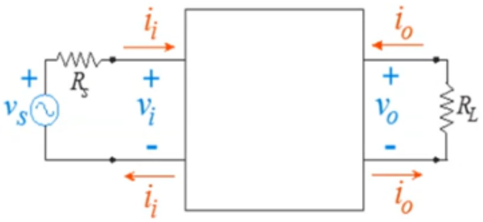
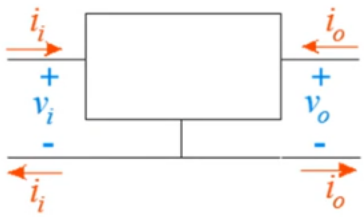
Quadripoli

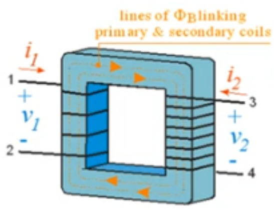
martedì 18 luglio 2023 14:27

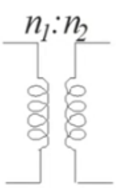
Vediamo in dettaglio lo schema:





In questo ultimo si ha un terminale in comune.
Vediamo ora il primo quadri polo (elemento passivo e semplice) : Trasformatore

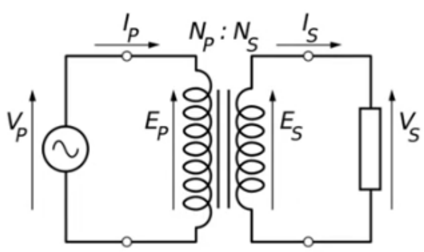




con n1 e n2 sono le spire . Nel primo nasce corrente, mentre nel secondo c'è la corrente indotta dal primo.

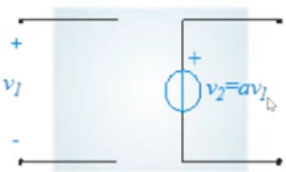
N=n1/n2
se n<1 step down -> abbasso tensione
V2=N*v1 se n>1 step up -> alzo tensione
se n=1 trasformatore di isolamento
i2=i1/n

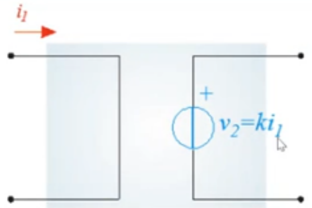
Vediamo il circuito :

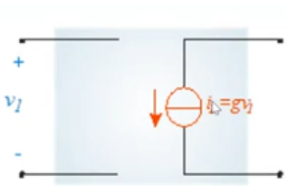


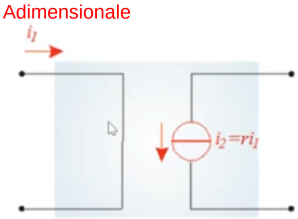
$$\frac{V_P}{V_S} = \frac{I_S}{I_P} = \frac{N_P}{N_S}$$

Vediamo ora un quadri polo composto: Generatore controllati : valore in uscita (ac/dc) dipende da altro parametro:

- VCVS (Generatore tensione controllato da tensione)
  - adimensionale
  - 

- CCVS (Generatore tensione controllato corrente)
  - Ohm
  - 

- VCCS (Generatore corrente controllato da tensione)
  - Siemens
  - 

- CCCS (Generatore corrente controllato da corrente)
  - Adimensionale
  - 

Andiamo ora a vedere come studiare l'uscita rispetto all'ingresso , ovvero in base alla coppia di variabili indipendenti in uscita studio il quadripolo con le variabili dipendenti dell'ingresso :

