Laiba Fatima
22K-5195
BSE-1B

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Cal - Assignment 2a

Q1

a) $\int x^2 2^x dx$

$u = x^2 \qquad dv = 2^x \, dx$

$du = 2x\,dx \qquad v = \dfrac{2^x}{\ln(2)}$

$$x^2 \times \frac{2^x}{\ln 2} - \int \frac{2^x}{\ln(2)} \times 2x\,dx$$

$$x^2 \times \frac{2^x}{\ln 2} - \frac{1}{\ln 2} \times 2 \times \int 2^x \times x \, dx$$

$$x^2 \times \frac{2^x}{\ln 2} - \frac{2}{\ln 2} \times \int x \, 2^x \, dx$$

$$x^2 \times \frac{2^x}{\ln(2)} - \frac{2}{\ln(2)} \times \left(x \times \frac{2^x}{\ln(2)} - \int \frac{2^x}{\ln(2)} dx \right)$$

$$x^2 \times \frac{2^x}{\ln(2)} - \frac{2}{\ln(2)} \times \left(x \times \frac{2^x}{\ln(2)} - \frac{1}{\ln(2)} \times \int 2^x \, dx \right)$$

$$x^2 \times \frac{2^x}{\ln(2)} - \frac{2}{\ln(2)} \times \left(x \times \frac{2^x}{\ln(2)} - \frac{1}{\ln(2)} \times \frac{2^x}{\ln 2} \right)$$

$$\frac{x^2 \times 2^x}{\ln(2)} - \frac{\ln(2) \times 2^{x+1} \times x - 2^{x+1}}{\ln(2)^3}$$

$$\frac{x^2 \times 2^x}{\ln(2)} - \frac{\ln(2) \times 2^{x+1} \times x - 2^{x+1}}{\ln(2)^3} + C$$

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b) $\int \frac{x^5}{\sqrt{x^2+2}} \, dx \qquad t = x^2 + 2$

$$\int \frac{t^2 - 4t + 4}{2\sqrt{t}} \, dt$$

$$\frac{1}{2} \int \frac{t^2 - 4t + 4}{t^{1/2}} \, dt$$

$$\frac{1}{2} \int \frac{t^2}{t^{1/2}} - \frac{4t}{t^{1/2}} + \frac{4}{t^{1/2}} \, dt$$

$$\frac{1}{2} \int t^{3/2} - 4t^{1/2} + \frac{4}{t^{1/2}} \, dt$$

$$\frac{1}{2} \times \left(\frac{2t^2\sqrt{t}}{5} - \frac{8t\sqrt{t}}{3} + 8\sqrt{t} \right)$$

$$\frac{1}{2} \left(\frac{2(x^2+2)^2\sqrt{x^2+2}}{5} - \frac{8(x^2+2)\sqrt{x^2+2}}{3} + 8\sqrt{x^2+2} \right)$$

$$\frac{\sqrt{x^2+2}\,(x^4+4x^2+4)}{5} - \frac{4(x^2+2)\sqrt{x^2+2}}{3} + 4\sqrt{x^2+2} + C$$

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c) $\int x \operatorname{cosec}^2 x \, dx$

$u = x \qquad dv = \int \csc x^2 \, dx$

$du = dx \qquad v = -\cot x$

$x \times -\cot x - \int -\cot x \, dx$

$-x \times \cot(x) + \int \cot x \, dx$

$-x\cot x + \int \frac{\cos x}{\sin x} \, dx$

$-x \times \cot x + \ln(\sin x) + C$

d) $\int \frac{\sqrt{x^2 - 9}}{x^3} \, dx$

$u = \sqrt{x^2 - 9} \qquad dv = \int \frac{1}{x^3} \, dx$

$du = \frac{1}{2\sqrt{x^2 - 9}} \times 2x \, dx \qquad v = \frac{-1}{2x^2}$

$\sqrt{x^2 - 9} \times -\frac{1}{2x^2} - \int -\frac{1}{2x^2} \times \frac{1}{2\sqrt{x^2-9}} \times 2x \, dx$

$\sqrt{x^2 - 9} \times -\frac{1}{2x^2} - 1 \times -\frac{1}{2} \times \frac{1}{2} \times 2 \times \int \frac{1}{x^2} \times \frac{1}{\sqrt{x^2-9}} \times x \, dx$

$\sqrt{x^2 - 9} \times \frac{-1}{2x^2} + \frac{1}{2} \times \int \frac{1}{x\sqrt{x^2-9}} \, dx$

$\sqrt{x^2 - 9} \times \left(\frac{-1}{2x^2}\right) + \frac{1}{2} \times \int \frac{1}{x\sqrt{x^2 - 3^2}} \, dx$

$\sqrt{x^2 - 9} \times \left(\frac{-1}{2x^2}\right) + \frac{1}{2} + \frac{1}{3} \times \operatorname{arcsec}\left(\frac{|x|}{3}\right)$

