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Assignment 2 2020

Linear Algebra (National University of Computer and Emerging Sciences)

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Assignment 3.

Question 1

i) Scalar multiplication:-

$$c\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} cx \\ y \end{bmatrix}$$

As vector addition is standard

So axioms 1-5 must hold.

vi) :-

$$c\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} cx \\ y \end{bmatrix} \in V$$

vii)

$$c\left[\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}\right] = c\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + c\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

$$c\begin{bmatrix} x_1+x_2 \\ y_1+y_2 \end{bmatrix} = \begin{bmatrix} cx_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} cx_2 \\ y_2 \end{bmatrix}$$

$$\begin{bmatrix} c(x_1+x_2) \\ y_1+y_2 \end{bmatrix} \neq \begin{bmatrix} c(x_1+x_2) \\ y_2 \end{bmatrix}$$

viii) $$(k+m)\begin{bmatrix} x \\ y \end{bmatrix} = k\begin{bmatrix} x \\ y \end{bmatrix} + m\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\cancel{\begin{bmatrix} x(k+m) \\ y \end{bmatrix} = \begin{bmatrix} x(k+m) \\ y \end{bmatrix}}$$

$$\begin{bmatrix} (k+m)x \\ y \end{bmatrix} = \begin{bmatrix} kx \\ y \end{bmatrix} + \begin{bmatrix} mx \\ y \end{bmatrix}$$

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$$\begin{bmatrix} kx^2 + mu \\ y \end{bmatrix} \neq \begin{bmatrix} kx + mx \\ 2y \end{bmatrix}$$

Hence Axiom 8 fails to hold.

ix)

$$k \begin{bmatrix} mu \\ y \end{bmatrix} = km \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} kmu \\ y \end{bmatrix} = \begin{bmatrix} kmx \\ y \end{bmatrix}$$

x) $$1 \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \cdot x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

Axiom 8 fails to hold hence given vector is not a vector space.

ii) All 2×2 matrix of form $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

i) $U, V \in V$ then $U+V \in V$

$$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix} \in V$$

ii)

$$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} + \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$$

$$\begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix} = \begin{bmatrix} a_2+a_1 & b_2+b_1 \\ c_2+c_1 & d_2+d_1 \end{bmatrix}$$

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iii) $\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \left(\begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} + \begin{bmatrix} a_3 & b_3 \\ c_3 & d_3 \end{bmatrix} \right) = \left(\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \right) + \begin{bmatrix} a_3 & b_3 \\ c_3 & d_3 \end{bmatrix}$

$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2+a_3 & b_2+b_3 \\ c_2+c_3 & d_2+d_3 \end{bmatrix} = \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix} + \begin{bmatrix} a_3 & b_3 \\ c_3 & d_3 \end{bmatrix}$

$\begin{bmatrix} a_1+a_2+a_3 & b_1+b_2+b_3 \\ c_1+c_2+c_3 & d_1+d_2+d_3 \end{bmatrix} = \begin{bmatrix} a_1+a_2+a_3 & b_1+b_2+b_3 \\ c_1+c_2+c_3 & c_1+c_2+c_3 \end{bmatrix}$ Holds

$U+0=U$

iv) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ Holds

$U-U=0$

v) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ Holds

vi)

$k\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix} \in V$

Holds

vii) $k(U+V) = kU + kV$

$k\left(\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \right) = k\begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix}$

$= \begin{bmatrix} ka_1+ka_2 & kb_1+kb_2 \\ kc_1+kc_2 & kd_1+kd_2 \end{bmatrix} = \begin{bmatrix} ka_1 & kb_1 \\ kc_1 & kd_1 \end{bmatrix} + \begin{bmatrix} ka_2 & kb_2 \\ kc_2 & kd_2 \end{bmatrix}$

$= kU + kV$ Holds

viii) $(k+m)U = kU + mU$

$(k+m)\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka+ma & kb+mb \\ kc+mc & kd+md \end{bmatrix}$

$= \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix} + \begin{bmatrix} ma & mb \\ mc & md \end{bmatrix}$

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$= ku + mu$ Holds.

ix) $k(mu) = (km)u$

$$k\begin{bmatrix} ma & mb \\ mc & md \end{bmatrix} = \begin{bmatrix} kma & kmb \\ kmc & kmd \end{bmatrix} = km\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

x) $1 \cdot u = u$

$$1 \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$ Holds

All 10 axioms hold true so this set of matrix is a vector space.

