Assignment 1

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1 Contributions of Each Member

Jie Wang: coded Hodgkin-Huxley model and wrote the corresponding part of the report

Ilana Zane: coded Izhikevich model, wrote corresponding part of the report, answered questions 1,2,3

Jennifer Rodriguez: coded LIF model, wrote corresponding part of the report, answered questions

Zain Ul-Abdin: helped debug and plot 3.2 (LIF model)

2 LIF Model

2.1 Introduction

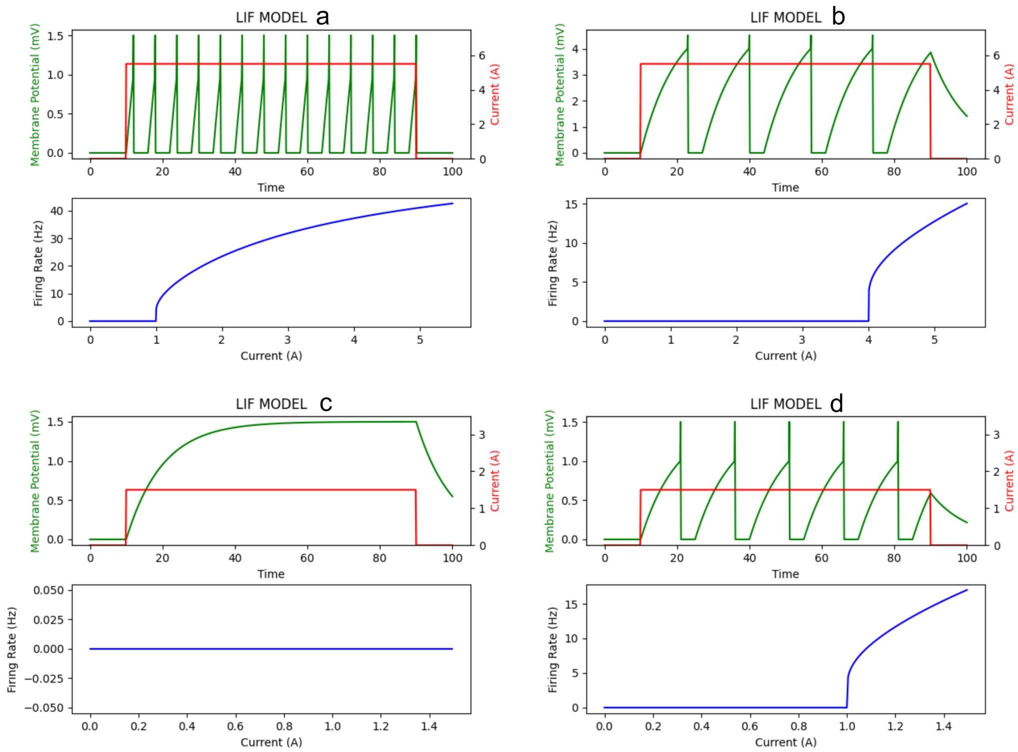
The goal of the Leaky integrate-and-fire model is to improve the Integrate-and-Fire model. An issue with the Integrate-and-fire model is it treats action potentials as events that happen at a specific point in time[1]. This is not an accurate representation of action potentials, therefore a better model is proposed. This model is the Leaky integrate-and-fire model.

2.2 Parameters & Equations

The LIF neuron shows the evolution of membrane potential over time as well as presents a mechanism to generate a spike[1]. The following equation is used to perform these actions:

$$\tau_m \frac{dv}{dt} = -V_m(t) + R_m I(t) \tag{1}$$

Where $\tau_m = R_m C_m$, and τ_m is the time constant.
The leak of charge in the cell membrane over time is characterized by a leaky resistor R_m [1]
C_m is the capacitance value
dt represents the change in time at each step of the simulation
dv represents the change in voltage at each step of the simulation.
$V_m(t)$ is the change in membrane potential over time
$I(t)$ is the change in current over time.
V_t represents the threshold value for the spike and V_r represents the value the voltage will reset to after a spike. All of these values are used to determine the membrane potential at a specific point at time t. Below we will demonstrate how a change in any of the input values can simulate different neurons:



In Figure 2.2a the input current is 1.5A and in Figure 2.2c the input current is 5.5A. For both of the figures the spike threshold is 1mV, and all of the other values are equal. In Figure 2.2b the input current is 1.5A but the spike threshold is 4mV. In Figure 2.2d the input current is 5.5A and the spike threshold is 4mV; all of the other values are equal to those in the previous figures. From these results we can see how different values for the parameters can simulate different types of neurons. We can also see the firing rate pattern change with different input currents and spike thresholds because as you increase the current, the value of $\frac{dv}{dt}$ gets larger, thus allowing for a quicker succession of spikes.

