CS300 HOMEWORK 4
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1-
The probability of the value being pivot : 1/M
The probability of the value being on the left : i / M
The probability of the value being on right : M-i-1 / M

$$Q(M) = M + \frac{1}{M}\sum_{i=0}^{M-1}\left(\frac{i}{M}Q(i) + \frac{M-i-1}{M}Q(M-i-1) + \frac{1}{M}0\right)$$

$$Q(M) = M + \frac{1}{M}\sum_{i=0}^{M-1}\left(\frac{2i}{M}Q(i)\right)$$

$$Q(M) = \frac{M^3}{M^2} + \frac{1}{M^2}\sum_{i=0}^{M-1}(2iQ(i))$$

$$M^2Q(M) = M^3 + \sum_{i=0}^{M-1}(2iQ(i))$$

$$(M-1)^2Q(M-1) = (M-1)^3 + \sum_{i=0}^{M-2}(2iQ(i))$$

Then If I subtract last two equations :

$$M^2Q(M) - (M-1)^2Q(M-1) = M^3 - (M-1)^3 + 2(M-1)Q(M-1)$$

$$M^2Q(M) = (M^2-1)Q(M-1) + 3M^2 - 3M + 1$$

And if I divide by M(M+1)

$$\frac{MQ(M)}{M+1} = \frac{(M-1)Q(M-1)}{M} + \frac{3(M-1)}{M+1} + \frac{1}{M(M+1)}$$

I say B(M) $= \frac{MQ(M)}{M+1}$

$$B(M) = B(M-1) + \frac{3(M-1)}{M+1} + \frac{1}{M} - \frac{1}{M+1}$$

$$B(M) = \sum_{i=1}^{M}\left(\frac{3(i-1)}{i+1} + \frac{1}{i} - \frac{1}{i+1}\right)$$

$$B(M) = \sum_{i=1}^{M}\left(\frac{3(i+1)-6}{i+1} + \frac{1}{i(i+1)}\right)$$

$$B(M) = \frac{M}{M+1} + 3M - 6\sum_{i=1}^{M}\frac{1}{i+1}$$

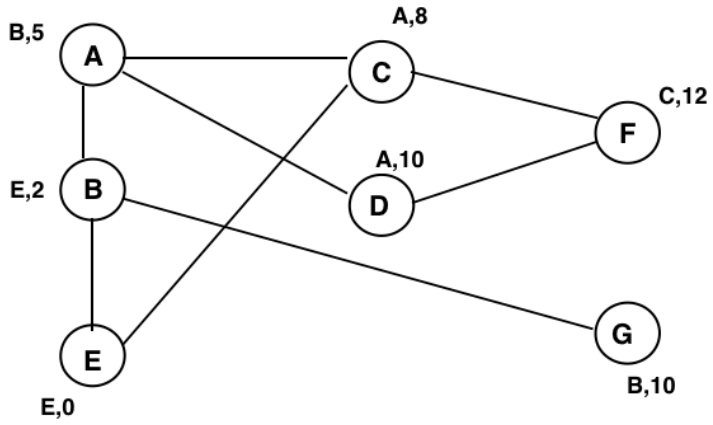
$$B(M) = \frac{M}{M+1} + 3M - 6 ln M$$

$$\frac{M T(N)}{M+1} = \frac{M}{M+1} + 3M - 6 ln M$$

$$Q(M) = 1 + 3(M+1) - \frac{6(M+1) ln M}{M}$$

And this proves Q(M) has O(M) complexity. Which I say if M=N it has O(N) complexity

2. **Trace the operation of Dijkstra's weighted shortest path algorithm for the graph given in Figure 1. Use vertex E as your start vertex.**




a. Known vertexes: -
Unknown vertexes: A(-,∞) B(-,∞) C(-,∞) D(-,∞)E(-,∞) F(-,∞) G(-,∞) H(-,∞)

b. Known vertexes: E(E,0)
Unknown vertexes: A(**-,∞**) B(E,2) C(E,9) D(**-,∞**) F(-,∞) G(-,∞)

c. Known vertexes: E(E,0) B(E,2) A(B,5)
Unknown vertexes: C(A,8) D(A**,10**) F(**-,∞**) G(B,10)

