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Planning of Optimal Collision Avoidance Trajectories with Timed Elastic Bands

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Abstract: This paper is concerned with the planning of optimal trajectories for vehicle collision avoidance with a Timed Elastic Band (TEB) framework. The avoidance trajectory is represented by a TEB which is optimized with respect to multiple partially conflicting objectives. The resulting trajectory constitutes the optimal compromise between a mere braking and a lane change maneuver that avoids the collision with the smoothest feasible path. The approach is applicable to general critical traffic situations as the TEB considers the constraints imposed by the vehicle dynamics, road boundaries, static obstacles and moving vehicles.

Keywords: Vehicles; Least squares; Optimization; Path planning; Trajectory planning.

1. INTRODUCTION

Advanced driver assistance systems are subject of many ongoing research and development projects. Collision avoidance or collision mitigation systems constitute an important subgroup of safety related driver assistance systems. Emergency braking systems are common in todays vehicles. Emergency steering constitutes a viable alternative to emergency braking. In critical scenarios in which a rear end collision is imminent, an emergency steering maneuver might still avoid the collision whereas an emergency braking maneuver is only able to mitigate the impact. Compared to emergency braking the emergency evading is a much more complex task as it requires the coordination of steering and braking in a lane change. The advanced driver assistance system calculates a feasible escape route. However, non-expert drivers are usually not able to execute such a maneuver properly in particular to stabilize the lateral movement of the vehicle. In order to safely guide the driver through the maneuver the emergency steering assistant (ESA) superimposes an additional torque on the steering wheel to compensate for the slow and insufficient steering actions of the driver. The optimal trajectory depends among others on the trajectories of other involved vehicles, in particular the vehicle ahead and rear traffic in the neighboring lanes. The traffic situation is perceived through sensors such as lidar, sonar and vision pointing in different directions. A map of object in the vehicles local environment is generated by sensor data fusion. Even though sensor fusion is no trivial task we assume for the remainder of the paper that the positions and velocities of other vehicles are known. Emergency steering and braking has already been subject to research. The approaches use rather simple functions like polynomials (Eskandarian et al. [2008]), sigmoidal functions (Schorn et al. [2006], Schorn [2007], Stählin [2008], Choi et al. [2011]) or trapezoidal acceleration profiles (Soudbakhsh et al. [2011], Choi et al. [2014]) for the evading trajectory or path. This paper proposes a novel approach to plan optimal trajectories for a combined braking and steering maneuver with timed elastic bands. The realization of the trajectory with a vehicle is not discussed within the paper. This requires further control systems. An interesting approach is presented in Schorn [2007]. Elastic Bands are proposed by Quinlan et al. [1993] in order to refine coarse trajectories in mobile robot navigation. Elastic bands are also considered to plan optimal emergency path in the approach by Hilgert et al. [2003] and Sattel et al. [2005]. However, they consider only pure steering maneuvers. Going from a path to a trajectory by explicitly considering the temporal aspect of a maneuver enables combined steering and braking actions. The timed elastic band approach has been originally applied to optimal trajectory planning for non-holonomic mobile robots Roesmann et al. [2012]. This approach is augmented to capture the specific constraints and objectives inherent to emergency maneuvers in critical traffic situations. Optimal trajectories for collision avoidance are investigated in Schmidt et al. [2006], in which the existence regions for obstacles are determined by propagation and a collision free trajectory with minimal curvature is calculated. The remainder of the paper is organized as follows: Section two describes the model of driving physics that underlies the dynamic and holonomic constraints of the vehicle. Section three introduces the timed elastic band. The optimization of the band is presented in section four. Section five shows the results of the optimization in four prototypical critical traffic situations. The last section concludes the work.

