OPT 4 - Taylor series

Vladislav Iliushin

November 11, 2014

1

$$f(x) = \frac{x^3}{3} + \frac{x^2}{2} - x$$

$$T_1(x^*) = f(x^*) + (x^{*2} + x^* - 1)\dot{(}x - x^*)$$

$$T_2(x^*) = T_1(x^*) + 0.5\dot{(}2\dot{x}^* + 1)\dot{(}x - x^*)^2$$

2

$$f(x) = \begin{bmatrix} \cos(x) \\ \sin(2x) \end{bmatrix}$$

$$T_1(x^*) = f(x^*) + \begin{bmatrix} -\sin(x^*) \\ 2\cos(2x^*) \end{bmatrix} \dot{(}x - x^*)$$

$$T_2(x^*) = T_1(x^*) + 0.5 \begin{bmatrix} -\cos(x^*) \\ -4\sin(2x^*) \end{bmatrix} \dot{(}x - x^*)^2$$

3

$$f(x_1, x_2) = 2e^{-\mathbf{x_1}^2 - \mathbf{x_2}^2}$$

$$T_1(x_1^*, x_2^*) = f(x_1^*, x_2^*) + \begin{bmatrix} -4x_1^*\dot{e}^{-(x_1^*)^2-(x_2^*)^2} \\ -4x_2^*\dot{e}^{-(x_1^*)^2-(x_2^*)^2} \end{bmatrix} \begin{bmatrix} x_1 - x_1^* \\ x_2 - x_2^* \end{bmatrix}$$

$$T_2(x_1^*, x_2^*) = T(x_1^*, x_2^*) + 0.5 \begin{bmatrix} x_1 - x_1^* \\ x_2 - x_2^* \end{bmatrix} \begin{bmatrix} (8x_1^{*^2} - 4)e^{-x_1^{*^2}-x_2^{*^2}} & 8x_1^*\dot{x}_2^*e^{-x_1^{*^2}-x_2^{*^2}} \\ 8x_1^*\dot{x}_2^*e^{-x_1^{*^2}-x_2^{*^2}} & (8x_2^{*^2} - 4)e^{-x_1^{*^2}-x_2^{*^2}} \end{bmatrix} \begin{bmatrix} x_1 - x_1^* \\ x_2 - x_2^* \end{bmatrix}^{\mathbf{T}}$$

4

$$f(x_1, x_2) = \begin{bmatrix} (R + r\cos x_2)\cos x_1 \\ (R + r\cos x_2)\sin x_1 \\ r\sin x_2 \end{bmatrix}$$

$$T_1(x_1^*, x_2^*) = f(x_1^*, x_2^*) + \begin{bmatrix} -(R + r\cos x_2^*)\sin(x_1^*) & -r\cos x_1^* \sin x_2^* \\ (R + r\cos x_2^*)\cos(x_1^*) & -r\sin x_1^* \sin x_2^* \\ 0 & r\cos x_2^* \end{bmatrix} \begin{bmatrix} x_1 - x_1^* \\ x_2 - x_2^* \end{bmatrix}$$

$$T_2(x_1^*, x_2^*) = T_1(x_1^*, x_2^*) + \begin{bmatrix} x_1 - x_1^* \\ x_2 - x_2^* \end{bmatrix} \begin{bmatrix} -(R + r\cos x_2^*)\cos x_1^* & r\sin x_1^* \sin x_2^* \\ r\sin x_1^* \sin x_2^* & -r\cos x_1 \cos x_2^* \end{bmatrix} +$$

$$+ \begin{bmatrix} x_1 - x_1^* \\ x_2 - x_2^* \end{bmatrix} \begin{bmatrix} -(R + r\cos x_2^*)\sin x_1^* & -r\cos x_1^* \sin x_2^* \\ -r\cos x_1^* \sin x_2^* & -r\cos x_2 \sin x_1^* \end{bmatrix} +$$

$$+ \begin{bmatrix} x_1 - x_1^* \\ x_2 - x_2^* \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & -r\sin x_2^* \end{bmatrix}$$

