# Trend-following and Momentum Strategies in Futures Markets\*

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#### **ABSTRACT**

Constructing a time-series momentum strategy involves the volatility-adjusted aggregation of univariate strategies and therefore relies heavily on the efficiency of the volatility estimator and on the quality of the momentum trading signal. Using a dataset with intra-day quotes of 12 futures contracts from November 1999 to October 2009, we investigate these dependencies and their relation to time-series momentum profitability and reach a number of novel findings. First, momentum trading signals generated by fitting a linear trend on the asset price path maximise the out-of-sample performance while minimizing the portfolio turnover, hence dominating the ordinary momentum trading signal in literature, the sign of past return. Second, the results show strong momentum patterns at the monthly frequency of rebalancing, relatively strong momentum patterns at the weekly frequency and relatively weak momentum patterns at the daily frequency. In fact, significant reversal effects are documented at the very short-term horizon. Finally, regarding the volatility-adjusted aggregation of univariate strategies, the Yang-Zhang range estimator constitutes the optimal choice for volatility estimation in terms of maximizing efficiency and minimizing the bias and the ex-post portfolio turnover.

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KEY WORDS: Trend-following; Momentum; Futures; Volatility Estimation; Trading Signals.

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## 1. Introduction

Financial markets historically exhibit strong momentum patterns. Until recently, the "cross-sectional momentum" effect in equity markets (Jegadeesh and Titman 1993, Jegadeesh and Titman 2001) and in futures markets (Pirrong 2005, Miffre and Rallis 2007) has received most of the academic interest. Moskowitz, Ooi and Pedersen (2011) offer the first concrete piece of empirical evidence on "time-series momentum", using a broad daily dataset of futures contracts. Time-series momentum refers to the trading strategy that results from the aggregation of a number of univariate momentum strategies on a volatility-adjusted basis. The univariate time-series momentum strategy relies heavily on the serial correlation/predictability of the asset's return series, in contrast to the cross-sectional momentum strategy, which is constructed as a longshort zero-cost portfolio of securities with the best and worst relative performance during the lookback period<sup>1</sup>. The purpose of this paper is to explore the profitability of time-series momentum strategies for a broad grid of lookback periods, holding horizons and frequencies of portfolio rebalancing, using a novel dataset of intra-day quotes of 12 futures contracts for the period between November 1999 and October 2009. Furthermore, we investigate the mechanics of the time-series momentum strategy and in particular focus (a) on the momentum trading signals and (b) on the volatility estimation that is crucial for the aggregation of the individual strategies. The choice between various available methodologies for these two components of the strategy heavily affects the ex-post momentum profitability and portfolio turnover and is therefore very important for a momentum investor.

Following the above, the aim of this paper is to address the following research topics. First, we focus on the information content of traditional momentum trading signals and also devise new signals that capture a price trend, in an effort to maximise the out-of-sample performance and to minimise the transaction costs incurred by the portfolio rebalancing. Second and most importantly, the significance of time-series momentum patterns is assessed for a broad grid of lookback periods, holding horizons and frequencies of portfolio rebalancing. Briefly, momentum patterns are indeed found to be strong and robust at the monthly and weekly frequencies, but relatively weaker at the daily frequency. In fact, there also exist some signs of very short-term reversal effects. Lastly, we investigate a family of volatility estimators and assess their efficiency from a momentum investing viewpoint. The availability of high-frequency data allows the examination of various range and high-frequency volatility estimators.

Regarding the first objective of the paper, the results show that the traditional momentum trading signal, that of the sign of the past return (Moskowitz et al. 2011) and denoted for convenience by SIGN, can only provide a rough indication of a price trend. The reason is that it merely constitutes a comparison between the farthest and most recent price levels, disregarding the information content of the price path itself throughout the lookback period. For that purpose, we introduce another four methodologies that focus on the trend behaviour of the price path: (i) a moving average indicator as, for instance, in Han, Yang and Zhou (2011) and Yu and Chen (2011), (ii) a signal related to the price trend that is extracted using

<sup>&</sup>lt;sup>1</sup>In the absence of transaction costs, a cross-sectional momentum strategy needs no capital to be constructed. The short portfolio finances the long portfolio and each of these two portfolios consists of a fraction of the available M instruments, for instance when decile portfolios are used, then each of these two portfolios consists of M/10 securities. Instead, a time-series momentum strategy always consists of M open positions, which in the extreme case can even simultaneously be M long or M short positions.

the Ensemble Empirical Mode Decomposition, introduced by Wu and Huang (2009), (iii) the t-statistic of the slope coefficient from a least-squares fit of a linear trend on the price path and (iv) a more robust version of the previous signal using the statistically meaningful trend methodology of Bryhn and Dimberg (2011). For convenience, we call these four signals using the shorthand notations MA, EEMD, TREND, SMT respectively. We refer to the last two signals, TREND and SMT, as the "trend-related" trading signals and it is stressed that only these two methodologies from the family of available signals offer a natural way to decide upon the type of trading activity (long/short position) or the absence of any trading activity for the forthcoming investment horizon, hence resulting in trading signals of long/inactive/short type. This is achieved by using the statistical significance of the extracted linear-trend and abstaining from trading when the significance is weak, in order to avoid eminent price reversals. To give an indication of the resulting trading activity in the sample, a 12-month lookback period leads to trading activity for about 87% of the time when using the TREND signal and for 63% of the time when using the SMT signal. This sparse trading activity gives by construction an advantage to the trend-related signals, because it significantly lowers the turnover of the momentum portfolio, which at times is halved.

For the above family of momentum signals, we study the profitability of the time-series momentum strategy using monthly, weekly and daily frequencies of portfolio rebalancing and a broad grid of lookback and holding periods. The results show strong momentum patterns at the monthly frequency for lookback and holding periods that range up to 12 months. With the passage of time the momentum profits diminish and the patterns partly reverse for longer holding periods in line with the findings of Moskowitz et al. (2011). Similar time periods are also associated with the cross-sectional momentum strategy in futures markets as in Pirrong (2005), Miffre and Rallis (2007), but the time-series momentum is not fully captured by the cross-sectional patterns following Moskowitz et al. (2011). Regarding higher frequencies of portfolio rebalancing, relatively strong momentum patterns are documented at the weekly frequency for lookback and holding periods that range even up to 8 weeks for some trading signals and lastly scarce momentum patterns on the daily frequency. In fact, significant reversal effects are documented at the very short-term horizon. Apart from a small number of exceptions, the trend-related trading signals offer the best out-of-sample momentum performance in terms of mean return, dollar growth and Sharpe ratio across all frequencies of portfolio rebalancing, all lookback and holding periods.

Quantitatively, the time-series momentum strategy with a 6-month lookback period and a 1-month holding period generates 28.36% annualised mean return using the SMT signal compared to the 15.97% of the traditional SIGN signal; both are strongly significant at the 1% level. The growth of an initial investment of \$1 at the beginning of the sample is \$11.1 for the SMT signal and only \$4 for the SIGN signal. The difference in the ex-post Sharpe ratio is not so pronounced (1.18 versus 1.00), because of the increased ex-post volatility of the momentum strategies due to the sparse trading activity of the trend-related signals. It appears, however, that the increased volatility is the result of successful trend capturing by the trend-related signals and therefore successful momentum bets, which lead to more positively skewed return distributions. This is desirable from an investment perspective. We compute for the above strategies the downside-risk Sharpe ratio (Ziemba 2005), which constitutes a modification of the ordinary Sharpe ratio and treats differently the negative and positive returns by penalizing more the negatively

skewed return distributions. We find it to be 1.83 for the SMT signal and only 1.38 for the SIGN signal. That, combined with the significant decline in the portfolio turnover -it is more than halved when using the SMT signal- renders the trend-related trading signals superior by all metrics.

Additionally, a strategy that is weekly rebalanced and uses a 3-week lookback period and a 1-week holding period generates annualised return, dollar growth and downside-risk Sharpe ratio equal to 19.99%, \$5.60 and 0.95 for the SMT signal compared to the 10.56%, \$2.49 and 0.70 for the SIGN signal, hence reinforcing our arguments. For the daily frequency of rebalancing, the momentum patterns are in general relatively weaker, hence the differences across signals regarding the ex-post momentum profitability are not so pronounced. Instead, the most interesting feature of the daily frequency of portfolio rebalancing is found for the strategy with a 3-day lookback period and a 1-day holding period. This is the only strategy across all strategies, signals and frequencies of rebalancing in this paper that generates statistically significant and economically important reversal effects. For instance, using the SMT signal, the time-series momentum strategy loses on average 13.38% annualised and an initial investment of \$1 shrinks to \$0.21 at the end of the 10-year period of our sample.

Finally, we show that traditional daily volatility estimators, like the standard deviation of daily past returns, provide relatively noisy volatility estimates, hence worsening the turnover of the time-series momentum portfolio. In fact, Moskowitz et al. (2011) acknowledge that there exist various more efficient volatility estimators than their simple exponentially-weighted moving average of past daily returns, without however exploring any of them. Using a 30-minute quote high-frequency dataset on 12 futures contracts, we provide more efficient volatility estimates by employing a high-frequency volatility estimator, the realized variance by Andersen and Bollerslev (1998) and a family of estimators, known as range estimators that make use of daily information on open, close, high and low prices. In particular, we employ the estimators by Parkinson (1980), Garman and Klass (1980), Rogers and Satchell (1991) and Yang and Zhang (2000). The term "range" refers to the daily high-low price difference and its major advantage is that it can even successfully capture the high volatility of an erratically moving price path intra-daily, which happens to exhibit similar opening and closing prices and therefore a low daily return<sup>2</sup>. Alizadeh, Brandt and Diebold (2002) show that the range-based volatility estimates are approximately Gaussian, whereas return-based volatility estimates are far from Gaussian, hence rendering the former estimators more appropriate for the calibration of stochastic volatility models using a Gaussian quasi-maximum likelihood procedure.

As expected the realized variance estimator is superior among the volatility estimators. Given the fact that it uses the complete high-frequency price path information leads to greater theoretical efficiency (Barndorff-Nielsen and Shephard 2002) and therefore is used as the benchmark for the comparison among the rest of estimators. It is found that the Yang and Zhang (2000) estimator dominates the remaining estimators and is therefore used throughout the paper for the construction of time-series momentum strate-

<sup>&</sup>lt;sup>2</sup>As an indicative example, on Tuesday, August 9, 2011, most major exchanges demonstrated a very erratic behaviour, as a result of previous day's aggressive losses, following the downgrade of the US's sovereign debt rating from AAA to AA+ by Standard & Poor's late on Friday, August 6, 2011. On that Tuesday, FTSE100 exhibited intra-daily a 5.48% loss and a 2.10% gain compared to its opening price, before closing 1.89% up. An article in the Financial Times entitled "Investors shaken after rollercoaster ride" on August 12 mentions that "...the high volatility in asset prices has been striking. On Tuesday, for example, the FTSE100 crossed the zero per cent line between being up or down on that day at least 13 times...".

gies. The reasons for this choice are: (a) it is theoretically the most efficient estimator (after the realized variance of course), (b) it exhibits the smallest bias when compared to the realized variance and (c) it generates the lowest turnover, hence minimising the costs of rebalancing the momentum portfolio. It can be argued that based on the above discussion the optimal choice for volatility estimation would be the realized variance estimator. It must be stressed that this is indeed the case. We choose to use the Yang and Zhang (2000) estimator, because it constitutes an optimal tradeoff between efficiency, turnover and the necessity of high-frequency data, since it can be satisfactorily computed using daily information on opening, closing, high and low prices. If anything, it is shown that the numerical difference between these two estimators is relatively small and consequently they lead to statistically indistinguishable results for the performance of the momentum strategies.

In a nutshell, the above findings document statistically strong and economically important time-series return predictability and therefore pose a substantial challenge to the random walk hypothesis and the efficient market hypothesis (Fama 1970, Fama 1991). The objective of this paper is not to explain which mechanism is at work<sup>3</sup>, but the fact that the source of this predictability is merely a single firm effect relates the findings to two strands of literature, namely the rational and the behavioural explanations to serial correlation in a firm's return series. Along the first strand and among others, Berk, Green and Naik (1999) argue that a firm's optimal investment choices can change its systematic risk and expected return and consequently allow for return predictability. Based on that, Chordia and Shivakumar (2002) link time-series momentum to time variation in expected returns that is captured by a set of macroeconomic variables, related to the business cycle. Johnson (2002) develops a single-firm partial equilibrium model, under which past performance is correlated with the expected growth rate of the dividend process, which in turn is monotonically related to risk. Sagi and Seasholes (2007) build a model for a single firm that is based on revenues, costs, growth options and shutdown options and show how the return autocorrelation depends on these firm-specific attributes. Finally, from a relatively different perspective, Christoffersen and Diebold (2006) and Christoffersen, Diebold, Mariano, Tay and Tse (2007) show that there exists a direct link between volatility predictability and return sign predictability even when there exists no return predictability. Obviously, return sign predictability is enough to generate time-series momentum trading signals.

Along the behavioural strand of literature, it should be noted that most theories that have been developed in order to explain cross-sectional momentum patterns are solely single-firm paradigms that manifest return predictability and therefore apply more to the time-series momentum explanation. Some indicative examples of these models of investor sentiment are Barberis, Shleifer and Vishny's (1998) model, which incorporates the representativeness heuristic and the conservatism bias and links return autocorrelation to underreaction effects and Daniel, Hirshleifer and Subrahmanyam's (1998) model, which incorporates the overconfidence effect and the biased self-attribution effect of investment outcomes and eventually links momentum to overreaction effects to private information. Finally, Hong and Stein (1999) justify momentum profitability by means of investor underreaction caused by the gradual information diffusion.

The rest of the paper is organized as follows. Section 2 provides an overview of the high-frequency

<sup>&</sup>lt;sup>3</sup>This would require various theoretical economic models with testable implications.

dataset, section 3 presents the mechanics of the time-series momentum strategy focusing explicitly on the trading signal and on the volatility estimation. The empirical results regarding the return predictability and the time-series momentum profitability are next presented in section 4 and finally section 5 concludes.

## 2. Data Description

The dataset to be used consists of intra-day futures prices for 6 commodities (Cocoa, Crude Oil, Gold, Copper, Natural Gas and Wheat), 2 equity indices (S&P500 and Eurostoxx50), 2 FX rates (US Dollar Index and EUR/USD rate) and 2 interest rates (Eurodollar and 10-year US Treasury Note) spanning a period of 10 years, from November 1, 1999 to October 30, 2009 (2610 days, 520 weeks, 120 months). The frequency of intra-day quotes is 30 minutes, hence leading to 48 observations per day and 125280 observations per contract for the entire 10-year period. The dataset is appropriately adjusted for rollovers<sup>4</sup> and is provided by a large financial institution. Since the contracts are traded in various exchanges each with different trading hours and holidays, the data series are appropriately aligned in order to avoid potential lead-lag effects by filling forward any missing asset prices, following Pesaran, Schleicher and Zaffaroni (2009), and quoted in US dollars using the appropriate exchange rates from Datastream. Figure 1 presents the time evolution of the futures prices and the respective 60-day running volatility (in annual terms) computed using the Yang and Zhang (2000) estimator (discussed in the next sections).

#### [Insert Figure 1 here.]

Using the end-of-month/end-of-Wednesday/end-of-day quotes, we build monthly/weekly/daily data series. We then construct return series for each contract and each frequency by computing the percentage change in the closing asset price level. The construction of a return data series for a futures contract does not have an objective nature and various methodologies have been used in the literature<sup>5</sup>. Among others, Bessembinder (1992), Bessembinder (1993), Gorton, Hayashi and Rouwenhorst (2007), Pesaran et al. (2009) and Fuertes, Miffre and Rallis (2010) compute returns similarly as the percentage change in the price level, whereas Pirrong (2005) and Gorton and Rouwenhorst (2006) also take into account interest rate accruals on a fully-collateralized basis. Miffre and Rallis (2007) use the change in the logarithms of the price level. Lastly, Moskowitz et al. (2011) use the percentage change in the price level in excess of the risk-free rate. In undocumented results, all the above return definitions have been tried without significant -qualitative or quantitative- changes in our conclusions; one reason for this is the fact that the interest rates have been kept to relatively lower historical levels during the period 1999-2009 (on average less than 3% annually).

<sup>&</sup>lt;sup>4</sup>This is the standard methodology when working with futures contracts; see for instance, de Roon, Nijman and Veld (2000) and Miffre and Rallis (2007).

<sup>&</sup>lt;sup>5</sup>As noted by Miffre and Rallis (2007), the term "return" is imprecise for futures contracts, because the mechanics of opening and maintaining a position on a futures contract involve features like initial margins, potential margin calls, interest accrued on the margin account and if anything, no initial cash payment at the initiation of the contract. Constructing a data series of percentage changes in the asset price level implies that initial cash payment takes places, which, in turn, is practically inaccurate. Following the above discussion, it must be stressed that the use of the term "return" throughout this paper should be interpreted as a holding period return on a fully-collateralized position (in the sense that the initial margin equals the settlement price at the initiation of the contract) without any interest rate accruals, hence leading to a more conservative estimate of the return.

