uc3m | Universidad Carlos III de Madrid
Departamento de Matemáticas

Numerical Calculus - 2022/2023 - Practice 2 Applied Mathematics and Computation 121

Name and Lastname: Group:

Hand in before 5pm on Wednesday the 2nd of November. Same basic rules from previous assignment still apply. Name your file practice02teamXX.m where XX is the number of your team. On the first few lines of code write as comment the names and NIAs of all the members of the team.

The aim of this practice is to better understand graphs at the same time as we learn how to implement the Newton method as a tool for optimization.

We are going to work on minimizing a function f over a set S, where:

$$f(x,y) = 2x^2 - 1.05x^4 + \frac{x^6}{6} + xy + y^2,$$

$$S = \{(x,y) \in \mathbb{R}^2 : x, y \in [-5,5]\}.$$

Any minima of this function, since it is a polynomial, must be at zeros of the gradient and thus, we can use the Newton-Raphson method to search for those zeros, so in some sense we know the minima already a priori, but what we want to understand is **from which points will we have faster convergence to the minimum?** From some points there may be no convergence at all or it may take more or less iterations to reach a reasonable accuracy around the minimum. Let's fix this accuracy at points that are less than 10^{-6} distance from the correct minimum. We will also pay attention at the conditioning of the problem at each step, expressed in the condition number of the Jacobian.

Problem 1. Write a program that implements the Newton-Raphson method in order to find zeros of the gradient of the function f above from a given starting point $x \in S$.

Plot the function together with a visual explanation of the regions from where there will be convergence to the minimum, or any other situation. For this you may do a grid search, that is, apply the method from many equispaced points in the defined region, and see where each point goes.

Also paint each point in the grid with a different colour (this may be in a separate plot) according to how many iterations does it take to arrive to the desired accuracy.

Draw a 3rd plot with the condition number of the Jacobian matrix used to iterate from each point. From your observations about these, is the process well-conditioned as a whole?

Think also about the following questions: (1) what happens if the point's orbit does, at some step, leave the square S? (2) what criteria did you choose for stopping apart from arriving to the correct spot?

Notice the condition number of a matrix is implemented in Matlab in the command cond.