Probability & STATS

Date:

Assignment # 3

K20-1052
S. Mohassem Ali
BSE-4B.

Q.1 Population 1

$\mu_1 = 6.5 \quad \sigma_1 = 0.9 \quad n_1 = 36$

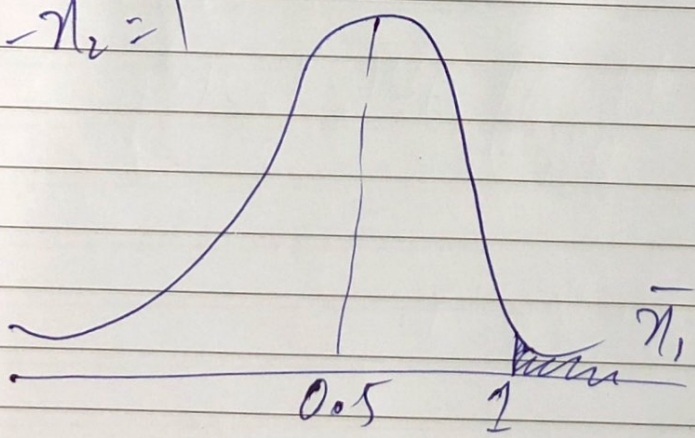
P-2.

$\mu_2 = 6 \quad \sigma_2 = 0.8 \quad n_2 = 49$

$\mu_{x_1} - \mu_{x_2} = 6.5 - 6 = 0.5$

$\sigma_{x_1} - \sigma_{x_2} = \sqrt{\frac{0.81}{36} + \frac{0.64}{49}} = 0.189$

$\bar{x}_1 - \bar{x}_2 = 1$



$z = \frac{\bar{x} - \mu}{\sigma}$

$= \frac{1 - 0.5}{0.189}$

$= 2.65$

$P(\bar{x}_1 - \bar{x}_2 \geq 1) = P(z > 2.65) = 1 - P(z < 2.65)$

$= 1 - 0.9960$

$= 0.0040$

Normally distributed hence CLT applied.

Date:

2. Mean and Sd using Cal.

$\bar{x} = 10 \quad S = 0.283$.

$n = 7$ (hence t-test to be used)

95% confidence interval ($\alpha = 0.5$)

$\nu = n - 1 = 7 - 1 = 6$

$100(1-\alpha) = 95\% \quad / \quad 1 - \alpha = 0.95$

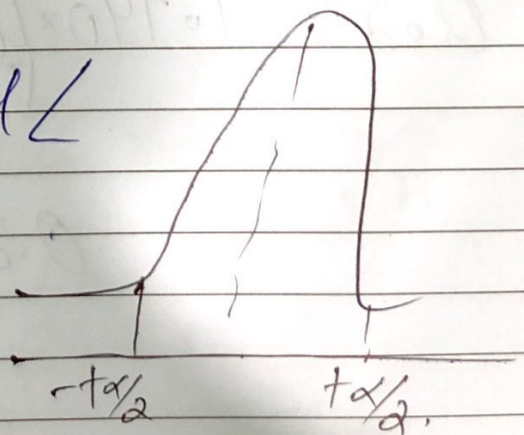
$\alpha = 0.5$

$$\bar{x} - t_{\alpha/2} S/\sqrt{n} < \mu < \bar{x} + t_{\alpha/2} S/\sqrt{n}$$

$C.V = t_{\alpha/2, 6} = t_{0.025, 6} = 2.447$

$$10 - 2.447\left(\frac{0.283}{\sqrt{7}}\right) < \mu <$$

$$10 + 2.447\left(\frac{0.283}{\sqrt{7}}\right)$$



$9.74 < \mu < 10.26$

Date:

Q.3 $\bar{x} = 0.29$ $S = 0.074$

$n = 18$ 99% confidence level.

$100(1-\alpha) = 99$; $\nu = n-1$

$(1-\alpha) = 0.99$ $= 18-1 = 17$

$\alpha = 0.01$

C.V

$= t_{\alpha/2, \nu} = t_{0.05, 17} = 1.740$

$$\bar{x} - t_{\alpha/2} S/\sqrt{n} < \mu < \bar{x} + t_{\alpha/2} S/\sqrt{n}$$

$$0.29 - 1.740 \times \left(\frac{0.074}{\sqrt{18}}\right) < \mu <$$

$$0.29 + 1.740 \left(\frac{0.074}{\sqrt{18}}\right)$$

$$0.26 < \mu < 0.32$$

ALBA

Date:

.4 S1:

$n_1 = 15 \quad \bar{x}_1 = 3.084 \quad S = 3.07$

S2:

$n_2 = 12 \quad \bar{x}_2 = 1.049 \quad S = 0.80.$

95% confidence interval. $\mu_1 - \mu_2$.

