Math Assignment 1

SET 13

SECTION 13

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1) $\int_{-\infty}^{\infty} \frac{e^{-t}}{1+e^{-2t}} \, \partial t$

we can write it as, $\int_{-\infty}^{0} \frac{e^{-t}}{1+e^{-2t}} \partial t + \int_{0}^{\infty} \frac{e^{-t}}{1+e^{-2t}}$

① For the $\int_{\infty}^{0} \frac{e^{-t}}{1+e^{-2t}} \partial t$ we get,

$$= \lim_{n \to -\infty} \int_{n}^{0} \frac{e^{-t}}{1+e^{-2t}} \partial t$$

$$= \lim_{n \to -\infty} \left[-\tan^{-1}(e^{-t}) \right]_{n}^{0}$$

$$= \lim_{n \to -\infty} -\left[\frac{\pi}{4} - \tan^{-1}(e^{-n}) \right]$$

$$= -\left[\frac{\pi}{4} - \frac{\pi}{2} \right]$$

$$= \frac{\pi}{4} \quad \text{——} \quad (1)$$

For the $\int_{0}^{\infty} \frac{e^{-t}}{1+e^{-2t}} \partial t$ we get,

$$\lim_{n \to \infty} \int_{0}^{n} \frac{e^{-t}}{1+e^{-2t}}$$

$$= \lim_{n \to \infty} \left[-\tan^{-1}(e^{-t}) \right]_0^n$$

$$= \lim_{n \to \infty} - \left[\tan^{-1}(e^{-n}) - \frac{\pi}{4} \right]$$

$$= - \left[0 - \frac{\pi}{4} \right] \quad \text{— (II)}$$

$$= \frac{\pi}{4}$$

$\therefore$ we can say ~~that~~ from (I) and (II) ~~tha~~ that,

$$\int_{-\infty}^{0} \frac{e^{-t}}{1+e^{-2t}} dt + \int_{0}^{\infty} \frac{e^{-t}}{1+e^{-2t}} dt$$

$$= \frac{\pi}{4} + \frac{\pi}{4}$$

$$= \frac{\pi}{2}$$

$\therefore$ ~~integral con~~

$\therefore$ integral is convergent

(Ans)

~~Sol~~

2) $I_n = \int \cos^n(x)\,dx$

$= \int \cos^{n-1}(x)\,dx \cdot \cos(x)\,dx$

$= \cos^{n-1}(x)\sin(x) + \int (n-1)\cos^{n-2}(x)\sin^2(x)\,dx$

$= \cos^{n-1}(x)\sin(x) + \int (n-1)\cos^{n-2}(x)(1-\cos^2 x)\,dx$

$I_n = \cos^{n-1}(x)\sin(x) + \int (n-1)\cos^{n-2}(x)\,dx - \int (n-1)\cos^n(x)\,dx$

$= \cos^{n-1}(x)\sin(x) + \int (n-1)\cos^{n-2}(x)\,dx - I_n(n-1)$

$\Rightarrow \cancel{I_n + I_n}$

$\Rightarrow I_n + I_n(n-1) = \cos^{n-1}(x)\sin(x) + \int (n-1)\cos^{n-2}(x)\,dx$

$\Rightarrow n I_n = \cos^{n-1}(x)\sin(x) + \int (n-1)\cos^{n-2}(x)\,dx$

$\Rightarrow I_n = \frac{1}{n}\cos^{n-1}(x)\sin(x) + \frac{n-1}{n}\int \cos^{n-2}(x)\,dx$

Now we can say that, Reduction Formula of $\int \cos^n x$ is,

$$\int \cos^n x \, dx = \frac{1}{n}\cos^{n-1}x \sin x + \frac{n-1}{n}\int \cos^{n-2}x \, dx$$

$$\therefore \cos^8 x \, dx = \frac{1}{8}\cos^7 x \sin x + \frac{7}{8}\int \cos^6 x \, dx$$

