## 11.3 STATIC ECONOMIC-ORDER-QUANTITY (EOQ) MODELS

This section presents three variations of the economic-order-quantity model with static (constant) demand. These models are characteristically simple from the analytic standpoint.

### 11.3.1 Classic EOQ Model

The simplest of the inventory models involves constant-rate demand with instantaneous order replenishment and no shortage. Define

$$y = \text{Order quantity (number of units)}$$

$$D = \text{Demand rate (units per unit time)}$$

$$t_0 = \text{Ordering cycle length (time units)}$$

The inventory level follows the pattern depicted in Figure 11.1. An order of size $y$ units is placed and received instantaneously when the inventory reaches zero level. The stock is then depleted uniformly at the constant demand rate $D$. The ordering cycle for this pattern is

$$t_0 = \frac{y}{D} \text{time units}$$

The cost model requires two cost parameters.

$$K = \text{Setup cost associated with the placement of an order (dollars per order)}$$

$$h = \text{Holding cost (dollars per inventory unit per unit time)}$$

Given that the average inventory level is $\frac{y}{2}$, the total cost *per unit time* (TCU) is thus computed as

$$\begin{aligned}
\text{TCU}(y) &= \text{Setup cost per unit time} + \text{Holding cost per unit time} \\
&= \frac{\text{Setup cost} + \text{Holding cost per cycle } t_0}{t_0} \\
&= \frac{K + h\left(\frac{y}{2}\right)t_0}{t_0}
\end{aligned}$$

$$= \frac{K}{\left(\frac{y}{D}\right)} + h\left(\frac{y}{2}\right)$$

The optimum value of the order quantity $y$ is determined by minimizing $\text{TCU}(y)$ with respect to $y$. Assuming $y$ is continuous, a necessary condition for finding the optimal value of $y$ is

$$\frac{d\,\text{TCU}(y)}{dy} = -\frac{KD}{y^2} + \frac{h}{2} = 0$$

The condition is also sufficient because $\text{TCU}(y)$ is convex.

The solution of the equation yields the EOQ $y^*$ as

$$y^* = \sqrt{\frac{2KD}{h}}$$



Thus, the optimum inventory policy for the proposed model is

$$\text{Order} \quad y^* = \sqrt{\tfrac{2KD}{h}} \quad \text{units every} \quad t_0{}^* = \tfrac{y^*}{D} \quad \text{time units}$$

Actually, a new order need not be received at the instant it is ordered. Instead, a positive **lead time**, $L$, may occur between the placement and the receipt of an order as Figure 11.2, demonstrates. In this case, the **reorder point** occurs when the inventory level drops to $LD$ units.

Figure 11.2 assumes that the lead time $L$ is less than the cycle length $t_0^*$, which may not be the case in general. To account for this situation, we define the **effective lead time** as

$$L_e = L - nt_0^*$$

where $n$ is the largest integer not exceeding $\frac{L}{t_0^*}$. This result is justified because after $n$ cycles of $t_0^*$ each, the inventory situation acts as if the interval between placing an order and receiving another is $L_e$. Thus, the reorder point occurs at $L_eD$ units, and the inventory policy can be restated as

Order the quantity $y^*$ whenever the inventory level drops to $L_eD$ units



## Example 11.3-1

Neon lights on the U of A campus are replaced at the rate of 100 units per day. The physical plant orders the neon lights periodically. It costs $100 to initiate a purchase order. A neon light kept in storage is estimated to cost about $.02 per day. The lead time between placing and receiving an order is 12 days. Determine the optimal inventory policy for ordering the neon lights.

From the data of the problem, we have

$$D = 100 \text{ units per day}$$

$$K = \$100 \text{ per order}$$

$$h = \$.02 \text{ per unit per day}$$
$$L = 12 \text{ days}$$

Thus,

$$y^* = \sqrt{\frac{2KD}{h}} = \sqrt{\frac{2 \times \$100 \times 100}{.02}} = 1000 \text{ neon lights}$$

The associated cycle length is

$$t_0^* = \tfrac{y^*}{D} = \tfrac{1000}{100} = 10 \text{ days}$$

Because the lead time $L = 12$ days exceeds the cycle length $t_0^*$ ($= 10$ days), we must compute $L_e$. The number of integer cycles included in $L$ is

$$\begin{aligned} n &= \left(\text{Largest integer} \leq \tfrac{L}{t_0^*}\right) \\ &= \left(\text{Largest integer} \leq \tfrac{12}{10}\right) \\ &= 1 \end{aligned}$$

Thus,



$$L_e = L - nt_0^* = 12 - 1 \times 10 = 2 \text{ days}$$

Thus,

$$L_e = L - nt_0^* = 12 - 1 \times 10 = 2 \text{ days}$$

The reorder point thus occurs when the inventory level drops to

$$L_eD = 2 \times 100 = 200 \text{ neon lights}$$

The inventory policy for ordering the neon lights is

*Order 1000 units whenever the inventory level drops to 200 units.*

The daily inventory cost associated with the proposed inventory policy is

$$\begin{aligned} \text{TCU}(y) &= \frac{K}{\left(\frac{y}{D}\right)} + h\left(\tfrac{y}{2}\right) \\ &= \frac{\$100}{\left(\frac{1000}{100}\right)} + \$.02\left(\tfrac{1000}{2}\right) = \$20 \text{ per day} \end{aligned}$$

## PROBLEM SET 11.3A

1. In each of the following cases, no shortage is allowed, and the lead time between placing and receiving an order is 30 days. Determine the optimal inventory policy and the associated cost per day.
   - **(a)** $K = \$100, h = \$.05, D = 30$ units per day
   - **(b)** $K = \$50, h = \$.05, D = 30$ units per day
   - **(c)** $K = \$100, h = \$.01, D = 40$ units per day
   - **(d)** $K = \$100, h = \$.04, D = 20$ units per day

*2. McBurger orders ground meat at the start of each week to cover the week's demand of 300 lb. The fixed cost per order is \$20. It costs about \$.03 per lb per day to refrigerate and store the meat.
   - **(a)** Determine the inventory cost per week of the present ordering policy.
   - **(b)** Determine the optimal inventory policy that McBurger should use, assuming zero lead time between the placement and receipt of an order.

