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LA Assignment 1

Ex 1.1 (10, 20)

10.

$$x + 2y - 2z = 3$$
$$3x - y + z = 1$$
$$-x + 5y - 5z = 5$$

a) $\Rightarrow (5/7) + 2(8/7) - 2(1) = 3$

$5/7 + 16/7 - 2 = 3$

$3 - 2 = 3$

$1 \neq 3$ ×

$\therefore$ a is not a solution $(5/7, 8/7, 1)$

b) $\Rightarrow (5/7) + 2(8/7) - 2(0) = 3$

$5/7 + 16/7 - 0 = 3$

$3 - 0 = 3$

$3 = 3$ ✓

$\Rightarrow 3(5/7) - (8/7) + 0 = 1$

$15/7 - 8/7 + 0 = 1$

$1 + 0 = 1$

$1 = 1$ ✓

$\Rightarrow -(5/7) + 5(8/7) - 5(0) = 5$

$-5/7 + 40/7 - 0 = 5$

$5 - 0 = 5$

$5 = 5$ ✓

$\therefore$ b is a solution $(5/7, 8/7, 0)$

c) $\Rightarrow (5) + 2(8) - 2(1) = 3$

$5 + 16 - 2 = 3$

$21 - 2 = 3$

$19 \neq 3$

$\therefore$ c is not a solution $(5, 8, 1)$

d) $\Rightarrow (5/7) + 2(10/7) - 2(2/7) = 3$

$5/7 + 20/7 - 4/7 = 3$

$3 = 3$ ✓

$\Rightarrow 3(5/7) - (10/7) + (2/7) = 1$

$15/7 - 10/7 + 2/7 = 1$

$1 = 1$ ✓

$\Rightarrow -(5/7) + 5(10/7) - 5(2/7) = 5$

$-5/7 + 50/7 - 10/7 = 5$

$5 = 5$ ✓

$\therefore$ d is a solution

$(5/7, 10/7, 2/7)$

e) $\Rightarrow (5/7) + 2(22/7) - 2(2) = 3$

$5/7 + 44/7 - 4 = 3$

$7 - 4 = 3$

$3 = 3$ ✓

$\Rightarrow 3(5/7) - (10/7) + (2/7) = 1$

$15/7 - 10/7 + 2/7 = 1$

$1 = 1$ ✓

$\Rightarrow -(5/7) + 5(22/7) - 5(2) = 5$

$-5/7 + 110/7 - 10 = 5$

$15 - 10 = 5$

$5 = 5$ ✓

$\therefore$ e is a solution

$(5/7, 22/7, 2)$

$\Rightarrow$ b), d), e) are solutions

a), c) are not solutions

20 a)

$2R_1 + R_2 \quad \begin{bmatrix} 3 & -4 & k \\ 0 & 0 & 2k+5 \end{bmatrix}$

$3x - 4y = k$

$0 = 2k + 5$

if $k = -5/2$, $0 = 0$ makes system consistent

if $k \neq -5/2$ then system becomes inconsistent

b) $R_1 + R_2$ $\begin{bmatrix} k & 1 & -2 \\ 4+k & 0 & 0 \end{bmatrix}$

constant

$$kx + y = -2$$
$$(4+k)x = 0$$

constant is zero so system is consistent for all values of k

Ex. 1.2 (8, 21, 26)

8.
$$-2b + 3c = 1$$
$$3a + 6b - 3c = -2$$
$$6a + 6b + 3c = 5$$

$$\begin{bmatrix} 0 & -2 & 3 & 1 \\ 3 & 6 & -3 & -2 \\ 6 & 6 & 3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 6 & -3 & -2 \\ 0 & -2 & 3 & 1 \\ 6 & 6 & 3 & 5 \end{bmatrix} \quad R_1 \Leftrightarrow R_2$$

$$\begin{bmatrix} 1 & 2 & -1 & -2/3 \\ 0 & -2 & 3 & 1 \\ 6 & 6 & 3 & 5 \end{bmatrix} \quad 1/3 * R_1$$

