20F-0441 BS-SE-3B Linear Algebra Assignment 2 Talha Zahid

Linear Algebra (National University of Computer and Emerging Sciences)

20F-0441

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BS(SE) 3B

Assignment No 02

Linear Algebra

Q No 1

i) False

It can be any real number either +ve or -ve

ii) True

It will be scalar multiple of other vector

iii) True

It is due to trivial sol of $Ax = 0$

iv) False

In case of homogenius equation, we may have non trivial sol as well.

v) True

Correct by definition

vi) True

Definition of span

vii) True

Vector in $\mathbb{R}^2$ can be xpressed as combination of two ~~non~~ non ~~[illegible]~~ parallell vectors in $\mathbb{R}^2$.

viii) True

ix) True Definition of span
Definition of span

x) True

If $Ax = \underline{v}$ is inconsistent so $\underline{v}$ does not belongs to $\{a_1, a_2, \ldots a_n\}$ by theorem 4.

xi) True
It is bcz system will only have unique sol and that will be trivial sol

xii) True
$A\underline{x} = 0$ has trivial sol

xiii) True
Theorem 4

xiv) False

$\underline{x} = p + t\underline{v}$ is paralell to $\underline{v}$

xv) False

A non trivial sol can have some non zero entries not all.

Q No 02

(a)

let $\underline{u} = u_1 + u_2 + \dots + u_n$

$\underline{w} = w_1 + w_2 + \dots + w_n$

$\underline{v} = v_1 + v_2 + \dots v_n$

$(\underline{u} + \underline{v}) + \underline{w} = u + (v + w)$

L.H.S

$\underline{u} + \underline{v} = (\underline{u}_1 + \underline{v}_1 + u_2 + v_2 + \dots u_n + v_n)$

$(\underline{u} + \underline{v}) + \underline{w} = (u_1 + v_1 + u_2 + u_2 + \dots u_n + v_n) + (w_1 + w_2 + \dots w_n)$

$= (u_1 + v_1 + w_1 + u_2 + v_2 + w_2 + \dots u_n + v_n + w_n)$

R.H.S

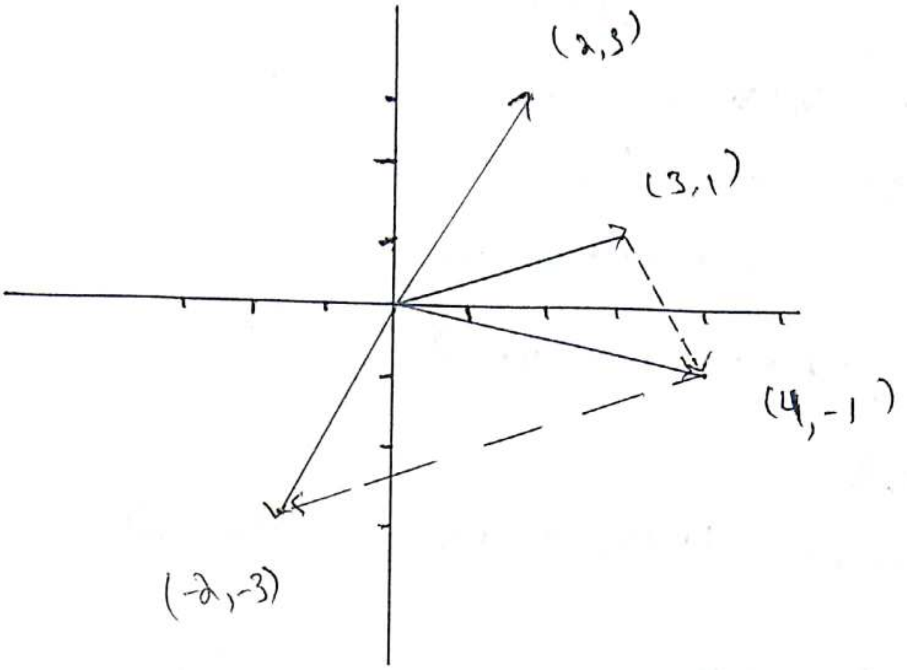
$\underline{v} + \underline{w} = (v_1 + w_1 , v_2 + w_2 + , \dots v_n + w_n)$

