Tutorial sheet - 6

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Solution 1) Minimum spanning tree is a subset of the edge of a connected edge-weighted undirected graph that connects all the vertices together without any cycle & with the minimum possible total edge weighted.

Applications:

(1) Consider n stations are to be linked using a communication network and lying of communication link b/w any two stations involves a cost.

The ideal solution would be to extract or subgraph termed as minimum cost spanning tree.

(2) Suppose you want to construct highways or rail roads spanning several cities then we can use the concept of spanning tree.

(3) Digning LAN

(4) Laying pipelines connecting offshore drilling sites, refineries & consumer markets

(5) Suppose you meant to apply a set of houses with:

=> Electric power
=> Water
=> Telephone lines
=> Sewage lines

PTO

Solution 2) :→ Time complexity of prim's algorithm : $O(|E|$

space complexity of ~~kruskal~~ prim's algorithm : $O(|V|)$

⇒ Time complexity of kruskal's algorithm : $O(|E| \log E)$

⇒ space complexity of kruskal's algorithm : $O(|V|)$

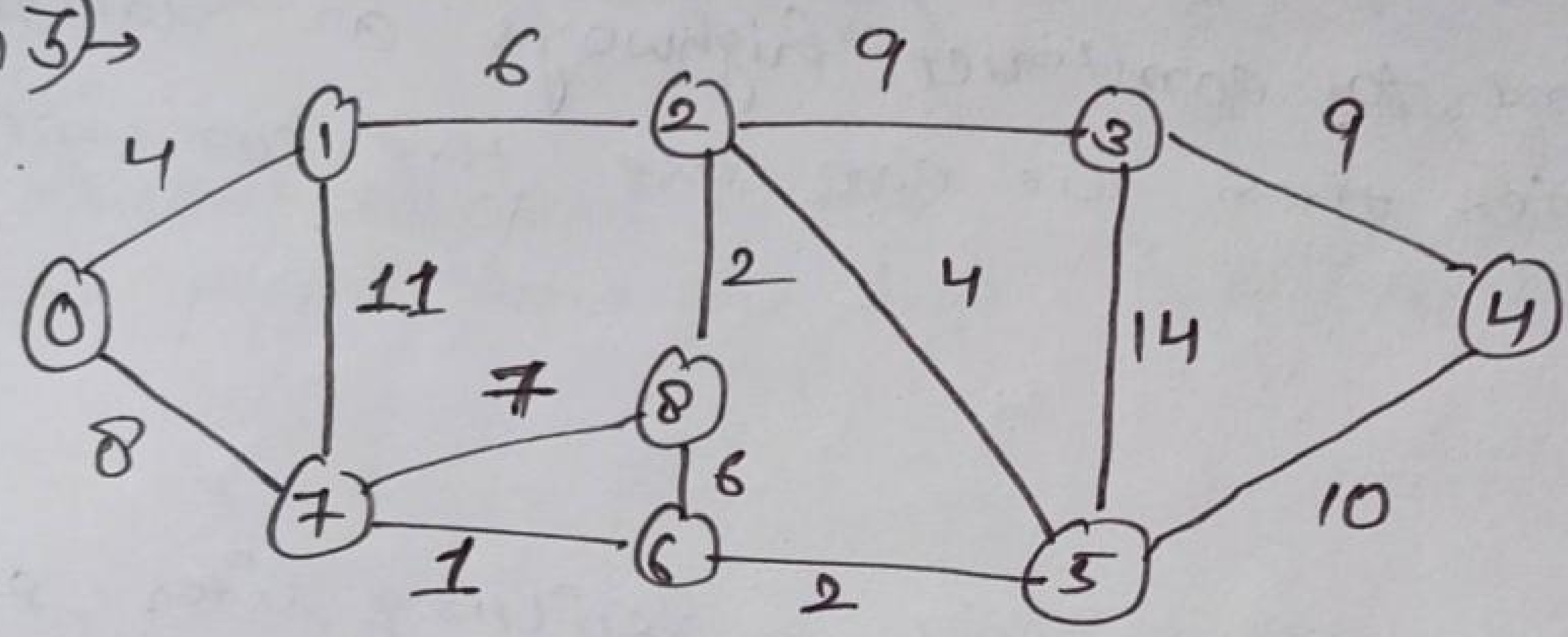
⇒ Time complexity of Dijkstra's algorithm : $O(V^2)$

