Lab Math 6BL

UCSB Physics

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1 Introduction

In 1932 Max Kleiber publish a paper titled "Body Size and Metabolism". He was interested in the relationship between an animal's size and the amount of energy it uses. For mammals, the rate at which an animal uses energy is known as the Basal Metabolic Rate (BMR). To determine the size of the animals Kleiber focused on the animal's mass. It was known that smaller animals have lower BMRs while larger animals have higher BMRs, but the relationship between size and BMR is not linear. The relationship took the form of a power law. That is to say,

$$BMR = aW^b, \tag{1}$$

where a and b are both unknown constants and W is the mass of the animal. There was a theory based on the surface area to volume ratio of animal that suggested that the value for b is $2/3$. This was called the surface law.

We will look at the collection of data in Kleiber's original paper and use our knowledge of logarithms to see what value of b was reported by Kleiber.

2 Data

TABLE 1

BASAL METABOLISM PER SQUARE METER OF BODY SURFACE AND PER UNIT OF POWERS OF BODY WEIGHT

Group No.	Animal	W Average weight, kilograms	Cals. per 24 hrs. per animal	Formula for surface area	Heat production in 24 hours in Calories per unit of: Body surface (sq. meter)	$W^{2/3}$	$W^{0.7}$	$W^{0.73}$	$W^{0.74}$	$W^{0.75}$	$W^{0.76}$	$W^{0.77}$	$W^{0.8}$	W
1	Steer	679	8,274	$0.1081 \times W^{5/8}$	1,300	107.1	86.3	71.0	66.3	62.2	58.3	54.6	44.8	12.2
2	Steer	342	6,255	$0.1081 \times W^{5/8}$	1,465	127.9	105.5	88.5	83.5	79.2	74.2	70.0	58.8	18.3
3	Cow	388	6,421	$0.1081 \times W^{5/8}$	1,387	120.7	99.1	82.8	77.9	73.6	69.2	65.2	53.2	16.5
4	Man	64.1	1,632	$71.84 \times W^{0.425} \times L^{0.725}$*	926	101.9	88.7	78.3	75.1	72.0	69.1	66.3	58.5	25.5
5	Woman	56.5	1,349	$71.84 \times W^{0.425} \times L^{0.725}$	848	91.6	80.1	71.0	68.2	65.5	63.6	61.1	53.5	23.9
6	Sheep	45.6	1,219.9	$0.124 \times W^{0.561}$	1,163	104.8	84.1	74.9	72.1	69.4	66.8	64.3	57.3	26.7
7	Male dog	15.5	525	$0.112 \times W^{2/3}$	776	84.5	77.2	70.8	69.1	67.2	65.4	63.6	58.5	33.8
8	Female dog	11.6	443	$0.112 \times W^{2/3}$	772	86.5	79.7	74.0	72.2	70.5	68.8	67.1	62.4	38.2
9	Hen	1.96	106	$5.86 \times W^{0.5} \times L^{0.6}$	676	67.7	66.2	65.0	64.2	63.8	63.6	63.1	61.8	54.1
10	Pigeon	0.300	30.8	$0.0985 \times W^{2/3}$	697	68.7	71.5	74.1	75.0	75.9	76.9	77.8	80.6	102.6
11	Male rat	0.226	25.5	$0.1136 \times W^{2/3}$	600	68.7	72.2	75.4	76.6	77.7	79.0	80.1	83.9	112.9
12	Female rat	0.173	20.2	$0.1136 \times W^{2/3}$	572	65.1	68.9	72.7	74.0	75.3	76.6	78.0	82.4	116.6
13	Ring dove	0.150	19.5	$0.0985 \times W^{2/3}$	701	69.1	73.6	77.9	79.4	80.9	82.5	84.0	88.9	130.0
Average of all 13 groups, Calories:					914	89.6	81.0	75.1	73.3	71.8	70.3	68.9	65.0	54.7
Average of 9 groups (excluding ruminants), Calories:					730	78.2	75.3	73.2	72.6	72.1	71.7	71.2	70.1	70.8
					per cent	*per cent*	*per cent*	*per cent*	*per cent*	*per cent*	*per cent*	*per cent*	*per cent*	*per cent*
V, Coefficient of variability,† 13 groups, per cent:					±33.7	±23.9	±14.3	± 7.9	± 7.6	± 8.1	±10.0	±12.5	±21.5	±80.2
V, Coefficient of variability, 9 groups, per cent:					±16.0	±16.9	± 9.1	± 5.6	± 6.5	± 8.2	±10.0	±12.1	±19.4	±61.9
τ, Coefficient of tendency,‡ 13 groups, per cent:					+ 0.215	+ 0.163	+ 0.088	+ 0.024	+ 0.002	− 0.031	− 0.042	− 0.064	− 0.132	− 0.506
τ, Coefficient of tendency, 9 groups, per cent:					+ 0.701	+ 0.808	+ 0.355	− 0.056	− 0.187	− 0.328	− 0.456	− 0.593	− 1.003	− 3.270

