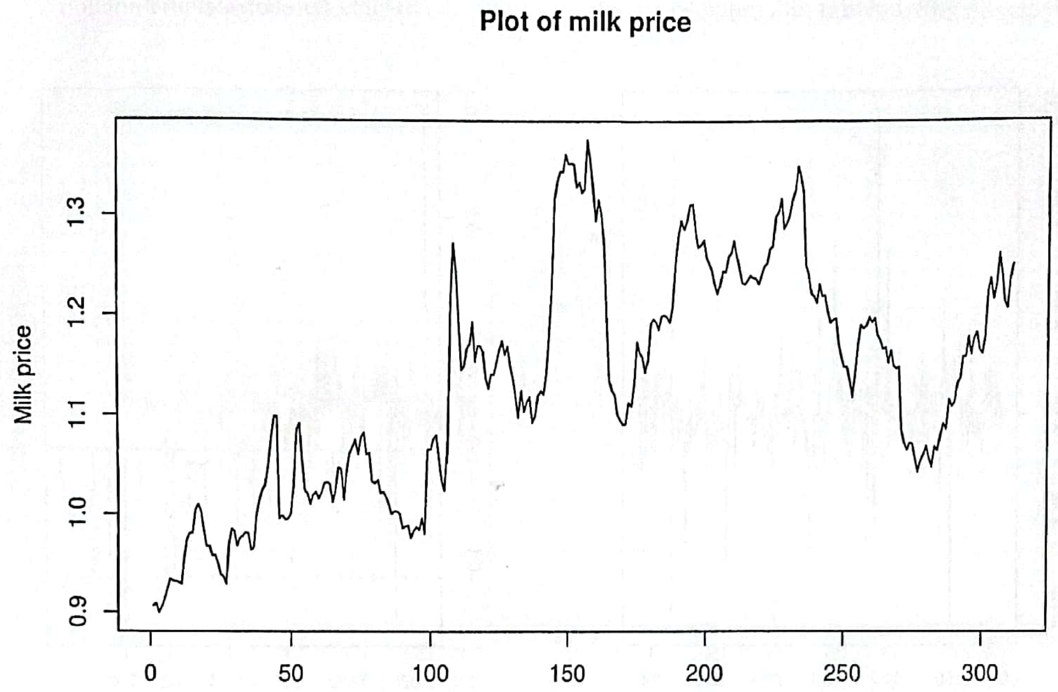

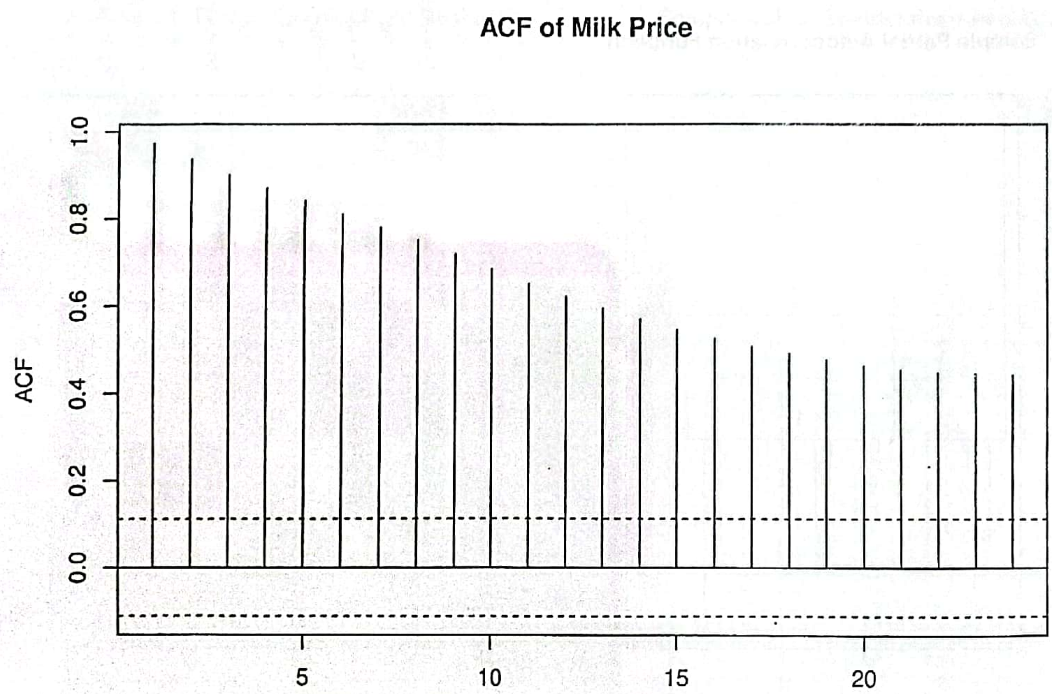

Figure 1: Milk prices and its ACF

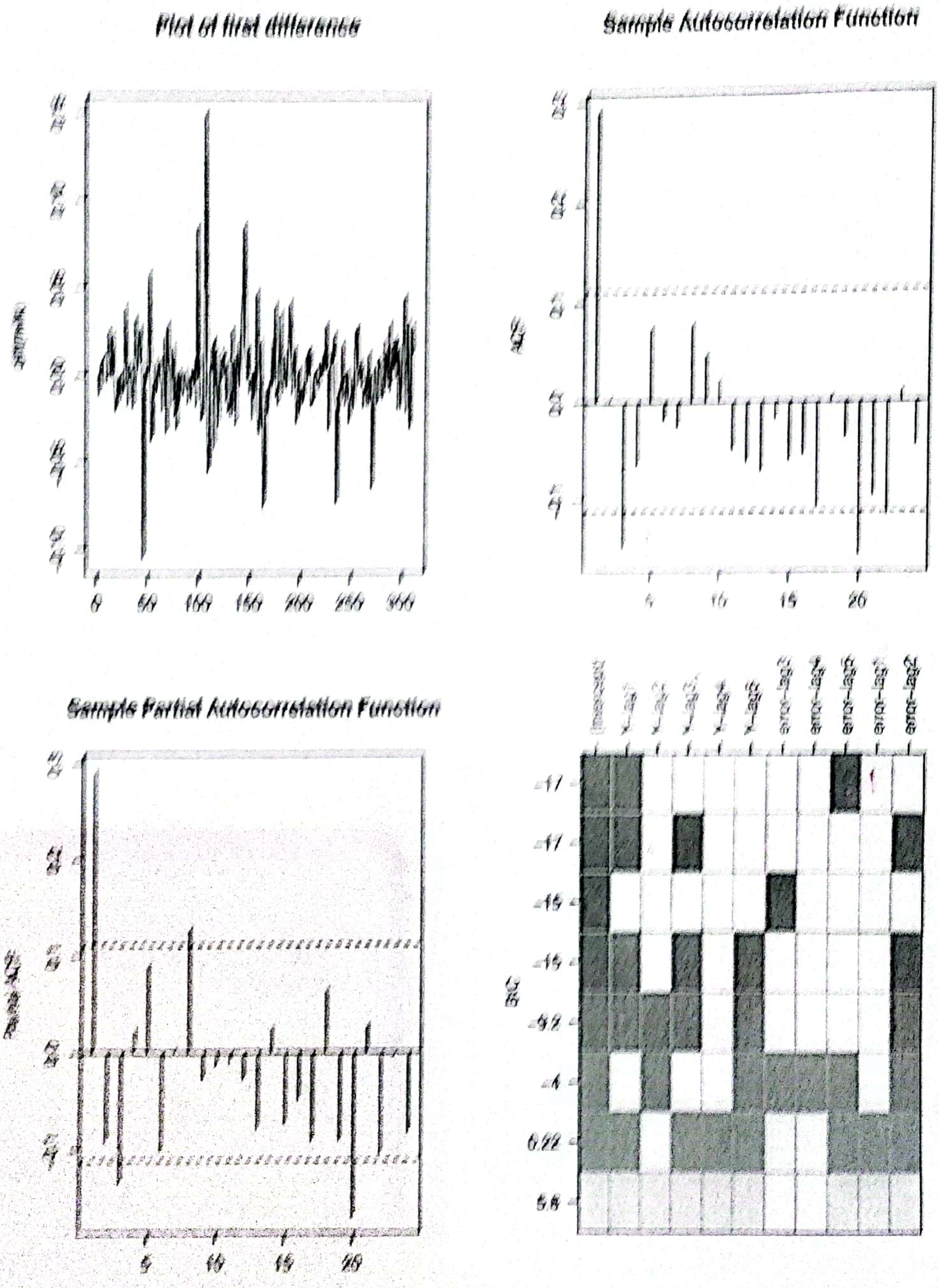

Figure 2: Plots for model specification

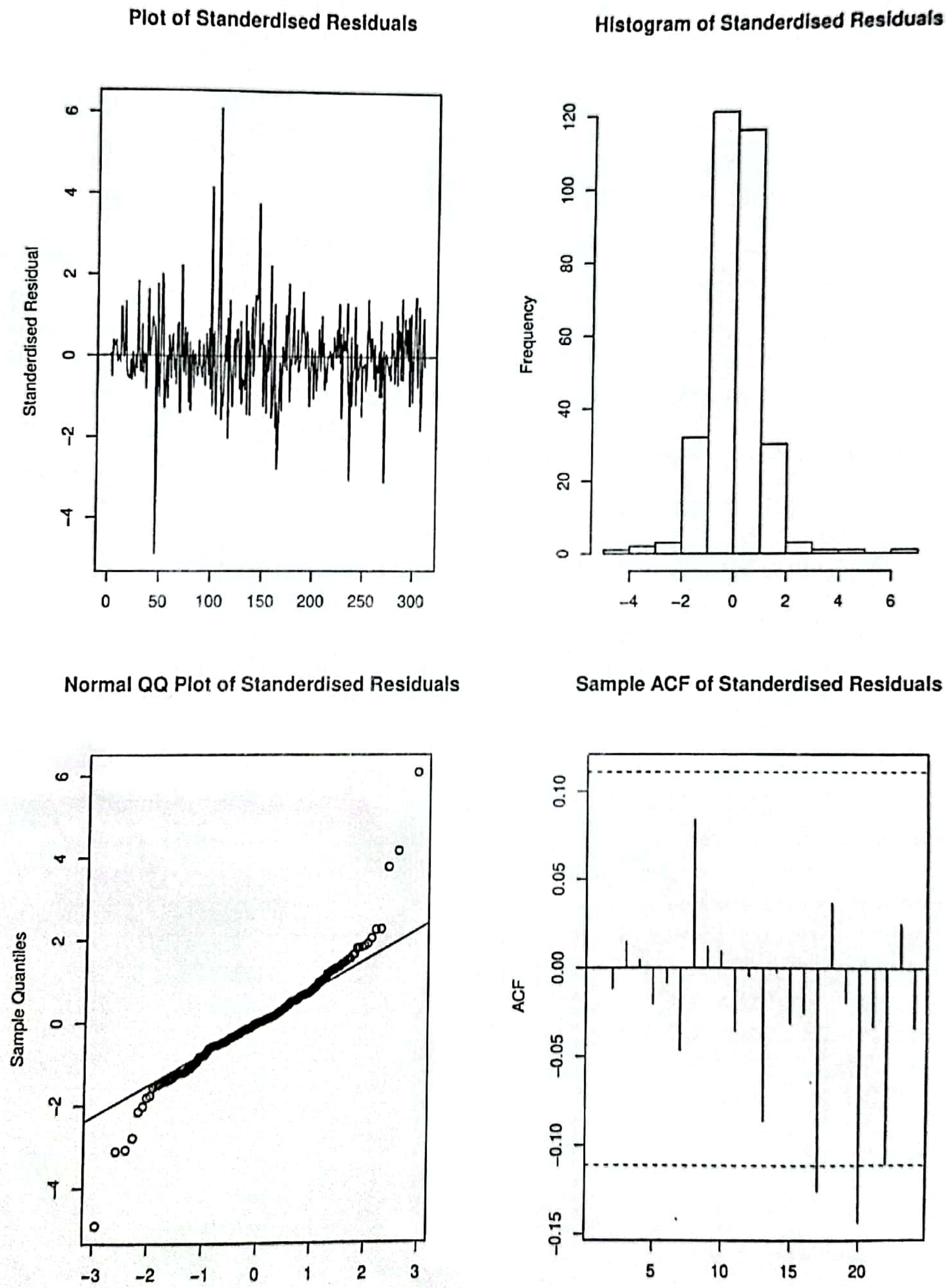

Figure 3: Plots for model diagnostics

PART 2: PROBLEMS

(a) **(5 points)** $\{e_t\}$ is a sequence of uncorrelated random variables having zero mean and variance σ_e^2

i. What are the requirements for a process to be weakly stationary?

ii. Write the following processes using the backshift operator notation then determine, with reasoning, which of them is second-order stationary and/or invertible and give the values of the parameters p, d, q of the correspondong ARIMA models:

