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Professor McDonnell | Assigned: February 16, 2017 | Due: February 28, 2017

Contents

4 Elementary number theory and methods of proof

4.1 10, 21, 34, 41, 53

10) There is an integer n such that $2n^2 - 5n + 2$ is prime.
A integer n is prime if and only if its greater than 1 and if $n = rs$ then either ($r = 1$ and $s = n$) or ($s = 1$ and $r = n$). This is the case due to the definition of a prime number.
To disprove this, the second half should be false, so if none of the components are equal to one or if the are both greater than one.

$$2n^2 - 5n + 2$$
$$(2n-1)(n-2) \text{ Let } (2n-1) = r \text{ and } (n-2) = s$$
$$2n - 1 > 1 \text{ and } n - 2 > 1$$
$$2n - 1 > 1 \text{ and } n - 2 > 1$$
$$n > 1 \text{ and } n > 3$$

If n is greater than three is it also greater than one. There are select integers greater than three that satisfy said condition. $Q.E.D$ $\boxed{\text{True}}$

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21) For all real numbers, if $x > 1$ then $x^2 > x$.
Proof: Given $\forall x \in \mathbb{R}$, $x > 1$ $\boxed{\text{TRUE}}$ because:

$$x(x > 1)$$
$$\equiv x^2 > x \quad Q.E.D$$

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34) If n is an odd integer than $(-1)^n = (-1)$ Proof:

$$(-1)^n = (-1)^{2n+1} \equiv ((-1)^2)^n + (-1)^1 \equiv (1)^n + (-1)^1 \equiv (1)^n + (-1)^1$$
$$\equiv -1$$

$\boxed{\text{True}}$

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41) $mn = (2p)(2q+1) =$ They assumed that $m \cdot n$ will be even it as $r = 4pq + 2p = 2(2pq + p) = even$
$\boxed{\text{The set of r was never defined/initialized}}$

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53) $\forall$ integers $n,\ n^2 - n + 11$ is prime
<u>Proof:</u> A integer n is prime if and only if its greater than 1 and if $n = rs$ then either ($r = 1$ and $s = n$) or ($s = 1$ and $r = n$). This is the case due to the definition of a prime number.

$$r : (n + 5 \geq 1) \text{ OR } s : (n - 6 \leq 1)$$

$$r : (n \geq -4) \text{ OR } s : (n \geq 7)$$

If n is less than 4 both conditions are violated. This makes the requirement for a prime number false. Making this argument $\boxed{\text{FALSE}}$ as it violates the "for all" premise. The counter example that makes is false is if n is less than -4.

4.2 18, 22, 28

18) If r and s are any two rational numbers, then $\dfrac{r+s}{2}$ is also rational.
<u>Proof:</u> Given that r and s are rational by definition of rationality it can be considered that $r = \dfrac{a}{b}$ and $s = \dfrac{c}{d}$ such that b and d both do not equal 0 and a,b,c and d are in the set of real integers.
Consider addition:

$$r + s = \frac{ad + bc}{bd}$$

ac and bd are integers because an integer times an integer is an integer. bd is one integer because an integer times an integer is an integer. By the definition of rationality if one were to divide the integers (such as: $r + s = \dfrac{ad+bc}{bd}$) the quotient of the two is rational. Thereby making r+s rational and the number 2 is just a constant integer so it can be added such that $\dfrac{r+s}{2} = \dfrac{ad+bc}{2bd}$ thus making it rational by definition of rationality.
$\boxed{\text{Q.E.D}}$

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22) True or False, if any integer a is odd then $a^2 + a$ is even
<u>Proof:</u> Let $a = 2r + 1$ such that r can be any integer in the set of all integers. (Definition of an odd number)
$a^2 + a = (2r+1)^2 + (2r+1) = 4r^2 + 4r + 1 + 2r + 1 = 4r^2 + 6r + 2 = 2(2r^2 + 3r + 1)$
Since $2r^2 + 3r$ is an integer through integer multiplication and integer addition yielding integers, we can let $(2r^2 + 3r + 1) = m$
Because of this, we can express $2(2r^2 + 3r + 1)$ as $2m$, this is indicative of the fact that it is even (By Definition).
$\therefore \boxed{\text{TRUE, Q.E.D.}}$

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28) Suppose a, b, c, d are integers in $\mathbb{Z}$ and $a \neq c$ suppose x is in $\mathbb{R}$ such that: $\dfrac{ax+b}{cx+d} = 1$

$$ax + b = cx + d$$

$$ax - cx = d - b$$

$$x(a - c) = (d - b)$$

$$\therefore x = \frac{d - b}{a - c}$$

This must mean that x is rational because $d - b$ and $a - c$ are integers in $\mathbb{Z}$ and the division of those two, x, yields in a rational number. Both these statements can be justified through the definition of integer subtraction and rationality. Yes x must be Rational. QED.

