Assignment #2
~~Discrete~~ Discrete Structures

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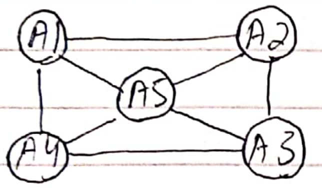
BSE-2B

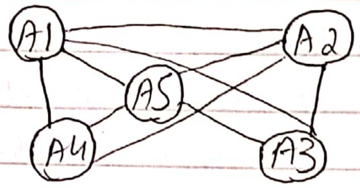
Q1 i, It has undirected edges
It has no loops
It has multiple edges
It has undirected multigraph

ii, It has undirected edges
It has no loops
It has no multiple edges
It is undirected simple graph

iii, It has undirected edges
It has three loops
It has multiple edges
It is undirected pseudo graph

iv, It has directed edges
It has two loops
It has multiple edges
It is directed multi graph.

Q2 i,


ii,


Q3 a i, No. of vertices - 5
No. of edges - 13

Degree of vertices = deg(a)=deg(b)=deg(c)=6 deg(d)=5 deg(e)=3
Neighbourhood vertices -
N(a)={a,b,e}, N(b)={a,b,d,e}, N(c)={b,c,d}
N(d)={b,c,e} N(e)={a,b,d}

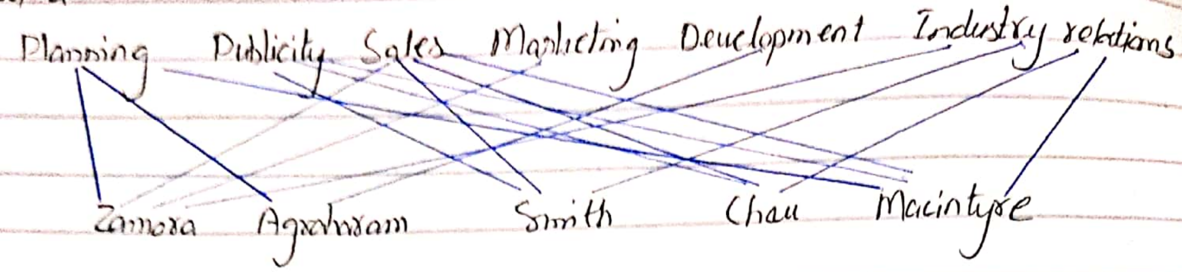
ii No. of vertices - 9
No. of edges - 12

Degree of vertices - deg(a)=3 deg(b)=2 deg(c)=4 deg(d)=0
deg(e)=6 deg(f)=0 deg(g)=4 deg(h)=2 deg(i)=3
Neighbourhood vertices - N(a)={c,e,i}, N(b)={e,h}, N(c)={a,e,g,i}
N(d)=ϕ N(e)={a,b,c,g} N(f)=ϕ N(g)={c,e} N(h)={b,i} N(i)={a,c,h}

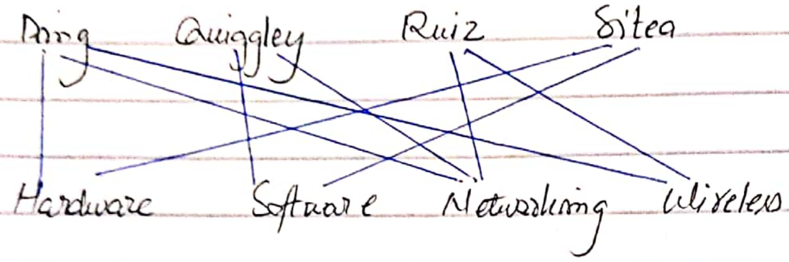
b i, In degree of vertices - $deg^-(a)=6$ $deg^-(b)=1$ $deg^-(c)=2$ $deg^-(d)=4$ $deg^-(e)=0$
Outdegree of vertices - $deg^+(a)=1$ $deg^+(b)=5$ $deg^+(c)=5$ $deg^+(d)=2$ $deg^+(e)=0$
no. of vertices = 5 edges = 13

ii, Indegree of vertices - $deg^-(a)=2$ $deg^-(b)=3$ $deg^-(c)=2$ $deg^-(d)=1$
Out degree of vertices - $deg^+(a)=2$ $deg^+(b)=4$ $deg^+(c)=1$ $deg^+(d)=1$
no. of vertices = 4 edges = 10