$\frac{-\sqrt{x^2-9}}{2x^2} + \frac{\operatorname{arcsec}\left(\frac{|x|}{3}\right)}{6} + C$

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e) $\int \sin x \ln(\cos x)\, dx$

$u = \ln(\cos x)$ $\qquad dv = \sin x\, dx$

$du = \frac{1}{\cos x} \times -\sin x\, dx$ $\qquad v = -\cos x$

$\ln\cos x \times -\cos x - \int -\cos x \times \frac{1}{\cos x} \times -\sin x\, dx$

$-\ln(\cos x)\cos x - \int \sin x\, dx$

$-\ln(\cos x)\cos x + \cos x + C$

f) $\int \sqrt{1-4x^2}\, dx$

$\int \sqrt{1-4\times\left(\frac{1}{2}\times\sin t\right)^2} \times \frac{1}{2}\times\cos t\, dt)$

$\frac{1}{2} \int \sqrt{1-4\left(\frac{1}{2}\times\sin t\right)^2} \times \cos t\, dt)$

$\frac{1}{2} \times \int \sqrt{1-4\times\frac{1}{4}\times\sin^2 t} \times \cos t\, dt)$

$\frac{1}{2} \times \int \sqrt{1-\sin^2 t} \times \cos t\, dt)$

$\frac{1}{2} \times \int \sqrt{\cos^2 t} \times \cos t\, dt$

$\frac{1}{2} \int \cos t \cos t\, dt$

$\frac{1}{2} \int \frac{1+\cos(2t)}{2}\, dt$

$\frac{1}{2} \times \frac{1}{2} \int 1 + \cos 2t\, dt$

$\frac{1}{4} \times \int 1\, dt + \int \cos 2t\, dt$

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f) $\frac{1}{4} \times \left(t + \frac{\sin(2t)}{2}\right)$

$\frac{1}{4} \times \left(\arcsin\left(\frac{x}{1/2}\right) + \frac{\sin\left(2\arcsin\left(\frac{x}{1/2}\right)\right)}{2}\right)$

$\frac{\arcsin 2x}{4} + \frac{x\sqrt{1-4x^2}}{2} + c$

g) $\int \sqrt{x-1}\ x^2\ dx \qquad t = x-1$

$\int \sqrt{t}\ t^2 + 2t\sqrt{t} + \sqrt{t}\ dt$

$\int t^{1/2} \times t^2 + 2t \times t^{1/2} + t^{1/2}\ dt$

$\int t^{5/2} + 2t^{3/2} + t^{1/2}\ dt$

$\frac{2t^3\sqrt{t}}{7} + \frac{4t^2\sqrt{t}}{5} + \frac{2t\sqrt{t}}{3}$

$\frac{2(x-1)^3\sqrt{x-1}}{7} + \frac{4(x-1)^2\sqrt{x-1}}{5} + \frac{2(x-1)\sqrt{x-1}}{3} + c$

~~2√x-1 (x3~~

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h) $\int \sqrt{5+4x-x^2}\ dx$

$\int \sqrt{-x^2+4x+5}\ dx$

$\int \sqrt{-(x^2-4x+4-9)}\ dx$

$\int \sqrt{-((x^2-2)^2-9)}\ dx$

$\int \sqrt{9-(x-2)^2}\ dx \quad ; \ t = x-2$

$\int \sqrt{9-t^2}\ dt$

$\int \sqrt{9-3\sin(u)^2} \times 3\cos u\ du$

$3 \times \int \sqrt{9-(3\sin(u))^2}\ \cos u\ du$

$3 \times \int \sqrt{9-9\sin^2 u}\ \cos u\ du$

$3 \times \int \sqrt{9-(1-\sin^2 u)}\ \cos u\ du$

$3 \times \int \sqrt{9\cos^2 u}\ \cos u\ du$

$3 \times \int 3\cos u \cos u\ du$

$3 \times 3 \int \cos u^2\ du$

$9 \times \int \frac{1+\cos 2u}{2}\ du$

$\frac{9}{2} \times \int 1+\cos 2u\ du$

$\frac{9}{2} \times u + \frac{\sin 2u}{2}$

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h) $\frac{9}{2} \times \arcsin \frac{t}{3} + \frac{\sin\left(2\arcsin\left(\frac{t}{3}\right)\right)}{2}$

$\frac{9}{2} \times \arcsin \frac{x-2}{3} + \frac{\sin\left(2\arcsin\left(\frac{x-2}{3}\right)\right)}{2}$

$\frac{9\arcsin\left(\frac{x-2}{3}\right) + (x-2)\sqrt{5-x^2+4x}}{2} + C$

i) $\int x \sin x \cos x \, dx$

$\int x \times \frac{\sin(2x)}{2} \, dx$

$\frac{1}{2} \int x \times \sin 2x \, dx$

$\frac{1}{2} \times \left(x \times \left(-\frac{\cos 2x}{2}\right) - \int -\frac{\cos 2x}{2} \, dx\right.$

