# Wind Energy

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# Outline

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# History of Wind Energy

De Westermolens in Den Haag (1757). Paulus Constantijn la Fargue.

Understanding Wind Power Technology: Theory, Deployment and Optimisation, First Edition. Edited by Alois Schaffarczyk. Translated by Gunther Roth.

# History of Wind Energy

• 600–1890: Classical period. Classic windmills for mechanical drives; more than 100,000 windmills in northwestern Europe. The period ended after the discovery of the steam engine and because of the ready availability of wood and coal.

• 1890–1930: Development of electricity-generating wind turbines. The development of electricity as a source of energy available to everyone leads to the use of windmills as an additional possibility for generating electricity. Basic developments in the field of aerodynamics. The period ended due to cheaper fossil oil.

• 1930–1960: First phase of innovation. The necessity of electrifying rural areas and the shortage of energy during the Second World War stimulated new developments. Advances in the field of aerodynamics. The period ended because of cheaper gas and fossil oil.

• From 1973: Second innovation phase and mass production. The energy crisis and environmental problems in combination with technological advances ensure a commercial breakthrough.

# Power and Energy of Wind

- Power is the rate at which energy is used or produced. Light bulbs use power; WTs generate power.
- Units of power are expressed in work (energy) over an amount of time:

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{Energy}}{\text{Time}} = \frac{\text{N} \cdot \text{m}}{\text{s}} = \frac{\text{J}}{\text{s}} = \text{W}$$

- WTs are rated in terms of power. Large commercial wind turbines have power ratings in the 1–5 MW range, whereas a WT used for a single home is likely rated in the 3–15 kW range.

# Power and Energy of Wind

- Energy, is a measure of how much work can be done by a force. Units of energy are joules, J, the product of force and distance:

$$\text{Energy} = \text{N} \cdot \text{m} = \text{J}$$

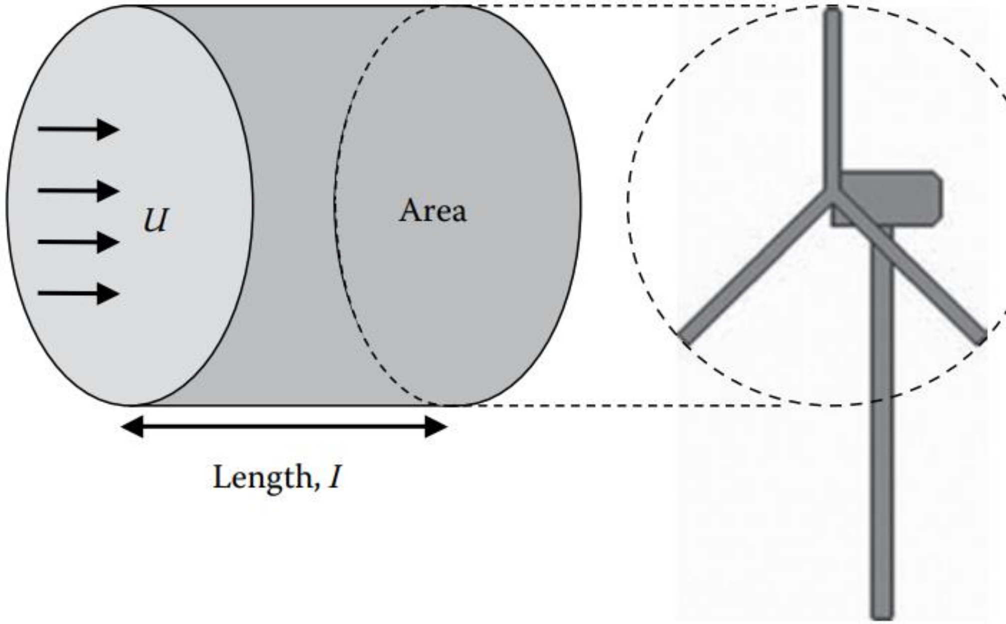
- The link between power and energy comes from the rate of power usage. Energy equals the product of power and time or power is energy divided by time.
- If a 100-W light bulb is turned on for 24 hours, the energy usage is 100 W × 24 h = 2.4 kWh.
- Turbines are denoted by their rated power production (e.g., 100 kW, 2.5 MW).
- We pay for electricity based on how much energy we use (e.g., $0.06 per kWh).

# Calculation of Wind Power

- To determine the power contained in the wind moving toward a WT, consider the kinetic energy, KE, of a mass of air, *m* (in kg), moving at speed, *U* (in m/s):

$$KE = \frac{1}{2} mU^2$$

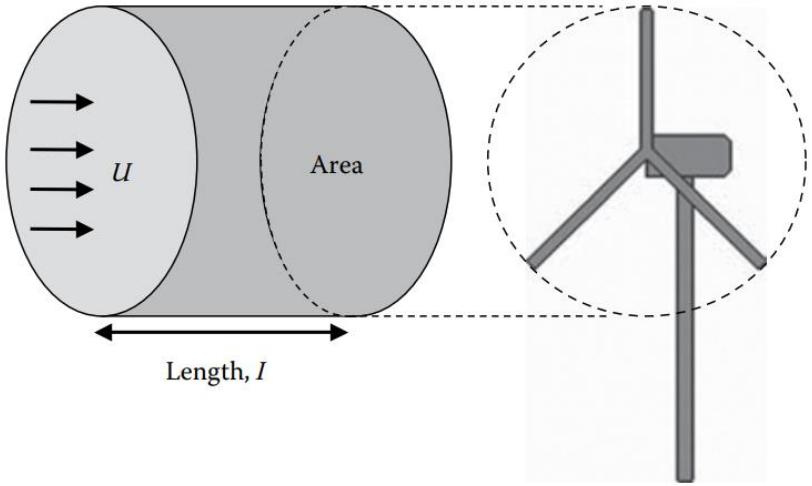
- A horizontal cylinder of air of area *A* (in m$^2$) and velocity *U*, moving toward a WT.
- The area, *A*, corresponds to the area swept out by the rotating WT blades.



# Calculation of Wind Power

- The mass $m$ of the air column is volume $V$ times the density, where $\rho$ represents the air density in kg/m3.

$\rho$ is the air density, $kg/m^3$
$V$ is the volume, $m^3$
$A$ is the area, $m^2$
$l$ is the length, m
$U$ is the wind speed, m/s
$t$ is the time, s




$$m = \rho V = \rho A l = \rho A U t$$

- The length of the column of air, $l$, is equal to the distance the wind travels in a given time interval, $t$, and is found by multiplying the wind speed, $U$, by the time.

# Calculation of Wind Power

$$KE = \frac{1}{2}mU^2 = \frac{1}{2}(\rho AUt)U^2 = \frac{1}{2}\rho AtU^3$$

- Dividing the KE by time yields the power of a moving column of air:

$$\text{Power} = \frac{KE}{t} = \frac{1}{2}\rho AU^3$$

- The power in a column of wind is linearly proportional to the air density and area of the air column.
- The power is also proportional to the cube of the wind speed.

# Calculation of Wind Power

- The circular area swept out by the turbine blades is proportional to the blade radius squared ($R^2$), turbine power is also proportional to the blade radius squared ($R^2$):

$$\text{Power} = \frac{1}{2}\rho A U^3 = \frac{1}{2}\rho\pi R^2 U^3$$

- Longer blade lengths and taller turbines (providing access to higher wind speeds) are options to increasing WT power output.
- Longer blades need more sophisticated manufacturing techniques to build blades strong enough to withstand wind gusts and extreme wind speeds (in excess of 100 mph).
- Taller towers must be strong enough to withstand the forces transmitted from the blades, weight of the nacelle, and other parts while still being cost-effective.
- Transporting turbine blades from manufacturing facilities to installation sites also becomes more challenging and more costly.

# Coefficient of Performance

- The coefficient of performance, $C_P$, which is a ratio of the aerodynamic power extracted by the WT, $P_{aero}$, divided by the total power in the wind, $P_{wind}$, and is calculated as:

$$C_p = \frac{P_{aero}}{P_{wind}} = \frac{P_{aero}}{1/2\,\rho A U^3} = \frac{P_{aero}}{1/2\,\rho \pi R^2 U^3}$$

- The theoretical maximum turbine efficiency was derived by Albert Betz in 1920 and is:

$$C_p = \frac{16}{27} \approx 0.59 = \text{Betz limit}$$

# Coefficient of Performance

- $C_P$ is a function of how fast the tips of the WT blades are rotating compared to the incoming wind.
- The ratio of the blade's rotational speed divided by the incoming wind speed is known as the tip speed ratio (TSR):

$$\text{Turbine tip speed ratio} = \frac{\Omega R}{U}$$

Where, $\Omega$ is the turbine blade rotational speed, rad/min
$R$ is the rotor radius, m
$U$ is the wind speed, m/s