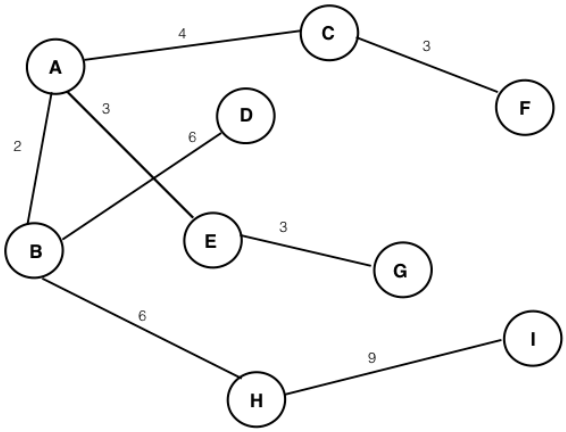
d. Known vertexes: E(E,0) B(E,2) A(B,5) C(A,8)
Unknown vertexes: D(A**,10**) F(**C,12**) G(B,10)

e. Known vertexes: E(E,0) B(E,2) A(B,5) C(A,8) D(A**,10**)
Unknown vertexes: F(**C,12**) G(B,10)

f. Known vertexes: E(E,0) B(E,2) A(B,5) C(A,8) D(A**,10**) G(B,10)
Unknown vertexes: F(**C,12**)

g. Known vertexes: E(E,0) B(E,2) A(B,5) C(A,8) D(A**,10**) G(B,10) F(**C,12**)
Unknown vertexes:-

3.Trace the operation of Prim's minimum spanning tree algorithm for the graph in Figure 2. Use vertex E as your start vertex.




1. Vertexes in tree: -
   Edges in tree:-
   Vertexes not in tree: A B C D E F G H,I
   Possible edges:-
   Thrown edges due to cycle:-

2. Vertexes in tree: E
   Edges in tree:-
   Vertexes not in tree: A B C D F G H,I
   Possible edges: DE(8) GE(3) AE(3)
   Thrown edges due to cycle:

3. Vertexes in tree: E, A
   Edges in tree: AE(3)
   Vertexes not in tree: B C D F G H,I
   Possible edges: DE(8) GE(3),CA(4),BA(2)
   Thrown edges due to cycle: -

4. Vertexes in tree: E, A,B
   Edges in tree: AE(3),BA(2)
   Vertexes not in tree: C D F G H,I
   Possible edges: DE(8) GE(3),CA(4),DB(6),HB(6)
   Thrown edges due to cycle: -

5 Vertexes in tree: E, A,B,G
Edges in tree: AE(3),BA(2), GE(3)
Vertexes not in tree: C D F H,I
Possible edges: DE(8),CA(4),DB(6),HB(6),FG(5),IG(10)
Thrown edges due to cycle: -

6 Vertexes in tree: E, A,B,G,C
Edges in tree: AE(3),BA(2), GE(3),CA(4)
Vertexes not in tree: D F H,I
Possible edges: DE(8),DB(6),HB(6),FG(5),IG(10),DC(8),FC(3),HC(6)
Thrown edges due to cycle: -

7 Vertexes in tree: E, A,B,G,C,F
Edges in tree: AE(3),BA(2), GE(3),CA(4),FC(3)
Vertexes not in tree: D, H,I
Possible edges: DE(8),DB(6),HB(6),FG(5),IG(10),,HC(6)
Thrown edges due to cycle: -

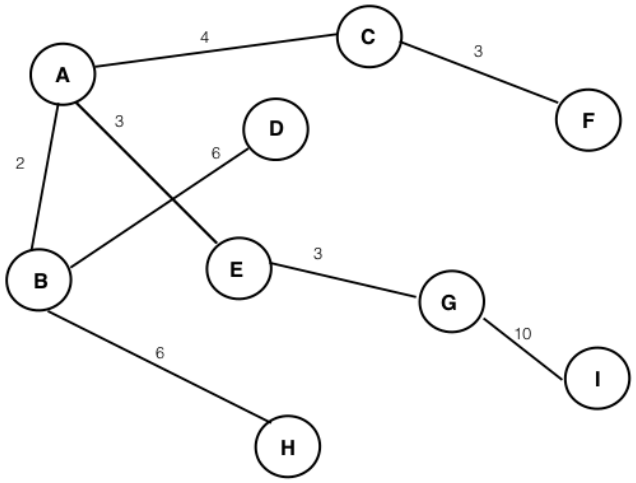
9 Vertexes in tree: E, A,B,G,C,F,D
Edges in tree: AE(3),BA(2), GE(3),CA(4),FC(3),DB(6)
Vertexes not in tree: H,I
Possible edges: DE(8),HB(6),IG(10),DC(8),HC(6),DC(8)
Thrown edges due to cycle: FG(5)

