National University of Computer & Emerging Sciences, Karachi
FAST School of Computing
Final Exam (FALL-2023)
3rd January, 2023 Wednesday 12:30pm -3:30pm

Course Code:MT2005	Course Name: Probability and Statistics
Instructor Names: Mr. Jamil Usmani and Mr.Moheez Ur Rahim	
Student Roll No:	Section No:

Instructions:

- Solve all question and return the question paper.
- Read each question completely before answering it. **There are 05 Questions and 04 pages.**
- Display your answer correct up to three decimal places. Show all necessary steps.
- Formula sheet and Statistical tables are attached.

Time Allowed: 180 minutes **Max Marks :100**

Question 01: [CLO-1] [5+5=10]

a) The following data represent the length of life in years, measured to the nearest tenth, of 30 similar fuel pump:

2.0	3.0	0.3	3.3	1.3	0.4	1.5	4.0	5.9	1.8
0.2	6.0	5.5	6.5	0.2	2.3	4.5	0.3	1.5	0.5
4.7	0.7	1.0	6.0	5.6	2.5	5.0	1.2	0.2	6.0

i) Construct a stem-and-leaf plot for the life in years of the fuel pumps, using the digit to the left of the decimal point as the stem for each observation
ii) Discuss five number summary.
iii) Compute the sample mean and sample standard deviation. (use calculator)

b) A shipment of two boxes, each containing 6 telephones, is received by a store. Box one contains 1 defective phone, and box two contains 2 defective phones. After the boxes are unpacked, a phone is selected and found to be defective. Find the probability that it came from box two. (Use Baye's rule)

Question 02: [CLO-2] [8+3+6+3=20]

a) The fraction X of male runners and the fraction Y of female runners who compete in marathon races are described by the joint density function

$$f(x,y) = \begin{cases} \frac{x(1+3y^2)}{4}, & 0 < x < 2, 0 < y < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

i) Find the covariance of X and Y
ii) Find the correlation coefficient of X and Y

b) The probability that a student pilot passes the written test for a private pilot's license is 0.7 Find the probability that a given student will pass the test
i) on the third try;
ii) before the fourth try

c) The finished inside diameter of a piston ring is normally distributed with a mean of 10 centimetres and a standard deviation of 0.03 centimetre

   i) What proportion of rings will have inside diameters exceeding 10.075 centimetres?
   ii) What is the probability that a piston ring will have an inside diameter between 9.97 and 10.03 centimetres?

d) The probability that a student at a local high school fails the screening test for scoliosis (curvature of the spine) is known to be 0.004. Of the next 1875 students at the school who are screened for scoliosis, using binomial approximation to Poisson to find the probability that

   i) fewer than 5 fail the test;
   ii) 8,9, or 10 fail the test.

Question 03: [CLO-3] [10+5+10=25]

a) A dietitian wishes to see if a person's cholesterol level will change if the diet is supplemented by a certain mineral. Six subjects were pretested, and then they took the mineral supplement for a 6-week period. The results are shown in the table. (Cholesterol level is measured in milligrams per decilitre.) Assume the variable is approximately normally distributed.
Calculate the 95% confidence interval of the mean difference.

Subject	1	2	3	4	5	6
Before (X_1)	210	235	208	190	172	244
After (X_2)	190	170	210	188	173	228

b) Students may choose between a 3-semester-hour physics course without labs and a 4-semester-hour course with labs. The final written examination is the same for each section. If 12 students in the section with labs made an average grade of 84 with a standard deviation of 4 and 18 students in the section without labs made an average grade of 77 with a standard deviation of 6. Find a 99% confidence interval for the difference between the average grades for the two courses. Assume the populations to be approximately normally distributed with equal variances.

c) A researcher wishes to try three different techniques to lower the blood pressure of individuals diagnosed with high blood pressure. The subjects are randomly assigned to three groups; the first group takes medication, the second group exercises. and the third group follows a special diet. After four weeks, the reduction in each person's blood pressure is recorded.

Medication	Exercise	Diet
10	6	5
12	8	9
9	3	12
15	0	8
13	2	4

   i) Construct ANOVA table
   ii) Test the claim that there is no difference among the means at $\alpha = 0.05$

Question 04: [CLO-3] [5+10=15]

a) A researcher claims that the average cost of men's athletic shoes is less than $\$ 80$. He selects a random sample of 36 pairs of shoes from a catalog and finds the following costs (in dollars). Is there enough evidence to support the researcher's claim at $\alpha = 0.10$? Assume $\sigma = 19.2$.

60	70	75	55	80	55	120	90	75	85	80	60
50	40	80	70	50	95	110	65	80	85	85	45
75	60	90	90	60	95	110	85	45	90	70	70

b) A survey found that the average hotel room rate in New Orleans is $\$ 88.42$ and the average room rate in Phoenix is $\$ 80.61$. Assume that the data were obtained from two samples of 50 hotels each and that the standard deviations of the populations are $\$ 5.62$ and $\$ 4.83$, respectively. At $\alpha = 0.01$, can it be concluded that there is a significant difference in the rates?

Question 05: [CLO-3] [5+15+10=30]

a) A study was made by a retail merchant to determine the relation between weekly advertising expenditures and sales.

Advertising cost	40	20	25	20	30	50	40	20	50	40	25	50
Sales	385	400	395	365	475	440	490	420	560	525	480	510

i) Use calculator to find regression equations for quadratic polynomial.

ii) Estimate $\mu_{Y|35}$ using quadratic regression equation $\mu_{Y|x} = \beta_0 + \beta_1 x + \beta_2 x^2$.

b) A study was performed on a type of bearing to find the relationship of amount of wear y to $x_1 =$ oil viscosity and $x_2 =$ load. The following data were obtained.

y	x_1	x_2
193	1.6	851
172	22.0	1058
113	33.0	1357
230	15.5	816
91	43.0	1201
125	40.0	1115

i) Compute $\sum y, \sum x_1, \sum x_2, \sum x_1 x_2, \sum x_1 y, \sum x_2 y, \sum x_1^2$ and $\sum x_2^2$

ii) Estimate the unknown parameters of the multiple linear regression equation using Normal equations. $\mu_{Y|x_1,x_2} = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.

iii) Predict wear when oil viscosity is 20 and load is 1200.

iv) Find the multiple co-efficient of correlation $\boldsymbol{R}$ and co-efficient of determination $\boldsymbol{R^2}$

c) Construct a scatter plot for the data shown for car rental companies in the United States for a recent year.

Company	Cars (in ten thousands)	Revenue (in billions)
A	63.0	7.0
B	29.0	3.9
C	20.8	2.1
D	19.1	2.8
E	13.4	1.4
F	8.5	1.5

i) Find the regression coefficient and equation of the regression line $\hat{y} = a + bx$

ii) Graph the regression line on the scatter plot.

iii) Compute the correlation coefficient.

iv) Test the significance of the correlation coefficient , $\rho = 0$ against $\rho \neq 0$ at level of significance $\alpha = 0.05$

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