$\underline{u} + (\underline{v} + \underline{w}) = (u_1 + v_1 + w_1 , u_2 + v_2 , w_2 , + \dots , u_n + v_n + w_n)$

Hence Proved

L.H.S = R.H.S

(b)



It is a linear combination

$$\begin{bmatrix} 4 \\ -1 \end{bmatrix} = x_1 \begin{bmatrix} 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$2x_1 + 3x_2 = 4$$
$$3x_1 + x_2 = -1$$

$$\left[\begin{array}{cc|c} 2 & 3 & 4 \\ 3 & 1 & -1 \end{array}\right] R_1 - R_2$$

$$\begin{bmatrix} -1 & 2 & 5 \\ 3 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & -5 \\ 3 & 1 & -1 \end{bmatrix} -R_1$$

(2)

$$\begin{bmatrix} 1 & -2 & -5 \\ 3 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & -5 \\ 0 & 7 & 14 \end{bmatrix} R_2 - 3R_1$$

$$\begin{bmatrix} 1 & -2 & -5 \\ 0 & 7 & 14 \end{bmatrix} \frac{R_2}{7}$$

$$\begin{bmatrix} 1 & -2 & -5 \\ 0 & 1 & 2 \end{bmatrix} R_1 + 2R_2$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$$

$x_1 = -1$ $\qquad$ $x_2 = 2$

(c)

$A = \begin{bmatrix} 3 & 2 \\ -3 & -2 \end{bmatrix}$, $x \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$Ax = 0$

$$\begin{bmatrix} 3 & 2 & 0 \\ -3 & -2 & 0 \end{bmatrix} R_2 + R_1$$

$$\begin{bmatrix} 3 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(4)

x_1 is basic variable
x_2 is free variable

So it has infinite solutions

(d)

S. $x_1 - 3x_2 + 5x_3 = 0$

a: $x_1 = 3x_2 - 5x_3$

x_2 & x_3 are free variable

$$S.S = x\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_2 - 5x_3 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= \begin{bmatrix} 3x_2 \\ x_2 \\ 0 \end{bmatrix} + \begin{bmatrix} -5x_3 \\ 0 \\ x_3 \end{bmatrix}$$

$$= x_2\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix}$$

$= t\underline{v} + r\underline{u}$

b: $x_1 - 3x_2 + 5x_3 = 4$

$x_1 = 4 + 3x_2 - 5x_3$

x_2 & x_3 are free

(3)

e

Any vector can be written as linear combination of standard vectors

$e_1 (1,0)$
$e_2 = (0,1)$ are standard vectors in R^2

Generally,

$$e_1 = (1,0,0,\dots)$$
$$e_2 = (0,1,0,\dots)$$
$$e_n = (0,000,\dots,1) \text{ in } R^n$$

(f)

A is 2×3
B is 3×1

$$\begin{bmatrix} 3 & -1 & 4 \\ -2 & 6 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 0\begin{bmatrix} 3 \\ -2 \end{bmatrix} + 1\begin{bmatrix} -1 \\ 6 \end{bmatrix} + 0\begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

(g)

$$S = \{(1,-1)\}$$

Span of single vector contains all linear combination of that vector

$$\text{Span } S_1 = \begin{bmatrix} c \\ -c \end{bmatrix} = c\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$ which is a line with equation

$$y = -x.$$

(b) Span of S_2 will contain all the linear combination of vector given in S_2

$v(1,-1)$ &
$(-2,2)$

are scalar multiple of each other where as $(1,-1)$ and $(2,1)$ are not parallell to each other

Hence

$$S_2 = a(1,-1) + b(2,1)$$

$$a(1,-1) + 0(-2,2) + b(2,1)$$

Any vector in $\mathbb{R}^2$ is a linear combination of vector in S_2 so ans is Yes

$$\text{Span of } S_2 = \mathbb{R}^2$$

(4)

(h)

$x_1 + 2x_2 - x_3 = 1$

$x_1 = 1 - 2x_2 + x_3$

x_2 & x_3 are free

$$S.S = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 - 2x_2 + x_3 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \underline{p} + (t\underline{v} + r\underline{u})$$

S.S involves free variable, so system is consistent with given solution given.