* L = Body length. † V = Standard deviation in per cent of the mean. ‡ τ = Term explained on p. 320.

Sources of data:

Group 1: *Benedict and Ritzman (1927)*: The value of 1,300 Cals. per square meter given in the summary (Moulton formula) was used, and the average weight of the two steers calculated from the table given by Benedict and Ritzman (p. 60).
Group 2: *Forbes, Kriss, and Braman (1927, p. 176, table 2)*: Average of 18 determinations on 4 steers, third to tenth day of fasting.
Group 3: *Forbes, Kriss, and Braman (1927)*: Average of 10 determinations on 4 cows, second to ninth day of fast.
Groups 4 and 5: *Harris and Benedict (1919, p. 57, 65, and 67)*: Average of 136 men and 103 women.
Group 6: *Ritzman and Benedict (1931, p. 31)*: Average of 7 determinations on 7 sheep lying 18 to 50 hours after food, 21° to 25° C.
Group 7: *Kunde and Steinhaus (1926, p. 129)*: Average of 10 determinations on 10 male dogs, 8.75 to 26.8 kilograms in weight, 18 to 20 hours after food.
Group 8: *Lusk and DuBois (1924, p. 213)*: Average of 16 determinations on 11 female dogs, 9.1 to 15.8 kilograms in weight.
Group 9: *Mitchell and Haines (1927, p. 932)*: Average of 44 determinations on 14 hens of 1.63 to 3.51 kilograms weight at 54° to 99° F in darkness.
Group 10: *Benedict and Riddle (1929, p. 523)*: Average of 3 determinations on 3 pigeons of pure race at 30° C: average of day and night runs.
Groups 11 and 12: *Mitchell and Carman (1926, p. 392)*: Average of results on 23 male and 18 female rats. Periods of rest only. 28° to 31° C.
Group 13: *Benedict and Riddle (1929, p. 523)*: Average of 21 determinations on 9 ring doves of pure race, at 30° C; average of day and night runs.

Figure 1: Kleiber's Data

Above in Fig. (1) is the raw data that was printed in Kleiber's report. Because it is hard to parse the important information we did that for you. Fig. (2) is the data table you should reference. The first column is the mass of each type of animal in kilograms. The second column is how much energy the animal used in one day (the BMR) and uses Kleiber's preferred units of Cal/24hrs. Since Calories are not the SI unit of energy, we converted the second column to Joules/24hrs in the third column for you.

Mass[kg]	BMR [Cal/24hrs]	BMR [J/24hrs]
679	8274	34618416
342	6255	26170920
388	6421	26865464
64.1	1632	6828288
56.5	1349	5644216
45.6	1219.9	5104062
15.5	525	2196600
11.6	443	1853512
1.96	106	443504
0.3	30.8	128867
0.226	25.5	106692
0.173	20.2	84517
0.15	19.5	81588

Figure 2: This is the simplified table.