⇒ space complexity of Dijkstra's algorithm : $O(V^2)$

⇒ Time complexity of Bellman ford's algorithm : $O(VE)$

⇒ space " " " " " = $O(e)$

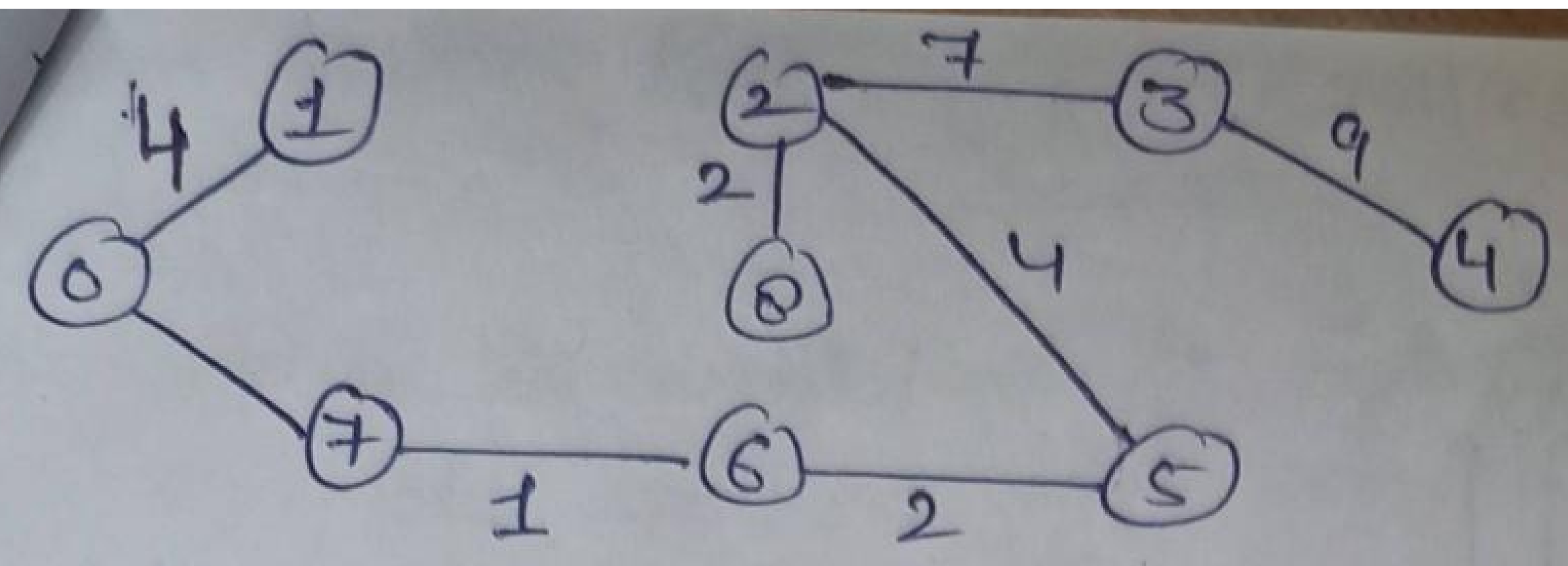
Solution 3) →



Kruskal's algorithm :