Table 1 presents in Panels A, B and C various summary statistics of monthly, weekly and daily return series respectively for all contracts. The return series essentially represent the performance of a buyand-hold or equivalently a long-only strategy. The mean return, the volatility and the Sharpe ratios are annualised to allow comparison across panels. The first observation is that there exists a great amount of cross-sectional variation in mean returns and volatilities, with the commodities being historically the most volatile contracts, in line with Pesaran et al. (2009) and Moskowitz et al. (2011). The distribution of the buy-and-hold return series exhibits, except for very few instances, fat tails as deduced by the kurtosis and the maximum-likelihood estimated degrees of freedom for a Student t-distribution; a normal distribution is almost universally rejected by the Jarque and Bera (1987) and the Lilliefors (1967) tests of normality. Interestingly, the fatness of the tails and the departure from normality becomes more pronounced and aggressive in higher frequencies. The conclusions about potential first-order time-series autocorrelation using the Ljung and Box (1978) test are mixed, with the daily frequency though exhibiting stronger rejection of the null hypothesis suggesting serial independence. This could constitute an indirect, elementary indication of stronger momentum patterns in lower (than daily) frequencies. Additionally, very strong evidence of heteroscedasticity is apparent across all frequencies with only one and four exceptions in the weekly and monthly frequency respectively, as deduced by the ARCH test of Engle (1982); this latter effect of time-variation in the second moment of the return series is also apparent in the volatility plots of Figure 1.

#### [Insert Table 1 here.]

The last column of Table 1 presents a modification of the ordinary Sharpe ratio (SR), known as the downside-risk Sharpe ratio (DR-SR) and introduced by Ziemba (2005), which treats differently the negative and positive returns<sup>6</sup>. From an investment perspective, increased volatility generated by positive returns is desired, however the ordinary SR offers a reward-to-risk ratio that treats equally positive and negative returns. Ziemba (2005) suggests a reward-to-risk ratio that uses as a measure of the asset variance (risk) twice the variance generated only by the negative returns. The two ratios are summarized in the formulas below:

SR = 
$$\frac{\bar{R}}{\sigma}$$
, where  $\sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} (R_i - \bar{R})^2$  (1)

DR-SR = 
$$\frac{\bar{R}}{\sqrt{2}\sigma_{(-)}}$$
, where  $\sigma_{(-)}^2 = \frac{1}{N-1} \sum_{j=1}^{N} \left( R_j \cdot \mathbf{1}_{\{R_j < 0\}} \right)^2$ , (2)

where N denotes the number of trading periods and  $\bar{R} = \frac{1}{N} \sum_{j=1}^{N} R$  is the average return over these N periods. Clearly, DR-SR and SR will be very similar for a symmetric distribution, but DR-SR will be substantially larger for a positively skewed distribution. This is apparent in Table 1, where for instance for the Eurodollar contract in the monthly frequency, DR-SR is more than twice the ordinary SR. DR-SR will prove to be a very useful statistic to evaluate the performance of time-series momentum strategies in

<sup>&</sup>lt;sup>6</sup>Knight, Satchell and Tran (1995) also acknowledge the necessity for handling positive and negative shocks to returns differently and introduce a model for asset returns that accounts for such asymmetries.

the next sections.

## 3. Methodology

This section presents the building blocks of the methodology: (i) the definition of time-series momentum, (ii) the family of methodologies that we employ, in order to estimate the realized volatility of the assets and (iii) the family of methodologies that we employ, in order to capture a price trend and therefore generate momentum trading signals.

#### 3.1. Time-Series Momentum

Univariate time-series momentum is defined as the trading strategy that takes a long/short position on an asset based on a metric of the recent asset performance. Let J denote the lookback period over which the asset's past performance is measured and K denote the holding period. Throughout the paper, both J and K are measured in months, weeks or days depending on the rebalancing frequency of interest; for convenience, this strategy is denoted by the pair (J, K).

In line with Moskowitz et al. (2011), we subsequently construct the return series of the (aggregate) *time-series momentum* strategy as the inverse-volatility weighted average return of all available individual momentum strategies:

$$R^{TS}(t, t+K) = \sum_{i=1}^{M} X_i(t-J, t) \cdot \frac{10\%/\sqrt{M}}{\sigma_i(t; D)} \cdot R_i(t, t+K),$$
(3)

where M is the number of available assets and  $\sigma_i(t;D)$  denotes an estimate at time t of the realized volatility of the  $i^{th}$  asset computed using a window of the past D trading days.  $X_i(t-J,t)$  is the trading signal for the  $i^{th}$  asset which is determined during the lookback period and in general takes values in the set  $\{-1,0,1\}$ , which in turn translates to  $\{\text{short},\text{inactive},\text{long}\}$ . The scaling factor  $10\%/\sqrt{M}$  is used in order to achieve an ex-ante volatility equal to 10% as in Moskowitz et al.  $(2011)^7$ . The families of volatility estimators and trading signals that are used in this paper are described in the following subsections.

<sup>&</sup>lt;sup>7</sup>This scaling is arguably simplistic as it ignores any covariation among the individual momentum strategies and also ignores any potential changes in the individual volatility processes. In the case that the individual time-series strategies are mutually independent the resulting portfolio conditional variance at time t is  $\operatorname{Var}_t\left[R^{TS}(t,t+K)\right] = \sum_{i=1}^M X_i^2(t-J,t) \cdot \frac{(10\%)^2/M}{\sigma_i^2(t;D)}$ .  $\operatorname{Var}_t\left[R_i(t,t+K)\right] = \sum_{i=1}^M \frac{(10\%)^2}{M} = (10\%)^2$ , since  $X_i^2(t-J,t) = 1$  and also it can be assumed that  $\operatorname{Var}_t\left[R_i(t,t+K)\right] \approx \sigma_i^2(t;D)$ , due to the persistence of the volatility process. Consequently the ex-ante portfolio volatility is the desired 10%. In practice, the ex-post volatility is not 10% due to time-varying volatility conditions among the portfolio constituents and also due to potential covariation among them. Nevertheless, such a scaling easies the interpretation of the results as it offers reasonable, real-life ex-post volatilities. Besides, note that there might exist trading periods when  $X_i(t-J,t) = 0$  for some trading signal X and some asset i, but the frequency of such events is relatively small, as it is documented later in the paper, to affect the above argument.

#### 3.2. Volatility Estimation

The time-series momentum strategy is defined in equation (3) as an inverse-volatility weighted average of individual time-series momentum strategies. This risk-adjustment (in other words, the use of standardized returns) across instruments is very common in the futures literature (see e.g. Pirrong (2005) and Moskowitz et al. (2011)), because it allows for a direct comparison and combination of various asset classes with very different return distributions (see cross-sectional variation in mean returns and volatilities in Table 1) in a single portfolio and safeguards against dominant assets in a portfolio with non-standardized constituents.

The momentum literature to date has used simple ways to estimate asset volatilities, the reason being that the available data series most frequently consist of daily data and consequently no further efficiency can be gained out of using intra-day information. Pirrong (2005) uses the standard estimate of volatility, which is the -equally weighted- standard deviation of past daily returns, whereas Moskowitz et al. (2011) use an exponentially-weighted measure of squared daily past returns. In fact, Moskowitz et al. (2011) do insist that "...while all of the results in the paper are robust to more sophisticated volatility models, we chose this model due to its simplicity...". Let D denote the number of past trading days that are used to estimate the volatility and C(t) denote the closing log-price at the end of day t. The above two estimators are given below.

• Standard Deviation of Daily Returns (STDEV):

The daily log-return at time t is R(t) = C(t) - C(t-1). Hence, the annualised D-day variance of returns is given by:

$$\sigma_{\text{STDEV}}^{2}(t;D) = \frac{261}{D} \sum_{i=0}^{D-1} \left[ R(t-i) - \bar{R}(t) \right]^{2}, \tag{4}$$

where  $\bar{R}(t) = \frac{1}{D} \sum_{i=0}^{D-1} R(t-i)$  and 261 is the number of trading days per year.

• Exponentially-Weighted Moving Average estimator (EWMA):

Moskowitz et al. (2011) use an exponentially-weighted moving average measure of lagged squared daily returns with the center of mass of the weights being equal to 60 days:

$$\sigma_{\text{EWMA}}^{2}(t;D) = 261 \sum_{i=0}^{\infty} (1 - \delta) \,\delta^{i} \left[ R(t - i) - \bar{R}(t) \right]^{2}. \tag{5}$$

where  $\bar{R}(t) = \sum_{i=0}^{\infty} (1-\delta) \, \delta^i R(t-i)$  and  $\delta$  is chosen so that  $\sum_{i=0}^{\infty} (1-\delta) \, \delta^i i = \frac{\delta}{1-\delta} = 60$  (note that  $\sum_{i=0}^{\infty} (1-\delta) \, \delta^i = 1$ ).

The availability of intra-day data allows the employment of volatility estimators that make use intraday information for more efficient volatility estimates. As mentioned in section 2, the dataset includes 48 30-minute intra-day data points per contract. Not all of them constitute transaction quotes, since for some hours during the day the contracts are not traded in the respective exchange or in the respective online trading platform<sup>8</sup>. As it has been mentioned, these entries are filled forward during the construction of the dataset in order to avoid potential lead-lag effects. Our purpose is to estimate running volatility at the end of each trading day, after trading in all exchanges has been terminated.

For that purpose, we employ six different methodologies that make use of intra-day information. We also generate for each contract and trading day four daily price series using this intra-day information, namely the opening, closing, high and low log-price series. Let  $N_{\rm day}(t)$  denote the number of active price quotes during the trading day t, hence the intra-day quotes are denoted by  $S_1(t)$ ,  $S_2(t)$ ,  $S_2(t)$ , ...,  $S_{N_{\rm day}(t)}$ . Then:

Opening price: 
$$O(t) = \log S_1(t)$$
 (6)

Closing price: 
$$C(t) = \log S_{N_{\text{day}}}(t)$$
 (7)

High price: 
$$H(t) = \log \left( \max_{j=1,\dots,N_{\text{day}}} S_j(t) \right)$$
 (8)

Low price: 
$$L(t) = \log \left( \min_{j=1,\dots,N_{\text{day}}} S_j(t) \right)$$
 (9)

Normalized Closing price: 
$$c(t) = C(t) - O(t) = log(S_{N_{day}}(t)/S_1(t))$$
 (10)

Normalized High price: 
$$h(t) = H(t) - O(t) = log\left(\max_{j=1,\dots,N_{\text{day}}} S_j(t)/S_1(t)\right)$$
 (11)

Normalized Low price: 
$$l(t) = L(t) - O(t) = log \left( \min_{j=1,\dots,N_{\text{day}}} S_j(t) / S_1(t) \right)$$
 (12)

Using the above definitions we describe below the six methodologies of interest.

#### • Realized Variance/Volatility (RV):

Andersen and Bollerslev (1998) and Barndorff-Nielsen and Shephard (2002) use the theory of quadratic variation, introduce the concept of integrated variance and show that the sum of squared high-frequency intra-day log-returns is an efficient estimator of daily variance in the absence of price jumps and serial correlation in the return series. In fact, theoretically, in the absence of market microstructure noise effects (lack of continuous trading, bid/ask spread, price discretization), the daily variance can be estimated arbitrarily well, as long as one can get ultra high-frequency data. However, the above effects swamp the estimation procedure and in the limit, microstructure noise dominates the result<sup>9</sup>. Among others, Hansen and Lunde (2006) show that microstructure effects start to significantly affect the accuracy of the estimation when the sampling interval of observations becomes smaller than 5 minutes. On the other hand, intervals between 5 to 30 minutes tend to give satisfactory volatility estimates, even if the variance of the estimation increases for lower

<sup>&</sup>lt;sup>8</sup>For example, the Wheat Futures contract is traded in the Chicago Mercantile Exchange (CME) trading floor from Monday to Friday between 9:30am and 1.15pm Central Time (CT) and in the electronic platform (CME Globex) from Monday to Friday between 9:30am and 1.15pm and between 6:00pm and 7:15pm CT. Instead, the Cocoa Futures contract is traded in the Intercontinental Exchange (ICE) between 4:00am and 2:00pm New York Time, and the Eurostoxx50 Index Futures contract is traded in Eurex between 07:50am to 10:00pm Central European Time (CET).

<sup>&</sup>lt;sup>9</sup>The research on high-frequency volatility estimation and the effects of microstructure noise is currently extremely active. Among others, see Aït-Sahalia, Mykland and Zhang (2005), Bandi and Russell (2006), Hansen and Lunde (2006), Bandi and Russell (2008), Andersen, Bollerslev and Meddahi (2011) and Bandi and Russell (2011).

frequencies.

Following the above, the availability of 30-minute quotes allows the estimation of the daily variance that is virtually free of microstructure frictions as:

$$\sigma_{\text{RV}}^{2}(t) = \sum_{j=2}^{N_{\text{day}}} \left[ \log S_{j}(t) - \log S_{j-1}(t) \right]^{2}.$$
(13)

• Parkinson (1980) estimator (PK):

Parkinson (1980) is the first to propose the use of intra-day high and low prices in order to estimate daily volatility as follows:

$$\sigma_{PK}^{2}(t) = \frac{1}{4\log 2} [h(t) - l(t)]^{2}. \tag{14}$$

This estimator assumes that the asset price follows a driftless diffusion process and is shown (Parkinson 1980) to be theoretically around 5 times more efficient than STDEV (Garman and Klass (1980) compute the efficiency with respect to STDEV to be 5.2 times larger).

• Garman and Klass (1980) estimator (GK):

Garman and Klass (1980) extend Parkinson's (1980) estimator and include opening and closing prices in an effort to increase the efficiency of the PK estimator. However, like the PK estimator, their estimator assumes that the asset price follows a driftless diffusion process and also does not take into account the opening jump. The GK estimator is given by:

$$\sigma_{\text{GK}}^{2}(t) = 0.511 \left[ h(t) - l(t) \right]^{2} - 0.019 \left\{ c(t) \left[ h(t) + l(t) \right] - 2h(t) l(t) \right\} - 0.383 c^{2}(t) \tag{15}$$

Garman and Klass (1980) show that the GK estimator is 7.4 times more efficient than STDEV. The authors also offer a computationally faster expression that eliminates the cross-product terms, but still achieves virtually the same efficiency:

$$\sigma_{GK}^{2}(t) = 0.5 \left[ h(t) - l(t) \right]^{2} - (2\log 2 - 1) c^{2}(t)$$
(16)

• Yang and Zhang (2000) modification of Garman and Klass (1980) estimator (GKYZ):

Yang and Zhang (2000) modify the GK estimator by incorporating the difference between the current opening log-price and the previous day's closing log-price. This estimator becomes robust to the opening jump, but still assumes a zero drift in the price process. The estimator is given by:

$$\sigma_{\text{GKYZ}}^{2}(t) = \sigma_{\text{GK}}^{2} + [O(t) - C(t-1)]^{2}$$
 (17)

• Rogers and Satchell (1991) estimator (RS):

Rogers and Satchell (1991) are the first to introduce an unbiased estimator that allows for a non-

zero drift in the price process. However, the RS estimator does not account for the opening jump. The estimator is given by:

$$\sigma_{RS}^{2}(t) = h(t) [h(t) - c(t)] + l(t) [l(t) - c(t)]$$
(18)

The RS estimator is not significantly worse in terms of efficiency when compared to the GK estimator. Rogers and Satchell (1991) show that GK is just 1.2 times more efficient than RS. Besides, Rogers, Satchell and Yoon (1994) show that the RS estimator can also efficiently deal with timevariation in the drift component of the price process.

The last five estimators, RV, PK, GK, GKYZ and RS provide daily estimates of variance/volatility. An annualised *D*-day estimator is therefore given by the average estimate over the past *D* days.

$$\sigma_{\text{meth}}^{2}(t;D) = \frac{261}{D} \sum_{i=0}^{D-1} \sigma_{\text{meth}}^{2}(t-i), \text{ where meth} = \{\text{RV, PK, GK, GKYZ, RS}\}.$$
 (19)

• Yang and Zhang (2000) estimator (YZ):

Yang and Zhang (2000) are the first to introduce an unbiased volatility estimator that is independent of both the opening jump and the drift of the price process. By construction, such an estimator has to have a multi-period specification. This estimator is a linear combination of the STDEV estimator, the RS estimator and an estimator in the nature of STDEV that uses opening prices instead of closing prices. The YZ estimator is given by:

$$\sigma_{YZ}^{2}(t;D) = \sigma_{OPEN}^{2}(t;D) + k\sigma_{STDEV}^{2}(t;D) + (1-k)\sigma_{RS}^{2}(t;D)$$
 (20)

where  $\sigma_{\text{OPEN}}^2(t;D) = \frac{261}{D} \sum_{i=0}^{D-1} \left[ O(t-i) - O(t-1-i) - \frac{1}{D} \sum_{i=0}^{D-1} \left[ O(t-i) - O(t-1-i) \right] \right]^2$  and k is chosen so that the variance of the estimator is minimised. Yang and Zhang (2000) show that this is in practice achieved for  $k = \frac{0.34}{1.34 + (D+1)/(D-1)}$ .

The YZ estimator can optimally achieve efficiency of around 14 for D=2 (i.e. a 2-day estimator) in comparison to STDEV. Throughout the paper, we use 30-day or 60-day estimates of volatility. The efficiency of the YZ estimator for these windows is around 8 and  $8.1^{10}$ .

Loosely speaking, the only estimator that uses high-frequency intra-day data is the RV<sup>11</sup>, whereas the remaining estimators (PK, GK, GKYZ, RS and YZ), also known as "range" estimators<sup>12</sup> only need

 $<sup>^{10}</sup>$ Yang and Zhang (2000) show that the efficiency of the YZ estimator in comparison to the STDEV estimator is given by  $\mathrm{Eff_{YZ}}=1+\frac{1}{k}$ . Hence, for D=30,  $k=\frac{0.34}{1.34+(D+1)/(D-1)}=0.14$  and consequently  $\mathrm{Eff_{YZ}}\approx 8.1$ . For D=60,  $\mathrm{Eff_{YZ}}\approx 8$ .

