Do the following tasks using Mathematica.

y

2 = 2x + 6

y = x – 1

(a) Plot the curves in a single graph for $-5 \le x \le 10$ and shade the bounded

region only. Then find the area of the bounded region using Area[ ] function.

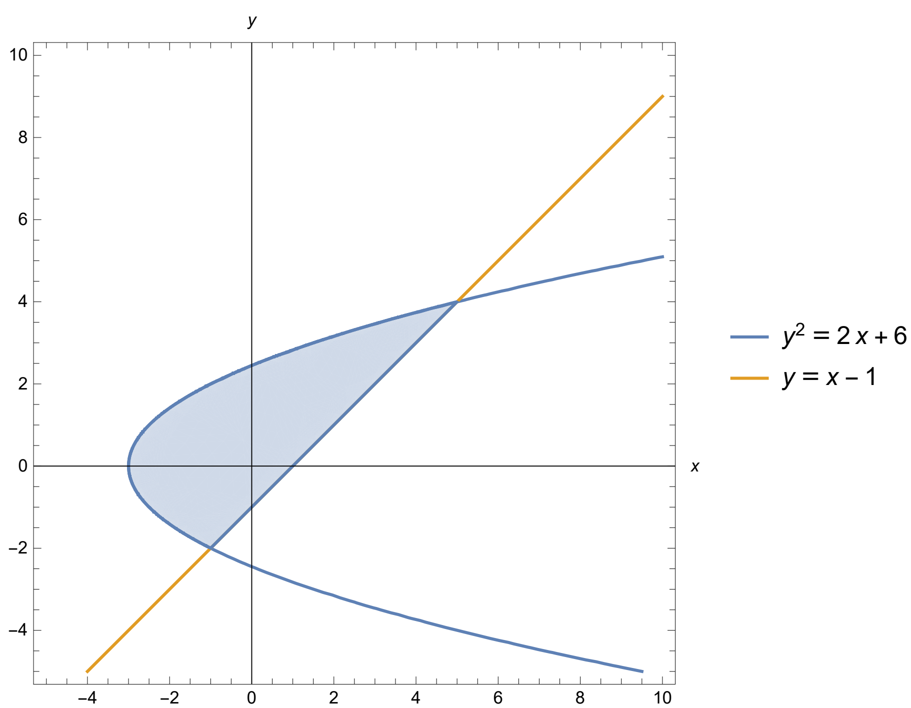
Ans:

In[14]:=
```
plot1 = ContourPlot[{y^2 == 2 x + 6, y == x - 1}, {x, -5, 10}, {y, -5, 10},
   Axes → True, AxesLabel → Automatic, PlotLegends → "Expressions"];
```

In[15]:=
```
region1 = ImplicitRegion[x < y + 1 && x > (y^2 - 6)/2, {{x, -5, 10}, {y, -5, 10}}];
```

In[16]:=
```
Show[plot1, RegionPlot[region1]]
```

Out[16]=



In[17]:=
```
Area[region1]
```

Out[17]= 18

(b) Find the length of the curve $y = \frac{1}{x}$ for $1 \le x \le 2$. Do not use the built-in ArcLength[ ] function.

Ans:

In[18]:=
```
ClearAll["Global`*"]
```

In[19]:= $D\left[\frac{1}{x}, x\right]$

Out[19]= $-\frac{1}{x^2}$

In[20]:= $L = \int_1^2 \sqrt{1 + \left(-\frac{1}{x^2}\right)^2}\, dx$ // N

Out[20]= $1.13209 - 4.44089 \times 10^{-16}\, i$

As $4.44089 \times 10^{-16}\, i$ is close to 0 we can say the length of the curve $y = \frac{1}{x}$ is 1.13209

So the answer is 1.13209

(c) Find the surface area obtained by rotating the curve

i. $y = x^3$, $0 \le x \le 1$ about y axis.

ii. $y = \cos\left(\frac{x}{2}\right)$, $0 \le x \le \pi$ about x axis.

Ans:

In[21]:=

$D\left[y^{\frac{1}{3}}, y\right]$

Out[21]= $\frac{1}{3\, y^{2/3}}$

In[22]:= $S1 = \int_0^1 2\, \pi\, y^{\frac{1}{3}} \sqrt{1 + \left(\frac{1}{3\, y^{2/3}}\right)^2}\, dy$ // N

Out[22]= 5.91943

In[23]:= $D\left[\mathrm{Cos}\left[\frac{x}{2}\right], x\right]$

Out[23]= $-\frac{1}{2}\, \mathrm{Sin}\left[\frac{x}{2}\right]$

In[24]:= $S2 = \int_0^{\pi} 2\, \pi\, \mathrm{Cos}\left[\frac{x}{2}\right] \sqrt{1 + \left(-\frac{1}{2}\, \mathrm{Sin}\left[\frac{x}{2}\right]\right)^2}\, dx$ // N

Out[24]= 13.0719