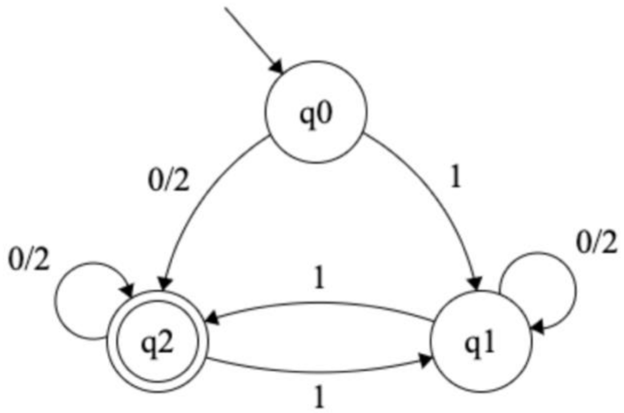
Exercise 3

Submit by Wednesday 14/04/21

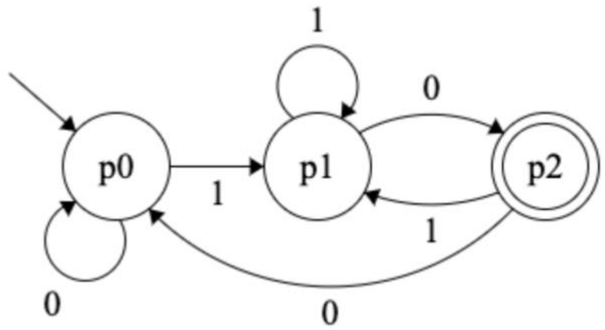
Question 1 (20 pts)

Define **DFA** for the following languages. It is enough to draw a state diagram for each language. For items c,d,e use the product automaton and specify the accepting states.

a. $L_1 = \{w \in \{0,1,2\}^+ \mid \text{the number of } w \text{ in basis 3 is divisible by 2, leading 0's are permitted}\}$



b. $L_2 = \{w10 \mid w \in \{0,1\}^*\}$



c. $L_3 = \{w \mid w \in L_1 \; or \; w \in L_2\}$

We add a state to L_2, call it p3, which handles {2} and the general product automaton without the unreachable states is:

