F distribution (is a continuous distribution)

1. The F-dist. is continuous and skewed to the right
2. The F-dist has two degree of freedom
   i- df for the numerator
   ii- df for the denominator } df(8, 14) num, denom
3. The units (the value) of the F-dist are non-negative

ANOVA (Analysis of variance)

ANOVA is a procedure that is used to test the null hypothesis that the mean of three or more population are all equal.

Assumptions of One-way ANOVA

1. The population from which the samples are drawn are approximately normal distributed
2. The population from which the sample are drawn have the same variance (or standard deviation)
3. The samples drawn from different populations are random and independent

Test Statistics

$$F = \frac{\text{Variance between samples}}{\text{Variance within samples}}$$

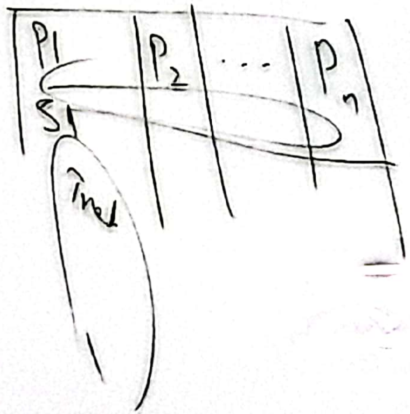
mple i

ll sample.

$H_0 : \mu_1 = \mu_2 = \mu_3 = \cdots = \mu_n$

H_1 : atleast two population mean are different

Assume that $\sigma_1^2 = \sigma_2^2 = \cdots = \sigma_n^2$



$$\frac{\text{Varian between Sample}}{\text{Varian within Sampl}}$$

Sum of square between sample

$$SSB = \left(\frac{T_1^2}{n_1} + \frac{T_2^2}{n_2} + \cdots\right) - \frac{\left(\sum x\right)^2}{n}$$

where x : data value

k : # of sample

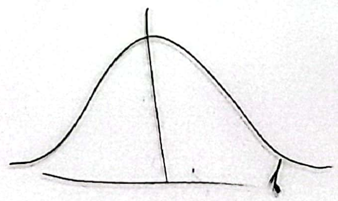
n_i : size of sample i

T_i = the sum of values in sample i

n := the no. of values in all sample.

Sum of square within sample

$$SSW = \sum x^2 - \left(\frac{T_1^2}{n_1} + \frac{T_2^2}{n_2} + \cdots\right)$$



Total sum of square $SST = SSB + SSW$

Sum of square between sample

$$SSB = \left(\frac{T_1^2}{n_1} + \frac{T_2^2}{n_2} + \cdots\right) - \frac{\left(\sum x\right)^2}{n}$$

Variance between sample

$$MSB = \frac{SSB}{k-1}$$

where
x : data value
k : # of sample
n_i : size of sample i
T_i = the sum of values of sample i
n = the no of values in all sample

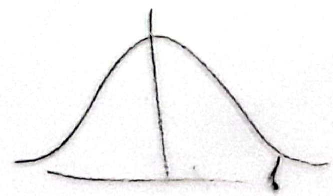
Sum of square within sample

$$SSW = \sum x^2 - \left(\frac{T_1^2}{n_1} + \frac{T_2^2}{n_2} + \cdots\right)$$

Variance within sample

$$MSW = \frac{SSW}{n-k}$$

$$F = \frac{MSB}{MSW}$$



Total sum of square $SST = SSB + SSW$

Example Find F-value

Sample 1	Sample 2	Sample 3
48	55	84
73	85	68
51	70	95
65	69	74
87		67
$n_1 = 5$	$n_2 = 5$	$n_3 = 5$
$T_1 = 324$	$T_2 = 369$	$T_3 = 388$

$$\sum x = T_1 + T_2 + T_3 = 1081$$

$$\sum x^2 = (48)^2 + \cdots + (67)^2 = 80{,}709$$

$$K = 3, \quad n = 15$$

■ Since.

Sum of square between sample

$$SSB = \left(\frac{T_1^2}{n_1} + \frac{T_2^2}{n_2} + \cdots\right) - \frac{\left(\sum x\right)^2}{n}$$

$$SSB = \left(\frac{(324)^2}{5} + \frac{(369)^2}{5} + \frac{(388)^2}{5}\right) - \frac{(1081)^2}{15}$$

$$= 432.1333$$

Sum of square within sample

$$SSW = \sum x^2 - \left(\frac{T_1^2}{n_1} + \frac{T_2^2}{n_2} + \cdots\right)$$

$$= 80709 - \left(\frac{(324)^2}{5} + \frac{(369)^2}{5} + \frac{(388)^2}{5}\right)$$

$$= 2372.8000$$

Total sum of square $SST = SSB + SSW$

$$= 2804.9333$$

Varience between sample

$$MSB = \frac{SSB}{k-1} = \frac{432.1333}{3-1}$$

$$= 216.0667$$

Varience within sample

$$MSW = \frac{SSW}{n-k} = \frac{2372.8}{15-3}$$

$$= 197.7333$$

$$F = \frac{MSB}{MSW}$$

$$= \frac{216.0667}{197.7333}$$

$$= 1.09$$

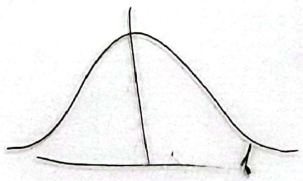
where x : data value

k : # of sample

n_i : size of sample i

T_i = the sum of value of sample i

n := the no. of value in all sample



ANOVA Table

Source of Variation	Degree of freedom	Sum of square	Mean square	Value of Test statistic
Between	$k-1$	SSB	MSB	$F = \dfrac{MSB}{MSW}$
Within	$n-k$	SSW	MSW	
Total	$n-1$	SST		

