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Professor McDonnell | Assigned: April 25, 2017 | Due: May 4, 2017

Contents

8 Relations

8.1 6b, 6c, 11, 18, 20

6 $X = \{a, b, c\}$ with Relation $\mathbf{J}$ such that $A\mathbf{J}B \Leftrightarrow A \cap B \neq \varnothing$
b) Yes, $\{a, b\} \cap \{b, c\} \neq \varnothing$
c) Yes, $\{a, b\} \cap \{a, b, c\} \neq \varnothing$

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11) $A = \{3, 4, 5\}$ and $B = \{4, 5, 6\}$ with Relation $\mathbf{S}$ such that $x\mathbf{S}y \Leftrightarrow x|y$, and such that $x \in A$ and $y \in B$ (Definition of Cartesian Product).
$\mathbf{S} = \{(3, 6), (4, 4), (5, 5)\}$ and $\mathbf{S}^{-1} = \{(4, 4), (5, 5)\}$

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18) $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ and the relation $x\mathbf{V}y$ means that $5|(x^2 - y^2)\ \forall x \forall y \in A$

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20)
$\mathbf{R} = \{(-1, 1), (1, 1), (2, 2)\}$ and $\mathbf{S} = \{(-1, 1), (1, 1), (2, 2), (4, 2)\}$ and $\boxed{\mathbf{R} \cap \mathbf{S} = = \{(-1, 1), (1, 1), (2, 2)\}}$

8.2 7, 17, 33, 36, 40

7) $\mathbf{A} = \{0, 1, 2, 3\}$ and $\mathbf{B} = \{(0, 3), (2, 3)\}$
Graph:

b) Not Reflexive , any of them are counter examples and don't point to themselves.
c) Not Symmetric, 0 points to 3 but 3 does not point to 0, it should be all of them to be symmetric, but a counter example was found.
d) It is Transitive, vacuous truth, the left is never true in the definition for transitivity.

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17) Relation $\mathbf{P}$ $\forall m \forall n \in \mathbb{Z} m\mathbf{P}\exists$ a prime number such that $p|m$ and $p|n$

Reflexive, **yes** because $(n\mathbf{P}n)$ because if $p|n$ then that is the same as $p|n$.

Symmetric, **yes** because (if $m\mathbf{P}n$ then $n\mathbf{P}m$) is the first is true, a conjunction is commutative so the latter holds true as well therefore making this relation symmetric.

Transitive, (if $m\mathbf{P}n$ and $n\mathbf{P}o$ then $m\mathbf{P}o$) **yes** because if the first part is satisfies, m is in the factors of n, and if n is in the factors of o, m is also divisible by definition of transitivity. that means that the factors on m coincide within o. This means that $m|o$, which satisfies the relation.

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33) $\mathbf{A} = \{\text{all lines on the plane}\}\ \forall l_1 \forall l_2,\ l_1\mathbf{P}l_2 \Leftrightarrow l_1$ is perpendicular to l_2.

Reflexive, **NO**, lines are not perpendicular to themselves, that breaks the definition of being perpendicular (90 degrees), this is 0 degrees.

Symmetric, **YES**, when line 1 is perpendicular to line 2, line 2 is perpendicular to line 1.

Transitivity, **NO**, if line 1 is perpendicular to line 2, and line 2 is perpendicular to line 3, line 1 is NOT perpendicular to line 3 because line 3 can either be 180 degrees from line 1 or the same line, in both cases the definition of perpendicularity is violated.

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36) $\boxed{\textbf{TRUE}}$. if the relation $\mathbf{R}$ is transitive, this means that if $x\mathbf{R}y$ and if $y\mathbf{R}z$ then $x\mathbf{R}z$ must hold true. Now if we apply the definition of an inverse we get, if $y\mathbf{R}^{-1}x$ and $z\mathbf{R}^{-1}y$ then $z\mathbf{R}^{-1}x$ which we can do due to law of transitivity. and $z\mathbf{R}^{-1}x$ is the inverse of $x\mathbf{R}z$ due to the law of inverses. This proves that it is transitive.

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40) True because $\mathbf{R} \cup \mathbf{S}$ is just combining the reflexive tuples of each and they are all reflexive because reflexive requires the relations to be true for all possible values, thus including those within $\mathbf{S}$ for $\mathbf{R}$ and $\mathbf{R}$ for $\mathbf{S}$

8.3 10, 20, 26

10) $[a] = \{x \in A | x = \sqrt{3k + a^2}$, For Some $k \in \mathbb{Z}\}$
$[0] = \{-3, 0, 3\}$
$[1] = \{-5, -4, -2, -1, 1, 2, 4, 5\}$
All other equivalence classes are equal to the ones above.