$$\begin{bmatrix} 1 & 2 & -1 & -2/3 \\ 0 & -2 & 3 & 1 \\ 0 & -6 & 9 & 9 \end{bmatrix} \quad -6R_1 + R_3$$

$$\begin{bmatrix} 1 & 2 & -1 & -2/3 \\ 0 & 1 & -3/2 & -1/2 \\ 0 & -6 & 9 & 9 \end{bmatrix} \quad -1/2 \times R_2$$

$$\begin{bmatrix} 1 & 2 & -1 & -2/3 \\ 0 & 1 & -3/2 & -1/2 \\ 0 & \overset{0}{\cancel{-6}} & \overset{0}{\cancel{9}} & \overset{6}{\cancel{9}} \end{bmatrix} \quad 6R_2 + R_3$$

$$\begin{bmatrix} 1 & 2 & -1 & -2/3 \\ 0 & 1 & -3/2 & -1/3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad 1/6\, R_3$$

$\therefore 0 = \underline{1}$ inconsistent system

21. $$\begin{bmatrix} 2 & -1 & 3 & 4 & 9 \\ 1 & 0 & -2 & 7 & 11 \\ 3 & -3 & 1 & 5 & 8 \\ 2 & 1 & 4 & 4 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 2 & -1 & 3 & 4 & 9 \\ 3 & -3 & 1 & 5 & 8 \\ 2 & 1 & 4 & 4 & 10 \end{bmatrix} \quad R_1 \Leftrightarrow R_2$$

$\cancel{-21}$

$$\begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & -1 & 3 & 4 & 9 \\ 3 & -3 & 1 & 5 & 8 \\ 2 & 1 & 4 & 4 & 10 \end{bmatrix} \quad -2R_1 + R_2$$

$$\begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & -1 & 7 & -10 & -13 \\ 0 & -3 & 7 & -16 & -25 \\ 0 & 1 & 8 & -10 & -12 \end{bmatrix} \quad \begin{matrix} \\ \\ -3R_1 + R_3 \\ -2R_1 + R_4 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & -7 & 10 & 13 \\ 0 & -3 & 7 & -16 & -25 \\ 0 & 1 & 8 & -10 & -12 \end{bmatrix} \quad -1R_2$$

$$\begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & -7 & 10 & 13 \\ 0 & 0 & -14 & 14 & 14 \\ 0 & 0 & 15 & -20 & -25 \end{bmatrix} \quad \begin{matrix} \\ \\ -3R_2 + R_3 \\ -1R_2 + R_4 \end{matrix}$$

$$\begin{array}{c} \\ \\ -\frac{1}{14}R_3 \\ \\ \end{array}\begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & -7 & 10 & 13 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 15 & -20 & -25 \end{bmatrix}$$

$$\begin{array}{c} \\ \\ \\ -15R_3+R_4 \end{array}\begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & -7 & 10 & 13 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & -5 & -10 \end{bmatrix}$$

$$\begin{array}{c} \\ \\ \\ -\frac{1}{5}R_4 \end{array}\begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & -7 & 10 & 13 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & \cancel{0}1 & +2 \end{bmatrix}$$

$$\begin{array}{c} -7R_4+R_1 \\ -10R_4+R_2 \\ R_4+R_3 \\ \\ \end{array}\begin{bmatrix} 1 & 0 & -2 & 0 & -3 \\ 0 & 1 & -7 & 0 & -7 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{array}{c} 2R_3+R_1 \\ 7R_3+R_2 \\ \\ \\ \end{array}\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$I_1 = -1$, $I_2 = 0$, $I_3 = 1$, $I_4 = 2$

26. $\left[\begin{array}{cccc} 1 & 2 & 1 & 2 \\ 2 & -2 & 3 & 1 \\ 1 & 2 & -(a^2-3) & a \end{array}\right]$

$-2R_1+R_2$
$-1R_1+R_3$
$\left[\begin{array}{cccc} 1 & 2 & 1 & 2 \\ 0 & -6 & 1 & -3 \\ 0 & 0 & -a^2+2 & a-2 \end{array}\right]$