$$V = \frac{\left(S_1^2/n_1 + S_2^2/n_2\right)^2}{\left(S_1^2/n_1\right)/n_1 - 1 + \left(S_2^2/n_2\right)/n_2 - 1} = \frac{\left(3.07^2/15 + 0.80^2/12\right)^2}{\left(3.07^2/15\right)/14 + \left(0.80^2/12\right)/11} = 16.3 \rightarrow 16$$

$\bar{x}_1 - \bar{x}_2 = 3.084 - 1.049 = 2.35$

$1 - \alpha = 0.95 \quad , \quad \alpha = 0.5$

$C.V = t_{\alpha/2, v} = t_{(0.025, 16)} = 2.120$

$$\bar{x} - t_{\alpha/2} \frac{S}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2} \frac{S}{\sqrt{n}}$$

$$2.35 - 2.120 \sqrt{\frac{3.07^2}{15} + \frac{0.80^2}{12}} < \mu_1 - \mu_2 <$$

$$2.35 + 2.120 \sqrt{\frac{3.07^2}{15} + \frac{0.80^2}{12}}$$

$$0.60 < \mu_1 - \mu_2 < 4.10$$

ALBA

→ Test on single mean.

Date:

Q.5 $H_0: \mu = 46$ kwh $\alpha = 0.05$

$H_1: \mu < 46$ kwh (left tail test)

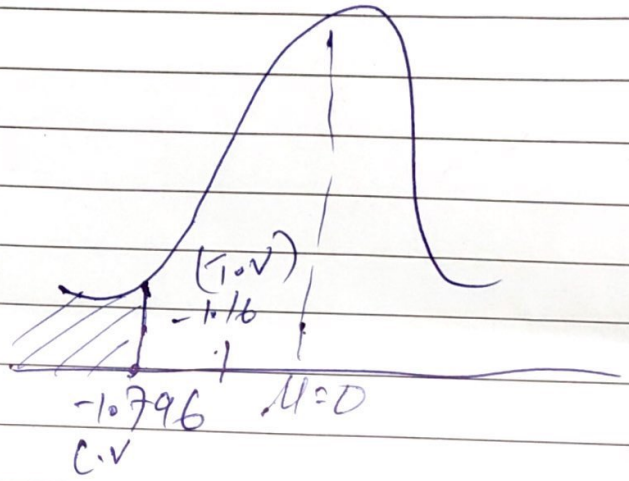
$n = 12$ $Sd = 11.9$ $\bar{x} = 42$.

$$t = \frac{\bar{x} - \mu}{S/\sqrt{n}} = \frac{42 - 46}{11.9/\sqrt{12}} = -1.16 \quad (T.V)$$

$v = 12 - 1 = 11$

$C.V = t_{\alpha/2, v} = t(0.05, 11)$

$= -1.796$



T.V lies outside critical region so do not reject h_0.

Which means that avg no of kwh of vacuum is less than 46.

→ Difference b/w means.

Date:

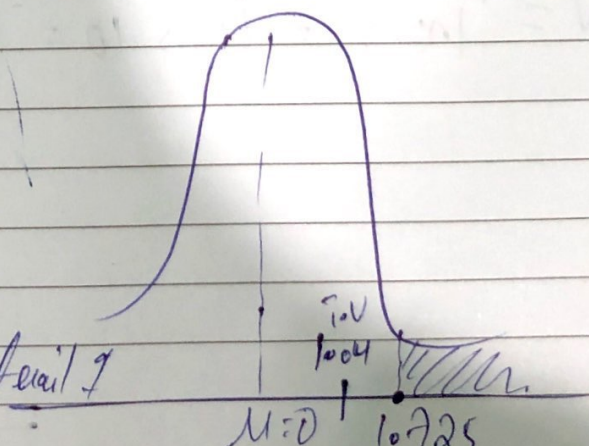
9.6 $H_0: \mu_1 - \mu_2 = 2$ $\alpha = 0.05$
$H_1: \mu_1 - \mu_2 > 2$ (right tail test)

$\bar{x}_1 = 85$ $s_1 = 4$ $n_1 = 12$ $v = 12+10-2 = 20$
$\bar{x}_2 = 81$ $s_2 = 5$ $n_2 = 10$

$$sp^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{v} = \frac{11(4)^2 + 9(5)^2}{20}$$
$$= 4.477$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{sp\sqrt{1/n_1 + 1/n_2}} = \frac{4-2}{4.477\sqrt{1/12 + 1/10}} = 1.04 \quad (T.V)$$

$$C.V = t(\alpha, v) = t(0.05, 20) = 1.725$$



∴ T.V lies outside so do not reject H_0.

