Laiba Fatima
22K-5195
BSE-1B

Date______20___

Cal - Assignment 2b

$\Rightarrow \quad f_1(x) = x^2(x-1)^{2/3} \quad : \quad [-1, 2]$

a) $f'(x)$ $\quad u = x^2 \quad v = (x-1)^{2/3}$

$u' = 2x \quad v' = \frac{2}{3}(x-1)^{-1/3}$

$$2x(x-1)^{2/3} + \frac{2x^2}{3(x-1)^{1/3}} = \frac{2x^2 + 6x(x-1)}{3(x-1)^{1/3}}$$

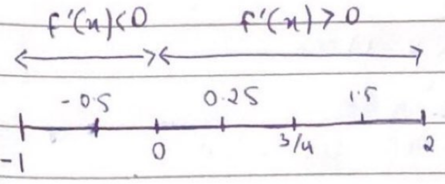
$f'(x) = 0$

$2x^2 + 6x(x-1) = 0$

$x(2x + 6x - 6) = 0$

$x = 0 \qquad 8x - 6 = 0$

$x = \frac{3}{4}$



$f'(-0.5) = -1.16 \qquad f'(0.25) = 0.367 \qquad f'(1.5) = 3.78$

$f'(x)$ is maximum during the interval $(0, 2]$

b) $f'(x)$ is minimum during interval $[-1, 0)$

c) at $x = 0$: $\frac{2(0)^2 + 6(0)(0-1)}{3(0-1)^{1/3}} = 0 \rightarrow$ stationary point

at $x = 3/4$: $\frac{2(3/4)^2 + (6 \times \frac{3}{4})(\frac{3}{4} - 1)}{3(3/4 - 1)^{1/3}} = 0 \rightarrow$ stationary point

at $x = 1$: $\frac{2(1) + 6(1-1)}{3(1-1)^{1/3}} = \frac{2}{0} \rightarrow$ undefined

Laiba Fatima
22K-5195

Date______20___

d) $f''(x)$

$u = 8x^2 - 6x \qquad v = 3(x-1)^{1/3}$

$u' = 16x - 6 \qquad v' = (x-1)^{-2/3}$

$$\frac{3(x-1)^{1/3}(16x-6) - (8x^2-6x)(x-1)^{-2/3}}{(3(x-1)^{1/3})^2}$$

$$= \frac{(48x - \cancel{18}\,18)(x-1)^{1/3} - (8x^2-6x)(x-1)^{-2/3}}{9(x-1)^{2/3}}$$

$f''(x) = 0$

$(48x-18)(x-1)^{1/3} = (8x^2-6x)(x-1)^{-2/3}$

$(24x-9)(x-1)^{1/3} = \dfrac{4x^2-3x}{(x-1)^{2/3}}$

$(24x-9)(x-1) = 4x^2 - 3x$

$24x^2 - 24x - 9x + 9 = 4x^2 - 3x$

$20x^2 - 30x + 9 = 0 \rightarrow \dfrac{-(-30) \pm \sqrt{(-30)^2 - 4(20)(9)}}{2(20)}$

$x = \dfrac{30 \pm 6\sqrt{5}}{40} = \dfrac{15 \pm 3\sqrt{5}}{20}$

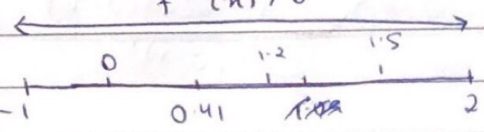
$x = 1.09 \quad \& \quad x = 0.41$

$f''(0) = 2$

$f''(1.2) = 3.42$

$f''(1.5) = 2.92$

$f''(x) > 0$



e) No interval in which f(x) is concave down

f) $(45x - 18)(x-1)^{1/3} - (8x^2-6x)(x-1)^{-2/3} = 0$

No inflexion point

Laiba Fatima
22K-5195

Date ______ 20 ___

g) $f(-1) = -1.59$ (min) $\qquad f(3/4) = 0.22$

$f(0) = 0 \qquad f(2) = 4$ (max)

