Transportation Model

INSTRUCTOR: DR. KHUSRO MIAN

Transportation Model:

The transportation model is a special class of linear programming that deals with shipping a commodity from sources to destination. The objective is to determine the shipping schedule that minimizes the total shipping cost while satisfying supply and demand limits.

Mapping Method:

Formation Transportation Model:

There is single item that has to be transported from '**m**' sources to '**n**' destinations. The supply in source '**i**' is 'a_i' and the demands in destination '**j**' is 'b_j'. The unit cost of transportation from sources '**i**' to destination '**j**' is 'C_{ij}'.

Let X_{ij} be the quantity of the item transported from sources to destination. The objective function is:

Min $W = {}_i\sum^m {}_j\sum^n C_{ij} X_{ij}$

Subject to ${}_i\sum^n X_{ij} \leq a_i$ Supply

${}_j\sum^m X_{ij} \leq b_j$ Demand

$X_{ij} \geq 0$

Transportation Table:

Destination					
Sources		**D_1**	**D_2**	**D_3**	**Supply**
	S_1	C_{11}	C_{12}	C_{13}	**a_1**
	S_2	C_{21}	C_{22}	C_{23}	**a_2**
	S_3	C_{31}	C_{32}	C_{33}	**a_3**
	Demand	b_1	b_2	b_3	Supply = Demand

$a_1+a_2+a_3 = b_1+ b_2+ b_3$

Supply = Demand

Transportation Matrix:

$$X_{i\,j} = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \end{bmatrix}$$

Problem 1:

ABC Auto has three plants in A, B and C city and four major distribution center D1, D2, D3 and D4. The capacities of the three plants during the next quarter are 6, 1 and 10 cars. The quarterly demands at the four distribution center are 7, 5, 3 and 2 cars. The transportation costs per car in the table in different rates in $ are:

Destination						
Sources		**D1**	**D2**	**D3**	**D4**	**Supply**
	A	2	3	11	7	6
	B	1	0	6	1	1
	C	5	8	15	9	10
	Demand	7	5	3	2	17

Solution:

Let L.P model of the problem is given by:

Min: $W = 2x_{11} + 3x_{12} + 11x_{13} + 7x_{14} + 1x_{12} + 0x_{22} + 6x_{23} + 1x_{24} + 5x_{31} + 8x_{32} + 15x_{33} + 9x_{34}$

Subject to: $x_{11} + x_{12} + x_{13} + x_{14} \leq 6$

$x_{21} + x_{22} + x_{23} + x_{24} \leq 1$ Supply

$x_{31} + x_{32} + x_{33} + x_{34} \leq 10$

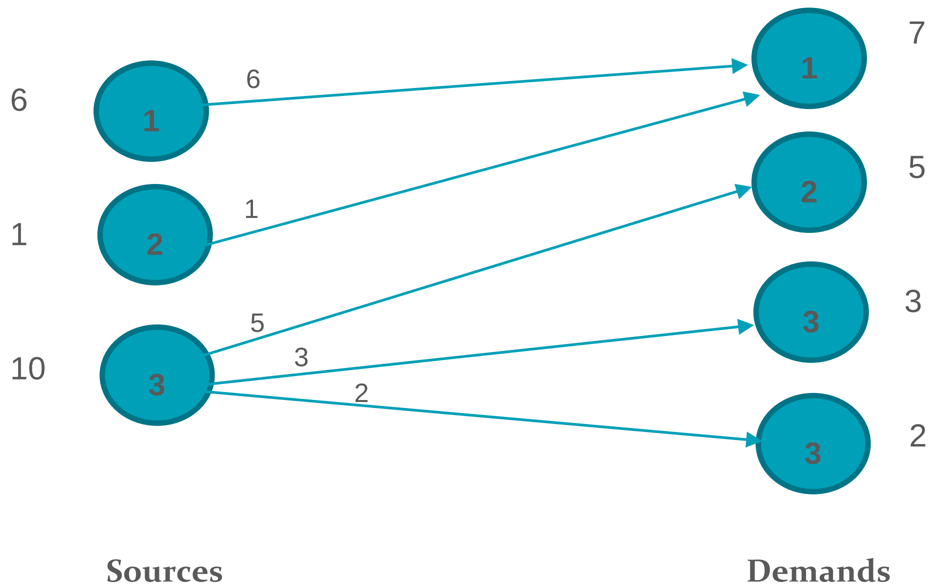
$x_{11} + x_{21} + x_{31} \leq 7$

$x_{12} + x_{22} + x_{32} \leq 5$ Demand

$x_{13} + x_{23} + x_{33} \leq 3$

$x_{14} + x_{24} + x_{34} \leq 2$

Transportation Map:



Transportation Matrix:

$$X_{ij} = \begin{bmatrix} 6 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 5 & 3 & 2 \end{bmatrix}$$

Putting the corresponding values of the matrix into W we get:

W = 2 x 6+ 0 + 0 + 0 + 1 x 1+ 0 + 0 + 0 + 0 + 8 x 5 + 3 x 5 + 9 x 2

W = 116 Answer

North West Corner Method:

There are three steps to the solution of transportation model:

1. **Starting with the cell at upper left corner of the transportation matrix**
   $X_{11} = \text{Min}\ (a_1, b_1)$

2. **If $\text{Min}(a_1 , b_1) = a_1$ then $X_{11} = a_{11}$**
   $X_{21} = \text{Min}\ (a_2, b_1 - X_{11})$

   If $\text{Min}\ (a_1, b_1) = b_1$ then $X_{11} = b_{11}$
   $X_{12} = \text{Min}\ (a_1\text{-}X_{11}, b_2)$

3. **If $a_1 = b_1$, $X_{11} = a_1 = b_1$**
   $X_{12} = \text{Min}\ (a_1 - a_1, b_1) = 0$
   OR
   $X_{21} = \text{Min}\ (a_2, b_1 - b_1) = 0$

Transportation Table:

Destination					
Sources		**D_1**	**D_2**	**D_3**	**Supply**
	S_1	C_{11}	C_{12}	C_{13}	**a_1**
	S_2	C_{21}	C_{22}	C_{23}	**a_2**
	S_3	C_{31}	C_{32}	C_{33}	**a_3**
	Demand	b_1	b_2	b_3	=

Problem 1:

Determine the basic feasible solution to the following transportation problem.

Destination						
Sources		**D_1**	**D_2**	**D_3**	**D_4**	**Supply**
	A	2	3	11	7	6
	B	1	0	6	1	1
	C	5	8	15	9	10
	Demand	7	5	3	2	17

Solution:

The objective function of the given transportation model.

Min: $W = 2x_{11} + 3x_{12} + 11x_{13} + 7x_{14} + 1x_{12} + 0x_{22} + 6x_{23} + 1x_{24} + 5x_{31} + 8x_{32} + 15x_{33} + 9x_{34}$

By North West Corner Method:

$X_{11} = \text{Min}(a_1, b_1)$
$X_{11} = \text{Min}(6, 7)$
$\mathbf{X_{11} = 6}$

$X_{21} = \text{Min}(a_2, b_1 - X_{11})$
$X_{21} = \text{Min}(1, 7 - 6)$
$X_{21} = \text{Min}(1, 1)$
$\mathbf{X_{21} = 1}$

$\mathbf{X_{31} = 0}$
$\mathbf{X_{22} = 0}$

$X_{32} = \text{Min}(a_3, b_2)$
$X_{32} = \text{Min}(10, 5)$
$\mathbf{X_{32} = 5}$

$X_{33} = \text{Min}\ (a_3 - X_{32}, b_3)$
$X_{33} = \text{Min}\ (10\text{-}5, 3)$
$X_{33} = \text{Min}\ (5, 3)$
$\mathbf{X_{33} = 3}$

$X_{34} = \text{Min}\ (a_3 - X_{33}, b_4)$
$X_{34} = \text{Min}\ (10\text{-}3, 2)$
$X_{34} = \text{Min}\ (7, 2)$
$\mathbf{X_{34} = 2}$

Transportation Matrix:

$$X_{ij} = \begin{bmatrix} 6 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 5 & 3 & 2 \end{bmatrix}$$

And we know that $X_{12} = X_{13} = X_{14} = X_{22}\ X_{23} = X_{24} = 0$. Hence:

The objective function:

W = 2 x 6+ 0 + 0 + 0 + 1 x 1+ 0 + 0 + 0 + 0 + 8 x 5 + 3 x 5 + 9 x 2

W = 116 Answer

Problem 2:

Solve the objective function of transportation model by North West Corner Method.

Destination						
Sources		**D_1**	**D_2**	**D_3**	**D_4**	**Supply**
	A	19	30	50	10	7
	B	70	30	40	60	9
	C	40	8	70	20	18
	Demand	5	8	7	14	34