Sapienza University of Rome

Master in Artificial Intelligence and Robotics

Machine Learning

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12. Convolutional Neural Networks

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with contributions from Valsamis Ntouskos

Overview

- Convolution
- Convolutional layers
- Pooling
- Example: LeNet
- CNNs for Images
- “Famous” CNNs
- Transfer learning
- Resources

References
Ian Goodfellow and Yoshua Bengio and Aaron Courville. Deep Learning - Chapter 9. `http://www.deeplearningbook.org`

Motivation

Up to now we treated inputs as general feature vectors

In some cases inputs have special structure:

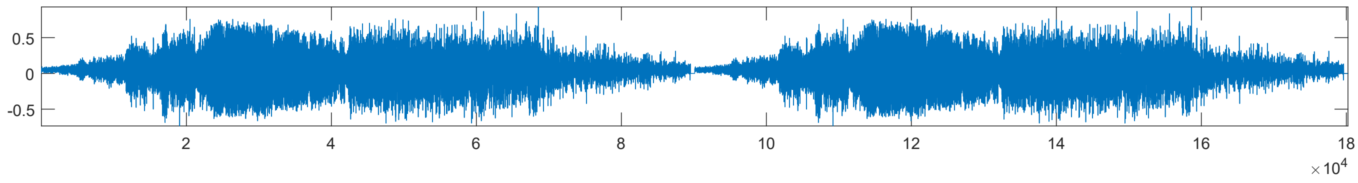
- Audio
- Images
- Videos

Signals: Numerical representations of physical quantities

Deep learning can be directly applied on signals by using suitable operators

Motivation - Examples

Audio

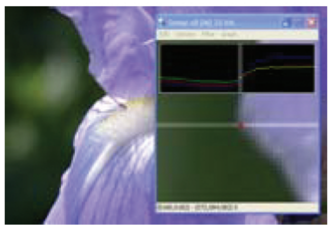


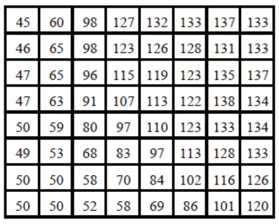
...	0.0468	0.0468	0.0468	0.0390	0.0390	0.0390	0.0546	0.0625	0.0625	0.0390	0.0312	0.0468	0.0625	...

Note: variable length 1D vectors (1D tensor)

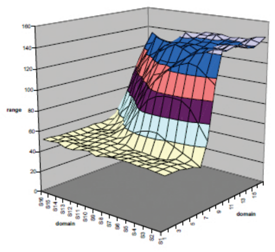
Motivation - Examples

Images





45	60	98	127	132	133	137	133
46	65	98	123	126	128	131	133
47	65	96	115	119	123	135	137
47	63	91	107	113	122	138	134
50	59	80	97	110	123	133	134
49	53	68	83	97	113	128	133
50	50	58	70	84	102	116	126
50	50	52	58	69	86	101	120



Note: multi-channel 2D matrices (3D tensor)

Video

A sequence of color images sampled through time
Note: sequence of multi-channel 2D matrices (4D tensor)

Convolution

Continuous functions

$$(x * w)(t) \equiv \int_{a=-\infty}^{\infty} x(a)\, w(t-a)\, da$$

Discrete functions

$$(x * w)(t) \equiv \sum_{a=-\infty}^{\infty} x(a)\, w(t-a)$$

2D Convolution

Discrete limited 2D functions:

$$(I * K)(i,j) \equiv \sum_{m \in S_1} \sum_{n \in S_2} I(m,n) K(i-m, j-n)$$

I: 2D input, K: 2D kernel, S_i: finite sets.

Commutative

$$(I * K)(i,j) = (K * I)(i,j) = \sum_{m \in S_1} \sum_{n \in S_2} I(i-m, j-n) K(m,n)$$

Note: requires horizontal and vertical flipping of K (or I)

Cross-correlation

Cross-correlation

$$(I * K)(i,j) = \sum_{m \in S_1} \sum_{n \in S_2} I(i+m, j+n) K(m,n)$$

actually implemented in machine learning libraries (more efficient)

3D Convolution

Discrete limited 3D functions:

$$(I * K)(i,j,k) \equiv \sum_{m \in S_1} \sum_{n \in S_2} \sum_{u \in S_3} I(m,n,u) K(i-m, j-n, k-u)$$

I: 3D input, K: 3D kernel, S_i: finite sets.

Cross-correlation

$$(I * K)(i,j,k) = \sum_{m \in S_1} \sum_{n \in S_2} \sum_{u \in S_3} I(i+m, j+n, k+u) K(m,n,u)$$

Different types of Convolutions

1D Convolution
Convolution kernel moves in one direction.

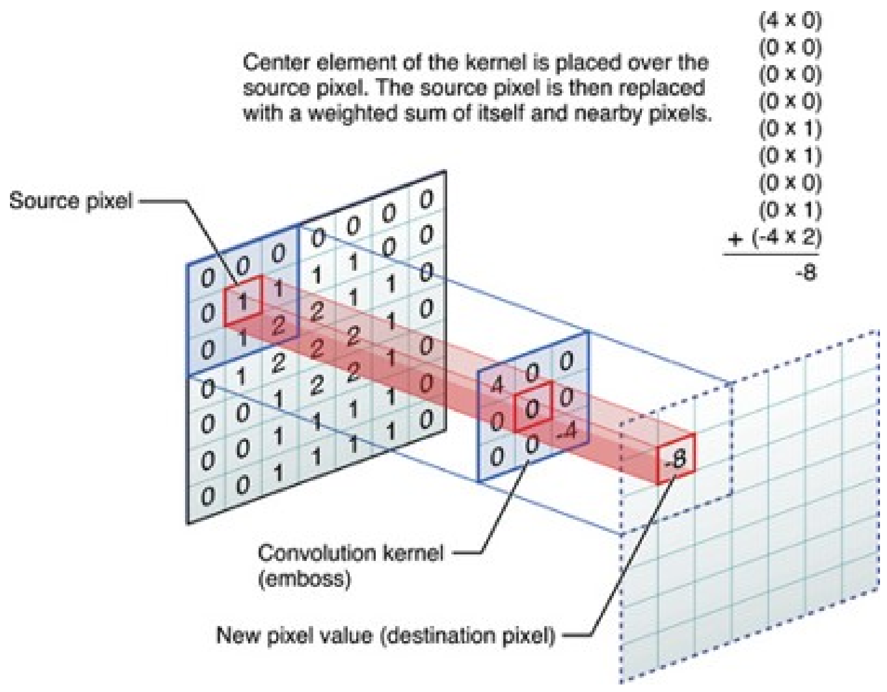
2D Convolution
Convolution kernel moves in two directions.

