Signal Classifications and Properties*

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Abstract

Describes various classifications of signals.

1 Introduction

This module will begin our study of signals and systems by laying out some of the fundamentals of signal classification. It is essentially an introduction to the important definitions and properties that are fundamental to the discussion of signals and systems, with a brief discussion of each.

2 Classifications of Signals

2.1 Continuous-Time vs. Discrete-Time

As the names suggest, this classification is determined by whether or not the time axis is **discrete** (countable) or **continuous** (Figure 1). A continuous-time signal will contain a value for all real numbers along the time axis. In contrast to this, a discrete-time signal[1], often created by sampling a continuous signal, will only have values at equally spaced intervals along the time axis.

*Version 2.21: Jun 11, 2010 9:24 am GMT-5
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[1] "Discrete-Time Signals" <http://cnx.org/content/m0009/latest/>

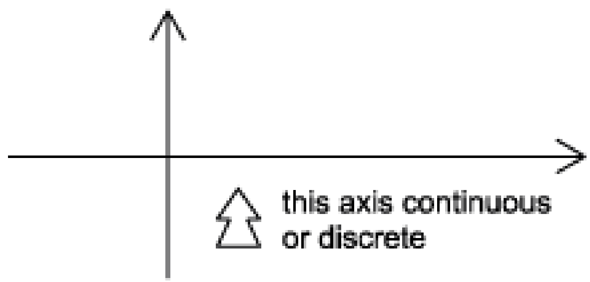


Figure 1

2.2 Analog vs. Digital

The difference between **analog** and **digital** is similar to the difference between continuous-time and discrete-time. However, in this case the difference involves the values of the function. Analog corresponds to a continuous set of possible function values, while digital corresponds to a discrete set of possible function values. An common example of a digital signal is a binary sequence, where the values of the function can only be one or zero.

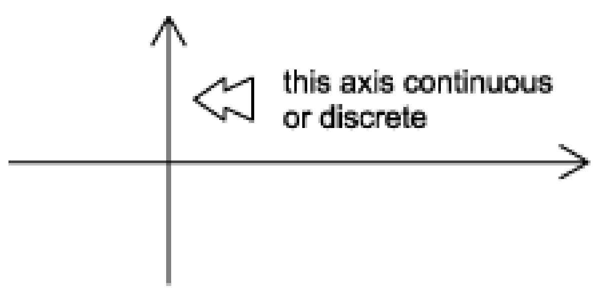


Figure 2

2.3 Periodic vs. Aperiodic

Periodic signals[2] repeat with some **period**T, while aperiodic, or nonperiodic, signals do not (Figure 3). We can define a periodic function through the following mathematical expression, where t can be any number and T is a positive constant:

$$f(t) = f(T+t) \tag{1}$$

[2]"Continuous Time Periodic Signals" <http://cnx.org/content/m10744/latest/>

The **fundamental period** of our function, $f(t)$, is the smallest value of T that the still allows (1) to be true.

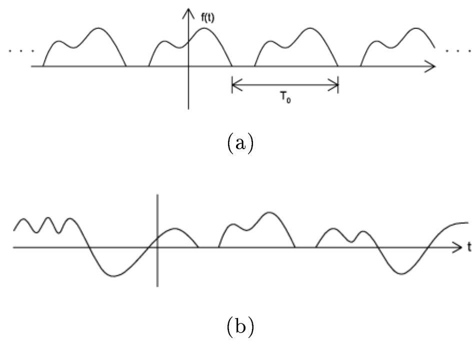


Figure 3: (a) A periodic signal with period T_0 (b) An aperiodic signal

2.4 Finite vs. Infinite Length

As the name implies, signals can be characterized as to whether they have a finite or infinite length set of values. Most finite length signals are used when dealing with discrete-time signals or a given sequence of values. Mathematically speaking, $f(t)$ is a **finite-length signal** if it is **nonzero** over a finite interval

