

# Using mixture models for distance sampling detection functions

David Lawrence Miller & Len Thomas

NCSE Summer meeting 2011  
Bath

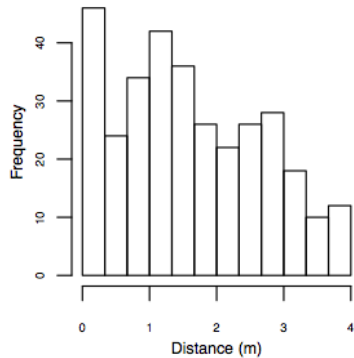
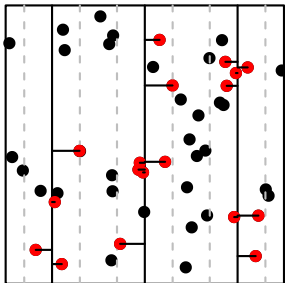
Distance sampling detection functions

Mixture detection functions

Real Data

Further work & conclusion

# Distance sampling



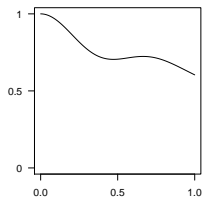
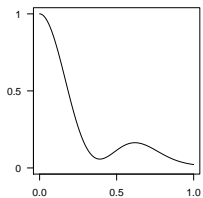
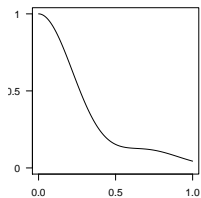
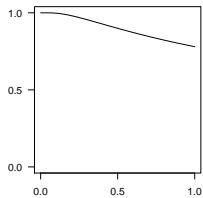
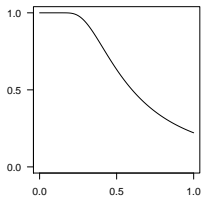
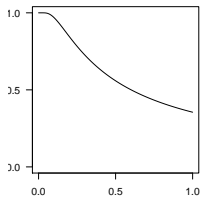
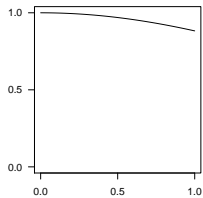
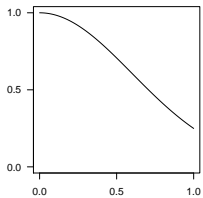
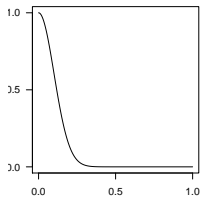
# Distance sampling detection functions

Conventional distance sampling:

$$g(x) = \text{key}(x)(1 + \text{adjustment series}(x))$$

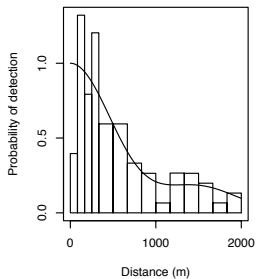
$$\text{key}(x) = \exp\left(-\frac{x^2}{2\sigma^2}\right) \quad \text{or} \quad 1 - \exp\left(-\left(\frac{x}{2\sigma}\right)^{-b}\right)$$

adjustment series( $x$ ) = cosines, Hermite polynomials or  
simple even polynomials

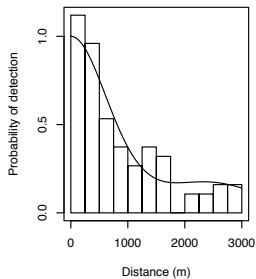


# Monotonicity

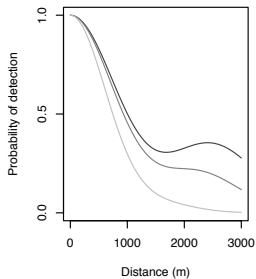
**Humpback**



**Long-finned pilot whale**



**Long-finned pilot whale  
Quantiles of Beaufort sea state**

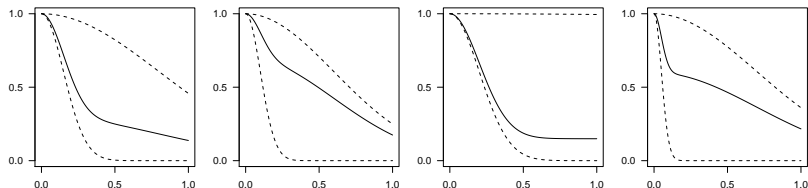


# Mixture models

Capture-recapture guys are doing it. . . Byron, Shirley Pledger, Andy Royle. . .

Built in monotonicity.

Mixture of **detection functions**.

