



# Basics of signal processing (4)

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# Material for this section of the course

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- Matlab notebooks available here:
  - <https://drive.matlab.com/sharing/d5ad1819-5e50-442a-81fc-6017505d91f3>
    - NEng\_1920\_03\_Spectr.mlx (cont'd)
    - NEng\_1920\_04\_Filt.mlx
- Not a textbook, but readings for those who want to have some context:
  - Steven W. Smith  
The Scientist and Engineer's Guide to Digital Signal Processing  
<https://www.dspguide.com/pdfbook.htm>
- See also:
  - John L Semmlow  
Biosignal and medical image processing (3rd Ed.)  
CRC Press  
Chapter 3-4

# Neuroengineering - Spectral analysis

Fourier Analysis, i.e. decomposition of signals into sum of sinewaves. Since each sinewave carries power at exactly one frequency, the decomposition can be used to analyze the signal in the frequency domain. Specifically Discrete Fourier Transform can be used to transform the (time-limited and sampled) time-domain representation of a signal into its (bandwidth limited and sampled) representation in the frequency domain.

Using the DFT to analyze the spectral content of a (stochastic) signal may limit the ability to interpret the results. Specific techniques are commonly in use:

- Zeropadding and windowing are techniques aimed at compensating the effect on the spectrum of a limited number of samples in the time domain (low resolution, sidelobes)
- Power Spectral Density (PSD) can be estimated techniques such as the *averaged periodogram* which limit the variability of the power estimate at the expense of a loss of spectral resolution.

See also:

- Semmlow, Biosignal and medical image processing, Chapter 3
- (<https://www.dspguide.com/pdfbook.htm>, Chapters 6 and 31)

A sinewave is a function of time, with parameters: frequency  $f$ , amplitude  $A$ , and initial phase  $\phi$

```
sinewave = @(t, f,A,phi) A * cos(2*pi*f*t + phi);
t = (0:.005:1)'; % seconds

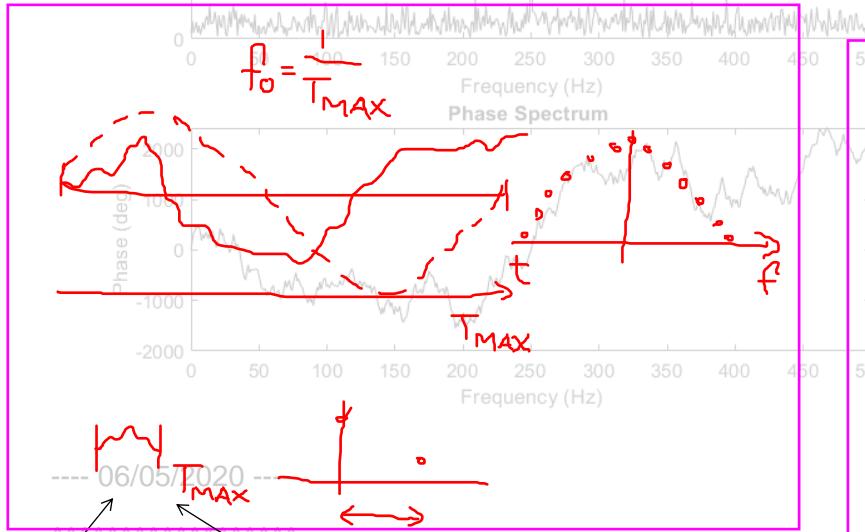
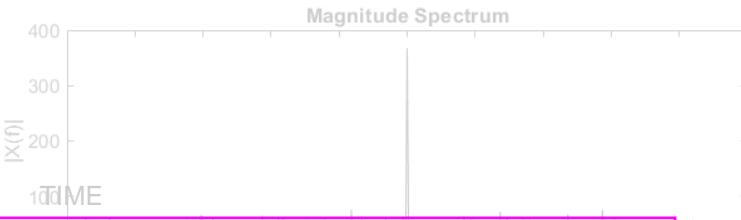
f = 10; % Hz
A = 0.5;
phi = 0.4 * pi; % radians

figure(1)
clf
subplot 211
plot(t, sinewave(t, f,A,phi))
xlim([0 1])
ylim([-2 2])
grid on
xlabel("time [s]")
```

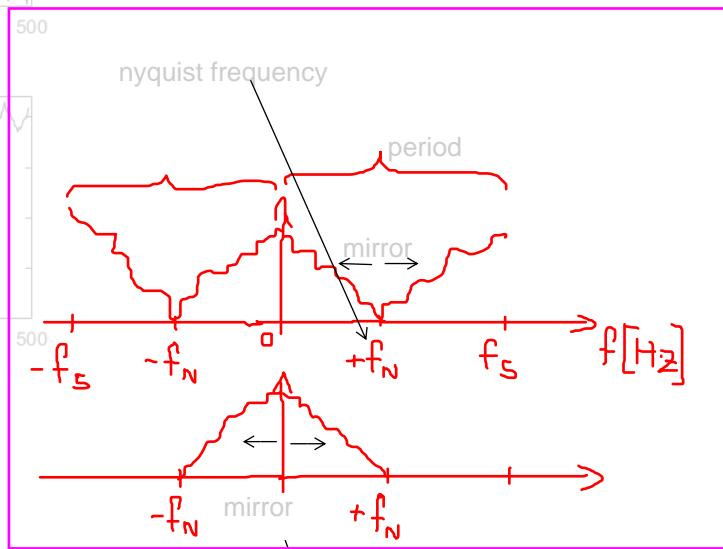
$$C = Ae^{j\phi} \Leftrightarrow A = |C|, \phi = \angle C$$

```
C = A*exp(1j*phi); % Complex amplitude
subplot 212
plot(real(C), imag(C), '.', 'MarkerSize', 20)

axis square equal
xlim([-2 2])
ylim([-2 2])
grid on; grid minor
title("Complex coefficient, f = " + f + " Hz")
```



FREQUENCY



we can consider only the positive part of the plane

if the signal is shorter when you make the transformation, in the frequency domain we have that the distance between 2 adjacent samples in the frequency domain is much longer

## Zero padding

Duality in time vs. frequency domain

Higher sampling rate in  $t \Leftrightarrow$  Broader spectrum in  $f$

$f_{\text{samp}} = 1 \text{ kHz} \Leftrightarrow T_s = 0.001 \text{ s} \Leftrightarrow f_{\text{min}} = 0 \text{ Hz}, f_{\text{max}} = 1000 \text{ Hz}$  (even if useful info only up to 500 Hz)

Longer signal in  $t \Leftrightarrow$  Higher resolution in  $f$

$x[t]$  from  $t=0$  to  $T=T_{\text{Max}}$  ||  $\Delta f = 1/T_{\text{max}}$  if the signal is short the resolution is poor

Zeropadding (i.e. adding a row of zeros at the end of the signal) allows to increase (artificially) the spectral resolution.

the question is what is the maximum frequency that appears in the spectrum?