$-\frac{1}{6}R_2$ $\left[\begin{array}{cccc} 1 & 2 & 1 & 2 \\ 0 & 1 & -1/6 & 1/2 \\ 0 & 0 \cancel{a+2} & -a^2+2 & a-2 \end{array}\right]$

when
$a=\sqrt{2}$ / $a=-\sqrt{2}$ ∴ no solution

* for remaining values, ∴ one solution

no values in which there are infinite solutions

Ex 1.5 (6, 12)

6. a)
EA
$-6R_1$ $\left[\begin{array}{cccc} 6 & 12 & -30 & 6 \\ 3 & -6 & -6 & 6 \end{array}\right]$

b) EA

$$\begin{array}{c c} -4R_1 + R_2 \to & \left[\begin{array}{ccc|cc} 2 & -1 & 0 & -4 & -4 \\ -7 & 1 & -1 & 21 & 19 \\ 2 & 0 & 1 & 3 & -1 \end{array}\right] \end{array}$$

c) EA

$$\begin{array}{c c} 5R_2 & \begin{bmatrix} 1 & 4 \\ 10 & 25 \\ 3 & 6 \end{bmatrix} \end{array}$$

12.a) $\left[\begin{array}{ccc|ccc} 1/5 & 1/5 & -2/5 & 1 & 0 & 0 \\ 1/5 & 1/5 & 1/10 & 0 & 1 & 0 \\ 1/5 & -4/5 & 1/10 & 0 & 0 & 1 \end{array}\right]$

$$\begin{array}{c c} \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 1 & 1 & 1/2 & 0 & 5\cancel{0} & 0 \\ 1 & -4 & 1/2 & 0 & 0 & 5 \end{array}\right] & \begin{array}{c} 5R_1 \\ 5R_2 \\ 5R_3 \end{array} \end{array}$$

$$\begin{array}{c c} \begin{array}{c} \\ -1R_1 + R_2 \\ -1R_1 + R_3 \end{array} & \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & 0 & 5/2 & -5 & 5 & 0 \\ 0 & -5 & 5/2 & -5 & 0 & 5 \end{array}\right] \end{array}$$

$$\begin{array}{c c} R_2 \Leftrightarrow R_3 & \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & -5 & 5/2 & -5 & 0 & 5 \\ 0 & 0 & 5/2 & -5 & 5 & 0 \end{array}\right] \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & 1 & -1/2 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 & 2 & 0 \end{array}\right] \begin{array}{l} \\ -1/5\, R_2 \\ 2/5\, R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 4 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -2 & 2 & 0 \end{array}\right] \begin{array}{l} 2R_3 + R_1 \\ \frac{1}{3}R_3 + R_2 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & \overset{3}{\cancel{4}} & \overset{1}{\cancel{0}} \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -2 & 2 & 0 \end{array}\right] \begin{array}{l} -1R_2 + R_1 \\ \\ \\ \end{array}$$

inverse :
$$\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & -1 \\ -2 & 2 & 0 \end{bmatrix}$$

b)
$$\left[\begin{array}{ccc|ccc} 1/5 & 1/5 & -2/5 & 1 & 0 & 0 \\ 2/5 & -3/5 & -3/10 & 0 & 1 & 0 \\ 1/5 & -4/5 & 1/10 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 2 & -3 & -3/2 & 0 & 5 & 0 \\ 1 & -4 & 1/2 & 0 & 0 & 5 \end{array}\right] \begin{array}{l} 5R_1 \\ 5R_2 \\ 5R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & -5 & 5/2 & -10 & 5 & 0 \\ 0 & -5 & 5/2 & -5 & 0 & 5 \end{array}\right] \begin{array}{l} \\ -2R_1 + R_2 \\ -1R_1 + R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & -5 & 5/2 & -10 & 5 & 0 \\ 0 & 0 & 0 & 5 & -5 & 5 \end{array}\right] \quad -1R_2+R_3$$

$\therefore$ row of zeroes

matrix is not invertible

Ex: 1.6 (5, 10, 19)

