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Professor McDonnell | Assigned: March 28th, 2017 | Due: April 6th, 2017

Contents

6 Set Theory

6.1 7, 12b, 12d, 12f, 12h, 12j, 15b, 25, 33a, 33c, 35c, 35d

7) Given that

$$A = \{x \in \mathbb{Z} | x = 6a + 4 \text{ for some int } a\}$$
$$B = \{y \in \mathbb{Z} | y = 18b - 2 \text{ for some int } b\}$$
$$C = \{z \in \mathbb{Z} | x = 18c + 16 \text{ for some int } c\}$$

a) Prove that $A \subseteq B$ This means that
$$\{\forall a | (a \in A) \Rightarrow (b \in B)\}$$
$$\text{Let: } 6a + 4 = 18b - 2$$
$$\therefore 6a + 6 = 18b$$
$$\therefore 6(a+1) = 6(3b)$$
$$a + 1 = 3b$$
$$\boxed{\text{TRUE } \{\exists a \in \mathbb{Z}, \exists b \in \mathbb{Z} | a + 1 = 3b\}}$$

b) Prove $B \subseteq A$ This means that
$$\{\forall a | (a \in B) \Rightarrow (b \in A)\}$$
$$\text{Let: } 18b - 2 = 6a + 4$$
$$\therefore 18b = 6a + 6$$
$$\therefore 6(3b) = 6(a+1)$$
$$3b = a + 1$$
$$\boxed{\text{TRUE } \{\exists a \in \mathbb{Z}, \exists b \in \mathbb{Z} | 3b = a + 1\}}$$

c) Prove that $B = C$
This means that $B \subseteq C$ and $C \subseteq B$
Case One: $B \subseteq C$, This means that
$$\{\forall x | (x \to B) \Rightarrow (x \in C)\}$$
$$\text{Let: } 18b - 2 = 18c + 16$$
$$\therefore 18b = 18c + 18$$
$$\therefore 18(b) = 18(c+1)$$
$$b = c + 1$$
$$\boxed{\text{TRUE } \{\forall b \in \mathbb{Z}, \forall c \in \mathbb{Z} | b = c + 1\}}$$

Case Two: $C \subseteq B$, This means that

$$\{\forall x | (x \to C) \Rightarrow (x \in B)\}$$
$$\text{Let: } 18b - 2 = 18c + 16$$
$$\therefore 18b = 18c + 18$$
$$\therefore 18(b) = 18(c+1)$$
$$c + 1 = b$$
$$\boxed{\text{TRUE } \{\forall c \in \mathbb{Z}, \forall b \in \mathbb{Z} | c + 1 = b\}}$$

$$\boxed{B \subseteq C \text{ and } C \subseteq B \text{ so } \therefore B = C}$$

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12) Given that

$$A = \{x \in \mathbb{R} | -3 < x \leq 0\}$$
$$B = \{x \in \mathbb{R} | -1 < x < 2\}$$
$$C = \{x \in \mathbb{R} | 6 \leq x \leq 8\}$$
$$A^c = \{x \in \mathbb{R} | (-3 > x) \cup (x > 0)\}$$

b) $A \cap B = \{x \in \mathbb{R} | -1 < x < 0\}$
d) $A \cup C = \{x \in \mathbb{R} | (-3 \leq x \leq 0) \cup (6 < x \leq 8)\}$
f) $B^c = \{x \in \mathbb{R} | (-1 \geq x) \cup (x \geq 2)\}$
h) $A^c \cup B^c = \{x \in \mathbb{R} | (-1 \geq x) \cup (x > 0)\}$
j) $(A \cup B)^c = \{x \in \mathbb{R} | (-3 > x) \cup (x > 2)\}$

15b) Show that $A \subseteq B$, $C \subseteq B$ and $A \cap C \neq \emptyset$

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25) Let $R_i = \{x \in \mathbb{R} | -1 \leq x \leq \left(1 + \frac{1}{i}\right)\} = \left[1, 1 + \frac{1}{i}\right]$

a) $\bigcup_{i=1}^{4} R_4 = [1, 2]$
b) $\bigcap_{i=1}^{4} R_4 = \left[1, \frac{5}{4}\right]$
c) Yes because the big union notation scopes the entire set, A_1 would result in the biggest range and one that is the result of the union
d) [1,2]
e) [1,1]
f) [1,2]
g) [1,1]