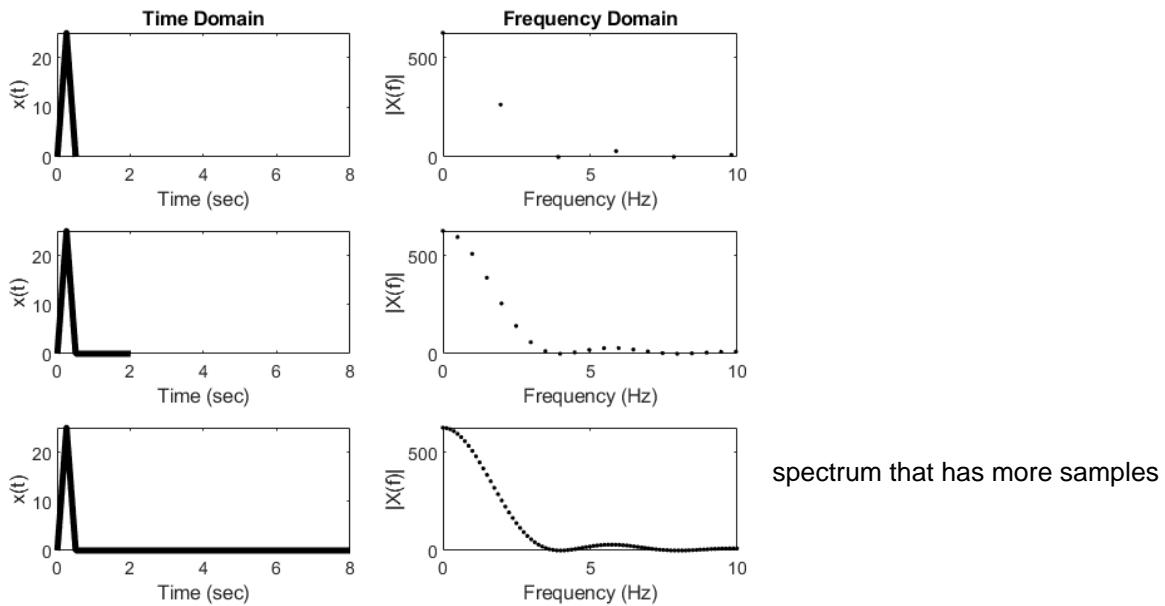
```
% Example 3.4 Generate a 1.0 second wave symmetrical triangle wave. Make fs = 100 Hz
% N = 100 points. Calculate and plot the magnitude spectrum.
% Zero pad so the period is extended to 2 and 8 sec. and recalculate and plot the
% magnitude spectrum. Since fs = 100 Hz the signal should be padded to 200 and 600 Points
%
% clear all, close all;
fs = 100; % Sample frequencies
N1 = [0 150 750]; % Padding added to the original 50 point signal
x = [(0:25) (24:-1:0)]; % Generate basic test signal, 50 pts long

figure(7); clf
for k = 1:3
    x1 = [x zeros(1,N1(k))]; % Zero pad signal
    N = length(x1); % Data length
    t = (1:N)/fs; % Frequency vector
    f = (0:N-1)*fs/N;
    subplot(3,2,k*2-1);
    % Plotting
end
```

```

plot(t,x1,'k', "LineWidth",3); % Plot test signal
xlabel('Time (sec)', 'FontSize',14);
ylabel('x(t)', 'FontSize',14);
if k == 1
    title('Time Domain', 'FontSize',14);
end
% xlim([0 t(end)]);
xlim([0 (length(x)+max(N1)) /fs]);
subplot(3,2,k*2);
X1 = abs(fft(x1)); % Calculate the magnitude spectrum
plot(f, X1,'.k'); % Plot magnitude spectrum
xlabel('Frequency (Hz)', 'FontSize',14);
ylabel('|X(f)|', 'FontSize',14);
xlim([0 10]);
% axis([0 10 0 max(X1)*1.2]);
if k == 1
    title('Frequency Domain', 'FontSize',14);
end
end

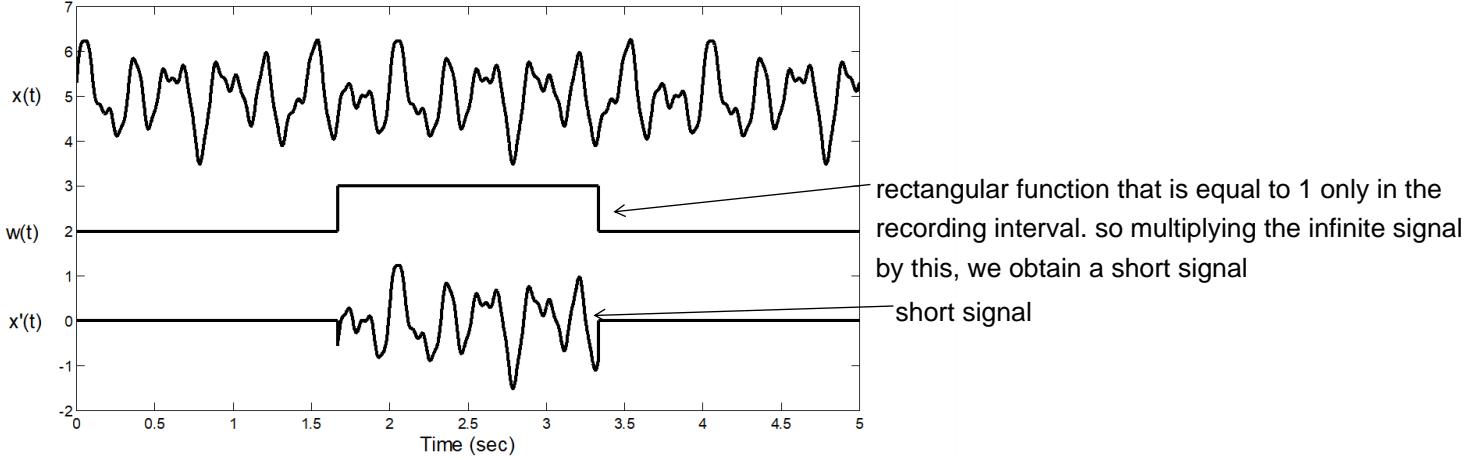
```



