Introduction

- Any project involves planning, scheduling and controlling a number of interrelated activities with use of limited resources, namely, men, machines, materials, money and time.
  - The projects may be extremely large and complex such as construction of a housing , a highway, a shopping complex etc.
  - introduction of new products and research and development projects.
- It is required that managers must have a dynamic planning and scheduling system to produce the best possible results and also to react immediately to the changing conditions and make necessary changes in the plan and schedule.

A convenient analytical and visual technique of **PERT** and **CPM** prove extremely valuable in assisting the managers in managing the projects.

PERT stands for **Project Evaluation and Review Technique** **developed during 1950's. The technique** was developed and used in conjunction with the planning and designing of the Polaris missile project.

CPM stands for **Critical Path Method** **which was developed by DuPont** Company and applied first to the construction projects in the chemical industry.

Though both PERT and CPM techniques have similarity in terms of concepts, the basic difference is, PERT is used for analysis of project scheduling problems. CPM has single time estimate and PERT has three time estimates for activities and uses probability theory to find the chance of reaching the scheduled time.

Project management generally consists of three phases.

Planning:

Planning involves setting the objectives of the project. Identifying various activities to be performed and determining the requirement of resources such as men, materials, machines, etc.

The cost and time for all the activities are estimated, and a network diagram is developed showing sequential interrelationships (predecessor and successor) between various activities during the planning stage.

Scheduling:

Basd on the time estimates, the start and finish times for each activity are worked out by applying forward and backward pass techniques, critical path is identified, along with the slack and float for the non-critical paths.

Controlling:

Controlling refers to analyzing and evaluating the actual progress against the plan. Reallocation of resources, crashing and review of projects with periodical reports are carried out.

Difference between CPM & PERT

CPM	PERT
• CPM works with fixed deterministic time	• PERT works with probabilistic time
• CPM is useful for repetitive and non complex projects with a certain degree of time estimates.	• PERT is useful for non repetitive and complex projects with uncertain time estimates.
• CPM includes time-cost trade off.	• PERT is restricted to time variable.
• CPM- for construction projects.	• PERT- used for R&D programs.

RULES IN CONSTRUCTING A NETWORK

1. No single activity can be represented more than once in a network. The length of an arrow has no significance.

2. The event numbered 1 is the start event and an event with highest number is the end event. Before an activity can be undertaken, all activities preceding it must be completed. That is, the activities must follow a logical sequence (or – interrelationship) between activities.

3. In assigning numbers to events, there should not be any duplication of event numbers in a network.

4. Dummy activities must be used only if it is necessary to reduce the complexity of a network.

5. A network should have only one start event and one end event.

Critical Path:

After determining the earliest and the latest scheduled times for various activities, the minimum time required to complete the project is calculated. In a network, among various paths, the longest path which determines the total time duration of the project is called the critical path. The following conditions must be satisfied in locating the critical path of a network.

An activity is said to be critical only if both the conditions are satisfied.

1. $TL - TE = 0$
2. $TLj - tij - TEj = 0$

Example :

A project schedule has the following characteristics as shown in Table

Table 8.5: Project Schedule

Activity	Name	Time	Activity	Name	Time (days)
1-2	A	4	5-6	G	4
1-3	B	1	5-7	H	8
2-4	C	1	6-8	I	1
3-4	D	1	7-8	J	2
3-5	E	6	8-10	K	5
4-9	F	5	9-10	L	7

i. Construct PERT network.
ii. Compute TE and TL for each activity.
iii. Find the critical path.

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4-9	F	5	9-10	L	7

(i) From the data given in the problem, the activity network is constructed as shown in Figure given below

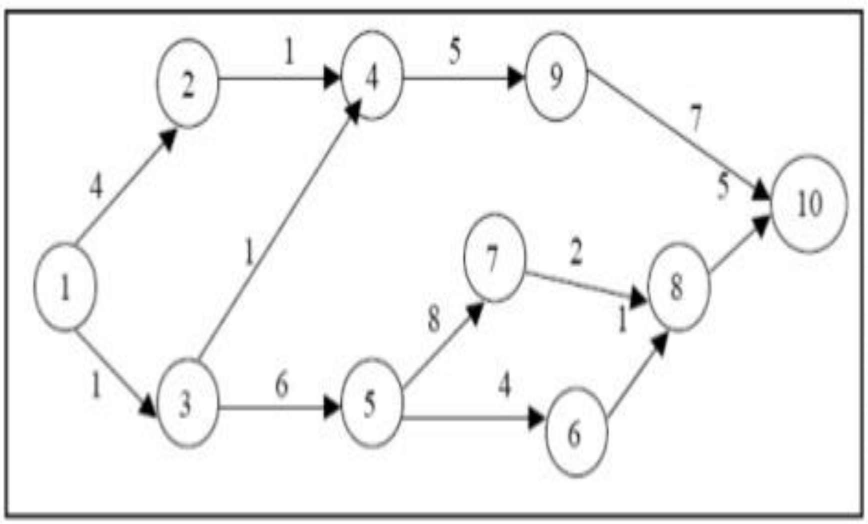


Figure 8.16: Activity Network Diagram

(iii) From the Table 8.6, we observe that the activities 1 – 3, 3 – 5, 5 – 7,7 – 8 and 8 – 10 are critical activities as their floats are zero.

