Why are voltage dividers universal to any circuit?

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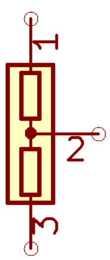
February 21, 2021

1 Introduction

Voltage divider is an ubiquitous and vesrsitaile electric component. It consists of two resistors connected in series with output terminal between them (See Figure 1. If the premise upon which the component works is manipulating two resistances, why cannot they be bought as a product?

Figure 1:



2 A voltage divider on its own

As mentioned in the introduction, a voltage divider are two resistors put in series and the desired output is across one of resistors. A very intuitive example of a voltage divider is a potentiometer, which generally is some kind of resistive material with slider on it where the resistance is proportional to the length of the material between slider and terminal (See Figure -). Potentiometer will be the basis of subsequent considerations.

3 Output voltage under no load

In order to calculate the output voltage divider will give, we need to use Ohm's law to derive the formula (use figure - as reference). Ohm's law states that $V = IR$, and since our output voltage is found across R_2, we can find V_{out} by doing the following (Equation (2))

$$V_{out} = IR_2, I = \frac{V}{R} \tag{1}$$

$$V_{out} = \frac{V_{in}}{R_1 + R_2} \cdot R_2 = V_{in} \frac{R_2}{R_1 + R_2} \tag{2}$$

From the calculations we could say that the output voltage V_{out} is dependent on the ratio of R_2 and the resistance of the whole circuit $R_1 + R_2$, and considering the case of potentiometer, where the net resistance always amounts to some constant value, the function of V_{out} with reference to R_2 is linear. From this point, it could be wrongly said that in order to step down the voltage to power some component, all that needs to be done is to set the dial to desider R_2 and plug in the load. This, however is not the case.

4 Output voltage under load

Figure - shows our potentiometer with load connected to the output terminal. It is already visible that the load, which has its own resistance is connected in parallel to R_2 rendering the net resistance of R_l and R_2 different, as the resistance calculated for parallel circuit (Equation (3)) will amount to less than the sum of the two resistances.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n} \tag{3}$$

We can derive the formula for this case from equation (2) (Equation 6)

$$R_p = (\frac{1}{R_2} + \frac{1}{R_l})^{-1} = \frac{R_2 R_l}{R_2 + R_l} \tag{4}$$

$$V_{out} = \frac{V_{in} \cdot R_p}{(R_1 + R_p)} \tag{5}$$

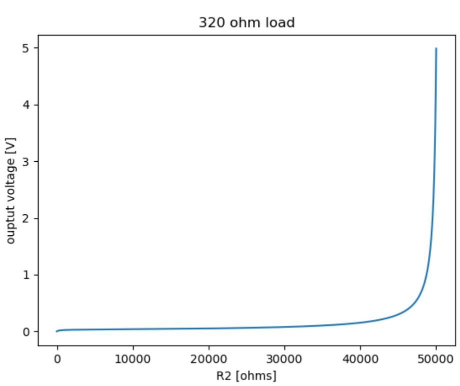
$$V_{out} = \frac{V_{in}(R_2 R_l)}{R_1(R_2 + R_l) + R_2 R_l} \tag{6}$$

We can see from equation (6) that the load connected to the potentiometer is an integral part to the circuit that cannot be neglected in any way and that it caused the linearity of the voltage output to disappear.

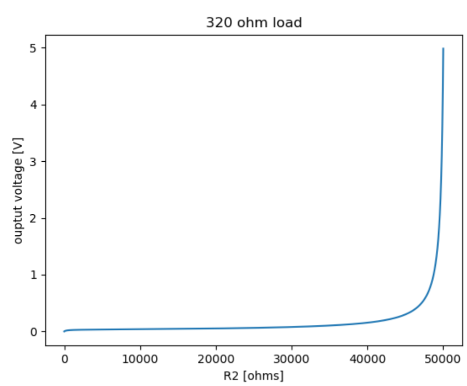
5 Voltage divider output with different loads