## Windowing

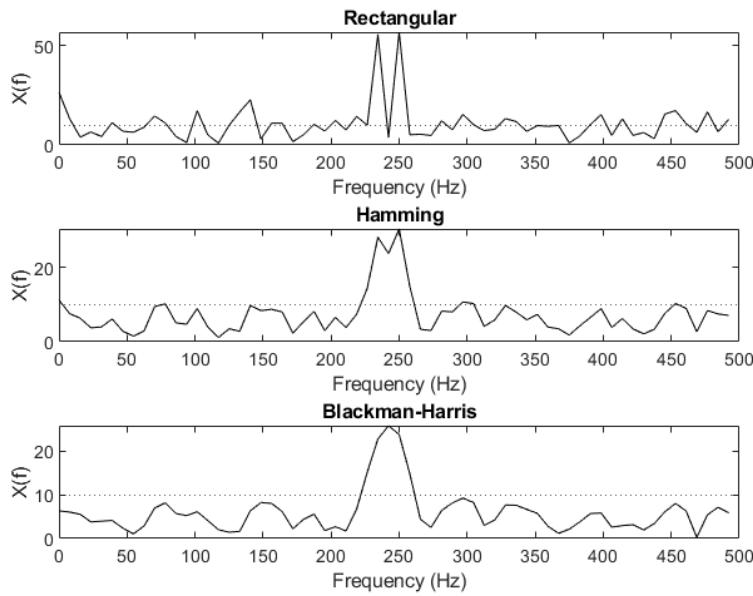
Taking a finite span of a signal is mathematically equivalent to multiplying by a `rect()` function.

Edge effects can be reduced using a tapered windowing function.



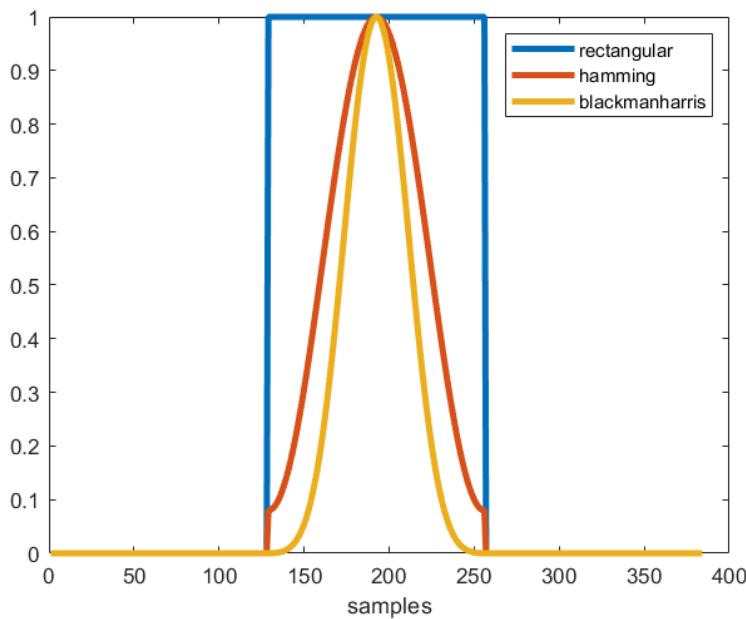
```
% Example 3.5 Application of several window functions
%
% clear all; close all;
fs = 1000; % Sampling freq assumed by sig_noise
N = 128;
x = sig_noise([235 250], -3, N); % Generate data
f = (0:N-1)*fs/N; % Frequency vector
%
X_mag = abs(fft(x)); % Mag. spect: rect. (no) window

figure(8); clf;
subplot(3,1,1);
plot(f(1:N/2), X_mag(1:N/2), 'k'); % Plot magnitude
% semilogy(f(1:N/2), X_mag(1:N/2), 'k'); % Plot magnitude
yline(10, ':');
xlabel('Frequency (Hz)', 'FontSize', 14); % Lables
ylabel('X(f)', 'FontSize', 14); % Lables
title('Rectangular', 'FontSize', 14); % Title
%
x1 = x .* hamming(N); % Apply Hamming window (Eq. 2.26)
X_mag = abs(fft(x1)); % Mag. spect: Hamming window
subplot(3,1,2);
plot(f(1:N/2), X_mag(1:N/2), 'k'); % Plot magnitude
% semilogy(f(1:N/2), X_mag(1:N/2), 'k'); % Plot magnitude
yline(10, ':');
xlabel('Frequency (Hz)', 'FontSize', 14); % Lables
ylabel('X(f)', 'FontSize', 14); % Lables
title('Hamming', 'FontSize', 14); % Title
%
x1 = x .* blackmanharris(N); % Apply Blackman-Harris (Eq. 2.27)
X_mag = abs(fft(x1)); % Mag. spect: Blackman-Harris window
subplot(3,1,3);
plot(f(1:N/2), X_mag(1:N/2), 'k'); % Plot magnitude
% semilogy(f(1:N/2), X_mag(1:N/2), 'k'); % Plot magnitude
yline(10, ':');
xlabel('Frequency (Hz)', 'FontSize', 14); % Lables
ylabel('X(f)', 'FontSize', 14); % Lables
title('Blackman-Harris', 'FontSize', 14); % Title
```



Edge effects determine the spectral leakage

```
figure(9); clf
plot([zeros(N,1); rectwin(N); zeros(N,1)], "LineWidth", 3)
hold on
plot([zeros(N,1); hamming(N); zeros(N,1)], "LineWidth", 3)
plot([zeros(N,1); blackmanharris(N); zeros(N,1)], "LineWidth", 3)
hold off
xlabel("samples")
legend(["rectangular" "hamming" "blackmanharris"])
```

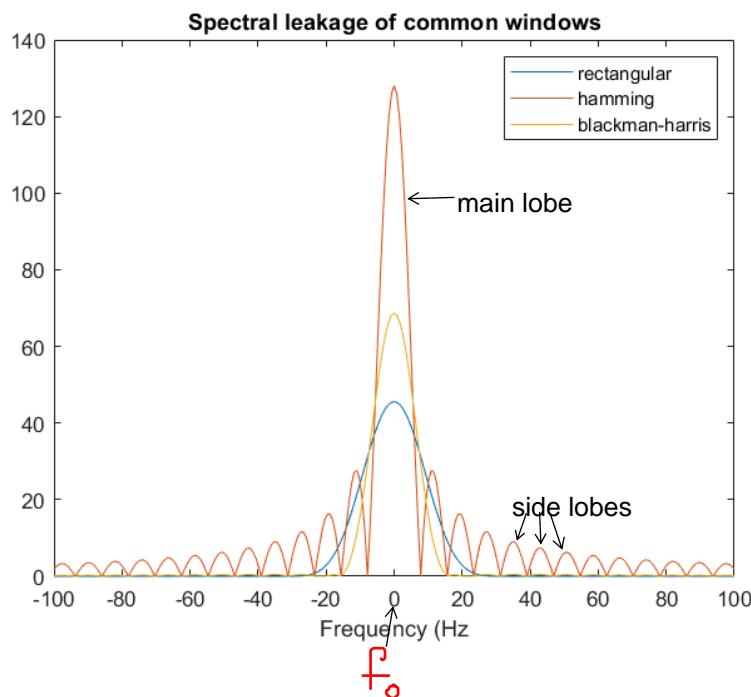


```
allwindows = [rectwin(N) hamming(N) blackmanharris(N)];
```

```

zeropadded_len = N*10;
allwindows(zeropadded_len, :) = 0; % zero pad
f_windows = fs/(zeropadded_len) * [0:zeropadded_len-1];
f_windows(f_windows>=fs/2) = f_windows(f_windows>=fs/2) - fs; % fold frequencies above fs/2
figure(10)
clf
Win_mag = abs(fft(allwindows));
% plot(f_windows(1:zeropadded_len/2),Win_mag(1:zeropadded_len/2, :)); % Plot magnitude
plot(fftshift(f_windows),fftshift(Win_mag)); % Plot magnitude
xlim([-100 100])
xlabel("Frequency (Hz")
title("Spectral leakage of common windows")
legend(["rectangular", "hamming", "blackman-harris"])

```



## Power Spectrum

Power spectral density describes the power of the signal in each frequency band of the spectrum.