2. VEHICLE MODEL

The vehicle dynamics are described by a basic point mass model. Although the vehicle reaches its stability limits in an emergency maneuver which suggests a single track model or an even more refined model, the point mass

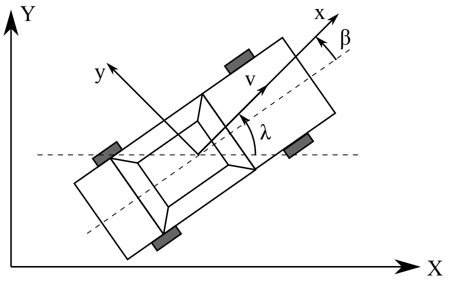

Fig. 1. Vehicle in the global and local reference frames.

model turns out to be sufficient to capture the most important dynamic properties such as lateral and longitudinal acceleration. The vehicle movement is assumed to be planar and the point mass model implies that the turn rate of the vehicle coincides with the curvature of the planned trajectory. This assumptions have also been made by Schmidt et al. [2006]. Fig. 1 shows the vehicle and the global reference frame as well as the vehicle reference frame. The trajectory is calculated in the global frame while the kinematic quantities are calculated in the vehicle frame. The point mass model neglects the vehicles side slip angle β.

3. TIMED ELASTIC BAND

A timed elastic band is composed of a fixed number n of geometric waypoints or vehicle poses P_i. The set of waypoints is described by

$$Q = \{P_i\}_{i=1\ldots n} \,. \tag{1}$$

where each waypoint consists of the tupel

$$P_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix}. \tag{2}$$

Two consecutive waypoints are separated by a time interval ΔT_i. Our approach considers these time intervals as constant as in contrast to robot navigation there is no objective of a fastest trajectory. In the context of collision the timed elastic bands merely optimizes the location of intermediate waypoints as there are no boundary conditions for the final vehicle state. The set of time intervals is given by.

$$\tau = \{\Delta T_i\}_{i=1\ldots n-1} \,. \tag{3}$$

The TEB consists of the two sets:

$$B := (Q, \tau) \,. \tag{4}$$

The optimal band is calculated by minimizing the objective function

$$f(B) = \sum_k \gamma_k \Gamma_k(B), \tag{5}$$

which is a weighted sum of multiple objectives and soft penalties for constraint violations. The objectives and their underlying cost functions Γ_k are described in the next section. The optimal trajectory B^* is given by

$$B^* = \min_B f(B). \tag{6}$$

The vehicle velocity, turn rate and accelerations are obtained from the finite differences between a pair or triple of consecutive waypoints

$$v_i = \frac{\sqrt{\left(x_i - x_{i-1}\right)^2 + \left(y_i - y_{i-1}\right)^2}}{\Delta T_i}. \tag{7}$$

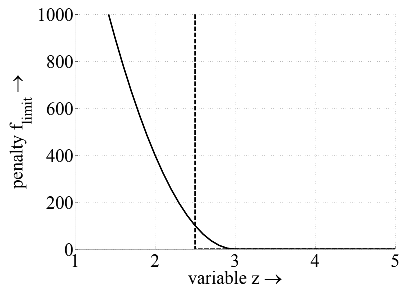

Fig. 2. Step function and its approximation by the penalty function

The change of the velocity yields the longitudinal acceleration

$$a_{x,i} = \frac{v_i - v_{i-1}}{\Delta T_i}. \tag{8}$$

The course angle is defined by

$$\lambda = \arctan\left(\frac{y_i - y_{i-1}}{x_i - x_{i-1}}\right). \tag{9}$$

The change of the course angle yields the course rate

$$\dot{\lambda}_i = \frac{\lambda_i - \lambda_{i-1}}{\Delta T_i}. \tag{10}$$

Based on the course rate and the velocity the lateral acceleration is given by

$$a_{y,i} = v_i \dot{\lambda}_i. \tag{11}$$

The total acceleration is described by

$$a_{tot} = \sqrt{a_{x,i}^2 + a_{y,i}^2}. \tag{12}$$

The change of the acceleration, also called jerk, in both lateral and longitudinal direction is given by

