Ishan Sethi | ID: 110941217 | CSE 215 | Homework 6 in LaTeX

Professor McDonnell | Assigned: April 13, 2017 | Due: April 25, 2017

Contents

7 Functions

7.1 3b, 3d, 10d, 10e, 10f, 12c, 12d, 41

3b)

$$g(1) = b,\ g(3) = b \text{ and } g(5) = b$$

3d) 3 is not the inverse image of a, 1 is the inverse image of b.

10d)

$$T(5) = 5 \text{ and } 1$$

10e)

$$T(18) = 1,\ 2,\ 3,\ 6,\ 9 \text{ and } 18$$

10f)

$$T(21) = 1,\ 3,\ 7 \text{ and } 21$$

12c) $G(3,\ 2) = ((2 \cdot (3) + 1) mod\ 5,\ (3 \cdot (2) - 2) mod\ 5) = \boxed{(2, 4)}$

12d) $G(1,\ 5) = ((2 \cdot (1) + 1) mod\ 5,\ (3 \cdot (5) - 2) mod\ 5) = \boxed{(3, 3)}$

---

41) $\forall$ subsets of C and D of Y, Justify that $F^{-1}(C - D) = F^{-1}(C) - F^{-1}(D)$

$$x \in F^{-1}(C - D) \Rightarrow F(x) \in (C - D) - \text{Due to the inverse image law}$$
$$F(x) \in (C - D) \Rightarrow F(x) \in C \text{ and } F(x) \notin D - \text{Set Difference Law}$$

Now proceed to the right side

$$x \in F^{-1}(C) - F^{-1}(D) \Rightarrow x \in F^{-1}(C) \text{ and } x \notin F^{-1}(D) - \text{Set Difference Law}$$
$$x \in F^{-1}(C) \text{ and } x \notin F^{-1}(D) \Rightarrow F(x) \in C \text{ and } F(x) \notin D - \text{Inverse Image Law}$$

Since fundamental element argument is the same for both sides, they are both subsets of each other - Definition of Subsets.

$$F^{-1}(C - D) \subseteq F^{-1}(C) - F^{-1}(D) - \text{Definition of Subsets}$$
$$x \in F^{-1}(C) - F^{-1}(D) \Rightarrow x \in F^{-1}(C) \text{ and } x \notin F^{-1}(D) - \text{Set Difference Law}$$
$$x \in F^{-1}(C) \text{ and } x \notin F^{-1}(D) \Rightarrow F(x) \in C \text{ and } F(x) \notin D - \text{Inverse Image Law}$$

Now proceed to the right side

$$x \in F^{-1}(C - D) \Rightarrow F(x) \in (C - D) - \text{Due to the inverse image law}$$
$$F(x) \in (C - D) \Rightarrow F(x) \in C \text{ and } F(x) \notin D - \text{Set Difference Law}$$

Since fundamental element argument is the same for both sides, they are both subsets of each other - Definition of Subsets.

$$F^{-1}(C) - F^{-1}(D) \subseteq F^{-1}(C - D) - \text{Definition of Subsets}$$

Since they are subsets of each other they are equal $\Rightarrow F^{-1}(C - D) = F^{-1}(C) - F^{-1}(D)$

7.2 7b, 9c, 9d, 12, 18, 23

7b) Since no points map to g in the set of Y, the definition of onto is violated and it is $\boxed{\text{not onto}}$.

9c) If you took all the points from the set X and mapped it to one point out of more than one in the set Y, you have a function that is not onto and not one-to-one.
9d) If the set of X are the values $\{1, 2, 3\}$ and the set Y are the values $\{1, 2, 3\}$ and $F : X \to Y$ would result in the mappings 1 to 2, 2 to 3 and 3 to 1, this would be one-to-one, onto and not the identity function.

12a) $F : \mathbb{Z} \to \mathbb{Z}$ by the rule $F(n) = 2 - 3n$ $\forall n \in \mathbb{Z}$
(i) Definition of one-to-one: $\forall x_1$ and $\forall x_2$, if $F(x_1) = F(x_2)$ then $x_1 = x_2$

$$\begin{aligned} F(x_1) &= 2 - 3(x_1) \\ F(x_2) &= 2 - 3(x_2) \\ 2 - 3(x_1) &= 2 - 3(x_2) \\ x_1 &= x_2 \end{aligned}$$

This means that the function is one to one.

(ii) Definition of onto: $\forall y \in Y$ and $\forall n \in X$, $F(x) = y$.

Suppose $y \in \mathbb{Z}$

$$\begin{aligned} y &= 2 - 3(n) \\ n &= \frac{y - 2}{3} \\ \text{When } y &= 0, \text{ because it can be any integer.} \\ n &= \frac{2}{3} \text{ not an integer.} \end{aligned}$$

This means that the function is not onto.

More on the next page.

12b) $G : \mathbb{R} \to \mathbb{R}$ by the rule $G(x) = 2 - 3x$
Definition of onto: $\forall y \in Y$ and $\forall x \in X$ such that if $F(x) = y$ then the function is onto.