DISCRETE FOURIER TRANSFORM

$PSD(f) = |DFT(x(t))|^2$  if you could have several estimates of the spectrum, you will go closer and closer to the ideal spectrum, the spectrum of the process which generated the several signals

## Spectral averaging - Welch's method

The PSD of a stationary stochastic process (i.e. non-deterministic signal) is an estimate subject to error (variability)

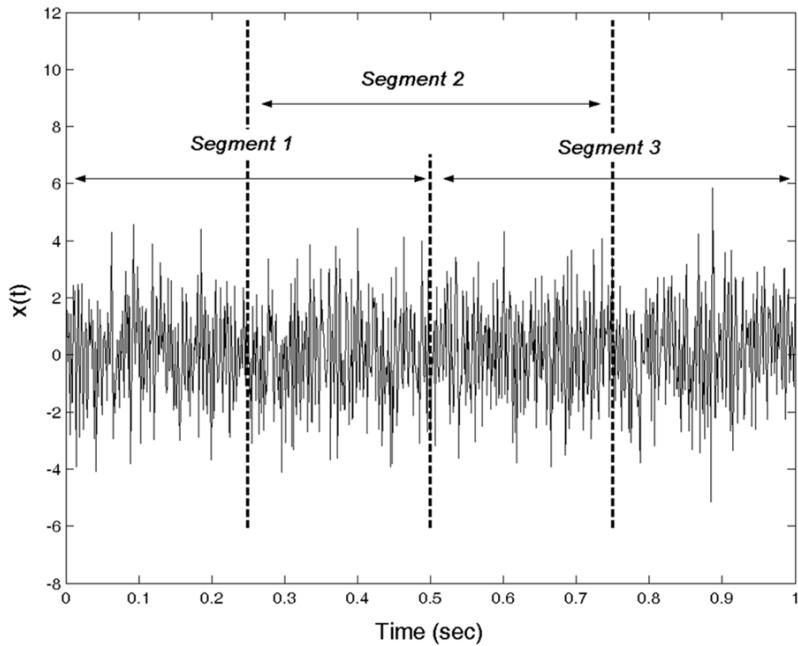
A long signal (many time samples) yield a very high spectral resolution (possibly way beyond the required resolution)

The *averaged periodogram* method trades-off spectral resolution with lower estimate variability. A windowing function can be applied (Welch's method).

0. starting from a long signal
1. Take a short segment of the signal of length nsamples
2. Apply a windowing function
3. Compute the periodogram, i.e.  $|DFT(win(t) \cdot x(t))|^2$ , and store the resulting PSD
4. Shift nsamples to the right (or a fraction of this amount, if overlapping segments are desired)
5. Repeat steps 1-4 until the end of the signal is reached (num\_segments)
6. Average all PSDs

The spectral resolution is determined by the length of the segment ( $1/\text{nsamples}$ )

The variability of the PSD is reduced by a factor  $\sqrt{\text{num\_segments}}$



```
% Example 3.7 Investigation of the influence of averaging to improve
% broadband spectral characteristics in the Power Spectrum.
% Loads file broadband1 that contains broadband and narrowband signals
% and noise.
% Calculates the standard Power Spectrum and one obtained using
% segment averaging. Use 126 sample segment length with a 99% overlap
% Assumes the data is variable x in the .mat file and was
% acquired at a sampling frequency of 1.0 kHz
%
close all; clear all;
```

Warning: Error occurred while executing the listener callback for event ObjectBeingDestroyed defined  
for class matlab.ui.Figure:  
Error using matlab.ui.container.Menu/horzcat

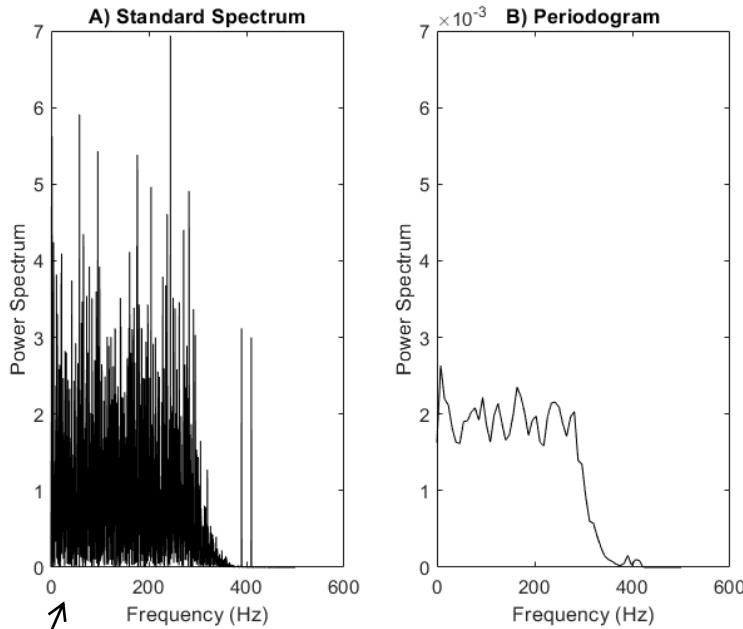
```

load broadband1;                                % Load data (variable x)
fs = 1000;                                     % Sampling frequency
nfft = 128;                                    % Segment size for averaging
PS = abs((fft(x)).^2)/length(x);               % Calculate un-averaged PS
half_length = fix(length(PS)/2);    % Find data length /2
f = (0:half_length-1)* fs/(2*half_length);     % Frequency vector for plotting

figure(11); clf;
ax1 = subplot(1,2,1);
plot(f,PS(1:half_length), 'k');                % Plot un-averaged Power Spectrum
xlabel('Frequency (Hz)', 'FontSize',14); ylabel('Power Spectrum', 'FontSize',14);
title('A) Standard Spectrum', 'FontSize',12);
%
[PS_avg,f] = pwelch(x,nfft,nfft-1,nfft,fs);   % Use 99% overlap
ax2 = subplot(1,2,2);
plot(f,PS_avg, 'k');                            % Plot averaged Power Spectrum
xlabel('Frequency (Hz)', 'FontSize',14); ylabel('Power Spectrum', 'FontSize',14);
ax2.YLim = 7/3*ax2.YLim;
title('B) Periodogram', 'FontSize',12);

```

estimating on a long signal



very high resolution but also very noisy

resolution is lower, because maybe we have that the distance between frequency sample is much larger

## Spectrogram

When the signal is not stationary, PSD should not be used to describe the signal's spectrum. If short segments of the signal can be considered stationary (the signal is pseudo-stationary), the evolution of the spectrum over time can be displayed using a spectrogram.