<sup>&</sup>lt;sup>11</sup>There exist several more high-frequency volatility estimators in the literature, most of which constitute improvements of the original RV estimator, in order to counteract potential market microstructure frictions, like for instance the Two-Scale RV (Zhang, Mykland and Aït-Sahalia 2005) and the Multi-Scale RV (Zhang 2006). These estimators however are designed for datasets with sampling intervals that go down to few minutes or even few seconds (these are the frequencies that microstructure effects are largely pronounced). Our 30-minute intra-day dataset is therefore inadequate for the employment of these techniques.

<sup>&</sup>lt;sup>12</sup>Martens and van Dijk (2007) follow the RV rationale and build a more efficient volatility estimator, the Realized Range ("RR") estimator, which instead of computing the sum of squared intra-day returns, it computes the sum of squared high-low

opening, closing, high and low price daily information. Strictly speaking though, the more high-frequent the dataset, the finer the discretization of the true price process and the more precise the estimation of the high and low prices. If anything, the discretization of a continuous price process will almost always lead to an estimate of the maximum (minimum) that resides below (above) the true maximum (minimum) of the continuous price path. Consequently, the approximated range h(t) - l(t) will always underestimate the true range and therefore the estimated volatility will be underestimated. See Rogers and Satchell (1991) for a discussion on this matter and an effort to bias-correct the RS and GK estimators.

On the other hand, the advantage of the range is that it can even successfully capture the high volatility of an erratically moving price path during a day that simply happens to exhibit similar opening and closing prices and therefore exhibits a low daily return (this applies for instance to the STDEV and EWMA estimators, but not to the RV estimator, because of its high-frequency nature). Furthermore, Alizadeh et al. (2002) show that the range-based volatility estimates are approximately Gaussian, whereas return-based volatility estimates are far from Gaussian, hence rendering the former estimators more appropriate for the calibration of stochastic volatility models using a Gaussian quasi-maximum likelihood procedure.

The above methodologies are first applied to the 12 futures contracts using a rolling window of D=60 trading days. The outcome is plotted in Figure 2 and serves as a visual inspection of the co-movement and the cross-sectional variation of the various estimators. The degree of co-movement appears to be large, which is also quantitatively certified by Panel A of Table 2, which presents the average correlation matrix of the volatility estimators across the 12 futures contracts.

[Insert Figure 2 here.]

[Insert Table 2 here.]

Nevertheless, there exists a great amount of cross-sectional variation in the absolute estimates of volatility, especially during the first half of the sample for some contracts (e.g. Cocoa, Dollar Index, Euro, Copper and T-note). In order to quantitatively assess the accuracy of the various estimators, the bias of the estimators is computed assuming that the true volatility process -given that we do not observe it- coincides with the RV estimator. The assumption that the RV estimator provides a good proxy of the volatility process is also made by Brandt and Kinlay (2005) and Shu and Zhang (2006), who present similar comparison studies for various volatility estimators. Panel B of Table 2 presents for each futures contract the annualised volatility bias computed as:

Bias = 
$$\frac{1}{2610 - D} \sum_{t=D}^{2610} \left[ \sigma_{RV}(t; D) - \sigma_{meth}(t; D) \right],$$
 (21)

where meth = {STDEV, EWMA, PK, GK, RS, GKYZ, YZ}, 2610 is the number of trading days in our sample and D is chosen to be 60 trading days (results for D = 30 are extremely similar).

ranges over the same intra-day intervals. Just as Parkinson's (1980) estimator (squared daily high-low range) improves the traditional STDEV estimator (squared daily returns), the RR estimator should theoretically improve the RV estimator. For the purposes of this paper, we cannot employ the RR estimator, because the 30-minute intra-day quotes do not allow the measurement of intra-day high-low ranges.

As it is expected, all five range estimators underestimate on average the RV estimator in all but three occasions (YZ for Cocoa, YZ and GKYZ for Eurostoxx50), while the two traditional estimators in all but two contracts (Eurodollar, Wheat) overestimate the RV estimator in line with the findings of Brandt and Kinlay (2005) and Shu and Zhang (2006). Since there exists a great amount of cross-sectional variation in the volatility level of the future contracts (see Figure 2), it would be inappropriate to compute the average bias of each estimator across all instruments. Instead, we sort the absolute biases per contract, hence assigning a rank score from 1 to 7 to each estimator per contract and then we average across contracts to deduce the last row of Panel B of Table 2. Clearly, the YZ estimator exhibits on average -and also for most contracts- the lowest absolute bias followed by EWMA, GKYZ and STDEV estimators. This result gives the YZ estimator a practical advantage that, in conjunction with its theoretical dominance, renders it the best candidate for the sizing of our momentum strategies.

From a trading perspective, it is always important to limit a portfolio's turnover. Lower turnover means that a smaller part of the portfolio composition changes at each rebalancing date, which, in turn, lowers the transaction costs that are incurred during rebalancing. This is arguably desirable for the investor. From equation (3), it is clear that an important determinant of the portfolio turnover is the asset volatility. In fact, the intertemporal change of the ratio  $\frac{1}{\sigma}$  along with the momentum trading signal jointly determine the portfolio turnover. Clearly, the more persistent the volatility process, the lower the resulting turnover for the momentum portfolio. Given the fact that the true volatility process is unknown and is only estimated using various methodologies, the persistence of the estimated path is solely dependent on the noise that is introduced by the estimation procedure, or equivalently on the efficiency of the estimator<sup>13</sup>. The more efficient the estimator, the less noisy or in other words the more persistent the estimated volatility path and therefore the lower the turnover. Hence, it is expected to see the most efficient estimator, the RV estimator, which makes use of high-frequency data, to generate the most persistent volatility estimates, followed by the range estimators that use intra-day information for high and low prices, with the worst performing estimators being those that only use daily information in closing prices, i.e. the STDEV and EWMA estimators.

In order to empirically assess the persistence of the volatility estimates, we compute for each estimator the following expression:

$$VTO = \frac{1}{2610 - (D+1)} \sum_{t=D+1}^{2610} \left| \frac{1}{\sigma(t;D)} - \frac{1}{\sigma(t-1;D)} \right|,$$
 (22)

which we call for convenience as the volatility turnover (VTO). Panel C of Table 2 presents the VTO for each futures contract. Arguably, the large cross-sectional variation in the volatility levels leads to a great variation in the VTO estimates for each contract. As in Panel B, the last row presents the average rank for each volatility estimators after ranking the VTO's for each contract. As it is expected, the RV estimator generates the most persistent volatility estimates and therefore the lowest turnover. Putting aside the RV estimator, the YZ estimator is, both on a contract-by-contract basis and on average, the estimator that generates the smoother volatility paths, hence achieving the lowest turnover and subsequently incurring

<sup>&</sup>lt;sup>13</sup>we thank Filip Zikes for this observation.

the lowest transaction costs. On the other hand, the traditional EWMA estimator generates one of the largest turnovers across all contracts. It is almost universally 1.5 times larger than that of YZ, hence casting doubts on its practical use due the increased transaction costs.

[Insert Figure 3 here.]

In a nutshell, after conducting a series of tests, it is concluded that the RV estimator is in general superior to the other estimators. The use of intra-day information gives the RV the advantage of larger efficiency, because all intra-day price movements are taken into account in the estimation procedure. Figure 3 summarises the ranks of the remaining volatility estimators from Table 2 in a bar diagram (due to the fact that the RV estimator is used as the baseline measure for the bias estimation and therefore does not have a bias rank, we decide to exclude the RV rank estimate for VTO as well in the figure; the VTO ranks for the remaining estimators are then recomputed). Excluding RV, the YZ estimator dominates the family of estimators and therefore is used throughout the paper for the construction of momentum strategies. The reasons for this choice are: (a) it is theoretically the most efficient estimator, (b) it exhibits the smallest bias when compared to the RV, (c) it generates the lowest turnover, hence minimising the costs of rebalancing the momentum portfolio and (d) it can be satisfactorily computed using daily information on opening, closing, high and low prices. One could argue that the use of the RV estimator throughout the paper would be optimal based on the above discussion, which is indeed a fair point. Instead, we choose to use the YZ estimator, because it is believed that this estimator constitutes an optimal tradeoff between efficiency, turnover and the necessity of proper high-frequency data. If anything, our results are more conservative and in any case the small bias among the YZ and RV estimator leads to very similar results for the performance of the momentum strategies<sup>14</sup>.

### 3.3. Momentum Signals

Five different methodologies are employed, in order to generate momentum trading signals. All methodologies focus on the asset performance during the lookback period [t-J,t].

**Return Sign** (SIGN): The standard measure of past performance in the momentum literature as in Moskowitz et al. (2011) is the sign of the J-period past return. A positive (negative) past return dictates a long (short) position:

$$SIGN(t - J, t) = \begin{cases} +1, & \text{if } R(t - J, t) > 0\\ -1, & \text{otherwise} \end{cases}$$
 (23)

**Moving Average** (MA): The moving average indicator has been extensively used by practitioners as a way to extract price trends. For the purposes of this study, a long (short) position is determined when the J-period lagging moving average of the price series lies below (above) a 1-period leading moving average of the price series. Let S(t) denote the price level of an instrument at time t,  $N_J(t)$  denote the number of

<sup>&</sup>lt;sup>14</sup>In undocumented results, we have used the RV estimator for the simulations and the conclusions remain both quantitatively and quantitatively the same.

trading days in the period [t-J,t] and  $A_J(t)$  denote the average price level during the same time period:

$$A_{J}(t) = \frac{1}{N_{J}(t)} \sum_{i=1}^{N_{J}(t)} S(t - N_{J}(t) + i).$$
(24)

Hence, the trading signal that is determined at time *t* is:

$$MA(t-J,t) = \begin{cases} +1, & \text{if } A_J(t) < A_1(t) \\ -1, & \text{otherwise} \end{cases}$$
 (25)

The idea behind the MA methodology is that when a short-term moving average of the price process lies above a longer-term average then the asset price exhibits an upward trend and therefore a momentum investor should take a long position. The reverse holds when the relationship between the averages changes. Clearly, this comparison of the long-term lagging MA with a short-term leading MA gives the MA methodology a market-timing feature<sup>15</sup> that the other signals of our paper do not have. The choice of the 1-period for the short-term horizon is justified, because it captures the most recent trend breaks. In a similar fashion, Yu and Chen (2011) study the cross-sectional momentum anomaly and try to maximise the performance by building portfolios based on the comparison between the geometric average rate of return during the past 12 months and during a shorter period of time; in fact, the authors show that the expost momentum returns are maximised when using a short period of 1 month. Lastly, Harris and Yilmaz (2009) apply the MA methodology in order to form time-series momentum strategies with currencies.

**EEMD Trend Extraction** (EEMD): This trading signal relies on some extraction of the price trend during the lookback period. In order to extract the trend from a price series, we choose to use a recent data-driven signal processing technique, known as the *Ensemble Empirical Mode Decomposition* (EEMD), which is introduced by Wu and Huang (2009) and constitutes an extension of the *Empirical Mode Decomposition* (Huang, Shen, Long, Wu, Shih, Zheng, Yen, Tung and Liu 1998, Huang et al. 1999). The EEMD methodology decomposes a time-series of observations into a finite number of oscillating components and a residual non-cyclical long-term trend of the original series, without virtually imposing any restrictions of stationarity or linearity upon application<sup>17</sup>.

Following the above, the stock price process can be written as the complete summation of an arbitrary

<sup>&</sup>lt;sup>15</sup>Han et al. (2011) apply the MA methodology to volatility-sorted decile portfolios in order to take advantage of this market-timing feature and subsequently to maximise the performance of these portfolios.

<sup>&</sup>lt;sup>16</sup>The Empirical Mode Decomposition (EMD) (Huang, Shen, Long, Wu, Shih, Zheng, Yen, Tung and Liu 1998, Huang, Shen and Long 1999) is the first step of an adaptive two-step decomposition transform that captures the instantaneous properties of a signal, known by the name of *Hilbert-Huang Transform* (HHT) and introduced by Huang, Shen, Long, Wu, Shih, Zheng, Yen, Tung and Liu (1998). Further information on the functionality of the HHT can be found in Huang et al. (1999), Rilling, Flandrin and Gonçalves (2003), Huang and Shen (2005), Kizhner, Blank, Flatley, Huang, Petrick, Hestnes, Center and Greenbelt (2006), Huang and Wu (2007), Huang and Wu (2008) and Rato, Ortigueira and Batista (2008)

<sup>&</sup>lt;sup>17</sup>Examples of EMD/EEMD application to non-stationary and nonlinear datasets include blood pressure (Huang, Shen, Huang and Fung 1998, Yeh, Lin, Shieh, Chen, Huang, Wu and Peng 2008), ocean waves (Huang et al. 1999), climate variations (Wu, Schneider, Hu and Cao 2001), heart rate analysis (Echeverría, Crowe, Woolfson and Hayes-Gill 2001), earthquake motion (Asce, Ma, Asce and Hartzell 2003), molecular dynamics (Phillips, Gledhill, Essex and Edge 2003), ocean acoustic data (Oonincx and Hermand 2004), solar cycles (Coughlin and Tung 2004), and electrocardiogram (ECG) denoising (Weng, Blanco-Velasco and Barner 2006).

number, n, of oscillating components  $c_i(t)$ , for  $i = 1, \dots, n$  and a residual long-term trend p(t),

$$S(t) = \sum_{i=1}^{n} c_i(t) + p(t)$$
 (26)

The focus is on the extracted trend p(t) and therefore an upward (downward) trend during the lookback period determines a long (short) position:

$$EEMD(t - J, t) = \begin{cases} +1, & \text{if } p(t) > p(t - J) \\ -1, & \text{otherwise} \end{cases}$$
 (27)

**Time-Trend t-statistic** (TREND): Another way to capture the trend of a price series is through fitting a linear trend on the *J*-period price series using least-square. The momentum signal can them be determined based on the significance of the slope coefficient of the fit. Assume the linear regression model:

$$\frac{S(i)}{S(t-N_J(t))} = \alpha + \beta \cdot i + \varepsilon(i), \quad i = 1, 2, \dots, N_J(t).$$
(28)

Estimating this model for the asset using all  $N_J(t)$  trading days of the lookback period yields an estimate of the time-trend, given by the slope coefficient  $\beta$ . The significance of the trend is determined by the t-statistic of  $\beta$ , denoted as  $t(\beta)$ , and the cutoff points for the long/short position of the trading signal are chosen to be  $\pm 2/-2$  respectively:

TREND 
$$(t - J, t) =$$

$$\begin{cases}
+1, & \text{if } t(\beta) > +2 \\
-1, & \text{if } t(\beta) < -2 \\
0, & \text{otherwise} 
\end{cases}$$
(29)

In order to account for potential autocorrelation and heteroscedasticity in the price process, Newey and West (1987) t-statistics are used. Lastly, notice that the normalization of the regressand in equation (28) is done for convenience, since it allows for cross-sectional comparison of the slope coefficient, when necessary; the t-statistic of  $\beta$  is of course unaffected by such scalings.

Statistically Meaningful Trend (SMT): Bryhn and Dimberg (2011) study the statistical significance of a linear trend and claim that if the number of data points is large, then a trend may be statistically significant even if the data points are very erratically scattered around the trend line. For that purpose, they introduce the term of *statistical meaningfulness* in order to describe a trend that not only exhibits statistical significance, but also describes the behaviour of the data to a certain degree. The authors therefore show that a trend is informative and strong if, except for a significant t-statistic (or equivalently a small p-value), the  $R^2$  of the linear regression exceeds 65%. Furthermore, they proceed one step further and for more robust inference they suggest splitting the dataset of the regression in a certain number of sub-intervals (usually between 3 to 30 intervals) and re-estimate (28) using as new data points the average values of the regressand and the regressors (asset price and linear trend respectively) over each subinterval. This method essentially provides some sort of pre-smoothing in the data before the extraction of the trend. They conclude that "...if one or several regressions concerning time and values in a time

series, or time and mean values from intervals into which the series has been divided yields  $R^2 \ge 0.65$  and  $p \le 0.05$ , then the time series is statistically meaningful", where p is the p-value of the slope coefficient. Along these lines, we follow the above methodology and split the lookback period in 4 to 10 intervals (i.e. 7 regressions per lookback period per asset) and decide upon a long/short position only if at least one of the regressions satisfies the above criteria. Thus:

$$SMT(t-J,t) = \begin{cases} +1, & \text{if } t_k(\beta) > +2 \text{ and } R_k^2 \ge 65\%, & \text{for some } k \\ -1, & \text{if } t_k(\beta) < -2 \text{ and } R_k^2 \ge 65\%, & \text{for some } k \\ 0, & \text{otherwise} \end{cases}$$
(30)

where k denotes the k<sup>th</sup> regression with  $k = 1, 2, \dots, 7$ . Notice that SMT and TREND constitute the only signals in the family of available methodologies that allow for inactivity in the momentum strategy, i.e. periods when no position is taken due to the nonexistence of a strong price trend and consequently a strong momentum pattern. Clearly, SMT is a stricter signal than TREND and therefore would lead to more periods of inactivity.