$$\dot{a}_{x,i} = \frac{a_{x,i} - a_{x,i-1}}{\Delta T_i} \tag{13}$$

$$\dot{a}_{y,i} = \frac{a_{y,i} - a_{y,i-1}}{\Delta T_i} \tag{14}$$

4. OBJECTIVES FOR EMERGENCY TRAJECTORIES

The optimal emergency trajectory avoids a collision with minimal lateral and longitudinal acceleration of the vehicle, in other words it represents the smoothest collision free path. The requirement of a collision free path is a constraint, which complicates the numerical optimization with the Levenberg-Marquardt-Algorithm. Therefore the hard constraint is reformulated by a soft penalty function given by

$$f_{limit}(z, z_m, \epsilon, S, n) = \begin{cases} 0 & for \quad z \le z_m - \epsilon \\ \left(\dfrac{z - (z_m - \epsilon)}{S}\right)^n & for \quad z > z_m - \epsilon \end{cases} \tag{15}$$

This function approximates the discontinuous step function, for example it imposes a lower limit on the separation between the ego vehicle and the obstacle. The parameters ϵ, S, n and z_m are chosen such that for all realistic situations the violation of the hard constraint imposes a much higher penalty than the cost of ordinary objective functions. Figure 2 shows the penalty function and the corresponding step function exemplary. Obstacles are also

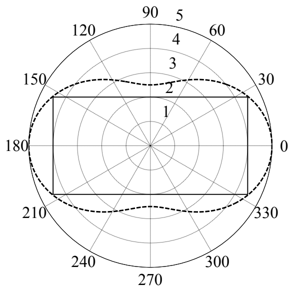

Fig. 3. An obstacle in the collision space and the circle plus figure-eight function.

considered as a point mass, with the constraint imposing a minimal separation between the two vehicles of d_1. In order to consider the rectangular shape of vehicles the distance is computed according to

$$d_{limit} = d_1 + d_2 \cdot \cos^2(\phi), \tag{16}$$

in which ϕ denotes the relative orientation between the ego vehicle and the obstacle vehicle. The combination of a circle with a figure-eight is shown in Fig. 3. Since the ego vehicle is considered as a point the obstacle diameter is doubled in the collision space to account for any intersection between the two vehicle bodies. Obstacle vehicles are assumed to travel at constant speeds along a straight lane with no lateral motion. The obstacle vehicles positions are calculated as

$$x_{o,tot} = x_o + v_{o,x} \cdot \sum_{i=1}^{n-1} \Delta T_i \tag{17}$$

$$y_{o,tot} = y_o + v_{o,y} \cdot \sum_{i=1}^{n-1} \Delta T_i \tag{18}$$

The Euclidean distance of an ego vehicle waypoint to an obstacle is given by

$$d_0 = \sqrt{(x_i - x_{o,tot})^2 + (y_i - y_{o,tot})^2}. \tag{19}$$

The first part of the objective function reads

$$\Gamma_{O,limit} = f_{limit}(-d_0, -d_{limit}, \epsilon, S, n). \tag{20}$$

Obstacles are either other vehicles or road boundaries and are represented by the same one-sided penalty function. The road boundary constraints limit the y-component of the waypoints to ensure that the vehicle stays on the road.

$$\Gamma_{Sl,limit} = f_{limit}(y, y_{max}, \epsilon, S, n) \tag{21}$$

$$\Gamma_{Sr,limit} = f_{limit}(-y, -y_{min}, \epsilon, S, n) \tag{22}$$

The concept of "soft" constraints is applied to the limits of vehicle dynamics imposed by physical laws as well. For example, according to Kamm's circle criterion a tire cannot simultaneusly transfer the maximum force in both lateral and longitudinal direction. Fig. 4 shows the lateral and longitudinal forces acting on a tire and Kamm's circle which defines the maximum reaction force of the tire. According to Newtons law a force limit implies a restriction on the maximum acceleration of the vehicle which depends on the vehicle mass and friction coefficient between tire and road surface.

$$\Gamma_{a_{tot},limit} = f_{limit}(a_{tot}, a_{limit}, \epsilon, S, n) \tag{23}$$

