Math Assignment 5

SET 13

SECTION 13

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1) a) $\frac{\partial x}{\partial t} = kx - \lambda x^2$

$\Rightarrow \frac{\partial x}{kx - \lambda x^2} = \partial t$

$\Rightarrow \int \frac{\partial x}{0.03x - \frac{3x^2}{10^8}} = \int \partial t$

$\Rightarrow 10^8 \int \frac{\partial x}{3\times10^6 x - 3x^2} = \int \partial t$

$\Rightarrow \frac{10^8}{3} \int \frac{\partial x}{10^6 x - x^2} = \int \partial t$

$\Rightarrow \frac{100}{3} \ln \frac{x}{x-10^6} = t + c$

$\Rightarrow \frac{x}{x-10^6} = ce^{\frac{3t}{100}}$

Given in the year 1980, $t=0$, $x(0) = 2\times10^5$

$\Rightarrow \frac{2\times10^5}{2\times10^5 - 10^6} = ce^0$

$$\Rightarrow \frac{2\times 10^5}{10^5(2-10)} = c$$

$$\Rightarrow \frac{2}{-8} = c$$

$$\therefore c = -\frac{1}{4}$$

Now we can write the equation as,

$$\frac{x}{x-10^6} = -\frac{1}{4}e^{\frac{3t}{100}}$$

$$\Rightarrow \frac{x-10^6}{x} = \frac{-4}{e^{\frac{3t}{100}}}$$

$$\Rightarrow 1-\frac{10^6}{x} = \frac{-4}{e^{3t/100}}$$

$$\Rightarrow \frac{10^6}{x} = \frac{e^{3t/100}+4}{e^{3t/100}}$$

$$\Rightarrow x = \frac{10^6 e^{3t/100}}{e^{3t/100}+4}$$

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$$\therefore \; x(t) = \frac{10^6 e^{\frac{3t}{100}}}{e^{3t/100} + 4}$$

(Ans-a)

b) From a) we get $x(t) = \dfrac{10^6 e^{\frac{3t}{100}}}{e^{3t/100} + 4}$

As the question told us to calculate the population in the year 2000 then $t = (2000 - 1980) = 20$

$$\therefore \; x(20) = \frac{10^6 e^{\frac{3\times 20}{100}}}{e^{\frac{3\times 20}{100}} + 4}$$

$$= 312964.89$$

(Ans-b)

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2) Newton's law of cooling,

$$\frac{\partial T}{\partial t} = -k(T - T_s)$$

$$\Rightarrow \frac{\partial T}{T - T_s} = -k\,\partial t$$

$$\Rightarrow \int \frac{\partial T}{T - T_s} = -\int k\,\partial t$$

$$\Rightarrow \ln|T - T_s| = -kt + c$$

$$\Rightarrow T - T_s = ce^{-kt}$$

$$\Rightarrow T = T_s + ce^{-kt}$$

we know, as the ~~is~~ thermometer was at 70°F so we can say the initial temparature of thermometer is 70°F

So, $T(0) = 70^{\circ}$, and the room tempanetune is $10^{\circ}F$

So, $T_s = 10^{\circ}$

$\Rightarrow T(0) = ce^{-k0} + T_s$

$\Rightarrow 70 = ce^{0} + 10$

$\Rightarrow c = 60$,

As the question stated after $\frac{1}{2}$ minutes the tempanetune $50^{\circ}F$

$\Rightarrow T(\frac{1}{2}) = 10 + 60e^{-k/2}$

$\Rightarrow 50 = 10 + 60e^{-k/2}$

$\Rightarrow 60e^{-k/2} = 40$

$\Rightarrow e^{-k/2} = \frac{2}{3}$

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$\Rightarrow -\frac{k}{2} = \ln\left|\frac{2}{3}\right|$

$\therefore k = 0.8109$

$\therefore \cancel{T(t) = 60e^{-\ln}}$

$\therefore T(t) = 60e^{-0.8109t} + 10$

$\Rightarrow T(1) = 36.67$

$\therefore$ The reading of the thermometer at $t = 1$ min is 36.67 °F (Ans 1)

Now,
As it is wanting us to find the time ~~at~~ it would take to reach 15 °F, we ~~cam v~~ can write,

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$$\Rightarrow 15 = 60e^{-0.8109t} + 10$$

$$\Rightarrow e^{-0.8109t} = \frac{1}{12}$$

$$\Rightarrow -0.8109t = \ln\left|\frac{1}{12}\right|$$

$$\therefore t = 3.064 \text{ min}$$

$\therefore$ It takes 3.064 min to reach ~~to~~ the thermometer $15^\circ F$.