3 Izhikevich Model

3.1 Introduction

The Integrate-and-fire model is computationally effective, but is not a realistic representation of firing patterns exhibited by neurons. The Hodgkin-Huxley model is biologically plausible, but is only able to represent a few neurons at a time. The Izhikevich model [2] is a medium between the two; with four parameters the model is able to recreate the firing patterns of several different types of cortical neurons

3.2 Parameters & Equations

a describes the tine scale of the recovery parameters u.
b describes the sensitivity of the recovery variable u to the subthreshold fluctuations of the membrane potential.
c describes the after-spike reset value of the membrane potential v caused by the fast high-threshold K^+ conductances.
d describes after-spike reset of the recovery variable u cause by slow high-threshold Na^+ and K^+ conductances.

The most typical neuron in the cortex is the regular spiking neuron. Typical values used to simulate a regular spiking neuron are as follows: a = 0.02, b = 0.2, c = -65, d= 8.

$$\begin{aligned} \frac{dv}{dt} &= 0.04v^2 + 5v + 140 - u + I \\ \frac{du}{dt} &= a(bv - u) \end{aligned} \tag{2}$$

Equation 2 represents the differential equations that are used to calculate the spikes, however these equations are non-linear and the Euler method is needed to approximate. In Equation 3 we represent the derivatives as the difference between voltage and time between two different moments in time.

$$
\begin{aligned}
\frac{v(t+h)-v(t)}{h} &= 0.04v^2+5v+140-u+I \\
\frac{u(t+h)-u(t)}{h} &= a(bv-u)
\end{aligned} \tag{3}
$$

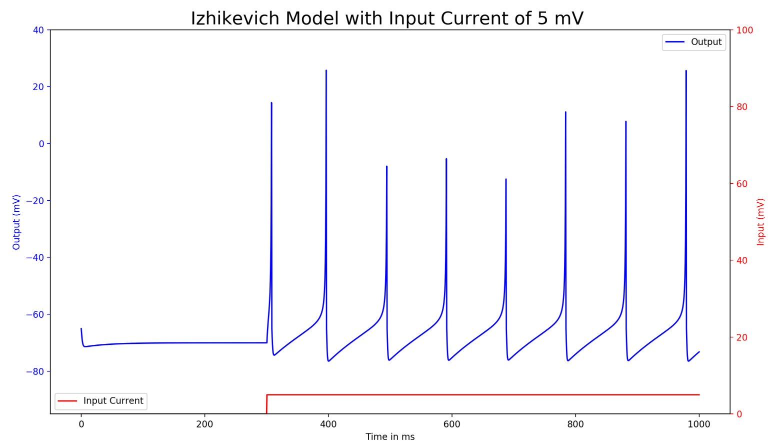


Figure 1: a=0.02,b=0.2,c=-65,d=8

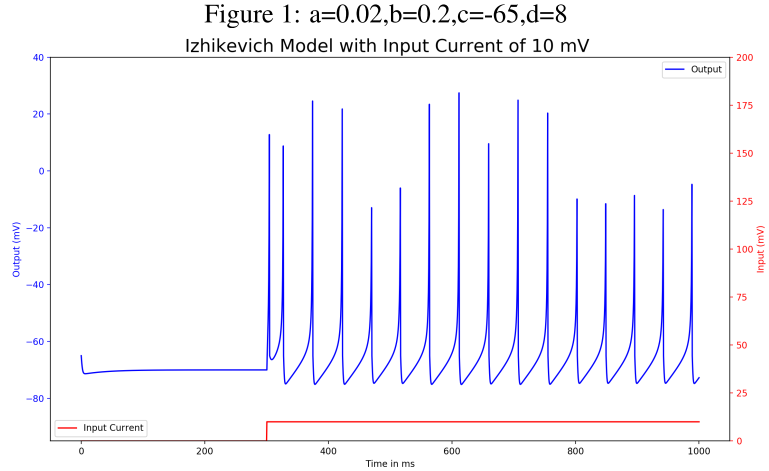


Figure 2: a=0.02,b=0.2,c=-65,d=8

Figure 1 represents a regular spiking neuron with a constant input current of 5A injected after 300 ms and the listed parameters. In Figure 2, the constant injected current of 10A,with the same parameters as before, results in an increased inter-spike frequency. If we change the parameters a,b,c,d we can simulate different types of neurons.