An actual tire is designed such that it transfers more longitudinal than lateral forces, such that Kamm's circle in fact becomes an ellipse rather than a circle. This effect

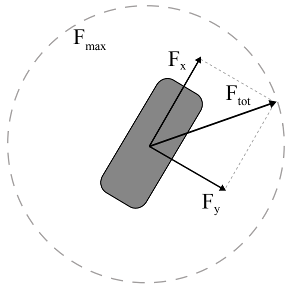

Fig. 4. Forces acting on a tire and Kamm's circle

is accounted for by introducing scaling factors g_x and g_y in the original equation (12) for the acceleration penalty function:

$$a_{tot} = \sqrt{\frac{a_{x,i}^2}{g_x} + \frac{a_{y,i}^2}{g_y}}. \tag{24}$$

Additionally the longitudinal and the lateral acceleration enter the objective function.

$$\Gamma_{a_x} = a_{x,i}^2 \tag{25}$$

$$\Gamma_{a_y} = a_{y,i}^2 \tag{26}$$

These objectives yield a smoother and therefore more comfortable and drivable path in case the emergency maneuver does not require to fully exploit the limits of the vehicle dynamics. The simple point mass model does not consider all limits of the vehicle dynamics, thus the optimal trajectory might include changes in accelerations and course rates that exceed the vehicles capacity. To avoid unrealistic changes the jerk in both directions is included by another objective function.

$$\Gamma_{\dot{a}_x} = \dot{a}_{x,i}^2 \tag{27}$$

$$\Gamma_{\dot{a}_y} = \dot{a}_{y,i}^2 \tag{28}$$

The overall cost functions is composed of the weighted sum of the above mentioned penalty and objective functions

$$\begin{gathered} f(B) = \sum_k \gamma_k \Gamma_k(B) = \\ \gamma_{O,limit} \cdot \Gamma_{O,limit}(B) + \\ +\gamma_{Sl,limit} \cdot \Gamma_{Sl,limit}(B) + \gamma_{Sr,limit} \cdot \Gamma_{Sr,limit}(B) + \\ +\gamma_{a_{tot,limit}} \cdot \Gamma_{a_{tot,limit}}(B) + \\ +\gamma_{a_x} \cdot \Gamma_{a_x}(B) + \gamma_{a_y} \cdot \Gamma_{a_y}(B) + \\ +\gamma_{\dot{a}_x} \cdot \Gamma_{\dot{a}_x}(B) + \gamma_{\dot{a}_y} \cdot \Gamma_{\dot{a}_y}(B) \end{gathered} \tag{29}$$

The optimization problem constitutes a nonlinear least squares problem, which is solved by the Levenberg-Marquardt algorithm. The weights γ_k are chosen by a trial and error approach such the resulting trajectory complies with the subjective preferences of the automotive engineer. Clearly collision avoidance is more important than low accelerations or jerks. Therefore the weights for the obstacle constraints are high. The ratio $\frac{\gamma_{a_x}}{\gamma_{a_y}}$ determines the engineers preference between evading the obstacle or mere braking. The selection of the weights $\gamma_{\dot{a}_x}$ and $\gamma_{\dot{a}_y}$ depend on the driving characteristics and power of the vehicle.

5. RESULTS

This section analyzes the trajectories optimized by the TEB in various emergency traffic situations. Fig. 5 and

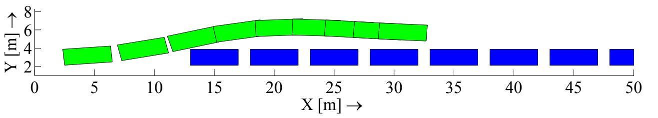

Fig. 5. Position of the obstacles and EGO vehicle at different times (first example).