# Example – TSR Calculation

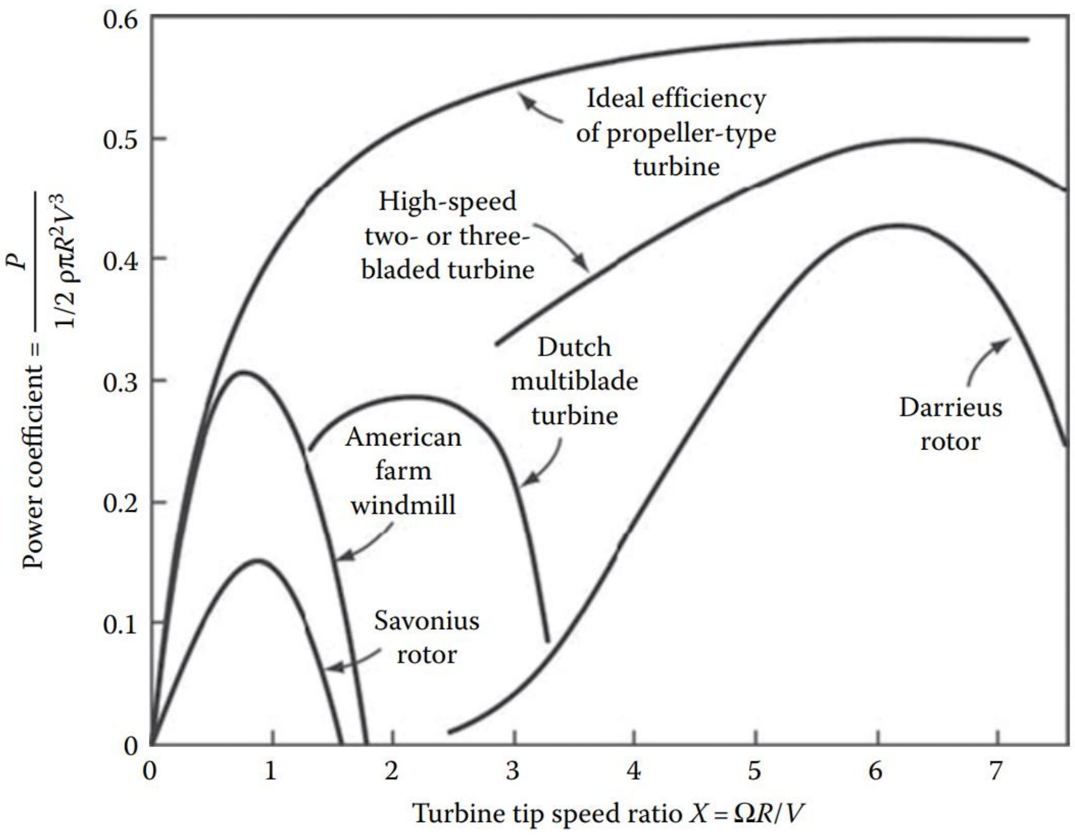
What is the TSR for a 90-m-diameter turbine rotating at 15 rpm at a wind speed of 10 m/s?

$$R = \frac{D}{2} = 45\,\mathrm{m} \quad \text{and} \quad U_{wind} = 10\,\mathrm{m/s}$$

$$\Omega = 15\,\mathrm{rpm} = 15\,\frac{\mathrm{rev}}{\mathrm{min}} \cdot 2\pi\,\frac{\mathrm{rad}}{\mathrm{rev}} \cdot \frac{\mathrm{min}}{60\,\mathrm{s}} = 1.57\,\mathrm{rad/s}$$

$$\mathrm{TSR} = \frac{\Omega R}{U_{wind}} = \frac{1.57\,\mathrm{rad/s} \cdot 45\,\mathrm{m}}{10\,\mathrm{m/s}} = 7.07$$

# Coefficient of Performance




- The highest line shows how the theoretical maximum power coefficient increases with increasing TSR.
- The Betz limit value of 0.59 is approached asymptotically as the TSR increases toward a value of seven (TSR = 7).
- The maximum theoretical performance coefficient for horizontal-axis two- or three-bladed turbines is approximately $C_P$ = 0.5.
- The Darrieus vertical-axis rotor has a maximum $C_P$ of approximately 0.40–0.45.

# Turbine Types


Darrieus VAWT

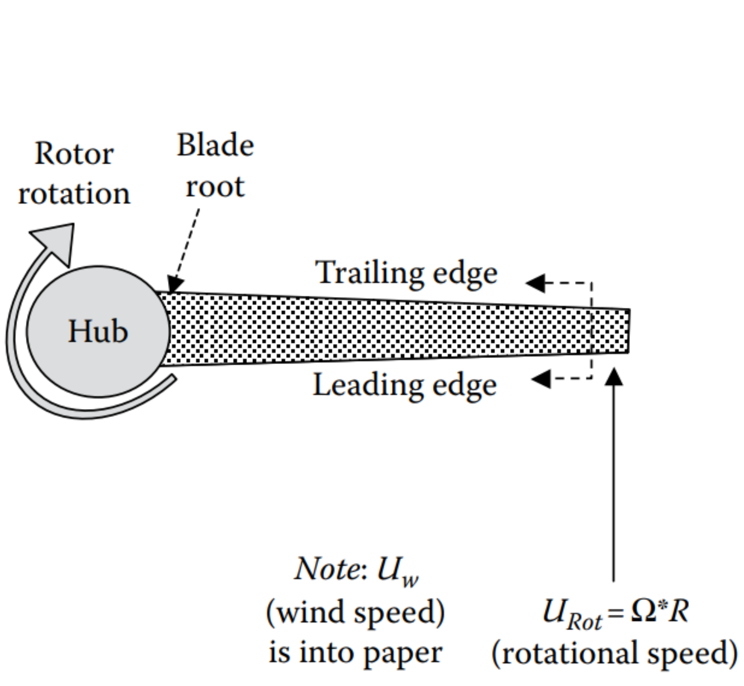
Three-bladed HAWT

Helical Savonius VAWT

https://www.eia.gov/energyexplained/wind/types-of-wind-turbines.php

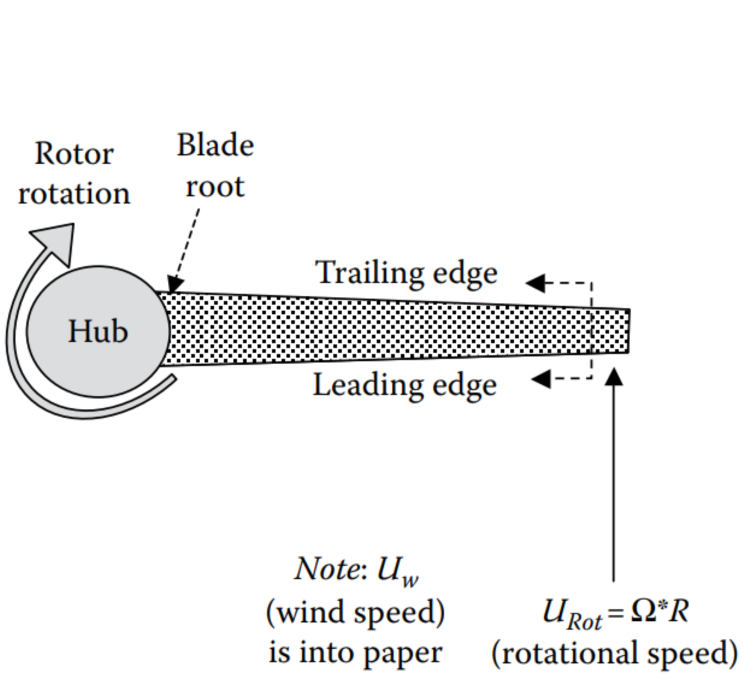
https://www.sciencedirect.com/topics/engineering/savonius-wind-turbine

# Aerodynamics



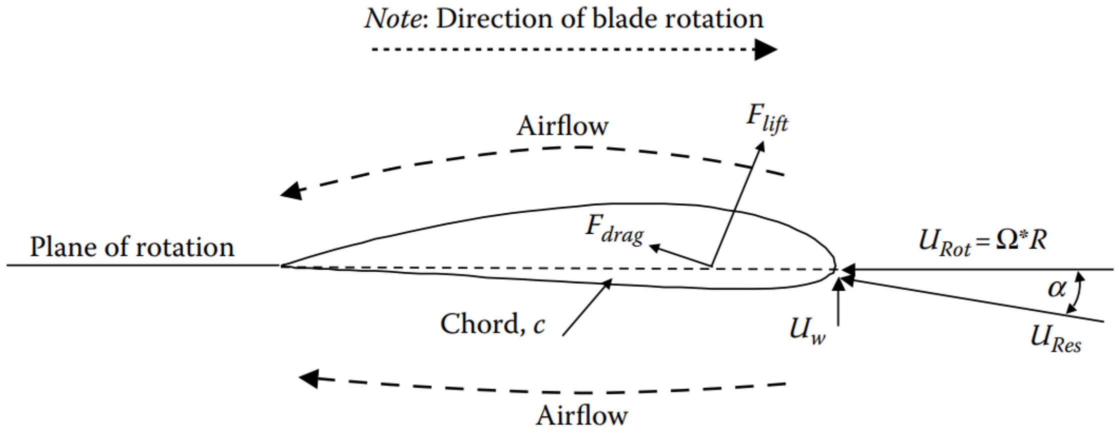

- A WT blade rotating (in the plane of the paper) in a clockwise direction.
- The atmospheric wind, $U_w$, is blowing toward the blade (i.e., into the plane of the paper).
- A section view of the blade is then taken near the tip of the blade looking in toward the root of the blade.

# Aerodynamics




- The two airflows acting on the blade are the incoming wind speed,
- $U_w$, which is perpendicular to the plane of the rotating blades, and the rotational wind
- speed, $U_{Rot} = \Omega R$, created by the turbine blade rotating at an angular speed, $\Omega$, at the radius of the cross section, $R$.

# Aerodynamics




- The line connecting the front (leading) and rear (trailing) edges of the airfoil is the chord line.
- The distance between the two points is designated as the chord, $c$.
- The resultant velocity component, $U_{Res}$, between the wind speed, $U_w$, and the wind arising from the blade's rotational speed, $U_{Rot}$.
- The angle between the resultant wind speed, $U_{Res}$, and the chord line, $c$, is the angle of attack, $\alpha$.

# Bernoulli’s Law

- Bernoulli’s law tells us that the pressure is the lowest where the velocity is the highest.

$$p + \frac{1}{2}\rho U^2 = \text{const}$$

- In the case of a WT blade, the pressure is lower over the rear (i.e., upper) surface of the airfoil compared to the front (i.e., lower) airfoil surface.
- The resultant forces on the airfoil are drawn perpendicular, $F_{lift}$, and parallel, $F_{Drag}$, to the resultant velocity, $U_{Res}$.
- The higher the lift force (and conversely the lower the drag force) along a WT blade, the greater the rotational force.
- The higher rotational force means the WT is capturing more of the wind’s energy and converting it to electricity.

