Noumankhan 21I-2744 La2

Linear Algebra (National University of Computer and Emerging Sciences)

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Q1. Write the solution set of the given homogenous $Ax = B$ .... homogenous system.

Sol:

To write the solution in parametric vector form.

$$\begin{bmatrix} 2 & 2 & 4 \\ -4 & -4 & -8 \\ 0 & -3 & -3 \end{bmatrix}$$

Writing the augmented form.

$$\begin{bmatrix} 2 & 2 & 4 & 0 \\ -4 & -4 & -8 & 0 \\ 0 & -3 & -3 & 0 \end{bmatrix}$$

Applying row operations.

$$\overset{R}{\sim} \begin{bmatrix} 1 & 1 & 2 & 0 \\ -4 & -4 & -8 & 0 \\ 0 & -3 & -3 & 0 \end{bmatrix} \begin{matrix} \frac{1}{2}R_1 \\ \\ R_2+4R_1 \end{matrix}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -3 & -3 & 0 \end{bmatrix}$$

$x_1 + x_2 + 2x_3 = 0 \Rightarrow x_1 = 1 - 2x_3$

$-3x_2 - 3x_3 = 0 \Rightarrow x_2 = -1.$

So

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_3 \\ -x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} x_3 .$$

$$= Px_3 .$$

Vectors whose span give solution

$$\begin{bmatrix} 2 & 2 & 4 \\ -4 & -4 & -8 \\ 0 & -3 & -3 \end{bmatrix} \begin{bmatrix} -2 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

So $x_1 = -2$, $x_2 = -2$, $x_3 = 2$

When $Ax = b$ in parameter form.

$$\begin{bmatrix} 2 & 2 & 4 & 8 \\ -4 & -4 & -8 & -16 \\ 0 & -3 & -3 & 12 \end{bmatrix} \begin{array}{l} \frac{1}{2} R_1 \\ R_2 + 4R_1 \end{array}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & -3 & -3 & 12 \end{bmatrix}$$

$$x_1 + x_2 + 2x_3 = 4 \rightarrow ①$$

$$-3x_2 \cancel{+} -3x_3 = 12 - (2)$$

$$-3x_2 = 12 + 3x_3$$

$$x_2 = \frac{-(12 + 3x_3)}{3} - (\mathrm{II})$$

$$x_1 = \frac{4 + 12 - 3x_3 - 2x_3}{3}$$

$$x_1 = 8 - x_3$$

$$x_2 = -4 - x_3$$

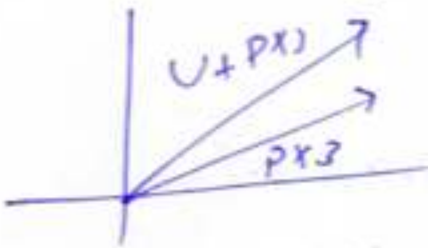
$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 8 - x_3 \\ -4 - x_3 \\ x_3 \end{pmatrix} = \begin{pmatrix} 8 \\ -4 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \qquad (2)$$

$$= v + p x_3 .$$

Geometric description:

Vector plane

translated and line is parrallel to main vector when their is non-homogenous eq. The plane is translated and parrallel to main vector.



Q2. Find the values of h if possible for which the vectors are linearly dependent.

$$\begin{bmatrix} 2 & 4 & -2 \\ -2 & -6 & h \\ 4 & 7 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & -2 \\ -2 & -6 & h \\ 4 & 7 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} 2 & 4 & -2 & 0 \\ 2 & -6 & h & 0 \\ 4 & 7 & 2 & 0 \end{pmatrix}$$

∵ Row operations.

$$\begin{bmatrix} 2 & 4 & -2 & 0 \\ 4 & 7 & 2 & 0 \\ -2 & -6 & h & 0 \end{bmatrix} \begin{matrix} R_2 - 2R_1 \\ R_3 + R_1 \end{matrix}$$

$$\begin{bmatrix} 2 & 4 & -2 & 0 \\ 0 & -1 & 6 & 0 \\ 0 & -2 & h-2 & 0 \end{bmatrix} R_3 - 2R_2$$

$$\begin{bmatrix} 2 & 4 & -2 & 0 \\ 0 & -1 & 6 & 0 \\ 0 & 0 & h+10 & 0 \end{bmatrix}$$

For the vectors to be linearly independent $h+10 = 0$ or $h \neq 10$

Q3: Let $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, y_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

.... is not linear.