Question 2:-

i) $V = R^3$, $W = \left\{ \begin{bmatrix} a \\ b \\ |a| \end{bmatrix} \right\}$

Axiom 1

$$x = \begin{bmatrix} a_1 \\ b_1 \\ a_1 \end{bmatrix} \quad y = \begin{bmatrix} -a_2 \\ b_2 \\ |-a_2| \end{bmatrix} = \begin{bmatrix} -a_2 \\ b_2 \\ a_2 \end{bmatrix}$$

$$x + y = \begin{bmatrix} a_1 - a_2 \\ b_1 + b_2 \\ a_1 + a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2b \\ 2a \end{bmatrix}$$

Here $0 \neq 2a$ so Axiom 1 doesn't hold.

This vector isn't a subspace.

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Q ii) $V = M_{22}$ $W = \begin{bmatrix} a & b \\ b & 2a \end{bmatrix}$

Axiom 1 $x, y \in V$

$x = \begin{bmatrix} a_1 & b_1 \\ b_1 & 2a_1 \end{bmatrix}$, $y = \begin{bmatrix} a_2 & b_2 \\ b_2 & 2a_2 \end{bmatrix}$

$x+y = \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ b_1+b_2 & 2a_1+2a_2 \end{bmatrix} \in V$ Holds

Axiom 6:-

$kx \in V$

$kx = k\begin{bmatrix} a & b \\ b & 2a \end{bmatrix} = \begin{bmatrix} ka & kb \\ kb & 2ka \end{bmatrix} \in V$ Holds

Hence given vector is vector space.

Spanning set

$W = \begin{bmatrix} a & b \\ b & 2a \end{bmatrix}$

$= a\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$W = \text{span}\left\{\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right\}$

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iii) $V = R^4$, $w = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : \begin{matrix} a - 2b = 4c \\ 2a = c + 3d \end{matrix}$

Finding a and c from equation

$a = 2b + 4c$

$c = 2a - 3d$

$W = \begin{bmatrix} 2b+4c \\ b \\ 2a-3d \\ d \end{bmatrix}$

$x = \begin{bmatrix} p \\ q \\ r \\ s \end{bmatrix}$ $y = \begin{bmatrix} t \\ u \\ v \\ w \end{bmatrix}$

Axiom 1 :-

$x, y, x+y \in W$

$x+y = \begin{bmatrix} p+b \\ q+u \\ r+v \\ s+w \end{bmatrix}$

equation for x ~~any~~ and y

$p - 2q = 4r \quad t - 2u = 4v$

$2p = r + 4s \quad 2t = v + 4w$

Adding respective eq.

$(p+t) - 2(q+u) = 4(r+v) \in V$

$2(p+t) = (r+v) + 4(s+w) \in V$

Thus $x+y \in W$

Axiom 6 :-

$kx \in V$

$kx = k \begin{bmatrix} p \\ q \\ r \\ s \end{bmatrix}$

$= \begin{bmatrix} kp \\ kq \\ kr \\ ks \end{bmatrix} \in V$

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Spanning set:-

$$W = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$a = 2b + 4c$

$c = 2a - 4d$

$c = 2(2b+4c) - \cancel{4}3d$

$c = 4b + 8c - \cancel{4}3d$

$$W = \begin{bmatrix} 2b+4c \\ b \\ c \\ \frac{4b}{3} + \frac{7}{3}c \end{bmatrix}$$

$3\cancel{4}d = 4b + 7c$

$d = \frac{4b}{3} + \frac{7}{3}c$

$$W = b\begin{bmatrix} 2 \\ 1 \\ 0 \\ 4/3 \end{bmatrix} + c\begin{bmatrix} 4 \\ 0 \\ 1 \\ 7/3 \end{bmatrix}$$

$$W = \text{span}\left\{\begin{bmatrix} 2 \\ 1 \\ 0 \\ 4/3 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 1 \\ 7/3 \end{bmatrix}\right\}$$