$$t_1 < f(t) < t_2$$

where $t_1 > -\infty$ and $t_2 < \infty$. An example can be seen in Figure 4. Similarly, an **infinite-length signal**, $f(t)$, is defined as nonzero over all real numbers:

$$\infty \leq f(t) \leq -\infty$$

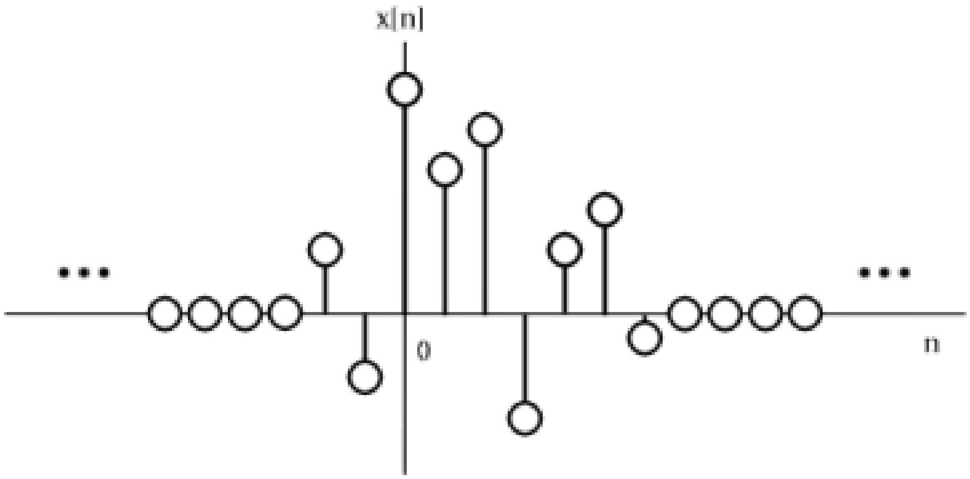
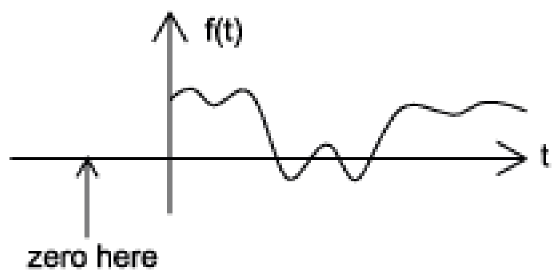


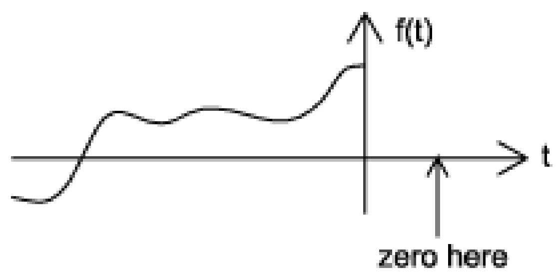
Figure 4: Finite-Length Signal. Note that it only has nonzero values on a set, finite interval.

2.5 Causal vs. Anticausal vs. Noncausal

Causal signals are signals that are zero for all negative time, while **anticausal** are signals that are zero for all positive time. **Noncausal** signals are signals that have nonzero values in both positive and negative time (Figure 5).



(a)



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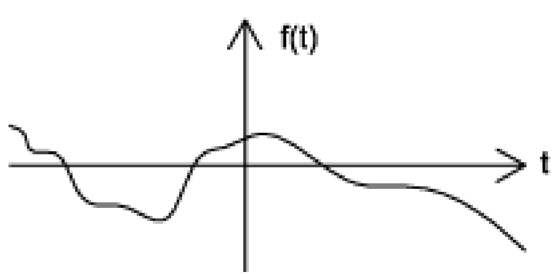
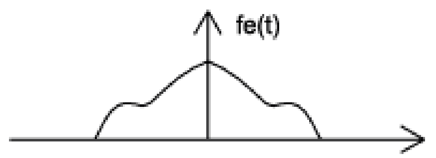


(c)

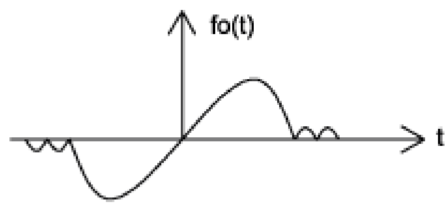
Figure 5: (a) A causal signal (b) An anticausal signal (c) A noncausal signal

2.6 Even vs. Odd

An **even signal** is any signal f such that $f(t) = f(-t)$. Even signals can be easily spotted as they are **symmetric** around the vertical axis. An **odd signal**, on the other hand, is a signal f such that $f(t) = -f(-t)$ (Figure 6).