Suppose $y \in \mathbb{R}$

$$
\begin{aligned}
y &= 2 - 3(x) \\
x &= \frac{y-2}{3} \\
F(x) &= 2 - 3\left(\frac{y-2}{-3}\right) \\
F(x) &= y
\end{aligned}
$$

This means that the function is onto as per the definition.

---

18) $f(x) = \dfrac{x+1}{x-1}$ $\forall x \in \mathbb{R}$ and $x \neq 1$
Prove if this is one-to-one
Definition of one-to-one: $\forall x_1$ and $\forall x_2$ and $x_1 \neq 1$ and $x_2 \neq 2$, if $F(x_1) = F(x_2)$ then $x_1 = x_2$

$$
\begin{aligned}
F(x_1) &= \frac{x_1+1}{x_1-1} \\
F(x_2) &= \frac{x_2+1}{x_2-1} \\
\frac{x_2+1}{x_2-1} &= \frac{x_1+1}{x_1-1} \\
x_1x_2 - x_1 + x_2 - 1 &= x_1x_2 + x_1 - x_2 - 1 \\
2x_1 &= 2x_2 \\
x_1 &= x_2
\end{aligned}
$$

This means that the function is one to one.

---

23) $H : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ by the rule $H(x, y) = (x + 1, 2 - y)$, $\forall (x, y) \in \mathbb{R} \times \mathbb{R}$

$$
\begin{aligned}
H(x_1, y_1) &= (x_1 + 1, 2 - y_1) \\
H(x_2, y_2) &= (x_2 + 1, 2 - y_2) \\
x_1 + 1 &= x_2 + 1 \\
2 - y_1 &= 2 - y_2 \\
x_1 &= x_2 \\
y_1 &= y_2
\end{aligned}
$$

Thus $(x_1, y_1) = (x_2, y_2)$This is means that it is one-to-one

---

7.3 7, 10, 17, 20, 24, 25

7) $H : \mathbb{Z} \to \mathbb{Z}$ and $K : \mathbb{Z} \to \mathbb{Z}$, $H(a) = 6a$ and $K(a) = a\ mod\ 4\ \forall a \in \mathbb{Z}$
$(K \circ H)(0) = K(H(0)) = K(6(0)) = K(0) = 0$
$(K \circ H)(1) = K(H(1)) = K(6(1)) = K(6) = 2$
$(K \circ H)(2) = K(H(2)) = K(6(2)) = K(12) = 0$
$(K \circ H)(3) = K(H(3)) = K(6(3)) = K(18) = 2$

---

10) $G : \mathbb{R}^+ \to \mathbb{R}^+$ and $G^{-1} : \mathbb{R}^+ \to \mathbb{R}^+$, $G(x) = x^2$ and $G^{-1}(x) = \sqrt{x}$
$(G \circ G^{-1})(x) = (\sqrt{x})^2 = x = I_x$
$(G^{-1} \circ G)(x) = (\sqrt{x^2}) = x = I_x$
We Gucci

---

17)

---

20) $f : W \to X$ and $g : X \to Y$ and $h : Y \to Z$ must $h \circ (g \circ f) = (h \circ g) \circ f$
Given a $W \in h \circ (g \circ f)$ this essentially means $W \in h(g(f(w)))$
Given a $W \in (h \circ g) \circ f$ this essentially means $W \in h(g(f(w)))$
These are the same making them subsets of each other therefore being equal by the definition of subsets.

---

24) $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$
$f(x) = x + 3$ $g(x) = -x$
$(g \circ f) = -x - 3$
$(g \circ f)^{-1} = -x - 3$
$g^{-1} = -x$
$f^{-1} = x - 3$
$(f^{-1} \circ g^{-1}) = -x - 3$
They are the same.

---

25) $f(x)$ is one-to-one

$f(x)$ is one-to-one

$$\begin{aligned} f(x) &= f(y) \\ g(f(x)) &= f(g(y)) \text{ Definition of composition} \\ x &= y \text{ by the given condition of } (g \circ f) = I_x \end{aligned}$$

$g(x)$ is one-to-one

$$\begin{aligned} g(x) &= g(y) \\ g(f(x)) &= f(g(y)) \text{ Definition of composition} \\ x &= y \text{ by the given condition of } (g \circ f) = I_y \end{aligned}$$

$f(x)$ is onto

$$\begin{aligned} \text{Suppose } x \in Y \text{ because } (f \circ g) &= I_y = y \\ (f \circ g)(x) &= y \\ f(g(x)) &= y \text{ and } x \in X \end{aligned}$$

While $x \in Y$ such that f(g(x)) results in x and $x \in X$ $\therefore$ it is ONTO

$g(x)$ is onto

$$\begin{aligned} \text{Suppose } x \in X \text{ because } (g \circ f)(x) &= x \\ (g \circ f)(x) &= x \\ g(f(x)) &= x \text{ and } x \in Y \end{aligned}$$

While $x \in X$ $g(f(x))$ gives x and $x \in Y$ $\therefore g(x)$ is ONTO

Because $f(g(y)) = y$, both $g^{-1} = f$ and $f^{-1} = g$... the co-domain of one equals the domain of the other.
