Noumankhan I212744 LA#3

Linear Algebra (National University of Computer and Emerging Sciences)

NATIONAL UNIVERSITY OF COMPUTER & EMERGING SCIENCES

NAME: Nouman Khan

ROLL No : 21I-2744

Section : N

Assignment # 03 (L.A).

SUBMITTED To : Hina Salman.

Q #1: Find the determinant of the following matrix.

Sol:

P#1

$$\begin{bmatrix} 6 & 2 & 1 & 0 & 5 \\ 2 & 1 & 1 & -2 & 1 \\ 1 & 1 & 2 & -2 & 3 \\ 3 & 0 & 2 & 3 & -1 \\ -1 & -1 & -3 & 4 & 2 \end{bmatrix}$$

By row operation reduction method.

$$\sim \begin{bmatrix} 1 & 1 & 2 & -2 & 3 \\ 2 & 1 & 1 & -2 & 1 \\ 6 & 2 & 1 & 0 & 5 \\ 3 & 0 & 2 & 3 & -1 \\ -1 & -1 & -3 & 4 & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 2 & -2 & 3 \\ 0 & -1 & -3 & 2 & -5 \\ 0 & -4 & -11 & 12 & -13 \\ 0 & -3 & -4 & 9 & -10 \\ 0 & 0 & -1 & 2 & 5 \end{bmatrix} \begin{matrix} \\ R_2-2R_1 \\ R_3-6R_1 \\ R_4-3R_1 \\ R_5+R_1 \end{matrix}$$

$$1 \begin{vmatrix} -1 & 3 & 2 & -5 \\ -4 & -11 & 12 & -13 \\ -3 & 4 & 9 & -10 \\ 0 & -1 & 2 & 5 \end{vmatrix}$$

$$\sim \begin{bmatrix} 1 & -3 & 2 & 5 \\ 0 & 1 & 4 & 7 \\ 0 & 5 & 3 & 5 \\ 0 & -1 & 2 & 9 \end{bmatrix} \begin{matrix} R_2 - 4R_1 \\ R_3 - 3R_1 \end{matrix}$$

$$1 \cdot 1 \begin{bmatrix} 1 & 4 & 7 \\ 5 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix}$$

Using co-factor expansion across Columns.

$$(-1)^{1+1} 1 \begin{vmatrix} 3 & 5 \\ 2 & 5 \end{vmatrix} (-1)^{1+2} (4) \begin{vmatrix} 5 & 5 \\ -1 & 5 \end{vmatrix}$$

$$+ (-1)^{1+3} (7) \begin{vmatrix} 5 & 3 \\ 4 & 2 \end{vmatrix}$$

$$= 1(15-10) - 4(25+5) + 7(10+3)$$

$$= 5 - 4(30) + 7(13)$$

$$= 5 - 120 + 91$$

$$= -24$$

So $\boxed{|A| = -24}$

Question #02:

Find the inverse of given matrix using determinant.

P#2

$$\begin{bmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{bmatrix}$$

Sol:

let

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{bmatrix}$$

$\det(A) = 3(12) - 2(-6) - 1(-16) = 64$

determinent is 64.

Now $A^{-1} = \dfrac{\operatorname{adj} A}{|A|}$.

To find adjoint.

$C11 = + \begin{vmatrix} 6 & 3 \\ -4 & 0 \end{vmatrix} = 6(0) - 3(-4) = 12$

$C12 = - \begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix} = -(1(0) - 3(2)) = 6$

$C13 = + \begin{vmatrix} 1 & 6 \\ 2 & -4 \end{vmatrix} = 6(2) - 1(-4) = 12 + 4$
$= 16$

$C_{21} = -\begin{vmatrix} 2 & -1 \\ -4 & 0 \end{vmatrix} = -(2(0)-(-1)(-4))$

$= -4$

$C_{22} = +\begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix} = 3(0)-2(-1)$

$= 2$

$C_{23} = -\begin{vmatrix} 3 & 2 \\ 2 & -4 \end{vmatrix} = 3(-4)-2(2) = -12-4$

$= 16$

$C_{31} = \begin{vmatrix} 2 & -1 \\ 6 & 3 \end{vmatrix} = 2(3)-6(-1) = 6+6 = 12$

$C_{32} = -\begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix} = -(3(3)-1(-1)) = -(9+1)$

$= -10$

$C_{33} = \begin{vmatrix} 3 & 2 \\ 1 & 6 \end{vmatrix} = 3(6)-2(1) = 18-2 = 16$

$Adj = \begin{bmatrix} 12 & 4 & 12 \\ 6 & 2 & -10 \\ -16 & 16 & 16 \end{bmatrix}$