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20) $P\mathbf{R}Q \Leftrightarrow$ P and Q have the same truth tables.

Reflexive - if x (a certain arrangement of p, q or r) has a certain truth table it is equivalent to itself so it related to itself $x\mathbf{R}x$. <u>It is reflexive</u>

Symmetric - if x is equivalent to y because they have the same truth tables, this means that $x\mathbf{R}y$, likewise, y will also relate to x due to their logical equivalences. <u>It is symmetric</u>

Transitivity - if x, y and z all arrangements or p, q or $r \in A$ such that $x \equiv y$ and $y \equiv z$ then this means that $x \equiv z$ due to the transitive property we saw in logic arguments in chapter one. <u>It is transitive</u>

Since this relation is reflexive, symmetric and transitive, it is an **equivalence relation**.

Distinct equivalent relations:

- $[x \in A] = \{$Every element that relates to $x\}$
- $[p] = \{$Every element that relates to $p\}$
- $[q] = \{$Every element that relates to $q\}$
- $[r] = \{$Every element that relates to $r\}$

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26) Given the relation $\mathbf{Q}$ on the set of $\mathbb{R} \times \mathbb{R}$, $\forall (w, x) \forall (y, z) \in \mathbb{R} \times \mathbb{R}$

Reflexive - Yes because $(w, x)\mathbf{Q}(w, x)$ and then $x = x \therefore$ it is reflexive.

Symmetric - $(w, x)\mathbf{Q}(y, z) = (y, z)\mathbf{Q}(w, x)$, true because x = z and z = x. This means it is symmetric.

Transitive - if $(w, x)\mathbf{Q}(y, z)$ and $(y, z)\mathbf{Q}(a, b)$ then $(w, x)\mathbf{Q}(a, b)$ because if x = z, and z = b, then x = b due to basic transitivity. This results in all of them relating and thus making the relation transitive.

Since this relation is reflexive, symmetric and transitive, it is an **equivalence relation**.
Distinct classes -

- $[(w, x), (y, z)] = \{\text{Where } x = z\}$
- $[(w, x), (y, z)] = \{\text{Where } x \neq z\}$

8.4 8, 9b, 15, 22

8a) $45 \equiv 3(mod\ 6)$, $6|45-3$, $6|42$ True because $6*7 = 42$ AND $104 = 2(mod\ 6)$, $6|104-2$, $6|102$ True because $6*17 = 102$
8b) $45+104 \equiv 5(mod\ 6)$, $6|(149-5)$, $6|144$ True because $6*19 = 144$
8c) $45-104 \equiv 1(mod\ 6)$, $6|-59-1$, $6|-60$ True because $6*-10 = -60$
8d) $45*104 \equiv (3*2)(mod\ 6)$, $6|4680-6$, $6|4674$ True because $6*769 = -60$
8e) $45^2 \equiv 3^2(mod\ 6)$, $6|2025-9$, $6|2016$ True because $6(336) = 2016$

9b)

$$\begin{aligned}
(a-b) \equiv (c-d)(mod\ n) - &\ \text{Given.}\\
(a-b)-(c-d) = kn - &\ \text{Definition of divisibility}\\
(a-b) = kn + (c-d) - &\ \text{By algebra}\\
qn + r = kn + (c-d) - &\ \text{Quoteint remainder theorem.}\\
(q-k)n + r = (c-d) - &\ \text{Same non-neg remainder } r \text{ as } a\\
(a-b)mod\ n = r \text{ and } (c-d)mod\ n = r - &\ \text{Definition of the MOD function}\\
n|(a-b) \text{ and } n|(c-d) - &\ \text{definition of the mod function and algebra}\\
a \equiv b(mod\ n) \text{ and } c \equiv d(mod\ n) - &\ \text{Definition of congruence modulo, QED}
\end{aligned}$$

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15) Units digit of $8^{100} = (8^2)^{50} = (64)^{50} = (2^6)^{50} = (2)^{300} = 6(mod\ 10)$
I was able to confirm this because $2^1 = 2$, $2^2 = 4$, power of 3 yields 8, power of 4 yields 6 in the ones spot, power of 5 yields 2 in the ones place, so it repeats here. What we can say now is that at 2^{300}, 300 is divisible by 4, $(4*75 = 300)$ so now the ones place is 6 at 2^{300} or it is $6(mod\ 10)$, QED.

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22) 75004900040, $7+0+4+0+0+0 = 11*3 = 33 + (5+0+9+0+4) = 51$, $60-51 = \boxed{9}$.

8.5 8, 12b, 12c, 15