minima $= -1$ & maxima $= 4$

$\Rightarrow f_2(x) = x^{3/5}(4-x)$ on $[-1, 6]$

a) $f'(x) \qquad u = x^{3/5} \qquad v = 4-x$

$u' = \frac{3}{5} x^{-2/5} \qquad v' = -1$

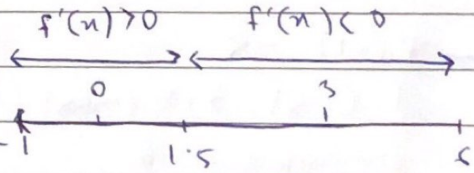
$$f'(x) = -x^{3/5} + \frac{3(4-x)}{5x^{-2/5}} = -5x + 3(4-x)$$

$f'(x) = 0$

$-5x = -12 + 3x$

$8x = +12$

$x = \frac{3}{2}$



$f'(0) = 12$

$f'(3) = -12$

$f'(x)$ is maximum during the interval $[-1, 1.5)$

b) $f(x)$ is minimum during the interval $(1.5, 6]$

c) $x = 3/2 \rightarrow -5(3/2) + 3(4 - 3/2) = 0 \rightarrow$ stationary points

d) $f''(x) = -x^{3/5} + \frac{12-3x}{5x^{2/3}}$

$f''(x) = 0 \rightarrow -3x^4 + 12x^{-3} - 3x^2 + 7.5x^2 = 0$

$= -3x^4 - 3x^2 + 7.5x^2 = -\frac{12}{x^3}$

Laiba Fatima

22K-5195

Date ______ 20__

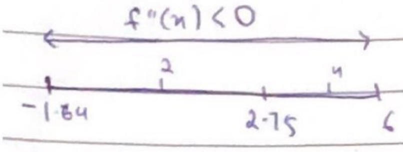
$= -3x^7 - 3x^5 + 7.5x^5 = -12$

$= \cancel{\varnothing}\, x^6(-3x^2 + 4.8) = -12$

$3x^2 = 16.5$

$x = 2.35 \qquad x = -1.64$

$f''(x) < 0$



$f''(2) = -0.089$ No interval

$f''(4) = -0.034$

e) $[-1, 6)$ concave down

f) No inflexion point

g) $f(-1) = -5$ $\qquad f(6) = -5.86$ (min

$f(1.5) = 3.19$ (max)

minima $= 6$ $\qquad$ maxima $= 1.5$

$\Rightarrow \quad f_3(x) = \dfrac{3x}{x^2+4}$ on $[-5, 5]$

a) $f'(x)$ $u = 3x \qquad v = x^2 + 4$

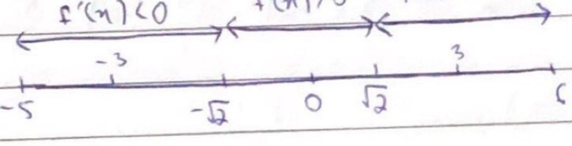
$u' = 3 \qquad v' = 2x$

$f'(x) = \dfrac{2x^2 + 8 - 6x^2}{(x^2+4)^2} = \dfrac{-4x^2 + 8}{x^4 + 8x^2 + 16}$