$A^{-1} = \dfrac{adj}{|A|} = \dfrac{\begin{bmatrix} 12 & 4 & 12 \\ 6 & 2 & -10 \\ -16 & 16 & 16 \end{bmatrix}}{64}$

$$= \begin{bmatrix} 3/16 & 1/16 & 3/16 \\ 3/32 & 1/32 & -5/64 \\ -1/4 & 1/4 & 1/4 \end{bmatrix} \text{ . Ans.}$$

Q#3: Find the volume $V(S)$ of the parallepiped S in R^3 determined by the vectors $v_1 = (1,1,0)$, $v_2(1,1,1)$, $V_3(0,2,3)$

Sol:

Volume of parrllelopiped $= |v_1 \cdot (v_2 \times v_3)|$

$$v_2 \times v_3 = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 0 & 2 & 3 \end{vmatrix}$$

$$= i(3-2) - j(3) + k(2)$$

$$= i - 3j + 2k$$

$$v_2 \times v_3 = (1, -3, 2)$$

Now

$$v_1 \cdot (v_2 \times v_3) = |(1 \times 1) + (1 \times (-3)) + 0 \times 2|$$

$$= |1 - 3 + 0|$$

$$= |-2| = 2 \text{ Ans.}$$

Q4 Use row operation to show that det $T = 0$

$$\begin{bmatrix} x^2 & 2x+1 & 4x+4 & 6x+9 \\ y^2 & 2y+1 & 4y+4 & 6y+9 \\ z^2 & 2z+1 & 4z+4 & 6z+9 \\ w^2 & 2w+1 & 4w+4 & 6w+9 \end{bmatrix}$$

Sol:

$$\begin{bmatrix} x^2 & 2x+1 & 4x+4 & 6x+9 \\ 0 & -\frac{y^2}{x^2}(2y+1) & -\frac{y^2}{x^2}(4y+4) & -\frac{y^2}{x^2}(6y+9) \\ 0 & & & \\ 0 & z^2/x^2(2z+1) & -\frac{z^2}{x^2}(4y+4 & -\frac{z^2}{x^2}(6y+9 \\ 0 & -w^2/z^2(2w+1 & -w^2/z^2(4z+4 & -\frac{w^2}{x^2}(6y+9 \end{bmatrix}$$

$R_2+(-\frac{y^2}{x^2})R_1$

$R_3+(\frac{z^2}{x^2})R_1$

$$-\frac{y^2}{x^2}-\frac{z^2}{x^2}\cdot\frac{w^2}{x^2}\begin{bmatrix} 2y+1 & 4y+4 & 6y+9 \\ 2z+1 & 4z-4 & 6z+9 \\ 2w+1 & 4w+4 & 6w+9 \end{bmatrix}$$

$R_4+(\frac{y^2}{x^2})R_1$

$$-\frac{y^2z^2w^2}{x^6}\begin{bmatrix} 2y+1 & 4y+4 & 6y+9 \\ 0 & \frac{-2z+1}{2y+1}(4z+4) & -\frac{(2z+1)}{2y+1}(6z+9) \\ 0 & -\frac{2(2w+1)}{2y+1}(4z+4) & -\frac{(2w+1)}{(2y+1)}(6x+9) \end{bmatrix}$$

$$-\frac{2z+1}{2y+1}$$

$$\begin{bmatrix} 4z+4 & 6z+9 \\ 4w+4 & 6w+9 \end{bmatrix} - \frac{y^2 z^2 w^2}{x^6}\left(\frac{2w+1}{2y+1}\right)\left(\frac{-2z+1}{2y+1}\right)$$

$(4z+4)(6w+9) - (6z+9)(4w+4)$

$24zw+36-24zw-36=0$

det $|A| = 0$ so inverse doesn't exists.

Q5: let $T: R^3 \to R^3$ be the linear transformation determined by the matrix.

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \quad \cdots\cdots$$

Sol a):

$$\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1$$

let $v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ and $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = Av$

Hence

$$U_1 = \frac{x_1}{a}, \quad U_2 = \frac{x_2}{b}, \quad U_3 = \frac{x_3}{c}$$

and U lies inside S.

and 2 lies lies inside $T(S) \left(\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} \leq 1 \right)$

(II):

According to theorem 10.

{volume of ellipsoid} = {volume of T(S)}

|det A| {volume of S}

$$|\det A| = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$|A| = abc$

volume of $S = 4\pi/3$

by putting volume in formula

we get

$\frac{4\pi abc}{3}$. Answer.