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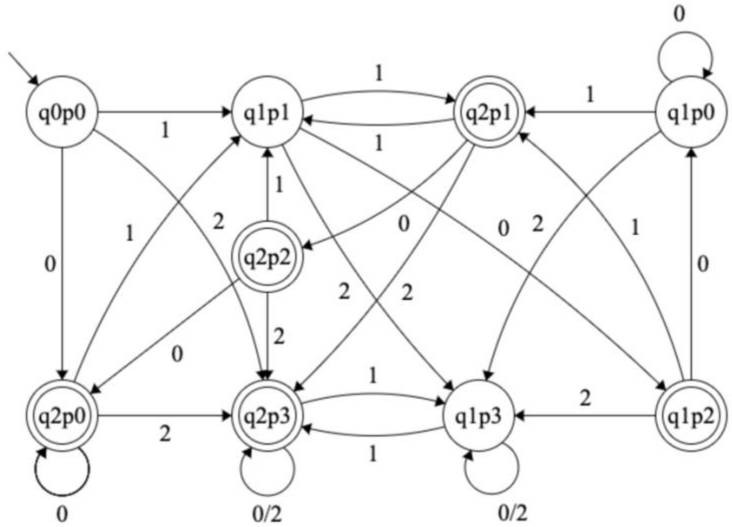
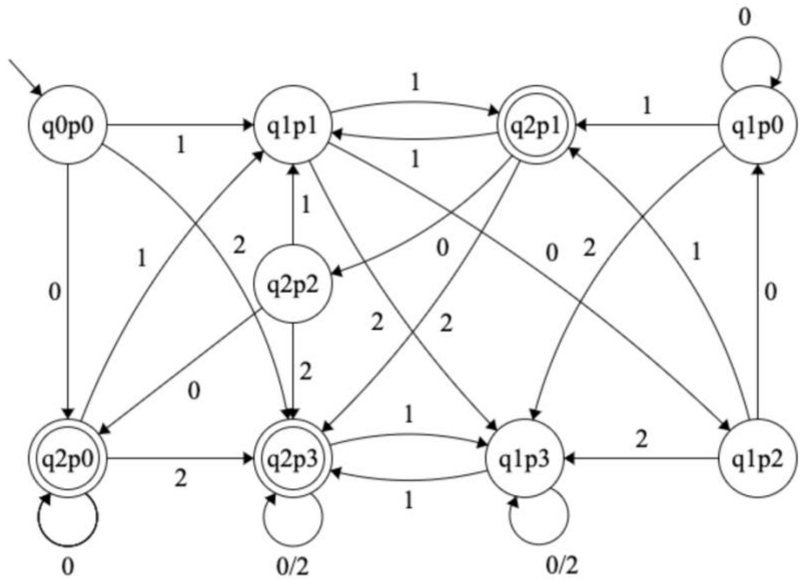
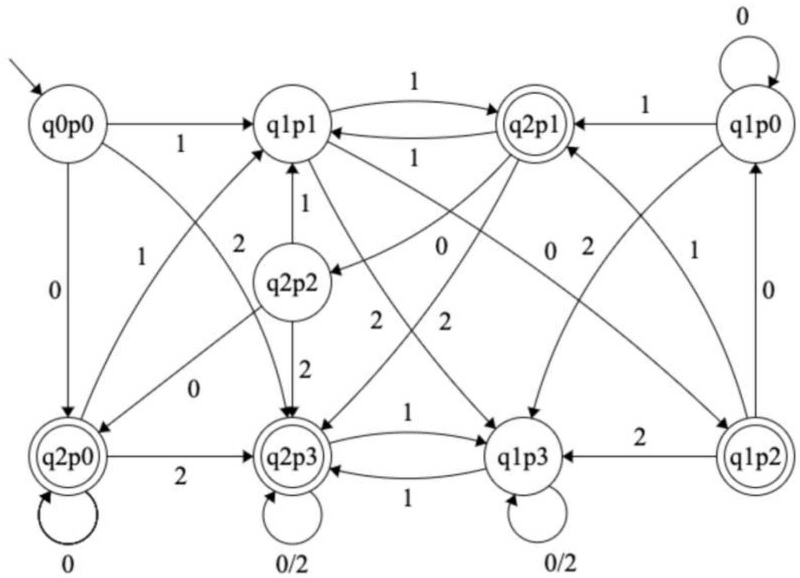


d. $L_4 = \{w \mid w \in L_1 \ \ and \ \ w \notin L_2\}$



e. $L_5 = \{w \mid w \in L_1 - L_2 \text{ or } w \in L_2 - L_1\}$

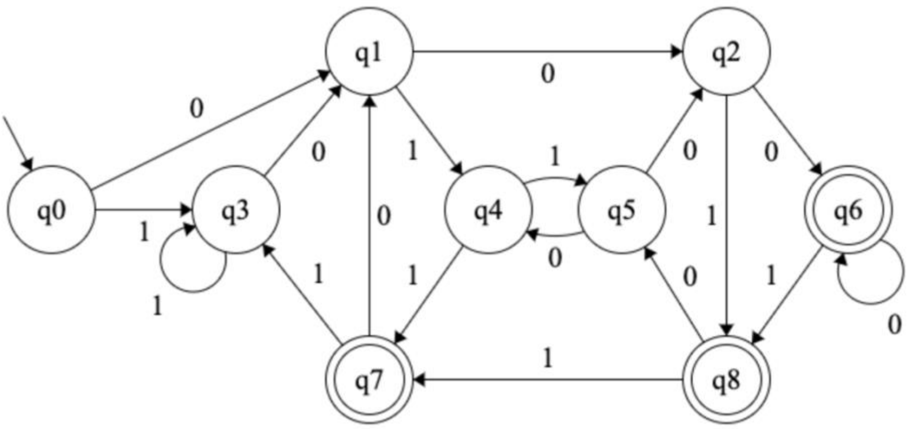


Question 2 (80 pts)