$f'(x) = 0$

$-4x^2 = -8$

$x = \pm\sqrt{2}$



$f(-3) = -0.32 \qquad f(3) = -0.16 \qquad f(0) = 1/2$

$f'(x)$ is maximum at $(-\sqrt{2}, \sqrt{2})$

Laiba Fatima
22K-5195

Date______20___

b) $f(x)$ is minimum in $(-5, -\sqrt{2})$ and $(\sqrt{2}, 5)$

c) at $x = -\sqrt{2}$, $f'(x) = 0 \rightarrow$ stationary point

at $x = \sqrt{2}$, $f'(x) = 0 \rightarrow$ stationary point

d) $f''(x) =$

$u = -4x^2 + 8 \qquad v = (x^2+4)^2$

$u' = -8x \qquad v' = 2(x^2+4) \times 2x = 4x^2(x^2+4)$

$$= \frac{-8x(x^2+4)\cancel{(4)} - (-4x^2+8)(4x^2)}{(x^2+4)^2}$$

$= -8x(x^2+4) = (-4x^2+8)$

$= -2x^2 + 8 = -4x^3 + 8x$

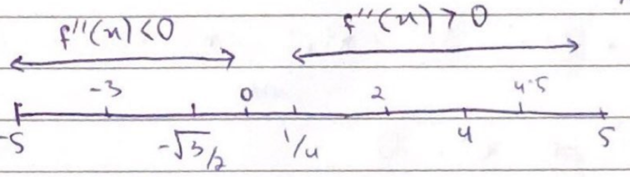
$= 2x(-2x^3 + x + 4) = 8$

$2x = 8 \qquad -2x^3 + x = 4$

$x = \frac{1}{4} \qquad x(-2x^2+1) = 4$

$x = 4 \qquad -2x^2 + 1 = 4$

$x = -\sqrt{3/2}$



$f''(-3) = -11.7$

$f''(0) = 0$

$f''(2) = 0.75$

$f''(4.5) = 0.353$

concave up $[-5, -\sqrt{3/2}]$

e) concave down $(1/4, 5]$

f) inflexion point $x = 1/4$

g) $f(-5) = 0.52 \qquad f(5) = 0.52$

$f(-\sqrt{2}) = -0.707$ (min) $\quad f(\sqrt{2}) = 0.707$ (max)

minimum $= -\sqrt{2} \qquad$ maximum $= \sqrt{2}$

Laiba Fatima
22K-5195

Date____20__

$\Rightarrow f_4(x) = x^2 e^x$

a) $f'(x) = \quad u = x^2 \quad v = e^x$

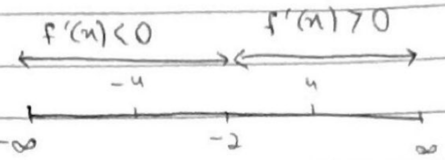
$u' = 2x \quad v' = e^x$

$f'(x) = x^2e^x + 2xe^x$

$f'(x) = 0$

$x^2e^x = -2xe^x$

$x = -2$



$f'(-4) = -0.44 \quad f'(4) = 1310.4$

$f(x)$ is maximum at $(-2, \infty)$

b) $f(x)$ is minimum $(-\infty, -2)$

c) at $x = -2 = f(x) = 0 \rightarrow$ stationary point

at $x = 0 \quad f(x) = 0 \rightarrow$ stationary point

d) $f''(x) \quad u = x^2 \quad v = e^x \qquad u = 2x \quad v = e^x$

$u' = 2x \quad v' = e^x \qquad u' = 2 \quad v' = e^x$

$f''(x) = x^2e^x + 2xe^x + 2xe^x + 2e^x = x^2e^x + 4xe^x + 2e^x$

$f''(x) = 0$

$x^2e^x + 4xe^x + 2e^x = 0 \Rightarrow e^x = 0$

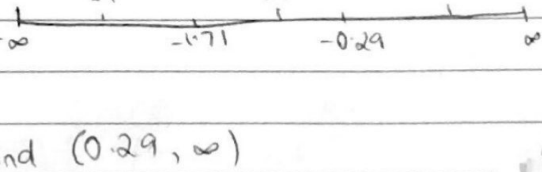
$\dfrac{-(4) \pm \sqrt{4^2 - 4(1)(2)}}{2(1)}$; $x = \dfrac{-2 \pm \sqrt{2}}{2}$ | $x = -0.29 \quad x = -1.71$

$x = -\infty$

$f''(-4) = -0.0371$

$f''(-1) = -0.37$

$f''(1) = 19.023$



concave up $(-\infty, 1.71)$ and $(0.29, \infty)$

Laiba Fatima
22K-5195

Date______ 20___

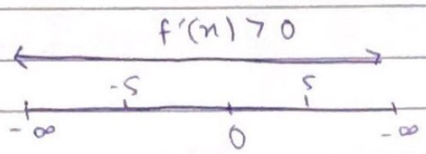
e) concave down $(-1.71, -0.29)$

f) inflexion point -1.71 & -0.29

g) $f(-2) = -0.54$ (min) $f(0) = 0$ (max)
max point $= 0$ min poi7 $= -2$

$\Rightarrow$ $f_5(x) = x^3$

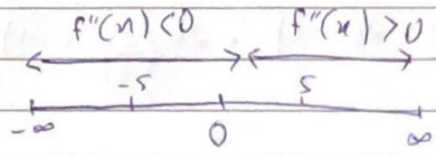
a) $f'(x) = 3x^2$
$f'(\cancel{0}x) = 0$
$3x^2 = 0$ $x = 0$