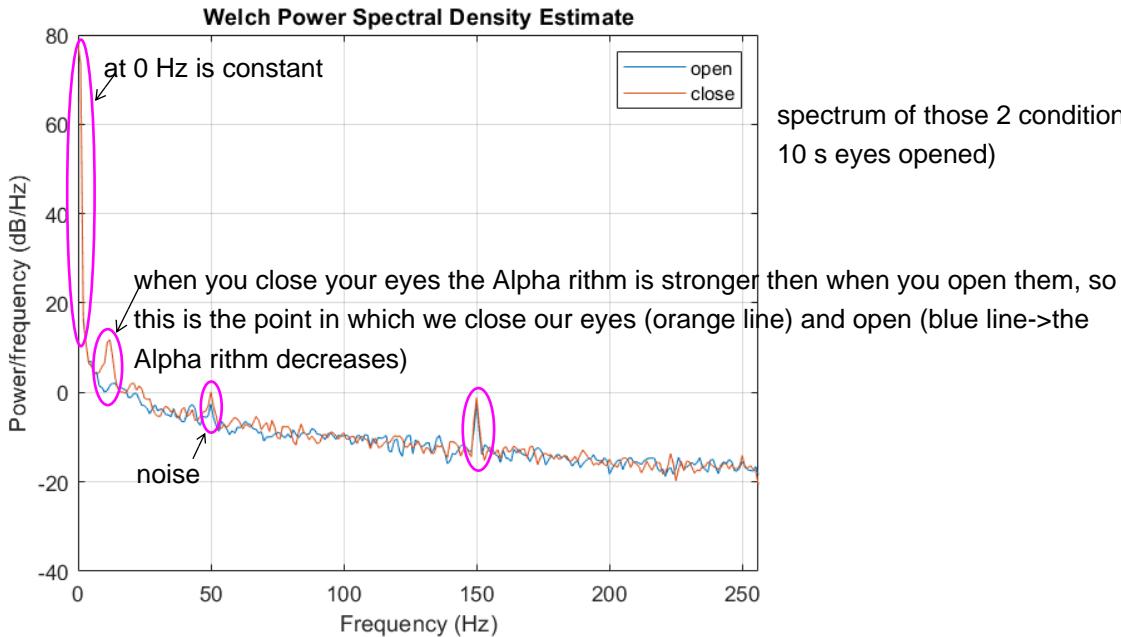
STFT (short-time fourier transform) is the simplest method to achieve this goal

```
clear
load eo_trial
load ec_trial
fs = 512;

fft_lenght = 1 * fs; % determines the spectral resolution
overlap_length = 0.5 * fs;
window = hamming(fft_lenght); % determines the spectral leakage

figure(1)
clf
hold on eyes open eyes close
pwelch([eo_trial ec_trial],window,overlap_length,fft_lenght,fs);

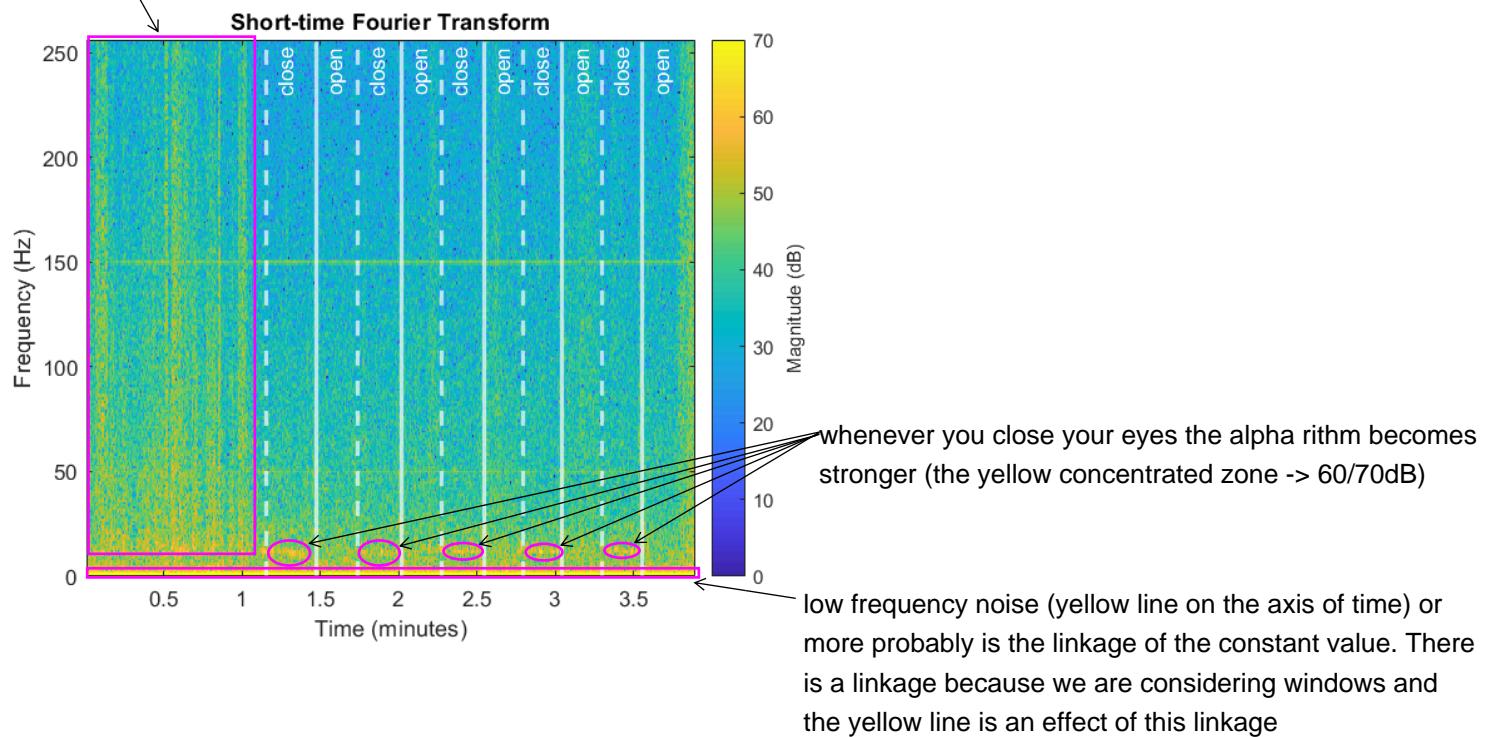
hold off
legend(["open", "close"])
```



```
%%
load eeg.mat eeg eo_start_samp ec_start_samp

figure(2)
clf
stft(eeg,fs,'Window',hamming(fs),'OverlapLength',fs/2,'FFTLength',fs);
ylim([0 fs/2])
caxis([0 70])
for samp = eo_start_samp(:)
    xline(samp/fs/60, 'w-', "open", "LineWidth", 2);
end
for samp = ec_start_samp(:)
    xline(samp/fs/60, 'w--', "close", "LineWidth", 2);
end
```