4 Hodgkin-Huxley Model

4.1 Introduction

Among the three models, The Hodgkin-Huxley model[3] is the most biologically plausible, it models the opening/closing of three ion channels (Na^+, K^+, Cl^-) in a neuron during the generation of an action potential.

4.2 Parameters & Equations

Change in voltage(mV) with respect to time(ms) is modeled using the following equation:

$$
C\frac{dv}{dt} = I - g_{Na}m^3h(V-V_{Na}) - g_K n^4(V-V_K) - g_L(V-V_L) \tag{4}
$$

C stands for membrane capacitance, which is usually ranging from 0.5 to 1 for most neurons.
I represents the injected current.
g_{Na}, g_K and g_L represent Sodium (Na),Potassium (K) and leaky maximum conductances, respectively.
m, n, h are the gating variables that control the opening and closing of Na^+ channel, K^+ channel and leaky channel.
V_{Na}, V_K and V_L represent Sodium (Na),Potassium (K) and leaky equilibrium potentials, respectively.
The resting membrane potential V is -65 mV, The initial state of all gating variables is set as 0 (m=h=n=0). The gating variables are updated on each timestep using the following equations:

$$\begin{aligned}\frac{dm}{dt} &= a_m V(V) * (1.0 - m) - b_m V(V) * m \\ \frac{dh}{dt} &= a_h V(V) * (1.0 - h) - b_h V(V) * h \\ \frac{dn}{dt} &= a_n V(V) * (1.0 - n) - b_n V(V) * n\end{aligned} \tag{5}$$

where the equations for the rate variables

$$\begin{aligned}a_m(V) &= \frac{0.1(V+40)}{1 - exp(-\frac{V+40}{10})} \\ b_m(V) &= 4exp(\frac{-V+65}{18}) \\ a_h(V) &= 0.07exp(-\frac{V+65}{20}) \\ b_h(V) &= 1/(1 + \frac{exp(-(V+35))}{10}) \\ a_n(V) &= 0.01(V+55)/(1 - exp(-\frac{(V+55)}{10})) \\ b_n(V) &= 0.125exp(-\frac{V+65}{80})\end{aligned} \tag{6}$$

Maximum conductances and equilibrium potentials are constants that are pre-determined according to online resources[4]:

$$g_{Na} = 120;\ g_K = 36;\ g_L = 0.3; V_{Na} = 60;\ V_K = -88;\ V_L = -61$$

Figure 3A shows the spikes generated over time in response to the injected current. When the injected current is 10, the membrane potential of the stimulated neuron will exceed the threshold and the neuron will generate one spike. When the injected current increases to 20, the Hodgkin-Huxley model predicts that the stimulated neuron will generate more spikes (three as shown in Figure 3A) and generate the spikes faster and earlier with respect to the onset the stimulus. This is because when the difference over the neuron membrane is less negative with more current injected, K^+ channel would open earlier and more potassium ions would flow out of the cell. Figure 3B shows the change in values of the gating variables over time that control the opening/closing of the ion channels.

