Probabilistic Inventory Models

Marginal cost of surplus per unit C_1=purchase cost -salvage value

Marginal cost of shortage per unit C_2=selling price - purchase cost

Let generalized probability distribution of the demand of the items be a discrete distribution as

Observation	i	1	2	…	n
Demand	D_i	D_1	D_2	…	D_n
Probability	P_i	P_1	P_2	…	P_n

The optimal order size D_i is determined by the relation

$$P_{i-1} < \frac{C_2}{C_1 + C_2} < P_i$$

Q1: the daily demand of bread at a bakery follows a discrete distribution as follow:

S No.	1	2	3	4	5	6	7	8	9	10	11
D	25	26	27	28	29	30	31	32	33	34	35
P	0.2	0.11	0.1	0.09	0.08	0.12	0.14	0.05	0.04	0.04	0.03

The purchase price of the bread is Rs. 8 per packet. The selling price of the bread is Rs. 11 per packet. If the bread packet nit sold within the day of purchase, they are sold at Rs. 4 per packet to hotels for secondary use. Find the optimal order size of the bread.

Solution:

purchase price of the bread is Rs. 8 per packet

selling price of the bread is Rs. 11 per packet

salvage price of the bread is Rs. 4 per packet

Marginal cost of surplus per unit $C_1 = 8 - 4 = 4$

Marginal cost of shortage per unit $C_2 = 11 - 8 = 3$

Therefore, cumulative probability

$$P = \frac{3}{4 + 3} = 0.43$$

Now we find the cumulative probability of demand.

S#	D	P	Cum P
1	25	0.2	0.2
2	26	0.11	0.31
3	27	0.1	0.41
4	28	0.09	0.5
5	29	0.08	0.58
6	30	0.12	0.7
7	31	0.14	0.84
8	32	0.05	0.89
9	33	0.04	0.93
10	34	0.04	0.97
11	35	0.03	1

From the table it follows that

$$P_3 < \frac{C_2}{C_1 + C_2} = 0.43 < P_4$$

$$0.41 < 0.43 < 0.5$$

Therefor, the optimal order size is D_4 which is equal to 28 breads

Q1: A probability Distribution of monthly sales of a certain item is as follows.

Monthly Sales (r)	0	1	2	3	4	5	6
Probability P(r)	0.02	0.05	0.3	0.27	0.2	0.1	0.06

The cost of carrying inventory is Rs. 10 per unit per month the current policy is to maintain a stock of four items at the beginning of each month. Assume that the cost of shortage is proportional to both time and quantity, obtain the computed cost of a shortage of one item for one unit of time.

Sol:

C_1= Rs. 10 per unit per month,

Q = a stock of four items = 4

Since the demand is uniformly distributed over the month, the least value of shortage cost C_2 can be determined using the relation.

$$\sum_{r=0}^{Q-1} P(r) + (Q-\tfrac{1}{2})\sum_{r=Q}^{\infty}\frac{P(r)}{r} \le \frac{C_2}{C_1+C_2} \le \sum_{r=0}^{Q} P(r) + (Q+\tfrac{1}{2})\sum_{r=Q+1}^{\infty}\frac{P(r)}{r}$$

LHS= $\sum_{r=0}^{Q-1} P(r) + (Q-\tfrac{1}{2})\sum_{r=Q}^{\infty}\frac{P(r)}{r} \le \frac{C_2}{C_1+C_2}$

$$\sum_{r=0}^{4-1} P(r) + (4-\tfrac{1}{2})\sum_{r=4}^{6}\frac{P(r)}{r} = 0.020 + .050 + .30 + .27 + (7/2)(0.2\tfrac{1}{4} + .1\tfrac{1}{5} + \tfrac{.06}{6})$$

$= 0.92 = \frac{C_2}{C_1+C_2}$

C_1=10

C_2=115

RHS= $\frac{C_2}{C_1+C_2} \le \sum_{r=0}^{Q} P(r) + (Q+\tfrac{1}{2})\sum_{r=Q+1}^{\infty}\frac{P(r)}{r}$

$$\sum_{r=0}^{4} P(r) + (4+\tfrac{1}{2})\sum_{r=5}^{6}\frac{P(r)}{r} = 0.020 + .050 + .30 + .27 + 0.2 + (9/2)(0.1\tfrac{1}{5} + \tfrac{.06}{6})$$

$= 0.975 = \frac{C_2}{C_1+C_2}$

C_1=10

C_2=390

$115 \le C2 \le 390$