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22K-5195

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Assignment 3

Q1 i) $a_n = 1 + \frac{(-1)^n}{n}$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left| 1 + \frac{(-1)^n}{n} \right|$

$= 1 + \frac{(-1)^\infty}{\infty}$

$= 1 - \frac{1}{\infty}$

$= 1 - 0 \Rightarrow 1$ converge

ii) $a_n = \frac{1-2^n}{2^n}$

$a_n = \frac{1}{2^n} - \frac{\cancel{2^n}}{\cancel{2^n}} = \frac{1}{2^n} - 1$

$\lim_{n\to\infty} \frac{1}{2^\infty} - 1 \Rightarrow 0 - 1 \Rightarrow -1$ converge

iii) $a_n = \frac{\ln(2n+1)}{n}$

using L'Hopital

$\frac{2}{n} \div 1 \Rightarrow \frac{2}{n}$

$\lim_{n\to\infty} = \frac{2}{\infty} \div 1 = 0$ converge

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Q1 iv) $a_n = \left(\frac{n+3}{n+1}\right)^n$

$\ln a_n = \ln\left(\frac{n+3}{n+1}\right)^n$

$= n \ln\left(\frac{n+3}{n+1}\right)$

$= \frac{\ln\left(\frac{n+3}{n+1}\right)}{1/n}$

$= \frac{\ln\left(\frac{1+3/n}{1+1/n}\right)}{1/n}$

$= \frac{\left(\frac{1}{1+3n}\right)(-3n^{-2}) - \frac{1}{1+1/n}(-n^{-2})}{-n^{-2}}$

$= \frac{3}{1+1/\infty} - \frac{1}{1+1/\infty}$

$\ln y = 2$

$y = e^2$

$a_n = e^2$ converge

Q2 i) $\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$

$\ln n \sum_{n=1}^{\infty} \frac{1}{n^3}$

p-value test

$p = 3$

$3 > 1$ converge

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Q2 ii) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$

$\frac{(-3)^{n+1}}{(n+1)n!} \times \frac{n!}{(-3)^n}$

$\frac{-3}{n+1} \Rightarrow 0$ converge

iii) $\sum_{n=2}^{\infty} \frac{(-3)^n}{n(\ln n)^2}$ acc to ratio test

$\rho = \lim_{n\to\infty} \frac{\ln+1}{\ln} = \frac{(-3)^{n+1}}{(n+1)(\ln(n+1))^2} \div \frac{(-3)^n}{n(\ln n)^2}$

$= \frac{(-3)^{n+1}}{(n+1)(\ln(n+1))^2} \times \frac{n(\ln n)^2}{(-3)^n} = \frac{(-3)\times(-3)^n \times n \times (\ln n)^2}{(n+1)\times \ln(n+1)^2 \times (-3)^n}$

$= \frac{n \times (\ln n)^2}{n(1+1/n)\times \ln(n+1)^2} = \frac{(\ln n)^2}{(1+\frac{1}{n})\times \ln(n+1)^2} = \frac{(\ln \infty)^2}{(1+\frac{1}{\infty})(\ln(\infty+1)}$

L'Hopitel

$\frac{2(1/n)}{\frac{-2}{n^2}\times 2\left(\frac{1}{n+1}\right)} = \frac{1/n}{\frac{-4}{n^2}\left(\frac{1}{n+1}\right)} = \frac{1}{2/n(n+1)}$