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iv) $V = \{ P(x) \text{ in } P_2 : xp'(x) = P(x) \}$

Axiom 1

$P(x), q(x) \in V$

$P(x) + q(x) \in V$

$P(x) = x\, p'(x)$

$q(x) = x\, q'(x)$

$P(x) + q(x) = xp(x) + xq(x)$

$= x(p'(x) + q'(x)) \in V$

Holds

Axiom 2:-

$k \cdot P(x) \in V$

$k \cdot p(x) = k \cdot x\, P'(x) \in V$

Holds

Hence $P(x)$ is a subspace.

Spanning set:-

let

$P(x) = ax^2 + bx + c$

$p'(x) = 2ax + b$

$x \cdot P'(x) = x(2ax + b)$

$x \cdot p'(x) = 2ax^2 + b$

$P(x) = xP'(x)$

$ax^2 + bx + c = 2ax^2 + bx$

$ax^2 - 2ax^2 + bx - bx + c = 0$

$c - ax^2 = 0$ compare coefficients

$c = 0,\ a = 0$

$P(x) = bx$

$P(x) = \text{span}\{x\}$

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v) $V = P_2$, $W = \{a + bx + cx^2 : abc = 0\}$

$P(x) = \cancel{a+bx}\ bx + c$ if $a = 0$

$q(x) = ax^2 + d$ if $b = 0$

Axiom 1

$P(x) + q(x)$

$bx + c + ax^2 + d = P(x) + q(x)$

$ax^2 + bx + (c+d) = P(x) + q(x)$

$(a)(b)(c+d) \neq 0$

Hence Axiom 1 doesn't hold and this vector isn't a subspace.

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Question 3:

$$V = \{A \text{ in } M_{22}\} \quad A \cdot B = B \cdot A \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in V$

$$A \cdot B = B \cdot A$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} a & a+b \\ c & c+d \end{bmatrix} = \begin{bmatrix} a+c & b+d \\ c & d \end{bmatrix}$$

$a = a+c \qquad a+\cancel{b} = \cancel{b}+d$

$c = c \qquad c+d = d$

$c = 0 \qquad a = d$

Put c and a in A

$$A = \begin{bmatrix} d & b \\ 0 & d \end{bmatrix}$$

$$A = d\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Basis of vector is $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$

~~only 2 dimensions~~

only d and b vector so

its 2 dimensional.

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Question 4:-

I $S: \{P_1, P_2, P_3\}$ is basis for P_2
$p_1 : 1 + 2x + x^2$, $P_2 = 2 + 9x$, $P_3 = 3 + 3x + 4x^2$

Writing in linear combination:-

$w(P_1, P_2, P_3) = c_1(1 + 2x + x^2) + C_2(2 + 9x) + C_3(3 + 3x + 4x^2)$

$= C_1 + 2C_1x + C_1x^2 + 2C_2 + 9C_2x + 3C_3 + 3C_3x + 4C_3x^2$

$(P_1, P_2, P_3) = (C_1 + 2C_2 + 3C_3, 2C_1x + 9C_2x + 3C_3x, C_1x^2 + 4C_3x^2)$

$P_1 = C_1 + 2C_2 + 3C_3 = 0$
$P_2 = 2C_1x + 9C_2x + 3C_3x = 0$
$P_3 = C_1 + 4C_3 = 0$

Taking det:-

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 9 & 3 \\ 1 & 0 & 4 \end{vmatrix}$$

$$= 1\begin{vmatrix} 9 & 3 \\ 0 & 4 \end{vmatrix} - 2\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} + 3\begin{vmatrix} 2 & 9 \\ 1 & 0 \end{vmatrix}$$

$= 1(36 - 0) - 2(8 - 3) + 3(9)$

$= 36 - 10 - 27 = -1 \neq 0$

Hence it is linearly independent and has unique solution for diff value of c_1, c_2 and c_3. $c_1 = c_2 = c_3 = 0$ will give trivial solution.

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