3D Convolution
Convolution kernel moves in three directions.

Note: 1D/2D/3D Convolutions does not refer to the dimensions of the input and of the kernel.
We can have 1D Convolutions for 2D input and kernels, 2D Convolutions for 3D input and kernels, etc.

Image 2D Convolutions

2D Convolution for 2D input image (gray scale) with 2D kernel


Center element of the kernel is placed over the source pixel. The source pixel is then replaced with a weighted sum of itself and nearby pixels.
(4 x 0)
(0 x 0)
(0 x 0)
(0 x 0)
(0 x 1)
(0 x 1)
(0 x 0)
(0 x 1)
+ (-4 x 2)
-8
Source pixel
Convolution kernel (emboss)
New pixel value (destination pixel)

Interactive example: `http://setosa.io/ev/image-kernels/`

2D Convolutions of 3D functions

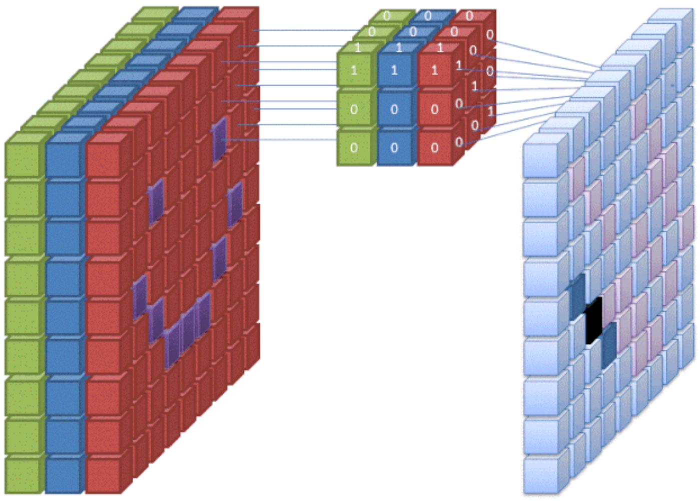
Given $I(i,j,k)$ and $K(i,j,k)$ (3D functions)

2D convolution (2D function)

$$(I * K)(i,j) = \sum_{m \in S_1} \sum_{n \in S_2} \sum_{u \in S_3} I(i+m, j+n, u) K(m,n,u)$$

Image 2D Convolutions

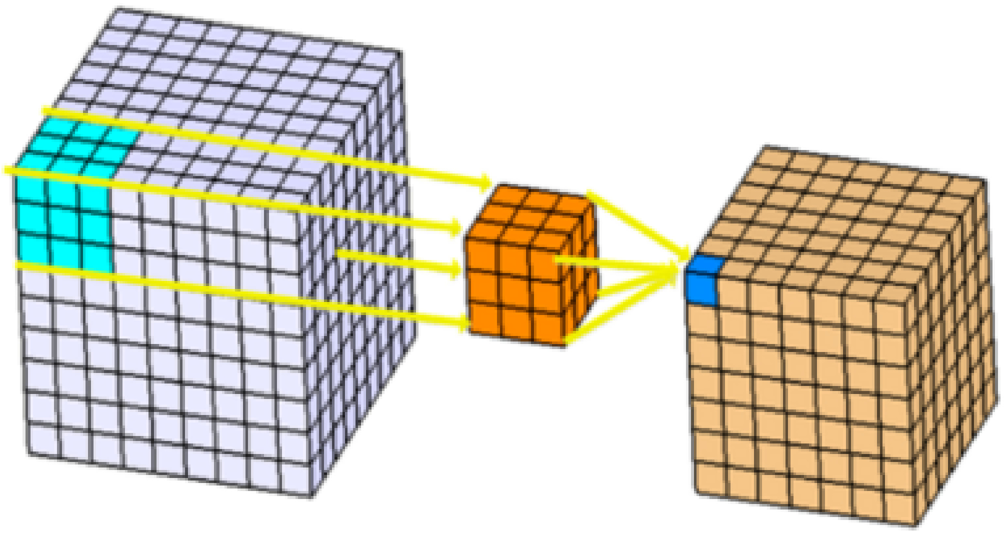
2D Convolution for 3D input image (RBG channels) with 3D kernel (3 channels). Output is 2D



Source: https://commons.wikimedia.org/wiki/File:Convolutional_Neural_Network_with_Color_Image_Filter.gif

3D Convolutions

3D Convolution with 3D input and 3D kernel



Source: https://www.kaggle.com/shivamb/3d-convolutions-understanding-use-case

Terminology

Input size $(w_{in} \times h_{in} \times d_{in})$ dimensions of the input

Kernel size $(w_k \times h_k \times d_k)$ dimensions of the kernel

Feature map or Depth slice output of convolution between an input and one kernel

Depth (d) number of kernels (i.e., of feature maps)

Padding (p) nr. of fillers for outer rows/columns (typically zeros)

Stride (s) step of sliding kernel (1 does not skip any pixel)

Receptive field region in the *input space* that a particular feature is looking at (i.e., is affected by)

2D Convolutions with multiple kernels

1 kernel applied to the input produces 1 feature map

Examples:
32 x 32 x 1 image * 5 x 5 x 1 kernel $\rightarrow$ 1 feature map 28 x 28
32 x 32 x 3 image * 5 x 5 x 3 kernel $\rightarrow$ 1 feature map 28 x 28

If we use d kernels, we can generate d feature maps (output of depth d)

Examples:
32 x 32 x 1 image * 6 kernels 5 x 5 x 1 $\rightarrow$ 6 feature maps 28 x 28
32 x 32 x 3 image * 6 kernels 5 x 5 x 3 $\rightarrow$ 6 feature maps 28 x 28

Note: 6 feature maps 28 x 28 are represented as a 28 x 28 x 6 tensor.

