2b.19

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ESERCIZIO 2b.19

Sia:

$$f(u,v) = 9e^{-3v}$$

dove $E = \{(u,v) \in \mathbb{R}^2 : 0 < u < v\}$

1) Calcolare $P(V > 1-U)$

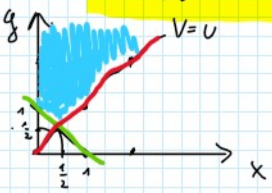
2) $f_U(u)$ e $f_V(v)$ e dire se U e V sono indipendenti

3) $Z = V - U$, $E(Z)$, $Var(Z)$ e $P(Z > \frac{1}{3})$

SVLG

1) $P(V > 1-U) = \int_0^{\frac{1}{2}} \int_{1-u}^{+\infty} f(u,v)\, dv\, du + \int_{\frac{1}{2}}^{+\infty} \int_u^{+\infty} f(u,v)\, dv\, du$

U	V
0	1
1	0
$\frac{1}{2}$	$\frac{1}{2}$



$V > U$

$V = U$

U	V
0	0
1	1
2	2

ORA, RISOLVIAMO L'INTEGRALE:

• PRIMA PARTE:

$$\int_{1-u}^{+\infty} 9e^{-3v}\, dv = 9\int e^{-3v}\, dv = 9\int \frac{-3}{-3} e^{-3v}\, dv = -3\int e^{-3v} \cdot -3\, dv$$

$$= -3\left[e^{-3v}\right]_{1-u}^{+\infty} = -3\left[0 - e^{-3+3u}\right] = \left[3e^{-3+3u}\right]$$

$$\int_0^{\frac{1}{2}} 3e^{-3+3u}\, du = 3\int e^{-3} \cdot e^{3u} = 3e^{-3} \cdot \int \frac{3}{3} e^{3u}\, du = e^{-3} \cdot \left[e^{3u}\right]_0^{\frac{1}{2}}$$

$$= e^{-3}\left[e^{\frac{3}{2}} - 1\right] = \underline{e^{-\frac{3}{2}} - e^{-3}}$$

• seconda parte

$$\int_u^{+\infty} 9e^{-3v}\, dv = -3\left[e^{-3v}\right]_u^{+\infty} = -3\left[0 - e^{-3u}\right] = 3e^{-3u}$$

$$\int_{\frac{1}{2}}^{+\infty} 3e^{-3u}\, du = -\int -3e^{-3u}\, du = -\left[e^{-3u}\right]_{\frac{1}{2}}^{+\infty} = -\left[0 - e^{-\frac{3}{2}}\right]$$

$$= \underline{e^{-\frac{3}{2}}}$$

$-\frac{3}{2}$ -3 $-\frac{3}{2}$ 3 2

$= e^{-2}$

ORA, SOMMIAMO TUTTO: $e^{-\frac{3}{2}} - e^{-3} + e^{-\frac{3}{2}} = 2e^{-\frac{3}{2}} - e^{-3}$ ✓

2) $$f_U(u) = \int_u^{+\infty} 9e^{-3v}\,dv = -3\left[e^{-3v}\right]_u^{+\infty} = \boxed{3e^{-3u}}$$ ✓

$$f_V(v) = \int_0^v 9e^{-3v}\,du = 9\int_0^v e^{-3v}\,du = 9e^{-3v}\cdot\int du$$

$$= 9e^{-3v}\cdot[u]_0^v = 9e^{-3v}\cdot v = \boxed{9ve^{-3v}}$$ ✓

$$f(u)\cdot f(v) = 3e^{-3u}\cdot 9ve^{-3v} = 27ve^{-3u}\cdot e^{-3v}$$

NON SONO INDIPENDENTI ! Δ

3) $Z = V - U$!

$V = y$

$U = x$

$$\phi = \begin{cases} U = U \\ Z = V - U \end{cases} \qquad \phi^{-1} = \begin{cases} U = U \\ V = Z + U \end{cases} \qquad J\phi^{-1} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \qquad |\Delta| = 1$$

$$f_{(U,Z)}(u,z) = f_{(U,V)}(u, z+u)\cdot\Delta$$

F. INDICATRICI:

$\boxed{u>0}$

$0 < u < v$

$0 < u < z+u$

$u < z+u$

$u - u < z$

$0 < z$

$$f_Z(z) = \int_0^{+\infty} 9e^{-3(z+u)}\,du \cdot 1_{[0,+\infty]}(u)$$

ORA, RISOLVIAMO L'INTEGRALE:

$$f_Z(z) = \int_0^{+\infty} 9e^{-3(z+u)}\,du = 9\int_0^{+\infty} e^{-3z}\cdot e^{-3u}\,du = 9e^{-3z}\int_0^{+\infty} e^{-3u}\cdot\frac{-3}{-3}\,du$$

$$= -3e^{-3z}\cdot\left[e^{-3u}\right]_0^{+\infty} = -3e^{-3z}[0-1] = \boxed{3e^{-3z}}$$

$$\exp(3) = \lambda\cdot e^{-\lambda z}$$

- $E(z) = \frac{1}{3}$

- $VAR(Z) = \frac{1}{9}$

- $P\left(Z > \frac{1}{3}\right) = \int_{\frac{1}{3}}^{+\infty} 3e^{-3Z}\, dZ = 3 \cdot \int e^{-3Z} \cdot \frac{-3}{-3} \cdot dZ$

$$= -\left[e^{-3Z}\right]_{\frac{1}{3}}^{+\infty} = -\left[0 - e^{-1}\right] = \boxed{\frac{1}{e}}$$