$f'(-5) = 75$ $f'(5) = 75$
maximum $(-\infty, \infty)$

b) No interval for minimum

c) $f'(0) = 0$ $\rightarrow$ stationary point



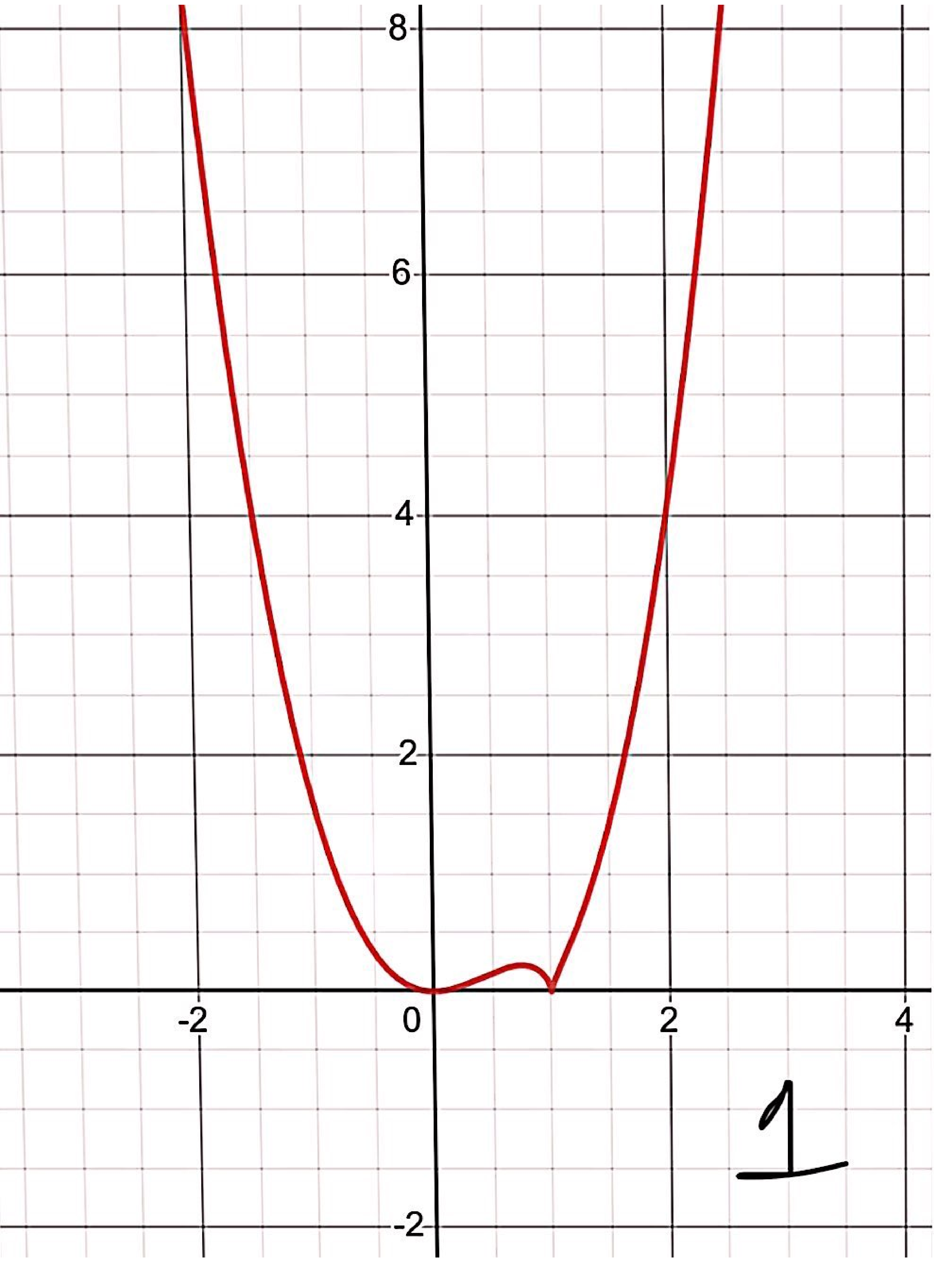
d) $f''(x) = 6x$
$f''(x) = 0$
$x = 0$

~~d)~~ $f''(-5) = -30$ $f''(5) = 30$
concave up $(0, \infty)$

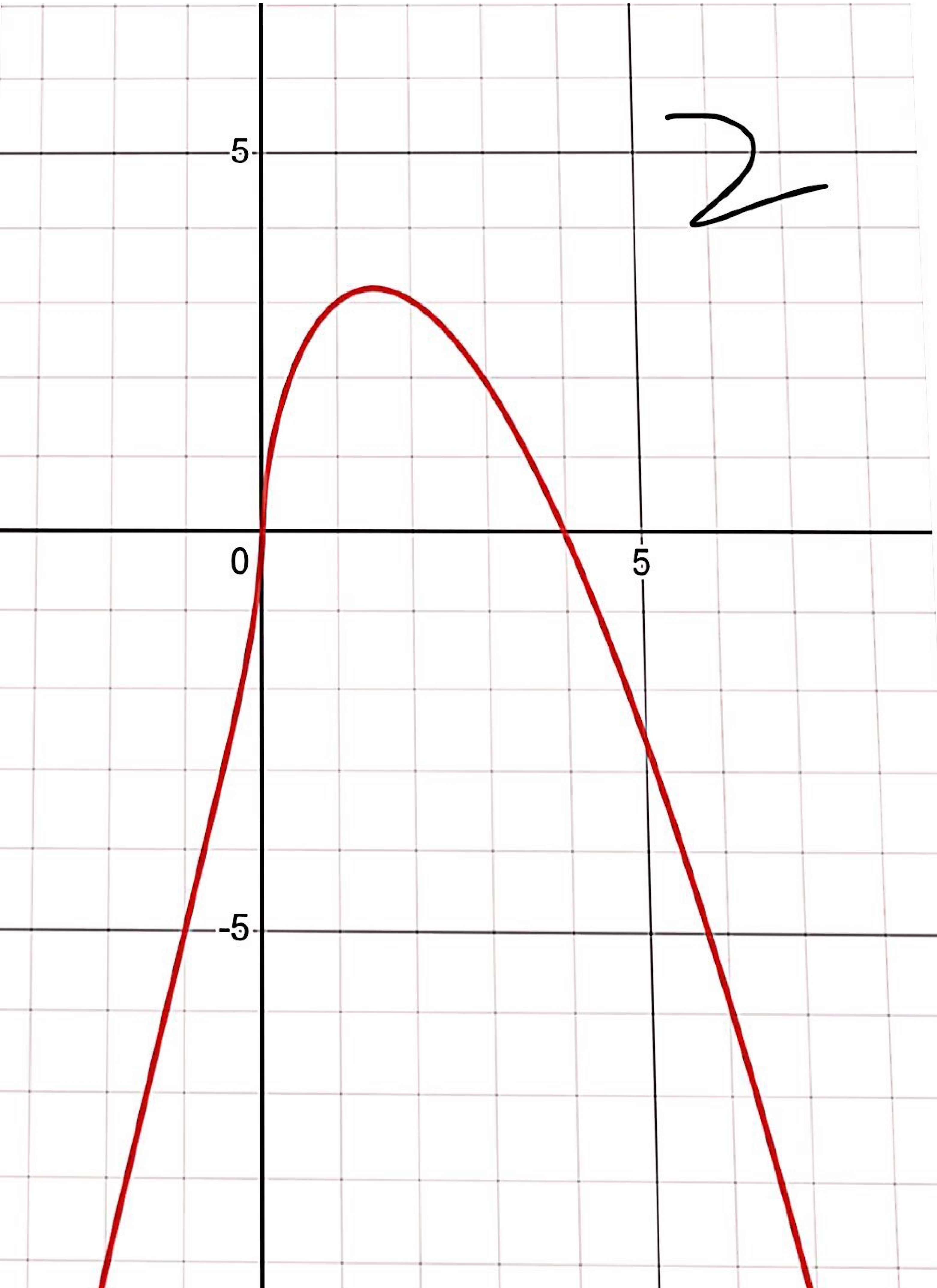