2D Convolutions with multiple kernels

In general

Input: $w_{in} \times h_{in} \times d_{in}$

Kernels: d_{out} of size $w_k \times h_k \times d_k$ (with $d_k = d_{in}$)

Output: feature maps $w_{out} \times h_{out} \times d_{out}$

with w_{out}, h_{out} computed according to stride and padding (see next slides)

Number of kernel parameters: $w_k \cdot h_k \cdot d_k \cdot d_{out}$

Note: for 3D convolutions $d_{in} > d_k$ and output with multiple kernels is a 4D tensor $w_{out} \times h_{out} \times z_{out} \times d_{out}$, with z_{out} computed according to slide and padding in the third dimension.

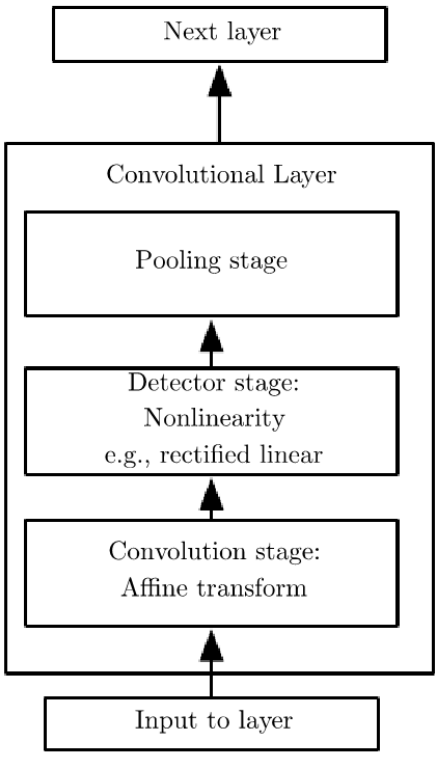
Convolutional Neural Networks

FNN with one or more convolutional layers.

Convolutional layer

Three stages:

- convolutions between input and kernel
- non-linear activation function (detector)
- pooling



Convolutional layer: Convolution

1. 2D Convolution stage

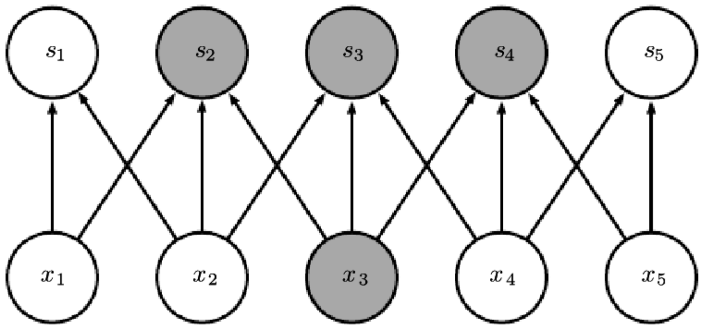
$$(I * K)(i,j) = \sum_m \sum_n I(i+m, j+n) K(m,n)$$

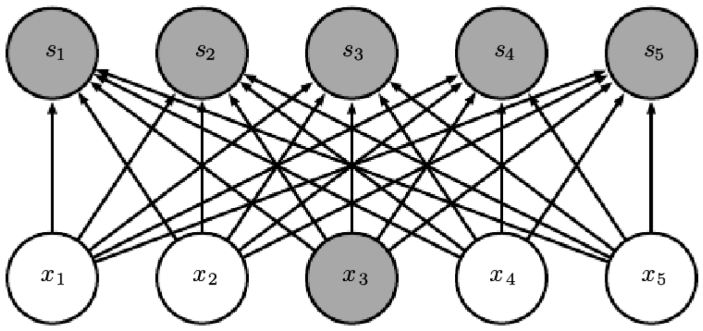
Very efficient in terms of memory requirements and generalization accuracy.

- Sparse connectivity
- Parameter sharing

Sparse connectivity

sparse interactions/ sparse connectivity: outputs depend only on a few inputs (as kernel is usually much smaller than input)

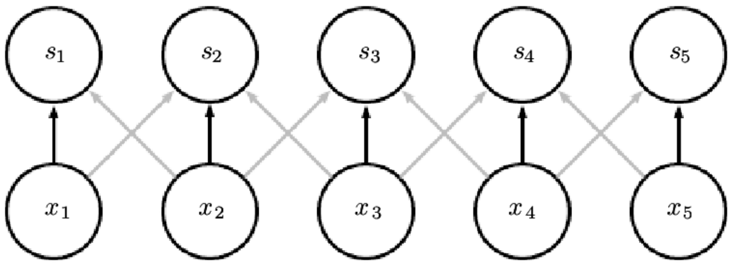


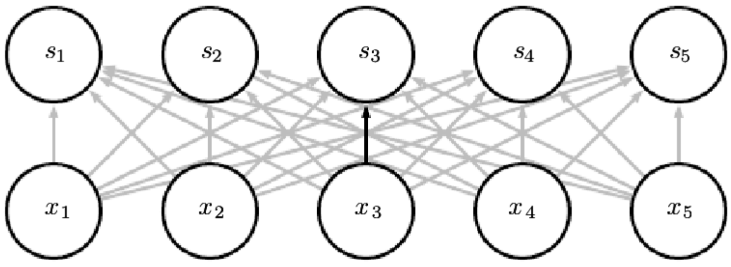


Parameter sharing

Learn only one set of parameters (for the kernel) shared to all the units.

k parameters instead of $m \times n$ (note: $k \ll m$)

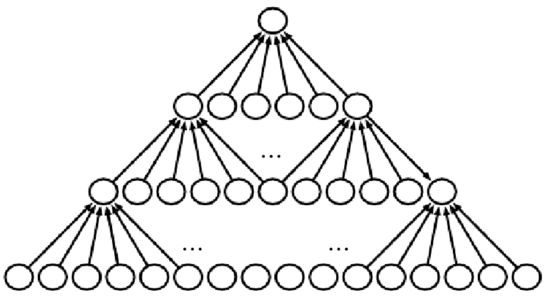


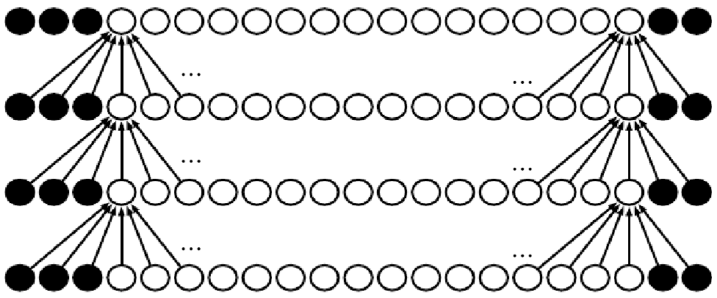


Padding

valid padding ($p = 0$): only valid values (output depends on kernel size)

same padding ($p = w_k/2$): output has same size of input





Convolutional layer: Detector

2. Detector stage

Use non-linear activation functions.