5

$$f(x_1, x_2) = \begin{bmatrix} \mathrm{x}_1 \log(1 + {\mathrm{x}_1}^2 + {\mathrm{x}_2}^2) \\ \mathrm{x}_2 \log(1 + {\mathrm{x}_2}^2 + {\mathrm{x}_2}^2) \end{bmatrix}$$

$$T_1(x_1^*, x_2^*) = f(x_1^*, x_2^*) + \begin{bmatrix} \log({x_1^*}^2 + {x_2^*}^2 + 1) + \frac{(2{x_1^*}^2}{{x_1^*}^2 + {x_2^*}^2 + 1}) & \frac{(2x_1 x_2)}{{x_1^*}^2 + {x_2^*}^2 + 1}) \\ \frac{(2x_1 x_2)}{{x_1^*}^2 + {x_2^*}^2 + 1}) & log({x_1^*}^2 + {x_2^*}^2 + 1) + \frac{(2{x_2^*}^2}{{x_1^*}^2 + {x_2^*}^2 + 1}) \end{bmatrix} \begin{bmatrix} x_1 - x_1^* \\ x_2 - x_2^* \end{bmatrix}$$

$$T_2(x_1^*, x_2^*) = T_1(x_1^*, x_2^*) + \begin{bmatrix} \begin{bmatrix} x_1 - x_1^* \\ x_2 - x_2^* \end{bmatrix} \\ \begin{bmatrix} x_1 - x_1^* \\ x_2 - x_2^* \end{bmatrix} \end{bmatrix} \cdot$$

$$\cdot \begin{bmatrix} \frac{(6x_1^*)}{({x_1^*}^2 + {x_2^*}^2 + 1)} - \frac{(4{x_1^*}^3)}{({x_1^*}^2 + {x_2^*}^2 + 1)^2} & \frac{(2x_2)}{(x_1^2 + x_2^2 + 1)} - \frac{(4x_1^2 x_2)}{(x_1^2 + x_2^2 + 1)^2} \\ \frac{(2x_2)}{({x_1^*}^2 + {x_2^*}^2 + 1)} - \frac{(4{x_1^*}^2 x_2)}{({x_1^*}^2 + {x_2^*}^2 + 1)^2} & \frac{(2x_1)}{(x_1^2 + x_2^2 + 1)} - \frac{(4x_1 x_2^2)}{(x_1^2 + x_2^2 + 1)^2} \\ \frac{(2x_2)}{(x_1^2 + x_2^2 + 1)} - \frac{(4x_1^2 x_2)}{(x_1^2 + x_2^2 + 1)^2} & \frac{(2x_1)}{(x_1^2 + x_2^2 + 1)} - \frac{(4x_1 x_2^2)}{(x_1^2 + x_2^2 + 1)^2} \\ \frac{(2x_1)}{(x_1^2 + x_2^2 + 1)} - \frac{(4x_1 x_2^2)}{(x_1^2 + x_2^2 + 1)^2} & \frac{(6x_2)}{(x_1^2 + x_2^2 + 1)} - \frac{(4x_2^3)}{(x_1^2 + x_2^2 + 1)^2} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} x_1 - x_1^* \\ x_2 - x_2^* \end{bmatrix}^T \\ \begin{bmatrix} x_1 - x_1^* \\ x_2 - x_2^* \end{bmatrix}^T \end{bmatrix}$$

6

$$f(x_1, x_2) = \begin{bmatrix} (R + r\cos x_2)\cos x_1 \\ (R + r\cos x_2)\sin x_1 \\ r\sin x_2 \end{bmatrix}$$

$$T_1(x_1^*) = \begin{bmatrix} \sin(\sin x_1^*) \cdot (R + r\cos(\cos x_1^*))\sin(\sin x_1^*) & -r\cos(\sin x_1^*)\sin(\cos x_1^1)\cos x_1^* \\ -\cos(\sin x_1^*) \cdot (R + r\cos(\cos x_1^*))\sin(\sin x_1^*) & -r\sin(\sin x_1^*)\cos(\cos x_1^1)\cos x_1^* \\ 0 & r\cos(\cos x_1^*)\cos x_1^* \end{bmatrix} \begin{bmatrix} -\sin x_1^* \\ 2\cos 2x_1^* \end{bmatrix} \begin{bmatrix}$$