e) concave down $(-\infty, 0)$

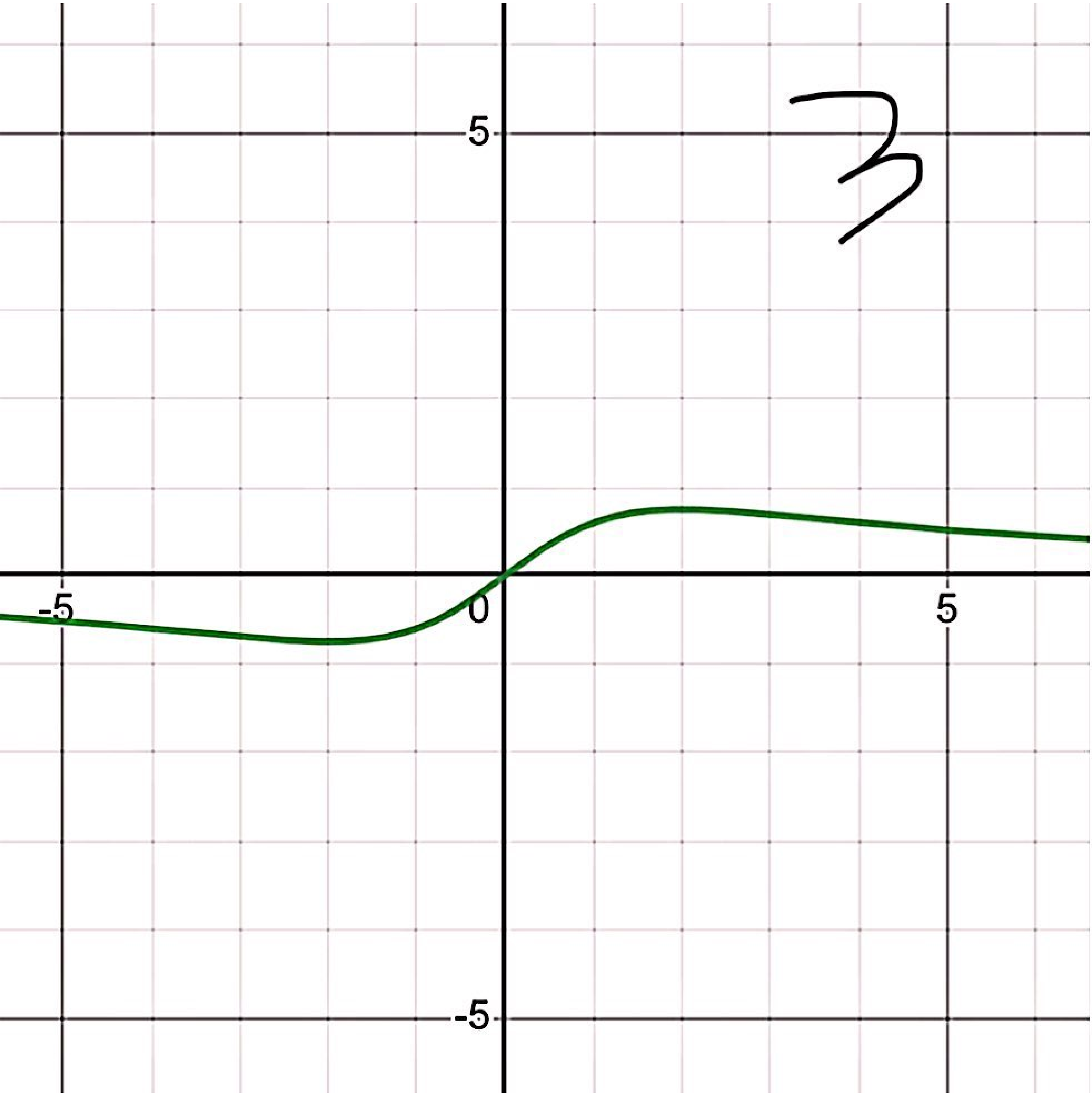
f) inflexion point $x = 0$

g) $f(0) = 0$ (max & min)



