

A.Y. 2020/21

Carefully read the following scenario and answer the questions listed below.

A study is conducted with the aim to **understand the brain organization** in a **neurological pathological condition** mostly affecting **elderly patients**.

The subjects enrolled in the study undergo an **EEG screening** during the **resting state**. They are seated in a comfortable position in a quiet, dim room and they are just asked to relax for the duration of the recordings.

**Six cortical regions** (3 for each hemisphere, 2 in the frontal lobe and 1 in the parietal lobe) are selected for a connectivity study of **causality in the statistical sense**. The goal is to describe the network properties in terms of **integration** and **segregation** between different modules.

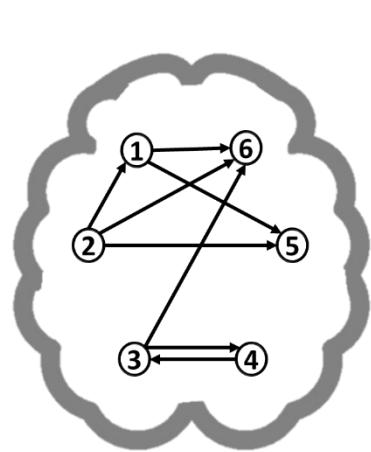


Fig. 1-A

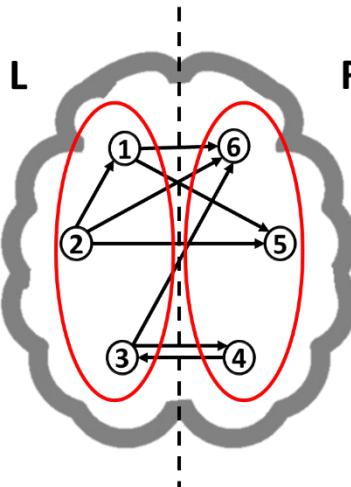


Fig. 1-B

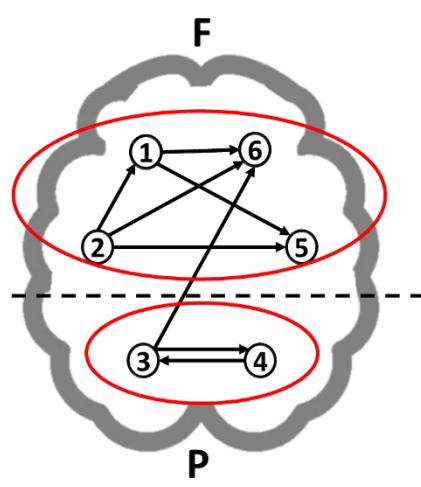


Fig. 1-C

## Questions

**Q1 – Indicate which connectivity estimator you would use to perform the network analysis.**  
**Motivate your choice.** NB: the answer and the motivation are independent from the networks depicted in Fig.1. (3 points)

(write the answers in the exam.net editor)

**Q2** – Assuming that the network analysis returns the functional network reported in **Fig. 1-A**:

**Q2.1:** Extract the corresponding **adjacency matrix** (*1 point*)

**Q2.2:** Compute the **Indegree** and the **Outdegree** for each node (*1 point*)

**Q2.3:** Compute the **Divisibility D** and the **Modularity Q** of the network, considering the two hemispheres as classes, like in **Fig. 1-B**:  $C=[1,1,1,2,2,2]$  (*2 points*)

**Q2.4:** Compute the **Divisibility D** and the **Modularity Q** of the network, considering the frontal lobe and the parietal lobe as classes, like in **Fig. 1-C**:  $C=[1,1,2,2,1,1]$  (*2 points*)

(*write the answers on paper*)

**Q3** – Given the results obtained at Q2, indicate which of the two divisions in classes (left-right or fronto-parietal) corresponds to a more segregated (=divided in two modules) network. **Motivate your answer.** (*2 points*)

(*write the answers in the exam.net editor*)

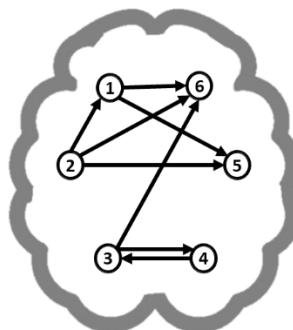
## Solutions

**Q1:** Since the study requires to compute **causality in the statistical sense**, the choice is between the **Granger Test (GT)** and **PDC**.

Since we are dealing with EEG data, we can assume that the **spectral information** is of relevance, so we can select the PDC (which is also more accurate as it mitigates the problem of the hidden source).

Alternate reasoning: since we are not sure that we have enough data to compute PDC (which is multivariate and therefore **not robust to data paucity**) we can also decide to use GT, to stay on the safe side at the expenses of the limitations typical of bivariate approaches.