# Lift-to-Drag Ratio

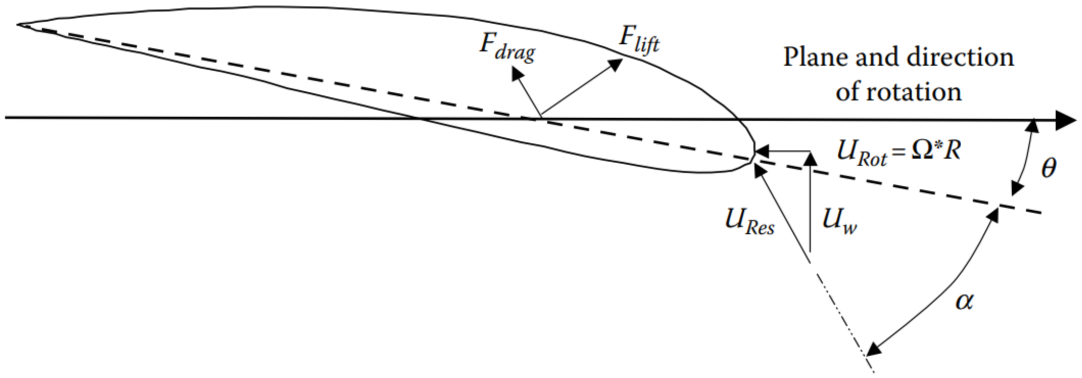
- The higher the lift-to-drag ratio $(L/D)$ is, the more efficient the blade will be at capturing the wind's energy.
- A Savonius VAWT has a poor $L/D$ ratio and thus a very low maximum $C_P$.
- The American farm windmill and Dutch multiblade turbines have higher $L/D$ ratios and hence higher maximum $C_P$ values.
- Machines with high $L/D$ ratios are the Darrieus VAWT rotor and the modern high-speed two- or three-bladed HAWT turbines, with the modern two- and three-bladed turbines having the highest $C_P$.

# Blade Geometry

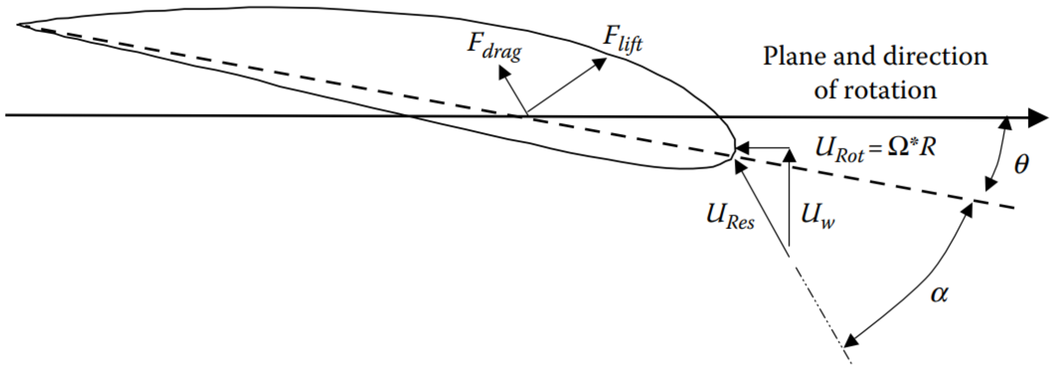
- WT blades have very complicated shapes that vary along the blade.
- The shape variation is needed not only to help start the WT rotating but also to operate very efficiently as the wind speed increases.
- The airfoil is twisted near the blade root to provide the required torque to start the turbine rotating at low wind speeds.
- Once rotating, the optimized blade profiles in the outer portions of the blade provide the necessary aerodynamic forces to keep the blade rotating at design speeds.

# Blade Geometry

- The blade is twisted, and the chord line now forms an angle $\theta$ with the plane of rotation of the blade.
- Twist angle, $\theta$, is a function of turbine blade radius and has a larger value near the blade root and a decreasing value with increasing blade radius. Because the resultant velocity, $U_{Res}$, has decreased approaching the blade root, this increases the angle between the chord line and $U_{Res}$, and results in an increased angle of attack.



# Blade Geometry



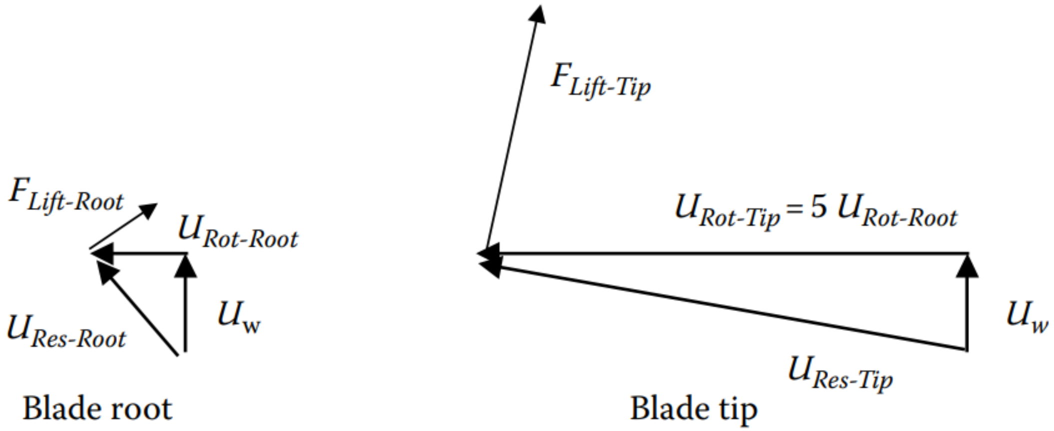

- The root portion of the blade encounters the same incoming wind speed, $U_w$, the rotational speed, $U_{Rot} = \Omega R_{Root}$, is lower. This occurs because, although the angular rate of rotation, $\Omega$, is the same along the blade, the radius at the blade root, $R_{Root}$, is smaller.
- The resulting lift force, $F_{Lift}$, per unit area is lower at the blade root compared to the blade tip. However, the direction of the force is better aligned to apply a torque to begin rotating the blades.

# Blade Geometry

- In addition, the surface area of the blade is considerably larger at the root. This provides sufficient area for the lower magnitude aerodynamic forces to act over, resulting in enough torque to start rotation at low wind speeds.
- The larger surface area is needed to accommodate the necessary structural strength to handle the cantilevered loads arising from wind forces on the blade and weight of the blade.

# Blade Geometry

- Assuming a constant wind speed at both the blade root and tip cross sections, we will consider the case where the root cross section is located at a radial location one-fifth of the blade tip cross section.
- This last assumption means that the rotation speed at the blade root, $U_{Rot-Root}$, is one-fifth the speed at the blade tip, $U_{Rot-Tip}$.



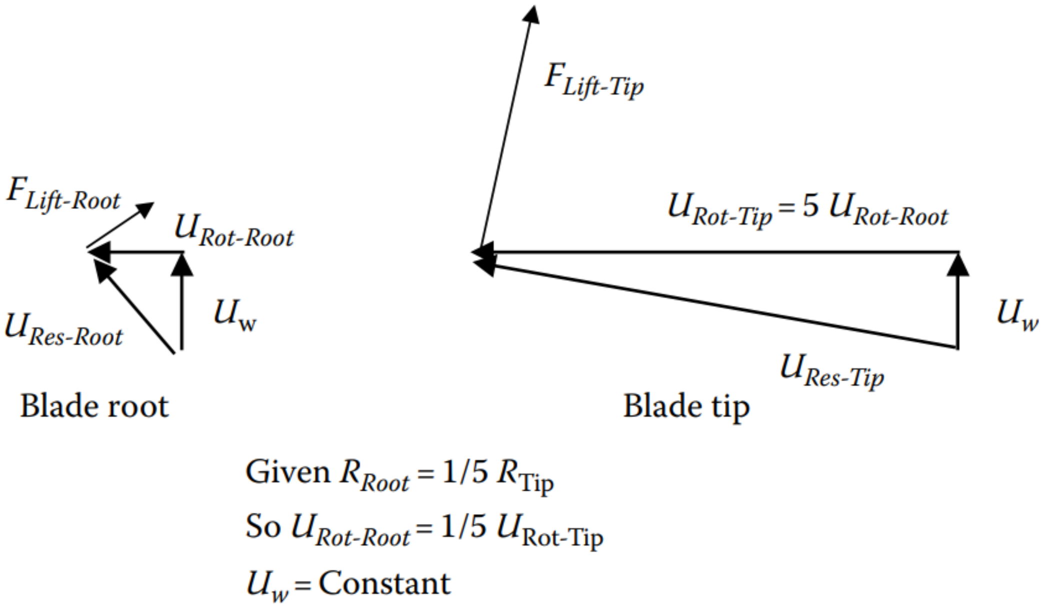

Given $R_{Root} = 1/5\ R_{Tip}$

So $U_{Rot\text{-}Root} = 1/5\ U_{Rot\text{-}Tip}$

$U_w = \text{Constant}$

# Blade Geometry

- The resultant velocity vector at the tip is much larger due to the contribution of the rotational wind speed. This results in a larger lift force per unit area at the tip.
- At the blade root, the lift force is better aligned to provide a start-up torque. While the blade root lift force is lower per unit area, it is applied over a large surface area.