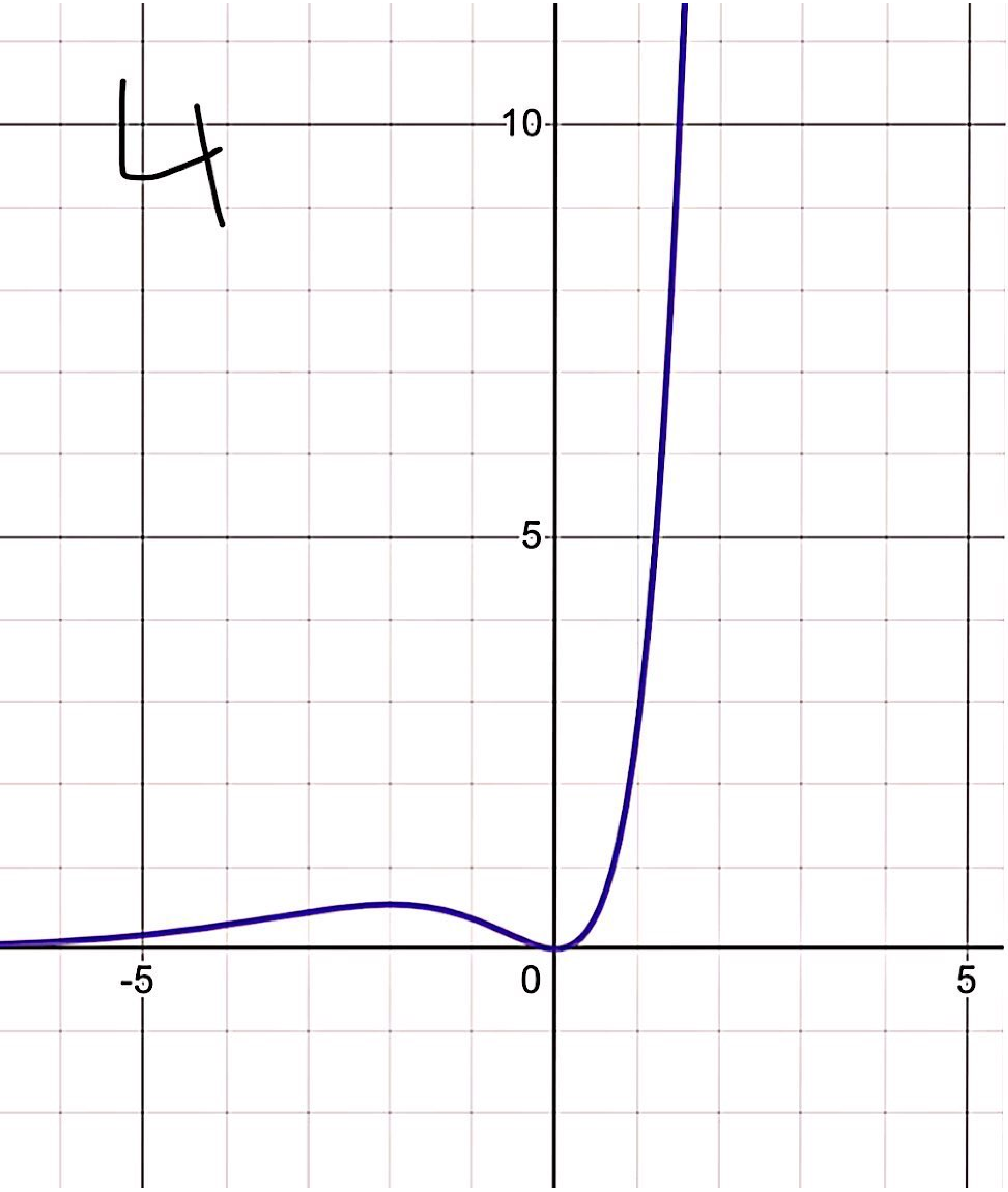
8
6
4
2
-2
-2
0
2
4
1