Sample 1	Sample 2	Sample 3
48	55	84
73	85	68
51	70	95
65	69	74
87	90	67
$n_1 = 5$	$n_2 = 5$	$n_3 = 5$
$T_1 = 324$	$T_2 = 369$	$T_3 = 388$

$\sum x = T_1 + T_2 + T_3 = 1081$

$\sum x^2 = (48)^2 + \dots + (67)^2 = 80{,}709$

$K = 3, \quad n = 15$

ANOVA Table

Source of variation	Degrees of freedom	Sum of squares	Mean square	Value of Test statistic
Between	$k-1$	SSB	MSB	$F = \frac{MSB}{MSW}$
Within	$n-k$	SSW	MSW	
Total	$n-1$	SST		

of sample i

in all sample

■ Since.

Sum of square between sample

$$SSB = \left(\frac{T_1^2}{n_1} + \frac{T_2^2}{n_2} + \cdots\right) - \frac{\left(\sum x\right)^2}{n}$$

$$SSB = \left(\frac{(324)^2}{5} + \frac{(369)^2}{5} + \frac{(388)^2}{5}\right) - \frac{(1081)^2}{15}$$

$$= 432.1333$$

Sum of square within sample

$$SSW = \sum x^2 - \left(\frac{T_1^2}{n_1} + \frac{T_2^2}{n_2} + \cdots\right)$$

$$= 80709 - \left(\frac{(324)^2}{5} + \frac{(369)^2}{5} + \frac{(388)^2}{5}\right)$$

$$= 2372.8000$$

Total sum of square $SST = SSB + SSW$

$$= 2804.9333$$

Variance between sample

$$MSB = \frac{SSB}{k-1} = \frac{432.1333}{3-1}$$

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Variance within sample

$$MSW = \frac{SSW}{n-k} = \frac{2372.8}{15-3}$$

$$= 197.7333$$

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$$= \frac{216.0667}{197.7333}$$

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where x : data value

k : # of sample

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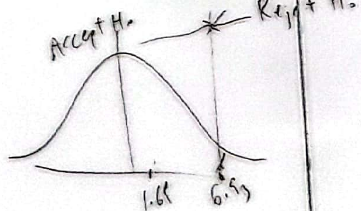

Example Find F-val...

$H_0 : \mu_1 = \mu_2 = \mu_3$

H_1 : at least two mean are different

Sample 1	Sample...
48	55
73	85
51	7...
65	6...
87	9...

$n_1 = 5$ $\quad n_2 = \ldots$

$T_1 = 324$ $\quad T_2 = \ldots$

$\sum x = T_1 + T_2 + T_3 = 1081$

$\sum x^2 = (48)^2 + \cdots + (67)^2 = 8\ldots$

$k = 3, \quad n = 15$

$\alpha = 0.01$

$H_0 : \mu_1 = \mu_2 = \mu_3$

H_1 : atleast two mean are different

Sample 1	Sample 2	Sample 3
48	55	84
73	85	68
51	70	95
65	69	74
87	90	67
$n_1 = 5$	$n_2 = 5$	$n_3 = 5$

$T_1 = 324$ $\quad T_2 = 369$ $\quad T_3 = 388$

$\sum x = T_1 + T_2 + T_3 = 1081$

$\sum x^2 = (48)^2 + \dots + (67)^2 = 80,709$

$K = 3, \quad n = 15$

ANOVA Table

Source of Variation	Degrees of freedom	Sum of squares	Mean square	Value of Test statistic
Between	$k-1$	SSB	MSB	$F = \frac{MSB}{MSW}$
Within	$n-k$	SSW	MSW	
Total	$n-1$	SST		

$df(k-1, n-k)$

$\alpha = 0.01$

$\alpha = 0.01, \; df(2, 12)$

$C.V = 6.93$

t H_0

F-value , $\alpha = 0.01$

$H_0 : \mu_1 = \mu_2 = \mu_3$

H_1 : atleast two mean are different

Sample 1	Sample 2	Sample 3
48	55	84
73	85	68
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$n_1 = 5$	$n_2 = 5$	$n_3 = 5$
$T_1 = 324$	$T_2 = 369$	$T_3 = 388$

$\sum x = T_1 + T_2 + T_3 = 1081$

$\sum x^2 = (48)^2 + \dots + (67)^2 = 80{,}709$

$K = 3, \quad n = 15$

+ H_0

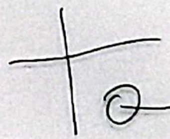
ANOVA Table

Source of Variation	Degree of freedom	Sum of square	Mean square	Value of Test statistic
Between	$k-1$	SSB	MSB	$F = \frac{MSB}{MSW}$
Within	$n-k$	SSW	MSW	
Total	$n-1$	SST		

$df(k-1, n-k)$

$\alpha = 0.01$

$\alpha = 0.01$, $df(2, 12)$



C.V $= 6.93$