10 Vertexes in tree: E, A,B,G,C,F,D,H
Edges in tree: AE(3),BA(2), GE(3),CA(4),FC(3),DB(6),HB(6),
Vertexes not in tree: I
Possible edges,IH(9),
Thrown edges due to cycle: HC(6),DE(8)

11 Vertexes in tree: E, A,B,G,C,F,D,H,I
Edges in tree: AE(3),BA(2), GE(3),CA(4),FC(3),DB(6),HB(6),IH(9)
Vertexes not in tree: -
Possible edges::IG(10)
Thrown edges due to cycle: -

12 Vertexes in tree: E, A,B,G,C,F,D,H,I
Edges in tree: AE(3),BA(2), GE(3),CA(4),FC(3),DB(6),HB(6),IH(9)
Vertexes not in tree: -
Possible edges:
Thrown edges due to cycle: IG(10)

Question 4 Trace the operation of Kruskal's minimum spanning tree algorithm for the graph in Figure 1.




Kruskal's Algorithm

1 . Vertexes in tree: -
Edges in tree: -
Vertexes not in tree: A B C D E F G H I
Possible edges: AB(2),AE(3),EG(3), FC(3), CA(4) ,GF(5),
BH(6),BD(6)CH(6),DE(8),DC(8),HI(9),IG(10)

2. Vertexes in tree: AB(2)
Edges in tree: tree1: A B
Vertexes not in tree: C D E F G H I
Possible edges: AE(3),EG(3), FC(3), CA(4) ,GF(5),
BH(6),BD(6)CH(6),DE(8),DC(8),HI(9),IG(10)

3. Vertexes in tree: AB(2),AE(3)
Edges in tree: tree1: A B,E
Vertexes not in tree: C D F G H I
Possible edges:EG(3), FC(3), CA(4) ,GF(5),
BH(6),BD(6)CH(6),DE(8),DC(8),HI(9),IG(10),

4. Vertexes in tree: AB(2),AE(3),EG(3)
Edges in tree: tree1: A B,E,G
Vertexes not in tree: C D F H I
Possible edges: FC(3), CA(4) ,GF(5),
BH(6),BD(6)CH(6),DE(8),DC(8),HI(9),IG(10),

5. Vertexes in tree: AB(2),AE(3),EG(3),FC(3)
Edges in tree: tree1: A B,E,G tree2: F, C
Vertexes not in tree: D H I
Possible edges:, CA(4) ,GF(5), BH(6),BD(6)CH(6),DE(8),DC(8),HI(9),IG(10)

6. Vertexes in tree: AB(2),AE(3),EG(3),FC(3), CA(4). //union
Edges in tree: tree1: A B,E,G, F, C
Vertexes not in tree: D H I
Possible edges: GF(5), BH(6),BD(6)CH(6),DE(8),DC(8),HI(9),IG(10),

7. Vertexes in tree: AB(2),AE(3),EG(3),FC(3), CA(4)
Edges in tree: tree1: A B,E,G, F, C
Vertexes not in tree: D H I
Possible edges: BH(6),BD(6)CH(6),DE(8),DC(8),HI(9),IG(10),
Thrown edges due to cycle: GF(5)

8. Vertexes in tree: AB(2),AE(3),EG(3),FC(3), CA(4),BH(6)
Edges in tree: tree1: A B,E,G, F, C,H
Vertexes not in tree: D, I
Possible edges:BD(6),CH(6),DE(8),DC(8),HI(9),IG(10),
Thrown edges due to cycle:

9. Vertexes in tree: AB(2),AE(3),EG(3),FC(3), CA(4),BH(6),BD(6)
Edges in tree: tree1: A B,E,G, F, C,H, D
Vertexes not in tree: I
Possible edges:HI(9),IG(10),
Thrown edges due to cycle: CH(6),DE(8),DC(8),

10. Vertexes in tree: AB(2),AE(3),EG(3),FC(3), CA(4),BH(6),BD(6)HI(9)
Edges in tree: tree1: A B,E,G, F, C,H, D,I
Vertexes not in tree: -
Possible edges:
Thrown edges due to cycle: IG(10)

5.**Find shortest unweighted path from G to all other vertices for the graph in Figure 1. Use breadth-first search algorithm in your answer. Do NOT forget to show the trace.**