- ReLU
- tanh
- ...

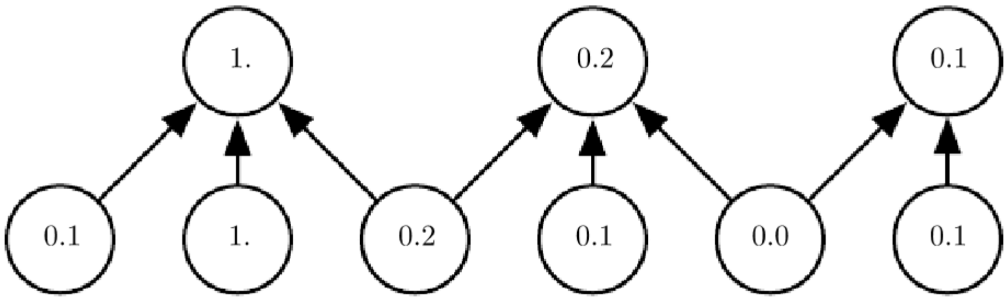
Convolutional layer: Pooling

3. Pooling stage
Implements *invariance to local translations*.

max pooling returns the maximum value in a rectangular region.
average pooling returns the average value in a rectangular region.

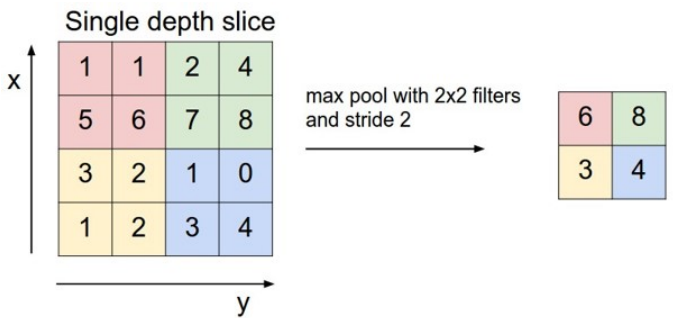
When applied with *stride*, it reduces the size of the output layer.

Example: max pooling with width 3 and stride 2

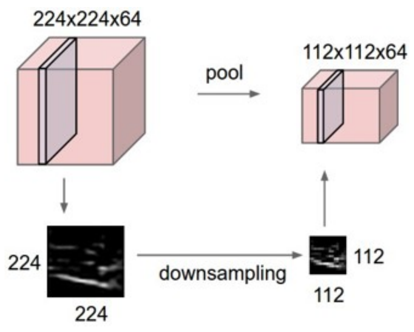


Convolutional layer: Pooling

Example of **max pooling**



Introduces subsampling:



Feature map size - Number of parameters

Consider input of size $w_{in} \times h_{in} \times d_{in}$, d_{out} kernels of size $w_k \times h_k \times d_{in}$, stride s and padding p

Dimensions of output feature map are given by:

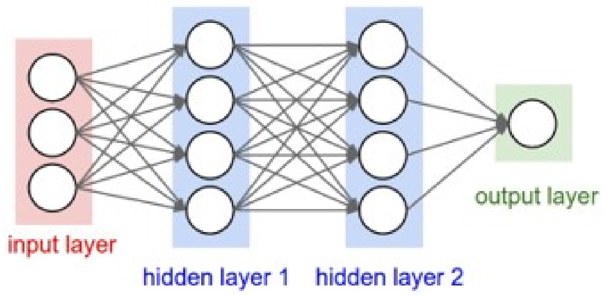
$$w_{out} = \frac{w_{in} - w_k + 2p}{s} + 1$$

$$h_{out} = \frac{h_{in} - h_k + 2p}{s} + 1$$

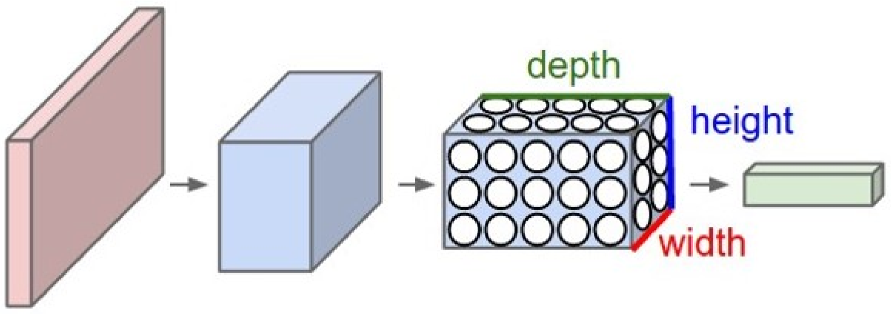
Number of trainable parameters of the convolutional layer is:

$$|\theta| = \underbrace{w_k \cdot h_k \cdot d_{in} \cdot d_{out}}_{\text{kernel weights}} + \underbrace{d_{out}}_{\text{bias}}$$

CNNs for Images (2D input)