Qu 4 a



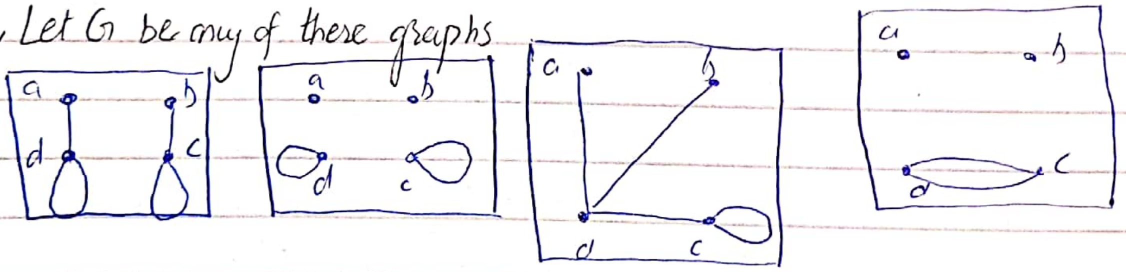
b,



Q5 i, Not bipartite (since a is adjacent to b+f vertices)
ii Bipartite (A (V_1, V_3, V_5) and B (V_2, V_4, V_6))
iii Not bipartite (since V_4, and V_5 are adjacent vertices)
iv, Not bipartite (since b is adjacent to d and e vertices)

Q6 i, Not such graph is possible. By Handshaking theorem, total degree of graph is even. But a graph of with four vertices of degrees 1,1,2 and 3 would have a total degree of $1+1+2+3=7$, which is odd.

ii, Let G be any of these graphs

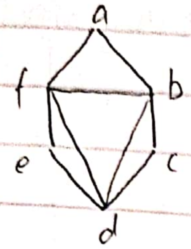


iii, There is no simple graph with four vertices of degrees 1,1,3 and 3

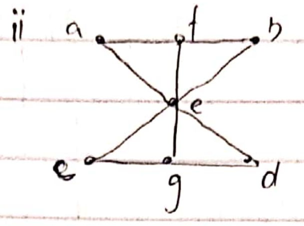
Q7 a, By using Handshaking theorem. No, there is no such graph possible, such that 15 vertices have degrees 3 (since $(15\times3)\neq 2e$

b, By using Handshaking theorem. Yes, there is a graph possible such that 4 vertices have degree 3. Since $(4\times3)=2e$

Q8 a i,



ii



b, - Regular graph of degree 4 with $m = 10$ edges

- n = vertices, each n vertices have 4 degree $\deg(v_i) = 4$ $(i = 1, 2)$.
- By handshaking theorem, the sum of degrees of all vertices is equal to twice the no. of edges
- $20 = 2(10) = 2m = \sum \deg(v_i) = \sum 4 = 4n$
- We then obtain the equation $20 = 4n$ $n = \frac{20}{4} = 5$

Q9 i, Both graph G and G' are satisfying all the invariant. Hence, they are ismorphic.
Function: $g(v_1) = w_2$ $g(v_2) = w_3$ $g(v_3) = w_1$ $g(v_4) = w_5$ $g(v_5) = w_4$

ii, Both graph G and G' are satisfying all the invariant. Hence, they are ismorphic.
Function $= g(v_1) = v_5$ $g(v_2) = v_2$ $g(v_3) = v_4$ $g(v_4) = v_3$ $g(v_5) = v_1$ $g(v_6) = v_6$

iii Both graph G and G' are satisfying all the invariant. Hence, they are isomorphic
Function $= g(v_1) = v_5$ $g(v_2) = v_4$ $g(v_3) = v_3$ $g(v_4) = v_2$ $g(v_5) = v_7$
$g(v_6) = v_1$ $g(v_7) = v_6$

iv, Graph G has no vertex of degree 4 where G' has vertex v_2 with degree 4. Hence, they are not ismorphic.