(a)



(b)

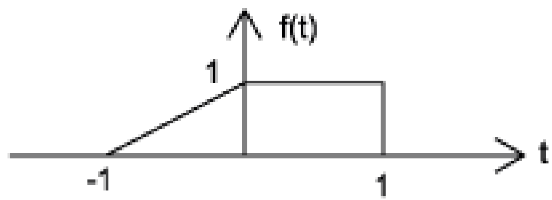
Figure 6: (a) An even signal (b) An odd signal

Using the definitions of even and odd signals, we can show that any signal can be written as a combination of an even and odd signal. That is, every signal has an odd-even decomposition. To demonstrate this, we have to look no further than a single equation.

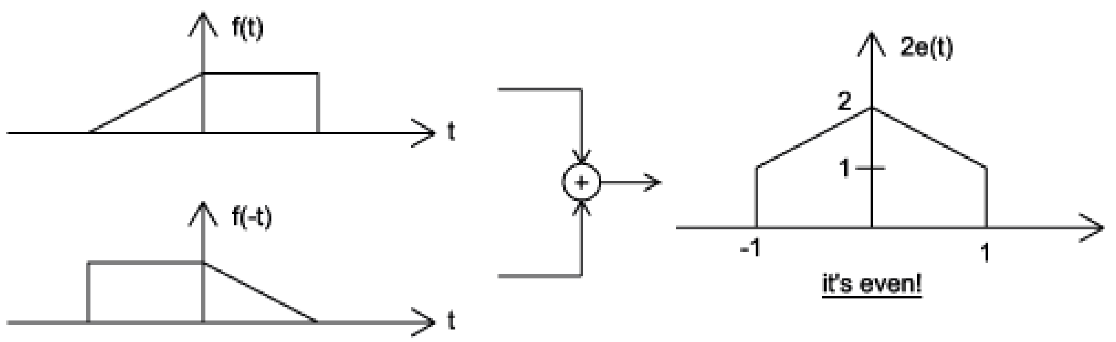
$$f(t) = \frac{1}{2}(f(t) + f(-t)) + \frac{1}{2}(f(t) - f(-t)) \tag{2}$$

By multiplying and adding this expression out, it can be shown to be true. Also, it can be shown that $f(t) + f(-t)$ fulfills the requirement of an even function, while $f(t) - f(-t)$ fulfills the requirement of an odd function (Figure 7).

Example 1



(a)