$\frac{1}{2}\left(x \times \left(-\left(\frac{\cos 2x}{2}\right)\right) - 1 \times \left(\frac{-1}{2} \times \int \cos 2x \, dx\right)\right)$

$\frac{1}{2} \times x \times \left(-\frac{\cos 2x}{2}\right) + \frac{1}{2} \times \frac{1}{2} \int \cos t \, dt$

$\frac{1}{2}\left(x\left(-\frac{\cos 2x}{2}\right) + \frac{1}{4} \times \sin 2x\right)$

$-\frac{x\cos 2x}{4} + \frac{\sin 2x}{8} + C$

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j) $\int \sqrt{2x-x^2}\ dx$

$\int \sqrt{-x^2+2x-1+1}\ dx$

$\int \sqrt{-(x^2-2x+1)+1}\ dx$

$\int \sqrt{1-(x-1)^2}\ dx$; $t = x-1$

$\int \sqrt{1-t^2}\ dt$

$\int \sqrt{1-\sin u^2} \times \cos u\ du$

$\int \sqrt{\cos u^2}\ \cos u\ du$

$\int \cos u \cos u\ du$

$\int \cos^2 u\ du$

$\int \frac{1+\cos 2u}{2}\ du$

$\frac{1}{2} \times \int 1+\cos 2u\ du$

$\frac{1}{2} \times \cancel{\int 1 + \cos}\ u + \frac{\sin 2u}{2}$

$\frac{1}{2}\ \arcsin t + \frac{\sin(2 \arcsin(t)}{2}$

$\frac{1}{2}\left(\arcsin(x-1) + \frac{\sin(2\arcsin(x-1))}{2}\right)$

$\frac{\arcsin(x-1) + (x-1)\sqrt{-x^2+2x}}{2} + c$

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K) $\int x^3 + \frac{3}{\sqrt{x}} + 5\sqrt{x} \quad dx$

$\int x^3 + \frac{3}{x^{1/2}} + \cancel{5} x^{1/5} \; dx$

$\frac{x^4}{4} + 6\sqrt{x} + \frac{5x\sqrt[5]{x}}{6} + c$

L) $\int \frac{x+1}{12x^2+24x+60} \, dx$; $t = 12x^2 + 24x + 60$

$\int \frac{1}{24t} \, dt$

$\frac{1}{24} \times \int \frac{1}{t} \, dt$

$\frac{1}{24} \times \ln t$

$\frac{1}{24} \times \ln(12x^2 + 24x + 60) + c$

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m) $\int \frac{x^2+1}{(x^2-2x+2)^2} dx$

$\int \frac{x^2}{(x^2-2x+2)^2} dx + \int \frac{1}{x^2-2x+2)^2} dx$

$= \frac{x(x+\arctan(x-1)(x^2-2x+2)-1)}{x^2-2x+\cancel{2}} - \frac{1}{2}\sin(2\arctan(x-1) -$

$\frac{(x-1)(x^2-2x+2)\left(\frac{x-1}{x^2-2x+2} + \arctan(x-1)\right)}{x^2-2x+2} + \frac{1}{2}(\arctan(x-1) +$

$\frac{1}{2}\sin(2\arctan(x-1)$

$= \frac{x(x+\arctan(x-1)(x^2-2x+2)-1)}{x^2-2x+2} - \frac{1}{4}\sin(2\arctan(x-1)) +$

$\frac{1}{2}\arctan(x-1) - \frac{(x-1)(x^2-2x+2)\left(\frac{x-1}{x^2-2x+2} + \arctan(x-1)\right)+1}{x^2-2x+2} + c$

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n) $\int e^{\sqrt{x}}\, dx$; $t = \sqrt{x}$

$\int 2t e^{t}\, dt$

$2 \int t e^{t}\, dt$

$2 (t e^{t} - \int e^{t}\, dt)$

$2 (t e^{t} - e^{t})$

$2 (\sqrt{x}\, e^{\sqrt{x}} - e^{\sqrt{x}})$

$2 e^{\sqrt{x}} \sqrt{x} - 2 e^{\sqrt{x}} + c$

o) $\int \sec^{3} x \tan x\, dx$; $t = \sec^{3} x$

$\int \frac{1}{3}\, dt$

$\frac{1}{3} t$

$\frac{1}{3} \sec^{3} x$

$\frac{\sec^{3} x}{3} + c$

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P) $\int \left(\frac{1}{x^2} + \sqrt{x} \right) dx$

$\int \frac{1}{x^2} + x^{1/2} \, dx$

$-\frac{1}{x} + \frac{2x\sqrt{x}}{3} + c$