For $\int \cos^6 x$ we get,

$$= \frac{1}{6}\cos^5 x \sin x + \frac{5}{6}\int \cos^4 x \, dx \quad \text{— (i)}$$

For, $\int \cos^4 x$ we get

$$= \frac{1}{4}\cos^3 x \sin x + \frac{3}{4}\int \cos^2 x \, dx$$

$$= \frac{1}{4}\cos^3 x \sin x + \frac{3}{8}\int (1+\cos 2x) \, dx$$

$$= \frac{1}{4}\cos^3 x \sin x + \frac{3}{8}\left[x + \frac{\sin 2x}{2}\right] + c \quad \text{— (ii)}$$

$$= \frac{1}{4}\cos^3 x \sin x + \frac{3}{8}$$

No we get, From (i) and (ii)

$$\int \cos^8 x \, dx = \frac{1}{8}\cos^7 x \sin x + \frac{7}{8}\left[\frac{1}{6}\cos^5 x \sin x + \frac{5}{6}\left[\frac{1}{4}\cos^3 x \sin x + \frac{3}{8}x + \frac{3\sin 2x}{16}\right]\right]$$

$$= \frac{1}{8}\cos^7 x \sin x + \frac{7}{48}\cos^5 x \sin x + \frac{35}{192}\cos^3 x \sin x + \frac{35}{128}x + \frac{35}{256}\sin 2x$$

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$$= \frac{1}{8}\cos^7 x \sin x + \frac{7}{48}\cos^5 x \sin x + \frac{35}{192}\cos^3 x \sin x$$

$$+ \frac{35}{128} x + \frac{35}{128} \sin x \cos x$$

(Ans)

3] $\int_0^1 \left(\frac{x}{\ln x}\right)^{\frac{1}{3}}$

$t=-\ln x$

$x=e^{-t}$

$dx=-e^{-t}$

x	1	0
t	0	∞

$\leftarrow$ limit

$$\int_\infty^0 \left(\frac{e^{-t}}{-t}\right)^{\frac{1}{3}}(-e^{-t})\,dt$$

$$=(-1)^{\frac{1}{3}}\int_\infty^0 \left(\frac{e^{-t}}{t}\right)^{\frac{1}{3}}(-e^{-t})\,dt$$

$$=(-1)^{\frac{1}{3}}\int_0^\infty e^{-\frac{4}{3}t}\,\cancel{\partial}\, t^{-\frac{1}{3}}\,dt$$

$$=(-1)^{\frac{1}{3}}\int_0^\infty t^{-\frac{1}{3}}\, e^{-\frac{4}{3}t}\,dt$$

$$\therefore (-1)^{\frac{1}{3}}\frac{\Gamma\frac{2}{3}}{\left(\frac{4}{3}\right)^{\frac{2}{3}}}$$ (Showed)

we know

$$\int_0^\infty e^{-kx}x^{n-1}\,dx=\frac{\Gamma n}{k^n}$$

4) $\int \frac{2x^2-10x+4}{(x+1)(x-3)^2}$

$$\frac{2x^2-10x+4}{(x+1)(x-3)^2} = \frac{A}{(x+1)} + \frac{B}{(x-3)} + \frac{C}{(x-3)^2}$$

$$\Rightarrow 2x^2-10x+4 = A(x-3)^2 + B(x+1)(x-3) + C(x+1)$$

$$= A(x^2-6x+9) + B(x^2-3x+x-3) + C(x+1)$$

$$= Ax^2 - 6Ax + 9A + Bx^2 - 3Bx + Bx - 3B + Cx + C$$

$$\Rightarrow 2x^2-10x+4 = x^2(A+B) + x(-6A-3B+B+C) + (9A-3B+C)$$

$\cancel{+9CA}$

$\therefore$ We can say,

$A+B=2$

$-6A-2B+C=-10$

$9A-3B+C=4$

After solving the equation we get,

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$A = 1$

$B = 1$

$C = -2$

$\therefore$ we can write that,

$$\int \frac{2x^2-10x+4}{(x+1)(x-3)^2} = \int \frac{1}{(x+1)}\partial x + \int \frac{1}{x-3}\partial x - \int \frac{2\,\partial x}{(x-3)^2}$$

$$= \ln|x+1| + \ln|x-3| + 2\left(\frac{1}{x-3}\right) + c$$

$$= \ln|x+1| + \ln|x-3| + \frac{2}{x-3} + c$$

(Ans)