5. $Ax = b$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -4 \\ -4 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & -4 & 0 & 1 & 0 \\ -4 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -5 & -1 & 1 & 0 \\ 0 & 5 & 5 & 4 & 0 & 1 \end{array}\right] \quad \begin{array}{l} -1R_1+R_2 \\ 4R_1+R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 5 & 5 & 4 & 0 & 1 \\ 0 & 0 & -5 & -1 & 1 & 0 \end{array}\right] \quad R_2 \Leftrightarrow R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 4/5 & 0 & 1/5 \\ 0 & 0 & 1 & 1/5 & -1/5 & 0 \end{array}\right] \begin{array}{l} \\ \frac{1}{5}R_2 \\ -\frac{1}{5}R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 4/5 & 1/5 & 0 \\ 0 & 1 & 0 & 3/5 & 1/5 & 1/5 \\ 0 & 0 & 1 & 1/5 & -1/5 & 0 \end{array}\right] \begin{array}{l} -1R_3 + R_1 \\ -1R_3 + R_2 \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/5 & 0 & -1/5 \\ 0 & 1 & 0 & 3/5 & 1/5 & 1/5 \\ 0 & 0 & 1 & 1/5 & -1/5 & 0 \end{array}\right] \begin{array}{l} -1R_2 + R_1 \\ \\ \end{array}$$

$$A^{-1} = \begin{bmatrix} 1/5 & 0 & -1/5 \\ 3/5 & 1/5 & 1/5 \\ 1/5 & -1/5 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/5 & 0 & -1/5 \\ 3/5 & 1/5 & 1/5 \\ 1/5 & -1/5 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 10 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}$$

$x = 1$, $y = 5$, $z = -1$

10.

$$\left[\begin{array}{ccc|c|c} -1 & 4 & 1 & 0 & -3 \\ 1 & 9 & -2 & 1 & 4 \\ 6 & 4 & -8 & 0 & -5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c|c} 1 & -4 & -1 & 0 & 3 \\ 1 & 9 & -2 & 1 & 4 \\ 6 & 4 & -8 & 0 & -5 \end{array}\right] \quad -1R_1$$

$$\left[\begin{array}{ccc|c|c} 1 & -4 & -1 & 0 & 3 \\ 0 & 13 & -1 & 1 & 1 \\ 0 & 28 & -2 & 0 & -23 \end{array}\right] \quad \begin{array}{l} -1R_1 + R_2 \\ -6R_1 + R_3 \end{array}$$

$$\left[\begin{array}{ccc|c|c} 1 & -4 & -1 & 0 & 3 \\ 0 & 1 & -1/13 & 1/13 & 1/13 \\ 0 & 28 & -2 & 0 & -23 \end{array}\right] \quad +1/13\, R_2$$

$$\left[\begin{array}{ccc|c|c} 1 & -4 & -1 & 0 & 3 \\ 0 & 1 & -1/13 & 1/13 & 1/13 \\ 0 & 0 & 2/13 & \cancel{0}\ \frac{-28}{13} & -\cancel{23}\ \frac{327}{13} \end{array}\right] \quad -28R_2 + R_3$$

$$\left[\begin{array}{ccc|c|c} 1 & -4 & -1 & 0 & 3 \\ 0 & 1 & -1/13 & 1/13 & 1/3 \\ 0 & 0 & \cancel{2/13}\ 1 & -\cancel{28/13}\ 14 & -\frac{327}{2} \end{array}\right] \quad 13/2\, R_3$$

$$\left[\begin{array}{ccc|c|c} 1 & -4 & 0 & -14 & -321/2 \\ 0 & 1 & 0 & -1 & -25/2 \\ 0 & 0 & 1 & -14 & -327/2 \end{array}\right] \quad \begin{array}{l} R_3 + R_1 \\ \frac{1}{13}R3 + R_2 \end{array}$$

$$\left[\begin{array}{ccc|c|c} 1 & 0 & 0 & -18 & -421/2 \\ 0 & 1 & 0 & -1 & -25/2 \\ 0 & 0 & 1 & -14 & -327/2 \end{array}\right] \quad 4R_2 + R_1$$

i) $x_1 = -18$, $x_2 = -1$, $x_3 = -14$

ii) $x_1 = \frac{-421}{2}$, $x_2 = \frac{-25}{2}$, $x_3 = \frac{-327}{2}$

19.

