**Hamming code – one error detection and one correction**

A = [1 0 1 0]
m – represents the number of information bits (in our case = 4)
k - represents the number of control bits (in our case = 3)

$2^k \geq m + k + 1$

k = 3

$2^0,\ 2^1,\ \ldots,\ 2^{k-1} \Rightarrow 1,\ 2,\ 4\ control\ bits\ possition$

$V = \left[c_1\, c_2\, a_3\, c_4\, a_5\, a_6\, a_7\right] - vector\ to\ be\ transmitted$

$a_3 = first\ information\ bit\ (\ = 1)$

$a_5 = second\ information\ bit\ (\ = 0)$

$a_6 = third\ information\ bit\ (\ = 1)$

$a7 = fourth\ information\ bit\ (\ = 0)$

The values of $C_1$, $C_2$, $C_4$ are calculated from:

$H \cdot V^T = 0$

$where: H - Hamming\ matrix\ (\ is\ generated\ by\ us)$

$it\ has\ n\ columns\ (\ m + k)\ \ and\ k\ rows$

$$H = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$H \cdot V^T = 0$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \\ a_3 \\ c_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} c_4 \oplus a_5 \oplus a_6 \oplus a_7 = 0 \\ c2 \oplus a_3 \oplus a_6 \oplus a_7 = 0\ ;\ c_4 = 1;\ c_2 = 0;\ c_1 = 1; \\ c_1 \oplus a_3 \oplus a_5 \oplus a_7 = 0 \end{matrix}$$

The sending station (Tx) will send the V vector/word, and the receiving station will receive the E vector/word.

To determine if the transmission was successful, we will be verifying as follows:

$H \cdot E^T = Z$

$Z - \ error\ vector$

If Z = 0, then the transmission was successful

If Z <> 0, the transmission had an error at the position given by Z. (transform from binary to decimal the Z vector, and you will get the erroneous position on the E vector).

Examples of error detection and correction:

a. $E = \left[0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 1\ \ 0\right]$

$H \cdot E^T = Z$

$$Z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow E_{corr} = \left[1\ \ 0\ \ 1\ \ 1\ \ 0\ \ 1\ \ 0\right]$$

b. $E = \left[0\ \ 1\ \ 1\ \ 1\ \ 0\ \ 1\ \ 0\right]$

$H \cdot E^T = Z$

$$Z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow E_{corr} = \left[0\ \ 1\ \ 0\ \ 1\ \ 0\ \ 1\ \ 0\right]$$

$correcting\ the\ wrong\ bit$

c. $E = \left[1\ \ 0\ \ 1\ \ 1\ \ 0\ \ 1\ \ 0\right]$

$H \cdot E^T = Z$

$$Z = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow succesful\ transmission,\ no\ bit\ correction$$

**Hamming code – two error detection and one error correction**

A = [1 0 1 0]
m – represents the number of information bits (in our case = 4)
k - represents the number of control bits (in our case = 3)

$2^k \geq m + k + 1$

k = 3

$2^0,\ 2^1,\ \ldots,\ 2^{k-1} \Rightarrow 1,\ 2,\ 4\ \textit{control bits possition}$

$V = \left[c_1\, c_2\, a_3\, c_4\, a_5\, a_6\, a_7\right] - \textit{vector to be transmitted}$

$a_3 = \textit{first information bit } (\ = 1)$

$a_5 = \textit{second information bit } (\ = 0)$

$a_6 = \textit{third information bit } (\ = 1)$

$a7 = \textit{fourth information bit } (\ = 0)$

$c_0 - \textit{parity bit}$

The values of $C_0$, $C_1$, $C_2$, $C_4$ are calculated from:

$H_2 \cdot V_2^{\,T} = 0$

$\textit{where}: H_2 - \textit{Hamming matrix (is generated by us)}$

$\textit{it has n columns } (m + k + 1) \textit{ and } (k + 1) \textit{ rows}$

$$H_2 = \begin{bmatrix} 0 & H \\ \vdots & \\ 1 & 1 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$H_2 \cdot V_2^{\,T} = 0$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ a_3 \\ c_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{matrix} c_4 \oplus a_5 \oplus a_6 \oplus a_7 = 0 \\ c2 \oplus a_3 \oplus a_6 \oplus a_7 = 0 \\ c_1 \oplus a_3 \oplus a_5 \oplus a_7 = 0 \end{matrix} \quad ;\ c_4 = 1;\ c_2 = 0;\ c_1 = 1;\ c_0 = 0$$

$c_0 \oplus c_1 \oplus c_2 \oplus a_3 \oplus c_4 \oplus a_5 \oplus a_6 \oplus a_7 = 0;$

The sending station (Tx) will send the $V_2$ vector/word, and the receiving station will receive the $E_2$ vector/word.

To determine if the transmission was successful, we will be verifying as follows:

$H_2 \cdot E_2^{\,T} = Z_2$

$Z_2 - \textit{error vector} \rightarrow (k + 1) \textit{ rows and one column}$

$Z_2 = \begin{bmatrix} Z_c \\ Z_p \end{bmatrix};\ Z_c \rightarrow k \textit{ rows and one column};\ Z_p \rightarrow 1 \textit{ row and } 1 \textit{ column}$

$\textit{If } Z_c = 0 \textit{ and } Z_p = 0 \Rightarrow \nexists\ \textit{error}$

$\textit{If } Z_c = 0 \textit{ and } Z_p = 1 \Rightarrow \exists\ \textit{error and it is the parity bit} \Rightarrow E_{corr}$

$\textit{If } Z_c <> 0 \textit{ and } Z_p = 1 \Rightarrow \exists\ \textit{error and } Z_c \textit{ will give the error bit possition} \Rightarrow E_{corr}$

$\textit{If } Z_c <> 0 \textit{ and } Z_p = 0 \Rightarrow \exists\ \textit{two errors and it can't be corrected}$

Examples of error detection and correction:

$E_2 = [0\ 1\ 0\ 1\ 1\ 0\ 1\ 0]$

$H_2 \cdot E_2{}^T = Z_2$

a.

$$Z_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \textit{ no errors } [\textit{successful transmission}]$$

$E_2 = [1\ 0\ 0\ 1\ 1\ 0\ 1\ 0]$

$H_2 \cdot E_2{}^T = Z_2$

b.

$$Z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}; Z_p = 0; Z_c <> 0.\ \exists \textit{ two erros}$$

$[\textit{unsuccessful transmission; to many errors and it can't be corrected}]$

$E_2 = [1\ 1\ 0\ 1\ 1\ 0\ 1\ 0]$

$H_2 \cdot E_2{}^T = Z_2$

c.

$$Z_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}; Z_p = 1; Z_c = 0.\ \exists \textit{ one error } \Rightarrow E_{2\,corr} = [\,0\ \ 1\ \ 0\ \ 1\ \ 1\ \ 0\ \ 1\ \ 0\,]$$

$[\textit{unsuccessful transmission; one error to be corrected}]$

$E_2 = [0\ 1\ 1\ 1\ 1\ 0\ 1\ 0]$

$H_2 \cdot E_2{}^T = Z_2$

d.

$$Z_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}; Z_p = 1; Z_c <> 0.\ \exists \textit{ one error } \Rightarrow Z_c \textit{ gives the error position;}$$

$E_{2\,corr} = [\,0\ \ 1\ \ 0\ \ 1\ \ 1\ \ 0\ \ 1\ \ 0\,]$

$[\textit{unsuccessful transmission; one error to be corrected}]$