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Professor McDonnell | Assigned: February 7, 2017 | Due: February 16, 2017

Contents

3 Predicates, Statements, Arguments and Quantified Statements

3.1 - 12, 16b, 16d, 16f, 25d, 26a, 30a, 30c and 33d

12) $\forall$ real x and y, $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$
Let $x = 10$ and let $y = 10$, this would make $\sqrt{x+y}$ really just be $\sqrt{20}$ which is not equal to $2\sqrt{10}$, coming from $\sqrt{x} + \sqrt{y} = \sqrt{10} + \sqrt{10}$. This would make the latter part of the implied conditional false which would make the "$\forall$" part false which in the end makes this statement **false**.

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16 -
b) $\forall$ real x, $x > 0$, $x < 0$ or $x = 0$.
d) $\forall$ logicians x, $\sim$ (logicians, x, are lazy)
f) $\forall$ real x, $\sim (x^2 = -1)$

25d) i) $\forall$ irrational x, $-x$ is also irrational.
ii) $\forall$ irrational x, if x is irrational then $-x$ is also irrational.

26a) $\forall x$, if x is an integer then it is rational, but $\exists$rational x such that if x is rational then it is not an integer.

30a) $\exists x \in \mathbb{Z}$ such that Prime(x) $\wedge$ $\sim$Odd(x)
Let $x = 2$, 2 is prime and 2 is not odd, this makes the conditional/conjunction true and this makes the whole predicate true because the premises only requires one to satisfy to yield **true**

30c) $\exists x \in \mathbb{Z}$ such that odd(x) $\wedge$ square(x)
Let $x = 81$, 81 is odd and 81 is a perfect square ($9^2 = 81$). This makes the conditional/conjunction true and this makes the whole predicate true because the premises only requires one to satisfy to yield **true**

33d) $\mathbb{R} = \{a, b, c, d\}$ and $(a < b) \wedge [(c < d) \Rightarrow (ac < bd)]$
Let $a = 2$, $b = 3$, $c = -3$ and $d = -2$, this would yield in $(2 < 3) \wedge [(-3 < -2) \Rightarrow (-6 < -6)]$
$-6 \not< -6$ so this would yield in the conditional being $T \rightarrow F$ which outputs a false, which makes the conjunction $T \wedge F$ which then evaluates to a **false** predicate.

3.2 - 2, 14, 21, 31, 42, 44

2) Original Statement: "All dogs are loyal"
a) "All dogs are not loyal" (Negation)
b) "No dogs are loyal" (Negation)
c) "Some dogs are disloyal" (Negation)
d) "Some dogs are loyal" (Not a Negation)
e) "There is a disloyal animal that is not a dog" (Not a Negation)
f) "There is a dog that is loyal" (Negation)

g) "No animals that are not dogs are loyal" (Negation)
h) "Some animals that are not dogs are loyal" (Negation)

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14) **Incorrect**
Corrected: $\exists$ real x_1 and x_2 such that $x_1^2 = x_2^2$ and $x_1 \neq x_2$

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21) **Original**: $\forall$ integers n, if (n is divisible by 6), then (n is divisible by 2 and n is divisible by 3).
Negation: $\exists$ integer n such that (n is divisible by 6) and $\sim$ (n is divisible by 2 and n is divisible by 3)

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31) **Original**: $\forall$ integers n, if (n is divisible by 6), then (n is divisible by 2 and n is divisible by 3). **True**
Converse: $\forall$ integers n, if (n is divisible by 2 and n is divisible by 3), then (n is divisible by 6). **True**
Inverse: $\forall$ integers n, if (n is not divisible by 6), then (n is not divisible by 2 or n is not divisible by 3). **True**
Contra: $\forall$ integers n, if (n is not divisible by 2 or n is not divisible by 3), then (n is not divisible by 6). **True**

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42) {$\exists$ comprehensive exams c such that if c is not passed, then masters is not obtained.}

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44) {$\exists$ person p such that $\sim$ (if p has happiness, then p has a high salary.)}

3.3 - 10b, 10d, 10f, 11b, 11e, 11f, 19, 24b, 36, 41d and 41g

10
b) $\forall$ students s, $\exists$ a salad T such that s chose T.
This statement is **False** because Yuen didn't choose a salad.
d) $\exists$ a beverage b, such that $\forall$ students D, D chose b.
This statement is **False** because the three people didn't have one drink in common.
f) $\exists$ a station Z, $\forall$ students s, $\exists$ an item I such that s chose I from Z.
This statement is **True** because all three chose one dish from one station, pie.

11
b) If you are a student in S and then you have seen Star wars.
e) If student s is in S and student t is in S and s is not the same as t, t and s will watch the same movie m in M.
f) If student s is in S and student t is in S and s is not the same as t, they will watch the same but an undefined movie m in set M.