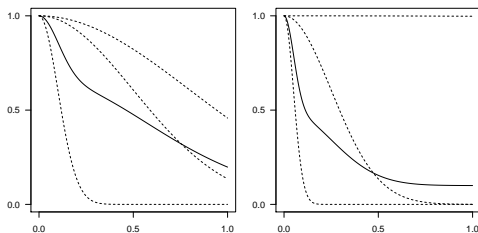


# Formulation

A  $J$ -point mixture detection function:

$$g(x) = \sum_{j=1}^J \phi_j g_j(x), \quad \text{where } \sum_{j=1}^J \phi_j = 1$$

Here,  $g_j(x)$  is half-normal,  $g_j(x) = \exp\left(-\frac{x^2}{2\sigma_j^2}\right)$





# Covariates

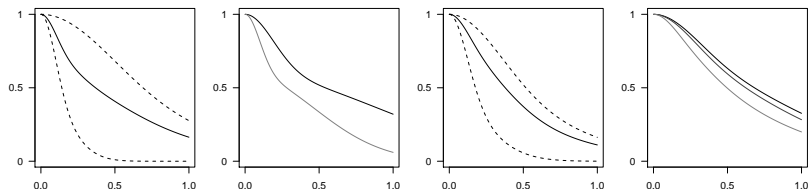
Conventional distance sampling:

$$\sigma_i = \exp(\beta_0 + \sum_{k=1}^K \beta_k z_{ik})$$

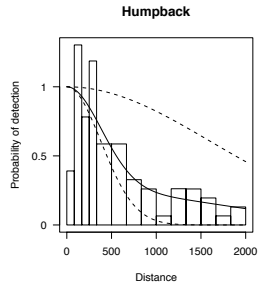
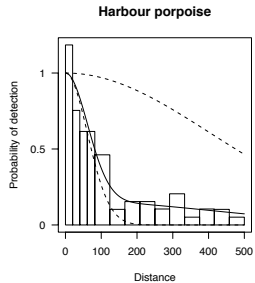
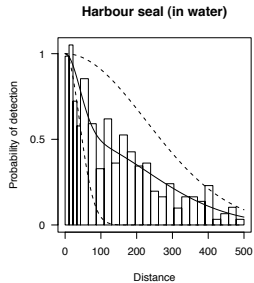
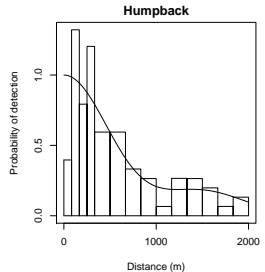
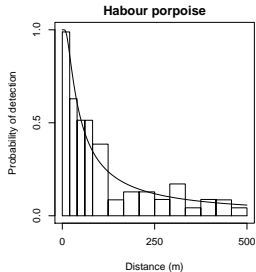
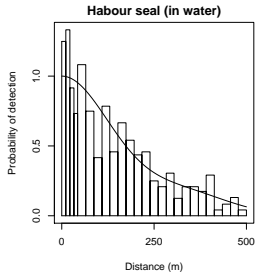
Mixtures:

$$\sigma_{ij} = \exp(\beta_{0j} + \sum_{k=1}^K \beta_k z_{ik})$$

Covariates the same in each mixture, only vary intercepts.

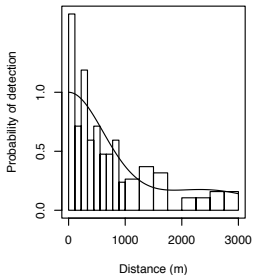


# Williams & Thomas (2007)

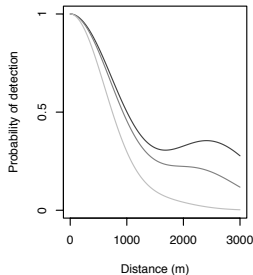


# Long-finned pilot whales

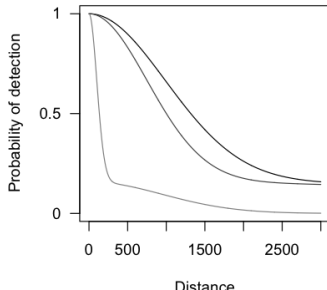
Long-finned pilot whale



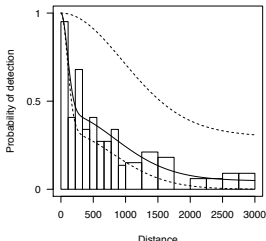
Long-finned pilot whale  
Quantiles of Beaufort sea state



Levels of Beaufort sea state



Average detection function



## Further work

Continuous mixtures, let  $J \rightarrow \infty \dots$

$$g(x) = \int_{\mathbf{R}} \varphi(\kappa) g_{\kappa}(x, \mathbf{Z}; \theta, \kappa) d\kappa$$

Mixtures of mixtures (*a la* Morgan and Ridout).

# Conclusion

Another option for the detection function.

Better AIC on most of the examples we tried.

Can always fall back to CDS via AIC selection.

There is an R package – `mmds`.

Available at [github.com/dill/mmds](https://github.com/dill/mmds).

Paper – soon!

