2b.18

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2b. 18

$$f(x,y) = \begin{cases} x+y & \text{se } 0<x<1, \quad 0<y<1 \\ 0 & \text{altrimenti} \end{cases}$$

1) $f_X(x)$ e $f_Y(y)$. Risultano V.A indipendenti?
2) $E(x)$, $E(y)$, $var(x)$, $var(y)$, $cov(x,y)$
3) $P(y-4x^2>0)$
4) $z = x+y$:
   a) $P(\ln(x+y)<0)$
   b) il quantile di z di ordine $\frac{1}{3}$, cioè il valore di q : $P(z \le q) = \left[\frac{1}{3}\right]$

SVLG

1) $f_X(x) = \int_0^1 x+y \, dy = \left[xy + \frac{y^2}{2}\right]_0^1 = \left[x + \frac{1}{2}\right]$

$f_Y(y) = \int_0^1 x+y \, dx = \left[\frac{x^2}{2} + yx\right]_0^1 = \left[\frac{1}{2}+y\right]$

$\left(x+\frac{1}{2}\right)\cdot\left(\frac{1}{2}+y\right) = \frac{1}{2}x + xy + \frac{1}{4} + \frac{1}{2}y$ [NON SONO INDIPENDENTI]

2) $E(x) = \int_0^1 \left(x+\frac{1}{2}\right)\cdot x = \int_0^1 x^2 + \frac{1}{2}x = \left[\frac{x^3}{3} + \frac{x^2}{4}\right]_0^1 = \frac{1}{3}+\frac{1}{4} = \frac{4+3}{12} = \frac{7}{12}$

$E(y) = \int_0^1 \left(y+\frac{1}{2}\right)\cdot y = \frac{7}{12}$

$Var(x) = E(x^2) - E^2(x) = \frac{5}{12} - \frac{49}{144} = \frac{60-49}{144} = \frac{11}{144}$

$\hookrightarrow E(x^2) = \int_0^1 x^2\cdot\left(x+\frac{1}{2}\right)dx = \int x^3 + \frac{1}{2}x^2 = \left[\frac{x^4}{4} + \frac{x^3}{6}\right]_0^1$

$= \frac{1}{4} + \frac{1}{6} = \frac{6+4}{24} = \frac{10}{24} = \frac{5}{12}$

$Var(y) = E(y^2) - E^2(y) = \frac{5}{12} - \frac{49}{144} = \frac{11}{144}$

$\hookrightarrow E(y^2) = \int_0^1 \left(y+\frac{1}{2}\right)\cdot y^2 \, dy = \int_0^1 y^3 + \frac{1}{2}y^2 = \left[\frac{y^4}{4} + \frac{y^3}{6}\right] = \frac{5}{12}$

$Cov(x,y) = E(x\cdot y) - E(x)\cdot E(y) = \frac{1}{3} - \frac{49}{144} = \frac{48-49}{144} = \left[-\frac{1}{144}\right]$

$\hookrightarrow E(xy) = \int\int f(x,y)\cdot x\cdot y \cdot dx\cdot dy$

$$E(XY) = \int\int f(x,y) \cdot x \cdot y \cdot dx \cdot dy$$

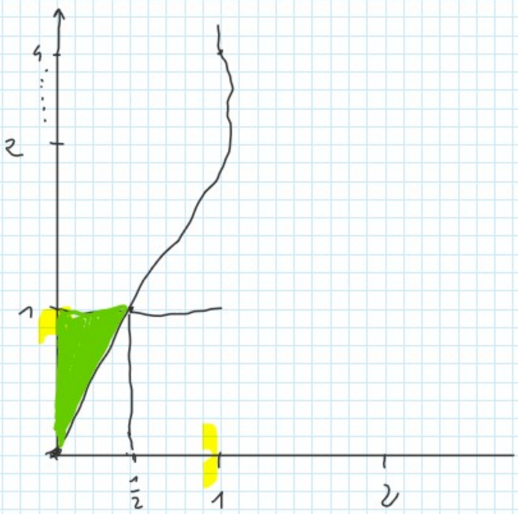
$$\int (x+y)\, y \cdot dy = \int_0^1 xy + y^2\, dy = \left[\frac{xy^2}{2} + \frac{y^3}{3}\right]_0^1 = \frac{x}{2} + \frac{1}{3}$$

$$\int_0^1 \left(\frac{x}{2} + \frac{1}{3}\right) \cdot x \cdot dx = \int_0^1 \frac{x^2}{2} + \frac{1}{3}x\, dx = \left[\frac{x^3}{6} + \frac{x^2}{6}\right]_0^1 = \frac{1}{6} + \frac{1}{6}$$

$$\left[\frac{1}{3}\right]$$

3) $P(y - 4x^2 > 0) = P(y > 4x^2)$ $\qquad y = 4x^2$

X	y
0	0
1	4
$\frac{1}{2}$	1



$$= \int_0^{\frac{1}{2}} \int_{4x^2}^{1} f(x,y)\, dx\, dy = \frac{21}{80}$$

4) $Z = X + Y \qquad \in [0, 2]$

$$\phi = \begin{cases} X = x \\ z = x + y \end{cases} \qquad \phi^{-1} = \begin{cases} X = X \\ y = z - x \end{cases} \qquad J\phi^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \quad |\Delta| = 1$$

$$f_{x,z}(x,z) = f_{x,y}(x, z-x) \cdot 1$$

studiamo funz. indicatrici!

$0 < x < 1 \qquad 0 < y < 1$

$0 < z - x < 1$

$x \in [0, 1]$

$z - x > 0$
$-x > -z$
$x < z$

$z - x < 1$
$-x < 1 - z$
$x > z - 1$

$x \in [z, z-1]$

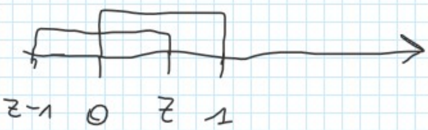
1 $\qquad$ 1

$$f_Z(z) = \int_0 x + z - x \, dx = \int_0 z \, dx$$

ORA: $[0,1] \cap [z, z-1]$

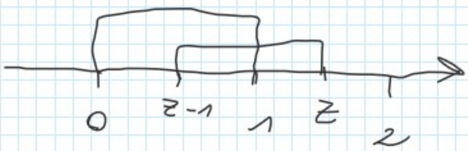
se $z < 0$, $= 0$

se $z \in [0,1]$:



$$\int_0^z z \, dx = [zx]_0^z = z^2$$

se $z \in [1,2]$:



$$\int_{z-1}^1 z \, dx = z[x]_{z-1}^1 = z(2-z)$$

• se $z > 2$, $= 0$!

$$f_Z(z) = \begin{cases} 0 & \text{se } z \le 0 \\ z^2 & \text{se } z \in [0,1] \\ z(2-z) & \text{se } z \in [0,2] \\ 0 & \text{se } z \ge 2 \end{cases}$$

a) $P(\ln(x+y) < 0) = P(\ln(z) < 0)$

$= P(z < 1) = F(1) = \int_0^1 z^2 dz = \left[\frac{z^3}{3}\right]_0^1 = \boxed{\frac{1}{3}}$

b) $P(z \le q) = \frac{1}{3}$

$$\int z^2 dz = \frac{z^3}{3}$$

$$\frac{z^3}{3} = \frac{1}{3}$$

$$z^3 = 1$$

$$\boxed{z = 1}$$