3. A company stocks an item that is consumed at the rate of 50 units per day. It costs the company \$20 each time an order is placed. An inventory unit held in stock for a week will cost \$.35.
   - **(a)** Determine the optimum inventory policy, assuming a lead time of 1 week.
   - **(b)** Determine the optimum number of orders per year (based on 365 days per year).



**1**

$y^* = \sqrt{\frac{2KD}{h}}$, $t_0 = \frac{y^*}{D}$, $TCU(y^*) = \sqrt{2KDh}$

a) $y^* = \sqrt{\frac{2\times100\times30}{.05}} = 346.4$ units

$t_0 = \frac{346.4}{30} = 11.55$ days

$TCU(y^*) = \frac{100\times30}{346.4} + \frac{.05\times346.4}{2} = \$17.32$

Policy: order 346.4 units whenever inventory drops to 207.2 units
Effective lead time = 6.91 days

b) $y^* = \sqrt{\frac{2\times50\times30}{.05}} \approx 245$ units

$t_0^* = \frac{245}{30} = 8.16$ days

$L_e = 5.51$ days

$TCU(y^*) = \frac{50\times30}{245} + \frac{.05\times245}{2} = \$12.25$

Policy: order 245 units whenever inventory drops to 165.15 units

c) $y^* = \sqrt{\frac{2\times100\times40}{.01}} = 894.4$ units

$t_0 = \frac{894.4}{40} = 22.36$ days

$L_e = 7.64$ days

$TCU(y^*) = \frac{100\times40}{894.4} + \frac{.01\times894.4}{2} = \$8.94$

Policy: Order 894.4 units whenever inventory drops to 305.57 units.

$L_e = 0$ days

Policy: Order 239 lb whenever inventory drops to zero level.

c) Cost difference $= 51.50 - 50.20$
$= \$1.30$

**3**

a) $h = \frac{.35}{7} = \$.05$/unit/day

$D = 50$ units/day, $K = \$20$

$y^* = \sqrt{\frac{2\times20\times50}{.05}} = 200$ units

$t_0 = \frac{200}{50} = 4$ days

$L = 7$ days, $L_e = 3$ days

$R = 3\times50 = 150$ units

Policy: Order 200 units whenever inventory drops to 150 units.

b) Optimum number of orders $= \frac{365}{4}$
$\cong 91$ orders

**4**

(a) Policy 1: $D = \frac{R}{L_e} = \frac{50}{10} = 5$ units/day

Cost/day $= \frac{KD}{y} + \frac{hy}{2}$
$= \frac{20\times5}{150} + \frac{.02\times150}{2} = \$2.17$

Policy 2: $D = \frac{75}{[illegible]} = 5$ units/day

EXAMPLE–5.1 :

ONLINE SMART

A company requires 16000 units of raw material costing ₹ 2 per unit. The cost of placing an order is ₹ 45 and the carrying costs are 10% per year per unit of the average inventory. Determine :

(i) the economic order quantity

(ii) cycle time

(iii) total variable cost of managing the inventory.

① EOQ

② Cycle time

SOLUTION :

Given Data : $D = 16000$ units

$C_u = 2$ ₹/unit

$C_o = 45$ ₹/order

$i = 10\%$ per year per unit

**(1) Economic order quantity**

$$Q^* = \sqrt{\frac{2 \times D \times C_O}{C_C}}$$

$$C_C = Cu \times i = 2 \times 0.10 \text{ ₹/unit/year}$$

$$\therefore \quad Q^* = \sqrt{\frac{2 \times 16000 \times 45}{2 \times 0.10}}$$

$$\therefore \quad Q^* = 2684 \text{ units}$$

**(2) No. of orders per year and Cycle Time**

$$N^* = \frac{\text{Annual Demand}}{\text{EOQ}} = \frac{D}{Q^*}$$


$$C_C = Cu \times i = 2 \times 0.10 \text{ ₹/unit/year}$$

$$\therefore \quad Q^* = \sqrt{\frac{2 \times 16000 \times 45}{2 \times 0.10}}$$

$$\therefore \quad Q^* = 2684 \text{ units}$$

**(2) No. of orders per year and Cycle Time**

$$N^* = \frac{\text{Annual Demand}}{\text{EOQ}} = \frac{D}{Q^*}$$


time.

$N = \frac{A.D}{EOQ}$.

$$\therefore \quad N^* = \frac{16000}{2684} = 6 \text{ No. of orders/year}$$

$$T^* = \frac{Q^*}{D} = \frac{1}{N^*}$$

$$\therefore \quad T^* = \frac{1}{6} \text{ year}$$

$$\therefore \quad T^* = 2 \text{ months Order Cycle Time}$$

**(3) Total variable cost of managing the inventory**

$$TVC^* = \frac{D}{Q^*} \times C_O + \frac{Q^*}{2} \times C_C$$

$$TVC^* = \frac{16000}{2684} \times 45 + \frac{2684}{2} \times (2 \times 0.10)$$

$$\therefore \quad TVC^* = 268.4 + 268.4$$

$$\therefore \quad TVC^* = 536.8 \text{ ₹}$$