal force for the small ~~force~~ star = $m\omega^2 r_2$

We know, $r_1 + r_2 = d$

$$\Rightarrow \frac{r_2}{2} + r_2 = d$$

$$\Rightarrow \frac{3r_2}{2} = d$$

$$\therefore r_2 = \frac{2}{3}d$$

So we can write the equation,

$$m\omega^2 r_2 = \frac{G m M}{d^2}$$

[As small star getting the centripetal force from gravitational force

$$m\omega^2 \frac{2}{3} d = \frac{G m 2m}{d^2} \quad [M = 2m \rightarrow \text{Big Star}]$$

$$\Rightarrow \omega^2 \frac{2}{3} d = \frac{G 2m}{d^2}$$

$$\Rightarrow \frac{4\pi^2}{T_m^2} \frac{2}{3} d = \frac{2Gm}{d^2} \quad \left[\omega = \frac{2\pi}{T_m}\right]$$

$$\Rightarrow T_m^2 = \frac{4\pi^2 d^3}{3Gm}$$

$$\Rightarrow T_m = \frac{2\pi d^{3/2}}{\sqrt{3}\sqrt{Gm}}$$

~~Fo~~ For Big star,

The mass of Big star = M

~~we know~~

The distance from the center of mas

$= r_1$

we know from the question that $M = 2m$

As the Bigstar getting centripetal force from the gravitational force, we can write,

$$M w^2 r_1 = \frac{GMm}{d^2} \qquad \left[r_1 + r_2 = d \right.$$

$$\Rightarrow r_1 + 2r_1 = d$$

$$\Rightarrow \cancel{2m} w^2 \frac{1}{3} d = \frac{G \cancel{2m} m}{d^2} \qquad \left. \Rightarrow r_1 = \frac{1}{3} d \right]$$

$$\Rightarrow \frac{4\pi^2}{T_M^2} \frac{d}{3} = \frac{Gm}{d^2} \qquad \left[W = \frac{2\pi}{T_M} \right]$$

$$\Rightarrow T_M^2 = \frac{4\pi^2 d^3}{3Gm}$$

$$\Rightarrow T_M = \frac{2\pi d^{3/2}}{\sqrt{3}\sqrt{Gm}}$$

So, The period for both revolution of the stars ~~are~~ about their center of mass are same which is

$$T_m = T_M = \frac{2\pi d^{3/2}}{\sqrt{3}\sqrt{Gm}} \quad \text{(Ans)}$$

b) As we get from the question (a)

$$T_m = T_M = \frac{2\pi d^{3/2}}{\sqrt{3}\sqrt{Gm}}$$

we know,
$$w = \frac{2\pi}{T}$$

so, $$W_m = W_M = \frac{2\pi}{\frac{2\pi d^{3/2}}{\sqrt{3}\sqrt{Gm}}} = \frac{\sqrt{3Gm}}{d^{3/2}}$$

we know, angular momentum $L = mw^2 r^2$

As the question asked,

$$\frac{L_m}{L_M} = \frac{m W_m^2 r_2^2}{M W_M^2 r_1^2}$$

$$= \frac{m (2r_1)^2}{2m\ r_1^2}$$

$$= 2 \quad \text{(Ans)}$$

c] ~~Fo~~ For the small star,

As the star have centripetal force and it is getting it for the gravitational force between two stars. We can write,

$$\frac{GmM}{d^2} = \frac{m V_m^2}{r_2}$$

$$\Rightarrow \frac{Gm(2m)}{d^2} = \frac{m V_m^2}{r_2}$$

$$\cancel{\Rightarrow \frac{Gm^2}{}}$$

$$\Rightarrow \frac{Gm^2 r_2}{d^2} = \frac{m V_m^2}{2}$$

$$\Rightarrow K_m = \frac{Gm^2 \cdot 2d}{3d^2}$$ [From a we get, $r_2 = \frac{2}{3}d$]

$$= \frac{2Gm^2}{3d}$$ [$\frac{m(V_m)^2}{2} = K_m$]

For the big star we get,

As the big star have centripetal force and it is getting it for the gravitational force between two stars, we can write,

$$\therefore \frac{GmM}{d^2} = \frac{MV_M^2}{r_1}$$

$$\Rightarrow \frac{Gm(2m)}{d^2} = \frac{MV_M^2}{r_1}$$

$$\Rightarrow \frac{Gm^2 r_1}{d^2} = \frac{M \cdot V_M^2}{2}$$

$$\Rightarrow K_M = \frac{Gm^2 d}{3d^2} \quad [r_1 = \frac{1}{3}d]$$

$$= \frac{Gm^2}{3d} \quad [K_M = \frac{MV_M^2}{2}]$$

As the question asked,

$$\frac{K_m}{K_M} = \frac{\frac{2}{3}\frac{Gm^2}{d}}{\frac{1}{3} \times \frac{Gm^2}{d}} = 2$$

$$\therefore \frac{K_m}{K_M} = 2 \quad \text{(Ans)}$$

2) a) Given in the question that the mass of the block is M kg and the mass of the clay ball is M/4 kg,

