Estimate parameters of AR model

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1 Data initialization

Because WAVREAD will be removed in a future release we will use AUDIOREAD instead:

```
[y,Fs]=audioread('gong.wav');
```

2 Problem definition

2.1 $\mathbf{Ma} \approx \mathbf{b}$

$$M = \begin{pmatrix} 1 & y_{p-1} & \cdots & y_0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & y_{T-1} & \cdots & y_{T-p} \end{pmatrix} \; b = \begin{pmatrix} y_p \\ y_{p+1} \\ \vdots \\ y_T \end{pmatrix}$$

Or, considering Matlab indexing:

$$M = \begin{pmatrix} 1 & y_p & \cdots & y_1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & y_T & \cdots & y_{T-p+1} \end{pmatrix} \; b = \begin{pmatrix} y_{p+1} \\ y_{p+2} \\ \vdots \\ y_{T+1} \end{pmatrix}$$

2.2 $\min_{\mathbf{a}} \|\mathbf{Ma} - \mathbf{b}\|^2$

$$\min_{\mathbf{a}} \| \mathbf{Ma} - \mathbf{b} \|^2 = 0.0020867$$

3 Problem definition using QR decomposition

3.1 $\mathbf{x} = \mathbf{solve\_ls}(\mathbf{A}, \mathbf{b})$

```
function x = solve_ls(  A, b )
% size
    s = size(A, 2);
    % QR Decomposition
    [Q, R] = qr(A, 0);
    % Least squares
```

```
    bq = Q' * b;
    x = zeros(s, 1);
    for i = s:-1:1
        x(i) = (bq(i) - (R(i,:) * x)) / R(i, i);
    end
end
```

3.2 Distance $\|\widehat{a_1} - \widehat{a_2}\|$

$$\|\widehat{a_1} - \widehat{a_2}\| = 5.3075 \cdot 10^{-12}$$

4 Synthetic sound generation

4.1 Sound reconstruction

$$y_{syn\_t} = y_{orig\_t}, 0 \leq t < p$$

$$y_t = a_0 + \sum_{i=1}^{p} a_i y_{t-i} + \varepsilon_t, t < p$$

Or, considering Matlab indexing:

$$y_{syn\_t} = y_{orig\_t}, 1 \leq t < p + 1$$

$$y_t = a_1 + \sum_{i=2}^{p+1} a_i y_{t-i+1} + \varepsilon_t t \geq p$$

4.2 Sound comparation

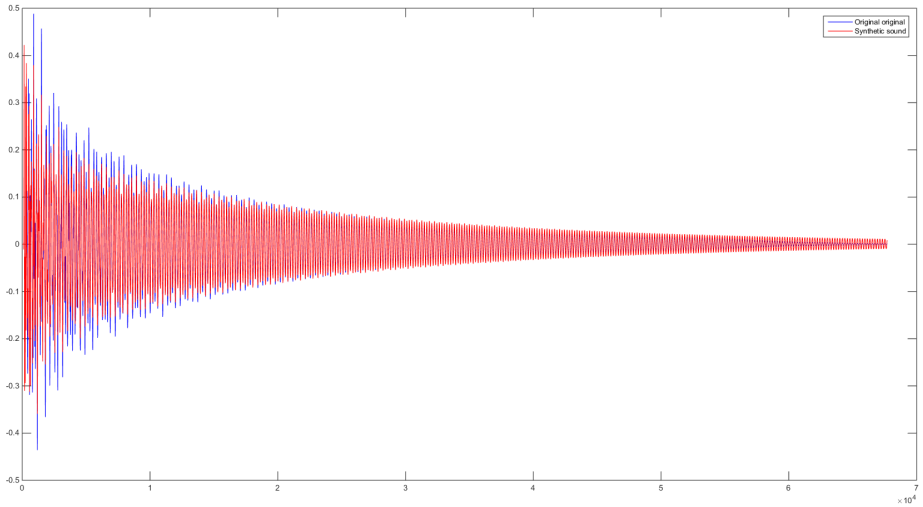

Figure 1: Comparation between original sound and synthetic sound, generated using least squares

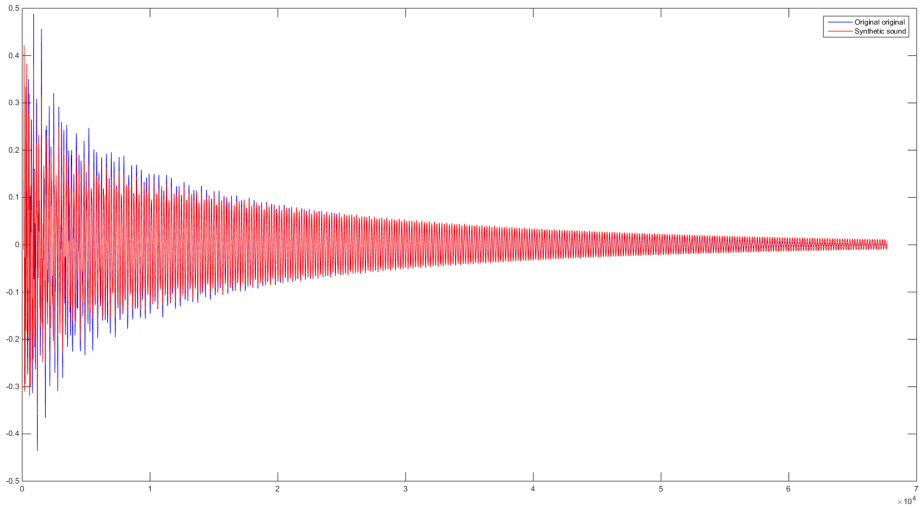

Figure 2: Comparation between original sound and synthetic sound, generated using QR decomposition