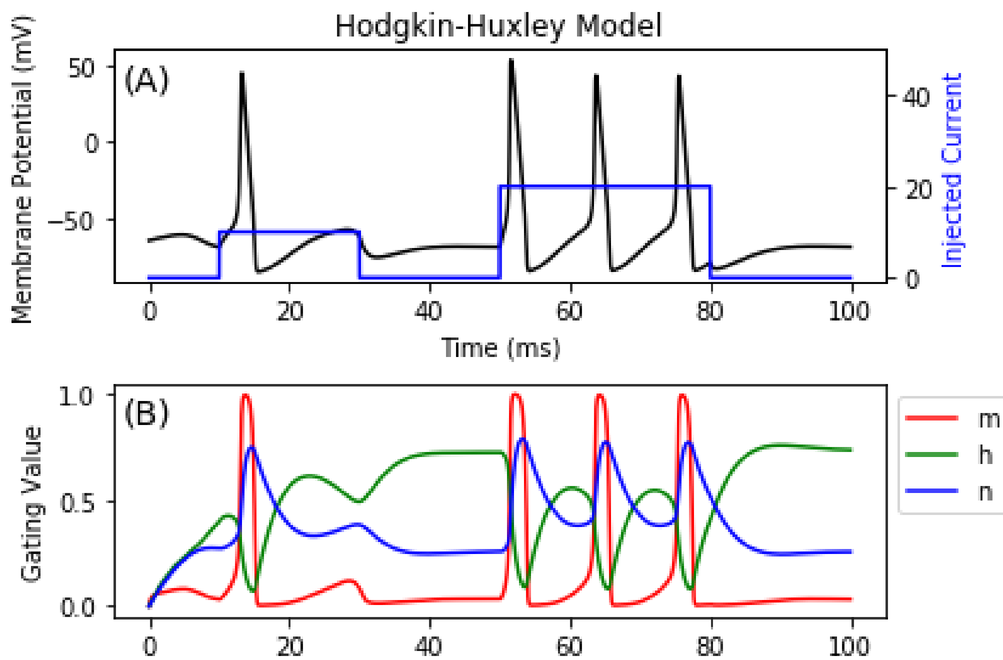


Figure 3: Hodgkin-Huxley Model: (A).firing rate as a function of injected current;(B).Change in gating value over time

5 Questions

1.IF neuron with low and high input currents

In order to determine whether an IF neuron will spike we need to consider the previous inputs to the system. The IF model is a continuous system that accumulates all of the input voltages and once that sum equals or surpasses a predetermined threshold, the neuron will spike. For example, if the threshold is 1 and our prior inputs sum to .9, then a small input of .1 will result in a spike. The small input current itself won't result in a spike, but when added to a build up of a sufficiently large charge, a spike will occur. When fed a sufficiently large current of, for example, .9 and there is an already accumulated charge of .1 in the system, then there will be a spike. In both situations we must consider the initial conditions of the system.

2. LIF neuron with low and high input currents

The LIF neuron is similar to the IF neuron except for the 'leaking' parameter that represents the loss of membrane potential over time at a rate that is dependent on resistance. When analyzing whether or not an input will result in a spike, we must consider two limiting cases for the resistance parameter. If R tends to infinity, the leaking voltage will become so small we will essentially be left with the same equation for an IF neuron (which we already know how it function with different input currents). If R tends to 0, then the leaking voltage will become sufficiently large enough that the current in the system will dissipate too quickly and all energy will be lost, requiring several input currents to keep energy in the system. In summary, a high R will result in a small amount of current leaving the system and a low R will result in a large loss of current. Assuming that we have a reasonable R, a small input current will not result in a spike because all of the energy will have left the system. A large enough input current will result in a spike, assuming that we have a reasonable R.

3. What are the limitations of an LIF neuron?

The most significant limitation of the LIF neuron is it's resistance parameter. If a moderate value for resistance is not chosen then we will have a model that simulates the two limiting cases that were previously mentioned in question 2. When R is too high, we will have to continuously supply input currents in order to keep energy in the system. If R is too low, we are essentially modelling the IF neuron, which is known to not be biologically plausible.

References

[1] Wulfram Gerstner Liam Paninski Werner M. Kistler, Richard Naud. 1.3 integrate-and-fire models. *Neuronal Dynamics: From Single Neurons to Networks and Models of Cognition*, 2014.

[2] Eugene M. Izhikevich. Simple model of spiking neurons. *IEEE Transactions on neural networks, 14(6):1569–1572*, 2003.

[3] Huxley Andrew F. Hodgkin, Alan L. A quantitative description of membrane current and its application to conduction and excitation in nerve. *The Journal of physiology, 117(4):500–544*, 1952.

[4] Wang K. Yuan Y. Sui D. Zhang H. Zhang, Y. Effects of maximal sodium and potassium conductance on the stability of hodgkin-huxley model. *Computational and mathematical methods in medicine*, 2014.