Var.	analytical notation	matrix notation	name	matrix, parameters
i_1 i_2	$\begin{cases} v_1 = z_{11}i_1 + z_{12}i_2 \\ v_2 = z_{21}i_1 + z_{22}i_2 \end{cases}$	$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$	Current controlled	$[Z]$, impedance
v_1 v_2	$\begin{cases} i_1 = y_{11}v_1 + y_{12}v_2 \\ i_2 = y_{21}v_1 + y_{22}v_2 \end{cases}$	$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$	Voltage controlled	$[Y]$, admittance
i_1 v_2	$\begin{cases} v_1 = h_{11}i_1 + h_{12}v_2 \\ i_2 = h_{21}i_1 + h_{22}v_2 \end{cases}$	$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$	hybrid 1	$[H]$, Hybrid
v_1 i_2	$\begin{cases} i_1 = gv_1 + g_{12}i_2 \\ v_2 = g_{21}v_1 + g_{22}i_2 \end{cases}$	$\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix}$	hybrid 2	$[G]$, Reverse hybrid
v_2 i_2	$\begin{cases} v_1 = a_{11}v_2 - a_{12}i_2 \\ i_1 = a_{21}v_2 - a_{22}i_2 \end{cases}$	$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$	Trasmission 1	$[A]$, transmission
v_1 i_1	$\begin{cases} v_2 = bv_1 + b_{12}i_1 \\ i_2 = b_{21}v_1 + b_{22}i_1 \end{cases}$	$\begin{bmatrix} v_2 \\ i_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ i_1 \end{bmatrix}$	Trasmission 2	$[B]$, Reverse transmission

La prima $\begin{bmatrix} ohm & ohm \\ ohm & ohm \end{bmatrix}$,

la seconda $\begin{bmatrix} siemens & siemens \\ siemens & siemens \end{bmatrix}$,

la terza $\begin{bmatrix} ohm & adimensionale \\ adimensionale & siemens \end{bmatrix}$, *la quarta* $\begin{bmatrix} siemens & adimensionale \\ adimensionale & ohm \end{bmatrix}$,

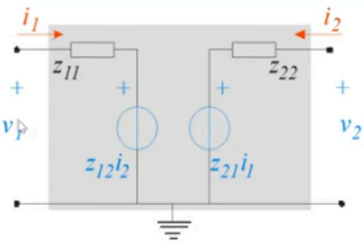
la quinta $\begin{bmatrix} adimensionale & siemens \\ siemens & adimensionale \end{bmatrix}$,

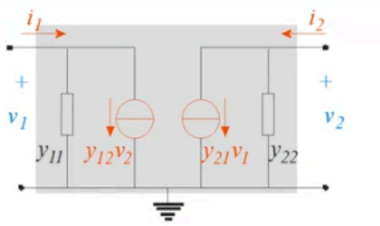
l'ultima $\begin{bmatrix} adimensionale & ohm \\ siemens & adimensionale \end{bmatrix}$.

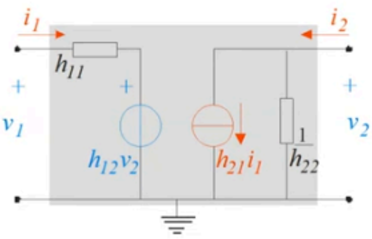
Non prendiamo le ultime 2 in quanto non esistono circuiti che le rappresentano .
Possibile esprimere tutte le matrici in funzioni delle altre :

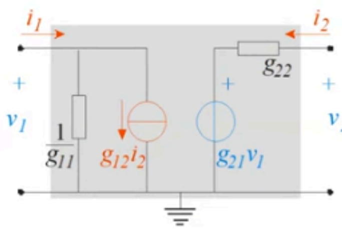
To: \ From:	$[Z]$	$[Y]$	$[H]$	$[A]$
$[Z]$	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta y} & -\frac{y_{12}}{\Delta y} \\ -\frac{y_{21}}{\Delta y} & \frac{y_{11}}{\Delta y} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{a_{11}}{a_{21}} & \frac{\Delta a}{a_{21}} \\ \frac{1}{a_{21}} & \frac{a_{22}}{a_{21}} \end{bmatrix}$
$[Y]$	$\begin{bmatrix} \frac{z_{22}}{\Delta z} & -\frac{z_{12}}{\Delta y} \\ -\frac{z_{21}}{\Delta y} & \frac{z_{11}}{\Delta y} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{a_{22}}{a_{12}} & -\frac{\Delta a}{a_{12}} \\ -\frac{1}{a_{12}} & \frac{a_{11}}{a_{12}} \end{bmatrix}$
$[H]$	$\begin{bmatrix} \frac{\Delta z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ -\frac{z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & -\frac{y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{a_{12}}{a_{22}} & \frac{\Delta a}{a_{22}} \\ -\frac{1}{a_{22}} & \frac{a_{21}}{a_{22}} \end{bmatrix}$
$[A]$	$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} -\frac{y_{22}}{y_{21}} & -\frac{1}{y_{21}} \\ -\frac{\Delta y}{y_{21}} & -\frac{y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} -\frac{\Delta h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{bmatrix}$	$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Vediamo i circuiti :