In order to assess the ability of the above five signals to capture a trending behaviour, they are first applied to the 12 futures contracts using a 12-month lookback period. For each asset, we construct the correlation table between the five resulting momentum signals and consequently the correlation matrices are averaged across all assets. The outcome is presented in Panel A of Table 3. The same panel also presents a similarly constructed matrix (average over the 12 futures contracts) that presents the number of time periods of agreement in the long/short position across all possible pairs of momentum signals divided by the number of active periods (i.e. either +1 or -1) of the signals across the vertical direction; for instance, the value 61.24 in the first row means that in almost 61% of the periods that the SIGN signal dictates a long or a short position, the SMT agrees (both in terms of trading activity and direction of trade).

#### [Insert Table 3 here.]

As expected, the pairwise correlations between the momentum signals are relatively large ranging between 0.68 up to 0.89. The smallest correlations with the rest of the signals are exhibited by MA and SMT; this was partly expected due to the ability to capture a trend break for the former and the strictness in the trend definition for the latter. Looking across the last row of the second table of Panel A, we realise that SMT, when active, almost fully agrees with all the rest of the signals. However, looking across the last column of the same table, it is observed that over the entire trading period, about 40% of the time, the signals SIGN/MA/EEMD capture a trend that SMT characterises as non statistically meaningful.

Panel B of Table 3 presents the percentage of time periods that each momentum signal dictates a long or a short position in each of the 12 futures contracts; note that by construction, the percentages of long and short positions sum up to one for the SIGN, MA and EEMD signals. On average, TREND generates activity for about 87% of the time and SMT for 63% of the time. This would give a practical advantage to the trend-related signals, since the sparse activity would lower the portfolio turnover, hence

the transaction costs. However, the sparse activity could potentially limit the ex-post portfolio mean return. For that reason, we next estimate for each contract and for each signal an *activity-to-turnover ratio*, which is called "signal speed" and is computed as the square root of the ratio between the time-series average of the squared signal value and the time-series average of the squared first-order difference in the signal value:

$$SPEED_{X} = \sqrt{\frac{\mathbb{E}[X^{2}]}{\mathbb{E}\left[(\Delta X)^{2}\right]}} = \sqrt{\frac{\frac{1}{T-J}\sum_{t=1}^{T}X^{2}(t-J,t)}{\frac{1}{T-J-1}\sum_{t=1}^{T}\left[X(t-J,t)-X(t-1-J,t-1)\right]^{2}}}.$$
 (31)

Clearly, the larger the signal activity and the smaller the average difference between consecutive signal values (in other words the smoother the transition between long and short positions), the larger the signal speed. Notice that for the SIGN, MA and EEMD signals the nominator is always equal to 1, because they constantly jump between long (+1) to short (-1) positions. Figure 4 presents the average speed of each signal across the 12 futures contracts for various lookback periods at daily (T=2610), weekly (T=520) and monthly (T=120) frequencies. The trend-related signals exhibit the largest activity-to-turnover ratio, drawing their advantage from the smoother transition between long and short positions, as there exist trading periods that these signals remain inactive, while the rest of the signals (SIGN, MA and EEMD) change erratically between +1 and -1. The worst performer is the MA signal which appears to be the most aggressively changing signal.

[Insert Figure 4 here.]

## 4. Time-Series Momentum Strategies

This section focuses on the evaluation of performance of time-series momentum strategies. This is first achieved by examining the time-series return predictability using a pooled panel regression analysis and consequently by constructing a series of momentum strategies on a grid of lookback and investment horizons. We first study the monthly momentum effects and subsequently investigate the weekly and daily frequencies of portfolio rebalancing.

## 4.1. Return Predictability

Before constructing momentum strategies, we first assess the amount of return predictability that is inherent in a series of predictors by running a pooled time-series cross-sectional regression of the contemporaneous standardized return on a lagged return predictor in line with Moskowitz et al. (2011):

$$\frac{R(t-1,t)}{\sigma_{YZ}(t-1;D)} = \alpha + \beta_{\lambda}Z(t-\lambda) + \varepsilon(t), \qquad (32)$$

<sup>&</sup>lt;sup>18</sup>we thank Yoav Git for sharing with us the practitioner's view regarding this measure.

where  $\lambda$  denotes the lag, D = 30 trading days and the regressor Z is chosen from a broad collection of momentum-related quantities:

$$Z(t) = \begin{cases} \frac{R(t-1,t)}{\sigma_{YZ}(t-1;D)}, & SIGN(t-1,t), EEMD(t-1,t), \\ \beta & \text{for } [t-1,t], \text{ t-stat } t(\beta) & \text{for } [t-1,t], \text{ TREND}(t-1,t), \end{cases}$$

$$SMT \beta & \text{for } [t-1,t], SMT \text{ t-stat } t_k(\beta) & \text{for } [t-1,t], SMT(t-1,t) \end{cases}. (33)$$

Notice that all possible choices are comparable across the various contracts and refer to a single period J=1, in order to avoid serial autocorrelation<sup>19</sup> in the error term of (32). Moreover, the second and third rows of regressor choices in (33) are solely related to the TREND and SMT methodologies respectively. Lastly, notice that the asset volatility, wherever used, is always lagged by one period in order to satisfy the real-life practice of risk-adjusting the asset position with the ex-ante volatility, as in equation (3).

The regression (32) is estimated for each lag and regressor by pooling all the futures contract together. Note that all regressor choices are normalized, in order to allow for the pooling across the instruments; the asset returns are normalized and the  $\beta$ 's have been estimated for normalized price paths in equation (28).

The quantity of interest in these regressions is the t-statistic of the coefficient  $\beta_{\lambda}$  for each lag. Large and significant t-statistics essentially support the hypothesis of time-series return predictability. Each regression stacks together  $T=120-\lambda$  monthly returns for each of the N=12 contracts therefore leading to  $1440-12\lambda$  data points. The t-statistics  $t(\beta_{\lambda})$  are computed using standard errors that are clustered by time and asset<sup>20</sup>, in order to account for potential cross-sectional dependence (correlation between contemporaneous returns of the contracts) or time-series dependence (serial correlation in the return series of each individual contract). Briefly, the variance-covariance matrix of the regression (32) is given by (see Cameron, Gelbach and Miller 2011, Thompson 2011):

$$V_{\text{TIME&ASSET}} = V_{\text{TIME}} + V_{\text{ASSET}} - V_{\text{WHITE}}, \tag{34}$$

where  $V_{\rm TIME}$  and  $V_{\rm ASSET}$  are the variance-covariance matrices of one-way clustering across time and asset respectively, and  $V_{\rm WHITE}$  is the White (1980) heteroscedasticity-robust OLS variance-covariance matrix. In fact, Petersen (2009) shows that when T >> N (N >> T) then standard errors computed via one-way clustering by time (by asset) are close to the two-way clustered standard errors; nevertheless, one-way clustering across the "wrong" dimension produces downward biased standard errors, hence inflating the resulting t-statistics and leading to over-rejection rates of the null hypothesis. We document that for

<sup>&</sup>lt;sup>19</sup>Notice that the regression choices do not include the MA signal, because by construction the MA signal compares a *J*-period average price level to the last period's average price level and therefore *J* must be larger than 1 for the MA signal to make sense. Choosing a larger *J* for this particular regressor would result in the error term of the regression (32) having an autoregressive structure. For that reason and for comparison purposes with the rest of the regressor choices, we refrain from reporting results for, say, Z(t) = MA(t-2,t), even if they are qualitatively very similar to those that are reported, due to the large commonality between the momentum signals.

<sup>&</sup>lt;sup>20</sup>Petersen (2009) and Gow, Ormazabal and Taylor (2010) study a series of empirical applications with panel datasets and recognise the importance of correcting for both forms of dependence.

our dataset where T > N two-way clustering or one-way clustering by time (i.e. estimating T cross-sectional regressions as in Fama and MacBeth (1973)) produces similar results, whereas clustering by asset produces inflated t-statistics that are similar to simple OLS t-statistics. One-way clustering by time is used by Moskowitz et al. (2011) in a similar setting of return predictability in futures markets.

#### [Insert Figure 5 here.]

Figure 5 presents for each one of the regressors Z, the two-way clustered t-statistics  $t\left(\beta_{\lambda}\right)$  for lags  $\lambda=1,2,\cdots,24$ . The t-statistics are almost always positive for the first twelve months for all regressor choices, hence indicating momentum patterns. However, exactly after the first year there exist strong signs of return reversal that subsequently attenuate and only seem to gain some significance for a lag of two years. The first two plots of the first column, where the regressor is the past standardized return and the sign of it are essentially a replication of the methodology in Moskowitz et al. (2011); the similarities between this figure and the respective figure in Moskowitz et al. (2011) are large, even if the t-statistics are generally larger in the latter case, the reason probably being that the dataset of Moskowitz et al. (2011) includes 58 futures contract over a 45-year period, whereas ours is substantially smaller, consisting of 12 futures contracts over a 10-year period.

The similarity of all plots of Figure 5 offers an additional piece of evidence on the commonality of all regressor choices to capture trending activity, in line with Panel A of Table 3. Focusing exclusively on the ability of the trading signals SIGN, EEMD, TREND and SMT to capture return continuation, it is observed that the first two exhibit stronger patterns for the most recent four months, whereas SMT has a more widespread ability to capture return continuation, which becomes stronger during the farthest half of the most recent 12-month period. This observation leads us to expect the momentum strategies with shorter lookback periods to be more profitable with the SIGN or EEMD signals, whereas the strategies with longer lookback periods to be more profitable with trend-related trading signals.

Lastly, it must be noted that part of this severe transition from a the largest positive t-statistic to the largest negative t-statistic after the lag of 12 months can be potentially attributed to seasonal patterns in the futures returns<sup>21</sup>. In undocumented results, we repeat the pooled panel regression after removing 4 contracts from the dataset that for various reasons might exhibit seasonality: the agricultural contracts (Cocoa, Wheat) and the energy-related contracts (Crude Oil, Gas). In general the patterns become relatively less pronounced, but our conclusions remain qualitatively the same and the momentum/reversal transition is still apparent.

## 4.2. Momentum Profitability

Having established the return predicability in futures markets, we proceed with the construction of timeseries momentum strategies at the monthly frequency for a grid of lookback (J) and investment periods (K) both measured in months and for all five different trading signals: SIGN, EEMD, MA, TREND

<sup>&</sup>lt;sup>21</sup>we thank Yoav Git for bringing that to our attention.

and SMT. The return of the aggregate time-series momentum strategy over the investment horizon is the volatility-adjusted weighted average of the individual time-series momentum strategies and is computed using equation (3), which is repeated below for convenience:

$$R^{TS}(t,t+K) = \sum_{i=1}^{M} X_i(t-J,t) \cdot \frac{10\%/\sqrt{M}}{\sigma_i(t,D)} \cdot R_i(t,t+K),$$

where M = 12 is the number of the futures contracts and  $\sigma_i(t;D)$  is chosen to be the YZ ex-ante volatility estimate for each contract i using a rolling window of D = 30 days. Instead of forming a new momentum portfolio every K months, when the previous portfolio is unwound, we follow the overlapping methodology of Jegadeesh and Titman (2001) and perform portfolio rebalancing at the last trading day of each month. The monthly return is then computed as the equally-weighted average across the K active portfolios during the month of interest<sup>22</sup>.

Table 4 presents in five panels -each for different momentum trading signal- various out-of-sample performance statistics for the (J,K) time-series momentum strategy with  $K,J = \{1,3,6,12,24\}$  months. The statistics are all annualised and include the mean portfolio return along with the respective Newey and West (1987) t-statistic, the portfolio volatility, the dollar growth, the Sharpe ration and the downsiderise Sharpe ratio (Ziemba 2005). For each panel and statistic, we present the largest value in bold and, especially, for the t-statistic we present in italic the t-statistics that generate p-values larger than 10% (hence, insignificant at the 10% level).

There appear several commonalities across panels, i.e. trading signals, but there also exist some interesting features that are next pointed out.

[Insert Table 4 here.]

[Insert Table 5 here.]

First, it is apparent that the time-series momentum strategy generates a statistically and economically significant mean return, especially, when both the lookback and holding periods are at most equal to 12 months. Except for very few occasions the significance is strong at the 1% significance level. In terms of mean return, it is maximised for a lookback period of 6 to 12 months and a holding horizon of 1 to 3 months, depending on the trading signal of interest, ranging from 19.34% for the SIGN signal up to the impressive 28.36% for the SMT signal. These conclusions are in line with Moskowitz et al. (2011), who base their empirical results solely on the SIGN signal and document similar windows of time-series momentum significance with their largest t-statistic being observed for the (12,1) strategy. Regarding the ex-post volatility of the time-series momentum strategies, there exists a general pattern of decreasing volatility for longer holding periods, and shorter lookback periods. The effects for the former pattern are

 $<sup>^{22}</sup>$ For example if K=3, at the end of January the Jan-Feb-Mar portfolio (built at the beginning of January) has been active for one month, the Dec-Jan-Feb portfolio has one more month to be held and the Nov-Dec-Jan portfolio is unwound and its place is taken by the newly constructed Feb-Mar-Apr. Hence, the January return is measured as the equally weighted average of the returns of the three portfolios Jan-Feb-Mar, Dec-Jan-Feb and Nov-Dec-Jan.

more pronounced for shorter lookback periods and for the latter pattern are more pronounced for longer holding periods. This is apparent from Figure 8, which is discussed later on in the next subsection.

Focusing on the most notable differences among the performance of the momentum strategies across the trading signals and in particular across the trend-related signals (TREND and SMT) and the other signals (SIGN, MA, EEMD), note that the former aim to capture only significant price trends and therefore, as shown in Panel B of Table 3, refrain from instructing trading activity in transient periods of trend reversals. In fact, SMT is a stricter version of TREND and generates trading activity only for the 63% of time in our sample using a 12-month lookback period (87% for TREND and by construction 100% for the rest). This sparse activity results in increased volatility for momentum strategies based on the TREND and SMT signals and consequently in limiting the resulting SR to similar values as for the rest of the signals. However, this "additional volatility" appears to be generated by successful momentum bets therefore leading to substantially larger mean return and a more positively-skewed return distributions<sup>23</sup>. The latter effect is captured by the DR-SR measure, which exceeds the value of 2 for the (3,3) strategy reaching for the SMT signal the value 2.33, which constitutes the largest value across all strategies of Table 3.

In order to shed light on the sources of the aggregate time-series momentum profitability, Figure 6 presents the t-statistic and the ex-post Sharpe ratio for the univariate (6,1) and (12,1) strategies for all trading signals. Except three and two occasions respectively, the time-series momentum patterns are apparent -though not always strongly significant- in the univariate strategies as well, hence further supporting the evidence in Moskowitz et al. (2011).

#### [Insert Figure 6 here.]

Further support to the dominance of the trend-related signals is offered by the growth of \$1 invested in a (6,1) time-series momentum strategy in April 2000; using the SMT signal \$1 grows to \$11.10 in October 2009, to \$7.92 using the TREND signal and to just \$4.02 using the SIGN signal. In order to visually inspect the wealth accumulation, Figure 7 presents the growth path of the \$1 for all five (6,1) strategies. Clearly, the SIGN signal generates the worst path and lies at all times below every other momentum strategy. On the other hand, the trend-related signals dominate with the SMT signal achieving almost three times larger final wealth compared to the SIGN signal.

## [Insert Figure 7 here.]

An interesting feature is revealed for the largest lookback period, that of 24 months, for which the MA and SMT signals do still capture momentum patterns. In particular, the (24,1), (24,3) and (24,6) strategies based on the MA signal generate relatively large and significant mean returns. The effects are weaker and remain relatively significant for the SMT signal, but the rest of the signals generate mostly insignificant mean returns. For example, the (24,1) strategy using the MA signal achieves an annualised

 $<sup>^{23}</sup>$ For instance the (12,1) strategy has a skewness of 0.36 for SIGN and 0.81 for SMT, while the (6,1) has a skewness of 0.35 and 0.79 respectively.

mean return of 15.72%, significant at the 1% level, with a SR of 0.97; the same strategy for the SMT signals generates a mean return of 14.96%, significant at the 5% level, with a SR of 0.76. All the other signals cannot generate significant returns, and even the point estimate of the mean annualised return does not exceed 7%.

#### 4.3. Investment Implications

Interpreting the above findings from an investment perspective, there exists a clear indication that the trend-related signals and especially the SMT signal succeed in appropriately filtering strong return continuation and consequently generating superior and strongly significant out-of-sample momentum performance in terms of mean return, DR-SR and dollar growth, even with trading activity that is substantially less frequent compared to standard trading signals like the SIGN or MA signals.

## [Insert Table 6 here.]

In order to further support this conclusion, the turnover of all strategies is presented in Table 5 and it is expected that the trend-related signals significantly lower the portfolio turnover and subsequently any transaction costs. The turnover of the momentum portfolio is computed as the equally weighted average of the turnover of all univariate time-series strategies. The turnover of a univariate strategy is measured as the percentage change in the number of open positions in the underlying futures contract, after ignoring positions that mutually cancel each other<sup>24</sup>. The evidence shows that the trend-related signals do indeed generate the lowest turnover among all signals, especially for holding horizons between 1 to 6 months, which are the most profitable periods of time-series momentum strategies. The remaining signals (SIGN, EEMD, MA) have in general very similar turnover estimates to each other, which are almost always twice as large as those for SMT for the afore mentioned holding horizons. For instance, the (6,1) strategy generates 19.3% turnover when the SMT signal is used and 44.0% when the SIGN signal is used. Remember that for the same strategy the two signals exhibit annualised mean returns of 28.36% and 19.34% respectively.