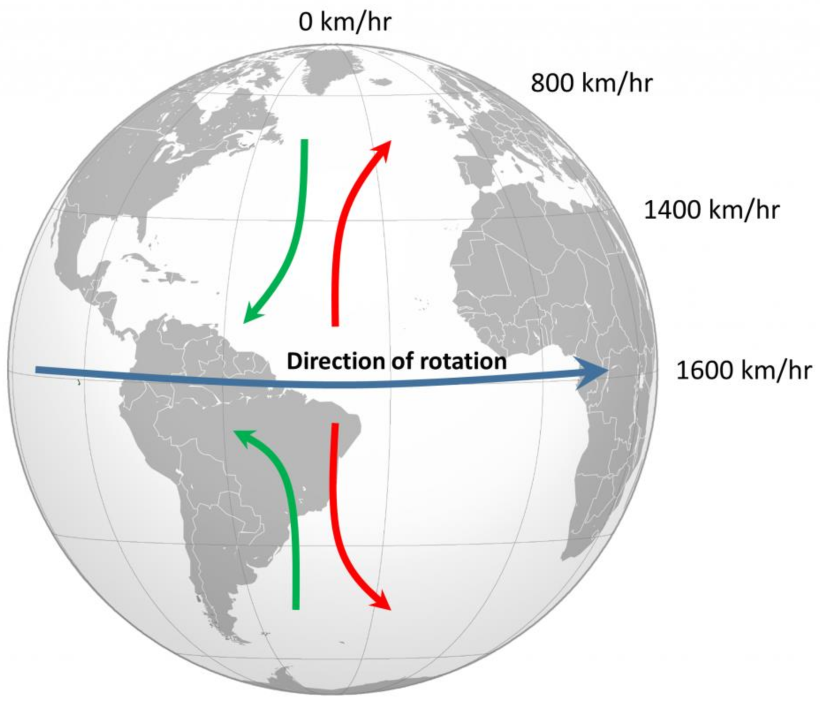


# The Coriolis Effect

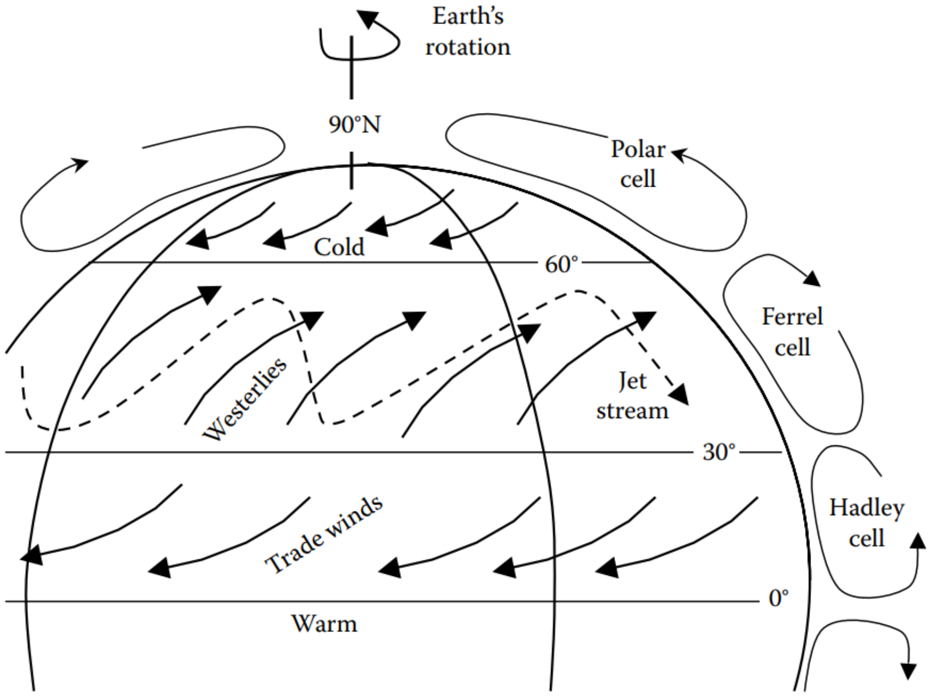
- The Coriolis Effect is a result of the fact that different latitudes on Earth rotate at different speeds. This is because every point on Earth must make a complete rotation in 24 hours, but some points must travel farther, and therefore faster, to complete the rotation in the same amount of time.
- In 24 hours a point on the equator must complete a rotation distance equal to the circumference of the Earth, which is about 40,000 km. A point right on the poles covers no distance in that time; it just turns in a circle. So the speed of rotation at the equator is about 1600 km/hr, while at the poles the speed is 0 km/hr. Latitudes in between rotate at intermediate speeds; approximately 1400 km/hr at 30° and 800 km/hr at 60°.
- As objects move over the surface of the Earth they encounter regions of varying speed, which causes their path to be deflected by the Coriolis Effect.

# The Coriolis Effect

- ❖ Objects moving from the equator towards the poles (red arrows) move into a region of slower rotational speed and their paths are deflected "ahead" of their point of origin.
- ❖ Movement from high latitudes to low latitudes (green arrows) goes from a region of low speed to a region of higher rotation speed, and there is deflection "behind" their point of origin.
- ❖ In the Northern Hemisphere this deflection is always to the right from the point of origin, and in the Southern Hemisphere the deflection is always to the left.

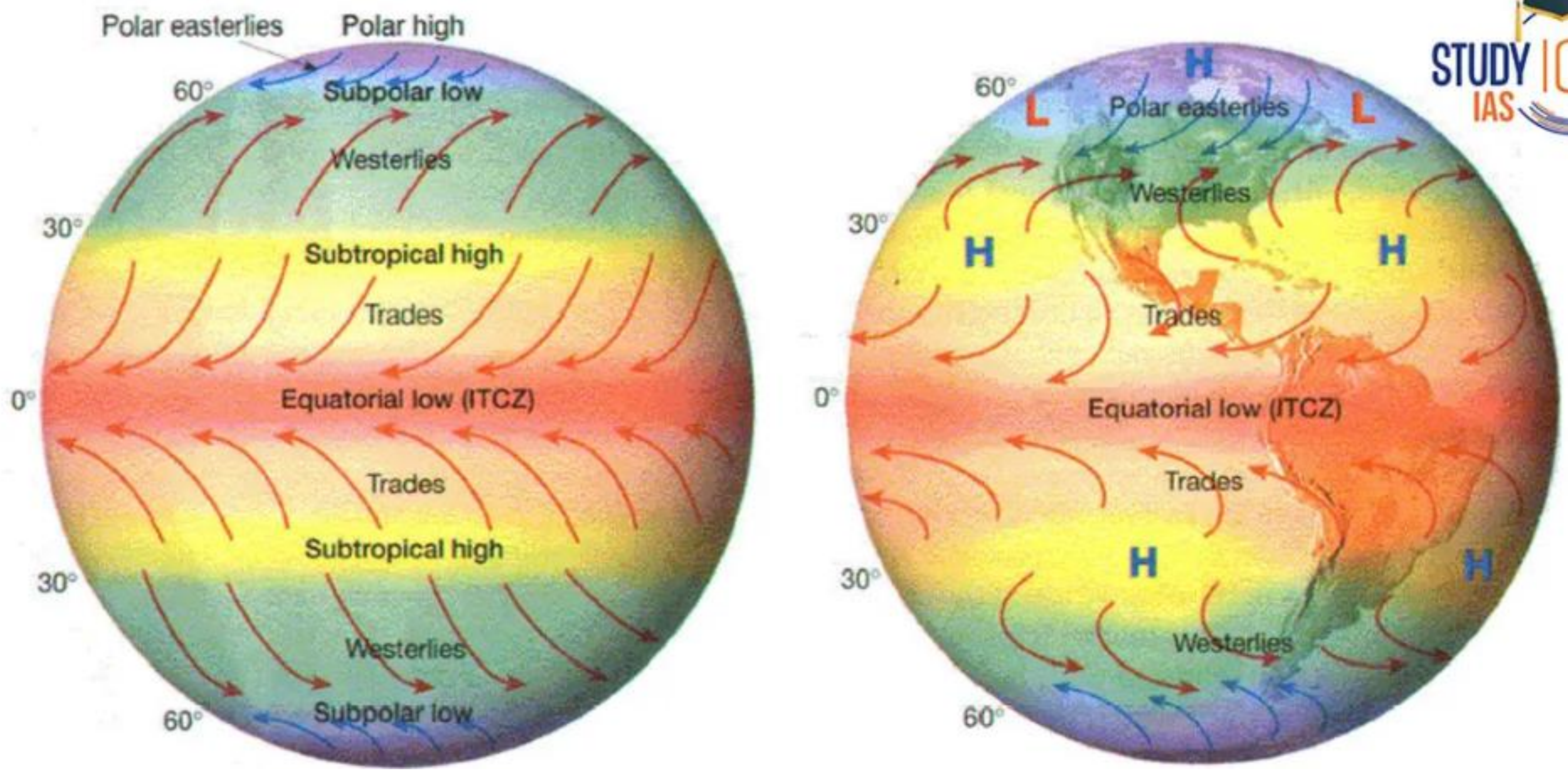


# Wind Characteristics



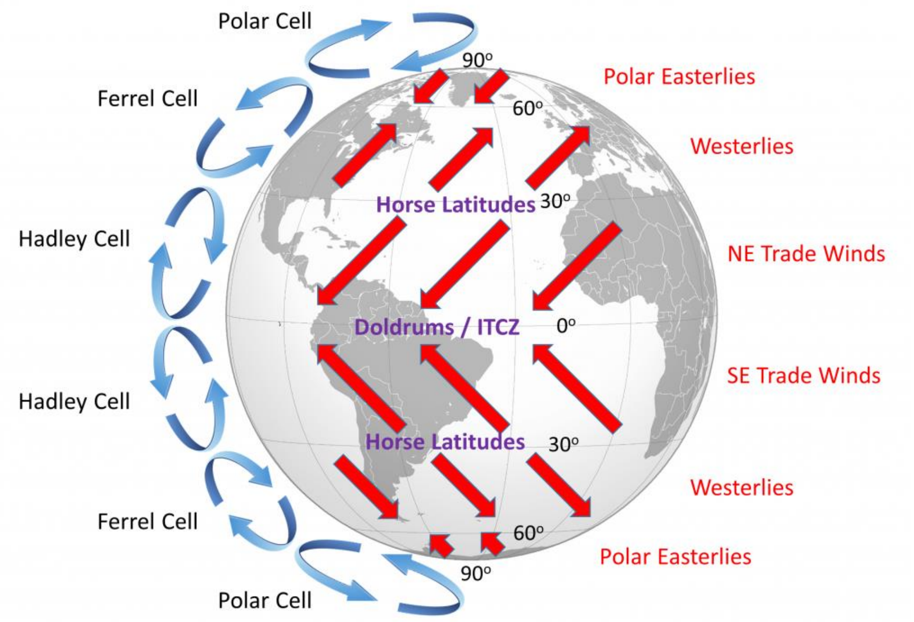

- The motion of air, otherwise known as wind, is caused by uneven heating of the Earth by solar radiation and the rotation of the Earth.
- First, Coriolis forces accelerate a particle of air to the right in the Northern Hemisphere (and to the left in the Southern Hemisphere).
- Second, each air particle has an angular momentum from west to east due to the Earth's rotational direction.