As the metal block stays at rest, so the initial ~~block~~ velocity is $v = 0\ ms^{-1}$,

But after clay hits the block with velocity v the speed of the block and clay ball sytem's speed becomes V_F

We can write the conservation of moment

$$Mu + \left(\frac{M}{4}v\right) = \left(M + \frac{M}{4}\right)V_F$$

$$\Rightarrow M.0 + \left(\frac{M}{4}v\right) = \left(\frac{5M}{4}\right)V_F$$

$$\Rightarrow V_F = \frac{\frac{M}{4}v}{5M/4}$$

$\therefore V_F = \frac{1}{5} V$

$\therefore$ The speed of the block and ball system immediately after the impact is

$\frac{1}{5} V$ (Ans)

b) As the block ~~com~~ with the spring get compressed so,

Energy at maximum compressed position

= Energy at uncompressed position,

so,

$$\frac{1}{2} k A^2 = \frac{1}{2} \left(M + \frac{M}{4} \right) (V_F)^2$$

$$\Rightarrow A = \frac{\sqrt{\frac{5}{4}M}\ V_F}{\sqrt{K}}$$

$$= \frac{\sqrt{5M}}{2\sqrt{K}} \frac{V}{5}$$

$$= \frac{\sqrt{M}\ V}{2\sqrt{5}\sqrt{K}}$$

So, amplitude is $A = \frac{\sqrt{M}\ V}{2\sqrt{5}\sqrt{K}}$

We know,

$$W = \sqrt{\frac{K}{M + \frac{M}{4}}}$$

$$= \frac{2\sqrt{K}}{\sqrt{5M}}$$

$\therefore$ Angular frequency $W = \frac{2\sqrt{K}}{\sqrt{5M}}$

$\therefore$

∴ We know,

$$x = A\sin(\omega t + \phi)$$

Here, as at $t=0$ the $x=0$ so the

$\phi = 0$,

so we can say,

$$x = A\sin(\omega t)$$

$$= \frac{\sqrt{M}\,V}{2\sqrt{5}\sqrt{K}} \sin\left(\frac{2\sqrt{K}}{\sqrt{5M}} t\right)$$

∴ The equation for the position

is $x = \frac{\sqrt{M}\,V}{2\sqrt{5}\sqrt{K}} \sin\left(\frac{2\sqrt{K}}{\sqrt{5M}} t\right)$

Amplitude $A = \frac{\sqrt{M}\,V}{2\sqrt{5}\sqrt{K}}$

Angular frequency $w = \frac{2\sqrt{K}}{\sqrt{5M}}$

(Ans.)

c) From question (b) we get,

$$w = \frac{2\sqrt{K}}{\sqrt{5M}}$$

$$A = \frac{\sqrt{M}\, V}{2\sqrt{5}\,\sqrt{K}}$$

we know,

$$w = \frac{2\pi}{T}$$

$$\Rightarrow T = \frac{2\pi}{w}$$

$$= \frac{2\pi\sqrt{5M}}{2\sqrt{K}}$$

$$= \frac{\pi\sqrt{5M}}{\sqrt{K}}$$

~~Calcu~~ Time period $T = \frac{\pi\sqrt{5M}}{\sqrt{K}}$

Maximum velocity $\Rightarrow V_{max} = wA$

$$\therefore V_{max} = \frac{\sqrt{M}\, V}{\cancel{2}\sqrt{5}\cancel{\sqrt{K}}} \times \frac{\cancel{2}\cancel{\sqrt{K}}}{\sqrt{5}\cancel{M}}$$

$$= \frac{V\sqrt{K}}{5\sqrt{K}}$$

$$= \frac{V}{5}$$

Maximum acceleration $a_{max} = w^2 A$

$$\therefore a_{max} = w^2 A$$

$$= \frac{4K}{5M} \times \frac{\sqrt{M}\, V}{2\sqrt{5}\sqrt{K}}$$

$$= \frac{2\sqrt{K}\, V}{5\sqrt{5}\sqrt{M}}$$

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$\therefore$ Time period $T = \dfrac{\pi\sqrt{5m}}{\sqrt{K}}$

$\therefore$ Maximum velocity $V_{max} = \dfrac{V}{5}$

$\therefore$ maximum acceleration $a_{max} = \dfrac{2\sqrt{K}\,V}{5\sqrt{5}\sqrt{m}}$

(Ans)