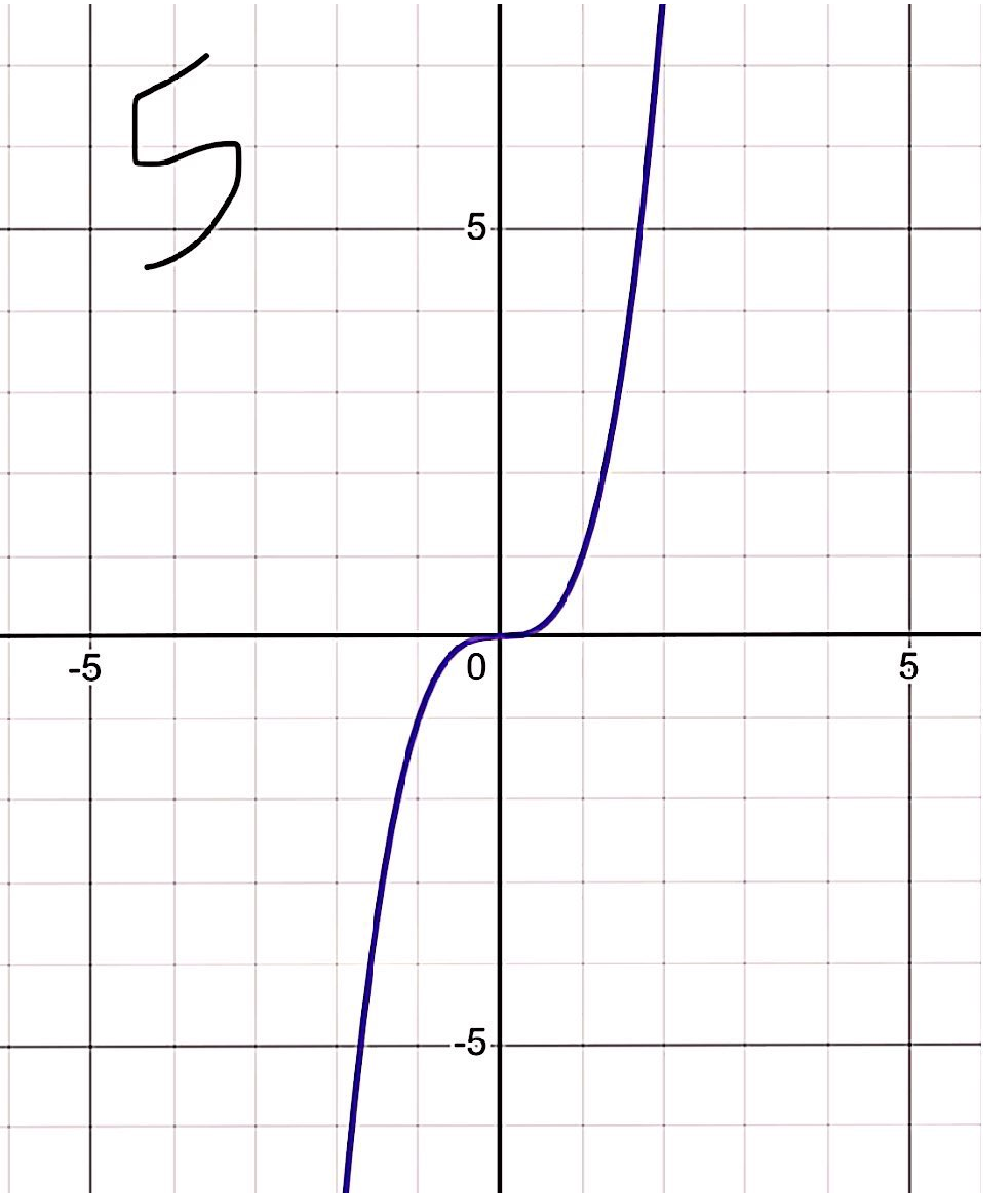

2
5
0
5
-5



3
5
-5
0
5
-5

4




5
5
-5
-5
0
5