Manifestly, not only does the SMT signal offer a means to superior momentum profitability, but it also achieves so with the least amount of transaction costs. Regarding the general turnover pattern, it is observed that in general the longer the lookback period and the shorter the holding horizon the lower the turnover of the portfolio. This is expected, because longer lookback periods offer a slower-moving characterisation of the intertemporal performance of the assets, therefore generating lower changes in the momentum portfolio. Similarly, shorter holding horizons offer the flexibility of maintaining a certain momentum position only for the period that the momentum pattern is strong.

#### [Insert Figure 8 here.]

<sup>&</sup>lt;sup>24</sup>If, for instance, at a given month there exist four long positions and two short positions on the contract, we ultimately have two open positions.

It is important at this point to emphasise that the interpretation of the turnover results has to be done with the appropriate caution. The fact that the trend-related signals generate lower turnover is partly due to the fact that, by construction, these signals only instruct trading at times when a significant trend is identified. Hence, a direct comparison in the turnover among the trend-related signals and the rest of signals is ex-ante biased towards the former. One could argue that SIGN signal can be altered, so that it is only instructing investment when the past return is larger/smaller than a certain positive/negative percentile of the ex-ante return distribution. Similar methodologies could be also devised for the EEMD and MA signals, but the key point is that these cutoff percentiles have to be ad-hoc chosen and should be based on historical information, therefore rendering these approaches not as straightforward as the TREND and SMT methodologies.

Lastly, focusing on the SMT signal and given its superiority across various directions, the methodology of Table 4 is repeated for lookback and holding periods ranging between 1 and 24 months. Figure 8 plots the various out-of-sample performance statistics across these two time dimensions. All the identified patterns that have been discussed so far in the previous and current subsections are visually apparent.

#### 4.4. Towards Weekly and Daily Effects

Having established the strong momentum patterns at the monthly frequency, we then study higher frequencies of portfolio rebalancing and in particular we explore momentum profitability at weekly and daily frequencies. For brevity, only the results for three momentum signals, SIGN, MA and SMT, are reported, since the EEMD signal behaves very similarly to the SIGN signal and the TREND signal behaves very similarly to the SMT signal.

Regarding the **weekly frequency**, the dataset consists of 520 weeks. Using the methodology of the time-series momentum strategy, we evaluate the performance of various (J,K) momentum strategies at the end of each Wednesday, where J and K are now measured in weeks. Table 6 shows that momentum patterns do exist at the weekly frequency. Across all three signals, the mean return is economically and statistically significant almost for every pair (J,K), except for the shortest lookback and holding periods of 1 to 2 weeks. The SIGN signal does capture large momentum patterns even for lookback and holding periods that range up to 8 weeks. The MA signal fails to follow across K=8, whereas the SMT signal fails to follow across J=8. Except for very few occasions, the significance is strong at the 1% level.

## [Insert Table 7 here.]

The mean return is in general maximised for a lookback period of 3 to 4 weeks and a holding period of 1 to 2 weeks. The SIGN signal achieves an annualised return of 15.84% for the (4,1) strategy, with a SR of 0.95 and a dollar growth of \$4.19 at the end of the 10-year period of our dataset. The SMT signal offers again the best performance across the signals, which is maximised for the (3,1) strategy, reaching 19.99% and a dollar growth of \$5.60. Overall, the SMT signal remains the dominant performer, offering the largest mean return (at times even twice as large as that of the SIGN signal) and SR for most of the cases, except for the strategies with an 8-week lookback period and a holding horizon of 1 to 3 weeks.

This latter effect was partly expected from the monthly momentum results, as the above strategies loosely resemble -except for the feature of weekly rebalancing- a monthly strategy with a 2-month lookback period and a 1-month holding period, i.e. in the region that the SMT signal performs poorly (see for instance the (1,1) monthly strategy in Table 4).

Lastly, Table 7 presents the performance statistics for time-series momentum strategies with **daily frequency** of rebalancing (*J* and *K* are now measured in days). A number of new patterns arise. First, there are very slight signs of daily momentum, which are interestingly scattered in various regions depending on the trading signal. Using the SIGN signal, significant -even at 1% significance level- momentum patterns are identified when the lookback period is equal to 30 days and the holding horizon is 1, 3 or 5 days. In contrast, using the SMT signal, momentum patterns are identified for shorter lookback periods that range between 3 to 15 days and longer holding horizons of 15 or 30 days. Lastly, the MA signal seems to loosely behave like the SMT signal, but it is indeed the weakest momentum signal for daily strategies leading on average to insignificant ex-post performance. Contrary to the monthly and weekly frequencies of portfolio rebalancing, it is not the SMT signal, but the SIGN signal the one that offers the largest mean return throughout the grid of daily strategies, equal to 13.75% and a SR of 0.83 for the (30,1) strategy. However, the SMT signal still generates better performance than the SIGN signal in terms of mean return in 21 out of 30 strategies in the grid, but the differences are relatively smaller compared to the monthly and weekly results and consequently the SMT strategies are penalized in the SR statistic due to the increased -by construction- ex-post volatility.

#### [Insert Table 8 here.]

Second and most importantly, the daily strategies reveal an important feature of time-series strategies, that of the very short-term reversal. In particular the (3,1) generates a statistically significant -at the 5% level for the SMT signal and at the 10% level for the SIGN and MA signals- negative return that reaches even -13.38% annualised for the SMT signal, resulting in an almost 80% decrease of the initial investment at the beginning of our 10-year period. This is the only instance across all trading signals and rebalancing frequencies that a statistically and economically significant reversal pattern is documented.

## 5. Concluding Remarks

The time-series momentum strategy refers to the trading strategy that results from the aggregation of various univariate momentum strategies on a volatility-adjusted basis. This paper builds on recent work by Moskowitz et al. (2011) that focuses on the profitability of time-series momentum strategies in futures markets and extends it by exploring monthly, weekly and daily frequencies of portfolio rebalancing for a broad grid of lookback and holding periods. The availability of a unique dataset of intra-day quotes for 12 futures contracts allows the investigation of two important aspects of the strategy that have not been studied in detail in the past, namely the efficiency of the volatility estimation procedure that is crucial for the aggregation of the univariate strategies and the trading signal that is used to build the univariate strategies.

The main findings of the paper are next summarized. First, a novel overview of momentum trading signals is presented and the results show that the information content of the price path throughout the lookback period can be used to provide more descriptive indicators of the intertemporal price trends and therefore to avoid eminent price reversals. Time-series momentum portfolios that are based on trend-related signals dominate -in terms of ex-post mean return, dollar growth, positive skewness and turnover-similar portfolios based on the traditional momentum signal, the sign of past return. Second, for a broad grid of lookback and holding periods, we document strong momentum patterns at the monthly frequency that partly reverse after the first year of investment, relatively strong momentum patterns at the weekly frequency for up to 8 weeks of investment and scarce momentum patterns at the daily frequency. In fact, significant reversal effects are documented for the momentum portfolio that is built using information from the last 3 days and held only for the following trading day. Finally, it is empirically shown that the volatility adjustment of the constituents of the time-series momentum is critical for the resulting portfolio turnover. The use of more efficient estimators like the Yang and Zhang (2000) range estimator can substantially reduce the portfolio turnover and consequently the transaction costs for the construction and rebalancing of the portfolio.

Time-series momentum profitability implies strong autocorrelation in the individual return series of the contracts. From a theoretical perspective, this finding poses a substantial challenge to the random walk hypothesis and the market efficiency. The objective of this study is not to explain which mechanism is at work, but the fact that the source of this predictability is merely a single-firm effect relates the empirical findings of the paper to two strands of literature, namely the rational and behavioural models that endogenise serial autocorrelation in the return series of an asset. For instance, Berk et al. (1999), Chordia and Shivakumar (2002) and Johnson (2002) justify the existence of time-series return predictability and consequently momentum profitability as compensation for bearing time-varying risk. Instead, Barberis et al. (1998), Daniel et al. (1998) and Hong and Stein (1999) develop theories of investor irrationality and attribute time-series return predictability to cognitive biases that affect investment decisions. Given the existence of this broad range of rational and behavioural attempts to explain the momentum patterns, the need for a unified theoretical explanation remains a fertile ground for future research.

From an investment perspective, the findings of the paper are clearly interesting. Future research on the appropriate sizing of the univariate time-series momentum strategies, instead of ordinary volatility-adjusted aggregation, appears fruitful and challenging. Similarly challenging appears to be the part of research that could optimally combine various frequencies of portfolio rebalancing as this would closer mimic investor's behaviour.

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Panel A: Usin	Panel A: Using Monthly Return Series [N=120 observations]												
	Mean (%)	NW t-stat	Vol. (%)	Skewness	Kurtosis	JB(p)	LF(p)	t-DoF	LB(p)	ARCH(p)	SR	DR-SR	
Cocoa	11.26	1.65	27.74	0.25	3.19	0.41	0.00	16.65	0.02	0.09	0.41	0.47	
Crude Oil	8.24	0.86	23.17	-0.40	6.62	0.00	0.32	4.95	0.00	0.00	0.36	0.36	
Dollar Index	-2.71	-0.94	8.34	0.26	4.22	0.02	0.18	6.05	0.25	0.43	-0.33	-0.31	
Euro	3.69	1.06	10.56	-0.05	4.47	0.01	0.08	4.26	0.48	0.46	0.35	0.38	
Eurodollar	0.94	2.71	0.92	2.25	10.66	0.00	0.00	2.35	0.08	0.84	1.03	2.07	
S&P500	-3.20	-0.57	15.32	-0.63	4.28	0.01	0.00	5.56	0.01	0.00	-0.21	-0.19	
Gold	9.00	2.33	14.15	-0.27	5.64	0.00	0.22	5.31	0.15	0.00	0.64	0.73	
Copper	36.75	1.95	60.32	1.06	5.92	0.00	0.00	2.82	0.04	0.06	0.61	0.83	
Gas	-8.37	-0.98	27.52	0.36	5.07	0.00	0.08	4.21	0.67	0.02	-0.31	-0.29	
T-Note 10Y	6.37	2.75	8.05	0.14	4.19	0.03	0.26	7.75	0.72	0.67	0.79	0.96	
Wheat	-4.19	-0.68	19.47	-0.05	5.24	0.00	0.00	2.31	0.89	0.00	-0.22	-0.20	
Eurostoxx50	-4.61	-0.56	23.24	-0.25	3.18	0.41	0.09	19.71	0.22	0.19	-0.20	-0.18	
Panel B: Using Weekly Return Series [N=522 observations]													
	Mean (%)	NW t-stat	Vol. (%)	Skewness	Kurtosis	JB(p)	LF(p)	t-DoF	LB(p)	ARCH(p)	SR	DR-SR	
Cocoa	10.85	1.43	25.79	0.08	3.93	0.00	0.04	6.60	0.33	0.05	0.42	0.44	
Crude Oil	8.20	1.02	23.19	-0.20	6.09	0.00	0.00	4.11	0.65	0.00	0.35	0.36	
Dollar Index	-2.68	-0.97	8.44	-0.30	6.09	0.00	0.21	8.43	0.66	0.05	-0.32	-0.30	
Euro	3.70	1.08	10.74	0.57	8.17	0.00	0.05	6.95	0.94	0.09	0.34	0.37	
Eurodollar	0.94	3.19	0.77	1.09	10.48	0.00	0.00	2.28	0.00	0.00	1.22	1.60	
S&P500	-2.70	-0.49	17.92	-0.60	8.46	0.00	0.00	3.52	0.44	0.00	-0.15	-0.14	
Gold	9.14	1.97	16.41	-0.01	10.84	0.00	0.00	3.06	0.44	0.00	0.56	0.59	
Copper	32.49	2.06	51.58	1.76	17.56	0.00	0.00	4.35	0.07	0.13	0.63	0.75	
Gas	-8.62	-1.10	26.25	0.51	8.90	0.00	0.00	2.75	0.04	0.06	-0.33	-0.32	
T-Note 10Y	6.30	2.46	7.91	-0.10	3.76	0.01	0.23	9.91	0.68	0.05	0.80	0.86	
Wheat	-3.81	-0.56	21.18	0.71	9.70	0.00	0.00	2.35	0.56	0.00	-0.18	-0.19	
Eurostoxx50	-3.54	-0.46	27.17	0.15	7.88	0.00	0.00	4.45	0.00	0.00	-0.13	-0.13	
Panel C: Using	g Daily Retur	n Series [N=2	2610 observ	ations]									
	Mean (%)	NW t-stat	Vol. (%)	Skewness	Kurtosis	JB(p)	LF(p)	t-DoF	LB(p)	ARCH(p)	SR	DR-SR	
Cocoa	10.63	1.29	26.07	-0.28	6.30	0.00	0.00	3.71	0.50	0.00	0.41	0.41	
Crude Oil	8.21	1.14	24.05	-0.20	6.79	0.00	0.00	3.59	0.02	0.00	0.34	0.34	
Dollar Index	-2.59	-1.00	8.45	-0.17	4.34	0.00	0.00	6.96	0.41	0.00	-0.31	-0.30	
Euro	3.57	1.11	10.41	0.11	4.65	0.00	0.00	5.69	0.49	0.00	0.34	0.35	
Eurodollar	0.91	3.62	0.77	0.53	13.28	0.00	0.00	1.62	0.00	0.00	1.18	1.31	
S&P500	-1.98	-0.36	21.36	0.53	17.77	0.00	0.00	2.52	0.00	0.00	-0.09	-0.09	
Gold	9.00	1.84	16.14	0.14	13.87	0.00	0.00	2.57	0.48	0.00	0.56	0.57	
Copper	31.16	2.13	48.64	0.46	7.53	0.00	0.00	3.88	0.12	0.00	0.64	0.68	
Gas	-8.59	-1.12	25.26	0.42	8.33	0.00	0.00	2.44	0.00	0.00	-0.34	-0.34	
T-Note 10Y	6.17	2.57	8.21	-0.07	5.03	0.00	0.00	5.67	0.18	0.00	0.75	0.77	
Wheat	-3.80	-0.61	20.49	-0.07	8.64	0.00	0.00	2.07	0.49	0.00	-0.19	-0.18	
Eurostoxx50	-2.43	-0.31	30.62	0.17	7.05	0.00	0.00	3.34	0.00	0.00	-0.08	-0.08	

**Table 1:** Summary Statistics for Futures Contracts

The table presents summary statistics for the 12 futures contracts of the dataset using monthly (120 observations; Panel A), weekly (522 observations; Panel B) and daily (2610 observations; Panel C) return series. The statistics are: mean return, Newey-West t-statistic, volatility, skewness, kurtosis, Jarque-Bera p-value ( $H_0$ : normal distribution), Lilliefors p-value ( $H_0$ : normal distribution), ML-estimated degrees of freedom for a Student-t distribution, Ljung-Box p-value ( $H_0$ : no first-order autocorrelation), p-value of Engle's ARCH test for heteroscedasticity ( $H_0$ : no conditional heteroscedasticity), Sharpe ratio (SR) and downside risk Sharpe ration (DR-SR) by Ziemba (2005). The mean return, the volatility and the Sharpe ratios are expressed in annual terms (using appropriate scaling) for comparison purposes. The dataset covers the period November 1, 1999 to October 30, 2009.

_	RV	STDEV	EWMA	PK	GK	RS	GKYZ	YZ
RV	1.000							
STDEV	0.937	1.000						
EWMA	0.932	0.994	1.000					
PK	0.986	0.923	0.918	1.000				
GK	0.983	0.899	0.895	0.995	1.000			
RS	0.978	0.886	0.881	0.988	0.998	1.000		
GKYZ	0.934	0.972	0.965	0.907	0.898	0.894	1.000	
YZ	0.934	0.973	0.966	0.907	0.896	0.893	1.000	1.00
Panel B: Avera	ige Bias a	ssuming RV	is the true	volatility				
	RV	STDEV	EWMA	PK	GK	RS	GKYZ	YZ
Cocoa	0.00	5.87	5.38	-3.86	-6.17	-6.90	-0.51	0.0
Crude Oil	0.00	2.13	1.70	-2.83	-4.32	-4.85	-2.40	-2.1
Dollar Index	0.00	0.62	0.47	-1.44	-1.89	-2.04	-0.63	-0.5
Euro	0.00	1.45	1.27	-1.18	-1.79	-2.03	-0.14	-0.0
Eurodollar	0.00	-0.01	-0.02	-0.16	-0.21	-0.23	-0.14	-0.1
S&P500	0.00	1.10	0.76	-2.25	-3.44	-3.83	-2.98	-2.6
Gold	0.00	0.86	0.58	-1.98	-2.90	-3.29	-1.82	-1.6
Copper	0.00	6.19	5.36	-6.96	-9.96	-10.91	-2.16	-1.4
Gas	0.00	3.06	2.54	-2.73	-4.39	-5.12	-2.14	-1.8
T-Note 10Y	0.00	0.69	0.55	-1.12	-1.70	-1.88	-1.04	-0.8
Wheat	0.00	-0.33	-0.69	-7.57	-9.65	-10.58	-1.59	-1.4
Eurostoxx50	0.00	7.71	7.20	-2.76	-4.70	-5.18	1.97	2.5
Avg. RANK	-	3.33	2.42	4.25	5.83	6.83	3.17	2.1
Panel C: Avera	$\log \left  \Delta \left( \frac{1}{\sigma} \right) \right $							
	RV	STDEV	EWMA	PK	GK	RS	GKYZ	Y
Cocoa	4.64	5.41	6.74	7.42	7.91	9.69	5.45	5.2
Crude Oil	3.91	6.10	7.73	6.09	6.57	8.18	5.42	5.3
Dollar Index	8.96	15.18	19.28	14.92	15.09	17.04	12.05	11.9
Euro	7.69	12.75	16.02	12.43	12.69	14.39	10.09	10.0
Eurodollar	176.49	362.01	430.50	344.58	354.88	375.79	285.62	279.9
S&P500	5.20	9.06	11.33	7.77	7.89	8.78	7.67	7.6
Gold	8.47	11.52	14.41	13.19	13.31	14.81	9.78	9.7
Copper	1.94	2.98	3.76	3.03	3.13	3.57	2.56	2.5
Gas	6.94	8.32	10.22	9.87	10.87	13.11	8.21	8.0
T-Note 10Y	10.48	16.57	20.80	15.08	15.62	18.40	13.77	13.7
Wheat	9.80	9.96	12.27	2.72	2.89	3.28	9.94	9.8
Eurostoxx50	4.32	5.28	6.59	6.64	6.85	8.16	4.70	4.6

 Table 2: Volatility Estimators

The table presents in Panel A the average (across the 12 futures contracts) correlation matrix of eight different volatility estimators: (a) Realized Volatility (RV), (b) standard deviation of past returns (STDEV), (c) exponentially-weighted average of past squared returns (EWMA), (d) Parkinson (1980) estimator (PK), (e) Garman and Klass (1980) estimator (GK), (f) Roger and Satchell (1991) estimator (RS), (g) Garman and Klass estimator adjusted by Yang and Zhang to allow for opening jumps (GKYZ) and (h) Yang and Zhang (2000) estimator (YZ). The estimation period is a rolling window of 60 trading days. The RV is estimated using intra-day 30min returns, whereas all the rest estimators use daily data for opening, closing, high and low futures prices. Panel B presents the average bias of all estimators for all contracts, assuming that the RV is the true volatility. The last row presents the average rank of each estimator across the futures contracts in terms of absolute bias (1: BEST, 7: WORST). Panel C presents the average change of the ratio  $\frac{1}{\sigma}$  for all futures contracts and volatility estimators. The last row presents the average rank of each estimator across the futures contracts in terms of VTO (1: BEST, 8: WORST). The dataset covers the period November 1, 1999 to October 30, 2009.