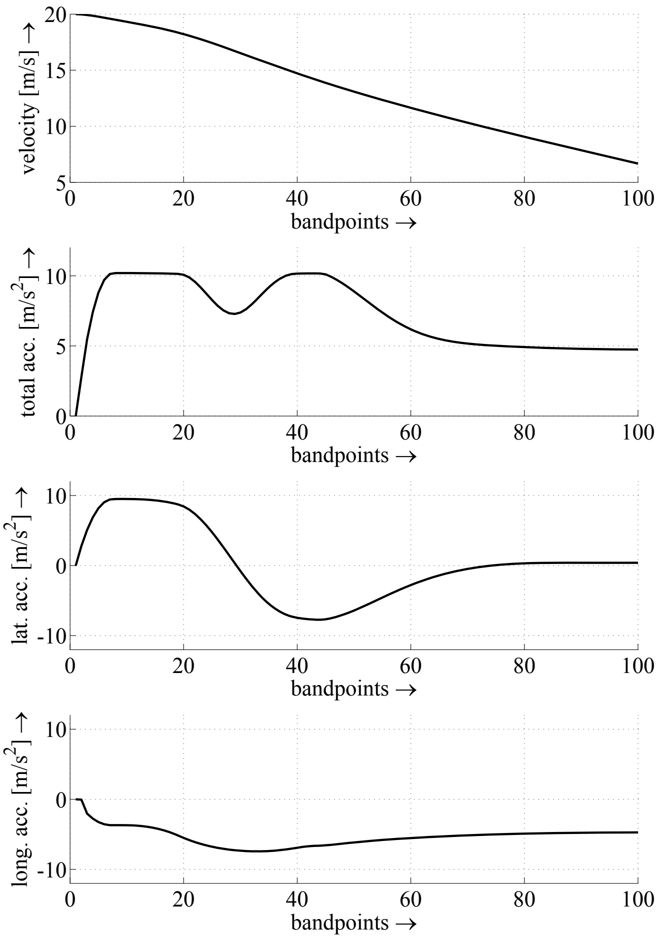

Fig. 6. Velocity and accelerations for the first example.

Fig. 6 show an example of a traffic congestion with a cue of standing vehicles ahead. Fig. 5 shows the ego vehicle passing the tail of the cue to the left. The situation is critical due to the limited acceleration to be allocated among lateral and longitudinal acceleration in an optimal compromise. Fig. 6 shows the temporal evolution of velocities and accelerations during the collision avoidance maneuver. The acceleration in both directions increases rapidly and soon saturates at the total acceleration limit of approximately 10 $\mathrm{m/s^2}$. The lane change requires a high lateral acceleration which leaves a small buffer to decelerate the vehicle. As the lateral acceleration is reversed the longitudinal deceleration temporarily increases. This results in a lower velocity for the counter steering maneuver after passing the obstacle, which in return reduces the demand for lateral acceleration. The close separation between ego and obstacle vehicle is acceptable for an emergency assistance system which neglects the drivers comfort during the maneuver. The collision is avoided with the minimum effort in acceleration and jerk which facilitates the safe execution of the maneuver along the planned trajectory. Fig. 8 and Fig. 7 show the trajectory in a scenario with rear traffic in the neighboring lane. In addition to the blocked ego lane another vehicle occupies the left lane and travels at the same speed as the ego vehicle at the beginning of the maneuver. In this case the initial distance to the