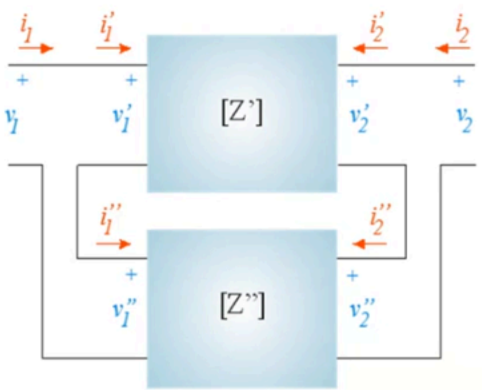






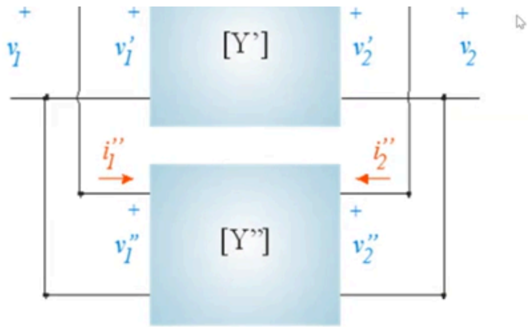


Mentre per quanto riguarda le connessioni tra bipoli:

- Serie-serie
  - 

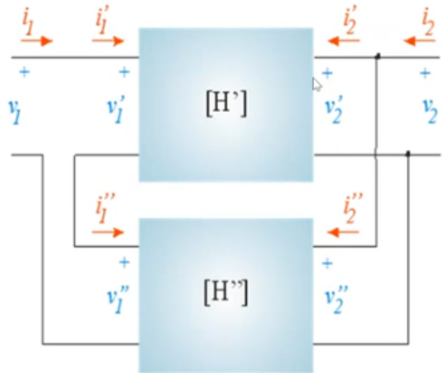
$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} z'_{11} & z'_{12} \\ z'_{21} & z'_{22} \end{bmatrix} + \begin{bmatrix} z''_{11} & z''_{12} \\ z''_{21} & z''_{22} \end{bmatrix}$$

- Parallelo-parallelo



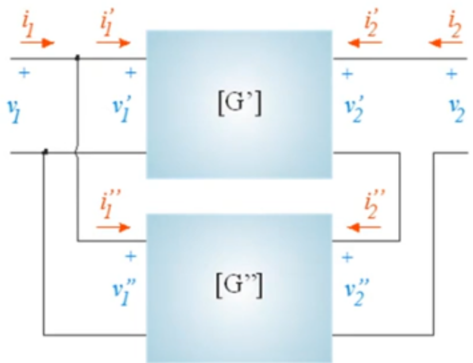
$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} y'_{11} & y'_{12} \\ y'_{21} & y'_{22} \end{bmatrix} + \begin{bmatrix} y''_{11} & y''_{12} \\ y''_{21} & y''_{22} \end{bmatrix}$$