Panel A: Momentum Signal Correlation and Position Agreement (%) Matrices **SIGN** MA **EEMD TREND** SMT **SIGN** MA **EEMD TREND** SMT SIGN 1.00 86.31 90.75 82.19 61.24 MA 0.68 1.00 86.31 87.00 79.43 60.55 **EEMD** 0.78 0.69 1.00 87.00 85.93 62.69 90.75 **TREND** 0.79 0.73 0.89 1.00 94.38 91.32 98.84 72.12 0.75 0.821.00 99.89 SMT 0.71 0.69 97.23 96.20 99.78

Panel B: Percentage of Long and Short Positions

		]	Long Activ	ity		Short Activity						
	SIGN	MA	EEMD	TREND	SMT		SIGN	MA	EEMD	TREND	SMT	
Cocoa	56.88	55.05	57.80	48.62	28.44		43.12	44.95	42.20	33.94	12.84	
Crude Oil	64.22	67.89	66.97	59.63	51.38		35.78	32.11	33.03	25.69	17.43	
Dollar Index	32.11	31.19	32.11	28.44	17.43		67.89	68.81	67.89	63.30	47.71	
Euro	68.81	68.81	70.64	62.39	46.79		31.19	31.19	29.36	26.61	15.60	
Eurodollar	65.14	63.30	65.14	59.63	51.38		34.86	36.70	34.86	30.28	23.85	
S&P500	49.54	47.71	52.29	48.62	34.86		50.46	52.29	47.71	45.87	37.61	
Gold	77.98	78.90	79.82	68.81	53.21		22.02	21.10	20.18	13.76	11.01	
Copper	57.80	65.14	61.47	53.21	41.28		42.20	34.86	38.53	25.69	14.68	
Gas	35.78	33.94	36.70	30.28	23.85		64.22	66.06	63.30	54.13	42.20	
T-Note 10Y	83.49	80.73	83.49	76.15	48.62		16.51	19.27	16.51	11.01	5.50	
Wheat	34.86	33.94	39.45	33.03	18.35		65.14	66.06	60.55	58.72	45.87	
Eurostoxx50	39.45	40.37	39.45	34.86	25.69		60.55	59.63	60.55	50.46	38.53	
Average	55.50	55.58	57.11	50.31	36.77		44.50	44.42	42.89	36.62	26.07	

**Table 3:** Momentum Signals

The table presents various properties of the five different momentum signals of interest: (a) SIGN: the sign of past return, (b) MA: the moving average crossovers between a 12-month lagging indicator and a 1-month leading indicator, (c) EEMD: the direction of the extracted price trend using the EEMD procedure, (d) TREND: the t-statistic of the slope coefficient in a regression of the price level on a time trend, (e) SMT: the t-statistic of a statistically meaningful time trend by Bryhn and Dimberg (2011). The lookback period is equal to 12 months for the entire table. Panel A presents the correlation and the position agreement matrices of the momentum signals. Both matrices constitute averages across all 12 futures contracts. The position agreement matrix denotes the number of time periods of agreement in the long/short positions across all possible pairs of momentum signals normalized by the number of active periods (i.e. either +1 or -1) of the signals across the vertical direction; for instance, the value 62.69 in the last column means that in 62.69% of the periods that the EEMD signals a long or a short position, the SMT agrees. Panel B presents the percentage of time periods that each momentum signal indicates a long or a short position in the underlying contract; by construction, the percentages of long and short positions sum up to one for the SIGN, MA and EEMD signals. The dataset covers the period November 1, 1999 to October 30, 2009.

K	1	3	6	12	24	1	3	6	12	24	1	3	6	12	24		
J		Annua	lized Me	an (%)			Annualia	zed Volat		NW t-statistic							
1	10.43	11.46	9.75	7.58	3.57	15.96	11.82	8.23	7.14	6.46	2.15	2.90	3.55	3.33	1.73		
3	15.03	16.57	13.07	8.82	4.59	16.37	14.01	12.48	9.86	8.66	2.90	3.80	3.32	2.83	1.78		
6	15.97	13.93	12.69	9.87	5.47	16.02	15.13	13.75	11.81	11.21	3.06	2.77	2.76	2.67	1.57		
12	19.34	18.19	16.24	13.44	7.66	18.23	16.49	15.39	14.67	15.26	3.05	3.21	3.41	2.93	1.56		
24	5.49	7.36	5.84	1.96	-1.38	15.06	15.05	15.69	16.57	15.99	1.08	1.54	1.24	0.35	-0.25		
J			llar Grov	vth			Sharpe ratio						Downside-Risk Sharpe ratio				
1	2.46	2.88	2.51	2.05	1.39	0.66	0.97	1.19	1.07	0.55	0.82	1.47	1.62	1.56	0.60		
3	3.79	4.54	3.30	2.25	1.51	0.92	1.19	1.05	0.90	0.53	1.30	1.83	1.49	1.12	0.57		
6	4.02	3.36	3.04	2.39	1.58	1.00	0.92	0.93	0.84	0.49	1.38	1.25	1.22	1.03	0.52		
12	4.88	4.52	3.85	3.03	1.79	1.07	1.11	1.06	0.92	0.50	1.50	1.59	1.51	1.20	0.54		
24	1.42	1.65	1.45	1.05	0.81	0.37	0.49	0.37	0.12	-0.09	0.43	0.59	0.44	0.13	-0.09		
Pan	el B: EEN	MD Tradi	ng Signa	1													
K	1	3	6	12	24	1	3	6	12	24	1	3	6	12	24		
J		Annualized Mean (%) Annualized Volatility (%)									NW t-statistic						
1	10.41	10.44	9.40	7.42	2.65	17.46	12.27	8.81	7.63	6.90	1.95	2.43	3.13	2.97	1.16		
3	12.38	14.86	12.02	7.87	3.53	16.10	12.55	11.03	8.97	8.15	2.53	3.76	3.25	2.78	1.47		
6	21.82	15.16	11.21	10.48	6.15	16.31	15.10	13.17	11.33	10.77	3.86	3.01	2.67	3.06	1.94		
12	17.86	17.24	14.43	11.17	6.48	17.07	16.04	15.12	14.34	14.61	3.07	3.03	2.97	2.51	1.39		
24	6.25	8.04	8.25	4.18	-1.09	15.18	15.00	15.50	16.23	16.21	1.23	1.76	1.85	0.78	-0.19		
J		Do	llar Grov	vth			Sharpe ratio						Downside-Risk Sharpe ratio				
1	2.40	2.59	2.42	2.01	1.27	0.60	0.85	1.07	0.98	0.39	0.72	1.24	1.52	1.35	0.38		
3	2.94	3.92	3.03	2.07	1.36	0.77	1.19	1.09	0.88	0.43	1.00	1.89	1.56	1.08	0.44		
6	6.93	3.77	2.67	2.54	1.70	1.34	1.01	0.86	0.93	0.57	2.18	1.46	1.15	1.20	0.64		
12	4.35	4.18	3.29	2.48	1.62	1.05	1.08	0.96	0.78	0.45	1.59	1.59	1.37	0.97	0.48		
24	1.50	1.74	1.76	1.26	0.82	0.41	0.54	0.53	0.26	-0.07	0.50	0.67	0.66	0.28	-0.07		
Pan	el C: MA	Trading	Signal														
K	1	3	6	12	24	1	3	6	12	24	1	3	6	12	24		
J		Annua	lized Me	an (%)			Annualiz	zed Volat	ility (%)		NW t-statistic						
1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
3	13.44	14.61	10.81	6.05	2.62	16.43	12.30	10.50	9.13	8.04	2.60	3.79	3.35	2.13	1.01		
6	19.23	16.81	11.27	8.92	5.42	18.04	14.71	12.93	11.07	10.45	3.24	3.45	2.65	2.51	1.55		
12	19.58	20.67	17.22	14.27	9.06	17.26	16.00	14.38	13.60	14.78	3.47	3.78	3.62	3.32	1.92		
24	15.72	13.30	10.43	6.72	0.44	16.29	15.66	15.32	15.78	16.84	2.71	2.42	2.11	1.29	0.07		
J		Do	llar Grov	vth			S	D	ownside	-Risk S	harpe ra	ıtio					
1	_	-	-	-	-	_	-	-	-	-	_	-	-	-	-		
3	3.24	3.84	2.71	1.73	1.25	0.82	1.19	1.03	0.67	0.33	1.09	1.81	1.36	0.75	0.32		
6	5.29	4.43	2.69	2.20	1.59	1.07	1.15	0.88	0.81	0.52	1.57	1.76	1.17	0.96	0.54		
12	5.06	5.67	4.26	3.31	2.04	1.14	1.30	1.20	1.05	0.62	1.76	1.96	1.73	1.45	0.65		
24	3.15	2.62	2.09	1.55	0.92	0.97	0.85	0.68	0.43	0.03	1.37	1.15	0.89	0.48	0.03		

(Continued on next page)

K	1	3	6	12	24	1	3	6	12	24	1	3	6	12	24
J		Annua	lized Me	an (%)			Annuali	zed Volat	ility (%)	NW t-statistic					
1	7.09	11.24	11.34	8.62	2.42	19.60	15.43	11.06	8.87	7.46	1.14	2.12	3.05	3.08	0.93
3	14.36	20.32	14.36	8.27	4.15	20.17	14.27	12.88	11.06	10.42	2.16	4.62	3.52	2.36	1.25
6	23.82	16.99	11.27	10.70	6.18	19.57	16.27	14.17	12.46	12.37	3.57	3.09	2.34	2.80	1.63
12	22.47	22.29	16.04	12.61	7.49	19.00	17.49	15.89	15.51	16.48	3.60	3.63	3.06	2.56	1.40
24	6.66	5.70	8.83	4.50	-1.73	16.84	16.16	15.98	17.08	17.37	1.27	1.14	1.87	0.76	-0.28
J		Do	llar Grov	vth			S	harpe rat	io		D	ownside	-Risk S	harpe ra	atio
1	1.67	2.69	2.86	2.24	1.23	0.36	0.73	1.03	0.98	0.33	0.43	1.02	1.38	1.27	0.32
3	3.32	6.49	3.72	2.11	1.42	0.72	1.43	1.12	0.75	0.40	0.88	2.26	1.54	0.86	0.39
6	7.92	4.40	2.65	2.56	1.67	1.22	1.05	0.80	0.86	0.50	1.84	1.41	1.03	1.07	0.52
12	6.36	6.40	3.76	2.78	1.73	1.19	1.28	1.01	0.82	0.46	1.79	1.89	1.37	1.00	0.47
24	1.52	1.42	1.83	1.27	0.77	0.40	0.35	0.56	0.26	-0.10	0.48	0.43	0.70	0.29	-0.10
Pan	el E: SM'	T Trading	g Signal												
Pan K	el E: SM'	T Trading	g Signal	12	24	1	3	6	12	24	1	3	6	12	24
		3			24	1		6 zed Volat		24	1		6 W t-stati		24
K		3	6		2.34	21.84				9.26	1.18				0.71
<i>К</i>	1	3 Annua	6 llized Me	an (%)			Annuali	zed Volat	ility (%)			N	W t-stati	stic	
<i>К J</i> 1	8.50	3 Annua 11.47	6 dized Me	an (%) 9.30	2.34	21.84	Annualii	zed Volat	ility (%)	9.26	1.18	NV 2.09	W t-stati 2.84	2.88	0.71
<i>K J</i> 1 3	8.50 14.10	3 Annua 11.47 22.17	6 dized Me 11.24 13.99	9.30 8.63	2.34 3.47	21.84 23.82	Annuali 16.74 15.96	zed Volat 12.40 13.43	10.11 12.08	9.26 11.94	1.18 1.86	2.09 <b>4.29</b>	2.84 3.15	2.88 2.29	0.71 0.90
<i>K J</i> 1  3  6	8.50 14.10 <b>28.36</b>	3 Annua 11.47 22.17 18.71	6 llized Me 11.24 13.99 12.72	9.30 8.63 11.01	2.34 3.47 4.64	21.84 23.82 <b>24.18</b>	Annuali: 16.74 15.96 19.04	12.40 13.43 16.16	10.11 12.08 13.43	9.26 11.94 13.12	1.18 1.86 3.51	2.09 <b>4.29</b> 2.94	2.84 3.15 2.50	2.88 2.29 2.66	0.71 0.90 1.18
<i>K J</i> 1  3  6  12	8.50 14.10 <b>28.36</b> 24.56	3 Annua 11.47 22.17 18.71 19.08 9.15	6 llized Me 11.24 13.99 12.72 16.11	9.30 8.63 11.01 12.69 6.22	2.34 3.47 4.64 6.55	21.84 23.82 <b>24.18</b> 23.44	Annuali 16.74 15.96 19.04 18.99 19.35	12.40 13.43 16.16 18.04	10.11 12.08 13.43 17.06 18.50	9.26 11.94 13.12 17.06	1.18 1.86 3.51 3.13 2.15	2.09 <b>4.29</b> 2.94 2.93 1.46	2.84 3.15 2.50 2.83 1.90	2.88 2.29 2.66 2.45	0.71 0.90 1.18 1.28 -0.18
K  J  1  3  6  12  24	8.50 14.10 <b>28.36</b> 24.56	3 Annua 11.47 22.17 18.71 19.08 9.15	6 llized Me 11.24 13.99 12.72 16.11 10.93	9.30 8.63 11.01 12.69 6.22	2.34 3.47 4.64 6.55	21.84 23.82 <b>24.18</b> 23.44	Annuali 16.74 15.96 19.04 18.99 19.35	12.40 13.43 16.16 18.04 17.91	10.11 12.08 13.43 17.06 18.50	9.26 11.94 13.12 17.06	1.18 1.86 3.51 3.13 2.15	2.09 <b>4.29</b> 2.94 2.93 1.46	2.84 3.15 2.50 2.83 1.90	2.88 2.29 2.66 2.45 0.95	0.71 0.90 1.18 1.28 -0.18
K  J  1  3  6  12  24  J	8.50 14.10 <b>28.36</b> 24.56 14.96	3 Annua 11.47 22.17 18.71 19.08 9.15	6 dized Me 11.24 13.99 12.72 16.11 10.93	9.30 8.63 11.01 12.69 6.22	2.34 3.47 4.64 6.55 -1.25	21.84 23.82 <b>24.18</b> 23.44 19.66	Annualii 16.74 15.96 19.04 18.99 19.35	12.40 13.43 16.16 18.04 17.91 harpe rati	10.11 12.08 13.43 17.06 18.50	9.26 11.94 13.12 17.06 19.48	1.18 1.86 3.51 3.13 2.15	2.09 <b>4.29</b> 2.94 2.93 1.46	2.84 3.15 2.50 2.83 1.90 2-Risk S	2.88 2.29 2.66 2.45 0.95 harpe ra	0.71 0.90 1.18 1.28 -0.18
<i>K</i> 1 3 6 12 24 <i>J</i> 1	8.50 14.10 28.36 24.56 14.96	3 Annua 11.47 22.17 18.71 19.08 9.15 Do	6 dized Me 11.24 13.99 12.72 16.11 10.93 ollar Grov 2.79	9.30 8.63 11.01 12.69 6.22 wth	2.34 3.47 4.64 6.55 -1.25	21.84 23.82 <b>24.18</b> 23.44 19.66	Annuali 16.74 15.96 19.04 18.99 19.35 S	12.40 13.43 16.16 18.04 17.91 harpe rat	10.11 12.08 13.43 17.06 18.50 io	9.26 11.94 13.12 17.06 19.48	1.18 1.86 3.51 3.13 2.15 D	2.09 <b>4.29</b> 2.94 2.93 1.46 ownside	2.84 3.15 2.50 2.83 1.90 2-Risk S	2.88 2.29 2.66 2.45 0.95 harpe ra	0.71 0.90 1.18 1.28 -0.18 atio
<i>K J</i> 1 3 6 12 24 <i>J</i> 1 3	8.50 14.10 <b>28.36</b> 24.56 14.96	3 Annua 11.47 22.17 18.71 19.08 9.15 Do 2.70 7.58	6 dized Me 11.24 13.99 12.72 16.11 10.93 ollar Grov 2.79 3.57	9.30 8.63 11.01 12.69 6.22 wth 2.37 2.16	2.34 3.47 4.64 6.55 -1.25	21.84 23.82 <b>24.18</b> 23.44 19.66	Annuali: 16.74 15.96 19.04 18.99 19.35 S 0.69 1.39	12.40 13.43 16.16 18.04 17.91 harpe rations	10.11 12.08 13.43 17.06 18.50 io 0.92 0.72	9.26 11.94 13.12 17.06 19.48	1.18 1.86 3.51 3.13 2.15 D 0.45 0.68	2.09 <b>4.29</b> 2.94 2.93 1.46 ownside 0.96 <b>2.33</b>	2.84 3.15 2.50 2.83 1.90 e-Risk S 1.24 1.42	2.88 2.29 2.66 2.45 0.95 harpe ra 1.13 0.79	0.71 0.90 1.18 1.28 -0.18 attio 0.23 0.28