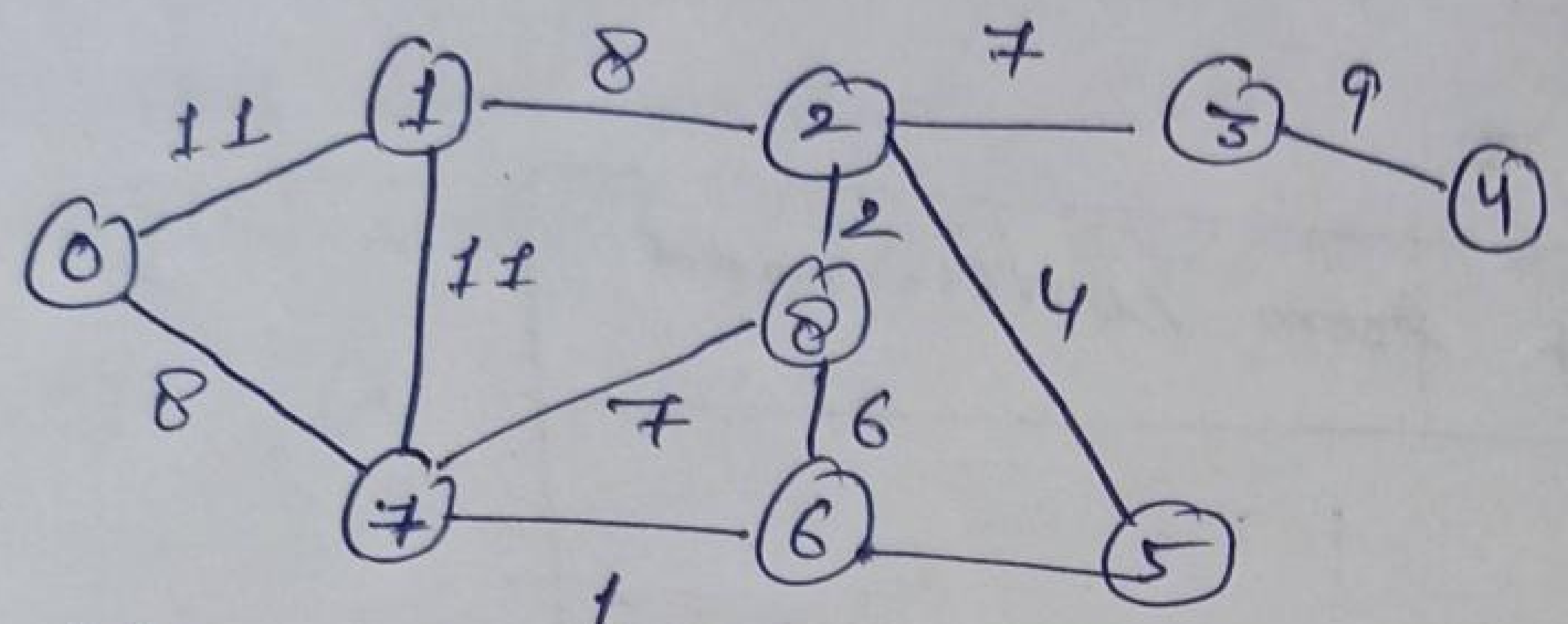
U	V	W	
6	7	1	✓
5	6	2	✓
2	8	2	✓
0	1	4	✓
2	5	4	✓
6	8	6	✗
2	3	7	✓
7	8	7	✗
0	7	8	✓
1	2	8	✗