$= \frac{n(n+1)}{2} = \infty$

series diverges

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Q2

iv) $\sum_{n=1}^{\infty} \frac{2}{n(2n+1)}$

$a_n = \frac{2}{2n^2+n}$

$2 \sum_{n=1}^{\infty} \frac{1}{2n^2+n}$

Applying Comparison tests

$b_n = 1/n^2$

using p-series

$2 > 1$ b_n converges

$\Rightarrow$ $a_n < b_n$

$\frac{2}{2n^2+n} < \frac{1}{n^2}$

$\sum_{n=1}^{\infty} \frac{2}{n(2n+1)}$ converges

v) $\sum_{n=1}^{\infty} \frac{2^n}{3^n+1}$

Applying Ratio test

$\lim_{n \to \infty} \frac{2^{n+1}}{3^{n+1}+1} \div \frac{2^n}{3^n+1}$

$= \frac{2^n \cdot 2}{3^n \cdot 3+1} \cdot \frac{3^n+1}{2^n}$

$\lim_{n \to \infty} \frac{2 \cdot 3^n + 1}{3 \cdot 3^n + 1}$

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v) $\dfrac{3^n(2+1/3^n)}{3^n(3+1/3^n)} = \dfrac{2}{3} < 1$ converges

vi) $\sum_{n=1}^{\infty} \dfrac{n!}{(2n)!}$

Applying Ratio test

$$\lim_{n\to\infty} \frac{(n+1)!}{(2n+2)!} \div \frac{n!}{(2n)!}$$

$$= \frac{(n+1)(n!)}{(2n+2)(2n+1)(2n)!} \cdot \frac{(2n)!}{n!}$$

$$\lim_{n\to\infty} \frac{n+1}{4n^2+4n+1} = 0 < 1 \text{ converges}$$

vii) $\sum_{n=1}^{\infty} \left(\dfrac{n}{n+1}\right)^{n^2}$

Applying Root test

$$\lim_{n\to\infty} \sqrt[n]{\left(\frac{n}{n+1}\right)^{n^2}}$$

$$\left(\frac{n}{n+1}\right)^{\frac{n^2}{2}} = \left(\frac{n}{n+1}\right)^{n}$$

$$= \left(\frac{\infty}{\infty}\right)^{\infty} \quad \text{L'Hopital}$$

$$\ln \lim_{n\to\infty} \sqrt[n]{\left(\frac{n}{n+1}\right)^{n^2}}$$

$$n \ln\left(\frac{n}{n+1}\right)$$

$$\lim_{n\to\infty} \frac{\ln(n) - \ln(n+1)}{1/n}$$

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$$= \frac{1/n - 1/n+1}{-1/n^2}$$

$$= \ln \lim_{n\to\infty} \frac{(n+1-n)}{n(n+1)} \cdot n^{-2}$$

$$= \ln \lim_{n\to\infty} \left(\frac{-n^2}{n(n+1)} \right)$$

$$= \ln \lim_{n\to\infty} \left(\frac{-1}{1+1/n} \right)$$

$$= \lim_{n\to\infty} e^{-1}$$

$$= e^{-1} < 1 \quad \text{converges}$$

viii) $\sum_{n=1}^{\infty} n^2 e^{-n}$

Root test

$$\lim_{n\to\infty} \frac{\left(\frac{n+1)^2}{e^{n+1}}\right.}{n^2/e^n}$$

$$= \frac{n^2+2n+1}{e^n \cdot e} \cdot \frac{e^n}{n^2}$$

$$\lim_{n\to\infty} \frac{n^2+2n+1}{en^2} \Rightarrow \frac{1}{e} < 1$$

converges

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ix) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \ldots$

Divergence test

$a_n = \frac{n}{n+1}$

$\sum_{n=1}^{\infty} \frac{n}{n+1}$

$\lim_{n \to \infty} \frac{n}{n+1} \Rightarrow \frac{\infty}{\infty}$

L' Hopital

$\lim_{n \to \infty} 1$ diverges

x) $1 - 3/2 + 9/4 - 27/8 + \ldots$

Geometric series test

$a_n = \left(\frac{-3}{2}\right)^{n-1}$

$\sum_{n=1}^{\infty} \left(\frac{-3}{2}\right)^{n-1}$

$|r| > 1 \therefore$ diverges

Q3.