Q10 i)

N	D(b)	D(c)	D(d)	D(e)	D(f)	D(g)	D(h)	D(i)	D(j)	D(k)	D(l)	D(m)	D(n)	D(o)	D(p)	D(q)	D(r)	D(s)	D(t)	D(z)
a	2,a	4,a	1,a																	
ad	2,a	4,a			6,d	5,d														
adb		4,a		3,b	6,d	5,d														
adbe		4,a			6,d	5,d	6,e													
adbeg					6,c	5,d	6,e													
adbegf					6,c		6,e			7,g										
adbegf							6,e	8,f	10,f	7,g										
adbegfh								8,f	10,f	7,g	7,h			14,h						
adbegfhk								8,f	10,f		7,h		11,h	14,h			9,h			
adbegfhkl								8,f	10,f			10,I	11,l	13,I			9,h			
adbegfhkl									10,f			10,I	11,k	13,I			9,k			
adbegfhkli									10,f			10,I	10,r	13,I		17,r			14,r	
adbegfhklir												10,I	10,r	13,I		17,r			14,r	
adbegfhklirj													10,r	13,I	12,m	17,r			14,r	
adbegfhklirjm														13,I	12,m	12,m			14,r	
adbegfhklirjmn														13,I		12,n		14,p	13,p	
adbegfhklirjmnp														13,I				14,p	13,p	
adbegfhklirjmnpq																		14,p	13,p	
adbegfhklirjmnpqo																		14,p		21,t
adbegfhklirjmnpqot																				16,s
adbegfhklirjmnpqots																				
adbegfhklirjmnpqotsz																				

ii)

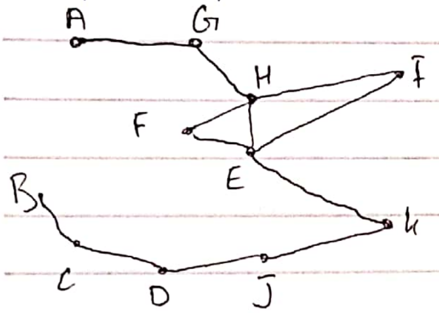
N	D(b)	D(c)	D(d)	D(e)	D(f)	D(g)	D(z)
a	4,a	3,a	∞	∞	∞	∞	∞
ac			6,c	9,c	∞	∞	∞
acb			6,c	9,c	∞	∞	∞
acbd				7,d	11,d	∞	∞
acbde					11,d	12,e	∞
acbdef						12,e	18,f
acbdefg							16,g
acbdefgz	4,a	3,a	6,c	7,d	11,d	12,e	16,g

Q11 i, Hamiltonian circuit ABCDA 125 ABDCA 140 ACBDA 155
~~Hamiltonian path~~ Hence ABCDA = 125 is the minimum distance travelled

Hamiltonian circuit ABCDA 97 ABDCA 108 ACBDA 141
~~Hamiltonian path~~. Hence ABCDA = 97 is the minimum distance travelled.

Q12 a, Yes, path A H -> G -> B -> C -> D -> G -> F -> F

b, floor plan



Each vertex of this graph has even edges except for A and B, each of which has degree 1. Hence by corollary 10.2.5 there is a euler path from A to B
AGFHEIHEKJDCB

Q13 i, Hamiltonian circuit $v_0, v_1, v_2, v_6, v_5, v_4, v_7, v_3$
Hamiltonian path $v_0, v_1, v_2, v_6, v_5, v_4, v_7$

ii Hamiltonian circuit does not exist
Hamiltonian path b, c, f, g, h, e, a, d

iii, Hamiltonian circuit: d, c, b, a, g, f, e, d
Hamiltonian path: d, c, b, a, g, f, e

Q14 a, All vertices have even degree so circuit exists
Euler circuit $v_1, v_2, v_5, v_4, v_5, v_2, v_3, v_4, v_1$

ii, Euler circuit does not exist because all vertices do not have even degree

b, Euler path does not exist because four vertices have odd degrees

b ii, Euler path exists because two vertices have odd degrees
Euler path $u, v_1, v_8, v_7, u, v_2, v_3, v_4, v_2, v_6, v_5, w, v_6, v_4, w$

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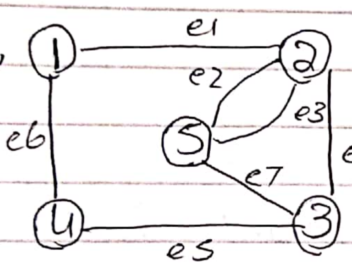
Q15.a