U	V	W	
4	3	9	✓
4	5	10	✗
1	7	11	✗
3	5	14	✗



Weight = 1 + 2 + 2 + 4 + 4 + 7 + 8 + 9 = 37

Prim's Algorithm



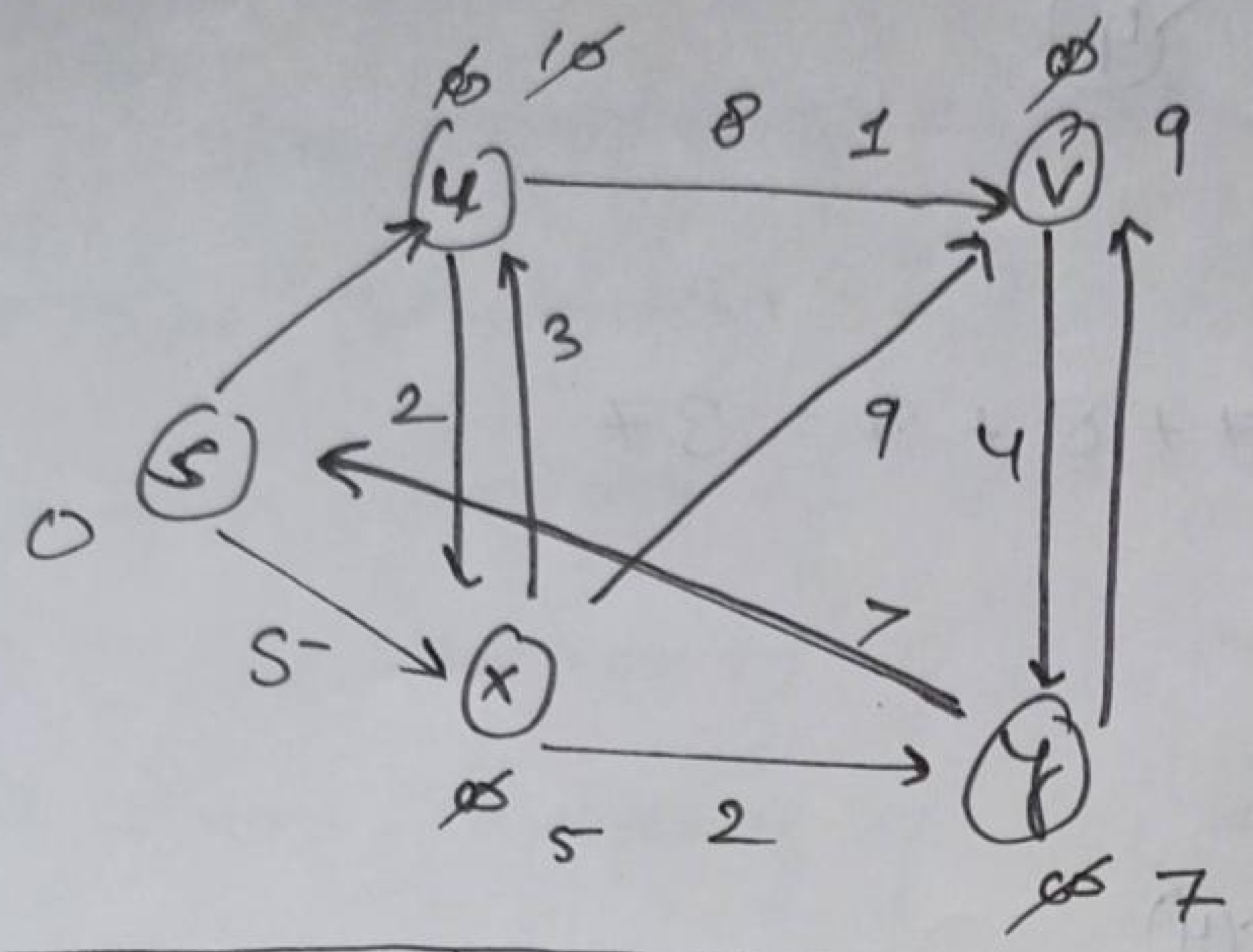
Weight = 4 + 8 + 2 + 4 + 2 + 7 + 9 = 37 Ans

Solution 4) → ① The shortest path may change. The reason is there may be different no. of edges in different path from 's' to 't'. for eg., let shortest path be of weight 15 and has edge 5 sides. let there be another path with 2 edges and total weight 25. The weight of the shortest path is increased by 2 ✓10 and become 15 + 50 weight of the other path is inc. by 2 ✓10 & become 25 + 20. So, the shortest path changes with weight as 45.

② If we multiply all edges weight by 60, the shortest path doesn't change. The reason is simple, weight of all paths from 's' to 't' get multiplied by some amount. The number of edges on a path doesn't matter. It is like changing units of weights.

PTO

Solutions → Dijkstra's Algorithm



node	shortest dist from source nodes
u	8
x	5
v	9
y	7

Bellman ford algorithm

1st → S^{0} $u^{\cancel{\infty}\,10}$ v^{∞} $x^{\cancel{\infty}\,5}$ y^{∞}

2nd → S^{0} u^{10} $v^{\cancel{\infty}\,11}$ x^{5} y^{∞}

graph doesn't have -ve cycle

3rd → S^{0} $u^{\cancel{10}\,8}$ $v^{\cancel{11}\,9}$ x^{5} $y^{\cancel{\infty}\,7}$

4th → S^{0} u^{8} v^{9} x^{5} y^{7}

final graph :-