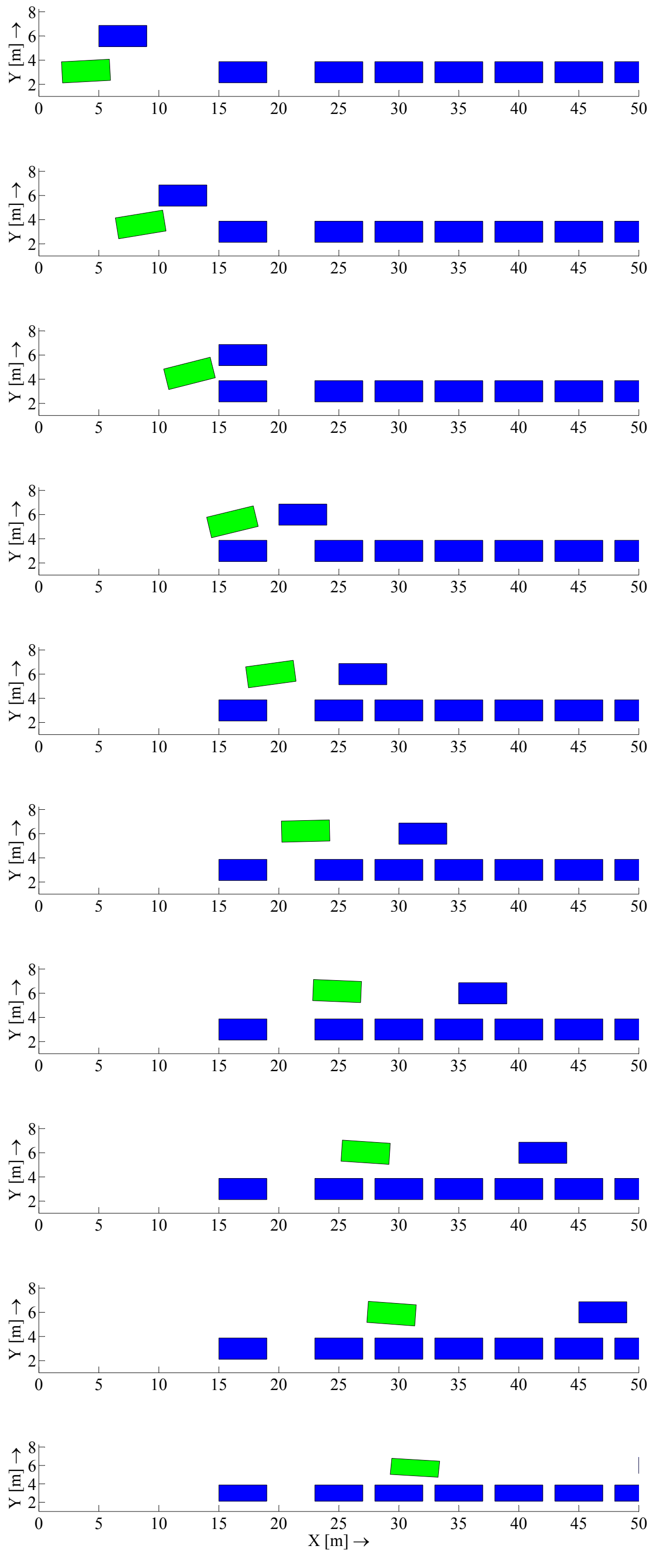

Fig. 7. Traffic jam with rear traffic (second example).

