

1 Organization and Introduction

- The Art of managing complexity
 - Abstraction: Hiding details when they are not important
 - Discipline: Intentionally restricting your design choices to that you can work more productively at higher abstraction levels
 - The three -Y's
 - * Hierarchy: A system is divided into modules of smaller complexity
 - * Modularity: Having well defined functions and interfaces
 - * Regularity: Encouraging uniformity, so modules can be easily re-used
- Bit: **B**inary **d**igit

2 Binary Numbers

- Powers of two:

$2^0 = 1$	$2^5 = 32$	$2^{10} = 1024$
$2^1 = 2$	$2^6 = 64$	$2^{11} = 2048$
$2^2 = 4$	$2^7 = 128$	$2^{12} = 4096$
$2^3 = 8$	$2^8 = 256$	$2^{13} = 8192$
$2^4 = 16$	$2^9 = 512$	$2^{14} = 16384$
- Binary to decimal conversion
$$\begin{aligned}10011_2 &= 2^4 \times 1 + 2^3 \times 0 + 2^2 \times 0 + 2^1 \times 1 + 2^0 \times 1 \\&= 16 \times 1 + 8 \times 0 + 4 \times 0 + 2 \times 1 + 1 \times 1 \\&= 16 + 0 + 0 + 2 + 1 = 19_{10}\end{aligned}$$
- Convert decimal to binary (roughly). Example with 47_{10} to binary

$2^6 = 64$	is $64 \leq 47$?	no	0	do nothing
$2^5 = 32$	is $32 \leq 47$?	yes	1	$47-32=15$
$2^4 = 16$	is $16 \leq 15$?	no	0	do nothing
$2^3 = 8$	is $8 \leq 15$?	yes	1	$15-8=7$
$2^2 = 4$	is $4 \leq 7$?	yes	1	$7-4=3$
$2^1 = 2$	is $2 \leq 3$?	yes	1	$3-2=1$
$2^0 = 1$	is $1 \leq 1$?	yes	1	$1-1=0$; done!

 $\Rightarrow 47_{10}$ to binary is 0101111_2
- Binary values and range
 - N -digit decimal number
 - * How many values: 10^N
 - * Range: $[0, 10^N - 1]$
 - * Example (3-digit number): $10^3 = 1000$ possible values, range: $[0, 999]$
 - N -bit binary number
 - * How many values: 2^N

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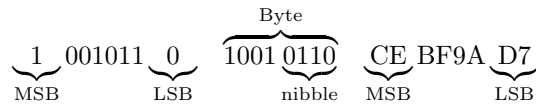
* Range: $[0, 2^N - 1]$

* Example (3-digit number): $2^3 = 8$ possible values, range: $[0, 7] = [000_2 \text{ to } 111_2]$

- Hexadecimal (Base-16) Numbers

Decimal	Hexadecimal	Binary		Decimal	Hexadecimal	Binary
0	0	0000		8	8	1000
1	1	0001		9	9	1001
2	2	0010		10	A	1010
3	3	0011		11	B	1011
4	4	0100		12	C	1100
5	5	0101		13	D	1101
6	6	0110		14	E	1110
7	7	0111		15	F	1111

- Bits, Bytes, Nibbles...



Where MSB=Most significant Bit and LSB=Least significant Bit

- Addition in base two works exactly the same as in base 10, using carries

- Overflow

- Digital systems operate on a fixed number of bits
- Addition overflows when the result is too big to fit in the available number of bits

- Signed Binary Numbers

- Sign/Magnitude Numbers

- * 1 sign bit, $N - 1$ magnitude bits
- * Sign bit is the most significant (left-most) bit
- * Example: 4-bit sign/mag repr. of ± 6 :
 - $+6 = 0110$
 - $-6 = 1110$
- * Range of an N -bit sign/magnitude number:
 - $[-(2^{N-1} - 1), 2^{N-1} - 1]$
- * Problems:
 - Addition doesn't work
 - Two representations of 0 (± 0): 1000 and 0000
 - Introduces complexity in the processor design

- One's Complement Numbers

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* A negative number is formed by reversing the bits of the positive number (MSB still indicates the sign of the integer)											
2 ⁷	2 ⁶	2 ⁵	2 ⁴	2 ³	2 ²	2 ¹	2 ⁰		One's Compl.	Unsigned	
0	0	0	0	0	0	0	0	=	0	0	
0	0	0	0	0	0	0	1	=	1	1	
0	0	0	0	0	0	1	0	=	2	2	
...	
0	1	1	1	1	1	1	1	=	127	127	
1	0	0	0	0	0	0	0	=	-127	128	
1	0	0	0	0	0	0	1	=	-126	129	
...	
1	1	1	1	1	1	0	1	=	-2	253	
1	1	1	1	1	1	1	0	=	-1	254	
1	1	1	1	1	1	1	1	=	-0	255	
* Range of n -bit number: $[-2^{n-1}-1, 2^{n-1}-1]$, 8 bits: $[-127, 127]$											
* Addition: Done using binary addition with end-around carry. If there is a carry out of the MSB of the sum, this bit must be added to the LSB of the sum											
- Two's Complement Numbers											
* Don't have same problems as sign/magnitude numbers:											
· addition works											
· Single representation for 0											
* Has advantages over one's complement:											
· Has a single 0 representation											
· Eliminates the end-around carry operation required in one's complement addition.											
* A negative number is formed by reversing the bits of the positive number (MSB still indicates the sign of the integer) and adding 1:											
2 ⁷	2 ⁶	2 ⁵	2 ⁴	2 ³	2 ²	2 ¹	2 ⁰		Two's Compl.	Unsigned	
0	0	0	0	0	0	0	0	=	0	0	
0	0	0	0	0	0	0	1	=	1	1	
0	0	0	0	0	0	1	0	=	2	2	
...	
0	1	1	1	1	1	1	1	=	127	127	
1	0	0	0	0	0	0	0	=	-128	128	
1	0	0	0	0	0	0	1	=	-127	129	
...	
1	1	1	1	1	1	0	1	=	-3	253	
1	1	1	1	1	1	1	0	=	-2	254	
1	1	1	1	1	1	1	1	=	-1	255	
* Same as unsigned binary, but the most significant bit (MSB) has value of -2^{N-1}											
· Most positive 4-bit number: 0111											
· Most negative 4-bit number: 1000											
* The most significant bit still indicates the sign (1=neg., 0=pos.)											
* Range of an N -bit two's comp. number: $[-2^{N-1}, 2^{N-1}-1]$, 8 bits: $[-128, 127]$											