# Pressure Belts & Wind Characteristics



# Wind Characteristics

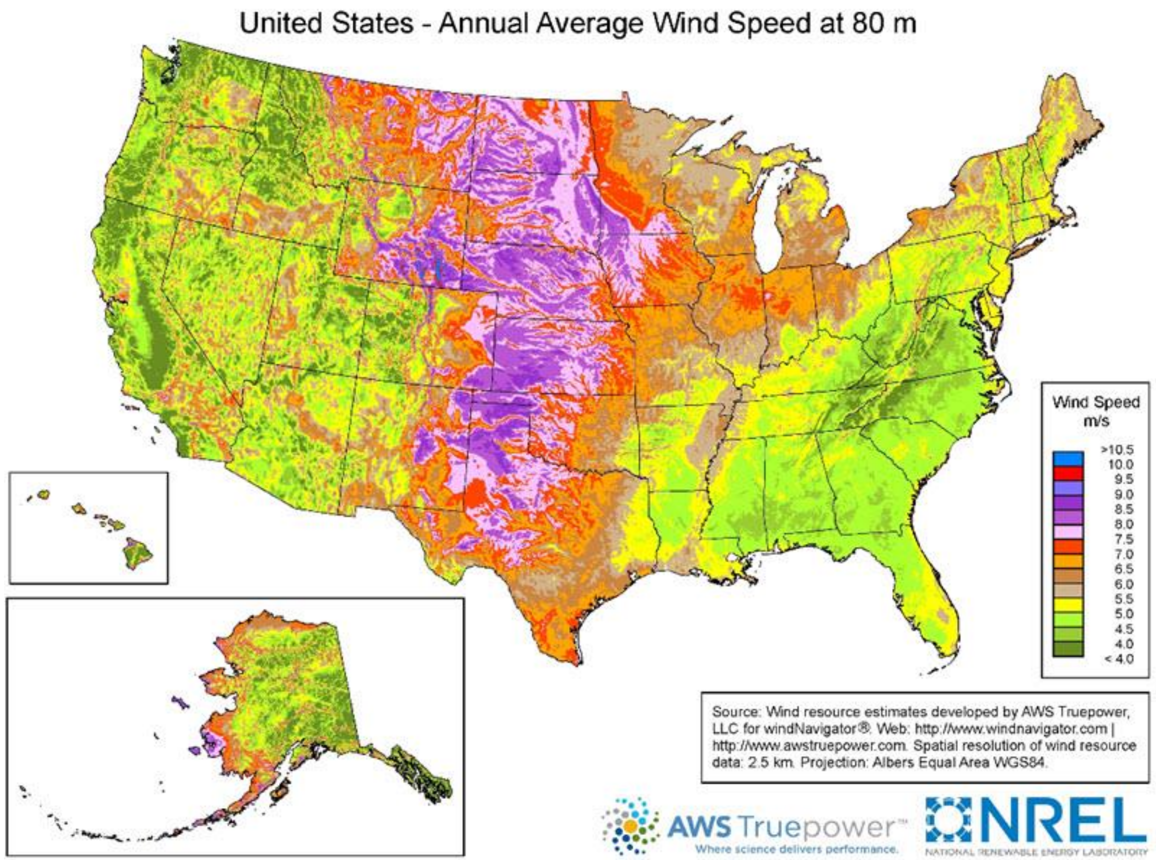
- In general, the westerlies are the best-suited region for wind energy applications.
- In addition, the polar jet stream (7–12 km or 20,000–40,000 ft above sea level) can have a profound impact on the westerlies, moving them north or south, resulting in lower or higher energy production from year to year.




Note: Along the equator the rising air creates a low pressure region called the doldrums, or the Intertropical Convergence Zone (ITCZ)(convergence zone because this is where the trade winds converge).

https://www.britannica.com/science/Ferrel-cell#/media/1/204996/107938

# Distribution of Wind




- The wind resource is the mean wind speed as well as how many hours per year the wind blows at a given speed.
- The 80 m height is a relevant starting point for current commercial installations, but taller turbines are increasingly used.
- The highest-class wind power (or highest mean wind speeds) regions are offshore along the East and West coasts.

# Distribution of Wind


https://p3tkebt.esdm.go.id/pilot-plan-project/energi_angin/potensi-energi-angin-indonesia-2020

# Wind Speed Profile

Log Law Wind Speed Profile

- The velocity profile as a function of height can be found to be: $U(z) = \frac{U^*}{\kappa} \ln\left(\frac{z}{z_0}\right)$
- Where,

  $U(z)$ is the wind speed at a height, $z$
  $U^*$ is the friction velocity arising from shear stress
  $z_0$ is the surface roughness length
  $\kappa$ is von Karman's constant ($\kappa = 0.4$)

- Values for $z_0$ are shown in this table:

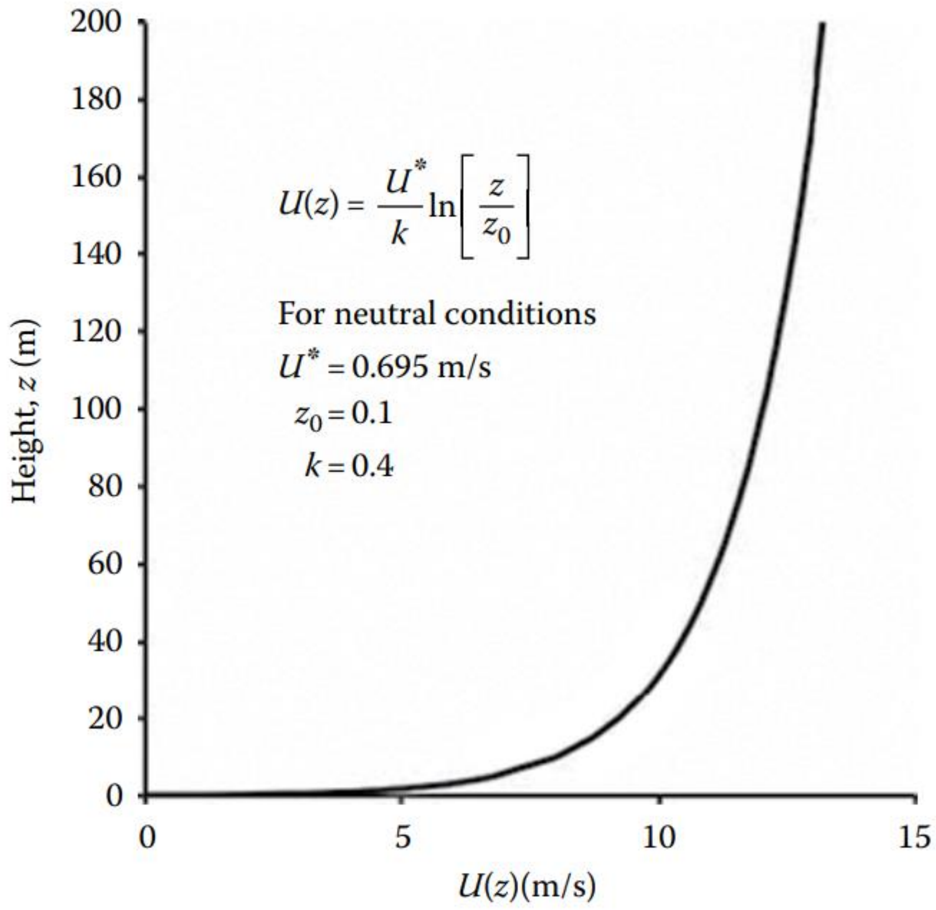
Surface Roughness for Various Outdoor Surfaces

| **Terrain Type** | **Roughness Length, $z_0$ (m)** |
|---|---|
| Cities, forests | 0.7 |
| Suburbs, wooded countryside | 0.3 |
| Countryside with trees and hedges | 0.1 |
| Open farmland | 0.03 |
| Flat grassy plains | 0.01 |
| Flat desert, rough sea | 0.001 |

*Source:* Burton, T. et al., 2001. *Wind Energy Handbook*, Wiley, Chichester, U.K. [34]; Manwell, J.F. et al., 2002. *Wind Energy Explained: Theory, Design and Application*, Wiley, Chichester, U.K. [53]

# Wind Speed Profile

- Substituting typical values for $U^*$ and $z_0$, the shape of the resulting atmospheric velocity profile for a neutrally stable atmosphere.
- One sees a predicted gradual change in wind speed through the first 20 m or so of the atmospheric boundary layer (ABL) and then an exponential growth approaching a steady-state value above 200 m.



