Single Period Problem with instantaneous demand.

- t is the const interval b/w order.
- Q is the stock for time t
- r is the estimated demand at a discontinuous rate with prob. $P(r)$.

C_1 is holding cost per item per t time unit

C_2 Shortag cost per item per t time unit

The optimal value of Q: $\int_0^Q f(r)\,dr = \frac{C_2}{C_1+C_2}$

Q. A baking company sells cake by the Kilogram. It makes a profit of 50 paise a Kilo on every ~~bag~~ sold on the day it is backed & it disposes of all cakes not sold on the date it is baked at a loss of 12 paise a kg. If demand is known to be rectangular b/w 2000 and 3000 kg. determine the optimum daily amount baked.

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Sol

$C_1 =$ Rs 0.12 (sale of left overs)

$C_2 =$ Rs 0.50 (Profit on sale)

Demand r is rectangular b/w 2000 and 3000 kg means the distribution $f(r)$ is continuous, given by

$$f(r) = \begin{cases} \frac{1}{b-a} & \text{if } a \le r \le b \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} \frac{1}{3000-2000} & \text{if } 2000 \le r \le 3000 \\ 0 & \text{otherwise} \end{cases}$$

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(b)

$= \frac{1}{1000}$ for $2000 \leq x \leq 3000$

otherwise

by P.d.f $\xrightarrow{0}$ $\int_0^Q f(x) = \frac{c_2}{c_1+c_2}$

$$\int_0^Q f(x)\,dx = \int_0^{2000} f(x)\,dx + \int_{2000}^{Q} f(x)\,dx.$$

$$= 0 + \int_{2000}^{Q} \frac{1}{1000}\,dx.$$

$$= \int_{2000}^{Q} \frac{1}{1000}\,dx$$

$$\int_0^Q f(x)\,dx = \frac{c_2}{c_1+c_2}$$

$$\int_{2000}^{Q} \frac{1}{1000}\,dx = \frac{0.50}{0.62}$$

$$\left.\frac{x}{1000}\right|_{2000}^{Q} = \text{''}$$

$$\left(\frac{Q}{1000} - \frac{2000}{1000}\right) = \frac{0.5}{0.62}$$

$$Q = 2.806.45 \text{ kg}$$

Example:- 3 G1 left 10/11/15

probabity distribution of monthly sales of a certain items as folows :-

Monthly Sales (r)	0	1	2	3	4	5	6
P(r) Probability	0.02	0.05	0.30	0.27	0.20	0.10	0.06

The cost of carrying inventory is Rs 10.00 per unit permonth. The current policy is to maintain a stock of four items at the beginning of each month. Assuming that the cost of shortage is proportional to both time and quantity short, obtain the imputed cost of a shortage of one item for one unit of time.

Solution:- Given, C_1 = Rs 10 per unit per month.
Q = Optimum Stock = 4 units.

Since the demand is uniformly distributed over the month, the least value of shortage cost C_2 can be determined using the relation.

$$\sum_{r=0}^{Q-1} p(r) + \left(Q-\frac{1}{2}\right) \sum_{r=Q}^{\infty} \frac{p(r)}{r} < \frac{c_2}{c_1+c_2} < \sum_{r=0}^{Q} p(r) + \left(Q+\frac{1}{2}\right) \sum_{r=Q+1}^{\infty} \frac{p(r)}{r}$$

Now,

$$\sum_{r=0}^{4-1} p(r) + \left(4-\frac{1}{2}\right) \sum_{r=4}^{6} \frac{p(r)}{r} = (0.64) + \frac{7}{2}\left[\frac{0.20}{4} + \frac{0.10}{5} + \frac{0.06}{6}\right] = 0.92$$

The least value of C_2 is given by

$$\frac{C_2}{C_1+C_2} \qquad \frac{C_2}{10+C_2} = 0.92$$

$C_2(1-0.92) = 9.2$ or, $C_2 = 115$

$$\frac{C_2}{C_1+C_2} = 0.92$$

$$\frac{C_2}{10+C_2} = 0.92$$

Or,

$0.92(10+C_2) = C_2 \Rightarrow 9.2 + 0.92C_2 = C_2$

$C_2 - 0.92C_2 = 9.2$

$C_2 = \frac{9.2}{0.08}$

$\boxed{C_2 = 115}$

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Similarly the greatest value of C_2 is obtained by considering

$$\frac{C_2}{C_1+C_2} = \sum_{r=0}^{4} p(r) + \left(4+\frac{1}{2}\right)\sum_{r=5}^{6}\frac{p(r)}{r}$$

$$= (0.84) + \frac{9}{2}\left[\frac{0.10}{5} + \frac{0.06}{6}\right]$$

$$= 0.975$$

$$C_2 = 0.975(10 + C_2) \Rightarrow C_2 = \text{Rs } 390$$

Hence, the range of shortage is given by

$$\text{Rs } 115 < C_2 < \text{Rs } 390.$$

* By the optimum value of Q.

$$\int_0^Q f(r)\,dr + \int_Q^\infty \frac{Q}{r} f(r)\,dr = \frac{C_2}{C_1+C_2}$$

Q.2 Same if $Q = 3$, $C_1 = 8$

$31 < C_2 < 92$

__Example 4.__

Let the probability density of demand of a certain item during a week be

$$f(r) = \begin{cases} 0.1 & 0 \le r \le 10 \\ 0 & \text{otherwise} \end{cases}$$

This demand is assumed to occur with a uniform pattern over the week. Let the unit carrying cost of the item in inventory be Rs 2.00 per week and unit shortage cost be Rs 8.00 per week. How will you determine the optimal order level of the inventory.

Solution: Since $f(r) = 0.1$, $0 \le r \le 10$, $C_1 = \text{Rs } 2.00$, $C_2 = \text{Rs } 8.00$, the optimum value of Q can be obtained by the relation.

*

$$\int_0^Q (0.1)\,dr + \int_Q^{10} \frac{Q(0.1)}{r}\,dr = \frac{8}{10} = 0.8$$

or, $(0.1)\left[Q + Q[\log r]_Q^{10}\right] = 0.8$

or, $(0.1)[Q + Q\log 10 - \log Q] = 0.8$ ln

or, $(0.1)[Q - Q\log Q + (2.3)Q] = 0.8$

or, $3.3Q - Q\log Q - 8 = 0$

The solution of this equation $Q = 4.5$ is obtained by trial and error method

	Fixed (Q)	Variable (S)
Variable (R)	(Q, R)	(S, R)
Fixed (t)	(Q, t)	(S, t)

Q1 The Production dep. for a require 3600 kg of raw material for manufacturing a Perticular item Per year. It has been estimated that the cost of Placing an order is Rs 36 and the cost of carrying inventory is 25 Percent of the investment in the inventories. The price is Rs 10 Per Kg. the Purchase manager wish To determine an ordering Policy for the raw material (Minimum yearly total cost).

$D = 3600$ Kg year

$C_s =$ Rs 36 Per order

$C_1 =$