A. $X_t = -\frac{1}{6}X_{t-1} + \frac{1}{6}X_{t-2} - \frac{1}{2}e_{t-1} + e_t$.

B. $X_t = \frac{16}{5}X_{t-1} - \frac{13}{5}X_{t-2} + \frac{2}{5}X_{t-3} - \frac{5}{2}e_{t-1} - e_{t-2} + e_t$.

iii. Consider the following process

$$X_t = \alpha X_{t-1} + e_t + \alpha e_{t-1}, \text{with } |\alpha| < 1.$$

Show that

$$var[X_t] = \frac{\sigma_e^2(1+3\alpha^2)}{1-\alpha^2}.$$

(b) **(5 points)** Let $\{a_t\}$ be a sequence of iid random variables, **distributed Uniformly** on [0, 1], and let $\{e_t\}$ be a sequence of independent Normal random variables with mean zero and variance one. Further, let the processes $\{a_t\}$ and $\{e_t\}$ be independent of each other and independent of past realisations of $\{X_t\}$. Define $\{X_t\}$ as a *random coefficient* AR(1) process:

$$X_t = a_t X_{t-1} + \sqrt{\frac{8}{9}}e_t$$

You may assume (without proof) that $\{X_t\}$ is second-order stationary.

i. Compute the autocovariance sequence of X_t.

ii. Compute the autocovariance sequence of the AR(1) process Y_t, defined by

$$Y_t = \frac{1}{2}Y_{t-1} + e_t.$$

iii. Compare (with discussion) the autocovariance functions of the processes X_t and Y_t.

(c) **(5 points)** We wish to model US produced milk prices. Our data set is the monthly milk price from 01/07/1997 to 01/05/2021 and our sample size is $N = 311$

i. Comment on the plots in Figure 1, and on why differencing has been carried out.

ii. From Figure 2 and by careful examination of the acf and pacf of the differenced data and BIC diagram, **state with reasons** which of the family of ARIMA models might be fitted to the data.

iii. In order to check the adequacy of the fitted model, the set of tests and diagrams have been produced (Figure 3). Comment on each of the results and discuss the adequacy of the model.

```
> shapiro.test(sres)
Shapiro-Wilk normality test
data:  sres
W = 0.92104, p-value = 9.565e-12

> runs(sres)
$`pvalue`
[1] 0.208
$observed.runs
[1] 168
$expected.runs
[1] 156.4212
```