# Wind Speed Profile

- At a height $z$, for the wind speed, $U(z)$, and then dividing it by the same equation for a reference height, $z_r$ at the reference wind speed, $U(z_r)$, eliminates the unknown friction velocity, $U^*$, and yields:

$$\frac{U(z)}{U(z_r)} = \frac{\ln(z/z_0)}{\ln(z_r/z_0)}$$

- Consider a meteorological tower taking measurements at 40 m ($z_r$ = 40 m), measuring a wind speed of $U(z_r)$ = 7.2 m/s at $z_r$, located in a wooded countryside, $z_0$ = 0.3, then the wind speed, $U(z)$, at a hub height of $z$ = 95 m is:

$$U(z) = 7.2 \cdot \frac{\ln(95/0.3)}{\ln(40/0.3)} = 8.5\,\mathrm{m/s}$$

# Wind Speed Profile

Power Law Wind Speed Profile

- The power law profile is used when one does not know the surface roughness, $z_0$, and/or stability information.

$$U(z) = U(z_r) \cdot \left[ \frac{z}{z_r} \right]^{\alpha}$$

where $\alpha$ is the power law or shear exponent.

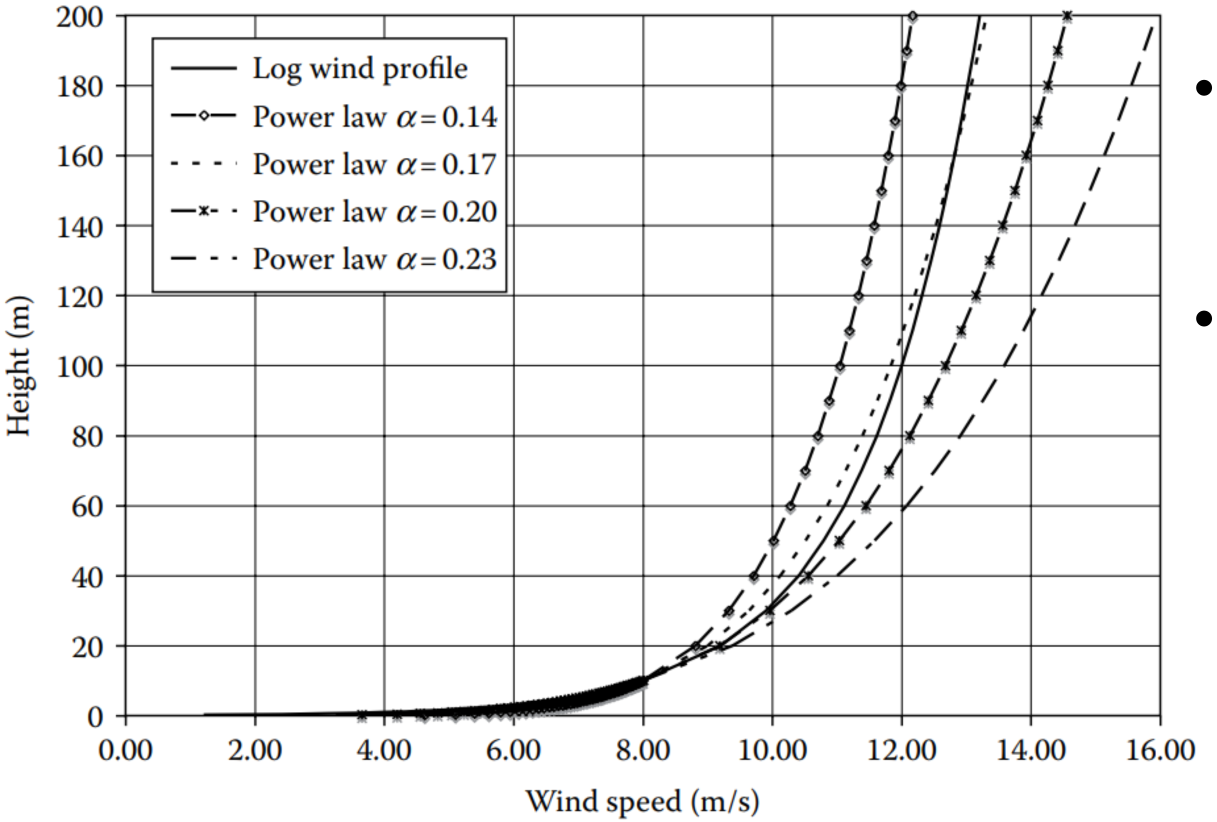
- Typical values for $\alpha$ are

$$\text{For neutral flows:} \quad \alpha \cong \frac{1}{7} = 0.143$$

$$\text{For unstable flows:} \quad \alpha < 0.1$$

$$\text{For strongly stable flows:} \quad \alpha > 0.3$$

# Wind Speed Profile




- The wind speed profile of the power law profile for a value of $\alpha$ = 0.17 is similar to the log law.
- The value of $\alpha$ is highly dependent on temperature, weather, geographic location (elevation and terrain), and time (of day and season).

# Wind Speed Profile

- The calculated wind speeds for two different values of $\alpha$ at a height of 100 m, which is a typical hub height for modern commercial WTs.
- The change in predicted power based on these wind speeds is the ratio $(13.6/11)^3 = 1.89$. This indicates that the predicted power can nearly double at 100 m if the value of $\alpha$ increases from 0.14 to 0.23.

Variation in Wind Speed with $\alpha$ at a Height of 100 m

| $\alpha$ | **Wind Speed $U$(z) (m/s)** |
|---|---|
| 0.14 | 11.0 |
| 0.23 | 13.6 |

# Probability of Observing a Given Wind Speed

- Determine the probability of a given wind speed occurring over a desired time period using a probability distribution function (PDF).
- A common PDF is the normal or Gaussian PDF. When graphed, this Gaussian PDF yields the commonly known "bell-shaped" curve.
- For all PDFs, the vertical axis is the observed frequency of a given event within a certain range or magnitude. The horizontal axis contains the range of all events. The peak of any PDF curve (where the curve is denoted by $f(x)$) is the mean magnitude of all occurring events.

# Probability of Observing a Given Wind Speed

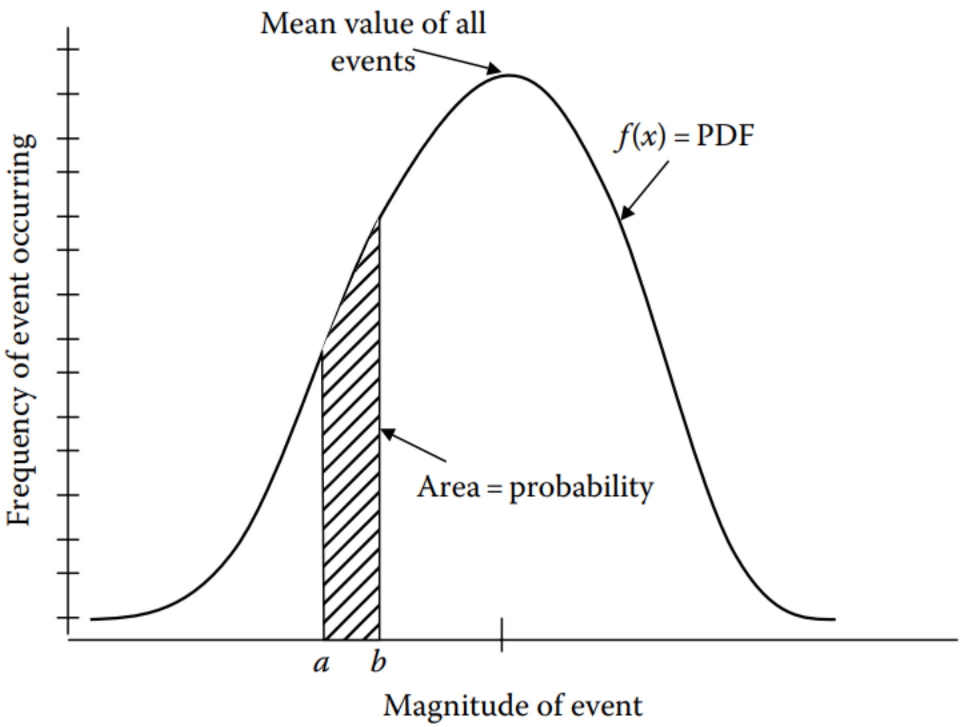
- The area under any section of a PDF curve corresponds to the probability, $P$, of a range of events occurring. The probability, $P$, of a range of events occurring is obtained by integrating the PDF, $f(x)$, between the range of the events, $a$ and $b$.

$$P = \int_a^b f(x)dx$$

- Integrating the PDF between the limits of $-\infty$ and $\infty$ yields a value of 1. In other words, the probability of having an event occur of any magnitude between the limits of all events is 100%.

$$P = \int_{-\infty}^{\infty} f(x)dx = 1$$

# Probability of Observing a Given Wind Speed




Gaussian or normal probability density function

- Two commonly used PDFs, Weibull and Rayleigh, are used to describe the probability of observing a given wind speed. These distributions are used because they more closely represent the distribution of wind speeds at a given location.
- These PDFs are NOT symmetric, as is the Gaussian distribution, but are skewed to the "left".
- A wind speed distribution skewed to the "left" means that the probability of seeing a lower wind speed is higher than the probability of seeing a higher wind speed.

# Probability of Observing a Given Wind Speed

- These PDFs are typically determined at a desired height, for example, hub height.
- Integrating the PDF will provide the probability, $P(U)$ of wind speed, $U$, occurring between two wind speeds, $U_a$ and $U_b$.

$$P = \int_{U_a}^{U_b} f(U)dU$$

- Mean wind speed:

$$\bar{U} = \int_{0}^{\infty} Uf(U)dU$$

# Probability of Observing a Given Wind Speed

- Standard deviation:

$$\sigma = \sqrt{\int_0^{\infty} \left(U - \bar{U}\right)^2 f(U)\,dU}$$

- If we want to know the probability (or the percent of time) that the wind will be equal to or less than a given wind speed. For example, what percent of a year will the wind speed be too low for the turbine to operate?