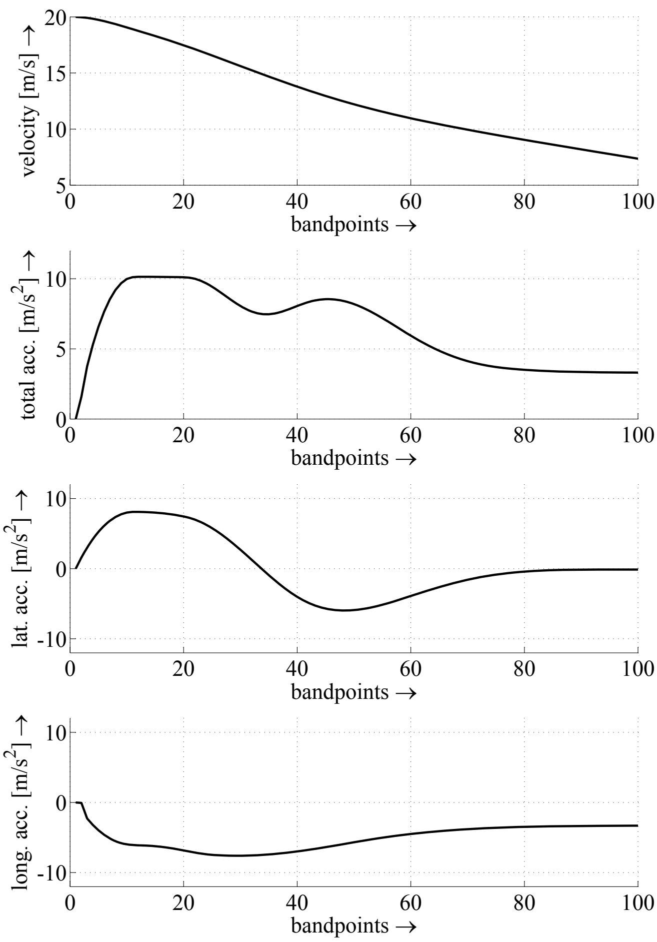

Fig. 8. Velocity and accelerations for the second example.

vehicle ahead is slightly larger than in the first case which enables the ego vehicle to clear the static vehicle after the rear vehicle has passed. Comparing Fig. 6 and Fig. 8 it is apparent that the ego vehicle decelerates stronger and executes the lane change in a slower manner until the rear vehicle has passed. The total acceleration still reaches the limitation, which shows the criticalitiy of the situation. In the two situations discussed beforehand the passing of obstacles is very close although the limits of vehicle dynamics are fully exploited. But there are also situations where a tradeoff has to be made between large distances to obstacles on the one hand and the smoothness of the trajectory on the other hand. Fig. 9 and Fig. 10 show a less critical scenario compared to the other situations before. Again the trajectory is collision free, smooth and driveable. The difference is, that the TEB in Fig. 10 was calculated on the basis of increased distance limits d_1 and d_2. According to that, the distance to the obstacle is larger, which might suit the driver better. But the accelerations reach higher values as can be seen in Fig. 11 and Fig. 12. Note that the total acceleration reaches the limitation in Fig. 12, which indicates a much more challenging task for the driver. So the automotive engineer has to decide which tradeoff is the best and this may vary among the OEMs.

6. CONCLUSION

The timed elastic band is a powerful method for path and/or trajectory planning and optimization in robotic as well as automotive applications. Collision free trajectories are generated that take multiple obstacles and different limitations on system dynamics into account. The sparse structure of the cost function enables utilization of efficient large scale optimization techniques which allows online planning and refinement of trajectories. A basic vehicle model suffices to capture the most important aspects and dynamic limitations in a combined braking and steering maneuver. The acceleration are allocated among steering and braking in a way that is compliant with the acceleration limits imposed by Kamm's circle. The available space is exploited in such that the collision is avoided with

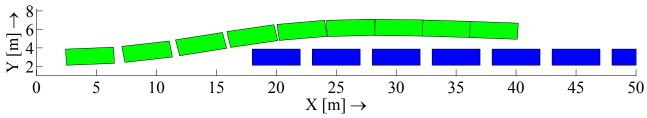

Fig. 9. Less critical traffic jam scenario (third example).

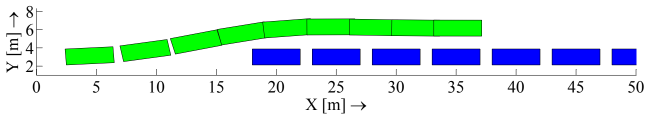

Fig. 10. A less critical scenario with increased distance limitations (fourth example).

