

- Sequence and Series YT

<https://www.youtube.com/watch?v=O4MdzWtOJto&list=PLHXZ9OQGMqxc4ySKTIW19TLrT91Ik9M4n>



<https://www.youtube.com/watch?v=wtlt1AU5bEI&list=PLHXZ9OQGMqxc4ySKTIW19TLrT91Ik9M4n>

$\frac{1}{32}, \frac{1}{16}, \frac{1}{8}, \frac{1}{4}, 1$

Sequence:  $\left\{ \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \dots \right\}$

$a_n = \left( \frac{1}{2} \right)^n$

Series:  $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots$

$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \left( \frac{1}{2} \right)^n = 1??$

<https://www.youtube.com/watch?v=G51Qc8XKtqM&list=PLHXZ9OQGMqxc4ySKTIW19TLrT91Ik9M4n>

Geometric series:

$\sum_{n=1}^{\infty} ar^{n-1} = \begin{cases} \text{Diverges} & |r| \geq 1 \\ \frac{a}{1-r} & |r| < 1 \end{cases}$

Ex 1:  $\sum_{n=1}^{\infty} \left( \frac{1}{2} \right)^n$

<https://www.youtube.com/watch?v=5ejmgwXVSqQ>

## Harmonic series:

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

$$s_2 = 1 + \frac{1}{2}$$

$$s_4 = 1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) > 1 + \frac{1}{2} + \left(\frac{1}{4} + \frac{1}{4}\right) = 1 + \frac{3}{4}$$

$$s_8 = 1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) > 1 + \frac{1}{2} + \left(\frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}\right) > 1 + \frac{3}{2}$$

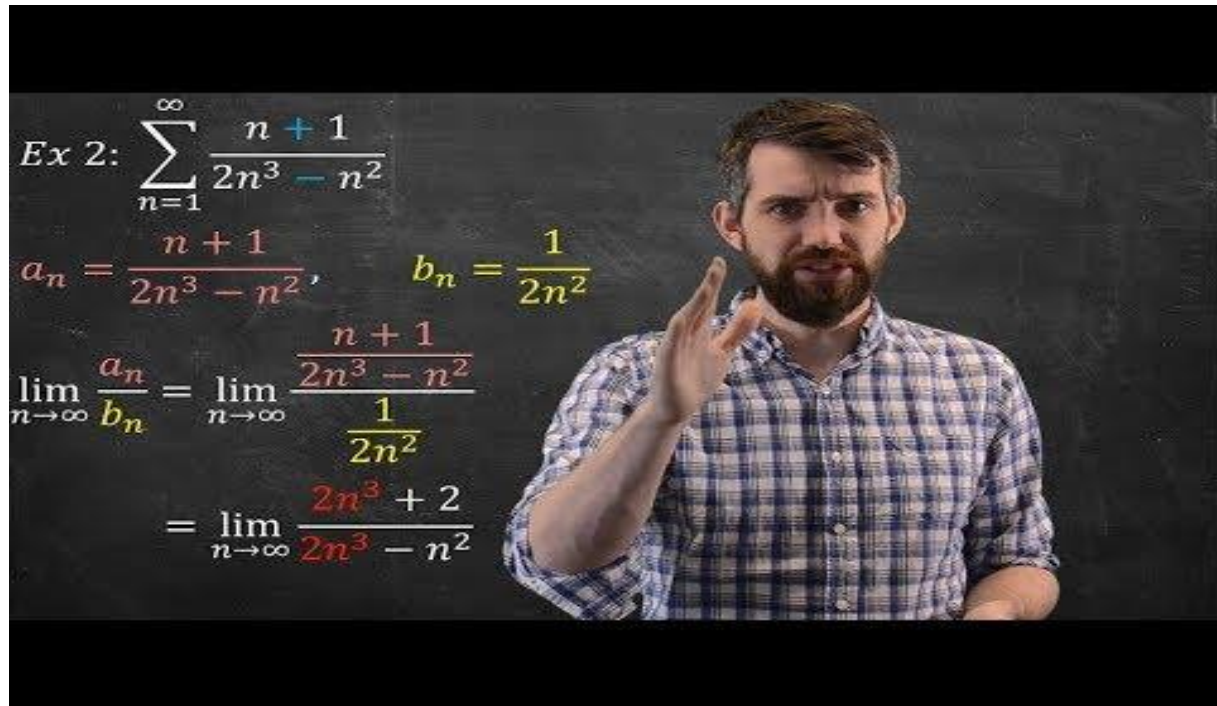
$$\text{Generally: } s_{2^n} > 1 + \frac{n}{2}$$



<https://www.youtube.com/watch?v=GBBg3ntsUDI&list=PLHXZ9OQGMqxc4ySKTIW19TLrT91lk9M4n>



<https://www.youtube.com/watch?v=7xhd8kdQFic&list=PLHXZ9OQGMqxc4ySKTIW19TLrT91Ik9M4n>



Ex 2:  $\sum_{n=1}^{\infty} \frac{n+1}{2n^3 - n^2}$

$a_n = \frac{n+1}{2n^3 - n^2}, \quad b_n = \frac{1}{2n^2}$

$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{2n^3 - n^2}}{\frac{1}{2n^2}}$

$= \lim_{n \rightarrow \infty} \frac{2n^3 + 2}{2n^3 - n^2}$

<https://www.youtube.com/watch?v=0wefqjpQyKM&list=PLHXZ9OQGMqxc4ySKTIW19TLrT91Ik9M4n>



$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

$$\sum_{n=1}^{\infty} \left( \frac{3n+1}{2n} \right)^n$$

$$\sum_{n=1}^{\infty} \frac{3^n}{n!}$$

$$\sum_{n=1}^{\infty} \frac{\sqrt{n^3+n}}{n^4-n^2}$$

**Limit Comparison Test:**  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c > 0$



<https://www.youtube.com/watch?v=we9xfbR8SwQ&list=PLHXZ9OQGMqxc4ySKTIW19TLrT91Ik9M4n>

Ex 1:  $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}} \right|$$

$$= \lim_{n \rightarrow \infty} \left| \frac{2^{n+1}}{2^n} \cdot \frac{n!}{(n+1)!} \right|$$