Report Requirements

For your report you should explicitly answer each question in the report. You should also include all your work for the exercises, which will consist of one table and two plots.

Exercise 1

Use Excel, Google Sheets, MATLAB, or any other data processing package of our choice to create your own version of the table in Fig. (2). You will want to include the "Average weight..." (Mass) column, the "Cals. per 24hrs..." (BMR) column, and the third BMR column in units of J/24hrs. **Note: In the third column, we have given you too many digits since it is an intermediary part of the calculation! In your own table you should adjust the third column to contain the correct number of significant figures.**

3 Units

Notice the units Kleiber used, Calories/24hrs. We need to convert these units to SI units. Calories are a unit of energy, and hours a unit of time. Energy per time is called power.

Note: Calories with a capital "C" are kilocalories or 1000 calories.

Furthermore 1 **Calorie** $= 1000$ **calories** $= 4184$ **Joules**

Exercise 2

Add another column to your table by converting the BMR column to the SI unit of power, Watts, which is also Joules per second. We have started the process for you by adding a J/24hrs column. Now convert that to J/s a.k.a. Watts.

4 Plot 1

Exercise 3

Now that you have converted the units, create a plot of BMR vs. Mass. Add a line of best fit to this plot. Make sure that it has all of the components of a good plot in order to receive full credit.

Question 1

Does this graph look linear, quadratic, exponential, or something else? How can you tell? Does it make sense to perform a linear fit with this data set, why or why not? Don't use R^2 in your answer. Does it look like the $b = 2/3$ theory could be correct?

5 Logarithm Review

Recall the following properties of logarithms. You will use these properties to determine the value of b

$$\log(AB) = \log(A) + \log(B) \tag{2}$$

$$\log(C^D) = D\log(C)` \tag{3}$$

$$10^{\log(F)} = F \tag{4}$$

Question 2

Take the log (base 10) of both sides of Eqn. (1) and use the logarithm properties above to write the equation in a linear form. Show your work.

Question 3

Your answer to Question 2 should look something like:

$$y = mx + b$$

Make a similar table to the one below into your report and fill in the missing column with the equivalent parts from you answer to Question 2.

Name	Variable	Variable
Dependent Variable	y	
Independent Variable	x	
Slope	m	
Y-Intercept	b	

Question 4

If you made a mistake above and did not take into account 1 Calorie is 1000 calories how would this affect your analysis going forward? Which of the constants a and b from Eqn. (1) would be affected by this mistake? Explain your answer using properties of logarithms.

Exercise 4

In the data table you created earlier add two more columns. These should be the logarithm of the mass and logarithm of the BMR. Fill in the table with the appropriate values.

Note: If using a computer for this calculation, the computer will often use the "log" command as the natural logarithm (Ln), or log base 2 by default. If you are using Excel or Google Sheets you will want the log10 function, not the log function.

6 Plot 2

Exercise 5

Create a new plot of the Log(BMR) vs Log(Mass). Again, make sure that it has all of the components of a good plot in order to receive full credit.

Question 5

Does this graph look linear, quadratic, exponential, or something else? Does it make sense to perform a linear fit with this data set?

Exercise 6

Create a best fit line for the new Log-Log plot.

Question 6

Does this graph look linear, quadratic, exponential, or something else? How can you tell? Does it make sense to perform a linear fit with this data set, why or why not? Don't use R^2 in your answer. Compare and contrast this plot to the previous one.

Question 7

What is the equation for your line of best fit? Using this equation, what is the value for b in Eqn. (1)?

Question 8

Does your value for b support or contradict the surface law?

7 Additional Info

While Kleiber published his report a long time ago, the exact relationship between BMR and size is still unknown. People still have many metrics in which they try to make a theory in order to predict and measure the value of b. We would like to note that this was just an exercise in some simple regression analysis and logarithm review in order to prepare you for the first lab. Please don't take anything in this document to be absolute or fact.