Do the following tasks using Mathematica.

If a projectile is fired with an initial velocity of v0 meters per second at an angle α above the horizontal and air resistance is assumed to be negligible, then it's position after t seconds is given by the parametric equations

x = (v_0 cos α)t and y = (v_0 sin α)t $-\frac{1}{2} g t^2$. Also v_x = dx/dt and v_y = dy/dt

(a) Find v_x and v_y . If a gun is fired with α = 300 and v0 = 500m/s when will the bullet hit the ground? How far from the gun will it hit the ground? What is the maximum height reached by the bullet?

Hint: The Bullet reaches the ground when y=0 and it is at maximum height when v_y = 0

In[1]:=
```
x = v0 Cos[α] t;
```

In[2]:=
```
y = v0 Sin[α] t - 1/2 g t^2;
```

In[3]:=
```
vx = D[x, t]
```

Out[3]=
```
Cos[α] v0
```

In[4]:=
```
vy = D[y, t]
```

Out[4]=
```
-g t + Sin[α] v0
```

In[5]:=
```
α = 30 °;
v0 = 500;
g = 9.8;
```

In[8]:=
```
Solve[y == 0 , t]
```

Out[8]=
```
{{t → 0. + 0. i}, {t → 51.0204}}
```

After 51.0204 sec bullet will hit the ground

In[9]:=
```
x /. t → 51.0204081632653`
```

Out[9]=
```
22092.5
```

Bullet hit the ground 22092.5 meters far from the gun

In[10]:=
```
Solve[vy == 0, t]
```

Out[10]=
```
{{t → 25.5102}}
```

In[11]:=
```
y /. t → 25.51020408163265`
```
Out[11]= 3188.78

Maximum height reached by the bullet is 3188.78 meters

(b) Plot the path of the projectile for v_0 = 500m/s and α = 30 °, 45° and 60° in a single graph.

In[12]:=

$$\text{Solve}\left[v_0 \,\text{Sin}[30\,°]\, t - \frac{1}{2} g\, t^2 == 0,\ t\right]$$

Out[12]= {{t → 0. + 0. ⅈ}, {t → 51.0204}}

In[13]:=

$$\text{Solve}\left[v_0 \,\text{Sin}[45\,°]\, t - \frac{1}{2} g\, t^2 == 0,\ t\right]$$

Out[13]= {{t → 0. + 0. ⅈ}, {t → 72.1538}}

In[14]:= $$\text{Solve}\left[v_0 \,\text{Sin}[60\,°]\, t - \frac{1}{2} g\, t^2 == 0,\ t\right]$$

Out[14]= {{t → 0. + 0. ⅈ}, {t → 88.3699}}

In[15]:=

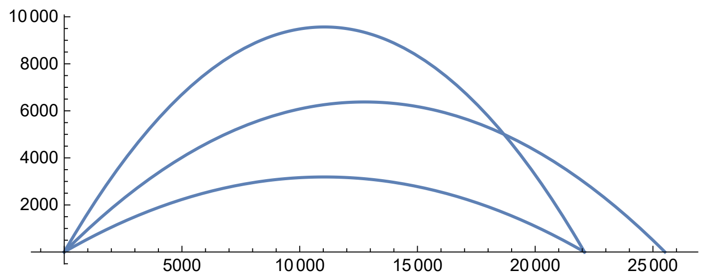
$$a = \text{ParametricPlot}\left[\left\{v_0 \,\text{Cos}[30\,°]\, t,\ v_0 \,\text{Sin}[30\,°]\, t - \frac{1}{2} g\, t^2\right\}, \{t, 0, 51.0204081632653\text{`}\}\right];$$

$$b = \text{ParametricPlot}\left[\left\{v_0 \,\text{Cos}[45\,°]\, t,\ v_0 \,\text{Sin}[45\,°]\, t - \frac{1}{2} g\, t^2\right\}, \{t, 0, 72.15375318230076\text{`}\}\right];$$

$$c = \text{ParametricPlot}\left[\left\{v_0 \,\text{Cos}[60\,°]\, t,\ v_0 \,\text{Sin}[60\,°]\, t - \frac{1}{2} g\, t^2\right\}, \{t, 0, 88.36993916167741\text{`}\}\right];$$

In[18]:=
```
Show[{a, b, c} , PlotRange → All]
```

Out[18]=



(c) A torus can be expressed parametrically as

x = (a + b cos v) cos u

y = (a + b cos v) sin u

z = b sin v

Plot the torus for a = 5 , b = 2 and $0 \leq u \leq 2\pi$ and $0 \leq v \leq 2\pi$ and use rainbow color function .

In[19]:=
```
Clear["Global`*"]
x[u_, v_] = (a + b Cos[v]) Cos[u];
y[u_, v_] = (a + b Cos[v]) Sin[u];
z[u_, v_] = b Sin[v];
a = 5;
b = 2;
```

In[25]:=
```
ParametricPlot3D[{x[u, v], y[u, v], z[u, v]},
 {u, 0, 2 π}, {v , 0, 2 π}, ColorFunction → "Rainbow"]
```

Out[25]=