Sol:

As given

$$T(e_1) = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, T(e_2) = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

So the matrix A is

$$A = \begin{bmatrix} 2 & -1 \\ 5 & 6 \end{bmatrix}$$

As we know that

$T(x) = A(x) = b$

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$$T\begin{bmatrix}5\\-3\end{bmatrix} = \begin{bmatrix}2 & -1\\5 & 6\end{bmatrix}\begin{pmatrix}5\\-3\end{pmatrix} = \begin{pmatrix}13\\7\end{pmatrix}$$

$$T\begin{bmatrix}x1\\x2\end{bmatrix} = \begin{bmatrix}2 & -1\\5 & 6\end{bmatrix}\begin{bmatrix}x_1\\x2\end{bmatrix} = \begin{pmatrix}2x_1 - x2\\5x_1 + 6x_2\end{pmatrix}$$

__Transformation:__

For the transformation to be linear this condition must be satisfied.

$T(x_1, x_2) = T(x_1) + T(x2)$

let $x_1(2,5)$, $x_2(-1,6)$

Given That

$(x_1 - 2x_2, 2x - 5x_2)$

$x1 + x2 = (1, 11)$

$T(\vec{x}1 + \vec{x}_2 = T(1,11) = (1-22, 2-55)$

$= (-21, -53)$

$T(x_1) = (2,5) = (2-10, 4-25) = (-8, -29)$

$T(x2) = (-1,6) = (-2-12, -2-36) = (-17, -32)$

$T(x_1) + T(x2) = (-8, -29) + (-17, -32)$

$= (-21, -53).$

+ condition is satisfied so it is linear transformable and statement is wrong.

Q4 let $T: R_2 \longrightarrow R_2$ be a linear transformation ..... one linear trans?

Sol.

$$\begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Augmented form becomes

$$\begin{bmatrix} 1 & -2 & 1 \\ 4 & 5 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 13 & -1 \end{bmatrix} R_2 - 4R_1$$

So $x_1 = 11/13$, $x_2 = -1/13$

$$y = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} 11/3 \\ -1/13 \end{bmatrix}$$

one to one:-

$$T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -2 & 1 \\ 4 & 5 & 3 \end{pmatrix}$$

For the transformation to be one solution must be trivial i-e $Ax = 0$, or vectors are linearly independent by theorem so this transformation is not one to one.

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Ontos:

$$\begin{bmatrix} 1 & -2 & 1 \\ 4 & 5 & 3 \end{bmatrix}$$

$$R_2 - 4R_1 = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 13 & -1 \end{bmatrix}$$

As there exists two pivots in R^2 or every column has its pivot so by theorem 4 it is onto because A spans $\mathbb{R}^2$.

Q5:

let $T: R^2 \longrightarrow R^2$ be a linear transformation .... $\phi = \pi/6$.

Sol:

1) Rotate by $\pi/4$:

The standard form is $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

When rotate by $\pi/2$

$$A_{rotate} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

2) Reflect by $y = x$:

When reflect general matrix is

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$S \cdot A = A \cdot ref \times A \cdot rot$

$$A = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & \sqrt{3}/2 \end{pmatrix}$$

$A = A_{ref}\ \varphi \times A_{rot}\ \theta$

$$A = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \times \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/\sqrt{2} & \sqrt{3}/2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\sqrt{3}+\sqrt{2}}{2\sqrt{2}} & \frac{\sqrt{3}-1}{2\sqrt{2}} \\ \frac{\sqrt{3}-\sqrt{2}}{2\sqrt{2}} & \frac{-\sqrt{3}-1}{2\sqrt{2}} \end{pmatrix}$$

Ans.

End :