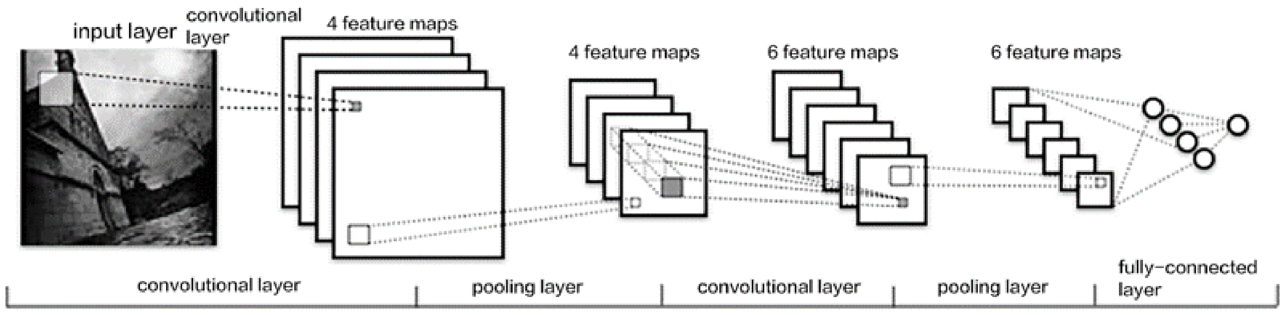
A regular 3-layer Neural Network

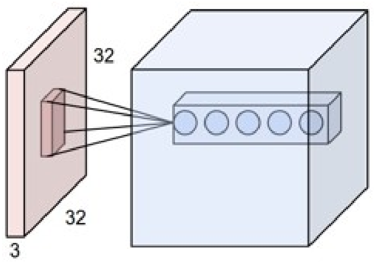


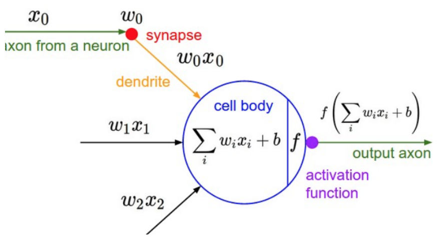
Every convolutional layer of a CNN transforms the 3D input volume to a 3D output volume of neuron activations.

Material from Fei-Fei's group

CNNs for Images (2D input)





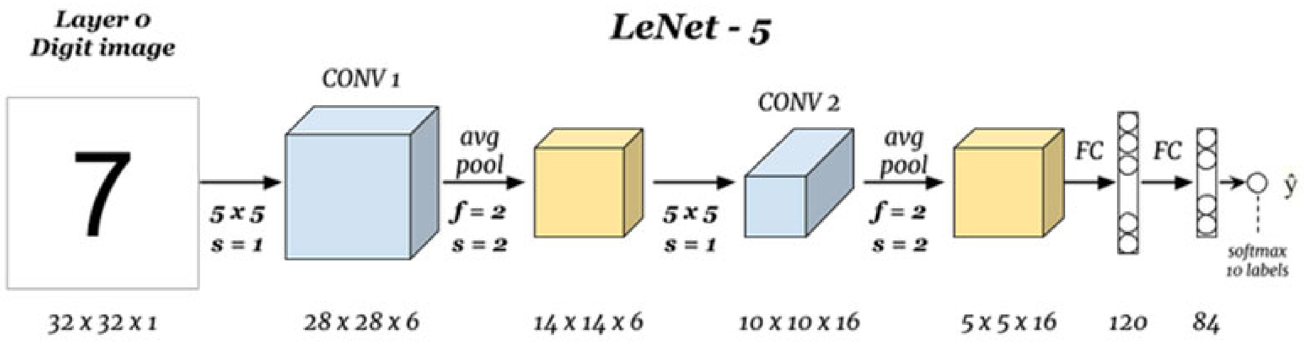


Each neuron is connected to a local 'horizontal' region of the input volume, but to **all** channels (depth)

The neurons still compute a dot product of their weights with the input followed by a non-linearity

Material from Fei-Fei's group

Example: LeNet



Y. LeCun, L. Bottou, Y. Bengio and P. Haffner: Gradient-Based Learning Applied to Document Recognition, Proceedings of the IEEE, 86(11):2278-2324, November 1998

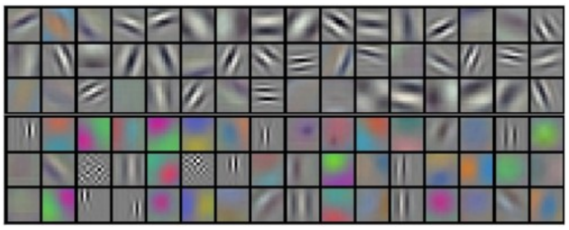
Handwritten recognition with LeNet

MNIST database

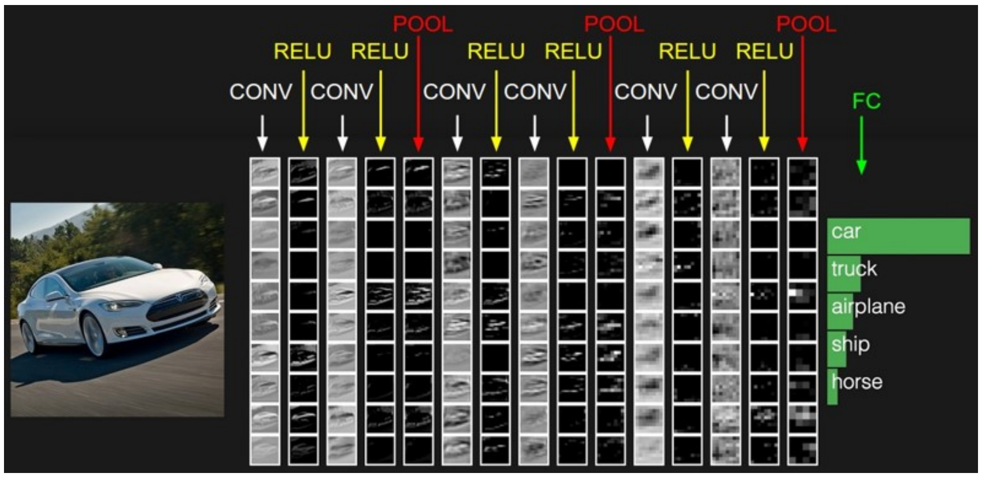
Accuracy up to 98 %.

Kernels and Feature maps - Example

Visualization of learned kernels of the first layer:



Computed activations during forward pass:



Material from Fei-Fei's group

Recent success of CNNs

- Theoretical advancements:
  - Dropout
  - ReLUs
  - Batch Normalization
  - Skip connections
  - ...
- Massively Parallel Computing (GPUs/TPUs)
- Very large training sets (ImageNet/MS COCO)
- International competitions (ILSVRC 2010-2017)
- Developing Frameworks (Tensorflow, PyTorch, ...)