1.Initial vertex : G
Known Vertex: G
Unknown Vertexes: A B C D E F H I
Vertexes adjacent : E F I

2.Vertex: E F I
Known vertex: G(0) E(1) F(1) I(1)
Unknown vertices: A B C D H
Vertexes adjacent to E: A(2) D(2)
Vertexes adjacent to F: C(2)
Vertexes adjacent to I: H(2)

3.Vertex: A D C H
Known vertex: G(0) E(1) F(1) I(1) D(2) A(2) C(2) H(2)
Unknown vertices: B
Vertexes adjacent to D: B(3)
Vertexes adjacent to A: -
Vertexes adjacent to C: -
Vertexes adjacent to H: -

i. Visit G

ii. Visit unvisited children of G (E,F,I)

iii. Visit unvisited children of E (A,D)

iv. Visit unvisited children of F (C)

v. Visit unvisited children of I (H)

vi. Visit unvisited children of A (B)

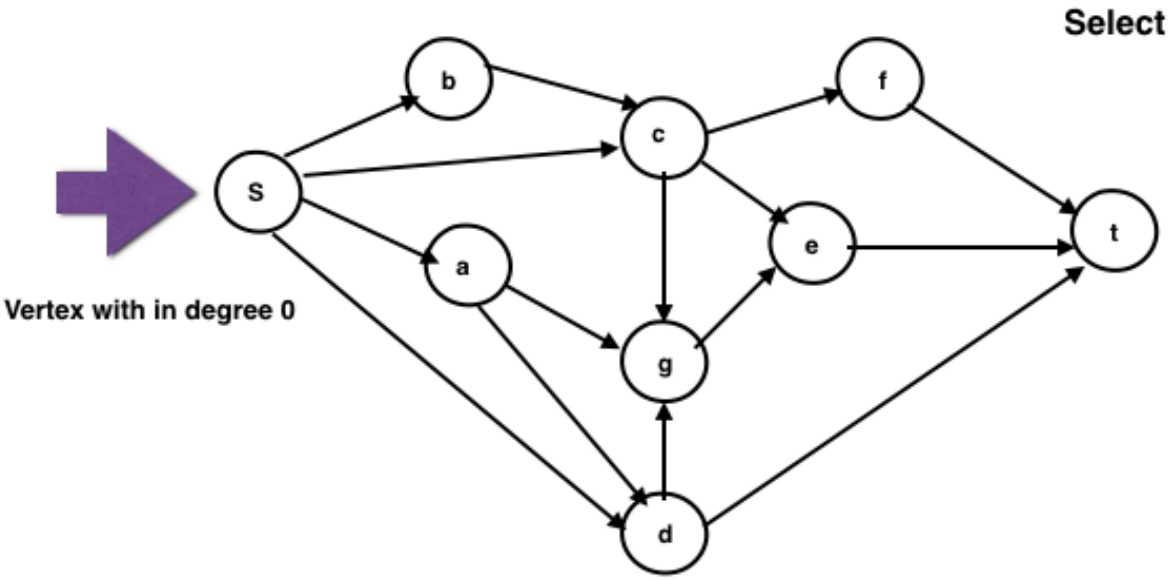
vii. Visit unvisited children of D (-)

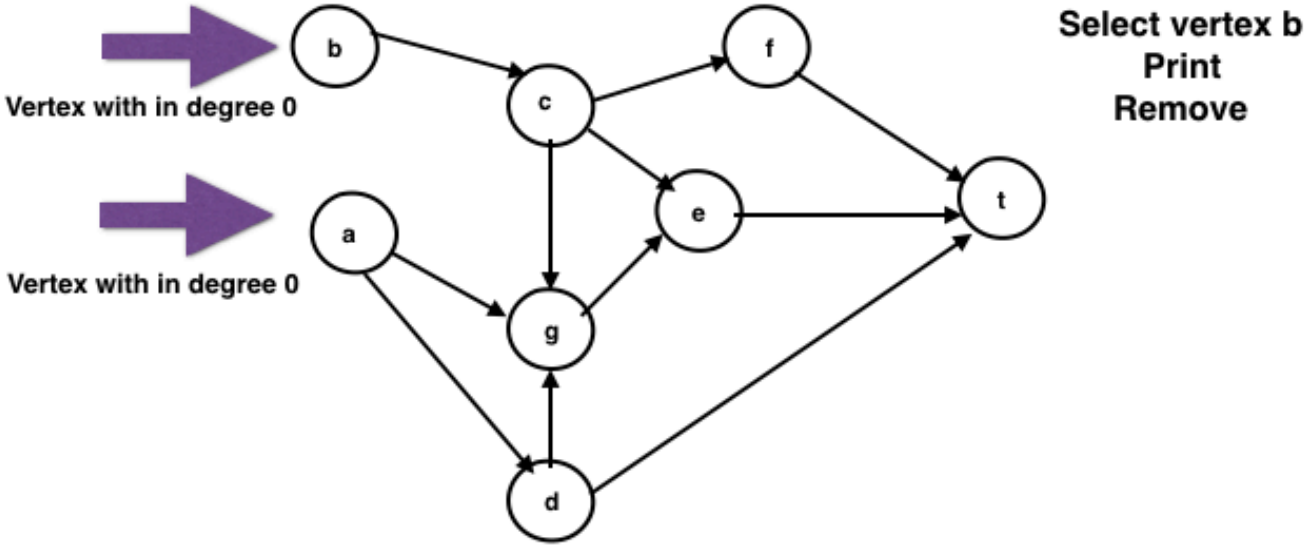
viii. Visit unvisited children of C (-)

