

CS 404 – Artificial Intelligence
HW 4 – 2020 Spring
150pt

Objective: Learning about Propositional Logic

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1) 20 pt - Decide whether each of the following sentences is valid, unsatisfiable, or satisfiable (not valid, but only satisfiable). Show how you come to that decision using truth table enumeration or logical equivalence rules.

- $\text{Smoke} \Rightarrow \text{Smoke}$ **VALID**.....10pt

Smoke	Smoke	$\text{Smoke} \Rightarrow \text{Smoke}$
1	1	1
0	0	1

Circle the correct answer: Valid, Satisfiable, Unsatisfiable

- $\text{Smoke} \Rightarrow \text{Fire}$ **SATISFIABLE**..... 10pt

Smoke	Fire	$\text{Smoke} \Rightarrow \text{Fire}$
1	1	1
1	0	0
0	1	1
0	0	1

Circle the correct answer: Valid, Satisfiable, Unsatisfiable
2) 40pt – Truth Table Enumeration for Inference

Assume we have the knowledge base **KB**: $\text{Rain} \wedge (\text{Rain} \Rightarrow \text{Wet}) \wedge (\text{Snow} \Rightarrow \text{Cold})$ and the given propositions are the only ones in the KB.

- a) **25 pts – How many *possible worlds*** (truth value assignments to the propositions) **are *models of the KB***? Show your work by filling the truth table for the KB.

There are 16 possible worlds but 3 possible worlds are models of KB.

Cold	Wet	Snow	Rain	Rain \Rightarrow Wet	Snow \Rightarrow C old	KB
0	0	0	0	1	1	0
0	0	0	1	0	1	0
0	0	1	0	1	0	0
0	0	1	1	0	0	0
0	1	0	0	1	1	0
0	1	0	1	1	1	1
0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	1	1	0
1	0	0	1	0	1	0
1	0	1	0	1	1	0
1	0	1	1	0	1	0
1	1	0	0	1	1	0
1	1	0	1	1	1	1
1	1	1	0	1	1	0
1	1	1	1	1	1	1

- b) **15pts** – Extend the above truth table (use truth table enumeration method) to show whether the knowledge base entails $\alpha = \text{Wet}$.

State your answer here: **Entails because because $\alpha = \text{Wet}$ is true in all worlds where KB is true.**.....

Cold	Wet	Snow	Rain	Rain \Rightarrow Wet	Snow= \Rightarrow Cold	KB	$\alpha =$ wet
0	0	0	0	1	1	0	0
0	0	0	1	0	1	0	0
0	0	1	0	1	0	0	0
0	0	1	1	0	0	0	0
0	1	0	0	1	1	0	1
0	1	0	1	1	1	1	1
0	1	1	0	1	0	0	1
0	1	1	1	1	0	0	1
1	0	0	0	1	1	0	0
1	0	0	1	0	1	0	0
1	0	1	0	1	1	0	0
1	0	1	1	0	1	0	0
1	1	0	0	1	1	0	1
1	1	0	1	1	1	1	1
1	1	1	0	1	1	0	1
1	1	1	1	1	1	1	1

3) 90pt – AIMA 3rd Ed. Q. 7.2

7.2 (Adapted from Barwise and Etchemendy (1993).) Given the following, can you prove that the unicorn is mythical? How about magical? Horned?

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

- a) **20pt** – First state the English paragraph as a set of Propositional Logic sentences. Please use the given two-letter proposition names below. Be careful about what propositions to use (especially for the the statement “mortal mammal”), considering all of the paragraph.

MY : Mythical
IM : Immortal
MM : Mammal
HO : Horned
MA : Magical.

1. $MY \Rightarrow IM$ (if the unicorn is mythical, then it is immortal)
2. $\neg MY \Rightarrow (\neg IM \wedge MM)$ (if it is not mythical, then it is a mortal mammal)
3. $(IM \vee MM) \Rightarrow HO$ (If the unicorn is either immortal or a mammal, then it is horned)
4. $HO \Rightarrow MA$ (The unicorn is magical if it is horned)

- b) **20pt** – Convert the above KB into Conjunctive Normal Form. Show your work clearly.

1. $\neg MY \vee IM$
2. $MY \vee (\neg IM \wedge MM)$.
3. $(\neg IM \wedge \neg MM) \vee HO$
 $(\neg IM \vee HO) \wedge (\neg MM \vee HO)$
4. $\neg HO \vee MA$.

- c) 50pt – See if the KB entails each of the following conclusions: “unicorn is mythical”, “unicorn is magical”, “unicorn is horned”. Use the indicated method, if available.

For each of them, please indicate the inference method you are using and clearly indicate your conclusion at the beginning:

I.10pts - “unicorn is mythical”:

Conclusion: Using **simple resolution**....., we can show
that we can conclude?/not conclude? (infer/not infer?) that the unicorn is mythical.

Show your work or state your argument: ...**Contrapositive of 1 is a unicorn is either not mythical or immortal. And solving 1 & 2 gives unicorn is immortal and mammal. So it is immortal but it is not mythical. We can not conclude that unicorn is mythical. .**
.....

II.30pt - “unicorn is horned”:

Use either:

- **simple resolution**: that is apply resolution many times to see whether you can infer Horned, or

- **resolution refutation** (aka proof by contradiction): that is, add the negated form of whatever you want to check for entailment (e.g. \neg Horned), and see if you can reach a contradiction. This would show that the KB+negative is unsatisfiable, hence the conclusion can be inferred).

I used simple resolution

1. $\neg IM \Rightarrow \neg MY$ (I took contrapositive of 1, a unicorn is either not mythical or immortal)

2. $\neg IM \Rightarrow \neg IM \wedge MM$

3. $IM \vee (\neg IM \wedge MM)$

4. $(IM \vee \neg IM) \wedge (IM \vee MM)$

5. $IM \vee MM$ (1 and 2 gives 5)

Solving 1 & 2 gives $(\text{Immortal}) \vee (\sim \text{Immortal} \wedge \text{Mammal}) \rightarrow 5. (\text{Immortal} \vee \text{Mammal})$

Solving 5 & 3 \rightarrow 6. Horned

Unicorn is horned true.

III.10pt - “unicorn is magical”:

Use Modus Ponens method and state your conclusion.

$MY \Rightarrow IM$

$\neg MY \Rightarrow (\neg IM \wedge MM)$

$(IM \vee MM) \Rightarrow HO$

$HO \Rightarrow MA$

MY can be true or false in both case unicorn is magical

If MY true then IM true and if IM true it means HO true and it means MA true.

If MY false then MM true and if MM true it means HO true and it means MA true.

Another solution for the same problem

1. $\neg IM \Rightarrow \neg MY$ (I took contrapositive of 1, a unicorn is either not mythical or immortal)

2. $\neg IM \Rightarrow \neg IM \wedge MM$

3. $IM \vee (\neg IM \wedge MM)$

4. $(IM \vee \neg IM) \wedge (IM \vee MM)$

5. $IM \vee MM$ (1 and 2 gives 5)

6. HO

7. MA (Solving P4 and 6 gives unicorn is MA)

