Math Assignment 3

SET 13

Section 13

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1] $x \sin y \, \partial x + (x^2+1) \cos y \, dy = 0$

$\Rightarrow x \sin y \, \partial x = -(x^2+1) \cos y \, \partial y$

$\Rightarrow \frac{x}{x^2+1} \partial x = -\frac{\cos y}{\sin y} \partial y$

$\Rightarrow \frac{x}{x^2+1} \partial x = -\cot(y) \, \partial y$

$\cancel{\Rightarrow \frac{1}{2} \ln}$

$\Rightarrow \int \frac{x}{x^2+1} \partial x = \int -\cot(y) \, \partial y$

$\Rightarrow \frac{1}{2} \ln|x^2+2| = -\ln|\sin(y)| + c$

Given that, $y(1) = \frac{\pi}{2}$

$\therefore \frac{1}{2} \ln|2| = -\ln|1| + c$

$\Rightarrow c = \ln|\sqrt{2}|$

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Putting the value of c in the equation we get,

$$\Rightarrow \frac{1}{2} \ln|x^2+1| = -\ln|\sin(y)| + \ln|\sqrt{2}|$$

$$\Rightarrow \sqrt{x^2+1} = \frac{1}{\sin(y)} + \sqrt{2}$$

$$\Rightarrow \operatorname{cosec}(y) = \sqrt{x^2+1} - \sqrt{2}$$

$$\Rightarrow y = \operatorname{cosec}^{-1}\left(\sqrt{x^2+1} - \sqrt{2}\right)$$

(Ans)

2) $x \frac{\partial y}{\partial x} + (3x+1)y = e^{-3x}$

$$\Rightarrow \frac{\partial y}{\partial x} + \left(3 + \frac{1}{x}\right) y = \frac{1}{x} e^{-3x}$$

So,

$$\mu(x) = e^{\int (3+\frac{1}{x})\,\partial x}$$

$$= e^{(3x + \ln x)}$$

Multiplying $\mu(x)$ to the both sides we get,

$$\frac{\partial y}{\partial x} e^{(3x+\ln x)} + \left(3+\frac{1}{x}\right) e^{(3x+\ln x)} y = \frac{1}{x} e^{-3x} e^{(3x+\ln x)}$$

$$\Rightarrow \frac{\partial}{\partial x}\left(e^{(3x+\ln x)} y\right) = \frac{1}{x} e^{\ln x}$$

$$\Rightarrow \frac{\partial}{\partial x}\left(e^{(3x+\ln x)} y\right) = 1$$

$$\Rightarrow e^{(3x+\ln x)} y = x + c$$

$$\Rightarrow e^{3x} e^{\ln x} y = x + c$$

$$\Rightarrow e^{3x} x\, y = x + c$$

$$\Rightarrow y = e^{-3x} + c\, e^{-3x} x^{-1}$$

$$3]\ \int_0^1 \int_0^{1-x^2} \int_3^{4-x^2-y} x\, \partial z\, \partial y\, \partial x$$

$$= \int_0^1 \int_0^{1-x^2} x\,[4-x^2-y-3]\, \partial y\, \partial x$$

$$= \int_0^1 \int_0^{1-x^2} (x - x^3 - xy)\, \partial y\, \partial x$$

$$= \int_0^1 \left[xy - x^3 y - \frac{1}{2} x y^2\right]_0^{1-x^2} \partial x$$

$$= \int_0^1 \left[x - x^3 - x^3 + x^5 - \frac{1}{2} x (1 - 2x^2 + x^4)\right] \partial x$$

$$= \int_0^1 (x - 2x^3 + x^5 - \frac{1}{2} x + x^3 - \frac{1}{2} x^5)\, \partial x$$

$$= \int_0^1 (\frac{1}{2} x - x^3 + \frac{1}{2} x^5)\, \partial x$$

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$$= \left[\frac{1}{4}x^2 - \frac{1}{4}x^4 + \frac{1}{12}x^6\right]_0^1$$

