20F-0441 BS-SE-3B Linear Algebra Assignment 1 Talha Zahid

Linear Algebra (National University of Computer and Emerging Sciences)

20F-0441

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SE - 3B

Linear Algebra

Assignment 01

Q No 1

1. False

The augmented matrix of linear system of equation can also be a square matrix eg

$$5x_1 + 2x_2 = 5$$
$$3x_1 + 7x_2 = 6$$
$$x_1 - 6x_2 = 11$$

The augmented matrix of above system is a 3×3 square matrix.

2. False

It's not mandatory for system of linear equations to hold a solution. There are numerous equation which have no solution. Such systems are called inconsistent.

3. False

It is not the mandatory condition for echelon form. It is compulsory for reduced echelon form.

4. True

Yes, if a system has unique or infinity solution then it is called consistent solution.

5. True

Yes, if a column contains leading entry / pivot position then it is called pivot column.

6. False

It is may not be true in every case. In some cases, third row may contains all zero elements.

7. ~~True~~ False

If matrix does not contain zero column then the leading entries 1 will be in diagonal but it may or may not be an identity matrix.

eg $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ Although it is reduced echelon form with no zero column, still it is not a identity matrix.

8. True

A non zero matrix can never be converted to a zero matrix

9. True

Equivalent system of equation has same solution.

10. False

The reduced echelon form is unique but the sequence of elementary row operation is not unique.

Q No 2.

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 1 \\ 2 & 7 & a & 3 \\ 1 & a & -7 & 0 \end{array}\right]$$

$R_2 - 2R_1 \;\; R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 1 \\ 0 & 1 & a+2 & 1 \\ 0 & a-3 & -6 & -1 \end{array}\right]$$

$R_3 / a-3$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 1 \\ 0 & 1 & a+2 & 1 \\ 0 & 1 & \frac{-6}{a-3} & \frac{-1}{a-3} \end{array}\right]$$

$R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & -2 \\ 0 & 1 & a+2 & 1 \\ 0 & 0 & \frac{a-a^2}{a-3} & \frac{2-a}{a-3} \end{array}\right]$$

$$\left(\frac{a-a^2}{a-3}\right) x_3 = \frac{2-a}{a-3}$$

$$x_3 = \frac{2-a}{a-a^2}$$

$a - a^2 \neq 0$

$a(a-1) \neq 0$

either $a \neq 0$ or $a-1 \neq 0$, $a \neq 1$

So $\boxed{a \in R - \{0, 1\}}$

Q No 03.

$$\begin{bmatrix} 1 & 3 & 0 & -2 & 6 \\ 0 & 0 & 1 & 4 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

x_1 and x_3 are basic variables

x_2 and x_4 are free variable

Matrix's right most column is not pivot column so it is consistent system.

It contains free variables so it has infinity solutions.

$$x_1 + 3x_2 - 2x_4 = 6$$

$$x_3 + 4x_4 = 7$$

$$\begin{cases} x_1 = 6 - 3x_2 + 2x_4 \\ x_2 \text{ is free variable} \\ x_3 = 7 - 4x_4 \\ x_4 \text{ is free variable} \end{cases}$$

Q No 4

1)

Augmented Matrix

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 5 \\ 4 & -6 & 0 & -2 \\ -2 & 7 & 2 & 9 \end{array}\right]$$

$R_2 - 2R_1$

$R_3 + R_1$

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 5 \\ 0 & -8 & -2 & -12 \\ 0 & 8 & 3 & 14 \end{array}\right]$$

$R_3 + R_2$

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 5 \\ 0 & -8 & -2 & -12 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Now the matrix is in echelon form

2, -8 and 1 are the pivots

M_{11}, M_{22} and M_{33} are pivot positions

C_1, C_2 and C_3 are pivot columns.

(ii)

Augmented Matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -2 \\ 3 & 3 & -1 & 6 \\ 1 & -1 & 1 & -1 \end{array}\right]$$

$R_2 - 3R_1$
$R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -2 \\ 0 & 0 & -4 & 0 \\ 0 & -2 & 0 & 1 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} \textcircled{1} & 1 & 1 & -2 \\ 0 & \textcircled{-2} & 0 & 1 \\ 0 & 0 & \textcircled{-4} & 0 \end{array}\right]$$

Echelon Form

Circled elements are pivots

C_1, C_2 and C_3 are columns

M_{11}, M_{22} and M_{33} are pivot positions.

Q No 05

Matrix

$$\begin{bmatrix} 1 & 2 & -1 & 2 & 1 & 2 \\ -1 & -2 & 1 & 2 & 3 & 6 \\ 2 & 4 & -3 & 2 & 0 & 3 \\ -3 & -6 & 2 & 0 & 3 & 9 \end{bmatrix}$$

$R_2 + R_1$, $R_3 - 2R_1$, $R_4 + 3R_1$

$$\begin{bmatrix} 1 & 2 & -1 & 2 & 1 & 2 \\ 0 & 0 & 0 & 4 & 4 & 8 \\ 0 & 0 & -1 & -2 & -2 & -1 \\ 0 & 0 & -1 & 6 & 6 & 15 \end{bmatrix}$$

$R_2 \div 4$, $R_3(-1)$

$$\begin{bmatrix} 1 & 2 & -1 & 2 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 2 & 1 \\ 0 & 0 & -1 & 6 & 6 & 15 \end{bmatrix}$$

$R_1 + R_3$, $R_4 + R_3$

$$\begin{bmatrix} 1 & 2 & 0 & 4 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 2 & 1 \\ 0 & 0 & 0 & 8 & 8 & 16 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$, $R_4 \div 8$

$$\begin{bmatrix} 1 & 2 & 0 & 4 & 3 & 3 \\ 0 & 0 & 1 & 2 & 2 & 1 \\ 0 & 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 & 2 \end{bmatrix}$$

$R_1 - 4R_3$, $R_2 - 2R_3$, $R_4 - R_3$

$$\begin{bmatrix} 1 & 2 & 0 & 0 & -1 & -5 \\ 0 & 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Reduced Echelon Form

$$x_1 + 2x_2 - x_5 = -5 \quad \text{—(i)}$$

$$x_3 = -3$$

$$x_4 + x_5 = 2$$

$$x_1 = -5 - 2x_2 + x_5$$

x_2 is free variable

$x_3 = -3$

$x_4 = 2 - x_5$

x_5 is free variable

As the right most column is not pivot column so according to existence and uniqueness theorem it is a **consistent** solution.

Furthermore, it also contains free variables ie x_2 & x_5 so this system ~~can~~ hold **infinity** solutions.