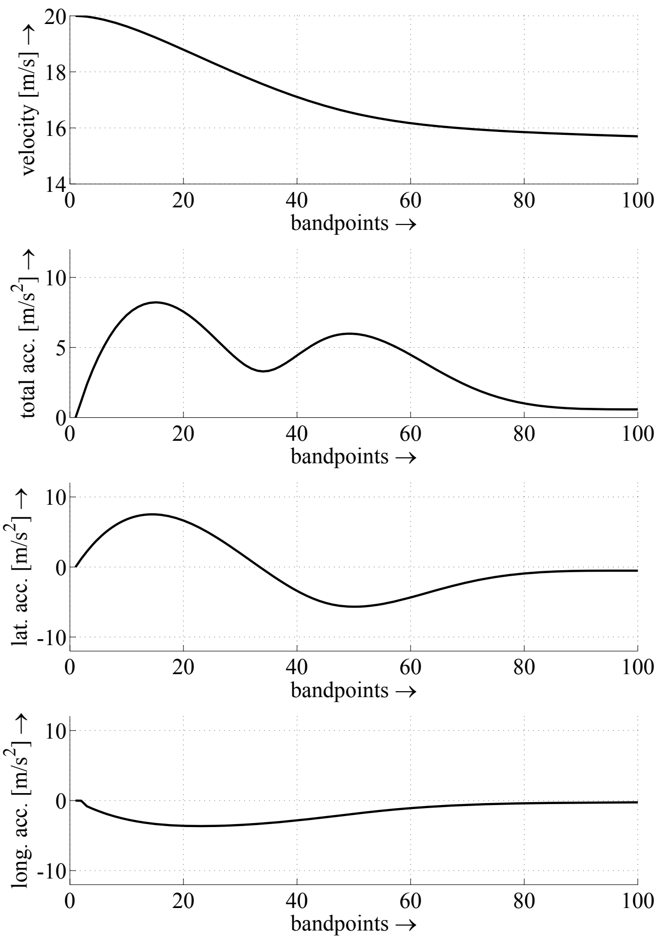

Fig. 11. Velocity and accelerations for the third example.

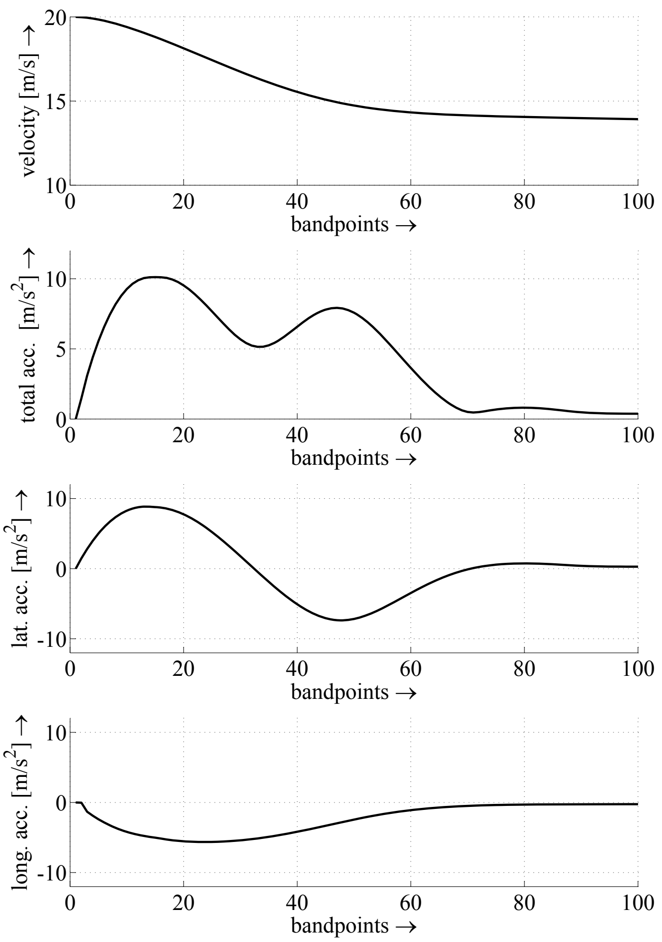

Fig. 12. Velocity and accelerations for the fourth example.

minimal jerk and acceleration throughout the maneuver. Future research is concerned with the integration of more refined models of vehicle dynamics to capture phenomena such as sliding in particular in low or split friction scenarios. Highly automated driving functions also contain trajectory planning algorithms that have to deal with complicated traffic situations as well. The TEB method seems to be a promising approach for this task.

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