$$= \frac{1}{12} \text{ (Ans)}$$

4) Given that,

$$x^2 + y^2 = 4$$

$$\therefore r^2 = 4$$

$$\therefore r = 2$$

and,

$$x + y + z = 4$$

$$\therefore z = 4 - x - y$$

$$= 4 - r\cos\theta - r\sin\theta$$

So we can write,

The region enclosed by the cylinder is,

$$\int_0^{2\pi}\int_0^{2}\int_0^{4-r\cos\theta-r\sin\theta} r\,\partial z\,\partial r\,\partial\theta$$

$$=\int_0^{2\pi}\int_0^{2}(4r-r^2\cos\theta-r^2\sin\theta)\,\partial r\,\partial\theta$$

$$=\int_0^{2\pi}\left[2r^2-\frac{1}{3}r^3\cos\theta-\frac{1}{3}r^3\sin\theta\right]_0^2\partial\theta$$

$$=\int_0^{2\pi}\left[8-\frac{8}{3}\cos\theta-\frac{8}{3}\sin\theta\right)\partial\theta$$

$$=\left[8\theta-\frac{8}{3}\sin\theta+\frac{8}{3}\cos\theta\right]_0^{2\pi}$$

$$=\left[16\pi-\frac{8}{3}\sin(2\pi)+\frac{8}{3}\cos(2\pi)\right]-0+\frac{8}{3}\sin(0)-\frac{8}{3}\cos(0)$$

$$=16\pi$$

(Ans)

5] Given that,

$\cancel{u = x}$

$u = x - 2y$ —— ①

$v = 2x + y$ —— ⑪

① × 2 − ⑪ we get,

$2u = 2x - 4y$

$-v = -(2x + y)$

$2u - v = -5y$

$\therefore y = \dfrac{v - 2u}{5}$

① + ⑪ × 2 we get,

$u = x - 2y$

$2v = 4x + 2y$

$u + 2v = 5x$

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$\therefore x = \frac{u+2v}{5}$

We know,

~~Juco~~ Fo Jacobian,

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{1}{5} & -\frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} \end{vmatrix}$$

$$= \frac{1}{25} + \frac{4}{25}$$

$$= \frac{5}{25}$$

$$= \frac{1}{5}$$

We can write the enclosed limits as,

$u = 1$

$u = 4$

$v = 1$

$v = 3$

We can write the following rectangular region as,

$$\cancel{\frac{1}{5}\int_{1}^{3}}$$

$$\frac{1}{5}\int_{1}^{3}\int_{1}^{4}\frac{u}{v}\,\partial u\,\partial v$$

$$= \frac{1}{5}\int_{1}^{3}\frac{1}{v}\left[\frac{1}{2}u^2\right]_1^4 \partial v$$

$$= \frac{1}{10}\int_{1}^{3}\frac{1}{v}\left[16-1\right]\partial v$$

$$= \frac{3}{2}\int_{1}^{3}\frac{1}{v}\,\partial v$$

$$= \frac{3}{2}\left[\ln|v|\right]_1^3$$

$$= \frac{3}{2}\ln|3|$$

(Ans)

Do the following tasks using Mathematica.

If a projectile is fired with an initial velocity of v0 meters per second at an angle α above the horizontal and air resistance is assumed to be negligible, then it's position after t seconds is given by the parametric equations $x = (v_0 \cos \alpha)t$ and $y = (v_0 \sin \alpha)t - \frac{1}{2} g t^2$. Also v_x = dx/dt and v_y = dy/dt

(a) Find v_x and v_y . If a gun is fired with α = 300 and v0 = 500m/s when will the bullet hit the ground? How far from the gun will it hit the ground?

What is the maximum height reached by the bullet?