33a) $\mathcal{P}(\emptyset) = \{\{\}\}$
33c) $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset))) = \{\{\{\{\}\}, \{\}\}, \{\{\}\}, \{\}\}$

35c) $A \times (B \cap C) = \{(a, 2), (b, 2)$
35d) $(A \times B) \cap (A \times C)) = \{(a, 2), (b, 2)$

6.2 14, 17, 23b, 32, 35

14) For all sets A, B and C if $(A \subseteq B)$ then $(A \cup C) \subseteq (B \cup C)$.

$$\text{Since } A \subseteq B, \text{then an integer } x \in A \text{ has to be in set B } - \text{Definition of a Subset}$$
$$\text{Suppose } x \in A, \text{then } x \in (A \cup C) - \text{Definition of Union}$$
$$\text{Suppose } x \in B, \text{ then } x \in (B \cup C) - \text{Definition of Union}$$
$$\text{Since } x \in (A \cup C) \text{ and } x \in (B \cup C) \text{ then } (A \cup C) \subseteq (B \cup C) - \text{By Definition of a subset}$$

$$\boxed{\text{TRUE, Q.E.D.}}$$

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17) For all sets A, B and C if $(A \subseteq B)$ and $(A \subseteq C)$ then $(A \cup B) \subseteq C$.

$$\text{Since } A \subseteq B, \text{then an integer } x \in A \text{ has to be in set B } - \text{Definition of a Subset}$$
$$\text{Since } A \subseteq B, \text{then an integer } x \in A \text{ has to be in set C } - \text{Definition of a Subset}$$
$$\text{Suppose } x \in A, \text{then } x \in (A \cup B) - \text{Definition of Union}$$
$$\text{Since } x \in (A \cup B) \text{ and } x \in (C) \text{ then } (A \cup B) \subseteq C - \text{By Definition of a subset}$$

$$\boxed{\text{TRUE, Q.E.D.}}$$

23b) Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ with a drawing.

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32) For all sets A, B and C, prove that if $(A \subseteq B)$ and $(B \cap C = \emptyset)$ then $(A \cap C = \emptyset)$

$$\text{Since } A \subseteq B, \text{then an integer } x \in A \text{ has to be in set B } - \text{Definition of a Subset}$$
$$\text{Since } (B \cap C = \emptyset), \text{ then an integer } x \in B \text{ cannot be in set C } - \text{Definition of a Subset and Union}$$
$$\text{Since } x \in A \text{ and } x \notin C, \text{ then } A \cap C = \emptyset - \text{Definition of Intersection and Null Set}$$

$$\boxed{\text{TRUE, Q.E.D.}}$$

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35) For all sets A, B and C, prove that if $(A \cap C = \emptyset)$ then $((A \times B) \cap (C \times D) = \emptyset)$

Consider (a_1, a_2) and (b_1, b_2), then assume $(A \times B) \cap (C \times D) = \emptyset$ – Definition of a Cartesian Product

Now $(a_1 \in A \text{ and } a_2 \in B) \cap (b_1 \in C \text{ and } b_2 \in D) = \emptyset$ – Definition of a Cartesian Product

Since $(a_1 \in A \text{ and } a_2 \in B) \cap (b_1 \in C \text{ and } b_2 \in D) = \emptyset$ then $(A \subseteq B) \cap (C \subseteq D) = \emptyset$ – Definition of subset (x2)

Since $A \cap C = \emptyset$ then $x \in A$ and $x \notin C$ – Definition of an intersection

Since $A \subseteq B$ then $x \in A$ and $x \in B$ – Definition of a Subset

Since $x \notin C$ and $C \subseteq D$ then $x \notin D$ – Definition of a Subset

We can now replace $(A \subseteq B) \cap (C \subseteq D)$ to be $((x \in A) \text{ and } (x \in B)) \cap ((x \notin C) \text{ and } (x \notin D)) - DefofSubset$

$$\boxed{\text{The final statement is true as 2 sets with no common values is a nullity , Q.E.D.}}$$

6.3 10, 20, 35, 73

10) For all the sets A, B and C if $(A \subseteq B)$ then $(A \cap B^c \neq \emptyset)$

$$\text{Since } A \subseteq B \text{ then } x \in A \text{ and } x \in B - \text{By definition of subset}$$
$$\text{Since } x \in A \text{ then } x \notin B^c - \text{By definition of compliment}$$
$$\text{Since } x \notin B^c = \emptyset - \text{By definition of intersection,}$$

$$\boxed{\text{The final statement is true as 2 sets with no common values is a nullity , Q.E.D.}}$$