there are also vertical yellow lines probably because it is a EMG so muscular artifacts, so the subject contracted his muscles



# Internal functions ↵ NO

```

function plot_complex_coefficient(C, f)
plot(real(C), imag(C), '.', 'MarkerSize', 20)
axis square equal
xlim([-2 2])
ylim([-2 2])
grid on; grid minor
text(-2,2, "f = " + f + " Hz", "VerticalAlignment", "top", "HorizontalAlignment", "left")
end% function

function [waveform_noise, time, waveform, snr_out] = sig_noise(freqsin, snr, N)
% [waveform_noise, time, waveform, snr_out] = sig_noise(freqsin, snr, npts);
% Function to generate test data Generates sinusoids in noise.
% Inputs
% freqsin           is a vector specifying the frequency of sinusoid(s)
%                   assuming a sample frequency of 1 KHz
% One sinusoid of amplitude 1 is generated for each entry
% snr               is a vector the SNR values in db of the associated sinusoid
%                   if snr is a scalar it is used for all frequencies
% npts              number of points in the array
% Outputs
% waveform_noise    is the output vector containing sinusoids and noise
% waveform          is the output containing only sinusoids (no noise)
% time              is the time vector useful in plotting the waveform
% i.e., plot(time, waveform)
%
%
fs = 1000;      % Assume a sampling freq of 1 kHz
Ts = 1/fs;
time = (0:(N-1))*Ts;
noise = randn(1,N);  % Generate noise and calculate RMS value
rms_noise = sqrt(mean(noise.^2));
if length(snr) < length(freqsin) && length(snr) == 1      % Check SNR vector length
    snr(2:length(freqsin)) = snr(1);    %If a scalar, use this value for all freq.
elseif length(snr) < length(freqsin)
    disp('Error: not enough SNR values')
    waveform_noise = rms_noise;
    return
end
%
for i = 1:length(freqsin)
    freq_scale = freqsin(i) * 2 * pi/fs;
    x = (1:N) * freq_scale;
    snr_n = 10^(snr(i)/20);            % Convert from dB
    A = snr_n * rms_noise * 1.414;    % Determine gain for appropriate SNR
    if i == 1
        component = sin(x) * A;
        waveform = component;
        rms_sig(i) = sqrt(mean(waveform.^2));
    else
        component = sin(x) * A;
        rms_sig(i) = sqrt(mean(component.^2));
        waveform = waveform + component;
    end
end

```

```
end
snr_out(i) = 20 * log10(rms_sig(i)/rms_noise); % Confirm SNR
end
waveform_noise = waveform + noise;
end% function
```

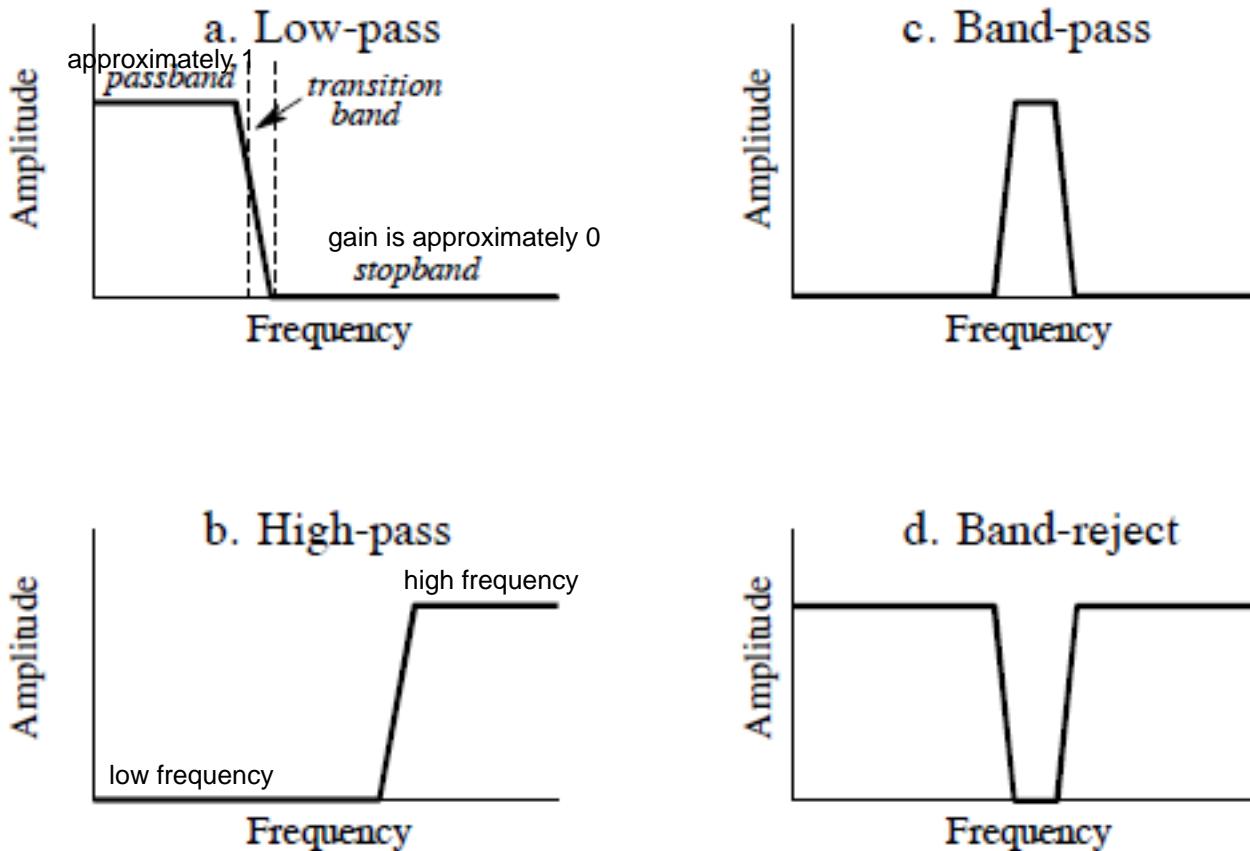
# Neuroengineering - Filters

The purpose a filter is to allow some spectral component of a signal to pass (almost) unaltered, while (almost) blocking other spectral components.

