Middle East Technical University | Department of Computer Engineering

CENG 223

Discrete Computational Structures

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Homework 3

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Question 1

By Fermat's Little Theorem,

$$2^{22} = 4^{11} \equiv 4 (mod\, 11)$$

$$4^{44} = (4^4)^{11} \equiv 4^4 \equiv 256 \equiv 3 (mod\, 11)$$

$$6^{66} = (6^6)^{11} \equiv 6^6 \equiv 36 * 36 * 36 \equiv 3 * 3 * 3 \equiv 5 (mod\, 11)$$

$$8^{88} = (8^8)^{11} \equiv 8^8 \equiv 2^{24} \equiv 4^{11} * 4 \equiv 4 * 4 \equiv 5 (mod\, 11)$$

$$10^{110} = (10^{10})^{11} \equiv 10^{10} \equiv 1 (mod\, 11)$$

Therefore,

$$4 + 3 + 5 + 5 + 1 \equiv 7 (mod\, 11)$$

Question 2

$$7n + 4 = (5n + 3) * 1 + 2n + 1$$

$$5n + 3 = (2n + 1) * 2 + n + 1$$

$$2n + 1 = (n + 1) * 1 + n$$

$$n + 1 = (n) * 1 + 1$$

$$n = (1) * n + 0$$

Therefore, $gcd(7n + 4, 5n + 3) = 1$.

Question 3

Given that $m^2 = n^2 + kx$,

$$m^2 - n^2 = kx$$

$$(m+n)(m-n) = kx$$

$$x \mid (m+n)(m-n)$$

Suppose $p \mid (m-n)$ is false, which results in gcd(m-n,p)=1 since p is prime. By Euclid's Lemma, which states if $a \mid bc$ and gcd(a,b)=1, then $a \mid c$ must be true where a,b,c are integers,

$$x \mid (m+n)$$

must be true. The case where $p \mid (m+n)$ is false is similar. Therefore $x \mid (m+n)$ or $x \mid (m-n)$ must be true.

Question 4

(Base case) For n=1, $\frac{n(3n-1)}{2} = 1$ is true. For some n=k, say

$$1 + 4 + 7 \dots 3k - 2 = \frac{k(3k-1)}{2}$$

If it is also true for n=k+1, this will prove our claim.

$$\frac{k(3k-1)}{2} + 3k + 1 \stackrel{?}{=} \frac{(k+1)(3k+2)}{2}$$

$$k(3k-1) + 6k + 2 \stackrel{?}{=} (k+1)(3k+2)$$

$$3k^2 - k + 6k + 2 = 3k^2 + 2k + 3k + 2$$

Therefore, our claim is true for n=k+1 which makes our claim true for all n≥1.