Now, let us check the calculations with an experiment. Circuit in figure - will be used to generate analog signal by changing the position of the dial in a $50k\Omega$ potentiometer with different loads connected to the output of the potential divider. The analog signal will be monitored and saved using a microcontroller. In order to make a theoretical prediction, a function $V_{out}(R_2)$ will be plotted with the same values using a Python script (See Appendix -)

5.1 320Ω load

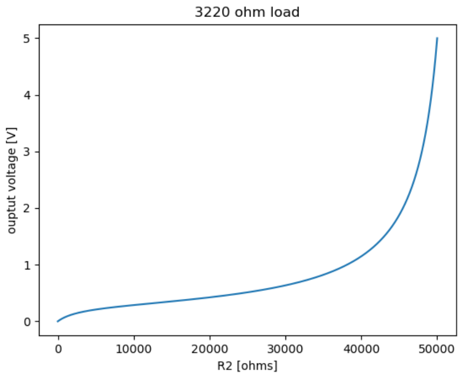


(a) prediction

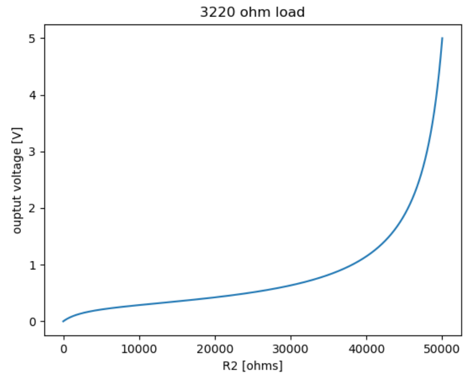


(b)

5.2 3220Ω load

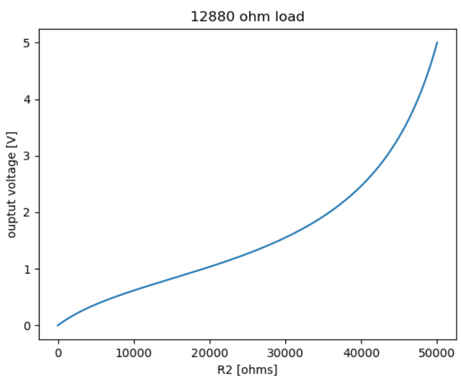


(c) prediction

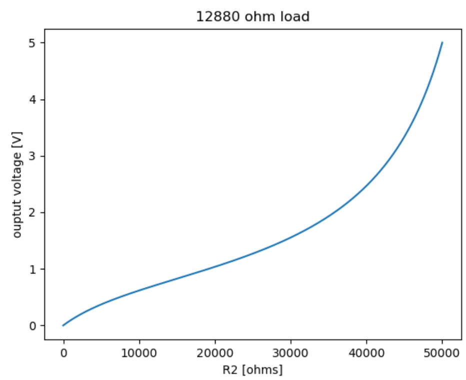


(d)

5.3 12880Ω load

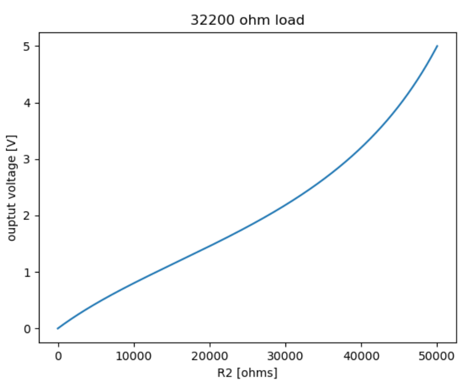


(e) prediction

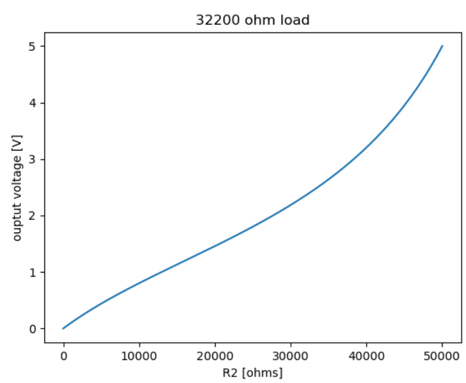


(f)

5.4 32200Ω load



(g) prediction



(h)