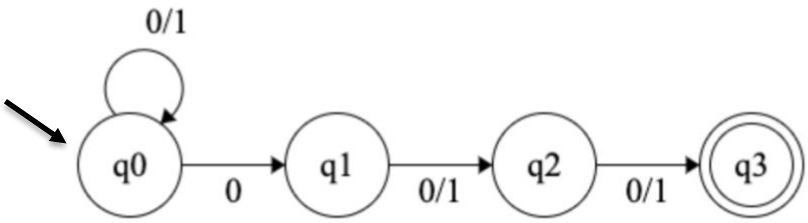
Define NFAs (non-deterministic automaton) for the following languages. It is enough to draw a state diagram for each language. Full credits will be given only for the NFA with the minimal possible states. For items a,b,c draw also the DFAs for these languages.

a. $L = \{w \in \{0,1\}^* \mid \text{the 3rd letter from the end is zero}\}$

DFA:

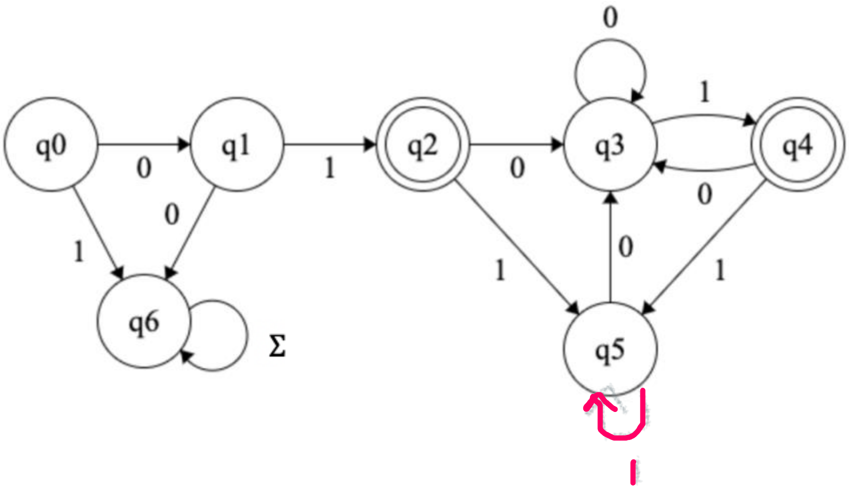


NFA:

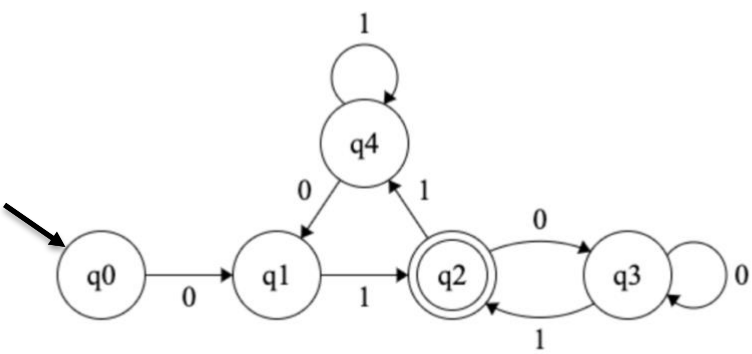


b. $L = \{w \in \{0,1\}^* \mid w \text{ starts with } 01 \text{ and ends with } 01\}$

DFA:

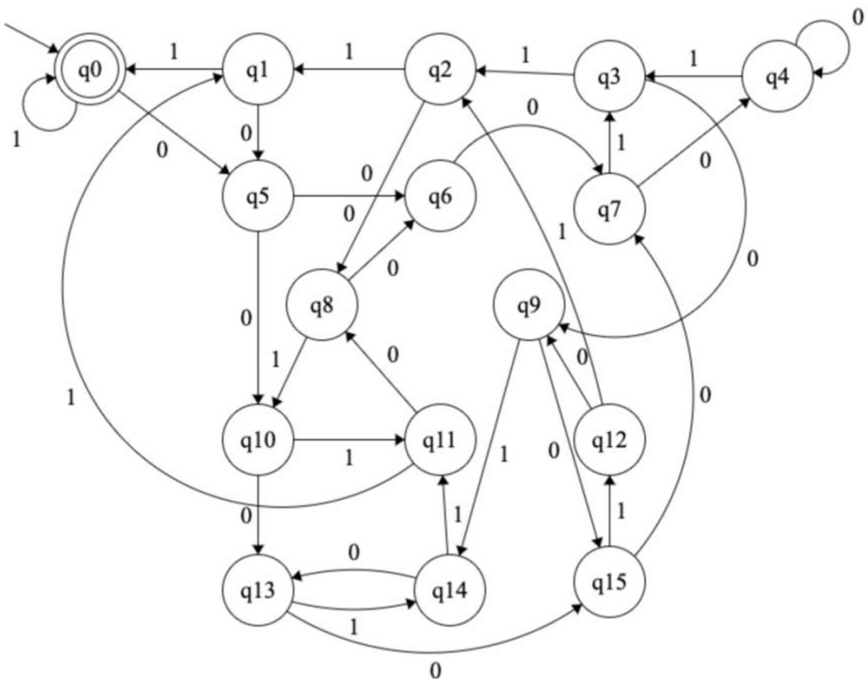


NFA:

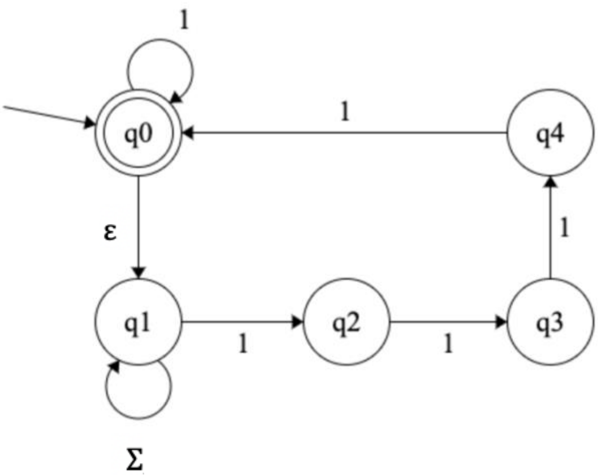


c. $L = \{w \in \{0,1\}^* \mid \text{none of the last 4 letters is 0}\}$

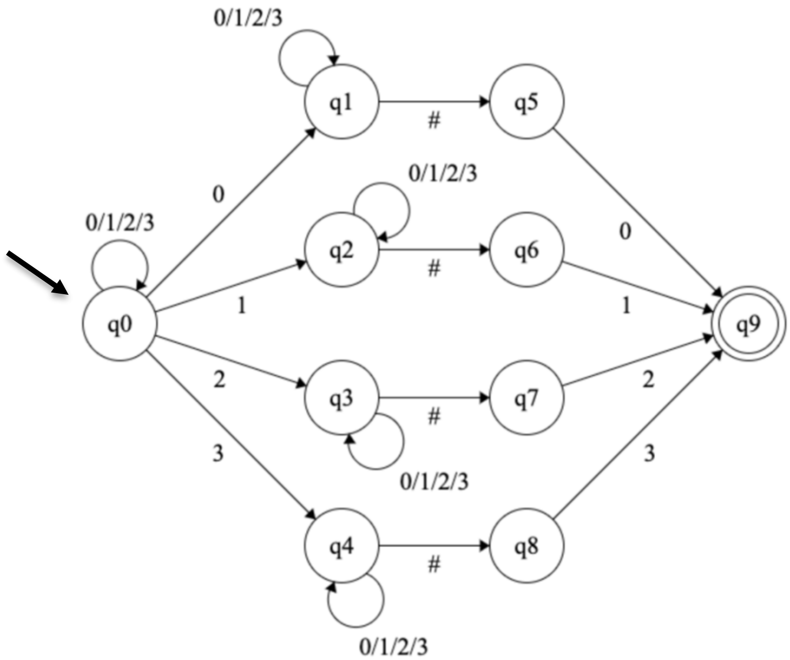
DFA:



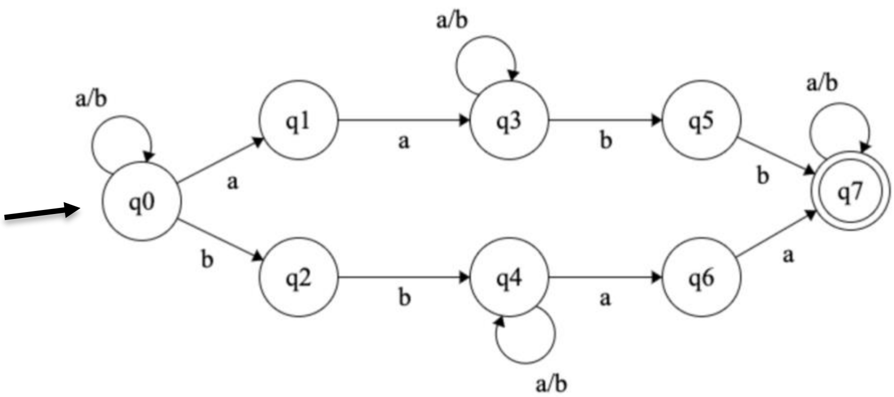
NFA:



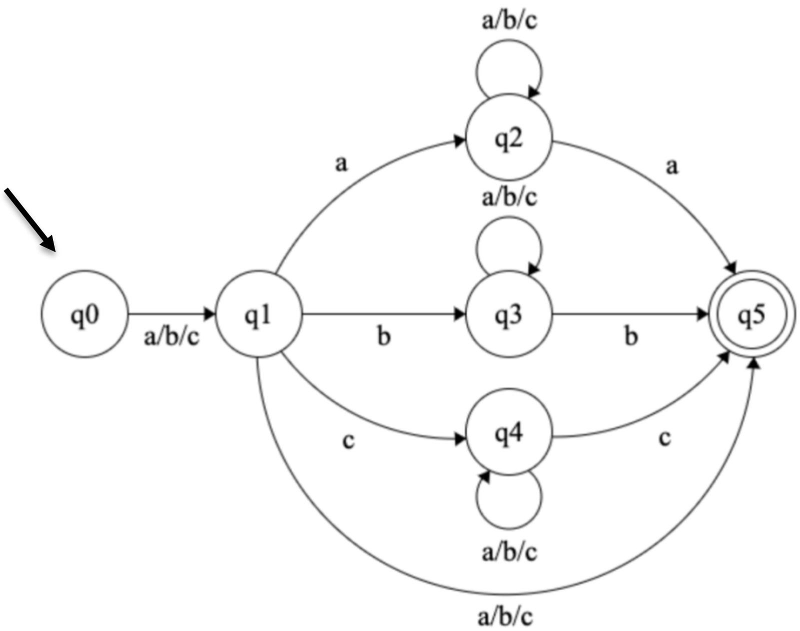
d. $L = \{w\#\sigma \mid \textit{where } \sigma \text{ occurs in } w, w \in \Sigma^*, \sigma \in \Sigma,\ \Sigma = \{0,1,2,3\}\ \}$, for example, 13031#3 is in L, but not 13031#2.



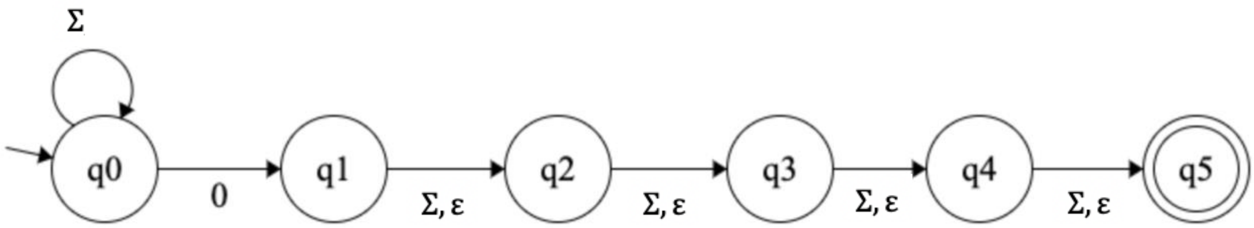
e. $L = \{w \in \{a,b\}^* \mid\ aa \text{ and } bb \text{ are substrings in } w\ \}$



f. $L = \{w \in \{a, b, c\}^* \mid \text{where the second letter in } w \text{ is identical to the last letter}\}$



g. $L = \{w \in \{0,1\}^* \mid \text{at least one of the last 5 letters is } 0\}$



h. $L = \{\sigma_1 u \sigma_2 v \sigma_3 \mid \sigma_1, \sigma_2, \sigma_3 \in \Sigma,\ u, v \in \Sigma^*,\ |u| = |v|,\ \sigma_2 = \sigma_1 \text{ or } \sigma_2 = \sigma_3 \text{ but not both}, \Sigma = \{0,1\}\}$

i.e. the middle letter is similar to the first letter or the last letter, but not both.