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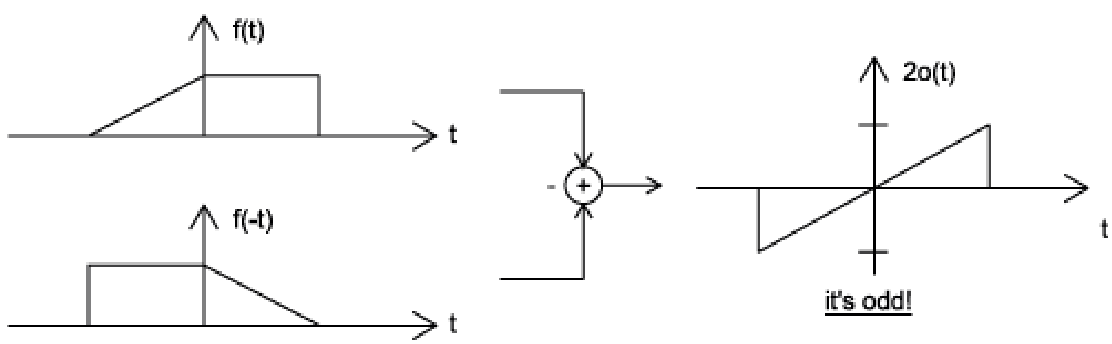
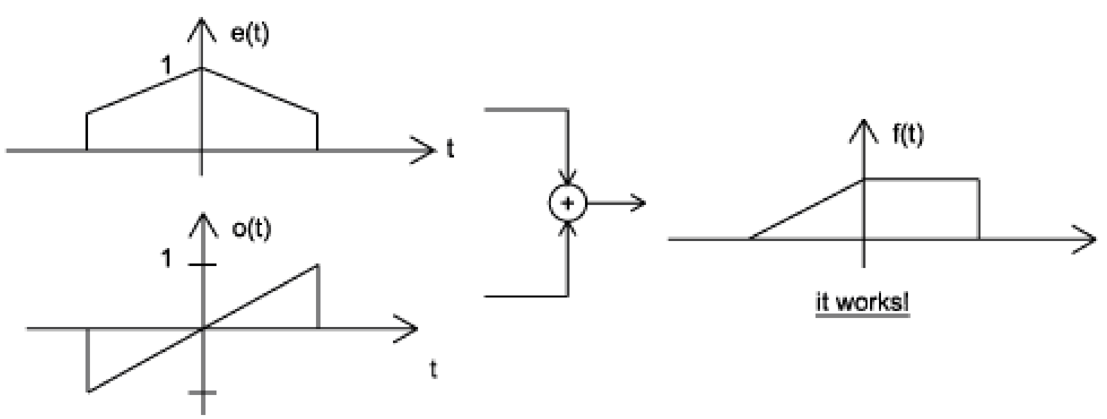


(c)



(d)

Figure 7: (a) The signal we will decompose using odd-even decomposition (b) Even part: $e(t) = \frac{1}{2}(f(t) + f(-t))$ (c) Odd part: $o(t) = \frac{1}{2}(f(t) - f(-t))$ (d) Check: $e(t) + o(t) = f(t)$

2.7 Deterministic vs. Random

A **deterministic signal** is a signal in which each value of the signal is fixed and can be determined by a mathematical expression, rule, or table. Because of this the future values of the signal can be calculated from past values with complete confidence. On the other hand, a **random signal**[3] has a lot of uncertainty about its behavior. The future values of a random signal cannot be accurately predicted and can usually only be guessed based on the averages[4] of sets of signals (Figure 8).

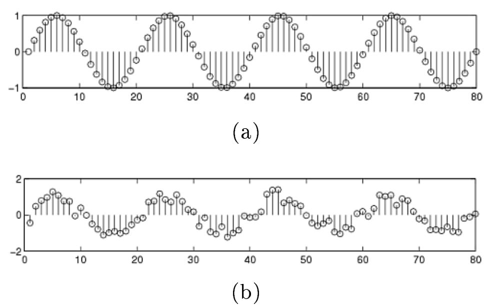


Figure 8: (a) Deterministic Signal (b) Random Signal

Example 2

Consider the signal defined for all real t described by

$$f(t) = \begin{cases} sin(2\pi t)/t & t \geq 1 \\ 0 & t < 1 \end{cases} \quad (3)$$

This signal is continuous time, analog, aperiodic, infinite length, causal, neither even nor odd, and, by definition, deterministic.

3 Signal Classifications Summary

This module describes just some of the many ways in which signals can be classified. They can be continuous time or discrete time, analog or digital, periodic or aperiodic, finite or infinite, and deterministic or random. We can also divide them based on their causality and symmetry properties. There are other ways to classify signals, such as boundedness, handedness, and continuity, that are not discussed here but will be described in subsequent modules.

[3]"Introduction to Random Signals and Processes" <http://cnx.org/content/m10649/latest/>
[4]"Random Processes: Mean and Variance" <http://cnx.org/content/m10656/latest/>