NUMERICAL MODELING AND SIMULATION FOR ACOUSTICS
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HOMEWORK 1

EXERCISE

Let us consider the following wave propagation problem in $\Omega = (0, L) \times (0, T]$:

$$
\begin{cases}
\dfrac{\partial^2 u}{\partial t^2}(x,t) - c^2 \dfrac{\partial^2 u}{\partial x^2}(x,t) = g(x,t) & \text{in } \ \Omega \times (0,T], \\
u(x,0) = u_0(x) & \text{in } \ \Omega, \\
\dfrac{\partial u}{\partial t}(x,0) = v_0(x) & \text{in } \ \Omega, \\
u(0,t) = c_1 & t \in (0,T], \\
u(L,t) = c_2 & t \in (0,T],
\end{cases}
\tag{1}
$$

where g, u_0, v_0 are given functions, regular enough, and c, c_1 and c_2 are positive constants. By supposign that the solution is time harmonic, i.e., $u(x,t) = u(x)e^{-i\omega t}$ and that the right is time harmonic, i.e., $g(x,t) = g(x)e^{-i\omega t}$, rewrite the equation (1) in the following form:

$$
\begin{cases}
\dfrac{\partial^2 u}{\partial x^2}(x) + n^2\omega^2 u(x) = f(x) & \text{in } \ \Omega, \\
u(0) = c_1, \\
u(L) = c_2,
\end{cases}
\tag{2}
$$

where f is a given function regular enough, ω, n, c_1 and c_2 are constants.

1. Write the weak formulation for problem (2). State precisely the functional spaces where the problem is formulated.

2. Write the Galerkin formulation of the problem at the previous point. Consider linear finite element space discretization.

3. Write the algebraic formulation in the form
$$
-A\mathbf{u} + n^2\omega^2 M\mathbf{u} = \mathbf{F}. \tag{3}
$$
Define precisely the entries of the mass and stiffness matrices M and A and of the right hand side $\mathbf{F}$.

4. Implement in Matlab the finite solution of (3). Test your implementation (convergence error) by considering the domain $\Omega = (0,1)$ and $n = \omega = 1$, by supposing that the exact solution to (2) $u_{ex}(x,t) = \sin(2\pi x)$ compute the forcing term f and the boundary data.

5. Starting from the solution obtained at the previous point compute the solution of (1) and compare it with the one obtained with linear finite element discretiazion coupled with the leap-frog scheme. In particular report the errors obatined at the final observation time $T = 4$.

6. Finally, solve (2) with linear finite elements by considering the following data

   - $\Omega = (0, 1)$ and $n = 1$, $\omega = 2\pi$ and $u_{ex}(x, t) = \sin(2\pi x)$
   - $\Omega = (0, 1)$ and $n = 1$, $\omega = 1000\pi$ and $u_{ex}(x, t) = \sin(2\pi x)$

   Comment on the results.