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Sec : 08

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1) a) For the spring, as it is compressed a distance x from its equilibrium position so we can write the energy conserved in the spring $U_s = \frac{1}{2}kx^2$

As the question told us to solve the problem using m, g, x and L,

we know, $\sin\theta = \frac{h}{L}$

$$\therefore h = L\sin\theta$$

So we can write,

$$\frac{1}{2}kx^2 = mgh$$

$$\Rightarrow \frac{1}{2}kx^2 = mgL\sin\theta$$

$$\Rightarrow \frac{1}{2}kx^2 = \frac{1}{2}mgL \quad [\because \sin 30^\circ = \frac{1}{2}]$$

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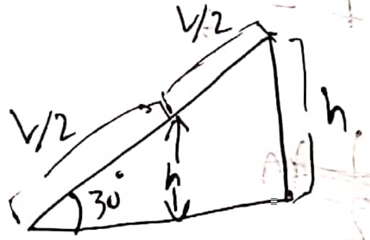
$\Rightarrow ku^2 = \cancel{\theta}\ mgL$

$\Rightarrow k = \dfrac{mgL}{u^2}$ (Ans)

b) So it is saying the mid point of the ramp.

So mid point of the ramp is $L/2$



we know,

$$\sin\theta = \frac{h'}{L/2}$$

$$\Rightarrow h' = L/2 \times \sin(30^\circ)$$

$$= \frac{L}{4}$$

Potential energy at midpoint of the ramp

$$U = mgh'$$

$$= \frac{mgL}{4} \text{ (Ans)}$$

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$\boxed{c}$ Translation kinetic work $= \frac{1}{2} mv^2$

Rotational kinetic work $= \frac{1}{2} I w^2$

Here, moment of intertia is,

$$I = \frac{2}{5} mr^2$$

$$\Rightarrow \cancel{\frac{1}{2} m}$$

we can write,

$$\cancel{\frac{1}{2} mv^2 + \frac{1}{2} \times \frac{2}{5}}$$

$$\frac{1}{2} mv^2 + \frac{1}{2} I w^2 = mgL \sin(30^\circ)$$

$$\Rightarrow \frac{1}{2} mv^2 + \frac{1}{2} \times \frac{2}{5} mr^2 \frac{v^2}{r^2} = \frac{1}{2} mgL$$

$$\Rightarrow \frac{1}{2} mv^2 + \frac{1}{5} mv^2 = \frac{1}{2} mgL$$

$$\Rightarrow \frac{7}{10} mv^2 = \frac{1}{2} mgL$$

$$\Rightarrow v^2 = \frac{5}{7} gL$$

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$$\therefore V = \frac{\sqrt{5}}{\sqrt{7}}\sqrt{gL}$$

(Ans)

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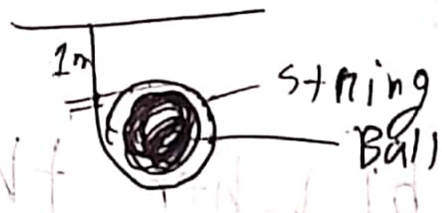
2](a] As the total length of



string is 4m and 1m is not wrapped with the sphere so ~~the~~ we can write,

$2\pi r = 4-1$

$\Rightarrow r = \dfrac{3}{2\pi}$

$= 0.4774\,m$

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we know,

moment of inertia of a sphere is,

$$I = \frac{2}{5} mr^2$$

$$= \frac{2}{5} \times 2 \times (0.4774)^2$$

$$= 0.1823 \text{ kgm}^2$$

(Ans)

b) when the sphere hits the left incline plane ~~to~~ with the potential energy is mgh than it converted into kinetic energy,

$$mgh = \frac{1}{2} I w^2 + \frac{1}{2} mv^2$$

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$$\Rightarrow mgh = \frac{1}{2} \times \frac{2}{5} mr^2w^2 + \frac{1}{2} mr^2w^2$$

$$\Rightarrow gh = \frac{1}{2} r^2w^2 + \frac{1}{5} r^2w^2$$

$$\Rightarrow gh = \left(\frac{1}{2} + \frac{1}{5}\right) r^2w^2$$

$$\Rightarrow w^2 = \frac{10}{7} \times \frac{gh}{r^2}$$

$$\therefore w = 13.575 \text{ rads}^{-1}$$

(Ans)

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c) ~~when~~ The sphere falls on the inclined plane with the energy mgh. But when it comes to stop at the inclined plane its potential energy becomes mgh'

As, given in the question that the energy lost during the motion is 10 J
we can write that,

