Tutorial -4

(Krishna Rathi F-62)

(1) $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n)$

$a \geq 1, b > 1$

on comparing

$a = 3, b = 2 \quad f(n) = n^2$

Now,

$C = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(n^2)$

(2) $T(n) = 4T(n/2) + n^2$

$a \geq 1, b > 1$

$a = 4, b = 2, f(n) = n^2$

$e = \log_2 4 = 2$

$\therefore n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \Theta(n^2 \log_2 n)$

(3) $T(n) = T(n/2) + 2^n$

$a = 1$

$b = 2$

$f(n) = 2^n$

$e = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = \Theta(2^n)$

(4) $T(n) = 2^n T(n/2) + n^n$

$a = 2^n$

$b = 2, \quad f(n) = n^n$

$e = \log_b a = \log_2 2^n$

$= n$

$n^c \Rightarrow n^n$

$\therefore f(n) = n^c$

$\therefore T(n) = \Theta(n^2 \log_2 n)$

(5) $T(n) = 16T(n/4) + n$

$a = 16, b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2$

$n^c = n^2$

$f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

(6) $T(n) = 2T(n/2) + n \log n$

$a = 2, b = 2$

$f(n) = n \log n$

$c = \log_2 2 = 1$

$\therefore n^c = n^1 = n$

Since, $n \log n > n$

$f(n) > n^c$

$\therefore T(n) = \Theta(n \log n)$

$T(n) = 2T(n/2) + n/\log n$

$a = 2, b = 2, f(n) = n/\log n$

$c = \log_2 2 = 1$

$\therefore n^c = n^1 = n$

Since, $\frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n)$

(8) $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, b = 4, f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$\therefore n^c = n^{0.5}$

Since, $n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \Theta(n^{0.51})$

(9) $T(n) = 0.5T(n/2) + 4n$

$a = 0.5, b = 2$

Since a<1 to Master theorem, $a \geq 1$, but here a is 0.5 so we can't apply master theorem

(10) $T(n) = 16T(n/4) + n!$

$a = 16, b = 4, f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

Now, $n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \Theta(n!)$

(11) $4T(n/2) + \log n$

$a = 4, b = 2, f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$\therefore n^c = n^2$

$f(n) = \log(n)$

Since $\log n < n^2$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n^c)$

$= \Theta(n^2)$

(12) $T(n) = \sqrt{n}\, T(n/2) + \log n$

$a = \sqrt{n}, b = 2$

$\therefore c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2}\log_2 n$

$\therefore \frac{1}{2}\log_2 n < \log(n)$

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(f(n))$

$= \Theta(\log(n))$

(13) $T(n) = 3T(n/2) + n$

$a = 3, b = 2, f(n) = n$

$c = \log_b a = \log_2 3 = 1.5849$

$\therefore n^c = n^{1.5849}$

$\therefore n < n^{1.5849}$

$\Rightarrow f(n) < n^c$

$\therefore T(n) = \Theta(n^{1.5849})$

$T(n) = 3T(n/3) + \text{sqrt}(n)$

$a = 3, \; b = 3$

$c = \log_b^a = \log_3^3 = 1$

$\because n^c = n^1 = n$

As $\text{sqrt}(n) < n$

$\because f(n) < n^c$

$\therefore T(n) = \Theta(n)$

(15) $T(n) = 4T(n/2) + cn$

$a = 4 ; b = 2$

$c = \log_b^a = \log_2^4 = 2$

$\because n^c = n^2$

$\because cn < n^2$ (for any constant)

$\because f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

(16) $T(n) = 3T(n/4) + n\log n$

$a = 3, \; b = 4, \; f(n) = n\log n$

$c = \log_b^a = \log_4^3 = 0.792$

$n^c = n^{0.792}$

$\because n^{0.792} < n\log n$

$\therefore T(n) = \Theta(n \log n)$

(17) $T(n) = 3T(n/3) + n/2$

$c = \log_b^a = \log_3^3 = 1$

$f(n) = n/2$

$n^c = n^1 = n$

As $n/2 < n$

$f(n) < n^c$

$T(n) = \Theta(n)$

(18) $T(n) = 6T(n/3) + n^2 \log n$

$a = 6 \; , b = 3$

$c = \log_b^a = \log_3^6 = 1.6309$

$n^c = n^{1.6309}$

As $n^{1.6309} < n^2 \log n$

$\therefore T(n) = \Theta(n^2 \log n)$

(19) $T(n) = 4T(n/2) + n\log n$

$a = 4, b = 2 . \; f(n) = n\log n$

$c = \log_b^a = \log_2^4 = 2$

$n^c = n^2$

$\because n\log n < n^2$

$\therefore T(n) = \Theta(n^2)$

(20) $T(n) = 64T(n/8) + n^2 \log n$

$a = 64 \; , b = 8$

$c = \log_b^a = \log_8^{64} = \log_8 (8)^2$

$c = 2$

$\because n^c = n^2$

$\because n^2 \log n > n^2$

$\therefore T(n) = \Theta(n^2 \log n)$

$T(n) = 7T(n/3) + n^2$

$a = 7, b = 3, f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$\Rightarrow n^{1.7712} < n^2$

$\therefore T(n) = \Theta(n^2)$

(22) $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, b = 2$

$c = \log_b a = \log_2 1 = 0$

$\therefore n^c = n^0 = 1$

$\therefore n(2 - \cos n) > n^c$

$\therefore T(n) = \Theta(n(2 - \cos n))$

The end