2b.26

mercoledì 8 dicembre 2021 15:44

# ESERCIZIO 2b.26

$$f(x,y) = \begin{cases} \lambda \dfrac{\sqrt{y}}{x^2} & \text{se } 1<x<2,\ 0<y<1 \\ 0 & \text{ALTRIMENTI} \end{cases}$$

1) calcolare $\lambda$ affinché la $f(x,y)$ sia una densità di probabilità

2) $P(y \le \frac{2}{3}x)$

3) $f_X(x)$ e $f_Y(y)$. sono indipendenti ?

SVLG

1) $$\int_1^2 \int_0^1 \lambda \frac{\sqrt{y}}{x^2}\, dx\, dy$$

$$= \int_0^1 \frac{\sqrt{y}}{x^2}\, dy = \frac{1}{x^2}\int \sqrt{y}\, dy = \frac{1}{x^2}\int y^{\frac{1}{2}} dy = \frac{1}{x^2}\cdot\left[\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_0^1 = \frac{1}{x^2}\left[\frac{2y}{3}^{\frac{3}{2}}\right]_0^1$$

$$= \frac{1}{x^2}\cdot\left[\frac{2}{3}\right] = \left[\frac{2}{3x^2}\right]$$

$$\int_1^2 \frac{2}{3x^2}\, dx = \frac{2}{3}\int x^{-2} dx = \frac{2}{3}\,\frac{x^{-1}}{-1} = -\frac{2}{3}\left[\frac{1}{x}\right]_1^2 = -\frac{2}{3}\left[\frac{1}{2}-\frac{1}{1}\right]$$

$$= -\frac{2}{3}\left[\frac{1-2}{2}\right] = -\frac{2}{3}\cdot -\frac{1}{2} = \frac{2}{6} = \left[\frac{1}{3}\right]$$

$$\frac{1}{3}\lambda = 1$$

$$\boxed{\lambda = 3}$$ ✓

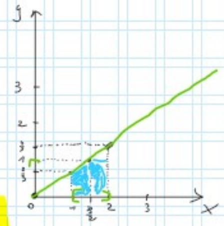
2) $P(y \le \frac{2}{3}x) =$

$x \in [1,2]$

$y \in [0,1]$

$y = \frac{2}{3}x$

| x | y |
|---|---|
| 0 | 0 |
| $\frac{3}{2}$ | 1 |
| 1 | $\frac{2}{3}$ |
| 2 | $\frac{4}{3}$ |




$$= \int_1^{\frac{3}{2}} \int_0^{\frac{2}{3}x} f(x,y)\, dx\, dy + \int_{\frac{3}{2}}^2 \int_0^1 f(x,y)\, dx\, dy$$

1° parte:

$$= \int_0^{\frac{2}{3}x} 3\frac{\sqrt{y}}{x^2}\, dy = \frac{3}{x^2}\int_0^{\frac{2}{3}x} \sqrt{y}\, dy = \frac{3}{x^2}\int y^{\frac{1}{2}} dy = \frac{3}{x^2}\,\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{3}{x^2}\cdot\left[\frac{2}{3}y^{\frac{3}{2}}\right]_0^{\frac{2}{3}x}$$

$$= \frac{3}{x^2}\cdot\left[\frac{2}{3}\sqrt{y^3}\right] = \frac{3}{x^2}\left[\frac{2}{3}\sqrt{\left(\frac{2}{3}x\right)^3} - \frac{2}{3}\sqrt{0}\right]$$

$$= \frac{3}{x^2}\left[\frac{2}{3}\cdot\left(\frac{2}{3}x\right)^{\frac{3}{2}}\right] =$$

$$= \frac{2}{x^2}\cdot\sqrt{\left(\frac{2}{3}x\right)^3}$$

$$\int_1^{\frac{3}{2}} \frac{2}{x^2}\sqrt{\left(\frac{2}{3}x\right)^3} = \int_1^{\frac{3}{2}} \frac{2}{x^2}\left(\frac{2}{3}x\right)^{\frac{3}{2}} dx = \underline{2\int_1^{\frac{3}{2}} x^{-2}\left(\frac{2}{3}x\right)^{\frac{3}{2}} dx}$$

