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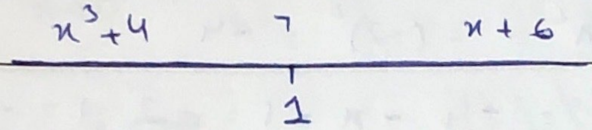
Date: Calculus Assignment 1

Q 1. $f(x) = \begin{cases} x^3+4 & , x<1 \\ 7 & , x=1 \\ x+6 & , x>1 \end{cases}$



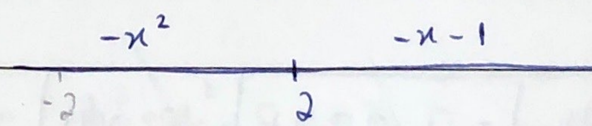
$\lim\limits_{x\to 1^+} f(x) : \lim\limits_{x\to 1^+} x+6 = (1)+6 = 7$

$\lim\limits_{x\to 1^-} f(x) : \lim\limits_{x\to 1^-} x^3+4 = (1)^3+4 = 5$

$\lim\limits_{x\to 1} f(x) :$ DNE RHS $\neq$ LHS

$f(1) : 7$

Q 2. $f(x) : \begin{cases} -x^2 & , x<2 \\ -x-1 & , x \geq 2 \end{cases}$



$\lim\limits_{x\to -2} f(x) : \lim\limits_{x\to -2^-} -x^2 = -(-2)^2 = -4$

$\lim\limits_{x\to 2} f(x) : \lim\limits_{x\to 2^-} = -x^2 = -(2)^2 = -4$

$\lim\limits_{x\to 2^+} = -x-1 = -2-1 = -3$

$\lim\limits_{x\to 2} f(x) =$ DNE

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Q2.

$\lim_{x \to 4} f(x) = \lim_{x \to 4} -x-1 = -4-1 = -5$

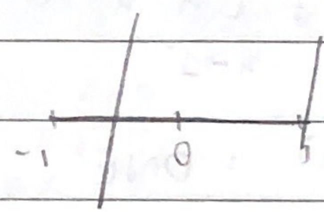
$f(-2) = \cancel{-4} - x^2 = -(-2)^2 = -4$

$f(2) = \cancel{-(-2)^2 = -4}\ -x-1 = -2-1 = -3$

$f(\cancel{2}4) = -x-1 = -4-1 = -5$

Q3.

i) $\lim_{x \to 0} \frac{3x+4}{x^2}$



x	-0.999	-0.9999
y		

x	-0.999999	-0.99999	-0.9999	-0.999
y	1.000005	1.000005	1.000501	1.00501

x	-0.00001	-0.0001	-0.001
y	-399997000	-3997	

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Q3.

i) $\lim_{x \to 0^-} \frac{3x+4}{x^2}$

-0.001	-0.01	-0.011
4×10^8	39700	32725

$= +\infty$

ii) $\lim_{x \to 3^+} \frac{x^2 - 9}{\sqrt{x-3}} = \frac{(x-3)(x+3)}{(x-3)^{1/2}}$

$= (x-3)^{1/2}(x+3)$

$= 0$

iii) $\lim_{x \to -2} \frac{x^2-4}{x^2-x-6}$ $\frac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}(x-3)} = \frac{x-2}{x-3}$

$= \frac{(-2)-2}{(-2)-3} = \frac{4}{5}$

iv) $\lim_{t \to 1} \frac{t^3+t^2-5t+3}{t^3-3t+2}$ improper

long division

$$\begin{array}{r|l} & 1 \\ \hline t^3-3t+2 & \cancel{t^3}+t^2-5\cancel{t}+3 \\ & -\cancel{t^3} \pm 0t^2 \mp \cancel{3}t \pm 2 \\ \hline & t^2-2t+1 \end{array}$$

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3 iv)

$$\cancel{1 + \frac{t^2 - 2t + 1}{t^3 + t^2 - 3t + 2}}$$

$$\cancel{1 + \frac{(t-1)(t-1)}{(t+2)(t-1)}}$$

$t = 1$

$$\frac{(1)^3 + (1)^2 - 5(1) + 3}{(1)^3 - 3(1) + 2} = \frac{0}{0}$$

derivative

$$\frac{3t^2 + 2t - 5}{3t^2 - 3} \quad \text{put } t = 1, \ \frac{0}{0}$$

differentiate

$$= \frac{6t + 2}{6t} = \frac{6(1) + 2}{6(1)}$$

$$= \cancel{\frac{8}{6}} = \frac{8}{6} = \frac{4}{3}$$

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v) $\lim\limits_{x \to 0} \dfrac{\sqrt{x+64} \cancel{x} - 8}{x}$

$= \dfrac{\sqrt{0+64} - 8}{0} = \dfrac{0}{0}$

Differentiate

$\frac{1}{2}(x+64)^{-1/2}$ put $x=0$

$\frac{1}{2}(0+64)^{-1/2}$

$= \dfrac{1}{16}$

vi) $\lim\limits_{y \to 0} \dfrac{5y^3 + 8y^2}{3y^4 - 16y^2} = \dfrac{5(0)+8(0)}{3(0)-16(0)} = \dfrac{0}{0}$

differentiate

$\dfrac{15y^2 + 16y}{12y^3 - \cancel{64y}\, 32y} \;\cancel{=}\; \neq \left(\dfrac{0}{0}\right)$

$\dfrac{30y + 16}{36y^2 - 32} = \dfrac{30(0)+16}{36(0)-32} = \dfrac{16}{-32}$

$= -\dfrac{1}{2}$

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Q4.