ImageNet and ILSVRC

ImageNet Huge dataset of images
over 14 M labelled high resolution images
about 22 K categories

ILSVRC Competitions of image classification at large scale (since 2010)
1.2 M images in 1 K categories
5 guesses about image label

`http://image-net.org`

“Famous” CNNs

- AlexNet (2012)
- GoogLeNet / Inception (2014)
- VGG (2014)
- ResNet (2015)

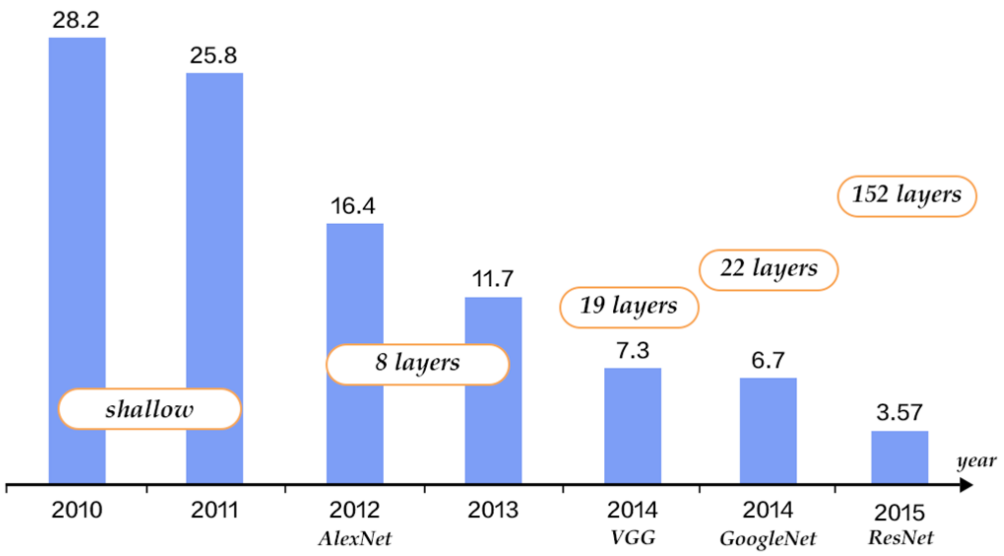
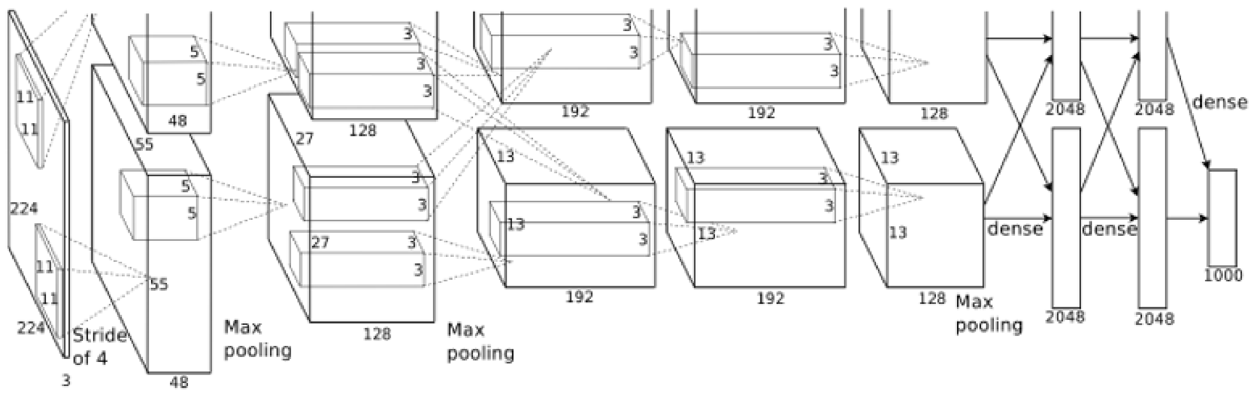


Image top-5 classification error rate on ILSVRC (%)

“Famous” CNNs

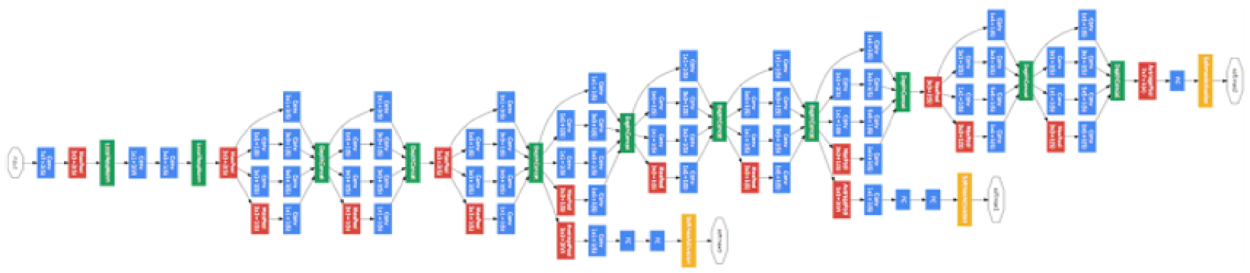
AlexNet - Winner of ILSVRC 2012



- Reached 16.4% top-5 image classification error on ILSVRC 2012
- Large gap from 25.8% top-5 error, best result of ILSVRC 2011
- Not commonly used anymore

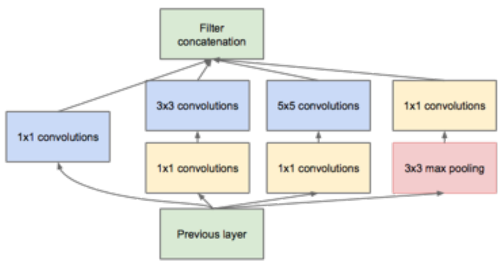
“Famous” CNNs

GoogLeNet/Inception - Winner of ILSVRC 2014



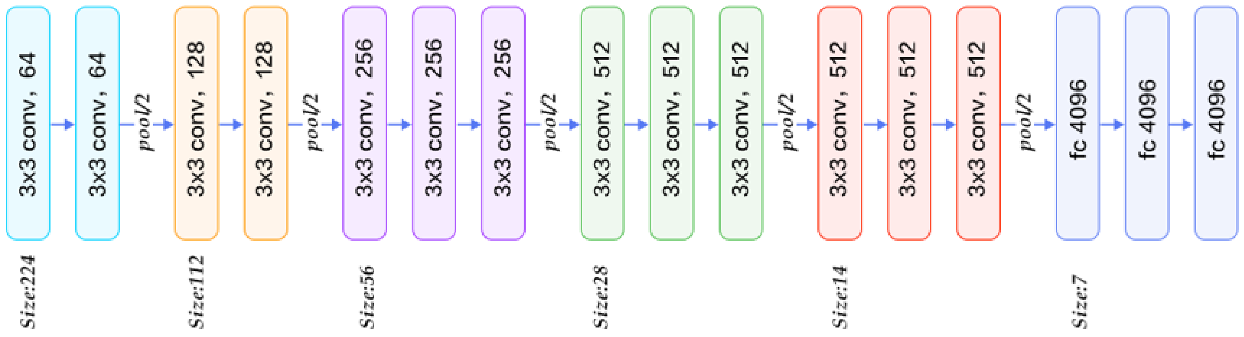
Inception module:

- Reached 6.7% top-5 image classification error on ILSVRC 2012
- Updated versions commonly used also today (Inception v3 and v4)



“Famous” CNNs

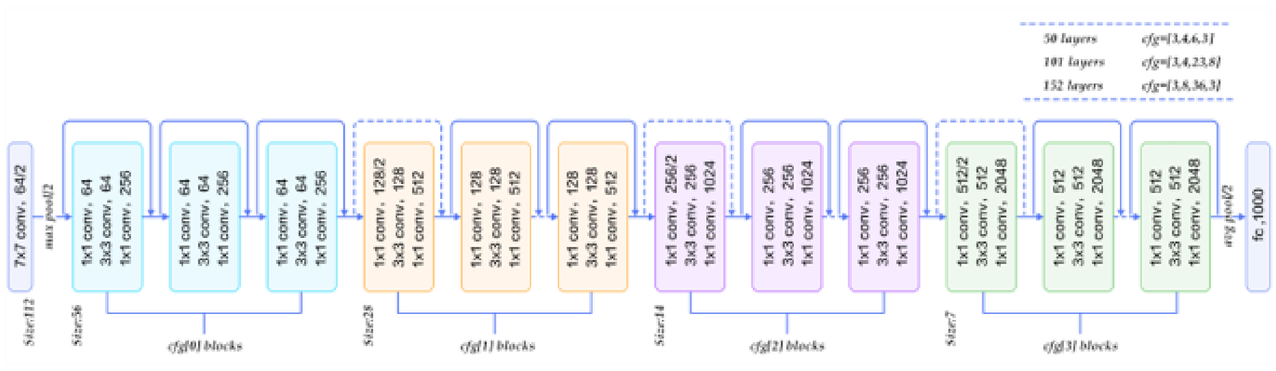
VGG - 1st Runner-Up of ILSVRC 2014



- Reached 7.3% top-5 image classification error on ILSVRC 2012
- Commonly used also today

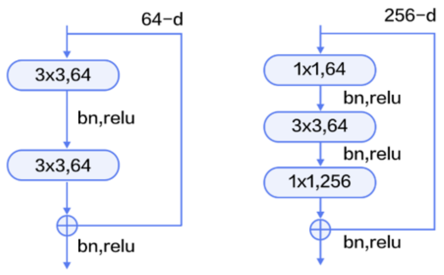
"Famous" CNNs

ResNet - Winner of ILSVRC 2015



- Reached 3.6% top-5 image classification error on ILSVRC 2012
- Commonly used also today (various versions e.g. 50, 101, 152 layers)

Residual Layers



(Use of skip connections)

Common uses of famous CNNs

- Train a new model on a dataset
- Use pre-trained models (e.g., trained on ImageNet) to
  - predict ImageNet categories for new images
  - extract features to train another model (e.g., SVM)
- Refine pre-trained models on a new dataset (new set of classes)

Examples: `https://keras.io/applications/`

Transfer Learning

Definitions

- D is a *domain* defined by data points $\mathbf{x}_i \in \mathbf{X}$ distributed according to $\mathcal{D}(\mathbf{x})$
- T is a *learning task* defined by labels $\mathbf{y} \in \mathbf{Y}$, a target function $f : \mathbf{X} \to \mathbf{Y}$, and distribution $P_{\mathcal{D}}(\mathbf{y}|\mathbf{x})$

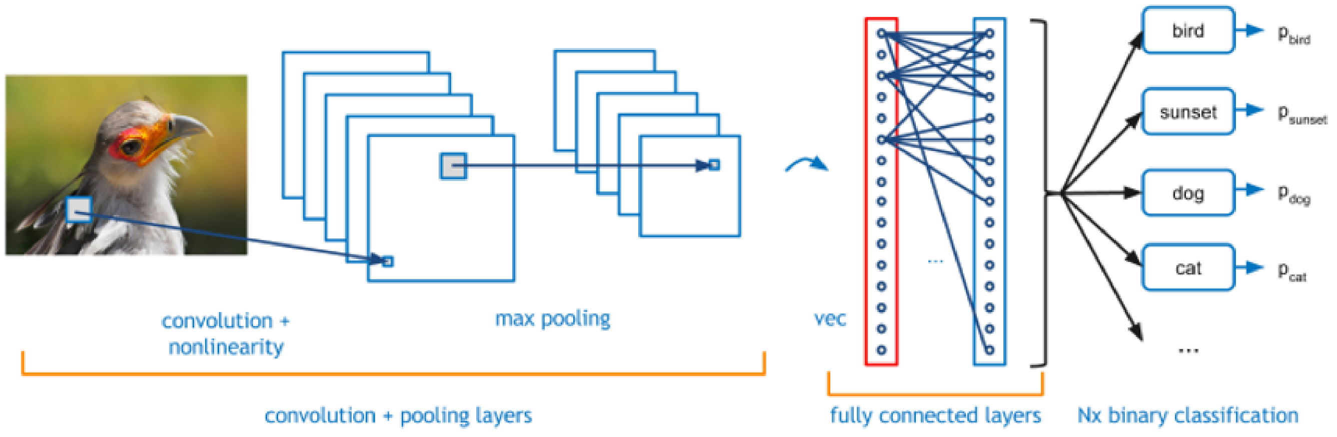
Given

- D_S and T_S a source domain and learning task
- D_T and T_T a target domain and learning task
- In general, $D_S \neq D_T$ and $T_S \neq T_T$

Goal
improve learning of $f_T : \mathbf{X}_T \to \mathbf{Y}_T$ using knowledge in D_S and T_S (i.e., after training $f_S : \mathbf{X}_S \to \mathbf{Y}_S$)

Transfer Learning - Example

Image classification using a CNN



CNN pre-trained on Imagenet

- millions of images (source domain)
- classification of 1000 classes (source learning task)