- This is known as the cumulative distribution function, $C(U)$, and is found by integrating from 0 m/s to a desired wind speed, $U$ m/s:

$$C(U) = \int_0^{U} f(U)\,dU$$

# Weibull PDF

- The Weibull PDF and cumulative distribution function are given by:

$$f(U) = \left(\frac{k}{c}\right)\left(\frac{U}{c}\right)^{k-1} \exp\left[-\left(\frac{U}{c}\right)^{k}\right]$$

$$C(U) = 1 - \exp\left[-\left(\frac{U}{c}\right)^{k}\right]$$

Where,

$k$ is the shape parameter
$c$ is the scale parameter

- $k$ and $c$ are functions of the mean wind speed, $U$, and the standard deviation of the wind speed, $\sigma$.

# Rayleigh PDF

- The Rayleigh PDF and cumulative distribution function are given by:

$$f(U) = \frac{\pi}{2}\left(\frac{U}{\bar{U}^2}\right)\exp\left[-\frac{\pi}{4}\left(\frac{U}{\bar{U}}\right)^2\right]$$

$$C(U) = 1 - \exp\left[-\frac{\pi}{4}\left(\frac{U}{\bar{U}}\right)^2\right]$$

- The Rayleigh PDF only depends on one parameter, the mean wind speed, $U$, making it much easier to use.
- Note that the Rayleigh distribution is a special case of the Weibull distribution when $k$ = 2 and $c = \bar{U}\left(\frac{4}{\pi}\right)^{1/2}$.

# Rayleigh Wind Speed Distribution Calculations

An analysis of wind speed data (10 min interval average, taken over a 1-year period) has yielded an average speed of $U$ = 6 m/s for a potential WT site. It has been determined that a Rayleigh wind speed distribution gives a good fit to the wind data.

A. Estimate the number of hours per year that the wind speed will be between $U_B$ = 10.5 and $U_A$ = 9.5 m/s during the year.

B. Estimate the number of hours per year that the wind speed is above 16 m/s.

# Rayleigh Wind Speed Distribution Calculations

A

To do this, we will find the probability the wind speed will be in a given range during the year, $P(U_A \leq U \leq U_B)$, and multiply the probability by the number of hours in a year.

$$P(U_A \leq U \leq U_B) = \int_{U_A}^{U_B} f(U)dU \quad = \int_{0}^{U_B} f(U)dU \quad - \int_{0}^{U_A} f(U)dU \quad = C(U_B) - C(U_A)$$

For a Rayleigh distribution $C(U) = 1 - \exp\left[-\frac{\pi}{4}\left(\frac{U}{\bar{U}}\right)^2\right]$

So

$$C(U_B) - C(U_A) = \left\{1 - \exp\left[-\frac{\pi}{4}\left(\frac{U_B}{\bar{U}}\right)^2\right]\right\} - \left\{1 - \exp\left[-\frac{\pi}{4}\left(\frac{U_A}{\bar{U}}\right)^2\right]\right\} = P(U_A \leq U \leq U_B)$$

# Rayleigh Wind Speed Distribution Calculations

Substituting $\bar{U} = 6$ m/s, $U_B = 10.5$ m/s, $U_A = 9.5$ m/s yields

$$P(U_A \le U \le U_B) = 0.049$$

In other words, the wind speed will be between $U_B$ and $U_A$ m/s 4.9% of the year. Thus, the total number of hours the wind speed is between $u_B$ and $u_A$ in a year is

$$\frac{\#\text{h}}{\text{year}} = 0.0494 \cdot \frac{24\,\text{h}}{\text{day}} \cdot \frac{365\,\text{day}}{\text{year}} = 432\,\text{h/year}$$

# Rayleigh Wind Speed Distribution Calculations

B This problem is the same as part A with the wind speed ranges being

$$U_B = \infty \quad \text{and} \quad U_A = 16\,\mathrm{m/s}.$$

$$P(16 \leq U \leq \infty) = \int_{16}^{\infty} P(U)dU = C(\infty) - C(16)$$

$$C(\infty) - C(16) = 1 - \exp[-\infty] - \left[1 - \exp\left[-\frac{\pi}{4}\left(\frac{U_A}{\bar{U}}\right)^2\right]\right] = \exp\left[-\frac{\pi}{4}\left(\frac{U_A}{\bar{U}}\right)^2\right] = 0.0038$$

# Rayleigh Wind Speed Distribution Calculations

Thus, the total number of hours the wind speed is greater than 16 m/s (i.e., between $U_B = \infty$ and $U_A = 16$ m/s) in a year is

$$\frac{\#\mathrm{h}}{\mathrm{year}} = 0.0038 \cdot \frac{24\,\mathrm{h}}{\mathrm{day}} \cdot \frac{365\,\mathrm{day}}{\mathrm{year}} = 33\,\mathrm{h/year}$$

# Turbine Performance

- The efficiency of a WT is the power it extracts compared to the power contained in the wind flowing through the area of the blades.
- The turbine does not start producing power until the wind speed reaches the cut-in wind speed.
- This is the lowest wind speed in which the rotor's aerodynamic power is greater than the sum of equipment loads and mechanical and electrical inefficiencies.
- The rated wind speed is the speed at which the turbine is operating near its peak efficiency.

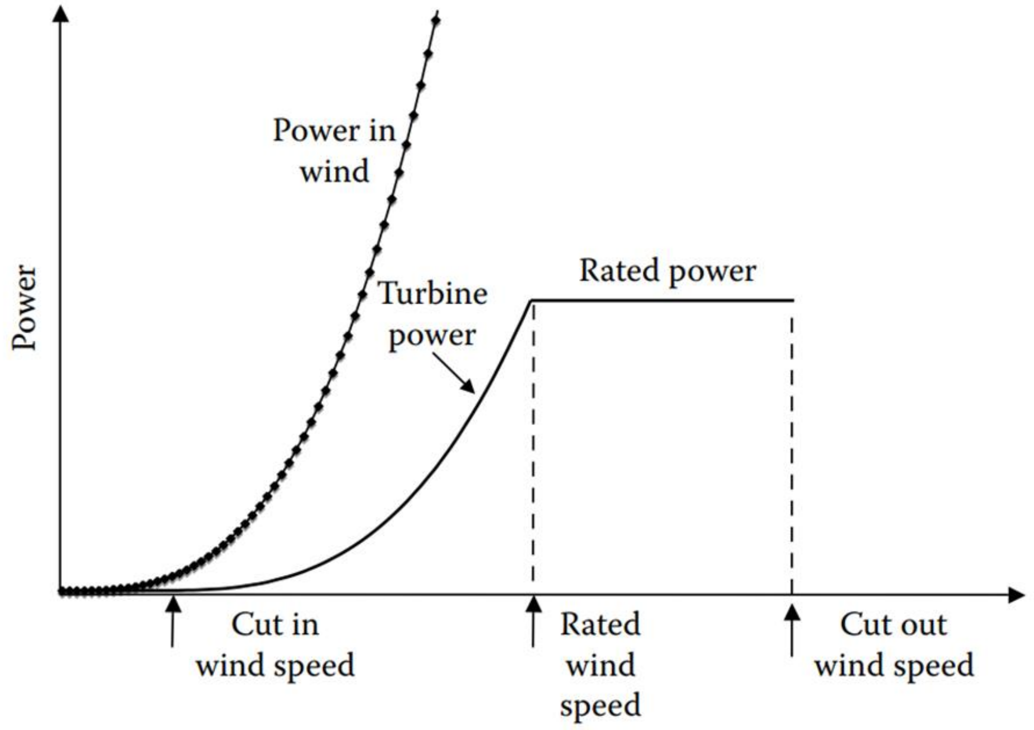
# Power Coefficient

- WT's efficiency is known as the power coefficient, $C_p$, and is defined as:

$$C_p = \frac{P_{aero}}{P_{wind}} = \frac{Q\Omega}{1/2\rho A U^3}$$

- Where,

$Q$ is the aerodynamic torque
$\Omega$ is the rotor rotational speed



# Example – Power Calculation

Given a WT with the following parameters, calculate the annual energy production:

$C_p = C_{p_{max}}$ = 0.48 ($C_p$ value assumed maximum and constant for all wind speeds).