(Ans-b)

3) a) $\frac{\partial n}{\partial t} = kn$

$\Rightarrow \frac{\partial n}{n} = k\,\partial t$

$\Rightarrow \ln|n| = kt + c$

$\Rightarrow n = ce^{kt}$

As ~~at t=0~~ the nuclei have the initial number of particle, we think the particle is n_0

$\therefore$ $t=0$ $n = n_0$

$\therefore n(0) = ce^{k0}$

$\Rightarrow n_0 = ce^0$

$\therefore c = n_0$

$\therefore \cancel{\frac{1}{2}N_0 = N_0 e^{kt}}$

$\therefore N(t) = N_0 e^{kt}$

$\therefore$ As half life in a period of 1500 years, we can write,

$\Rightarrow \frac{1}{2} N_0 = N_0 e^{k1500}$

$\Rightarrow e^{1500k} = \frac{1}{2}$

$\Rightarrow 1500k = \ln(\frac{1}{2})$

$\therefore K = -4.62 \times 10^{-4}$

$\therefore N(t) = N_0 e^{-4.62 \times 10^{-4} t}$

Now at $t = 4500$ years

$N(4500) = N_0 e^{-4.62 \times 10^{-4} \times 4500}$

$= \cancel{12.3}\ 0.125\, N_0$

$= 12.5\%$

$\therefore$ 12.5% of the original radioactive nuclei remain after 4500 years

(Ans-a)

b) As the question asked us to find the years when we get the one-tenth of the orginal number,

we can write,

$$\frac{N_0}{10} = N_0 e^{-4.62 \times 10^{-4} t}$$

$$\Rightarrow -4.62 \times 10^{-4} t = \ln\left|\frac{1}{10}\right|$$

$\therefore t = 4983.95$ years

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$\therefore$ In 4983.95 years one-tenth of the orginal number remain. (Ans-b)

4) we know LR series circuit equation,

$$L \frac{\partial i}{\partial t} + Ri = V$$

Given, $L = 0.1$ henry

$R = 50$ ohoms

$V = 30V$

$$\Rightarrow 0.1 \frac{\partial i}{\partial t} + 50i = 30$$

$$\Rightarrow \frac{\partial i}{\partial t} + 500i + 300$$

$\therefore$ intrigating factor $= e^{\int 500 \partial t}$

$= e^{500t}$

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Multipying the equation with intrigating factor we get,,

$$e^{500t}\frac{\partial i}{\partial t} + 500 i\, e^{500t} = 300 e^{500t}$$

$$\Rightarrow \frac{\partial}{\partial t}[e^{500t} i] = 300 e^{500t}$$

$$\Rightarrow e^{500t} i = \int \frac{300}{500} e^{500t}$$

$$\Rightarrow e^{500t} i = \frac{3}{5} e^{500t} + C$$

$$\Rightarrow e^{500t} i = \frac{3}{5} e^{500t} + C$$

$$\Rightarrow i(t) = \frac{3}{5} + C e^{-500t}$$

Given in the question, $i(0) = 0$

$$\Rightarrow i(0) = \frac{3}{5} + C e^{-500\times 0}$$

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$$\Rightarrow 0 = \frac{3}{5} + c e^{0}$$

$$\therefore c = -\frac{3}{5}$$

$$\therefore i(t) = \frac{3}{5} - \frac{3}{5} e^{-500t}$$

(Ans)

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5) a) Given equation,

$$\frac{\partial A}{\partial t} = kA$$

$$\Rightarrow \frac{\partial A}{A} = k\,\partial t$$

$$\Rightarrow \int \frac{\partial A}{A} = \int k\,\partial t$$

$$\Rightarrow \ln|A| = kt + C$$

$$\Rightarrow A = C e^{kt}$$

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As Given $A(0) = A_0$ we can write,