Table 4: Time-Series Momentum with Monthly Portfolio Rebalancing

The table presents the annualised mean return, the annualised volatility, the Newey and West (1987) t-statistic of the mean return, the dollar growth, the annualised Sharpe ratio and the annualised downside-risk Sharpe ratio by Ziemba (2005) for various (J,K) time-series momentum strategies, where J denotes the lookback period and K denotes the investment horizon, both measured in months. The portfolio rebalancing takes place at the end of each month and the momentum signals are: (a) SIGN: the sign of past return, (b) MA: the moving average crossovers between a J-month lagging indicator and a 1-month leading indicator, (c) EEMD: the direction of the extracted price trend using the EEMD procedure, (d) TREND: the t-statistic of the slope coefficient in a regression of the price level on a time trend, (e) SMT: the t-statistic of a statistically meaningful time trend by Bryhn and Dimberg (2011). The time-series momentum strategy is the volatility-adjusted (using the 30-day Yang and Zhang (2000) estimator) weighted average of the individual momentum strategies. The dataset covers the period November 1, 1999 to October 30, 2009.

Mor	nthly Port	tfolio Tu	mover (%	(b)												
K	1	3	6	12	24		1	3	6	12	24					
J			SIGN			_	EEMD									
1	102.2	109.7	803.1	412.1	571.2		105.0	125.4	587.4	394.8	76.9					
3	62.4	129.4	167.3	123.2	60.7		65.9	98.1	725.0	90.4	175.5					
6	44.9	98.7	439.0	87.6	50.2		43.2	66.0	153.1	81.8	80.0					
12	31.5	37.4	75.9	58.3	47.1		29.5	54.8	70.8	58.2	43.8					
24	23.0	30.5	39.0	53.7	162.1		20.5	25.1	50.2	31.3	115.5					
J			MA													
1	-	-	-	-	-											
3	74.05	93.2	363.3	102.1	104.4											
6	49.8	69.8	241.5	270.2	41.0											
12	33.2	48.4	111.6	42.4	76.8											
24	23.4	25.7	44.2	32.51	28.5											
J			TREND						SMT							
1	68.2	539.9	238.7	183.2	112.2		55.6	417.4	194.4	141.8	86.9					
3	41.6	351.3	147.2	90.8	72.7		30.4	344.7	377.6	223.8	693.5					
6	24.3	193.0	163.3	92.0	121.8		19.3	47.2	83.0	87.1	69.6					
12	18.0	62.2	57.7	70.0	47.1		15.6	22.5	38.3	58.4	85.6					
24	14.2	18.6	39.8	57.2	33.8		11.6	13.9	15.0	47.7	51.6					

**Table 5:** Monthly Portfolio Turnover for Time-Series Momentum Strategies

The table presents the monthly portfolio turnover for various (J,K) time-series momentum strategies, where J denotes the lookback period and K denotes the investment horizon, both measured in months. The portfolio turnover is computed as the percentage change in the aggregate number of open positions in futures contracts. The portfolio rebalancing takes place at the end of each month and the momentum trading signals are: (a) SIGN: the sign of past return, (b) MA: the moving average crossovers between a 12-month lagging indicator and a 1-month leading indicator, (c) EEMD: the direction of the extracted price trend using the EEMD procedure, (d) TREND: the t-statistic of the slope coefficient in a regression of the price level on a time trend, (e) SMT: the t-statistic of a statistically meaningful time trend by Bryhn and Dimberg (2011). The time-series momentum strategy is the volatility-adjusted (using the 30-day Yang and Zhang (2000) estimator) weighted average of the individual momentum strategies. The dataset covers the period November 1, 1999 to October 30, 2009.

K	1	2	3	4	8	1	2	3	4	8	1	2	3	4	8				
J		Annua	lized Me	an (%)			Annualiz	zed Volat	tility (%)			NW t-statistic							
1	3.33	4.38	9.04	8.24	6.67	16.46	11.58	10.22	9.36	7.78	0.69	1.25	3.18	2.95	2.67				
2	5.53	7.21	7.55	6.41	4.78	16.32	14.29	12.76	11.79	9.59	1.14	1.71	2.04	1.78	1.58				
3	10.56	10.41	9.40	8.00	5.94	16.38	14.78	13.95	13.29	10.85	2.18	2.39	2.23	1.98	1.72				
4	15.89	13.08	11.28	8.99	8.21	16.75	15.57	14.95	14.60	12.33	3.20	2.82	2.44	1.96	2.01				
8	12.03	10.18	10.46	9.69	10.62	17.15	16.82	16.42	16.07	14.49	2.31	1.93	2.01	1.89	2.25				
J		Do	llar Grov	vth			S	harpe rat	io		D	ownside	-Risk Sl	harpe ra	tio				
1	1.22	1.44	2.32	2.16	1.88	0.20	0.38	0.89	0.88	0.86	0.21	0.40	0.96	0.94	0.94				
2	1.51	1.85	1.95	1.76	1.53	0.34	0.51	0.59	0.54	0.50	0.35	0.52	0.60	0.56	0.52				
3	2.49	2.51	2.30	2.02	1.70	0.65	0.70	0.67	0.60	0.55	0.70	0.75	0.71	0.64	0.58				
4	4.19	3.24	2.74	2.19	2.09	0.95	0.84	0.76	0.62	0.67	1.03	0.90	0.82	0.67	0.74				
8	2.84	2.38	2.46	2.29	2.57	0.70	0.61	0.64	0.60	0.73	0.75	0.64	0.68	0.64	0.81				
Par	nel B: MA	A Trading	Signal																
K	1	2	3	4	8	1	2	3	4	8	1	2	3	4	8				
J		Annua	lized Me	an (%)			Annualia	zed Volat	tility (%)	NW t-statistic									
1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-				
2	6.12	9.61	8.88	7.19	4.58	16.35	13.92	12.21	11.19	8.89	1.29	2.32	2.57	2.18	1.66				
3	6.72	8.88	7.47	6.80	4.19	16.73	15.03	13.60	12.72	9.96	1.43	2.18	2.02	1.88	1.34				
4	8.28	9.37	7.31	6.49	4.06	16.49	15.37	14.28	13.70	11.15	1.81	2.21	1.82	1.63	1.14				
8	12.52	10.51	8.97	8.39	8.07	17.03	16.46	16.04	15.65	13.41	2.55	2.16	1.85	1.75	1.87				
J		Do	llar Grov	vth			S	harpe rat	io	Downside-Risk Sharpe ratio									
1	-	-	-	-	-	- 0.25	-	- 0.72	-	-	-		-	-	-				
2	1.60	2.35	2.24	1.92	1.51	0.37	0.69	0.73	0.64	0.52	0.38	0.72	0.77	0.69	0.55				
3	1.69	2.15	1.91	1.81	1.44	0.40	0.59	0.55	0.54	0.42	0.41	0.62	0.56	0.56	0.44				
4 8	1.98 <b>2.98</b>	2.25 2.47	1.86 2.14	1.73 2.03	1.41 2.03	0.50 <b>0.74</b>	0.61 0.64	0.51 0.56	0.47 0.54	0.36 0.60	0.52 <b>0.79</b>	0.63 0.68	0.53 0.59	0.49 0.57	0.38 0.65				
				2.03	2.03	0.74	0.04	0.50	0.54	0.00	0.77	0.00	0.59	0.57	0.03				
	nel C: SM			4		-			4										
<u>K</u>	1	2	3	4	8	1	2	3	4	8	1	2	3	4	8				
J		Annua	lized Me	an (%)			Annualized Volatility (%)						NW t-statistic						
1	2.60	7.26	9.62	8.67	7.40	21.79	16.27	14.27	12.66	10.32	0.41	1.51	2.16	2.11	2.17				
2	12.76	14.60	14.78	11.01	9.14	22.73	18.65	16.16	15.32	12.68	1.91	2.66	3.33	2.40	2.24				
3	19.99	18.29	13.31	9.70	10.74	22.58	19.18	17.17	16.26	13.55	3.07	3.29	2.62	1.98	2.40				
4	18.65	14.93	10.53	9.83	7.23	22.85	20.02	18.35	17.17	14.40	2.79	2.61	1.93	1.83	1.50				
8	9.76	6.21	8.32	9.79	12.24	24.34	21.76	20.30	19.32	15.91	1.24	0.90	1.31	1.67	2.28				
J		Do	llar Grov	vth			S	harpe rat	io			ownside	-Risk Sl	harpe ra	tio				
1	1.02	1.80	2.34	2.18	1.97	0.12	0.45	0.67	0.69	0.72	0.12	0.47	0.71	0.71	0.76				
2	2.73	3.57	3.79	2.65	2.28	0.56	0.78	0.92	0.72	0.72	0.59	0.86	0.98	0.76	0.77				
3	5.60	5.08	3.23	2.29	2.64	0.89	0.95	0.78	0.60	0.79	0.95	1.01	0.83	0.64	0.88				
4	4.89	3.60	2.40	2.29	1.85	0.82	0.75	0.57	0.57	0.50	0.92	0.82	0.63	0.65	0.56				
8	1.95	1.46	1.85	2.18	2.96	0.40	0.29	0.41	0.51	0.77	0.40	0.29	0.42	0.52	0.85				

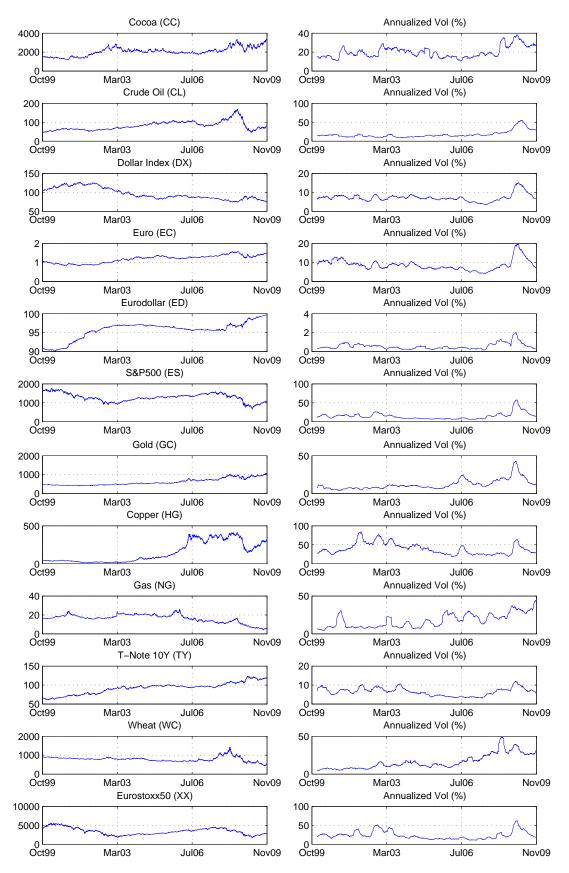
**Table 6:** Time-Series Momentum with Weekly Portfolio Rebalancing

The table presents the annualised mean return, the annualised volatility, the Newey and West (1987) t-statistic of the mean return, the dollar growth, the annualised Sharpe ratio and the annualised downside-risk Sharpe ratio by Ziemba (2005) for various (J,K) time-series momentum strategies, where J denotes the lookback period and K denotes the investment horizon, both measured in weeks. The portfolio rebalancing takes place at the end of each Wednesday and the momentum signals are: (a) SIGN: the sign of past return, (b) MA: the moving average crossovers between a J-month lagging indicator and a 1-month leading indicator, (c) SMT: the t-statistic of a statistically meaningful time trend by Bryhn and Dimberg (2011). The time-series momentum strategy is the volatility-adjusted (using the 30-day Yang and Zhang (2000) estimator) weighted average of the individual momentum strategies. The dataset covers the period November 1, 1999 to October 30, 2009.

K	1	3	5	10	15	30	1	3	5	10	15	30	1	3	5	10	15	30		
J		Ar	nnualized	Mean (%)	)			Annualized Volatility (%)							NW t-statistic					
3	-8.92	-0.54	1.04	2.01	3.04	2.84	16.08	11.67	9.65	7.17	6.18	5.09	-1.89	-0.16	0.36	0.98	1.62	1.84		
5	-1.60	2.76	3.07	3.59	5.00	3.81	16.47	14.19	12.30	9.71	8.49	6.97	-0.32	0.65	0.84	1.25	1.93	1.79		
10	3.39	3.48	3.28	4.60	5.87	3.89	16.19	15.08	14.31	12.86	11.71	9.72	0.69	0.79	0.76	1.17	1.65	1.29		
15	4.88	4.02	4.73	5.69	6.12	4.04	16.44	15.35	14.90	14.13	13.37	11.36	0.96	0.85	1.04	1.35	1.52	1.13		
30	13.75	12.73	10.05	7.24	7.29	6.04	16.62	16.23	16.07	15.79	15.44	13.96	2.71	2.57	2.06	1.50	1.52	1.35		
J			Dollar C	Growth					Sharp	e ratio				Down	side-Risl	s Sharpe	ratio			
3	0.35	0.88	1.06	1.20	1.34	1.32	-0.55	-0.05	0.11	0.28	0.49	0.56	-0.55	-0.05	0.11	0.28	0.50	0.56		
5	0.74	1.20	1.27	1.38	1.61	1.44	-0.10	0.19	0.25	0.37	0.59	0.55	-0.10	0.20	0.25	0.37	0.60	0.55		
10	1.24	1.27	1.26	1.47	1.70	1.42	0.21	0.23	0.23	0.36	0.50	0.40	0.21	0.23	0.23	0.36	0.50	0.40		
15	1.43	1.34	1.45	1.62	1.71	1.42	0.30	0.26	0.32	0.40	0.46	0.36	0.30	0.26	0.32	0.41	0.46	0.35		
30	3.54	3.22	2.45	1.85	1.87	1.68	0.83	0.78	0.63	0.46	0.47	0.43	0.86	0.81	0.64	0.46	0.47	0.43		
Pane	1 B: MA T	Frading S	ignal																	
K	1	3	5	10	15	30	1	3	5	10	15	30	1	3	5	10	15	30		
J		Ar	nnualized	Mean (%)	)			An	NW t-statistic											
3	-8.04	-0.68	2.89	3.25	3.99	3.53	15.68	12.32	10.71	8.21	7.14	5.92	-1.70	-0.19	0.91	1.35	1.85	1.94		
5	-2.69	4.27	5.20	4.76	5.74	4.13	15.93	13.54	12.16	10.01	8.94	7.51	-0.54	1.07	1.45	1.61	2.13	1.79		
10	1.47	3.28	3.62	4.46	6.21	4.17	16.12	14.61	13.65	12.22	11.34	9.72	0.29	0.75	0.89	1.21	1.81	1.3		
15	5.00	4.86	5.07	6.00	6.72	4.35	16.20	15.04	14.37	13.41	12.70	11.13	0.99	1.06	1.16	1.48	1.74	1.2		
30	8.88	7.83	7.70	7.62	8.04	6.82	16.32	15.66	15.39	14.91	14.45	13.18	1.73	1.62	1.64	1.68	1.80	1.62		
J			Dollar C	Growth				Downside-Risk Sharpe ratio												
3	0.39	0.86	1.27	1.35	1.47	1.41	-0.51	-0.05	0.27	0.40	0.56	0.60	-0.50	-0.06	0.27	0.39	0.56	0.60		
5	0.67	1.41	1.58	1.55	1.73	1.48	-0.17	0.32	0.43	0.48	0.64	0.55	-0.17	0.32	0.44	0.48	0.65	0.53		
10	1.02	1.25	1.32	1.46	1.77	1.46	0.09	0.22	0.27	0.37	0.55	0.43	0.09	0.23	0.27	0.36	0.55	0.4		
15	1.46	1.46	1.51	1.68	1.83	1.47	0.31	0.32	0.35	0.45	0.53	0.39	0.32	0.33	0.35	0.44	0.53	0.39		
30	2.17	1.97	1.95	1.95	2.05	1.84	0.54	0.50	0.50	0.51	0.56	0.52	0.56	0.51	0.50	0.51	0.55	0.5		
Pane	1 C: SMT	Trading	Signal																	
K	1	3	5	10	15	30	1	3	5	10	15	30	1	3	5	10	15	30		
J .	Annualized Mean (%)							Annualized Volatility (%)						NW t-statistic						
3	-13.38	-0.35	1.97	2.85	3.98	3.64	18.96	13.49	11.34	8.48	7.35	6.05	-2.31	-0.09	0.58	1.16	1.80	1.9		
5	0.77	3.90	3.54	4.89	6.83	5.63	21.66	17.67	15.10	12.36	11.08	9.46	0.11	0.71	0.76	1.29	2.02	1.9		
10	2.63	7.33	5.71	7.84	10.78	5.77	22.15	19.20	17.62	15.36	14.17	12.58	0.39	1.26	1.06	1.62	2.45	1.4		
15	7.02	7.42	5.79	10.59	9.83	7.59	22.88	20.43	18.97	16.87	15.65	13.82	1.01	1.18	0.99	2.09	2.06	1.7		
30	11.67	11.92	10.91	5.00	3.52	5.71	23.51	21.92	20.80	18.96	17.84	16.00	1.62	1.81	1.74	0.84	0.63	1.1		
J .			Dollar C	Growth						Down	side-Risl	Sharpe	ratio							
3	0.21	0.88	1.15	1.29	1.46	1.42	-0.71	-0.03	0.17	0.34	0.54	0.60	-0.71	-0.03	0.18	0.34	0.55	0.6		
5	0.85	1.27	1.28	1.53	1.89	1.70	0.04	0.22	0.23	0.40	0.62	0.60	0.04	0.22	0.24	0.40	0.63	0.6		
10	1.02	1.75	1.53	1.98	2.72	1.66	0.12	0.38	0.32	0.51	0.76	0.46	0.12	0.39	0.33	0.51	0.77	0.4		
15	1.57	1.73	1.50	2.56	2.41	1.97	0.31	0.36	0.31	0.63	0.63	0.55	0.31	0.37	0.31	0.64	0.63	0.5		
30	2.48	2.65	2.45	1.39	1.22	1.57	0.50	0.54	0.52	0.26	0.20	0.36	0.50	0.55	0.53	0.26	0.20	0.3		