5] $\int_0^1 x^4 (1-\sqrt{x})^5$

Let,

$x = t^2$

$\therefore \partial x = 2t\ \partial t$

Limit

x	1	0
t	1	0

~~Applyin~~

Converting x to t we get,

$2\int_0^1 t^8 (1-t)^5\ t\ \partial t$

$2\int_0^1 t^9 (1-t)^5\ \partial t$

We know, $\int_0^1 x^{m-1} (1-x)^{n-1}\ \partial x = \beta(m,n)$

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$$\therefore 2\int_0^1 t^9(1-t)^5\,dt$$

$$= 2\,\beta(10,6)$$

$$\therefore 2\,\beta(10,6) = 2 \times \frac{\Gamma 10\,\Gamma 6}{\Gamma 10+6} \qquad \left[\text{we know } \beta(m,n) = \frac{\Gamma m\,\Gamma n}{\Gamma m+n}\right]$$

$$\cancel{= 2 \times \frac{\Gamma 10\,\Gamma 6}{\Gamma 10 -}}$$

$$= 2 \times \frac{\Gamma 10\,\Gamma 6}{\Gamma 16}$$

$$= 2 \times \frac{9!\,5!}{15!}$$

$$= \frac{1}{15015} \quad (\text{Ans})$$

6. Do the following tasks using Mathematica.

(a) Plot the above functions in a single graph for -1 ≤ x ≤ 1.

Hint: Use Abs[ ] function to write absolute value

Ans:

In[1]:= $f[x\_] = x\, e^{x^2};$

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In[2]:= g[x_] = Abs[2 x];
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In[3]:= Plot[{f[x], g[x]}, {x, -1, 1}, PlotLegends → "Expressions"]
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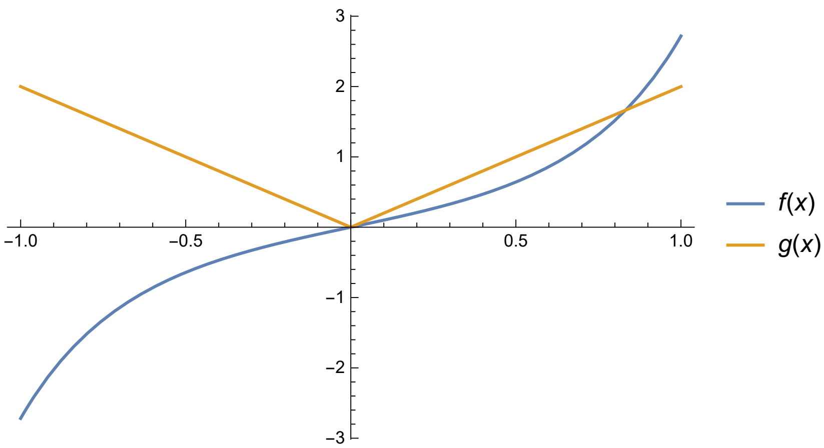
Out[3]=



(b) Find the limits of the integration for the area of the region enclosed by f(x) and g(x) for -1 ≤ x ≤ 1.

Hint: Solve equations to find the intersections.

Ans:

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In[4]:=

Solve[{f[x] == g[x]}]
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Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[4]= $\{\{x \to 0\}, \{x \to \sqrt{\text{Log}[2]}\}\}$

(c) Finally, do the integration to find the area

Ans:

In[5]:=

$\text{NIntegrate}\left[g[x] - f[x], \{x, 0, \sqrt{\text{Log}[2]}\}\right]$

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Out[5]= 0.193147
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