

# 1 Organization and Introduction

- The Art of managing complexity
  - Abstraction: Hiding details when they are not important
  - Discipline: Intentionally restricting your design choices to that you can work more productively at higher abstraction levels
  - The three -Y's
    - \* Hierarchy: A system is divided into modules of smaller complexity
    - \* Modularity: Having well defined functions and interfaces
    - \* Regularity: Encouraging uniformity, so modules can be easily re-used
- Bit: **B**inary **d**igit

# 2 Binary Numbers

- Powers of two:
 

$2^0 = 1$	$2^5 = 32$	$2^{10} = 1024$
$2^1 = 2$	$2^6 = 64$	$2^{11} = 2048$
$2^2 = 4$	$2^7 = 128$	$2^{12} = 4096$
$2^3 = 8$	$2^8 = 256$	$2^{13} = 8192$
$2^4 = 16$	$2^9 = 512$	$2^{14} = 16384$

- Binary to decimal conversion

$$\begin{aligned}
 10011_2 &= 2^4 \times 1 + 2^3 \times 0 + 2^2 \times 0 + 2^1 \times 1 + 2^0 \times 1 \\
 &= 16 \times 1 + 8 \times 0 + 4 \times 0 + 2 \times 1 + 1 \times 1 \\
 &= 16 + 0 + 0 + 2 + 1 = 19_{10}
 \end{aligned}$$

- Convert decimal to binary (roughly). Example with  $47_{10}$  to binary

$2^6 = 64$	is $64 \leq 47$ ?	no	0	do nothing
$2^5 = 32$	is $32 \leq 47$ ?	yes	1	$47-32=15$
$2^4 = 16$	is $16 \leq 15$ ?	no	0	do nothing
$2^3 = 8$	is $8 \leq 15$ ?	yes	1	$15-8=7$
$2^2 = 4$	is $4 \leq 7$ ?	yes	1	$7-4=3$
$2^1 = 2$	is $2 \leq 3$ ?	yes	1	$3-2=1$
$2^0 = 1$	is $1 \leq 1$ ?	yes	1	$1-1=0$ ; done!

$\Rightarrow 47_{10}$  to binary is  $0101111_2$

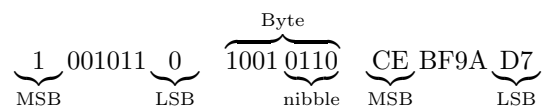
- Binary values and range

	How many values?	Range?	Example: 3-digit number
$N$ -digit decimal number	$10^N$	$[0, 10^N - 1]$	$10^3 = 1000$ possible values, range: $[0, 999]$
$N$ -bit binary number	$2^N$	$[0, 2^N - 1]$	$2^3 = 8$ possible values, range: $[0, 7] = [000_2 \text{ to } 111_2]$

- Hexadecimal (Base-16) Numbers

Decimal	Hexadecimal	Binary		Decimal	Hexadecimal	Binary
0	0	0000		8	8	1000
1	1	0001		9	9	1001
2	2	0010		10	A	1010
3	3	0011		11	B	1011
4	4	0100		12	C	1100
5	5	0101		13	D	1101
6	6	0110		14	E	1110
7	7	0111		15	F	1111

- Bits, Bytes, Nibbles...



Where MSB=Most significant Bit and LSB=Least significant Bit

- Addition in base two works exactly the same as in base 10, using carries

- Overflow

- Digital systems operate on a fixed number of bits
- Addition overflows when the result is too big to fit in the available number of bits

- Signed Binary Numbers

- Sign/Magnitude Numbers

- \* 1 sign bit,  $N - 1$  magnitude bits
- \* Sign bit is the most significant (left-most) bit
- \* Example: 4-bit sign/mag repr. of  $\pm 6$ :
  - $+6 = \mathbf{0110}$
  - $-6 = \mathbf{1110}$
- \* Range of an  $N$ -bit sign/magnitude number:
 
$$[-(2^{N-1} - 1), 2^{N-1} - 1]$$
- \* Problems:
  - Addition doesn't work
  - Two representations of 0 ( $\pm 0$ ): 1000 and 0000
  - Introduces complexity in the processor design