**Table 7:** Time-Series Momentum with Daily Portfolio Rebalancing

The table presents the annualised mean return, the annualised volatility, the Newey and West (1987) t-statistic of the mean return, the dollar growth, the annualised Sharpe ratio and the annualised downside-risk Sharpe ratio by Ziemba (2005) for various (J,K) time-series momentum strategies, where J denotes the lookback period and K denotes the investment horizon, both measured in days. The portfolio rebalancing takes place at the end of each day and the momentum signals are: (a) SIGN: the sign of past return, (b) MA: the moving average crossovers between a J-month lagging indicator and a 1-month leading indicator, (c) SMT: the t-statistic of a statistically meaningful time trend by Bryhn and Dimberg (2011). The time-series momentum strategy is the volatility-adjusted (using the 30-day Yang and Zhang (2000) estimator) weighted average of the individual momentum strategies. The dataset covers the period November 1, 1999 to October 30, 2009.



**Figure 1:** Futures Prices and Running Annualized Volatilities

The figure presents in the first column the time evol

The figure presents in the first column the time evolution of the futures prices for the 12 instruments of the dataset and in the second column the annualised estimates of the volatility for each instrument respectively. The volatilities are estimated using the Yang and Zhang (2000) estimator with a rolling window of 60 days. The sample period of the graphs is November 1, 1999 to October 30, 2009.

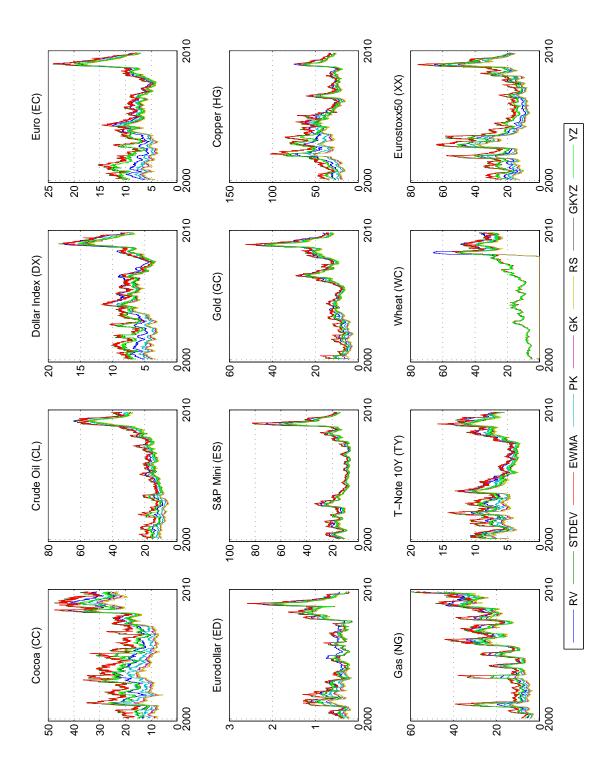
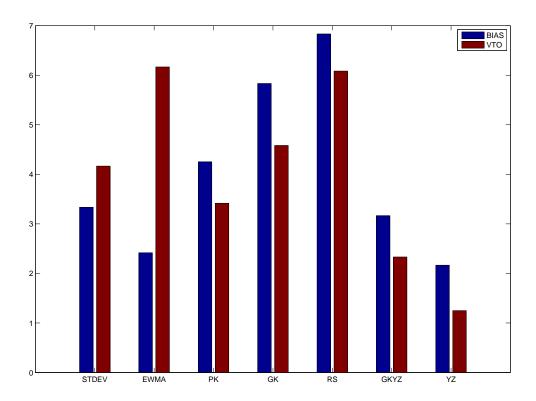


Figure 2: Running Annualized Volatilities using various Volatility Estimators

The figure presents the volatility estimates for the 12 futures contracts of the dataset using eight different volatility estimators: (a) Realized Volatility (RV), (b) standard deviation of past returns (STDEV), (c) exponentially-weighted average of past squared returns (EWMA), (d) Parkinson (1980) estimator (PK), (e) Garman and Klass (1980) estimator (GK), (f) Roger and Satchell (1991) estimator (RS), (g) Garman and Klass estimator adjusted by Yang and Zhang to allow for opening jumps (GKYZ) and (h) Yang and Zhang (2000) estimator (YZ). The estimation period is a rolling window of 60 trading days. The RV is estimated using intra-day 30min returns, whereas all the rest estimators use daily data for opening, closing, high and low futures prices. The dataset covers the period November 1, 1999 to October 30, 2009.



**Figure 3:** Ranks of Volatility Estimators

The bar diagram presents the average (across the 12 futures contracts) rank with respect to the absolute bias (BIAS) from the Realized Variance estimator (RV) and the volatility turnover (VTO) (i.e. the absolute change of  $\frac{1}{\sigma}$ ) for seven different volatility estimators: (a) standard deviation of past returns (STDEV), (b) exponentially-weighted average of past squared returns (EWMA), (c) Parkinson (1980) estimator (PK), (d) Garman and Klass (1980) estimator (GK), (e) Roger and Satchell (1991) estimator (RS), (f) Garman and Klass estimator adjusted by Yang and Zhang to allow for opening jumps (GKYZ) and (g) Yang and Zhang (2000) estimator (YZ). The estimation period is a rolling window of 60 trading days. The dataset covers the period November 1, 1999 to October 30, 2009.

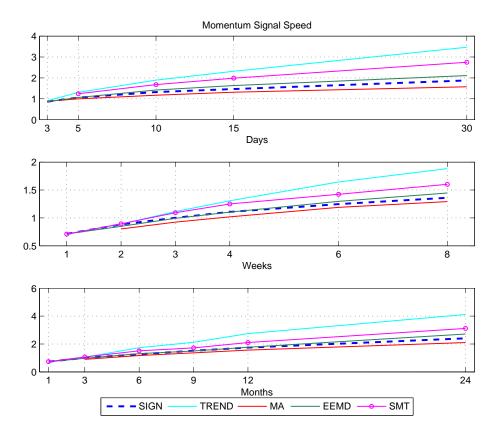


Figure 4: Speed Rank for Various Frequencies and Lookback Periods

The figure presents the speed of the five momentum signals of interest: (a) SIGN: the sign of past return, (b) MA: the moving average crossovers between a *J*-period lagging indicator and a 1-period leading indicator, (c) EEMD: the direction of the extracted price trend using the EEMD procedure, (d) TREND: the t-statistic of the slope coefficient in a regression of the price level on a time trend, (e) SMT: the t-statistic of a statistically meaningful time trend by Bryhn and Dimberg (2011). The lookback period (*J*) is measured in days, weeks and months in the first, second and third row respectively. The signal speed is computed for every of the 12 futures contracts of our dataset as SPEED<sub>X</sub> =  $\sqrt{\frac{1}{T-J}}\sum_{t=1}^{T}X^2(t-J,t)/\frac{1}{T-J-1}\sum_{t=1}^{T}[X(t-J,t)-X(t-1-J,t-1)]^2$ , where *T* is the number of trading periods and X(t-J,t) denotes the momentum signal taking values -1, 0 or +1 at the end of period *t*. The aggregate speed of each signal is computed as the average speed across the 12 futures contracts. The sample period is November 1, 1999 to October 30, 2009.

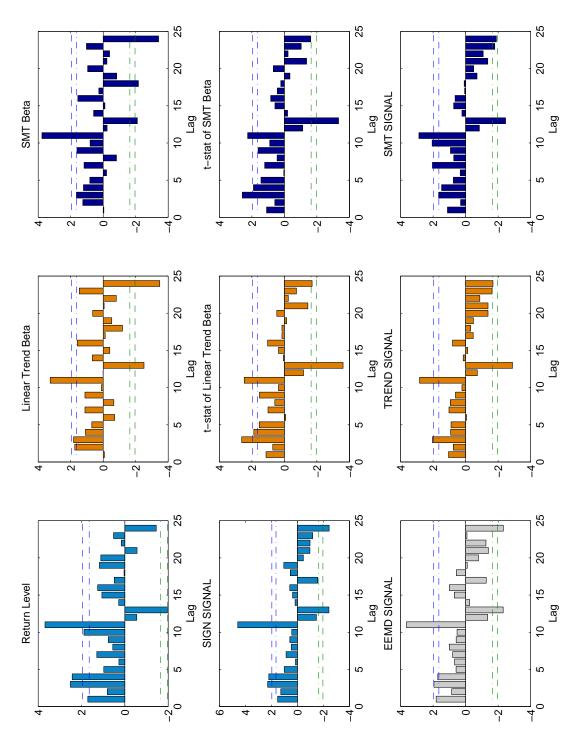
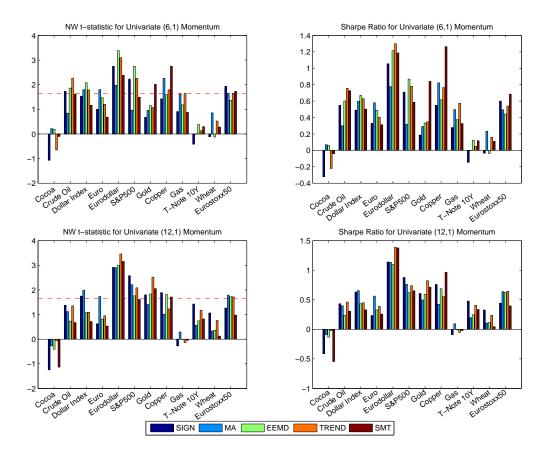


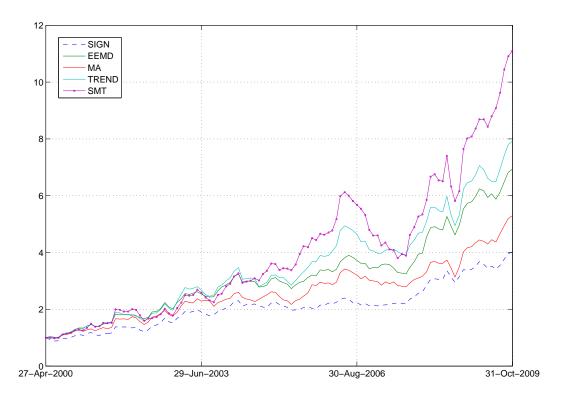
Figure 5: Return Predictability using a family of Predictors

The figure presents the t-statistics of the  $\beta_{\lambda}$  coefficient for the pooled panel linear regression  $\frac{R(t-1,t)}{\sigma_{YZ}(t-1;D)} = \alpha + \beta_{\lambda}Z(t-\lambda) + \epsilon(t)$  for lags  $\lambda = 1,2,\cdots,24$  months and a broad collection of regressors Z. The t-statistics are computed using standard errors clustered by asset and time (Cameron, Gelbach and Miller 2011, Thompson 2011). The volatility estimates are computed using the Yang and Zhang (2000) estimator on a D=30 day rolling window. The regressors are: column 1: (a) the standardized level of past return, (b) the trading signal SIGN, and (c) the trading signal EEMD; column 2: (a) the slope coefficient from fitting a time trend to the normalized asset path over the lagged month of interest, (b) the Newey and West (1987) t-statistic of the afore-mentioned slope coefficient and (c) the trading signal TREND; column 3: similar to column 2 but instead of using the simple linear fit of a time trend, the statistically meaningful trend (SMT) by Bryhn and Dimberg (2011) is employed. The colouring of the various figures is done in order to visually group together quantities that relate to each other on the basis of a certain methodology, e.g. the second column involves quantities that are related to the TREND signal methodology. The dashed (dot-dashed) lines represent significance at the 5% (10%) level. The dataset covers the period November 1, 1999 to October 30, 2009<sub>A</sub>



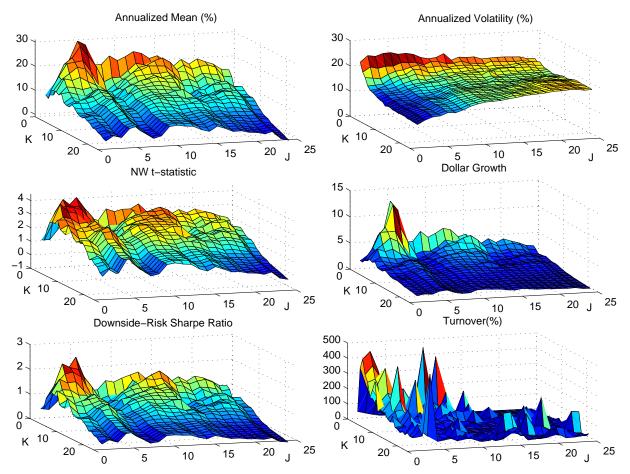
**Figure 6:** *Univariate* (6,1) *and* (12,1) *Time-Series Momentum Strategy* 

The figure presents the Newey-West t-statistics and annualised Sharpe ratios of the univariate (6,1) and (12,1) time-series momentum strategy for the 12 futures contracts of the dataset. The momentum strategies are built based on five trading signals: (a) SIGN: the sign of past return, (b) MA: the moving average crossovers between a *J*-month lagging indicator and a 1-month leading indicator, (c) EEMD: the direction of the extracted price trend using the EEMD procedure, (d) TREND: the t-statistic of the slope coefficient in a regression of the price level on a time trend, (e) SMT: the t-statistic of a statistically meaningful time trend by Bryhn and Dimberg (2011). All futures positions have been scaled by the 30-day ex-ante volatility that is estimated using the Yang and Zhang (2000) methodology. The red dashed line in the t-statistics plots represents significance at the 10% level. The dataset covers the period November 1, 1999 to October 30, 2009.



**Figure 7:** *Dollar Growth for the* (6,1) *Time-Series Momentum Strategy* 

The figure presents the growth of \$1 invested in five (6,1) time-series momentum strategies, each one of which is using a different trading signal: (a) SIGN: the sign of past return, (b) MA: the moving average crossovers between a 6-month lagging indicator and a 1-month leading indicator, (c) EEMD: the direction of the extracted price trend using the EEMD procedure, (d) TREND: the t-statistic of the slope coefficient in a regression of the price level on a time trend, (e) SMT: the t-statistic of a statistically meaningful time trend by Bryhn and Dimberg (2011). The dataset covers the period November 1, 1999 to October 30, 2009. The plot starts from the end of April 2000, since the first 6 months from November 1999 to April 2000 are used as the initial lookback period.



**Figure 8:** *Time-Series Momentum using SMT Signal with Monthly Portfolio Rebalancing*The plots present the annualised mean return, the annualised volatility, the Newey and West (1987) tstatistic of the mean return, the dollar growth, the annualised downside-risk Sharpe ratio by Ziemba (2005) and the monthly turnover for the (J,K) time-series momentum strategies, where  $J=1,2,\cdots,24$  denotes the lookback period and  $K=1,2,\cdots,24$  denotes the investment horizon, both measured in months. The portfolio rebalancing takes place at the end of each month and the momentum signal is the SMT, i.e. the sign of the t-statistic of a statistically meaningful time trend by Bryhn and Dimberg (2011). The time-series momentum strategy is the equally-weighted average of the return series across all futures contracts. All futures positions have been scaled by the 30-day ex-ante volatility that is estimated using the Yang and Zhang (2000) methodology. The dataset covers the period November 1, 1999 to October 30, 2009.