4.3 20, 27, 31, 38c

20) $\forall x$ s.t. $3|((x+0)+(x+1)+(x+2))$
This means that $3|(3x+3)$ is the claim that is claimed to be true for all x.
This can be simplified to $1|(x+1)$, through factorization.
The above statement is **True** as this is the identity postulate and $x+1$ is an Integer through integer addition and any integer divided by 1 is itself. $\boxed{\text{TRUE, QED}}$

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27) $\forall a \forall b \forall c$ in $\mathbb{Z}$ if $a|(b+c)$ then $(a|bora|c)$
This is **false**, Let a = 2, Let b = 5 and c = 1.

$$2|(5+1) \text{ but } 2 \nmid 1 \text{ and } 2 \nmid 5 \ \boxed{\text{QED}}$$

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31) $\forall$ integers a and b, if $a|(10b)$ then $(a|10)$ and $(a|b)$
FALSE because if we let a = 4 and b = 2 for example, $4|(20)$ holds true but $(4 \nmid 10)$ and $(4 \nmid 2)$

38c) $a = p_1^{e_1} \cdot p_2^{e_2} \cdots p_n^{e_n}$
$2^2 \cdot 3^5 \cdot 7^1 \cdot 11^1 \cdot m$
$m = 3^1 \cdot 7^1 \cdot 11^1 = \boxed{231}$
$\sqrt{2^2 \cdot 3^6 \cdot 7^2 \cdot 11^2} = 2^1 \cdot 3^3 \cdot 7^1 \cdot 11^1 = 4090$
$\boxed{(4090)^2}$

4.4 22, 25, 34, 37

22) $c \ mod\ 15 = 3$; $10c \ mod\ 15 = ?$;
$c - 15(c \ div\ 15) = 3$ Definition of modulus
$10c \ mod\ 15 = (10c) - 15(10c \ mod\ 15) = 10(c - 15(c \ mod\ 15)) = 10(3) = \boxed{30}$, Through definition of modulo and factorization.

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25) $a \ mod\ 7 = 5$ and $b \ mod\ 7 = 6$ then $ab \ mod\ 7 = 2$
$a - 7(a \ div\ 7) = 5$ Definition of modulus
$b - 7(b \ div\ 7) = 6$ Definition of modulus
$ab - 7(ab \ div\ 7) = a(b - 7(b \ div\ 7)) = a(6) = 2$ Definition of modulus
$ab - 7(ab \ div\ 7) = b(a - 7(a \ div\ 7)) = b(5) = 2$ Definition of modulus
This is only true for $a = \frac{1}{3}$ and $b = \frac{2}{5}$, any thing out side those values and it wouldnt work. This is $\boxed{\text{False}}$.

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34) **False**, Let $n = 4$ and its greater than three so it satisfies the base case.
$\exists$ positive integers r and s such that $n = rs$ and 1 < r < n and 1 < s < n. Definition/requirement for composite.
Since $n = 4$, 4 would be a composite number if we find that $r = 2$ and $s = 2$ and $2 \cdot 2 = 4$.
$n + 2 = 6$, 6 would be a composite number if we find that $r = 3$ and $s = 2$ and $3 \cdot 2 = 6$.
$n + 4 = 8$, 8 would be a composite number if we find that $r = 4$ and $s = 2$ and $4 \cdot 2 = 8$.
In this case we found a counter example that finds 3 composite integers so this statement (The question) is $\boxed{\text{False.}}$

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34) The square of any integer should take the form $4k$ or $4k+1$
Let n = 2m (Any even integer, by definition)
$n^2 = 4m^2$, let $m^2 = k$ so $n^2 = 4k$
Let n = 2q+1 (Any odd integer, by definition)
$n^2 = (2q+1)^2 = 4q^2 + 8q + 1 = 4(q^2 + 2q) + 1$, let $q^2 + 2q = p$ so $n^2 = 4p + 1$
In this we see that based on parity (Only two possible cases) n^2 can either be $4k$ or $4p+1$, QED., $\boxed{\text{True}}$

4.5 13, 22, 24

4.6 11, 15, 17