$$X = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$-2R_1 + R_2 \quad \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 5 & -2 & -2 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & -2 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \quad -2R_3 + R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & -2 \\ 0 & 0 & -1 & 4 & -2 & 5 \end{array}\right] \quad -2R_2 + R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & -2 \\ 0 & 0 & 1 & -4 & +2 & -5 \end{array}\right] \quad -1R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 5 & -2 & 5 \\ 0 & 1 & 0 & -2 & 1 & -2 \\ 0 & 0 & 1 & -4 & 2 & -5 \end{array}\right] \quad -1R_3 + R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1 & 3 \\ 0 & 1 & 0 & -2 & 1 & -2 \\ 0 & 0 & 1 & -4 & 2 & -5 \end{array}\right] \quad R_2 + R_3$$

$$X = \begin{bmatrix} 3 & -1 & 3 \\ -2 & 1 & -2 \\ -4 & 2 & -5 \end{bmatrix} \begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 12 & -3 & 27 & 26 \\ -6 & -8 & 1 & -18 & -17 \\ -15 & -21 & 9 & -38 & -35 \end{bmatrix}$$

Ex 1.8 (17, 20, 24, 31, 35, 38, 40)

17. a) $T(x_1, x_2) = \begin{bmatrix} -x_1 + x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$T(x) = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

~~$T(+1, 4, 4) = (5,$~~

$T(-1, 4) = (1 + 4, 4) = (5, 4)$

b) $T(x_1, x_2, x_3) = \begin{bmatrix} 2x_1 - x_2 + x_3 \\ x_2 + x_3 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$T(x) = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$

~~20.~~ $T(2, 1, -3) = (4 - 1 - 3, 1 - 3, 0) = (0, -2, 0)$

20 a) $T_A(x) = Ax = \begin{bmatrix} -2 & 1 & 4 \\ 3 & 5 & 7 \\ 6 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$= \begin{bmatrix} -2x_1 + x_2 + 4x_3 \\ 3x_1 + 5x_2 + 7x_3 \\ 6x_1 - x_3 \end{bmatrix}$

b) $T_A(x) = \begin{bmatrix} -1 & 1 \\ 2 & 4 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_1 + x_2 \\ 2x_1 + 4x_2 \\ 7x_1 + 8x_2 \end{bmatrix}$

24. a) $T(kx, ky) = (kx, ky+1)$ does not generally equal $kT(x,y) = k(x, y+1) = (kx, ky+k)$

b) $T(kx_1, kx_2, kx_3) = (kx_1, kx_2, \sqrt{kx_3})$ does not generally equal $kT(x_1, x_2, x_3) = k(x_1, x_2, \sqrt{x_3})$
$= (kx_1, kx_2, k\sqrt{x_3})$

31. a)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

b) $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

c) $$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

35. a) $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

b) $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

c) $$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

38.

a) $$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$= \begin{bmatrix} v_1\cos\alpha - v_2\sin\alpha \\ v_1\sin\alpha + v_2\cos\alpha \end{bmatrix}$$

b) $$\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$= \begin{bmatrix} v_1\cos\alpha + v_2\sin\alpha \\ -v_1\sin\alpha + v_2\cos\alpha \end{bmatrix}$$

40. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

a) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} k \\ 0 \end{bmatrix} = \begin{bmatrix} ka \\ kc \end{bmatrix}$

b) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} k & 0 \\ 0 & l \end{bmatrix}$

$= \begin{bmatrix} ka & lb \\ kc & l\cancel{k}d \end{bmatrix}$ $ka + lb$, $kc + ld$

Ex: 1.10 (4, 14)

4.

a)