$\Rightarrow A(0) = c e^0$

$\Rightarrow c = A_0$

$\therefore A(t) = A_0 e^{kt}$

Given that Half life is T.

so, we can write

$$\Rightarrow \frac{A_0}{2} = A_0 e^{kT}$$

$$\Rightarrow \frac{1}{2} = e^{kT}$$

$$\Rightarrow kT = \ln\left|\frac{1}{2}\right|$$

$$\therefore T = \frac{-\ln|2|}{k}$$

(Ans - a)

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b) From (a) we get, $T = \frac{-\ln|2|}{k}$

$$\therefore k = \frac{-\ln|2|}{T}$$

$$\therefore A(t) = A_0 e^{kt}$$

$$\Rightarrow A(t) = A_0 e^{-\frac{\ln|2|}{T} t}$$

$$= A_0 e^{-\frac{t}{T} \ln|2|}$$

$$= A_0 e^{\ln|2|^{-\frac{t}{T}}}$$

$$= A_0 2^{-\frac{t}{T}}$$

(showed)

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Do the following tasks using Mathematica.

(a) Solve the differential equation:

xy' = 4y

Plot multiple solutions of the differential equation with values of constants
c=-2,-1,0,1,2 in a single graph

In[1]:=
```
DSolve[x y'[x] == 4 y[x], y[x], x]
```

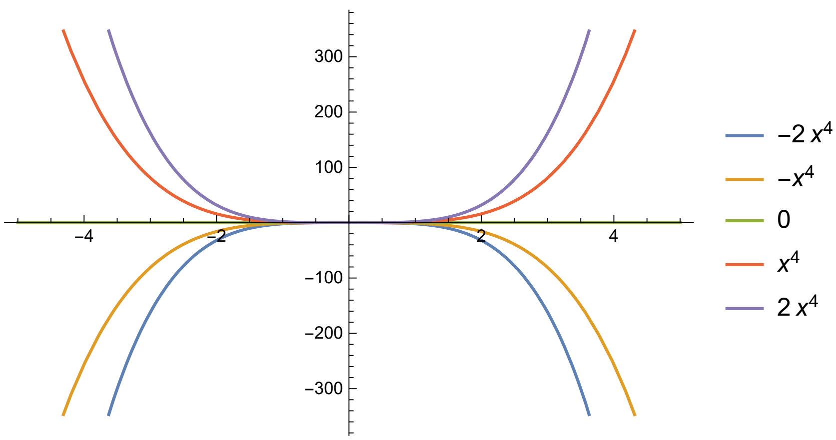
Out[1]= $\{\{y[x] \to x^4 C[1]\}\}$

In[2]:=
```
sol = y[x] /. %1 /. C[1] → a
```

Out[2]= $\{a\, x^4\}$

In[3]:=
```
Plot[Evaluate[Table[sol, {a, -2, 2}]], {x, -5, 5}, PlotLegends → "Expressions"]
```

Out[3]=



(b)

y'' – 10y' + 25y = 0 ; y(0) = 1 , y'(1) = 0 Find the value of y(2)

In[4]:=
```
DSolve[{y''[t] - 10 y'[t] + 25 y[t] == 0, y[0] == 1, y'[1] == 0}, y[t], t]
```

Out[4]= $\left\{\left\{y[t] \to -\frac{1}{6} e^{5t} (-6 + 5t)\right\}\right\}$

In[5]:=
```
y[t] /. %4 /. t → 2 // N
```

Out[5]= {-14 684.3}

In[6]:=
```
ClearAll["Global`*"]
```

(c) Plot the numerical solution of the differential equation for 0 ≤ t ≤ 50:

$x'' + 0.15x' - x + x^3 = 0.3\cos t\,,\ x(0) = -1\,,\ x'(0) = 1$

In[7]:=
```
NDSolve[{x''[t] + 0.15 x'[t] - x[t] + x[t]^3 == 0.3 Cos[t], x[0] == -1, x'[0] == 1},
 x[t], {t, 0, 50}]
```

Out[7]=
```
{{x[t] → InterpolatingFunction[ Domain: {{0., 50.}} Output: scalar ][t]}}
```

In[8]:=
```
Plot[x[t] /. %7, {t, 0, 50}, PlotRange → Full]
```

Out[8]=