National University of Computer & Emerging Sciences, Karachi
FAST School of Computing (BCY, BAI, & BCS)
Mid-Term 1 Examination, Spring-2023
Wednesday, March 01, 2023 11:30 am to 12:30 pm

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| Course Code: MT- 1004 | Course Name: Linear Algebra |
| Instructor Names : Ms Asma | Dr. Nazish Kanwal, Ms. Javeria Iftikhar, & Ms. Asma Masood |
| Student Roll No: 21K-4900 | Section No: B |

**Instructions:**

1. Answer all questions on answer script . Credit will be awarded for correct content and clarity of presentation.
2. There are 3 questions and 2 pages.

Time: 60 minutes. Max Points : 30

**Question 1:** ..... CLO 1 ..... *12 points*

(a) [7 points] For which values of $a$ will the following linear system has:
(i) no solution (ii) exactly one solution (iii) infinitely many solution

$$\begin{aligned} x + 5y - 4z &= 7 \\ 4x - 2y + 6z &= 6 \\ 2x + y + (a^2 - 8)z &= a - 3. \end{aligned}$$

(b) [5 points] Solve the linear system by inverting the coefficient matrix.

$$\begin{aligned} 4x + 5y &= 2 \\ 11x + y + 2z &= 3 \\ x + 5y + 2z &= 1 \end{aligned}$$

**Question 2:** ..... CLO 2 ..... *8 points*

(a) Let $\mathsf{T}(X) = AX$, where

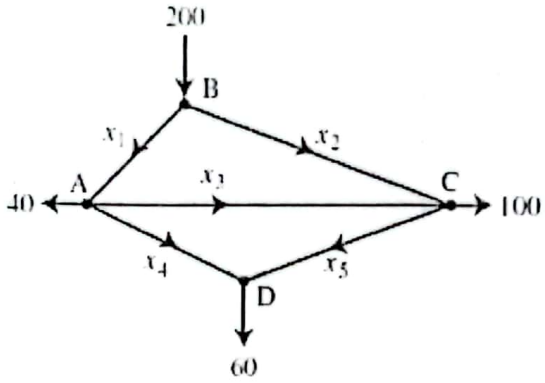
$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 4 \\ -1 & 0 & 2 \\ 4 & 1 & 1 \end{bmatrix}$$

i. [1 point] find the domain of T.
ii. [1 point] find the codomain of T.
iii. [2 points] write the tranformation in the form of linear equations.
iv. [1 point] compute $\mathsf{T}(-1, 2, 3)$.

(b) [3 points] Check whether $\mathsf{T}(x, y) = (3x + y,\ 2x + 5y)$ is a matrix transformation? If yes, prove that, if not, explain why.

**Question 3:** ..... CLO 3 ..... *10 points*

(a) Find the general traffic pattern in the freeway network shown in the figure. (Flow rates are in cars/minute.)




i. [4 points] Describe the general traffic pattern when the road whose flow is $x_4$ is closed.

ii. [1 point] When $x_4 = 0$, what is the minimum value of $x_1$?

(b) [5 points] Solve for $x$ by evaluating the determinant of the given matrices using some combinations of elementary row operations and cofactor expansion.

$$\begin{vmatrix} x & -1 \\ 3 & 1-x \end{vmatrix} = \begin{vmatrix} 1 & 0 & -3 \\ 2 & x & -6 \\ 1 & 3 & x-5 \end{vmatrix}$$

End of Exam