The Assignment Model Characteristics

- Special form of linear programming model similar to the transportation model.
- Supply at each source and demand at each destination limited to one unit.
- In a balanced model supply equals demand.
- In an unbalanced model supply does not equal demand.

ASSIGNMENT PROBLEMS

This group comprises problems where we are given a matrix of effectiveness, showing what happens when we associate each of a number of "origins" with each of the same number of "DESTINATIONS". Each origin is to be associated with ONE and ONLY ONE destination and we wish to make the associations in such a way as to minimize/maximize the summed effectiveness.

The assignment problem is a type of ALLOCATION problem in which n items are distributed among n boxes, one item to a box, in such a way that the return resulting from the distribution is optimized.

e.g.. A department head may have 5 people for assignment and 5 jobs to fill; which assignments will be best for the company?

e.g. A trucking company has an empty truck in each of cities 1,2,3,4,5,6 and needs an empty truck in each of cities 7,8,9,10,11,12. How should the trucks be dispatched so as to minimize the total mileage?

There is n! Such permutations (i.e. n! ways of assigning n items to n boxes). The method for choosing the optimal permutation or "assignment" is best explained with examples. Assignment problems are solved by a procedure called Hungarian Method (found by Hungarian mathematician called D. Koenig). The Hungarian Method involves what is called Matrix Reduction. Subtracting and adding appropriate values in the matrix yields an optimal solution to the assignment problems.

Problem-1

A department head has 4 subordinates, and 4 tasks to be performed. The subordinates differ in efficiency, and the tasks differ in their intrinsic difficulty. His estimate of the times each man would take to perform each task is given in the EFFECTIVENESS MATRIX below. How should the tasks be allocated, one to a man, so as to minimize the total man-hours?

Man

		I	II	III	IV
	A	8	26	17	11
Tasks	B	13	28	4	26
	C	38	19	18	15
	D	19	26	24	10

We choose the smallest number in row A and subtract it from each element in the row. The result is

0	18	9	3
13	28	4	26
38	19	18	15
19	26	24	10

Now suppose that we have assigned one task to each man. Whatever assignment we have made, the total man-hours for the new matrix will be 8 less than for the old matrix. Thus an assignment that minimizes the total for one matrix also minimizes the total for the other.
We now proceed to subtract the minimum element in each row from all the elements in its row, yielding

0	18	9	3
9	24	0	22
23	4	3	0
9	16	14	0

Next we subtract the minimum element in each column from all the elements in its column, obtaining

0	14	9	3
9	20	0	22
23	0	3	0
9	12	14	0

Our final step is that: we can choose an assignment that has a zero total; there cannot be an assignment with a lower total. Thus we make the following assignment.

A – I
B – III
C – II
D – IV

Problem-2

		New machines	
		M_1	M_2
Possible	P_1	78	68
Positions	P_2	82	62

Solution:

$$\begin{array}{cc} 10 & 0 \\ 20 & 0 \end{array} \Rightarrow \begin{array}{cc} 0 & 0 \\ 10 & 0 \end{array} \Rightarrow \begin{array}{cc} \boxed{0} & \cancel{0} \\ 10 & 0 \end{array}$$

Machine 1 to the P_1 78

Machine 2 to the P_2 62

140 MU

Problem-3

(minimize material handling costs)

		Possible positions			
		A	B	C	D
	Y_1	18	10	12	11
machines	Y_2	15	No work flow	13	20
	Y_3	5	7	10	6

Solution

$$\begin{array}{cccc} 18 & 10 & 12 & 11 \\ 15 & m & 13 & 20 \\ 5 & 7 & 10 & 6 \\ 0 & 0 & 0 & 0 \end{array} \Rightarrow \begin{array}{cccc} 8 & 0 & 2 & 1 \\ 2 & m- & 0 & 7 \\ 0 & 2 & 5 & 1 \\ 0 & 0 & 0 & 0 \end{array}$$

$$\Rightarrow \begin{array}{cccc} 8 & \boxed{0} & 2 & 1 \\ 2 & m- & \boxed{0} & 7 \\ \boxed{0} & 2 & 5 & 1 \\ \cancel{0} & \cancel{0} & \cancel{0} & \boxed{0} \end{array}$$

Problem-4

Possible Places

		A	B	C	D	E
	M_1	16	22	13	21	25
	M_2	9	7	12	6	15
Machines	M_3	13	16	15	12	16
	M_4	21	24	17	28	26
	M_5	14	10	12	11	15

<u>Solution:</u>

3	9	0	8	12		2	9	0	8	8
3	1	6	0	9		2	1	6	0	5
1	4	3	0	4	$\Rightarrow$	0	4	3	0	0
4	7	0	11	9		3	7	0	11	5
4	0	2	1	5		3	0	2	1	1

2	9	$\boxed{0}$	8	8
2	1	6	$\boxed{0}$	5
$\boxed{0}$	4	3	~~0~~	~~0~~
3	7	~~0~~	11	5
3	$\boxed{0}$	2	1	1

We have the maximal assignment so use the rule: starting with a maximal assignment.
a). Mark all rows for which assignments have not been made.

2	9	$\boxed{0}$	8	8	
2	1	6	$\boxed{0}$	5	
$\boxed{0}$	4	3	0	0	
3	7	0	11	5	√
3	$\boxed{0}$	2	1	1	

b). Mark columns <u>not</u> already marked which have <u>zeros</u> in marked rows.