- One's Complement Numbers

- \* A negative number is formed by reversing the bits of the positive number (MSB still indicates the sign of the integer)

$2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$		One's Compl.	Unsigned
0	0	0	0	0	0	0	0	=	0	0
0	0	0	0	0	0	0	1	=	1	1
0	0	0	0	0	0	1	0	=	2	2
...	...	...	...	...	...	...	...	...	...	...
0	1	1	1	1	1	1	1	=	127	127
1	0	0	0	0	0	0	0	=	-127	128
1	0	0	0	0	0	0	1	=	-126	129
...	...	...	...	...	...	...	...	...	...	...
1	1	1	1	1	1	0	1	=	-2	253
1	1	1	1	1	1	1	0	=	-1	254
1	1	1	1	1	1	1	1	=	-0	255

\* Range of  $n$ -bit number:  $[-2^{n-1} - 1, 2^{n-1} - 1]$ , 8 bits:  $[-127, 127]$

\* Addition: Done using binary addition with end-around carry. If there is a carry out of the MSB of the sum, this bit must be added to the LSB of the sum

– Two's Complement Numbers

\* Don't have same problems as sign/magnitude numbers:

- addition works
- Single representation for 0

\* Has advantages over one's complement:

- Has a single 0 representation
- Eliminates the end-around carry operation required in one's complement addition.

\* A negative number is formed by reversing the bits of the positive number (MSB still indicates the sign of the integer) and adding 1:

$2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$		Two's Compl.	Unsigned
0	0	0	0	0	0	0	0	=	0	0
0	0	0	0	0	0	0	1	=	1	1
0	0	0	0	0	0	1	0	=	2	2
...	...	...	...	...	...	...	...	...	...	...
0	1	1	1	1	1	1	1	=	127	127
1	0	0	0	0	0	0	0	=	-128	128
1	0	0	0	0	0	0	1	=	-127	129
...	...	...	...	...	...	...	...	...	...	...
1	1	1	1	1	1	0	1	=	-3	253
1	1	1	1	1	1	1	0	=	-2	254
1	1	1	1	1	1	1	1	=	-1	255

\* Same as unsigned binary, but the most significant bit (MSB) has value of  $-2^{N-1}$

- Most positive 4-bit number: 0111
- Most negative 4-bit number: 1000

\* The most significant bit still indicates the sign (1=neg., 0=pos.)

\* Range of an  $N$ -bit two's comp. number:  $[-2^{N-1}, 2^{N-1} - 1]$ , 8 bits:  $[-128, 127]$

- Increasing bit width (assume from  $N$  to  $M$ , with  $M > N$ ):

- Sign-extension
  - \* Sign bit is copied into MSB
  - \* Number value remains the same
  - \* Give correct result for two's compl. numbers
  - \* Example 1:
    - 4-bit representation of  $3 = 0011$
    - 8-bit sign-extended value: **00000011**
  - \* Example 2:
    - 4-bit representation of  $-5 = 1011$
    - 8-bit sign-extended value: **11111011**
- Zero-extension
  - \* Zeros are copied into MSB
  - \* Value will change for negative numbers
  - \* Example 1:
    - 4-bit value:  $0011_2 = 3_{10}$
    - 8-bit zero-extended value: **00000011<sub>2</sub> = 3<sub>10</sub>**
  - \* Example 2:
    - 4-bit value:  $1011_2 = -5_{10}$
    - 8-bit zero-extended value: **00001011<sub>2</sub> = 11<sub>10</sub>**

### 3 Short Introduction to Electrical Engineering (EE Perspective)

- The goal of circuit design is to optimize:
  - Area: Net circuit area is proportional to the cost of the device
  - Speed/Throughput: We want circuits that work faster, or do more
  - Power/Energy
    - \* Mobile devices need to work with a limited power supply
    - \* High performance devices dissipate more than  $100\text{W}/\text{cm}^2$
  - Design time
    - \* Designers are expensive
    - \* The competition will not wait for you
- (Frank's) Principles for engineering
  - Good engineers are lazy: They do not want to work unnecessarily, be creative
  - They know how to ask the question “why”? : take nothing for granted
  - Engineering is not a religion: Use what works best for you
  - Keep it simple and stupid: Engineers' job is to manage complexity