i) $\sum_{n=0}^{\infty} \frac{(n-1)(2x+1)^n}{(2n+1)2^n}$

$\frac{(0-1)(2x+1)^0}{(2(0)+1) \times 2^0} \Rightarrow a_1 = -1$

$\frac{(1-1)(2x+1)^1}{(3)(2)} \Rightarrow a_2 = 0$

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$$\frac{(2-1)(2x+1)^2}{(5)(4)} \Rightarrow a_3 = \frac{(2x+1)^2}{20}$$

$$\frac{(3-1)(2x+1)^3}{(7)(8)} \Rightarrow a_4 = \frac{(2x+1)^3}{28}$$

$$\frac{(4-1)(2x+1)^4}{(9)(16)} \Rightarrow a_5 = \frac{(2x+1)^4}{48}$$

ii) $\sum_{n=1}^{\infty} \frac{(x-1)^{2n-2}}{(2n-1)!}$

$$\frac{(x-1)^{2(1)-2}}{(2(1)-1)!} \Rightarrow a_1 = 1$$

$$\frac{(x-1)^2}{3!} \Rightarrow a_2 = \frac{(x-1)^2}{6}$$

$$\frac{(x-1)^4}{5!} \Rightarrow a_3 = \frac{(x-1)^4}{120}$$

$$\frac{(x-1)^6}{7!} \Rightarrow a_4 = \frac{(x-1)^6}{5040}$$

$$\frac{(x-1)^8}{9!} \Rightarrow a_5 = \frac{(x-1)^8}{362880}$$

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Q4. i) $f(x) = \sqrt{3+x^2}$; $x = -1$;

$f(x) = f'(c)(x-c) + f''\frac{(c)(x-2)^2}{2!} + \frac{f'''(c)(x-2)^3}{3!}$

$f(c) = \sqrt{3+x^2} \Rightarrow f(-1) = 2$

$f'(c) = \frac{x}{\sqrt{3+x^2}} \Rightarrow f'(-1) = \frac{-1}{2}$

$f''(c) = \frac{3}{(3+x^2)^{5/2}} \Rightarrow f''(-1) = \frac{3}{8}$

$f'''(c) = \frac{-9x}{(3+x^2)} \Rightarrow f'''(-1) = \frac{9}{32}$

$f(x) = 2 - \frac{1}{2}(x-(-1)) + \frac{3}{8}(x-(-1)^2 \times \frac{1}{2!}$

$= \frac{9}{32}(x-(-1))^3 \times \frac{1}{3!} + \frac{9}{128}(x-(-1))^4 \times \frac{1}{4!}$

ii) $f(x) = \frac{1}{1-x}$, $x = 2$

$f(c) = \frac{1}{1-x} \Rightarrow f(2) = -1$

$f'(c) = \frac{1}{(1-x)^2} \Rightarrow f'(2) = 1$

$f''(c) = \frac{-2}{(1-x)^3} \Rightarrow f''(2) = 2$

$f'''(c) = \frac{\cancel{24}\ 6}{(1-x)^4} \Rightarrow f'''(2) = 6$

$f''''(c) = \frac{-24}{(1-x)^5} \Rightarrow f''''(2) = 24$

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ii) $f(x) = -1 + 1(x-2) + \frac{2(x-2)^2}{2!} + \frac{6(x-2)^3}{3!} + \frac{24(x-2)^4}{4!}$

$f(x) = -1 + (x-2) + (x-2)^2 + (x-2)^3 + (x-2)^5$

Q5 . i) $\cos\sqrt{5x}$

as $f(x) = \cos\sqrt{5x}$

* As McLaurin series is a special case of taylor series where $a = 0$

$$\sum_{n=0}^{\infty} \frac{f^n(0)(x-0)^n}{n!}$$

Note: For McLaurin series, the series must be converging; if the series is diverging, the maclaurin series does not exist

as $f(x) = \cos\sqrt{5x}$

$$\lim_{x\to\infty} \cos\sqrt{5x}$$

$\cos(\infty) = DNE$

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ii) $f(x) = e^{-x^2}$

as $f(x) = e^{-x^2}$

$\lim_{x \to \infty} e^{-x^2}$

$= e^{-\infty}$

$= \frac{1}{e^{\infty}} = 0$ (converging)

* mclaurin exists