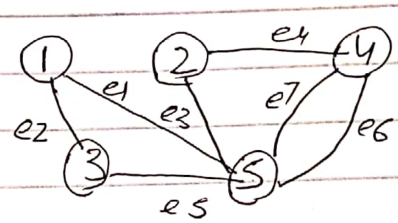
i) $\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

ii) $\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$

b i)



ii



Q16 i)

Initial Vertex	Terminal Vertices
a	a,b,c,d
b	d
c	a,b
d	b,c,d

$\begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$,

ii

Initial vertex	Terminal Vertices
a	b,d
b	a,c,d,e
c	b,c
d	a,e
e	c,e

$\begin{array}{c|c} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$

iii)

Vertex	Adjacent Vertices
a	b,d
b	a,c
c	b,d
d	a,c

$\begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$

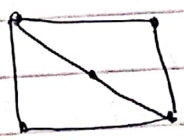
iv)

Vertex	Adjacent vertices
a	a,c,d
b	b,c,d
c	a,b,c
d	a,c,d

$\begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$

Q26

a)



b) Non-planar because we can form a $K_{3,3}$ graph with vertices {a,d,f} and {b,c,e}

Q27

a, R is reflexive because R contains (a,a), (b,b), (c,c) and (d,d).

b, R is not symmetric because R contains (a,c) but not (c,a) ∈ R

c, R is not anti symmetric because both (b,c) ∈ R and (c,b) ∈ R but b = c

d, R is not Transitive because both (a,c) ∈ R and (c,b) ∈ R, but not (a,b) ∈ R

e, R is not irreflexive R contains (a,a), (b,b) (c,c) and (d,d)

f, R is not assymetric because R is not antisymmetric

Q28

a, {(0,0), (1,1), (2,2), (3,3)}

b, {(1,3), (2,2), (3,1), (4,0)}

c, {(1,0) (2,0), (3,0), (4,0), (2,1), (3,1), (3,2), (4,1), (4,2), (4,3)}

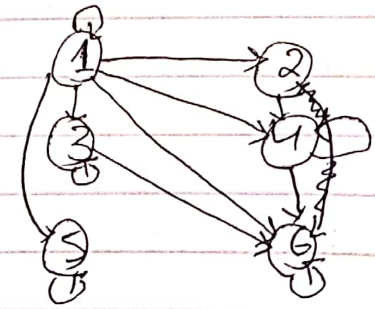
d, {(1,0), (2,0), (3,0), (4,0), (1,1), (1,2), (2,2), (1,3) (3,3)}

e, {(1,0), (0,1), (1,1), (1,2), (1,3), (2,1) (3,1) (4,1), (2,3) (3,2), (4,3)}

f {(1,2), (2,1), (2,2)}

Q29 R = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,4), (2,6), (3,3) (3,6), (4,4), (5,6) (6,6)}

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$



Q30

a, 1 R is not reflexive ; it doesn't contain (1,1) and (4,4)

2 R is not symmetric because R contains (2,4) but not (4,2) ∈ R

3 R is not antisymmetric, we have (2,3) and (3,2) but 2 ≠ 3

4 R is transitive because for any numbers a,b and c, if (a,b)(b,c) ∈ R then (a,c) ∈ R

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b 1 R is reflexive, it contains (1,1), (2,2), (3,3) and (4,4)
2 R is symmetric because (a,b) and (b,a) ∈ R
3 R is not antisymmetric, we have (1,2) and (2,1) but 1 ≠ 2
4 R is Transitive because for any number a,b and c, if (a,b), (b,c) ∈ R then (a,c) ∈ R

c 1 R is not reflexive, It doesn't contain (1,1) (2,2) (3,3) and (4,4)
2 R is symmetric because because R contains (2,4) and (4,2) ∈ R
3 R is not anti-symmetric we have (2,4) and (4,2) but 2 ≠ 4
4 R is not transitive because (2,4) and (4,2) ∈ R but not (2,2) ∈ R

d 1 R is not reflexive It does not contain (1,1), (2,2) (3,3) and (4,4)
2 R is not symmetric because (1,2) ∈ R but not (2,1) ∈ R
3 R is antisymmetric we have (a,b) but not (b,a) ∈ R
4 R is not transitive because (1,2) (2,3) ∈ R but not (1,3) ∈ R

e 1 R is reflexive, It contains (1,1), (2,2) (3,3) and (4,4)
2 R is symmetric because R contains (a,b) and (b,a) ∈ R
3 R is anti-symmetric we have (a,b) and (b,a) ∈ R then a = b
4 R is transitive because for any numbers a, b, and c, if (a,b), (b,c) ∈ R then (a,c) ∈ R

f 1 R is not reflexive, it does not contain (1,1), (2,2), (3,3) and (4,4)
2 R is not symmetric because (1,4) ∈ R but not (4,1) ∈ R
3 R is not antisymmetric, we have (1,3) and (3,1) ∈ R but 1 ≠ 3
4 R is not transitive because we have (1,3) and (3,1) ∈ R but not (1,1) ∈ R