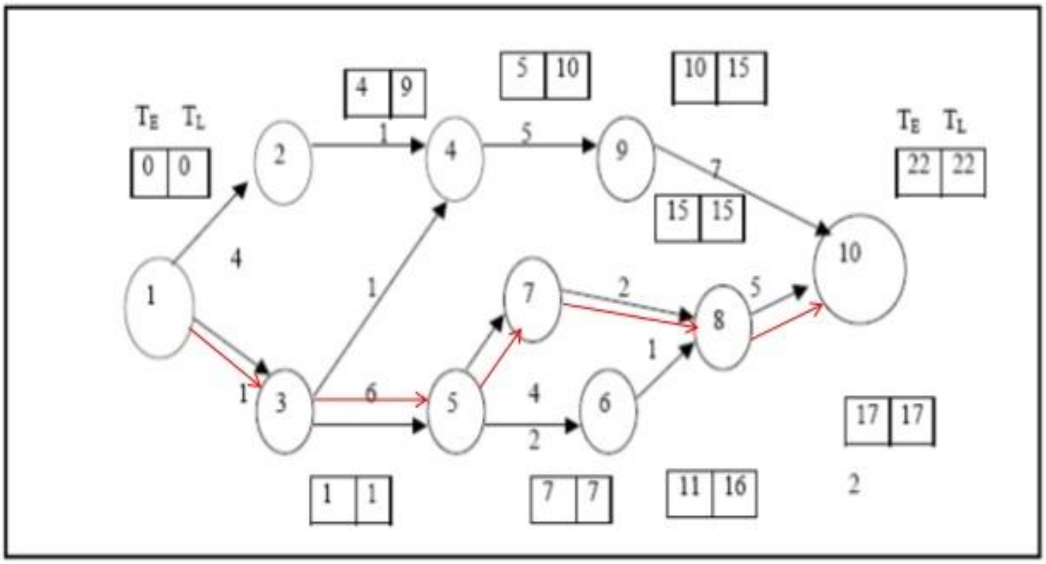


Figure 8.17: Critical Path of the Project

PROJECT EVALUATION REVIEW TECHNIQUE, (PERT)

In the critical path method, the time estimates are assumed to be known with certainty. In certain projects like research and development, new product introductions, it is difficult to estimate the time of various activities.

Hence PERT is used in such projects with a probabilistic method using three time estimates for an activity, rather than a single estimate, as shown in Figure 8.22.

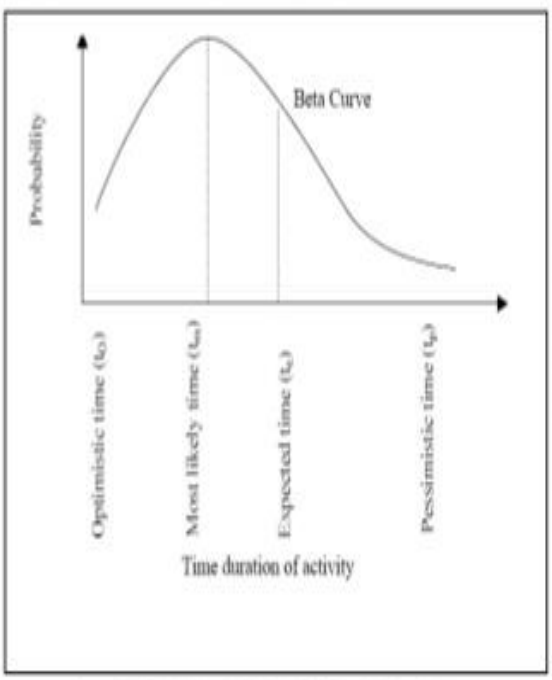


Figure 8.22: PERT Using Probabilistic Method with 3 Time Estimates

Optimistic time to:

It is the shortest time taken to complete the activity. It means that if everything goes well then there is more chance of completing the activity within this time.

Most likely time tm:

It is the normal time taken to complete an activity, if the activity were frequently repeated under the same conditions.

Pessimistic time tp:

It is the longest time that an activity would take to complete. It is the worst time estimate that an activity would take if unexpected problems are faced.

