To Use Simplex Method:

STEP 1: **Convert constraints (linear inequalities) into linear equations using SLACK VARIABLES.**

Slack variables:
s_1, s_2, s_3, etc.

For example:
If $x_1 + x_2 \le 10$
then $x_1 + x_2 + s_1 = 10$

$s_1 \ge 0$ and "takes up any slack"

Example 1: Convert each inequality into an equation by adding a slack variable.

a) $2x_1 + 4.5x_2 \le 8$

$\boxed{2x_1 + 4.5x_2 + s_1 = 8}$

b) $x_1 + 3x_2 + 2.5x_3 \le 100$

$\boxed{x_1 + 3x_2 + 2.5x_3 + s_1 = 100}$

Example 2:
a) Determine the number of slack variables needed
b) Name them
c) Use slack variables to convert each constraint into a linear equation

Maximize $z = 3x_1 + 2x_2 + x_3$

Subject to
$$2x_1 + x_2 + x_3 \le 150$$
$$2x_1 + 2x_2 + 8x_3 \le 200$$
$$2x_1 + 3x_2 + x_3 \le 320$$

with $x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$

a) 3 (one per constraint)

b) s_1, s_2, s_3

c)
$$\boxed{\begin{aligned} 2x_1 + x_2 + x_3 + s_1 &= 150 \\ 2x_1 + 2x_2 + 8x_3 + s_2 &= 200 \\ 2x_1 + 3x_2 + x_3 + s_3 &= 320 \end{aligned}}$$

STEP 2: REWRITE the objective function so all the variables are on the left and the constants are on the right.

$z = 3x_1 + 2x_2 + x_3 \longrightarrow z - 3x_1 - 2x_2 - x_3 = 0$

$$\boxed{-3x_1 - 2x_2 - x_3 + z = 0}$$

← Put variables in alpha order

STEP 3: WRITE the modified constraints (from step 1) and the objective function (from step 2) as an augmented matrix. This is called the "simplex tableau."

There should be a row for each constraint.
The last row is the objective function.
EVERY variable used gets a column.

	x_1	x_2	x_3	s_1	s_2	s_3	z	
constraints	2	1	1	1	0	0	0	150
	2	2	8	0	1	0	0	200
	2	3	1	0	0	1	0	320
objective	-3	-2	-1	0	0	0	1	0

Example: Introduce slack variables as necessary, then write the initial simplex tableau for each linear programming problem.

Ex 3) Find $x_1 \geq 0$, $x_2 \geq 0$, and $x_3 \geq 0$ such that

$$10x_1 - x_2 - x_3 \leq 138$$
$$13x_1 + 6x_2 + 7x_3 \leq 205$$
$$14x_1 + x_2 - 2x_3 \leq 345$$

and $z = 7x_1 + 3x_2 + x_3$ is maximized.

$$10x_1 - x_2 - x_3 + S_1 = 138$$
$$13x_1 + 6x_2 + 7x_3 + S_2 = 205$$
$$14x_1 + x_2 - 2x_3 + S_3 = 345$$
$$-7x_1 - 3x_2 - x_3 + z = 0$$

x_1	x_2	x_3	s_1	s_2	s_3	z	
10	-1	-1	1	0	0	0	138
13	6	7	0	1	0	0	205
14	1	-2	0	0	1	0	345
-7	-3	-1	0	0	0	1	0

Ex. 4) Find $x_1 \geq 0$ and $x_2 \geq 0$ such that

$$2x_1 + 12x_2 \leq 20$$
$$4x_1 + x_2 \leq 50$$

and $z = 8x_1 + 5x_2$ is maximized.

$$2x_1 + 12x_2 + s_1 = 20$$
$$4x_1 + x_2 + s_2 = 50$$
$$-8x_1 - 5x_2 + z = 0$$

$$\left[\begin{array}{ccccc|c} x_1 & x_2 & s_1 & s_2 & z & \\ \hline 2 & 12 & 1 & 0 & 0 & 20 \\ 4 & 1 & 0 & 1 & 0 & 50 \\ \hline -8 & -5 & 0 & 0 & 1 & 0 \end{array}\right]$$

When you are looking at a simplex tableau, you may be able to spot **basic variables**.
A **basic variable** is a variable that only has all zeros except one number in its column in the tableau.

What are the basic variables in this simplex tableau?

$$\left[\begin{array}{cccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & z & \\ 3 & 0 & 1 & 5 & 1 & 0 & 12 \\ 2 & 2 & 0 & 0 & 1 & 0 & 12 \\ - & - & - & - & - & - & - \\ -2 & 0 & 0 & 1 & 0 & 1 & 16 \end{array}\right]$$

basic basic basic "MAX"

x_2, x_3, z

One **basic feasible solution** can be found by finding the value of any basic variables and then setting all remaining variables equal to zero.

Example 6: Read a solution from the given simplex tableau.

a)

$$
\begin{array}{ccccccc}
 & * & * & & & * & \\
x_1 & x_2 & x_3 & s_1 & s_2 & z & \\
\end{array}
$$

$$
\left[\begin{array}{cccccc|c}
3 & 0 & 1 & 5 & 1 & 0 & 12 \\
2 & 2 & 0 & 0 & 1 & 0 & 12 \\
- & - & - & - & - & - & - \\
-2 & 0 & 0 & 1 & 0 & 1 & 16
\end{array}\right]
$$

$2x_2 = 12 \Rightarrow x_2 = 6$

$x_3 = 12$

$z = 16$

$x_1 = 0$	$s_1 = 0$
$x_2 = 6$	$s_2 = 0$
$x_3 = 12$	$z = 16$

b)

$$
\begin{array}{cccccccc}
x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & \\
\end{array}
$$

$$
\left[\begin{array}{ccccccc|c}
2 & 0 & 10 & -2 & 0 & 0 & 0 & 30 \\
0 & 5 & 0 & 10 & 0 & 0 & 0 & -30 \\
0 & 0 & 0 & 0 & \boxed{2} & 20 & 0 & \boxed{4} \\
\hline
-6 & 0 & 0 & 8 & 0 & 2 & 3 & -39
\end{array}\right]
$$

$5x_2 = -30$ $2s_2 = 4$

$x_2 = -6$ $s_2 = 2$

$10x_3 = 30$ $3z = -39$

$x_3 = 3$ $z = -13$

$x_1 = 0$	$s_1 = 0$
$x_2 = -6$	$s_2 = 2$
$x_3 = 3$	$s_3 = 0$
	$z = -13$