To Use Simplex Method:

STEP 1: Convert constraints (linear inequalities) into linear equations using SLACK VARIABLES.

Slack variables:

s₁, s₂, s₃, etc.

For example:

if
$$x_1 + x_2 \le 10$$

then $x_1 + x_2 + s_1 = 10$

 $s_1 \ge 0$ and "takes up any slack"

Example 1: Convert each inequality into an equation by adding a slack

a)
$$2x_1 + 4.5x_2 \le 8$$

b)
$$x_1 + 3x_2 + 2.5x_3 \le 100$$

$$2x_1 + 4.5x_2 + s_1 = 8$$

Example 2:

- a) Determine the number of slack variables needed
- b) Name them c) Use slack variables to convert each constraint into a linear equation

c)
$$2x_1 + x_2 + x_3 + 5_1 = 150$$

 $2x_1 + 2x_2 + 8x_3 + 5_2 = 200$
 $2x_1 + 3x_2 + x_3 + 5_3 = 320$

Maximize
$$z = 3x_1 + 2x_2 + x_3$$

$$2x_1 + x_2 + x_3 \le 150$$

Subject to
$$2x_1 + 2x_2 + 8x_3 \le 200$$

 $2x_1 + 3x_2 + x_3 \le 320$

with
$$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$$

STEP 2: REWRITE the objective function so all the variables are on the left and the constants are on the right.

$$z=3x_1+2x_2+x_3$$

$$Z-3x_1-2x_2-x_3=0$$

$$-3x_1-2x_2-x_3+2=0$$
In alpha order

WRITE the modified constraints (from step 1) and the objective function (from step 2) as an augmented matrix. This is called the "simplex tableau."

There should be a row for each constraint. The last row is the objective function. EVERY variable used gets a column.

Example: Introduce slack variables as necessary, then write the initial simplex tableau for each linear programming problem.

Ex 3) Find
$$x_1 \ge 0$$
, $x_2 \ge 0$, and $x_3 \ge 0$ such that
$$10x_1 - x_2 - x_3 \le 138$$

$$13x_1 + 6x_2 + 7x_3 \le 205$$

$$14x_1 + x_2 - 2x_3 \le 345$$
and $z = 7x_1 + 3x_2 + x_3$ is maximized.

$$10x_1 - x_2 - x_3 + 5_1 = 138$$

 $13x_1 + 6x_2 + 7x_3 + 5_2 = 205$
 $14x_1 + 4x_2 - 2x_3 + 5_3 = 345$
 $-7x_1 - 3x_2 - x_3 + 2 = 0$

X,	X2	X3	51	52	53	圣	
10	-1	-1	1	0	0	0	138
13	6	7	0	1	0	0	138 205 345
14	ı	-2	0	0	1	0	345
-7	-3	-1	O	0	0	1	U

Ex. 4) Find
$$x_1 \ge 0$$
 and $x_2 \ge 0$ such that
$$2x_1 + 12x_2 \le 20$$
$$4x_1 + x_2 \le 50$$
and $z = 8x_1 + 5x_2$ is maximized.

$$2x_1 + 12x_2 + 5_1 = 20$$

 $4x_1 + x_2 + 5_2 = 50$
 $-8x_1 - 5x_2 + 7 = 0$

When you are looking at a simplex tableau, you may be able to spot <u>basic variables</u>.

A basic variable is a variable that only has all zeros except one number in its column in the tableau.

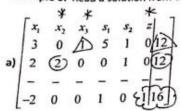
What are the basic variables in this simplex tableau?

$$X_2, X_3, Z$$

$$\begin{bmatrix} x_1 & x_2 & x_3 & s_1 & s_2 & z \\ 3 & 0 & 1 & 5 & 1 & 0 & 12 \\ 2 & 2 & 0 & 0 & 1 & 0 & 12 \\ - & - & - & - & - & - \\ -2 & 0 & 0 & 1 & 0 & 1 & 16 \end{bmatrix}$$
basic basic basic basic basic basic

One basic feasible solution can be found by finding the value of any basic variables and then setting all remaining variables equal to zero.

Example 6: Read a solution from the given simplex tableau.



$$2X_{2}=12 \Rightarrow X_{2}=6$$
 $X_{1}=0$ $X_{1}=0$ $X_{2}=6$ $X_{2}=6$ $X_{3}=12$ $X_{3}=12$ $X_{4}=16$

b)
$$\begin{bmatrix} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z \\ 2 & 0 & 10 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 & 0 & -30 \\ 0 & 0 & 0 & 0 & 2 & 20 & 0 & 14 \\ \hline -6 & 0 & 0 & 8 & 0 & 2 & 3 & -39 \end{bmatrix}$$

$$5X_{2}=-30$$
 $3S_{2}=4$ $X_{1}=0$ $S_{1}=0$ $X_{2}=-6$ $X_{2}=-6$ $X_{2}=-6$ $X_{2}=-6$ $X_{3}=-30$ $X_{3}=3$ X_{3