See also:

- Semmlow, Biosignal and medical image processing, Chapter 4
- (<https://www.dspguide.com/pdfbook.htm>, Chapter 14)

The figure below shows the four basic frequency responses.

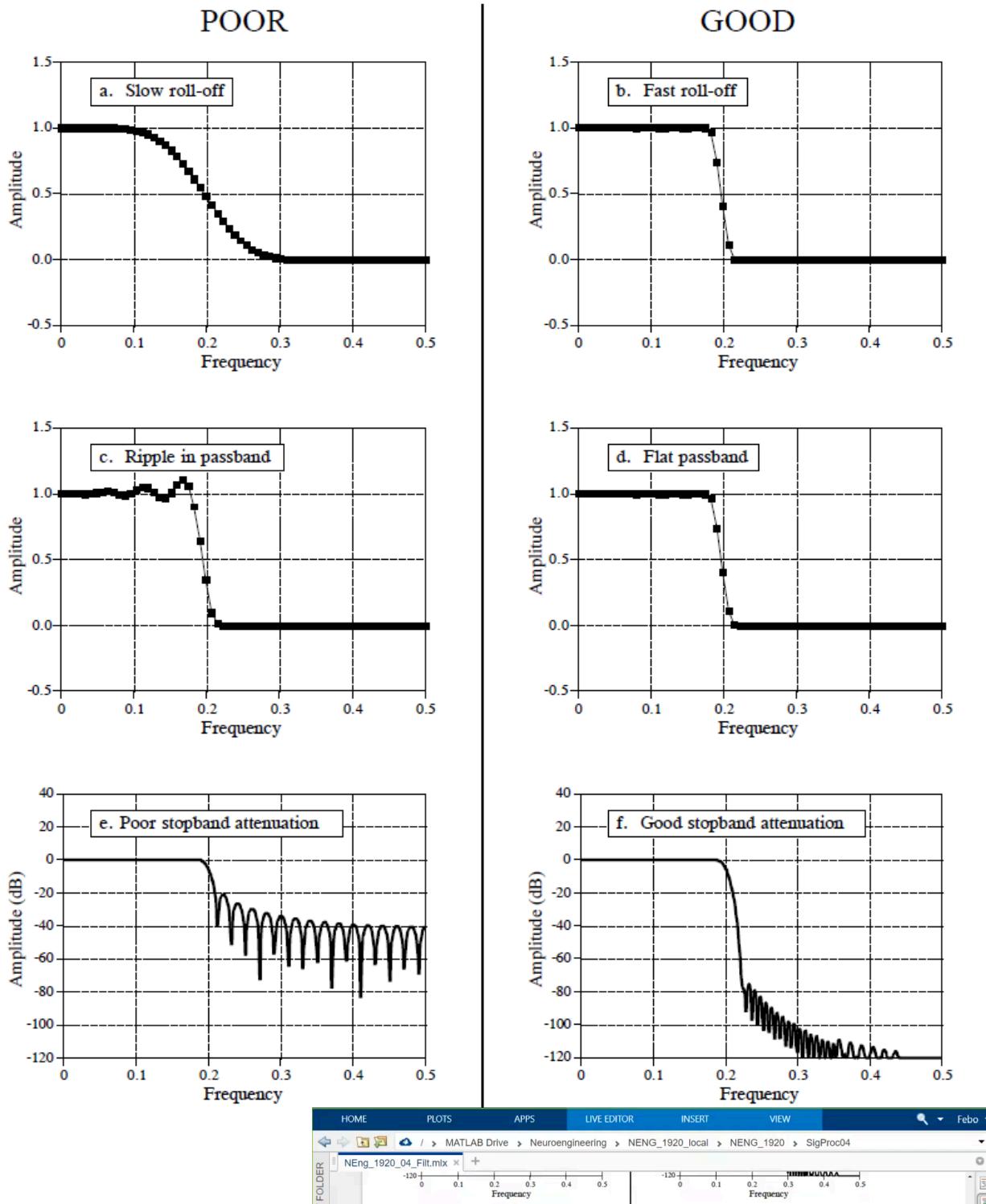


slope of the frequency response in the transition band

The **passband** refers to those frequencies that are passed, while the **stopband** contains those frequencies that are blocked. The **transition band** is between. A fast roll-off means that the transition band is very narrow. The frequency in which there is a change division between the passband and transition band is called the **cutoff frequency**. In analog filter design, the cutoff frequency is usually defined to be where the amplitude is reduced to 0.707 times the input (i.e.,  $-3dB$ ). Digital filters often specify different attenuation at the cutoff frequency, depending on the synthesis process.

The figure below shows three parameters that measure how well a filter performs in the frequency domain. To separate closely spaced frequencies, the filter must have a **fast roll-off**, (top row). For the passband

frequencies to move through the filter unaltered, there must be **no passband ripple**, (middle row). Lastly, to adequately block the stopband frequencies, it is necessary to have **good stopband attenuation**, (bottom row).



FIR vs. IIR design ...

this filter not distort the signal

#### FIR vs. IIR design

Finite impulse response and Infinite impulse response are the two main families of filter types.

In the former, the output  $y[i]$  is computed by combining samples of the input:  $y[i] = \sum_{k=0}^N b_k x[i-k]$

In the latter, the output is computed by combining samples of the input and previous samples of the output:  $y[i] = \sum_{k=0}^N b_k x[i-k] - \sum_{k=1}^M a_k y[i-k]$

The order  $N$  of a filter indicates how many recent samples are used to compute the output.

Examples of design methods for FIR filters:

- windowed sinc (using the same window types used in spectral analysis)
- minimax (or Parks-McClellan)

```

load ec_trial
fs = 512;
num_samp = length(ec_trial);
time = (0:num_samp-1) / fs;
ec_trial = ec_trial - mean(ec_trial); % fix overall baseline

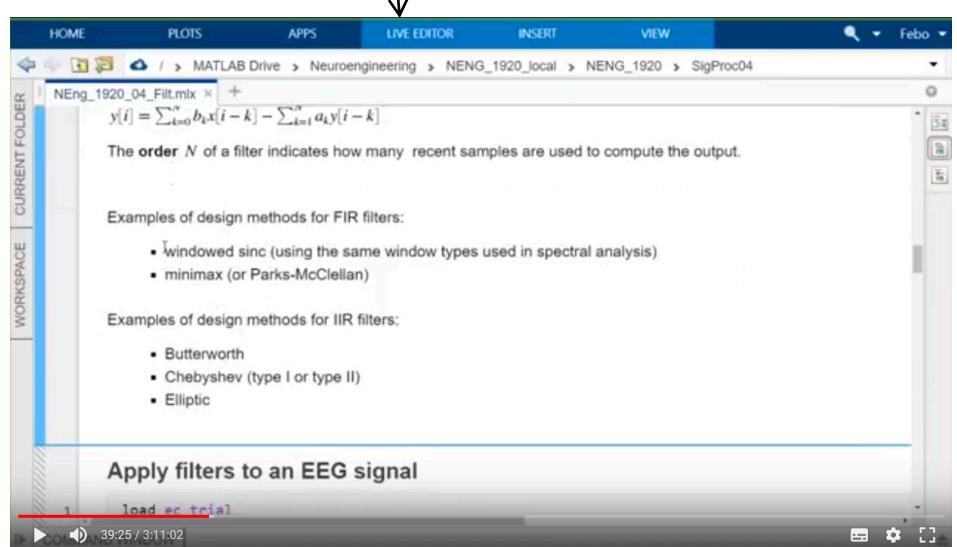
RESPONSE = "highpass" ↪ here there is also other options

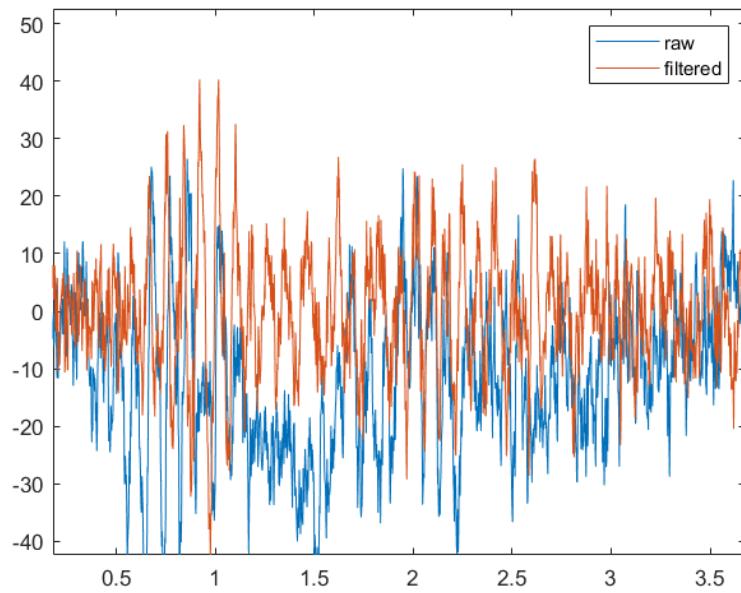
ORDER = 256;
FC_LOW = 1; % Hz
FC_HIGH = 30; % Hz
F_NOTCH = 50; % Hz
switch RESPONSE
    case "lowpass"
        filt = designfilt('lowpassiir', 'FilterOrder', ORDER, 'HalfPowerFrequency', FC_
                           FC_HIGH, 'SampleRate', fs);
    case "highpass"
        filt = designfilt('highpassiir', 'FilterOrder', ORDER, 'HalfPowerFrequency', FC_
                           FC_LOW, 'SampleRate', fs);
    case "bandpassiir"
        filt = designfilt('bandpassiir', 'FilterOrder', ORDER, 'HalfPowerFrequency1', FC_
                           FC_LOW, 'HalfPowerFrequency2', ...);
    case "bandpassfir"
        filt = designfilt('bandpassfir', 'FilterOrder', ORDER, 'CutoffFrequency1', FC_
                           FC_LOW, 'CutoffFrequency2', ...);
    case "notch"
        filt = designfilt('bandstopiir', 'FilterOrder', ORDER, 'HalfPowerFrequency1', F_NO_
                           F_NOTCH-.5, 'HalfPowerFrequency2', ...);
end

% fvtool(filt)

ec_trial_filtered = filter(filt, ec_trial);
    ↪ filter that you have just syntetized
figure(2)
clf
plot(time, [ec_trial, ec_trial_filtered])
legend ("raw", "filtered")

```

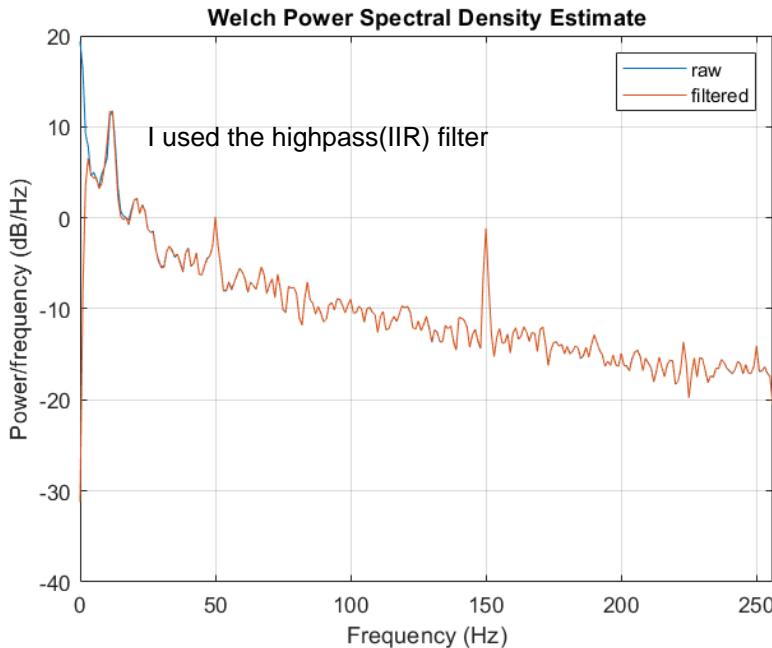




```
fft_lenght = 1 * fs; % determines the spectral resolution
overlap_length = 0.5 * fs;
window = hamming(fft_lenght); % determines the spectral leakage

figure(2)
clf
hold on
pwelch([ec_trial ec_trial_filtered],window,overlap_length,fft_lenght,fs);

hold off
legend(["raw", "filtered"])
```



```

load eeg.mat eeg ec_start_samp eo_start_samp
fs = 512;
num_samp_eeg = length(eeg);
time_eeg = (0:num_samp_eeg-1) / fs;
ec_trial = eeg - mean(eeg); % fix overall baseline

alpha_filt = designfilt('bandpassfir', 'FilterOrder', 256, 'CutoffFrequency1', 8, 'CutoffFrequency2', 12);
alpha = filtfilt(alpha_filt, eeg); % filtfilt introduces no delay

figure(3)
clf
plot(time_eeg, alpha)
for smp = eo_start_samp(:)
    xline(smp/fs, 'r-', "open", "LineWidth", 2);
end
for smp = ec_start_samp(:)
    xline(smp/fs, 'k--', "close", "LineWidth", 2);
end

```