**Q2.1:** Adjacency matrix:

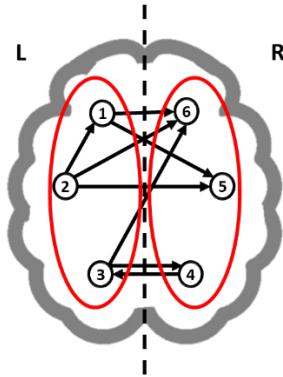


0	0	0	0	1	1
1	0	0	0	1	1
0	0	0	1	0	1
0	0	1	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

**Q2.2:** In- and out-degrees:

		$g_{IN}$						
		1	0	1	1	2	3	
$g_{OUT}$		2	0	0	0	0	1	1
		3	1	0	0	0	1	1
A=		2	0	0	0	1	0	1
		1	0	0	1	0	0	0
		0	0	0	0	0	0	0
		0	0	0	0	0	0	0

**Q2.3:**



		$g_{IN}$						
		$g_{OUT}$						
A=		1	0	1	1	2	3	
		2	0	0	0	0	1	1
		3	1	0	0	0	1	1
		2	0	0	0	1	0	1
		1	0	0	1	0	0	0
		0	0	0	0	0	0	0
		0	0	0	0	0	0	0
		1	1	1	2	2	2	

$$\begin{aligned} N &= 6 \\ L &= 8 \\ C &= [1 \ 1 \ 1 \ 2 \ 2 \ 2] \end{aligned}$$

$$D = \frac{L}{L + \sum_{i,j=1}^N a_{ij} [1 - \delta(C_i, C_j)]}$$

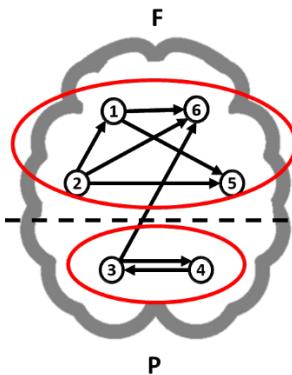
$$D = \frac{8}{8+7} = 0.53 \quad D \in [0.5, 1]$$

$$Q = \frac{1}{L} \sum_{i,j=1}^N (a_{ij} - \frac{g_i^{OUT} g_j^{IN}}{L}) \delta(C_i, C_j)$$

Existence of intra-community links      Probability of random links between the nodes

$$\begin{aligned} Q &= \frac{1}{8} \left[ 1 - \frac{(g_1^{OUT} + g_2^{OUT} + g_3^{OUT})(g_1^{IN} + g_2^{IN} + g_3^{IN})}{8} + 0 - \frac{(g_4^{OUT} g_5^{OUT} + g_6^{OUT})(g_4^{IN} g_5^{IN} + g_6^{IN})}{8} \right] = \\ &= \frac{1}{8} \left[ 1 - \frac{(2+3+2)(1+0+1)}{8} + 0 - \frac{(1+0+0)(1+2+3)}{8} \right] = \frac{1}{8} [1 - 1.75 + 0 - 0.75] = -0.19 \end{aligned}$$

**Q2.4:**



		$g_{IN}$						
		$g_{OUT}$						
A=		1	0	1	1	2	3	
		2	0	0	0	0	1	1
		3	1	0	0	0	1	1
		2	0	0	0	1	0	1
		1	0	0	1	0	0	0
		0	0	0	0	0	0	0
		0	0	0	0	0	0	0
		1	1	2	2	1	1	

$$\begin{aligned} N &= 6 \\ L &= 8 \\ C &= [1 \ 1 \ 2 \ 2 \ 1 \ 1] \end{aligned}$$

$$D = \frac{L}{L + \sum_{i,j=1}^N a_{ij} [1 - \delta(C_i, C_j)]}$$

$$D = \frac{8}{8+1} = 0.89 \quad D \in [0.5, 1]$$

$$Q = \frac{1}{L} \sum_{i,j=1}^N (a_{ij} - \frac{g_i^{OUT} g_j^{IN}}{L}) \delta(C_i, C_j)$$

Existence of intra-community links      Probability of random links between the nodes

$$\begin{aligned} Q &= \frac{1}{8} \left[ 5 - \frac{(g_1^{OUT} + g_2^{OUT} + g_5^{OUT} + g_6^{OUT})(g_1^{IN} + g_2^{IN} + g_5^{IN} + g_6^{IN})}{8} + 2 - \frac{(g_3^{OUT} g_4^{OUT})(g_3^{IN} g_4^{IN})}{8} \right] = \\ &= \frac{1}{8} \left[ 5 - \frac{(2+3+0+0)(1+0+2+3)}{8} + 2 - \frac{(2+1)(1+1)}{8} \right] = \frac{1}{8} [5 - 3.75 + 2 - 0.75] = 0.31 \end{aligned}$$

**Q3:** Given the results obtained, the fronto-parietal division in classes corresponds to a more segregated and less integrated network. In fact, both divisibility and modularity are higher than in the left-right division. In particular, L-R shows a divisibility close to its minimum value (=0.5) and a negative modularity, indicating that the two classes don't act as modules, while F-P shows a divisibility close to its maximum (=1) and a positive modularity, indicating that the two classes are more internally organized than an average group of nodes randomly chosen in the network.

	Divisibility	Modularity	
Left-Right	0.53	-0.19	Lower segregation/higher integration
Fronto-Parietal	0.89	0.31	Higher segregation/lower integration