- Increasing bit width (assume from N to M , with $M > N$):

- Sign-extension

- * Sign bit is copied into MSB
- * Number value remains the same
- * Give correct result for two's compl. numbers
- * Example 1:

- 4-bit representation of $3 = 0011$
- 8-bit sign-extended value: **00000011**

- * Example 2:

- 4-bit representation of $-5 = 1011$
- 8-bit sign-extended value: **11111011**

- Zero-extension

- * Zeros are copied into MSB
- * Value will change for negative numbers
- * Example 1:

- 4-bit value: $0011_2 = 3_{10}$
- 8-bit zero-extended value: **00000011**₂ = 3_{10}

- * Example 2:

- 4-bit value: $1011_2 = -5_{10}$
- 8-bit zero-extended value: **00001011**₂ = 11_{10}

3 Short Introduction to Electrical Engineering (EE Perspective)

- The goal of circuit design is to optimize:

- Area: Net circuit area is proportional to the cost of the device
- Speed/Throughput: We want circuits that work faster, or do more
- Power/Energy

- * Mobile devices need to work with a limited power supply
- * High performance devices dissipate more than $100\text{W}/\text{cm}^2$

- Design time

- * Designers are expensive
- * The competition will not wait for you

- (Frank's) Principles for engineering

- Good engineers are lazy: They do not want to work unnecessarily, be creative
- They know how to ask the question “why”?: take nothing for granted
- Engineering is not a religion: Use what works best for you
- Keep it simple and stupid: Engineers' job is to manage complexity

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<ul style="list-style-type: none"> • Building blocks for microchips <ul style="list-style-type: none"> – Conductors: Metals (Aluminium, Copper) – Insulators: Glass (SiO_2), Air – Semiconductors: Silicon (Si), Germanium (Ge) • N-type Doping: Add extra electron (negatively charged), zone becomes negatively charged • P-type Doping: Remove electron, zone becomes positively charged • Semiconductors: <ul style="list-style-type: none"> – You can “Engineer” its properties, i.e. <ul style="list-style-type: none"> * Make it P type by injecting type-III elements (b, Ga, In) * Make it N type by injecting elements from type-V (P, As) – You can combine P and N regions to each other, from a pure semiconductor – Allows you to make interesting electrical devices (Diodes, Transistors, Thyristors) • pMOS is a P type transistor, nMOS an N type transistors; combined they are a CMOS • CMOS (Properties) <ul style="list-style-type: none"> – No input current: Capacitive input, no resistive path from the input – No current when output is at logic levels: Little static power, current is needed only when switching – Electrical properties determined directly by geometry: A transistor that is 2 times larger drives twice the current – Very simple to manufacture: pMOS and nMOS can be manufactured on the same substrate • CMOS Gate Structure <ul style="list-style-type: none"> – The general form used to construct any inverting logic, such as: NOT, NAND, NOR <ul style="list-style-type: none"> * The networks may consist of transistors in series or parallel * When transistors are in parallel, the network is ON if either transistor is ON * When transistors are in series, the network is ON only if all transistors are ON – In a proper logic gate: One of the networks should be ON and the other OFF at any given time – Use the rule of conduction complements: <ul style="list-style-type: none"> * When nMOS transistors are in series, the pMOS transistor must be in parallel 	<div>Maybe add a definition or a better explanation</div>
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- Manufacturing smaller structures: some structures are already a few atoms in size
- Developing materials with better properties
- Optimizing the manufacturing steps
- New technologies

- Power consumption

- Power = Energy consumed per unit time
- Two types of power consumption:
 1. Dynamic power consumption: Power to charge transistor gate capacitances

$$P_{\text{dynamic}} = \frac{1}{2}CV_{DD}^2f$$

2. Static power consumption: Power consumed when no gates are switching, caused by the leakage current

$$P_{\text{static}} = I_{DD}V_{DD}$$

4 Combinational Circuits: Theory

- Circuit elements. A circuit consists of:

- Inputs
- Outputs
- Nodes (wires): Connections between I/O and circuit elements. To count them, look at
 - * Outputs of every circuit elements
 - * Inputs to the entire circuit
- Circuit elements

- Types of Logic Circuits

- Combinational Logic
 - * Memoryless
 - * Outputs determined by current values of inputs
 - * In some books called Combinatorial Logic
- Sequential Logic
 - * Has Memory
 - * Outputs determined by previous and current values of inputs

- Rules of Combinational Composition

- Every circuit element is itself combinational
- Every node of the circuit is either
 - * Designated as an input to the circuit
 - * Connects to exactly one output terminal of a circuit element

- The circuit contains no cyclic paths: Every path through the circuit visits each node at most once
- Boolean Equations¹

¹For a more in depth look, use the material from Diskrete Mathematik