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19) **Negation:** Any x in $\mathbb{R}^+$ will have one y in $\mathbb{R}^+$ such that $x > y$.

24b)

$$\sim (\exists x \in D(\exists y \in E(P(x,y))))$$

$$\equiv \forall x \in D \sim (\exists y \in E(P(x,y)))$$

$$\equiv \forall x \in D(\forall y \in E(\sim P(x,y)))$$

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36) "Somebody trusts everybody", Let $t(x, y)$ = "x trusts y"
Original: $\{\exists s \in S, \forall e \in E | t(s, e)\}$
Negation: $\{\forall s \in S, \exists e \in E | \sim t(s, e)\}$

41d) **Original:** $\{\forall x \in \mathbb{R}^+, \exists y \in \mathbb{R}^+ | xy = 1\}$
Yes this is **True** because if you have a positive real number, the reciprocal is of that is indeed a real number and that times the original = 1 $Q.E.D$ (the QED is a joke, if it doesn't apply, dont take points off pls)
g) **Original:** $\{\forall z \in \mathbb{Z}, \forall y \in \mathbb{Z} | z = x - y\}$
This is **True** because any real number subtracted from any real number will yield in 1 specific real number.

Extra Problem)
"For every object x, there is an object y such that $x \neq y$, then x and y have different colors"
Let $dC(x, y)$ = "x and y have different colors"
a) True
b) **Original:** $\{\forall \text{ objects } x, \exists \text{ object } y, |(x \neq y) \Rightarrow (dC(x, y))\}$
c) **Negation:** $\{\exists \text{ object } x, \forall \text{ objects } y, |(x \neq y) \wedge (\sim dC(x, y))\}$

3.4 - 12, 17, 18, 22, 27, 32

12)
All honest people pay taxes
Darth is not honest
$\therefore$ Darth does not pay taxes

This argument is **invalid** because this is an **inverse error**.

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17)
If an infinite series converges, then its terms go to 0.
The terms of the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ go to 0.
$\therefore$ The infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ converges.

This argument is **invalid** because this is a **converse error**.

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18)
If an infinite series converges, then its terms go to 0.
The terms of the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n-1}$ do not go to 0.
$\therefore$ The infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n-1}$ doesn't converge.

This argument is **valid** because **universal modus tollens.**

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22) Under the Proof (Number 32)

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27) Under the Proof (Number 32)

Number 32) - Main Proof
I) Let $G(x)$ = "Grumble from x" and $U(x)$ = "Understand x"
Statement: $\{\exists x \in L_e | \sim G(x) \Rightarrow U(x)\}$
II) Let $Ar(x)$ = "x arranged like I am used to"
Statement: $\{\forall x \in A_r | \sim Ar(x)\}$
III) Let $E(x)$ = "Easy Examples" and $H(x)$ = "Headache from x"
Statement: $\{\forall x \in L_e | E(x) \Rightarrow \sim H(x)\}$
IV) Phrases already defined
Statement: $\{\forall x \in A_r, \exists e \in L_e | \sim Ar(x) \Rightarrow \sim U(e)\}$
V) Phrases already defined
Statement: $\{\exists x \in L_e | \sim H(x) \Rightarrow \sim G(x)\}$
$\therefore \{\forall e \in L_e | \sim E(x)\}$

A) $\{\forall x \in A_r, \exists e \in L_e \| U(e) \Rightarrow Ar(x)\}$	**Contrapositive of Statement 4**
A) $\{\forall x \in A_r, \exists e \in L_e \| U(e) \Rightarrow Ar(x)\}$ 2) $\{\forall x \in A_r \| \sim Ar(x)\}$ $\therefore$ $\{\exists e \in L_e \| \sim U(e)\}$	From Statement A Statement 2 **B) Universal Modus Tollens**
5) $\{\exists x \in L_e \| \sim H(x) \Rightarrow \sim G(x)\}$ 3) $\{\forall x \in L_e \| E(x) \Rightarrow \sim H(x)\}$ $\therefore \{\forall x \in L_e \| E(x) \Rightarrow \sim G(x)\}$	Statement 5 Statement 3 **C) Universal Transitivity**
1) $\{\exists x \in L_e \| \sim G(x) \Rightarrow U(x)\}$ B) $\{\exists e \in A_r \| \sim U(e)\}$ $\therefore \{\exists e \in A_r \| \sim (\sim G(e))\}$	Statement 1 From Statement B **D) Universal Modus Tollens**
B) $\{\exists e \in A_r \| \sim (\sim G(e))\}$ $\therefore \{\exists e \in A_r \| G(e)\}$	From Statement D **E) Universal Double Negation Law**
C) $\{\forall x \in L_e \| E(x) \Rightarrow \sim G(x)\}$ E) $\{\exists e \in A_r \| G(e)\}$ $\therefore \{\forall e \in L_e \| \sim E(x)\}$	From Statement C From Statement E **Universal Modus Tollens**