Rated power = 5000 kW = 5.0 MW
Annual average wind speed = 8.0 m/s
Rated wind speed = 13.0 m/s
Cut-out wind speed = 25 m/s
Rotor radius = 48.0 m
Area = 7238.2 $m^2$
Air density = 1.20 $kg/m^3$

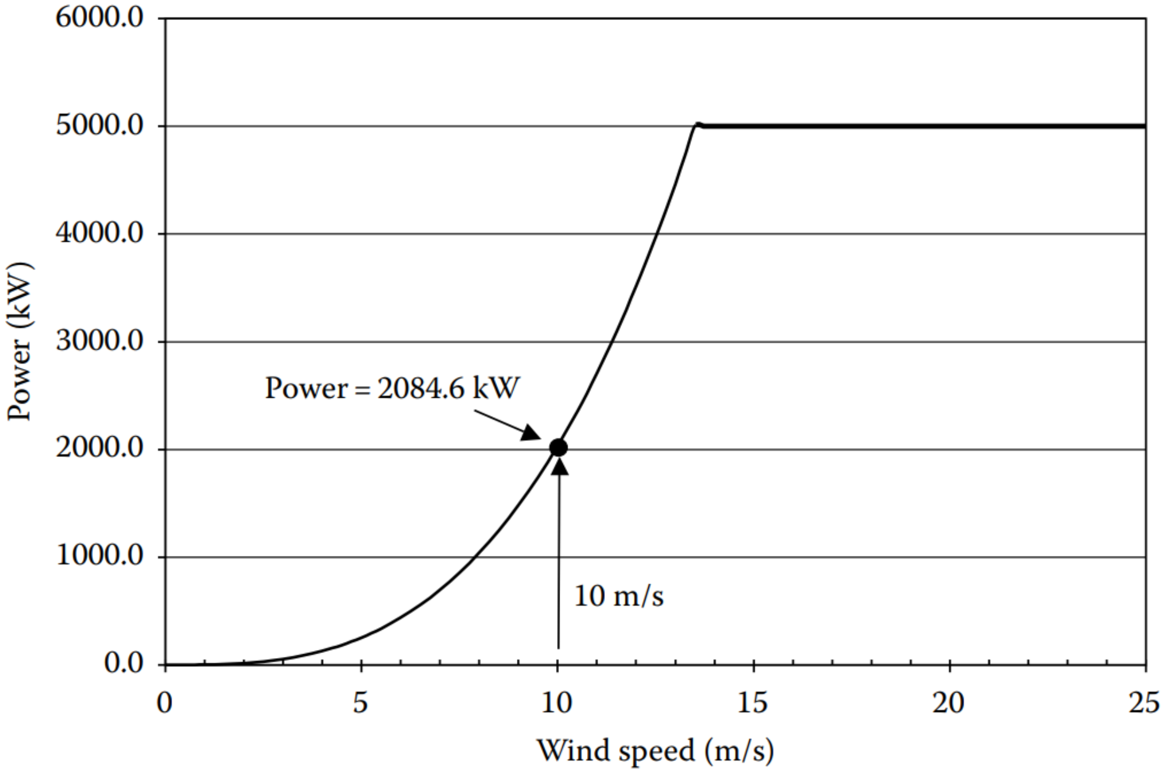
# Example – Power Calculation

- The annual energy production for all wind speeds will be illustrated by first calculating the annual energy production at a single wind speed. Then, the methodology will be applied to all wind speeds to arrive at the annual energy production for all wind speeds.
- First, determine the power production at a single wind speed, say 10 m/s. Then, a Rayleigh distribution will be calculated to determine the number of hours the wind speed is at 10 m/s. The power and number of hours are then multiplied to determine the yearly energy production for a 10 m/s wind speed.
- For a wind speed of 10 m/s, the power produced for the values shown earlier is:

$$P_{aero} = \frac{1}{2} C_P \rho A U_{wind}^3 = \frac{1}{2}\left(0.48 \cdot 1.2 \cdot 7238.2 \cdot 10^3\right) = 2084.6\,\text{kW}$$

# Example – Power Calculation

- A plot of the power produced for all wind speeds is shown.
- Note that above the rated wind speed, from 13 m/s up to the cut-out wind speed, the power production is constant at the rated level of 5000 kW.



# Example – Power Calculation

- Number of hours at 10 m/s: Assuming a Rayleigh PDF with an average wind speed of 8 m/s, the annual probability of a 10 m/s wind, calculated for the interval from 9.5 to 10.5 m/s, is:

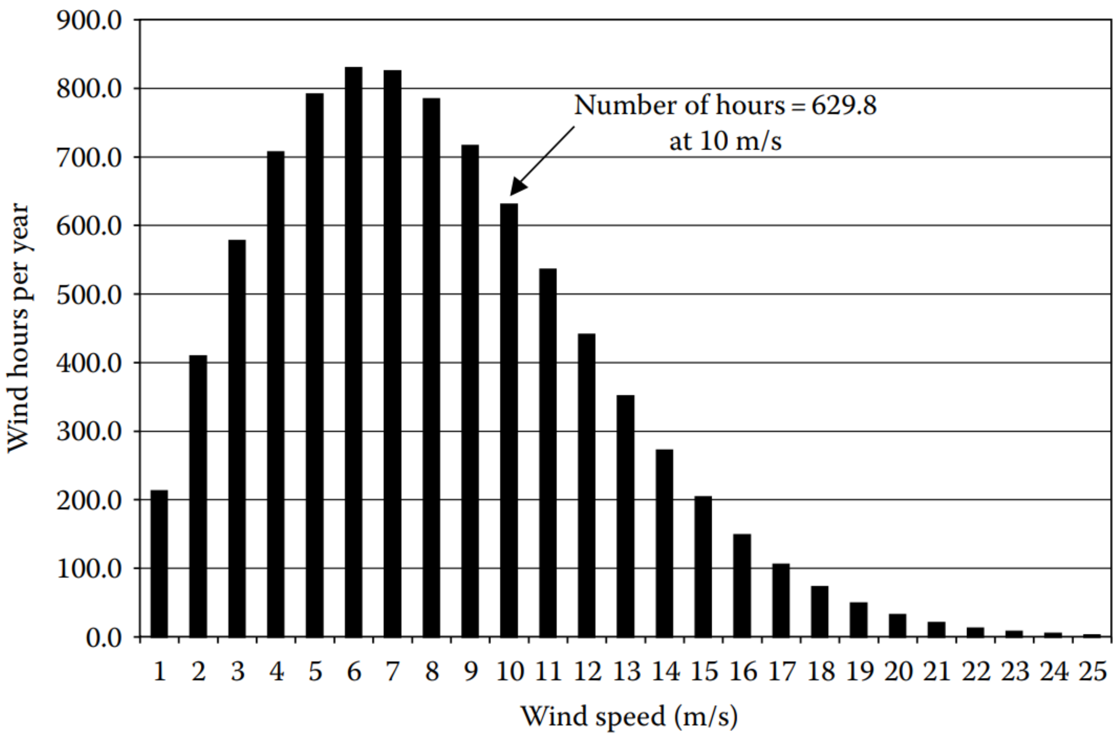
$$C(10.5) - C(9.5) = \exp\left[-\frac{\pi}{4}\left(\frac{10.5}{8}\right)^2\right] - \exp\left[-\frac{\pi}{4}\left(\frac{9.5}{8}\right)^2\right] = 0.0719 = 7.19\%$$

- Thus, in a given year, the number of hours the wind will blow at 10 m/s at this site is the probability times the number of hours in a year

$$0.0719 \cdot \frac{24\,\mathrm{h}}{\mathrm{day}} \cdot \frac{365\,\mathrm{day}}{\mathrm{year}} = 629.8\,\mathrm{h}$$

# Example – Power Calculation

- Finally, the annual energy production for this WT, at this site, at a wind speed of 10 m/s is found by multiplying the power production times the number of hours the power is produced.
- Annual energy production at 10 m/s = 2084.6 kW · 629.8 h = 1312.9 MWh



# Example – Power Calculation

The annual energy produced at this site is calculated to be 13,873 MWh. If electricity costs 10 cents/kWh ($0.10/kWh), the annual value of the energy produced in this example would be:

$$\text{Annual value} = 13{,}873{,}070\ \text{kWh} \cdot \frac{\$0.10}{\text{kWh}} = \$1.4\,\text{million}$$

# Example – Power Calculation

Power and Annual Energy Calculation

| Wind Speed (m/s) | Power (kW) | Probability (%) | # of Hours/Year | Energy (kWh) |
|---|---|---|---|---|
| 1 | 2.1 | 2.42 | 212.4 | 442.7 |
| 2 | 16.7 | 4.67 | 409.4 | 6,827.6 |
| 3 | 56.3 | 6.59 | 577.6 | 32,507.8 |
| 4 | 133.4 | 8.07 | 706.7 | 94,283.4 |
| 5 | 260.6 | 9.03 | 791.0 | 206,114.7 |
| 6 | 450.3 | 9.47 | 829.3 | 373,429.3 |
| 7 | 715.0 | 9.42 | 824.9 | 589,808.3 |
| 8 | 1067.3 | 8.95 | 784.2 | 837,017.8 |
| 9 | 1519.7 | 8.17 | 716.1 | 1,088,282.6 |
| 10 | 2084.6 | 7.19 | 630.2 | 1,313,739.7 |

# Example – Power Calculation

| | | | | |
|---|---|---|---|---|
| 11 | 2774.6 | 6.12 | 535.7 | 1,486,478.3 |
| 12 | 3602.2 | 5.03 | 440.7 | 1,587,567.2 |
| 13 | 4579.9 | 4.01 | 351.3 | 1,608,943.1 |
| 14 | 5000.0 | 3.10 | 271.6 | 1,358,130.2 |
| 15 | 5000.0 | 2.33 | 203.9 | 1,019,404.1 |
| 16 | 5000.0 | 1.70 | 148.7 | 743,288.9 |
| 17 | 5000.0 | 1.20 | 105.4 | 526,756.5 |
| 18 | 5000.0 | 0.83 | 72.6 | 362,992.5 |
| 19 | 5000.0 | 0.56 | 48.7 | 243,323.3 |
| 20 | 5000.0 | 0.36 | 31.7 | 158,710.6 |
| 21 | 5000.0 | 0.23 | 20.2 | 100,758.5 |
| 22 | 5000.0 | 0.14 | 12.5 | 62,274.8 |
| 23 | 5000.0 | 0.09 | 7.5 | 37,478.8 |
| 24 | 5000.0 | 0.05 | 4.4 | 21,967.4 |
| 25 | 5000.0 | 0.03 | 2.5 | 12,541.7 |
| | | **Total: 99.76** | | **Total: 13,873,070.1 kWh** |

# References

Understanding Wind Power Technology: Theory, Deployment and Optimisation, First Edition. Edited by Alois Schaffarczyk. Translated by Gunther Roth. © 2014 John Wiley & Sons, Ltd. Published 2014 by John Wiley & Sons, Ltd.

Principles of Sustainable Energy Systems, Third Edition. Charles F. Kutscher, Jana B. Milford and Frank Kreith. CRC Press. Taylor & Francis Group, 2019.