ix. Visit unvisited children of H (-)

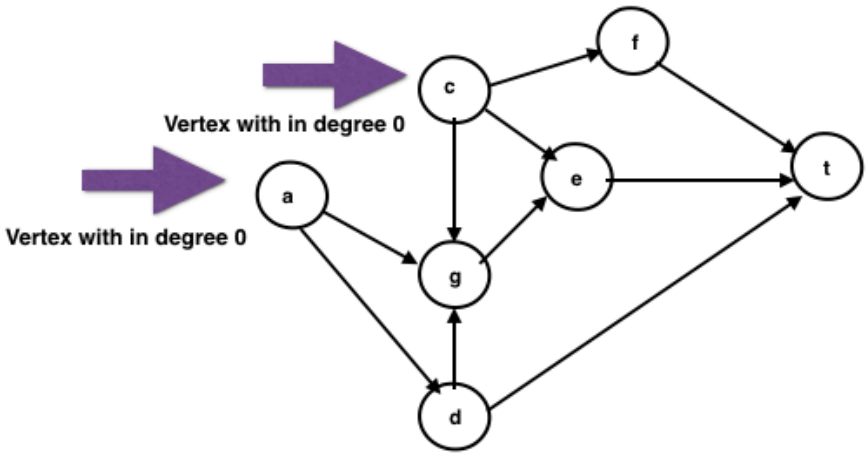
x. Visit unvisited children of B (-)