- Serie-parallelo



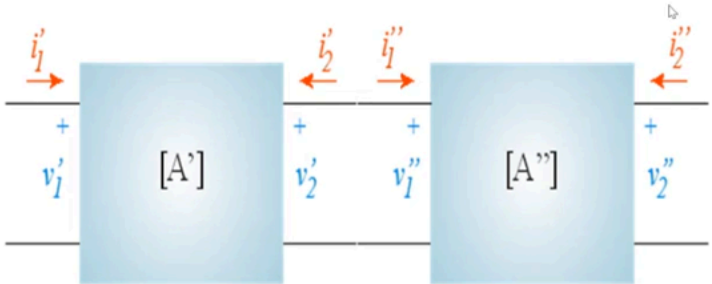
$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} h'_{11} & h'_{12} \\ h'_{21} & h'_{22} \end{bmatrix} + \begin{bmatrix} h''_{11} & h''_{12} \\ h''_{21} & h''_{22} \end{bmatrix}$$

- Parallelo-serie



$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \begin{bmatrix} g'_{11} & g'_{12} \\ g'_{21} & g'_{22} \end{bmatrix} + \begin{bmatrix} g''_{11} & g''_{12} \\ g''_{21} & g''_{22} \end{bmatrix}$$

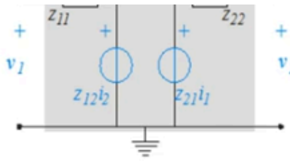
- Cascata



$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} a'_{11} & a'_{12} \\ a'_{21} & a'_{22} \end{bmatrix} \begin{bmatrix} a''_{11} & a''_{12} \\ a''_{21} & a''_{22} \end{bmatrix}$$

Vediamo un esempio di conversione: Da matrice Z a matrice H:

$$h_{11} = \frac{v_1}{i_1}\bigg| \quad \begin{cases} v_1 = z_{11} i_1 + z_{12} i_2 \\ 0 = z_{21} i_1 + z_{22} i_2 \end{cases}$$



$$\Rightarrow \quad v_1 = z_{11}i_1 + z_{12}\left(-\frac{z_{21}}{z_{22}}i_1\right)$$

$$\Rightarrow \quad h_{11} = \frac{z_{11}z_{22} - z_{12}z_{21}}{z_{22}}$$

$[H]=?$

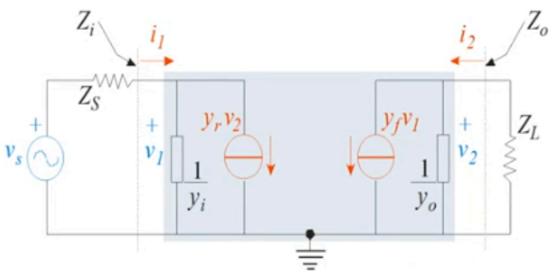
$$\begin{cases} v_1 = h_{11}i_1 + h_{12}v_2 \\ i_2 = h_{21}i_1 + h_{22}v_2 \end{cases} \qquad h_{12} = \left.\frac{v_1}{v_2}\right|_{i_1=0}$$

$$\begin{cases} v_1 = 0 + z_{12}i_2 \\ v_2 = 0 + z_{22}i_2 \end{cases}$$

$$\Rightarrow \quad h_{12} = \frac{z_{12}}{z_{22}}$$

$$[H] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{z_{11}z_{22} - z_{12}z_{21}}{z_{22}} & \dfrac{z_{12}}{z_{22}} \\ -\dfrac{z_{21}}{z_{22}} & \dfrac{1}{z_{22}} \end{bmatrix}$$

Vediamo ora le impedenze , ovvero impedenza vista dal generatore verso il quadri polo e quella che vede il carico. Si parla quindi di effetto di carico :



$$\begin{cases} i_1 = y_{11}v_1 + y_{12}v_2 \\ i_2 = y_{21}v_1 + y_{22}v_2 \end{cases}$$

$$Z_i \stackrel{\text{def}}{=} \frac{v_1}{i_1} = \frac{1}{y_i - \dfrac{y_f y_r Z_L}{1 + y_o Z_L}} \qquad Z_o \stackrel{\text{def}}{=} \frac{v_2}{i_2} = \frac{1}{y_o - \dfrac{y_f y_r Z_g}{1 + y_i Z_g}}$$