Q31 a The relation R is not reflexive, because a person cannot be taller than himself.
2 The relationship R is not symmetric because if person A is taller than person B, then person B is not taller than person A
3, The relation R is antisymmetric, because (a,b) ∈ R and (b,a) ∈ R cannot occur at the same time (as one person is always taller than the other, but not other way around)
4, The relation R is transitive because if person A ~~and person B~~ is taller than person B and if person B is taller than person C ~~as well~~ than person A needs to be taller than person C

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b 1 The relation R is ~~reflexive~~ because the person has the same first name.
2 The relation R is symmetric because if person A and person B are born on the same day, then person B is also born on the same day as person A
3 The relation R is not antisymmetric, because if person A and person B are born on the same day and person B and person A are born on the same day, then these two people are not necessarily the same person.
4 The relation R is transitive because if person A and person B are born on the same day and if person B and C are born on the same day, then person A and person C are born on the same day.

c 1 The relation R is reflexive because the person has the first name as themselves
2 The relation R is symmetric because if person A has the same first name as person B, then B also has the same first name as person A
3 The relation R is not anti-symmetric because if person A has the same first name as person B and ~~vice versa~~ person B and person has the same first name, then these two people are not necessarily the same person (different ppl with same name)
4 The relation R is transitive because if person A has the same first name as person B, and person B and person C have same names, then person A also has the same first name as person C

d 1 The relation ~~Re~~ is reflexive because the person has the same grandparents
2 The relation R is symmetric because if person A and person B have a common grandparent, then person B + person A have a common grandparent
3 The relation R is not antisymmetric because if person A and person B have the ~~same~~ common grandparent and person B and person A have the ~~same~~ common grandparent, then these two people are not necessarily the same people (as there are different people with same grandparent.
4 The relation R is not transitive because if person A and person B have a common grandparent and person B and person C have the common grandparent, then person A and person C do not necessarily have the common grandparent (common grandparent of A and B can be from B's paternal side, while the common grandparent of B and C could be from B's maternal side

Q32 a, {(1,1) (2,2), (3,3), (4,4)}
b, {(1,2), (2,1), (3,4)}

Q33
R1 {(2,1), (3,1), (3,2)}
R2 {(1,1), (2,2), (3,3), (2,1), (3,1), (3,2)}
R3 {(1,2), (1,3), (2,3)}
R4 {(1,1), (2,2), (3,3), (1,2), (1,3), (2,3)}
R5 {(1,1), (2,2), (3,3)}
R6 {(1,2), (1,3), (2,1), (2,3), (3,1), (3,2)}

a $R2 \cup R4$ = {(1,1), (2,2), (3,3), (2,1), (3,1), (3,2), (1,2), (1,3), (2,3)}
b $R3 \cup R6$ = {(1,2), (1,3), (2,1), (2,3), (3,1), (3,2)}
c $R3 \cap R6$ = {(1,2), (1,3), (2,3)}
d $R4 \cap R6$ = {(1,2), (1,3), (2,3)}
e $R3 - R6$ = { } OR ϕ
f $R6 - R3$ = {(2,1), (3,1), (3,2)}
g $R2 \oplus R6$ = {(1,1), (2,2), (3,3), (1,2), (1,3), (2,3)}
h $R3 \oplus R5$ = {(1,1), (2,2), (3,3), (1,2), (1,3), (2,3)}
i $R2 \circ R1$ = {(2,1), (3,1), (3,2)}
j $R6 \circ R6$ = {(1,1), (2,2), (3,3), (2,1), (3,1), (3,2), (1,2), (1,3), (2,3)}

Q34 a i, $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ ii, $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ iii, $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ iv, $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

b i, R = {(1,1), (1,3), (2,2), (3,1), (3,3)}

ii, R = {(1,2), (2,2), (3,2)}

iii, R = {(1,1), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2), (3,3)}

Q35a Show that all of the properties of an equivalence relation hold.