2	9	$\boxed{0}$	8	8	
2	1	6	$\boxed{0}$	5	
$\boxed{0}$	4	3	0	0	
3	7	0	11	5	√
3	$\boxed{0}$	2	1	1	
		√			

c). Mark rows not already marked which have assignments in marked columns.

2	9	$\boxed{0}$	8	8	√
2	1	6	$\boxed{0}$	5	
$\boxed{0}$	4	3	0	0	
3	7	0	11	5	√
3	$\boxed{0}$	2	1	1	
		√			

d). Repeat steps b and c until the chain of marking ends. (No change)

e). Draw lines through all unmarked rows and through all marked columns.

2	9	$\boxed{0}$	8	8	√
2	1	6	$\boxed{0}$	5	
$\boxed{0}$	4	3	0	0	
3	7	0	11	5	√
3	$\boxed{0}$	2	1	1	
		√			

e). Select the smallest element that has no line through it.

- 2 in the example
- Subtract it from all the elements that do not have a line through them
- Add to the element at the intersection of two lines
- Leave the remaining elements on the lines unchanged
- Make the assignment

Final step is as follows:

$\boxed{0}$	7	$\cancel{0}$	6	6
2	1	8	$\boxed{0}$	5
$\cancel{0}$	4	5	$\cancel{0}$	$\boxed{0}$
1	5	$\boxed{0}$	9	3
3	$\boxed{0}$	4	1	1

Problem-5

(Estimated Profit 1000 MU) for each department- location combination.)

Suppose ABC Co., has just leased a new store and is attempting to determine where various departments should be located within the store. The store manager has 4 locations that have not yet been assigned a department and is considering 5 departments that might occupy the 4 locations.

The departments under consideration are: SHOES, TOYS, AUTO PARTS, HOUSEWARES and VIDEOS. After a careful study of the layout of the store, the store manager has made estimates of the expected annual profit for each department in each location.

	Location									
Dept.	1	2	3	4						
Shoe	10	6	12	8		10	6	12	8	0
Toys	15	18	5	11		15	18	5	11	0
Auto Parts	17	10	13	16	⇒	17	10	13	16	0
House wares	14	12	13	10		14	12	13	10	0
Video	14	16	6	12		14	16	6	12	0

8	12	6	10	18		2	6	0	4	12
3	0	13	7	18		3	0	13	7	18
1	8	5	2	18	⇒	0	7	4	1	17
4	6	5	8	18		0	2	1	4	14
4	2	12	6	18		2	0	10	4	16

2	6	0	3	0			2	8	0	3	0	
3	0	13	6	6	√		1	0	11	4	4	√
0	7	4	0	5		⇒	0	9	4	0	5	
0	2	1	3	2			0	4	1	3	2	√
2	0	10	3	4	√		0	0	8	1	2	√
	√						√	√				

3	9	0	3	0
1	0	10	3	3
1	10	4	0	5
0	4	0	2	1
0	0	7	0	1

Total Profit & Assignment.

Shoe	→	No assignment	
Toys	→ 2		18 (000) MU
A.Parts	→ 4		16 (000) MU
H.Wares	→ 3		13 (000) MU
Video	→ 1		14 (000) MU
			61 (000) MU

Problem-6

Baseball umpiring crews are currently in four cities where three-game series are beginning. When these are finished, the crews are needed to work games in four different cities. The distances (kms) from each of the cities where the crews are currently working to the cities where the new games will begin are shown in the table below.

To → From	Kansas	Chicago	Detroit	Toronto
Seattle	1500	1730	1940	2070
Arlington	460	810	1020	1270
Oakland	1500	1850	2080	X
Baltimore	960	610	400	330

The X indicates that the crew in Oakland cannot to send to Toronto.

Determine which crew should be sent to each of each city to minimize the total distance travelled and how many kms. will be travelled if these assignments are made?

Solution:

To From	(A) Kansas	(B) Chicago	(C) Detroit	(D) Toronto
Seattle (x)	1500	1730	1940	2070
Arlington (y)	460	810	1020	1270
Oakland (z)	1500	1850	2080	X
Baltimore (q)	960	610	400	330

Steps:

1. **Check whether it is square matrix or not?**
2. **If not, Put dummy column which contains zeros**
3. **Row reduction: choose the smallest number in the cell and subtract from the other numbers in each cell.**
4. **e.g. first cell: 1500, second cell: 460, third cell: 1500: and forth cell: 330.**

0	230	440	570
0	350	560	810
0	350	580	M
630	280	70	0

M is the very large number

0	230	440	570
0	350	560	810
0	350	580	M
630	280	70	0

5. **5. Column Reduction: Choose the smallest number in each and subtract from the other numbers in each column.**

0	$\boxed{0}$	370	570
$\boxed{0}$	120	490	810
0	120	510	M
630	50	$\boxed{0}$	0

6. **It is time to assign and apply the line method.**

0	0	370	570	
$\boxed{0}$	120	490	810	✓
0	120	510	M	✓
630	50	$\boxed{0}$	0	
✓				

120	$\boxed{0}$	370	570	✓
$\boxed{0}$	0	370	690	✓
0	0	390	M	✓
750	50	$\boxed{0}$	0	
✓				

120	$\boxed{0}$	0	200	
0	0	$\boxed{0}$	320	
$\boxed{0}$	0	20	M	
1150	420	0	$\boxed{0}$	

It is vicious circle so that there exists more than one optimal solution.