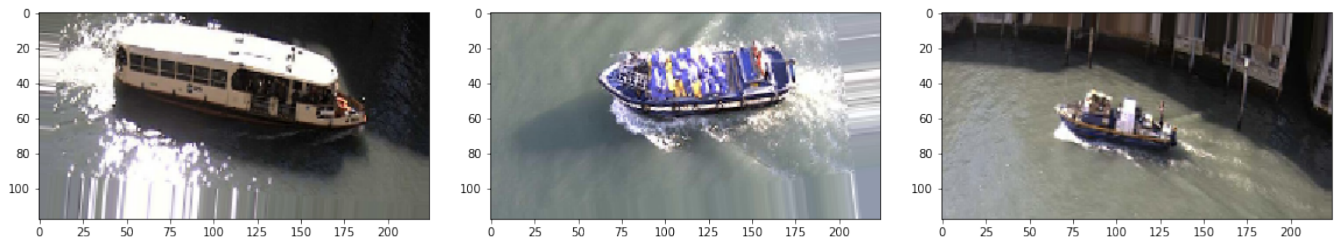
Slide from from Caffe framework tutorial @CVPR2015

Transfer Learning - Example

Image classification using a CNN

Use a pre-trained model for a different domain and/or learning task
E.g. Boat recognition in ARGOS dataset:

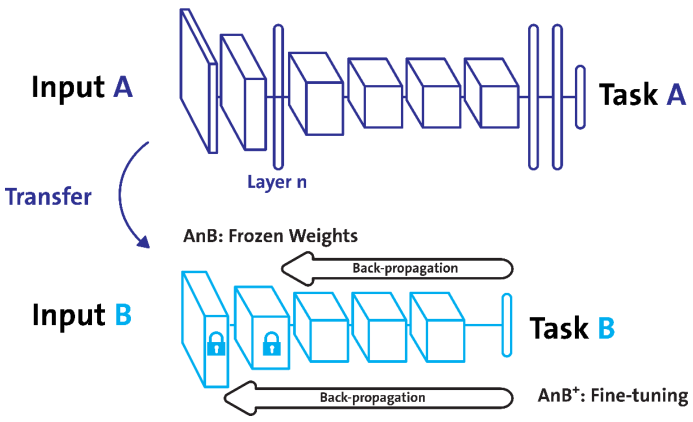
- thousands of boat images (target domain)
- classification of 20 boat classes (target learning task)



Transfer Learning - Example

1st solution - Fine-tuning

- use same network architecture with pre-trained model
- network parameters ‘copied’ from the pre-trained model
- no random initialization



Strategies

- training of all network parameters
- ‘freeze’ parameters of some layers (usually the first ones)

Pro Full advantage of the CNN!
Con ‘Heavy’ training

Image from P. Gudikandula’s blog

Transfer Learning - Example

2nd solution - CNN as feature extractor

1. extract features at a specific layer of CNN, usually:
   - last convolutional layer (flattened)
   - dense layers
2. collect extracted features $\mathbf{x}'$ of training/validation split and associate corresponding labels t in a new dataset $D' = \{(\mathbf{x}'_1, t_1), \dots, (\mathbf{x}'_N, t_n)\}$
3. train a new classifier C' using dataset D', e.g.
   - ANN (extreme case of fine-tuning)
   - SVM
   - linear classifier
   - ...
4. classify extracted features of test set using the classifier C'

Pro No need to train the CNN!

Con Cannot modify features, source and target domains should be as 'compatible' as possible

Transfer Learning - Example

2nd solution - CNN as feature extractor

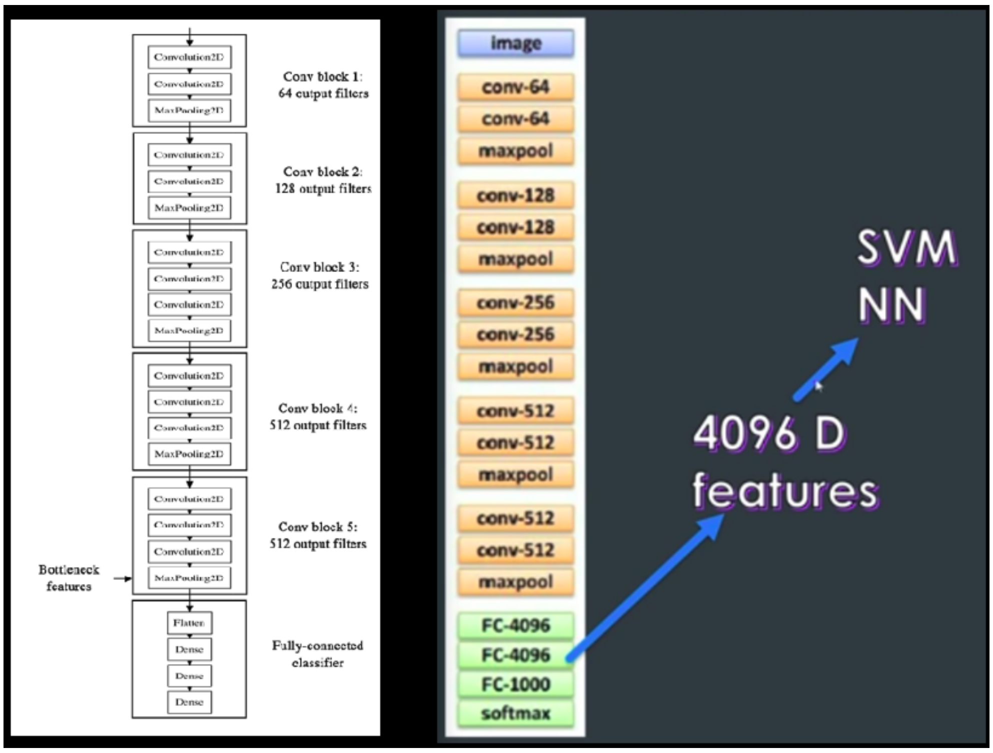


Image from P. Gudikandula's blog

Resources

Frameworks

- Caffe/Caffe 2 (UC Berkeley) - C/C++, Python, Matlab
- TensorFlow (Google) - C/C++, Python, Java, Go
- Theano (U Montreal) - Python
- CNTK (Microsoft) - Python, C++ , C#/.Net, Java
- Torch/PyTorch (Facebook) - Lua/Python
- MxNet (DMLC) - Python, C++, R, Perl, ...
- Darknet (Redmon J.) - C

High-level application libraries and models

- Keras
- TFLearn

Datasets

- `https://www.tensorflow.org/datasets`
- `https://www.kaggle.com/`

Summary

- CNNs are deep networks using convolution
- Very efficient (memory and generalization accuracy)
- Many successful examples
- Requires very large amount of data and very high computational resources