- Reflexivity: Because $I(a) = I(a)$, it follows that aRa for all strings a
- Symmetry: Suppose that aRb, Since $I(a) = I(b)$, $I(b) = I(a)$ also holds and bRa
- Transitivity: Suppose that aRb and bRc. Since $I(a) = I(b) = I(c)$, $I(a) = I(c)$ also holds and aRc

b, Recall that $a \equiv b \pmod{m}$ if and only if m divides $a-b$

- Reflexivity: $a \equiv a \pmod{m}$ since $a-a=0$ is divisible by m since $0 = 0 \cdot m$
- Symmetry: Suppose that $a \equiv b \pmod{m}$. Then $a-b$ is divisible by m, and so $a-b = km$, where k is integer. It follows $b-a = (-k)m$, so $b \equiv a \pmod{m}$
- Transitivity: Suppose that $a \equiv b \pmod{m}$ and $b \equiv c \pmod{m}$. Then m divides both $a-b$ and $b-c$. Hence there are integers k and l with $a-b = km$ and $b-c = lm$. We obtain by adding the equations $a-c = (a-b)+(b-c) = km + lm = (k+l)m$. Therefore, $a \equiv c \pmod{m}$

Q36a i 1, 2, 4, 8, 16 are the first five terms of the given sequence

ii $17/2$, $11/2$, 4, $5/2$ are the first five terms

iii $-1, -1/4, -1/9, -\frac{1}{16}, -\frac{1}{25}$ are the first five terms

iv 7, $10/3$, $13/5$, $16/7$, $19/9$ are the first five terms

b i, $d = -7$ $T_n = a + (n-1)d$; $T_{11} = -15 + (11-1)(-7) = -85$

ii $d = 3b$ $T_n = a + (n-1)d$; $T_{15} = a - 42b + (15-1)(3b) = a$

iii $r = 3/4$ $T_n = ar^{n-1}$; $T_{17} = 4(3/4)^{17-1} = \frac{3^{16}}{4^{15}}$

iv $r = 1/2$ $T_n = ar^{n-1}$; $T_9 = 32(1/2)^{9-1} = 1/8$

Scanned with CamScanner

Q37 a i, $T_n = ar^{n-1}$ $T_3 = ar^2 = 10$ $T_5 = ar^4 = 5/2$

$T_3 = ar^2$ $T_5 = ar^4$ $\frac{10}{r^2} = \frac{2.5}{r^4}$

$10 = ar^2$ $2\frac{1}{2} = ar^4$ $r^2 = 1/4$

$a = 10/r^2$ $a = 2.5/r^4$ $r = 1/2$ $a = 40$ G.P = 40, 20, 10, 5, 5/2

ii $T_n = ar^{n-1}$ $T_5 = ar^4 = 8$ $T_8 = ar^7 = 64/27$

$T_5 = ar^4$ $T_8 = ar^7$ $\frac{8}{r^4} = \frac{-64}{27r^7}$ $r = -2/3$

$8 = ar^4$ $\frac{-64}{27} = ar^7$ $r^3 = -64/216$ $a = 81/2$

$a = 8/r^4$ $a = -64/27r^7$ $r^3 = -8/27$ G.P = 81/2, -27, 18, -12, 8, $\frac{-16}{3}$...

$\frac{32}{9}$, $\frac{-64}{27}$

b i, $T_4 = a + 3d = 7$ $T_{16} = a + 15d = 31$ - ii $a = 7 - 3d$ $a = 31 - 15d$

i - ii = $d = 2$ $a = 1$ ~~d+3d=7~~ $7 - 3d = 31 - 15d$ $d = 2$

Solve simultaneously $7(a) + (3)(2)$ $a = 1$

A.P = 7 1, 3, 5, 7, 9 ...