Hint: The Bullet reaches the ground when y=0 and it is at

maximum height when $v_y = 0$

```
In[1]:= x = v0 Cos[α] t;

In[2]:= y = v0 Sin[α] t - 1/2 g t^2;

In[3]:= vx = D[x, t]
Out[3]= Cos[α] v0

In[4]:= vy = D[y, t]
Out[4]= -g t + Sin[α] v0

In[5]:= α = 30 °;
        v0 = 500;
        g = 9.8;

In[8]:= Solve[y == 0 , t]
Out[8]= {{t → 0. + 0. i}, {t → 51.0204}}
```

After 51.0204 sec bullet will hit the ground

```
In[9]:= x /. t → 51.0204081632653`
Out[9]= 22092.5
```

Bullet hit the ground 22092.5 meters far from the gun

```
In[10]:=
        Solve[vy == 0, t]
Out[10]= {{t → 25.5102}}
```

In[11]:=
```
y /. t → 25.51020408163265`
```
Out[11]= 3188.78

Maximum height reached by the bullet is 3188.78 meters

(b) Plot the path of the projectile for v_0 = 500m/s and α = 30 °, 45° and 60° in a single graph.

In[12]:=

$$\text{Solve}\left[v_0 \,\text{Sin}[30\,°]\, t - \frac{1}{2} g\, t^2 == 0\,,\ t\right]$$

Out[12]= {{t → 0. + 0. i}, {t → 51.0204}}

In[13]:=

$$\text{Solve}\left[v_0 \,\text{Sin}[45\,°]\, t - \frac{1}{2} g\, t^2 == 0\,,\ t\right]$$

Out[13]= {{t → 0. + 0. i}, {t → 72.1538}}

In[14]:= $$\text{Solve}\left[v_0 \,\text{Sin}[60\,°]\, t - \frac{1}{2} g\, t^2 == 0\,,\ t\right]$$

Out[14]= {{t → 0. + 0. i}, {t → 88.3699}}

In[15]:=

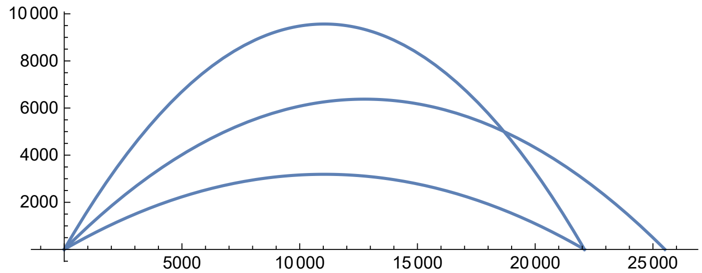
$$a = \text{ParametricPlot}\left[\left\{v_0 \,\text{Cos}[30\,°]\, t,\ v_0\, \text{Sin}[30\,°]\, t - \frac{1}{2} g\, t^2\right\}, \{t, 0\,, 51.0204081632653\text{`}\}\right];$$

$$b = \text{ParametricPlot}\left[\left\{v_0 \,\text{Cos}[45\,°]\, t,\ v_0\, \text{Sin}[45\,°]\, t - \frac{1}{2} g\, t^2\right\}, \{t, 0\,, 72.15375318230076\text{`}\}\right];$$

$$c = \text{ParametricPlot}\left[\left\{v_0 \,\text{Cos}[60\,°]\, t,\ v_0\, \text{Sin}[60\,°]\, t - \frac{1}{2} g\, t^2\right\}, \{t, 0\,, 88.36993916167741\text{`}\}\right];$$

In[18]:=
```
Show[{a, b, c} , PlotRange → All]
```

Out[18]=



(c) A torus can be expressed parametrically as

x = (a + b cos v) cos u

y = (a + b cos v) sin u

z = b sin v

Plot the torus for a = 5 , b = 2 and 0 ≤ u ≤ 2π and 0 ≤ v ≤ 2π and use rainbow color function .

In[19]:=
```
Clear["Global`*"]
x[u_, v_] = (a + b Cos[v]) Cos[u];
y[u_, v_] = (a + b Cos[v]) Sin[u];
z[u_, v_] = b Sin[v];
a = 5;
b = 2;
```

In[25]:=
```
ParametricPlot3D[{x[u, v], y[u, v], z[u, v]},
 {u, 0, 2 π}, {v , 0, 2 π}, ColorFunction → "Rainbow"]
```

Out[25]=