Assignment#02 - Solution

probability and statistics for engineering and the sciences (National University of Computer and Emerging Sciences)

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SOLUTION - ASSIGNMENT 02

①

Stratum	Cumulative Percentage	Percentage
1	17	17
2	40	23
3	71	31
4	95	24
5	100	5

n = 9% of 80,000 = 80,000 × 9% = 7,200

Sample from each stratum:

① 17% of total = 80,000 × 17% = 13,600

② 23% of total = 80,000 × 23% = 18,400

③ 31% of total = 80,000 × 31% = 24,800

④ 24% of total = 80,000 × 24% = 19,200

⑤ 5% of total = 80,000 × 5% = 4,000

① $\frac{13,600}{80,000} \times 7200 = 1224 \; (n_1)$

② $\frac{18,400}{80,000} \times 7200 = 1656 \; (n_2)$

③ $\frac{24,800}{80,000} \times 7200 = 2232 \; (n_3)$

④ $\frac{19,200}{80,000} \times 7200 = 1728 \; (n_4)$

⑤ $\frac{4000}{80,000} \times 7200 = 360 \; (n_5)$

7200

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2. $n_1 = 100$ $\quad n_2 = ?$

$\sigma_{\bar{x}_1} = 15$ $\quad \sigma_{\bar{x}_2} = 10$

$$\sigma_{\bar{x}_1} = \frac{\sigma}{\sqrt{n_1}} \qquad \sigma_{\bar{x}_2} = \frac{\sigma}{\sqrt{n_2}}$$

$$15 = \frac{\sigma}{\sqrt{100}} \qquad \sqrt{n_2} = \frac{\sigma}{\sigma_{\bar{x}_2}}$$

$$\sigma = 15 \times 10 \qquad \sqrt{n_2} = \frac{150}{10}$$

$$\sigma = 150$$

$$\sqrt{n_2} = 15 \Rightarrow n_2 = 225$$

3. $N = 1000$, $X = 600$ $> P = \frac{X}{N} = \frac{600}{1000} \Rightarrow P = 0.6$

$n = 100$

$$Q = 1 - P$$
$$= 1 - 0.6$$
$$Q = 0.4$$

① $\mu_{\hat{p}} = P$

Hence, $P = 0.6$ & $\mu_{\hat{p}} = 0.6$

② $\sigma^2_{\hat{p}} = \frac{PQ}{n} = \frac{(0.6)(0.4)}{100} = 0.0024$

$$\sigma_{\hat{p}} = \sqrt{0.0024} = 0.0489$$

$P(\hat{p} > 0.5)$

$$Z = \frac{0.5 - 0.6}{0.0489} \Rightarrow \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}}$$

$$= -2.04$$

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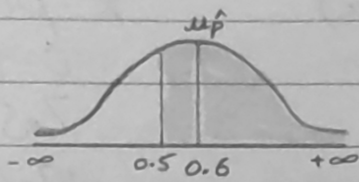

$$p(0.5 \le \hat{p}_1 \le \overset{(\mu_{\hat{p}})}{\underline{0.6}}) + p(\mu_{\hat{p}} \le \hat{p}_2 \le +\infty)$$

$$= p(2.04 \le z \le 0) + p(0 \le z \le +\infty)$$

$$= 0.47932 + 0.5$$

$$= 0.97932$$

$$= 97.93\%$$

4. $n = \dfrac{N}{1+Ne^2}$, $n_0 = \dfrac{Z^2 pq}{e^2}$, $n = \dfrac{x^2 NP(1-P)}{e^2(N-1)+x^2P(1-P)}$

5. $n = 55$ $\qquad$ $n = 165$

$\delta_{\bar{x}}^2 = 27$ $\qquad$ $\delta_{\bar{x}} = ?$

$\delta^2 = ?$

↳ $\delta_{\bar{x}} = \dfrac{\delta}{\sqrt{n}} \Rightarrow \delta_{\bar{x}}^2 = \dfrac{\delta^2}{n} \Rightarrow \delta^2 = \delta_{\bar{x}}^2 \times n$

$= 27 \times 55$

$\delta^2 = 1{,}485$

when $n = 165$, $\delta^2 = 1{,}485$, $\delta_{\bar{x}} = ?$

$$\delta_{\bar{x}} = \frac{\delta}{\sqrt{n}}$$

$$= \sqrt{\frac{1{,}485}{165}}$$

$$\delta_{\bar{x}} = 3$$

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6.

X	$P(x)$	$x \cdot p(x)$	$x^2 \cdot p(x)$
3	0.1	0.3	0.9
5	0.3	1.5	7.5
7	0.4	2.8	19.6
9	0.2	1.8	16.2
		$\Sigma x \cdot p(x) = 6.4$	$\Sigma x^2 \cdot p(x) = 44.2$

a) $\mu_{\bar{x}} = E(\bar{X}) = \Sigma x \cdot p(x) = 6.4$

b) $\sigma^2_{\bar{x}} = \Sigma x^2 \cdot p(x) - \{\Sigma x \cdot p(x)\}^2$

$= 44.2 - (6.4)^2$

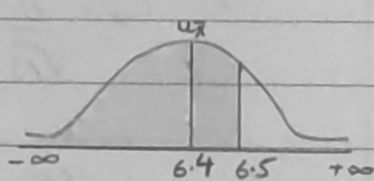
$= 3.24$

$\sigma_{\bar{x}} = 1.8$

c) $Z = \dfrac{\bar{X} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$

$= \dfrac{6.5 - 6.4}{1.8}$

$Z = 0.06$



$P(-\infty \le \bar{X} \le \mu_{\bar{x}}) + P(\mu_{\bar{x}} \le \bar{X} \le 6.5)$

$= P(-\infty \le Z \le 0) + P(0 \le Z \le 0.06)$

$= 0.5 + 0.02392$

$= 0.52392$

7.

a) Stratified Random Sampling is used.

b) Systematic Sampling is used.

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8. M, S, M, S, M

$N = 5$

Point of Interest : P = M

$P = \frac{X}{N} = \frac{3}{5} \Rightarrow 0.6 \qquad Q = 1 - P = 0.4$

Sample size = $n = 3$, w/o replacement

$^N C_n = {^5 C_3} = 10$

Distribution. fixate = $n - 1 = 2$

1 M, S, M
2 M, S, S
3 M, S, M
4 M, M, S
5 M, M, M
6 M, S, M
7 S, M, S
8 S, M, M
9 S, S, M
10 M, S, M

$\hat{p}$	$f(\hat{p})$	$\hat{p} \cdot f(\hat{p})$	$\hat{p}^2 \cdot f(\hat{p})$
1/3	3/10	3/30	3/90
2/3	6/10	12/30	24/90
3/3	1/10	3/30	9/90
		0.6	0.4

① $\mu_{\hat{p}} = E(\hat{p}) = \Sigma \hat{p} \cdot f(\hat{p})$

$= 0.6$

$\mu_{\hat{p}} = P$

$0.6 = 0.6$ Proved

② $\sigma^2_{\hat{p}} = \Sigma \hat{p}^2 \cdot f(\hat{p}) - \{\Sigma \hat{p} \cdot f(\hat{p})\}^2$

$= 0.4 - (0.6)^2$

$= 0.04$

$\sigma^2_{\hat{p}} = \frac{PQ}{n}\left(\frac{N-n}{N-1}\right) \Rightarrow 0.04 = \frac{(0.6)(0.4)}{3}\left(\frac{5-3}{5-1}\right)$

$0.04 = 0.04$

Proved

Maxim