$\lim_{x \to -4^-} f(x) = 3$

$\lim_{x \to -2^-} f(x) = 2$

$\lim_{x \to -1} f(x) = \lim_{x \to -1^+} = 1 \quad \lim_{x \to -1^-} = 2$

$\lim_{x \to -1} f(x) = DNE$

$\lim_{x \to 3} f(x) : \lim_{x \to 3^+} f(x) = 1 \quad \lim_{x \to 3^-} f(x) = \cancel{4} +\infty$

$\lim_{x \to 3} f(x) = DNE$

Q5. i) $g(x) = \dfrac{\tan 3x}{(x+7)^4}$ Quotient rule

$u = \tan 3x \quad v = (x+7)^4$

$u' = 3\sec^2(3x) \quad v' = 4(x+7)^3$

$$\frac{vu' - uv'}{v^2}$$

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5. i) $\dfrac{(x+7)^4 \times 3\sec^2(3x) - \tan 3x \times 4(x+7)^3}{(x+7)^8}$

$\dfrac{3(x+7)^4 \sec^2(3x) - 4(x+7)^3 \tan(3x)}{(x+7)^8}$

ii) $\cancel{y} = \tan\sqrt{\theta} \; \sec\left(\dfrac{1}{\theta}\right)$ Product rule

$u = \tan\sqrt{\theta}$ $\quad v = \sec \dfrac{1}{\theta}$

$u' = \dfrac{1}{2} \cancel{\ldots} \sec^2\sqrt{\theta}\ (\theta)$

$u = \tan\sqrt{\theta}$ $\quad v = \sec \dfrac{1}{\theta} = \dfrac{1}{\cos 1/\theta}$

$u' = \dfrac{1}{2} \sec^2\sqrt{\theta} \times \theta^{-1/2}$

$v = \cos^{-1}\left(\dfrac{1}{\theta}\right)$

$u' = \dfrac{\sec^2\sqrt{\theta}}{2\sqrt{\theta}}$ $\quad v' = +\sin^{-2}\left(\dfrac{1}{\theta}\right)$

$vu' + uv'$

$\cos^{-1}\left(\dfrac{1}{\theta}\right) \dfrac{\sec^2\sqrt{\theta}}{2\sqrt{\theta}} + \tan\sqrt{\theta}\ \sin^{-2}\dfrac{1}{\theta}$

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5ii) $r = \tan\sqrt{\theta}\,\sec\frac{1}{\theta}$ Product rule

$u = \tan\sqrt{\theta}$ $v = \sec\frac{1}{\theta}$

$u' = \frac{\sec^2(\sqrt{\theta})}{2\sqrt{\theta}}$ $v' = \sec\frac{1}{\theta}\tan\frac{1}{\theta} \times -\frac{1}{x^2}$

$vu' + uv'$

$\sec\frac{1}{\theta}\left(\frac{\sec^2\sqrt{\theta}}{2\sqrt{\theta}}\right) - \frac{\sec\frac{1}{\theta}\tan\frac{1}{\theta}}{x^2}$

iii) $y = \left[\frac{t^2}{t^{3-4t}\cancel{+}\cancel{4t}}\right]^3$ $y = \left(\frac{1}{t^{1-4t}}\right)^3$

$\cancel{3}\cancel{t}^{3(4t-1)}$ $y = \frac{1}{t^{3-12t}}$

$\cancel{d}y' = \frac{-\frac{d}{dt}(t^{3-12t})}{(t^{3-12t})^2}$

$y' = \frac{-\frac{d}{dt}((e^{\ln(t)})^{3-12t})}{(t^{3-12t})^2}$

$\cancel{y'}$ let $g = \ln(t) \times (3-12t)$

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iii) $y' = \dfrac{\frac{d}{dg} e^{g} \times \frac{d}{dt}(\ln(t) \times (3-12t))}{(t^{3-12t})^2}$

$y' = \dfrac{e^{g} \times \left(\frac{1}{t} \times (3-12t) + \ln t \times -12\right)}{(t^{3-12t})^2}$

$y' = -3t^{-4+12t} + 12t^{12t-3} + 12t^{12t-3} \times \ln(t)$

iv) $q = \tan\left[\dfrac{\cos t}{t}\right]$

$\dfrac{d}{dt} = \sec^2\left(\dfrac{\cos t}{t}\right) \dfrac{d}{dt}\left[\dfrac{\cos t}{t}\right]$

$\dfrac{d}{dt} \dfrac{\cos t}{t}$ * Quotient rule

$u = \cos t \qquad v = t$

$u' = -\sin t \qquad v' = 1$

$\dfrac{vu' - uv'}{v^2} = \dfrac{-t \sin t - \cos t}{t^2}$

$= \dfrac{\sec^2\left(\frac{\cos t}{t}\right)(-t \sin t - \cos t)}{t^2}$

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Q5.

v) $y = \left(t^{-3/4} \times \sin t\right)^{4/3}$

$y = t^{-1} \times \sin t^{4/3}$

$\cancel{\frac{d}{dt} = -1 t^{-2} \times \sin}$

$u = t^{-1}$ $\qquad v = \sin t^{4/3}$

$u'' = -t^{-2}$ $\qquad v'' = \frac{4}{3} \sin t^{1/3} \cos t$

$vu' + uv'$

$-t^{-2} \sin t^{4/3} + t^{-1} \times \frac{4}{3} \sin t^{1/3} \cos t$