ii $T_5 = 86$ $T_{10} = a + 9d$ $a = 86 - 4d$ $a = 146 - 9d$

$T_5 = a + 4d$ $146 = a + 9d$ $86 - 4d = 146 - 9d$

$86 = a + 4d$ $5d = 60$ $d = 12$

$86 = a + 4(12)$ $a = 38$

$a = 38$ $d = 12$ A.P = 38, 50, 62, 74, 86

Q38 a, $a = 259$ $d = 7$ b, $a = 1/n$ $T_n = \frac{n^2 - n + 1}{n}$

$T_n = a + (n-1)d$

$784 = 259 + (n-1)7$ $S_n = n/2\,[a + T_n]$

$n = 76$ $n/2\,[1/n + \frac{n^2 - n + 1}{n}]$

$n/2\,[\frac{n^2 - n + 2}{n}]$

$S_n = n/2\,[a + T_n]$

$\frac{76}{2}\,[259 + 784]$ $S_n = \frac{n^2 - n + 2}{2}$

$= 39{,}634$

~~b~~

Q39 a $\sum_{j=1}^{100} \frac{1}{j}$

b i) $(-1)^4 + (-1)^5 + (1)^6 + (-1)^7 + (-1)^8$

$1 + (-1) + 1 + (-1) + 1 = 1$

ii, $1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 1 + 4 + 9 + 16 + 25 = 55$

Q40

a, $a_0 = -1$

$a_1 = -2a_0 = -2(-1) = 2$

$a_2 = -2a_1 = -2(2) = -4$

$a_3 = -2a_2 = -2(-4) = 8$

$a_4 = -2a_3 = -2(8) = -16$

$a_5 = -2a_4 = -2(-16) = 32$

b, $a_0 = 2 \quad a_1 = -1$

$a_2 = a_1 - a_0 = -1 - 2 = -3$

$a_3 = a_2 - a_1 = -3 - (-1) = -2$

$a_4 = a_3 - a_2 = -2 - (-3) = 1$

$a_5 = a_4 - a_3 = 1 - (-2) = 3$

c, $a_0 = 1$

$a_1 = 3a_0^2 = 3(1^2) = 3$

$a_2 = 3a_1^2 = 3(3^2) = 3(9) = 27$

$a_3 = 3a_2^2 = 3(27)^2 = 3(729) = 2187$

$a_4 = 3a_3^2 = 3(2187)^2 = 3(4782969) = 14348907$

$a_5 = 3a_4^2 = 3(14348907^2) = 3(205891132094649) = 617673396283947$

d, $a_0 = -1 \quad a_1 = 0$

$a_2 = 2a_1 + a_0^2 = 2(0) + (-1)^2 = 1$

$a_3 = 3a_2 + a_1^2 = 3(1) + 0^2 = 3$

$a_4 = 4a_3 + a_2^2 = 4(3) + 1^2 = 13$

$a_5 = 5a_4 + a_3^2 = 5(13) + 3^2 = 74$

Q41

a, Propositional logic:

1, In computer science, it is used to design and evaluate the correctness of programs, to verify the correctness of hardware, and to build secure communications protocols

2, In legal reasoning, propositional logic is used to reason about the truth or falsity of legal propositions and to evaluate the validity of legal arguments

b, Predicates and Quantifiers

1, In database management systems, predicates and quantifiers are used to specify complex queries that involve multiple tables and conditions.

2, In mathematics, predicates and quantifiers are used to make statements about entire classes of objects, rather than just individual objects.

c, Sets

1, In statistics, sets are used to define populations and samples for hypothesis testing and data analysis.

2, In computer science, sets are used in algorithms such as searching and sorting, and in data structures such as hash tables and sets.

d, Functions

1, In economics, functions are used to model relationships between variables such as supply and demand

2, In engineering, functions are used to model physical systems and to design control systems.

e, Relations

1, In database management systems, relations are used to model the structure of databases and to specify relationships between tables.

2, In social network analysis, relations are used to model the connections between individuals or groups.

f, Sequence and Series

1, In finance, sequences and series are used to model financial transactions and investments

2, In physics, sequences and series are used to model time-dependent phenomena such as waveforms and ossicillations.

g, Graph theory

1, In computer sciene, graph theory is used to model networks such as social network, transportation networks, and communication networks

2, In biology, graph theory is used to model protein interaction network and gene regulation networks

h, Trees

1, File systems: Trees allow computer file systems to be organized efficiently, making it easy to find and manage files.

2, Decision Trees: Trees allow complex decision making processes to be modeled and visualized, and makes it easier for the organizations to make informed decisions based on potential risks.