(i)

$Ax = b$ $\qquad b = (b_1, b_2)$

$$\left[\begin{array}{cc|c} 1 & 2 & b_1 \\ 3 & 6 & b_2 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 2 & b_1 \\ 0 & 0 & b_2 - 3b_1 \end{array}\right] R_2 - 3R_1$$

System will be consistent when

$b_2 - 3b_1 = 0$

$b_1 = 3b_1$

will be inconsistent when

$b_2 \neq 3b_1$

QNo 03

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 2 & 1 & -1 & 0 \\ 3 & 4 & 1 & 5 \end{array}\right] \quad \underline{b} = (8, 0, 5)$$

If the equivalent system is consistent that is (unique sol/infinite sol) then $\underline{b}$ can be expressed as linear combination of remaining vectors.

$$\begin{bmatrix} 1 & 0 & 2 & 8 \\ 0 & 1 & -5 & -16 \\ 0 & 4 & -5 & -19 \end{bmatrix} \quad \begin{matrix} R_2 - 2R_1 \\ R_3 - 3R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 2 & 8 \\ 0 & 1 & -5 & -16 \\ 0 & 0 & \cancel{-15}\ -5 & 45 \end{bmatrix} \quad R_3 - 4R_2$$

$$\begin{bmatrix} 1 & 0 & 2 & 8 \\ 0 & 1 & -5 & -16 \\ 0 & 0 & 1 & 3 \end{bmatrix} \quad \cancel{2R_2}\ R_3/15$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & -5 & -16 \\ 0 & 0 & 1 & 3 \end{bmatrix} R_1 - 2R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix} R_2 + 5R_3$$

Due to consistency b can be expressed as linear combination of remaining vector.

Q No 04

$$S = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} = V \begin{bmatrix} 2 \\ r \\ -1 \end{bmatrix}$$

Aug Mat

$$\left[\begin{array}{cc|c} 1 & -1 & 2 \\ 0 & 3 & r \\ -1 & 2 & -1 \end{array}\right]$$

$R_3 + R_1$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & r \\ 0 & 1 & 1 \end{bmatrix}$$

$R_3 - \frac{R_2}{3}$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & r \\ 0 & 0 & 1 - r/3 \end{bmatrix}$$

$$1 - \frac{r}{3} = 0$$

$$\frac{r}{3} = 1$$

$$r = 3$$

For $r = 3$, system $\underline{v}$ is in span of S

⑥ Q No 5

$x_1 + 2x_2 - x_3 + 2x_5 - x_6 = 0$

$2x_1 + 4x_2 - 2x_3 - x_4 - 5x_6 = 0$

$-x_1 - 2x_2 + x_3 + x_4 + 2x_5 + 4x_6 = 0$

$x_4 + 4x_5 + 3x_6 = 0$

$$\left[\begin{array}{cccccc|c} 1 & 2 & -1 & 0 & 2 & -1 & 0 \\ 2 & 4 & -2 & -1 & 0 & -5 & 0 \\ -1 & -2 & 1 & 1 & 2 & 4 & 0 \\ 0 & 0 & 0 & 1 & 4 & 3 & 0 \end{array}\right]$$

$R_2 - 2R_1$, $R_3 + R_1$

$$\left[\begin{array}{cccccc|c} 1 & 2 & -1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & -4 & -3 & 0 \\ 0 & 0 & 0 & 1 & 4 & 3 & 0 \\ 0 & 0 & 0 & 1 & 4 & 3 & 0 \end{array}\right]$$

$-R_2$

$$\left[\begin{array}{ccccccc} 1 & 2 & -1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 & 4 & 3 & 0 \\ 0 & 0 & 0 & 1 & 4 & 3 & 0 \\ 0 & 0 & 0 & 1 & 4 & 3 & 0 \end{array}\right]$$

$R_3 - R_2$, $R_4 - R_2$

$$\left[\begin{array}{ccccccc} 1 & 2 & -1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 & 4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 + 2x_3 - x_3 + 2x_5 - x_6 = 0$

$x_1 = -2x_2 + x_3 - 2x_5 + x_6$

x_2 is free variable

x_3 is free variable

$x_4 \quad + 4x_5 + 3x_6 = 0$

$x_4 = -4x_5 - 3x_6$

x_5 is free

x_6 is free

General Solution

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} -2x_2 + x_3 - 2x_5 + x_6 \\ 0 \\ 0 \\ -4x_5 - 3x_6 \\ 0 \\ 0 \end{bmatrix}$$