Taking all these time estimates into consideration, the expected time of an activity is arrived at.

The average or mean (ta) value of the activity duration is given by,

$$T_a = \frac{t_0 + 4t_m + t_p}{6} \quad \text{.....................(5)}$$

The variance of the activity time is calculated using the formula,

$$\sigma_i^2 = \left(\frac{t_p - t_0}{6}\right)^2$$

Probability for Project Duration

The probability of completing the project within the scheduled time (Ts) or contracted time may be obtained by using the standard normal deviate where Te is the expected time of project completion.

$$Z_0 = \frac{T_s - T_e}{\sqrt{\Sigma\sigma^2} \text{ in critical path}}$$

Probability of completing the project within the scheduled time is,

$$P(T \le T_s) = P(Z \le Z_0) \text{ (from normal tables)} \quad ....$$

Example

An R & D project has a list of tasks to be performed whose time estimates are given in the Table 8.11, as follows.

Table 8.11: Time Estimates for R & D Project

Activity i j	Activity Name	T_0	t_m (in days)	t_p
1-2	A	4	6	8
1-3	B	2	3	10
1-4	C	6	8	16
2-4	D	1	2	3
3-4	E	6	7	8
3-5	F	6	7	14
4-6	G	3	5	7
4-7	H	4	11	12
5-7	I	2	4	6
6-7	J	2	9	10

a. Draw the project network.
b. Find the critical path.
c. Find the probability that the project is completed in 19 days. If the probability is less that 20%, find the probability of completing it in 24 days.

Time expected for each activity is calculated using the formula (5): Similarly, the expected time is calculated for all the activities.

$$T_a = \frac{t_0 + 4tm + tp}{6}$$

$$= \frac{4 + 4(6) + 8}{6} = \frac{36}{6} = 6 \text{ days for activity A}$$

The variance of activity time is calculated using the formula (6). Similarly, variances of all the activities are calculated.

$$\sigma_i^2 = \left(\frac{t_p - t_0}{6}\right)^2$$

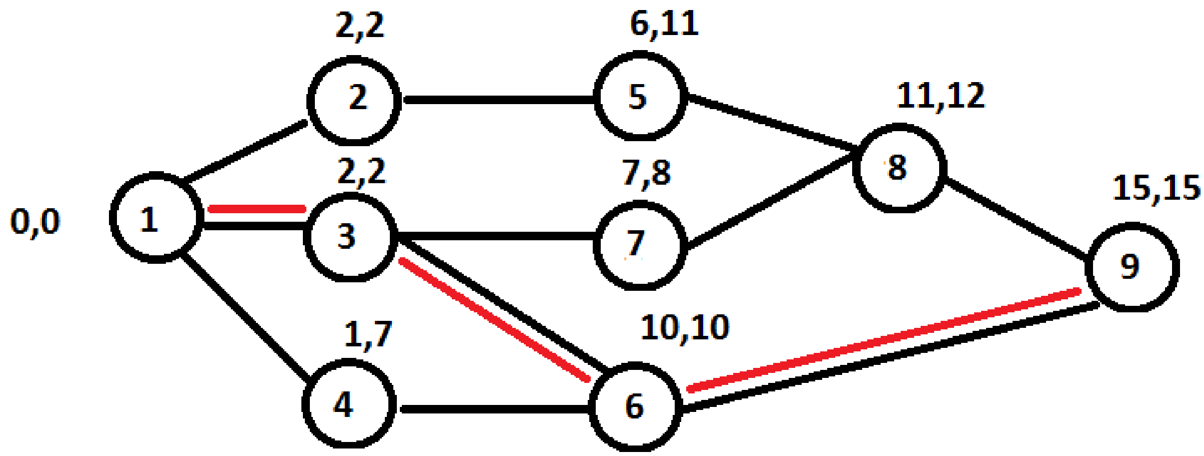
$$= \left(\frac{8-4}{6}\right)^2 = 0.444$$

Table 8.12: T_e & s^2 Calculated

Activity	T_o	T_m	T_p	T_a	σ^2
1-2	4	6	8	6	0.444
1-3	2	3	10	4	1.777
1-4	6	8	16	9	2.777
2-4	1	2	3	2	0.111
3-4	6	7	8	7	0.111
3-5	6	7	14	8	1.777
4-6	3	5	7	5	0.444
4-7	4	11	12	10	1.777
5-7	2	4	6	4	0.444
6-7	2	9	10	8	1.777

Q1.

Activity	1-2	1-3	1-4	2-5	3-6	3-7	4-6	5-8	6-9
Months	2	2	1	4	8	5	3	1	5
Activity	7-8	8-9							
months	4	3							



Activity	Months
1-3	
3-6	
6-9	