**Question 6**
**Find a topological ordering of the graph in Figure 3.**



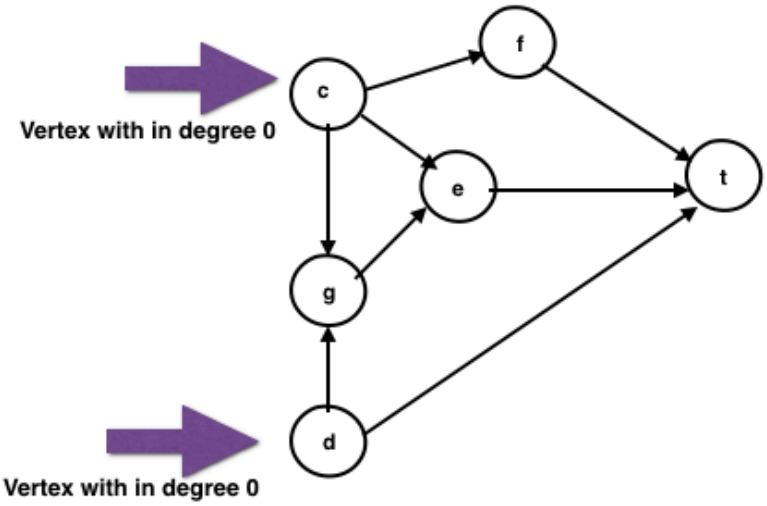




S,



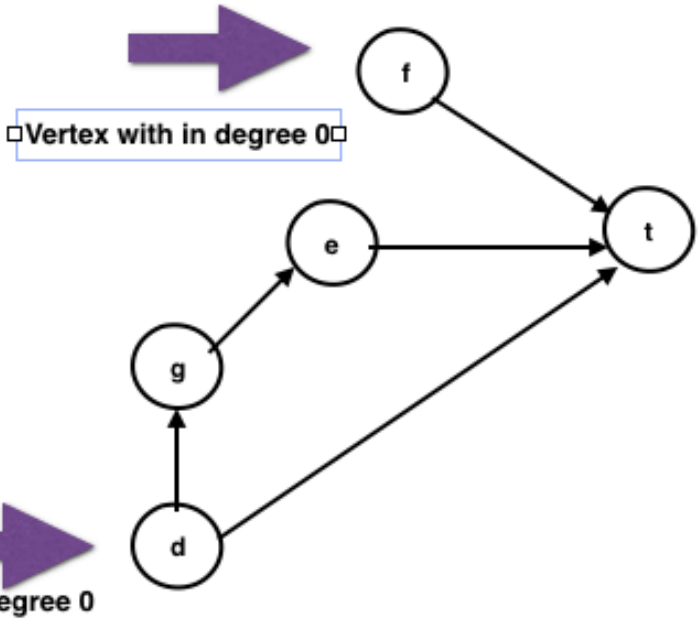

Select vertex a
Print
Remove

s,b,



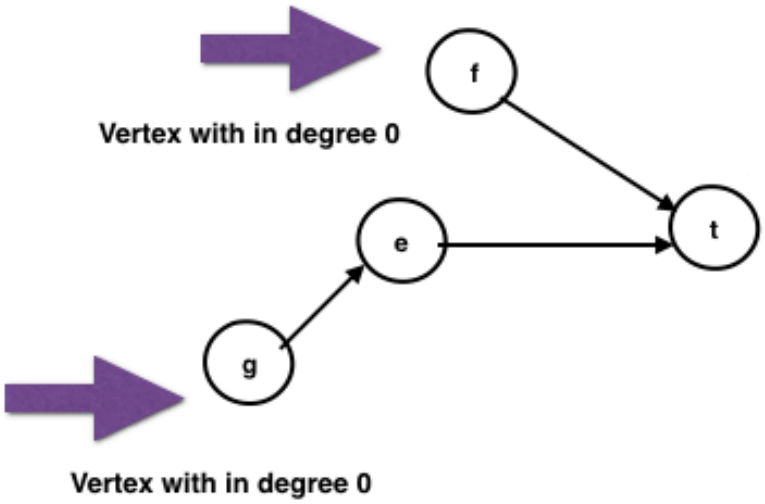

Select vertex c
Print
Remove

s,b,a

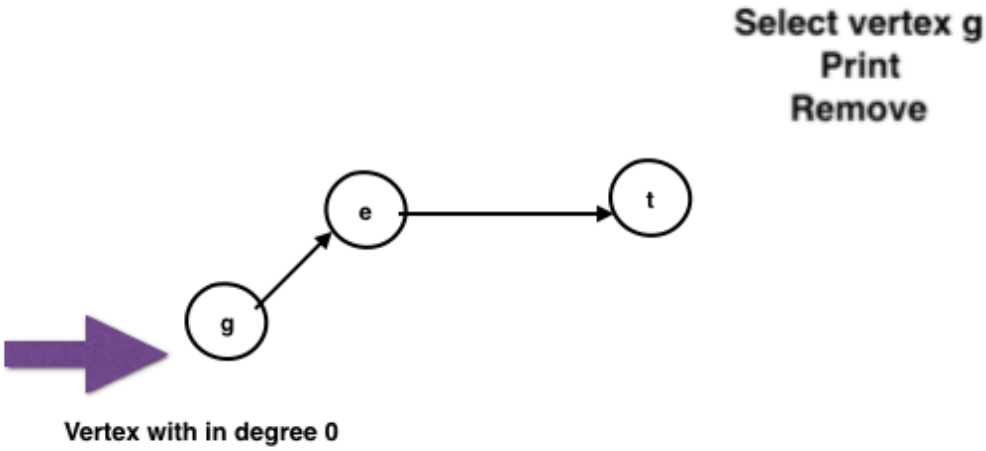



Select vertex d
Print
Remove

s,b,a,c




s,b,a,c,d




s,b,a,c,d,f

Select vertex e
Print
Remove

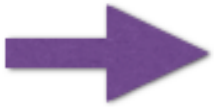

Vertex with in degree 0

s,b,a,c,d,f,g

Select vertex t
Print
Remove


Vertex with in degree 0

s,b,a,c,d,f,g,e



Vertex with in degree 0

s,b,a,c,d,f,g,e,t