$U_1 = 10 + U_2$

$\Rightarrow mgh = 10 + mgh'$

$\Rightarrow 2 \times 9.8 \times 3 = 10 + 2 \times 9.8\, h'$

h' is the height of the spheres center of mass

$\therefore \cancel{h' = 2.48}$

$\therefore h' = 2.49\,m$

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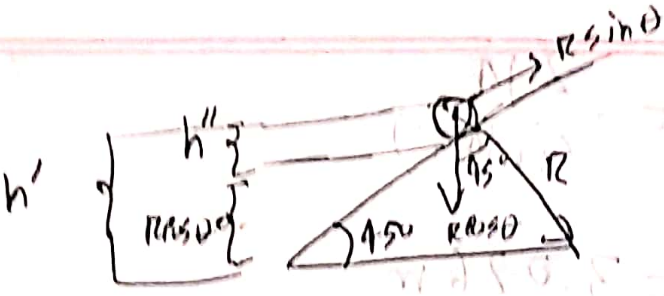

Here, h'' the height of the point at which the sphere touches the plane, when it is at rest on the inclined plane (right)

h' = height of its center of mass.

Here, $h'' = h' - R\cos\theta$

$= 2.48 - (0.4774)\cos 45$

$= 2.48 - 0.33$

$= 2.14$ m

So, distance travelled along the plane (at right)

$d = \dfrac{h''}{\sin\theta}$

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$$= \frac{2.14}{\sin(45^\circ)}$$

$$= 3.026\,m$$

(Ans)

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3] a] Moment of inertia of bullet I_b

" " " of board I_B

So we can write the equation,

$$I_b W_b + I_B W_B = (I_b + I_B) W_f$$

$$\therefore W_f = \frac{m\left(\frac{A}{2}\right)^2 \times \frac{V}{A/2} + 0}{I_b + I_B}$$

Here, $I_b = m\frac{A^2}{4}$

$I_B = \frac{1}{3} M A^2$

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$$w_f = \frac{m \frac{A}{2} \times V}{m\frac{A^2}{4} + \frac{1}{3}MA^2}$$

$$= 5.461 \text{ rad s}^{-1}$$

(Ans)

b) As the question want us to find the maximum height ~~so~~ does the center of the board reach from the equilibrium, we can write the equation as,

$$\frac{1}{2}(I_b + I_B)(w_f)^2 = (M+m)gh$$

$$\Rightarrow h = \frac{(\frac{1}{4}mA^2 + \frac{1}{3}MA^2)(w_f)^2}{2(M+m)g}$$

Here,

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$I_b = m\frac{A^2}{4}$ [Here A is the lenght of the board which is 0.25m]

$I_b = \frac{1}{3} MA^2$

$$h = \frac{(\frac{1}{4}mA^2 + \frac{1}{3}MA^2)(\omega_f)^2}{2(M+m)g}$$

$$= \frac{(\frac{1}{4}\times 1.9\times 10^{-3}\times 0.25^2 + \frac{1}{3}\times 0.75\times 0.25^2)(5.461)^2}{2(0.75+1.9\times 10^{-3})\times 9.8}$$

$= 0.0316\ m$

(Ans)

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c] The potential energy of the ~~board~~ bullet is,

$E = mgh$

$= 1.9 \times 10^{-3} \times 9.8 \times 0.25$

~~= 1.8~~

$= 1.842 J$

~~For the~~ We can write,

$E = \frac{1}{2} I w^2$ $\quad [I = \frac{1}{3} \times 0.75 \times 0.25^2$

$= 0.015625]$

$\Rightarrow \frac{1}{2} I w^2 = 1.842$

$\Rightarrow \frac{1}{2} \times 0.015625 \times w^2 = 1.842$

$\therefore w = 15.34 \ rads^{-1}$

So, minimum bullet speed is needed for the board to swing all the way over after

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the impact is,

$$\Rightarrow mv\left(\frac{A}{2}\right) = \left\{\frac{1}{3}MA^2 + m\left(\frac{A}{2}\right)^2\right\}w$$

Here,

$m = 1.9\times10^{-3}$ kg

$M = 0.75$ kg

$A = 0.25$m

$w = 15.34$ rad s^{-1}

$$\therefore mv\left(\frac{A}{2}\right) = \left\{\frac{1}{3}MA^2 + m\left(\frac{A}{2}\right)^2\right\}w$$

$$\therefore v = \frac{\left\{\frac{1}{3}MA^2 + m\left(\frac{A}{2}\right)^2\right\}w}{m\frac{A}{2}}$$

$= 1011.12\ ms^{-1}$

(Ans)

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