$$= 2\int_1^{\frac{3}{2}} x^{-2}\cdot\left(\frac{2}{3}\right)^{\frac{3}{2}}\cdot x^{\frac{3}{2}} = 2\cdot\left(\frac{2}{3}\right)^{\frac{3}{2}}\cdot\int_1^{\frac{3}{2}} x^{-\frac{1}{2}} dx = 2\cdot\left(\frac{2}{3}\right)^{\frac{3}{2}}\left[\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]$$

$$= 2\cdot\left(\frac{2}{3}\right)^{\frac{3}{2}}\cdot\left[2x^{\frac{1}{2}}\right]_1^{\frac{3}{2}} =$$

$$= \underbrace{2\cdot\left(\frac{2}{3}\right)^{\frac{3}{2}}}\cdot\left[2\cdot\left(\frac{3}{2}\right)^{\frac{1}{2}} - 2\right]$$

$$= 2\sqrt{\frac{8}{27}}\cdot\left[2\cdot\frac{\sqrt{3}}{\sqrt{2}} - 2\right]$$

$$= 4\,\frac{\sqrt{8}}{\sqrt{27}}\cdot\frac{\sqrt{3}}{\sqrt{2}} - 4\,\frac{\sqrt{8}}{\sqrt{27}}$$

$$= 4\cdot\frac{2\cancel{\sqrt{2}}}{3\cancel{\sqrt{3}}}\cdot\frac{\cancel{\sqrt{3}}}{\cancel{\sqrt{2}}} - 4\cdot\frac{2\sqrt{2}}{3\sqrt{3}} = \boxed{\frac{8}{3} - \frac{8}{3}\sqrt{\frac{2}{3}}}$$

$$\int^1 \frac{3\sqrt{y}}{x^2}\, dy = \frac{3}{x^2}\int^1 \sqrt{y}\, dy = \frac{3}{x^2}\cdot\left[\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]^1 = \frac{3}{x^2}\left[\frac{2}{3}\cdot y^{\frac{3}{2}}\right]^1$$

$$\int_0^1 \frac{3\sqrt{y}}{x^2}\,dy = \frac{3}{x^2}\int_0^1 \sqrt{y}\,dy = \frac{3}{x^2}\cdot\left[\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_0^1 = \frac{3}{x^2}\left[\frac{2}{3}\cdot y^{\frac{3}{2}}\right]_0^1$$

$$\frac{3}{x^2}\left[\frac{2}{3}\right] = \frac{2}{x^2}$$

$$\int_{\frac{3}{2}}^{2} \frac{2}{x^2}\,dx = 2\int_{\frac{3}{2}}^{2} x^{-2}\,dx = 2\left[\frac{x^{-1}}{-1}\right]_{\frac{3}{2}}^{2} = +2\cdot\left[-\left(\frac{1}{2}\right)-\frac{8}{3}\right]$$

$$2\left[-\frac{1}{2}+\frac{2}{3}\right]$$

$$2\left[\frac{-3+4}{6}\right] = \frac{2}{6} = \frac{1}{3}$$

sommando

$$\frac{8}{3}-\frac{8}{3}\sqrt{\frac{2}{3}}+\frac{1}{3} = \boxed{3-\frac{8}{3}\sqrt{\frac{2}{3}}}$$

x, al prof viene diverso! per 1 numero ??

3) $$\boxed{f_X(x)} = \int_0^1 \frac{3\sqrt{y}}{x^2}\,dy = \frac{3}{x^2}\int \sqrt{y}\,dy = \frac{3}{x^2}\int y^{\frac{1}{2}}\,dy$$

$$= \frac{3}{x^2}\cdot\left[\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_0^1 = \frac{3}{x^2}\left[\frac{2}{3}y^{\frac{3}{2}}\right]_0^1 = \frac{3}{x^2}\left[\frac{2}{3}\right] = \boxed{\frac{2}{x^2}}$$

$$\boxed{f_Y(y)} = \int_1^2 \frac{3\sqrt{y}}{x^2}\,dx = 3\sqrt{y}\int x^{-2} = 3\sqrt{y}\,\frac{x^{-1}}{-1} = -3\sqrt{y}\cdot\left[x^{-1}\right]_1^2$$

$$= -3\sqrt{y}\left[\frac{1}{2}-\frac{1}{1}\right] = -3\sqrt{y}\left[\frac{1-2}{2}\right] = \boxed{\frac{3}{2}\sqrt{y}}$$