Cannot conclude that abrasive wear of Material 1 exceeds Material 2 by more than 2.

ALBA

Date:

7) S1:
$\bar{x}_1 = 85$ $\sigma_1 = 162.8$ $sd_1 = 12.76$

S2:
$\bar{x}_2 = 80$ $\sigma_2 = 10.4$ $sd_2 = 3.22$

$\alpha = 0.05$ $n = 6$

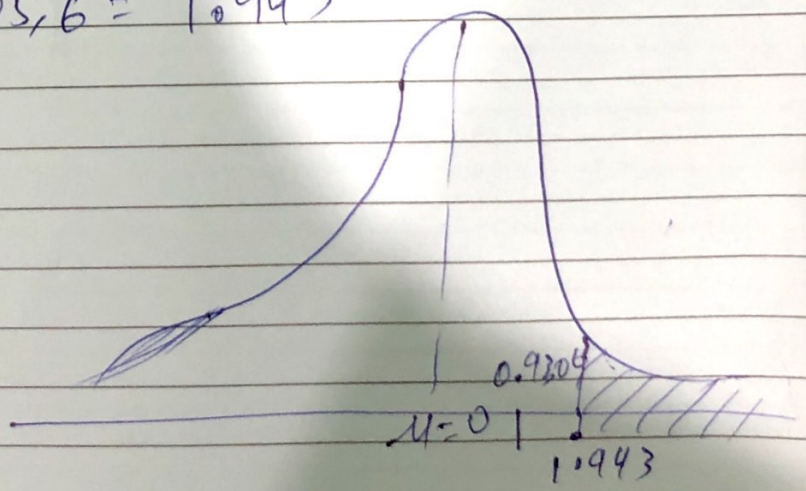
$H_0: \mu_1 - \mu_2 = 0$
$H_1: \mu_1 - \mu_2 > 0$ right tail test.

$$t = \frac{(\bar{x}_2 - \bar{x}_1) - d_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{85 - 80}{\sqrt{\frac{(12.76)^2}{6} + \frac{(3.22)^2}{6}}} = 0.9306 \quad (T.V)$$

$$\nu = \frac{\left(s_1^2/n_1 + s_2^2/n_2\right)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} \quad \frac{(s_2^2/n_2)^2}{n_2 - 1}} = \frac{\left(\frac{12.76^2}{6} + \frac{3.22^2}{6}\right)^2}{\frac{\left(12.76^2/6\right)^2}{5} + \frac{(3.22^2/6)^2}{5}}$$

$$= 5.63 \approx 6$$

$C.V = t_{\alpha,\nu} = t_{0.05,6} = 1.943$



ALBA

→ Independent samples ($\sigma_1 = \sigma_2$)

Date:

Q.8

Lab	Without lab
$n_1 = 11$	$n_2 = 17$
$\bar{x} = 85$	$\bar{x} = 79$
$s = 4.7$	$s = 6.0$

$H_0: \mu_2 - \mu_1 = 8$

$H_1: \mu_2 - \mu_1 > 8$ (right tail test)

$\alpha = 0.05$

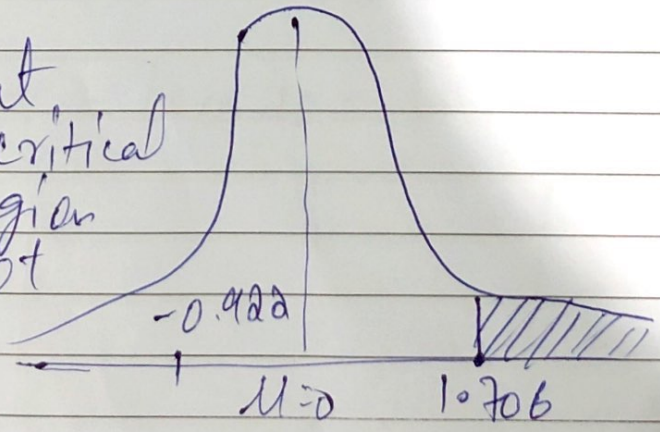
$$s_p = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{\nu} = \frac{10(85)^2 + (16)(79)^2}{26}$$

$$= 5.603$$

$$t = \frac{(\bar{x}_2 - \bar{x}_1) - d_0}{s_p\sqrt{1/n_1 + 1/n_2}} = \frac{(85-79)-8}{5.603\sqrt{1/17 + 1/11}}$$

$$t = -0.922 \rightarrow \text{T.V}$$

C.V $= t_{\alpha,\nu} = t_{0.05,26} = 1.706$ (from table)



T.V lies out side the critical reject region hence donot reject H_0.

Which means that if semester novel course with labs will not increase the avg grade by 8 points.