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20) For all the sets A and B prove that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

$$\text{Since } \mathcal{P}(A \cap B) \text{ then } x \in \mathcal{P}(A \cap B) \text{ therefore } x \in (A \cap B) - \text{Definition of Power Set} \tag{1}$$
$$\text{Since } x \in (A \cap B) \text{ then } x \in A \text{ and } x \in B - \text{Definition of intersection} \tag{2}$$
$$\text{Since } x \in A \text{ and } x \in B \text{ then } x \in \mathcal{P}(A) \text{ and } x \in \mathcal{P}(B) - \text{Definition of Power Set} \tag{3}$$
$$\text{Since } x \in \mathcal{P}(A) \text{ and } x \in \mathcal{P}(B) \text{ then } \mathcal{P}(A) \cap \mathcal{P}(B) - \text{Definition of intersection} \tag{4}$$
$$\text{Since } x \in \mathcal{P}(A) \cap \mathcal{P}(B) \text{ then } \boxed{\mathcal{P}(A \cap B) \subseteq \mathcal{P}(A) \cap \mathcal{P}(B)} - \text{Definition of Subset, (4)\& (1)} \tag{5}$$
$$\text{Suppose } \mathcal{P}(A) \cap \mathcal{P}(B) \text{ then } x \in \mathcal{P}(A) \cap \mathcal{P}(B) - \text{Definition of Power Set} \tag{6}$$
$$\text{Since } x \in \mathcal{P}(A) \cap \mathcal{P}(B) \text{ then } x \in \mathcal{P}(A) \text{ and } x \in \mathcal{P}(B) - \text{Definition of Intersection} \tag{7}$$
$$\text{Since } x \in \mathcal{P}(A) \text{ then } x \in A - \text{Definition of Power Set} \tag{8}$$
$$\text{Since } x \in \mathcal{P}(B) \text{ then } x \in B - \text{Definition of Power Set} \tag{9}$$
$$\text{Since } x \in A \text{ and } x \in B \text{ then } x \in (A \cap B) - \text{Definition of Intersection} \tag{10}$$
$$\text{Since } x \in (A \cap B) \text{ then } x \in \mathcal{P}(A \cap B) - \text{Definition of a Power Set} \tag{11}$$
$$\text{Since } x \in \mathcal{P}(A) \cap \mathcal{P}(B) \text{ and } x \in \mathcal{P}(A \cap B) \text{ then } \boxed{\mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B)} - \text{Definition of subset} \tag{12}$$

$$\boxed{\mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B) \text{ and } \mathcal{P}(A \cap B) \subseteq \mathcal{P}(A) \cap \mathcal{P}(B) \textbf{ then } \mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)}$$

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35) For all the sets A and B prove that $A - (A - B) = A \cap B$

$$\begin{aligned}
A - (A - B) &= A - (A \cap B^c)\text{- By the set difference law} \\
&= A \cap (A \cap B^c)^c\text{- By the set difference law} \\
&= A \cap (A^c \cup B)\text{- By DeMorgans Law} \\
&= (A \cap A^c) \cup (A \cap B)\text{- By the Distributive Property} \\
&= (\emptyset) \cup (A \cap B)\text{- By the complement Law} \\
&= \boxed{(A \cap B)} \text{ - By the Identity Law}
\end{aligned}$$

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43) Simplify the following - $((A \cap (B \cup C)) \cap (A - B)) \cap (B \cup C^c)$

$$\begin{gathered}
((A \cap (B \cup C)) \cap (A \cap B^c)) \cap (B \cup C^c) \text{ - Set Difference Law} \\
(A \cap ((B \cup C) \cap (A \cap B^c))) \cap (B \cup C^c) \text{ - Associativity} \\
(A \cap ((A \cap B^c) \cap (B \cup C))) \cap (B \cup C^c) \text{ - Communative} \\
((A \cap (A \cap B^c)) \cap (B \cup C)) \cap (B \cup C^c) \text{ - Associativity} \\
(A \cap (B \cup C)) \cap (B \cup C^c) \text{ - Absorption Law} \\
A \cap ((B \cup C) \cap (B \cup C^c)) \text{ - Associativity} \\
A \cap (B \cup (C \cap C^c)) \text{ - Distributive property} \\
A \cap (B \cup \emptyset)) \text{ - Compliment} \\
\boxed{A \cap B} \text{ - Identity}
\end{gathered}$$