	Flow In	Flow Out	
A	$500+300$	$x_1 + x_3$	$x_1 + x_3 = 800$
B	$x_1 + x_4$	$200 + x_2$	$x_2 - x_1 - x_4 = -200$
C	$x_2 + 100$	$x_5 + 600$	$x_5 - x_2 = -500$
D	$x_3 + x_6$	$400 + 350$	$x_3 + x_6 = 750$
E	$x_7 + 600$	$x_4 + x_6$	$x_4 + x_6 - x_7 = 600$
F	$x_5 + 450$	$x_7 + 400$	$x_7 - x_5 = 50$

~~$x_1 + x_3 = 800$~~

b)

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 800 \\ -1 & 1 & 0 & -1 & 0 & 0 & 0 & -200 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & -500 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 600 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 750 \end{array}\right]$$

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 800 \\ 0 & 1 & 1 & -1 & 0 & 0 & 0 & 600 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & -500 \\ 0 & 0 & 0 & 0 & -1 & 0 & +1 & 50 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 600 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 750 \end{array}\right] \quad R_1 + R_2$$

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 800 \\ 0 & 1 & 1 & -1 & 0 & 0 & 0 & 600 \\ 0 & 0 & \cancel{0}1 & \cancel{0}-1 & 1 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 600 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 750 \end{array}\right] \quad R_2 + R_3$$

$$\left[\begin{array}{cccccccc} 1 & 0 & 0 & 1 & -1 & 0 & 0 & 700 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 500 \\ 0 & 0 & 1 & -1 & 1 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 600 \\ 0 & 0 & 0 & 1 & -1 & 1 & 0 & 650 \end{array}\right] \begin{array}{l} -R_3 + R_1 \\ -R_3 + R_2 \\ \\ \\ \\ -R_3 + R_6 \end{array}$$

$$\left[\begin{array}{cccccccc} 1 & 0 & 0 & 0 & -1 & -1 & 1 & 100 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 500 \\ 0 & 0 & 1 & 0 & 1 & -1 & 1 & -500 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 600 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 50 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 50 \end{array}\right] \begin{array}{l} -R_4 + R_1 \\ \\ -R_4 + R_3 \\ R_4 \Leftrightarrow R_5 \\ \\ -R_4 + R_6 \end{array}$$

$$\left[\begin{array}{cccccccc} 1 & 0 & 0 & 0 & -1 & -1 & 1 & 100 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 500 \\ 0 & 0 & 1 & 0 & 1 & -1 & 1 & -500 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 600 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & -50 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 50 \end{array}\right] \begin{array}{l} \\ \\ \\ \\ -1 \times R_5 \\ \\ \end{array}$$

$$\left[\begin{array}{cccccccc} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 50 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 450 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 750 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 600 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & -50 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} R_5 + R_1 \\ R_5 + R_2 \\ -R_5 + R_3 \\ \\ \\ R_5 + R_6 \end{array}$$

$x_1 - x_6 = 50$
$x_2 - x_7 = 450$
$x_3 + x_6 = 750$
$x_4 + x_6 - x_7 = 600$
$x_5 - x_7 = -50$

$x_6 = a \quad , \quad x_7 = b$
$x_1 = 50 - a$
$x_2 = 450 + b$
$x_3 = 750 - a$
$x_4 = 600 - a + b$
$x_5 = -50 + b$

c) if $x_1 = 0$ then $a = -50$
hence it is not possible

14. $(0,0)$, $(-1,+1)$, $(1,1)$ $p(x) = a_0 + a_1 x + a_2 x^2$
$a_0 = 0$
$1 = a_0 - a_1 + a_2$
$1 = a_0 + a_1 + a_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{array}{l} \\ -1R_1 + R_2 \\ -R_1 + R_3 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 1 & +1 & 1 \end{bmatrix} \begin{array}{l} \\ -R_2 \\ \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 2 & 2 \end{bmatrix} \quad -R_2 + R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad R_3 \times \frac{1}{2}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cancel{0}1 & 1 \end{bmatrix} \quad R_3 + R_2$$

$a_0 = 0$, $a_1 = 0$, $a_2 = 1$

$P(x) = x^2$