Nuclear matter EoS with minimal length
($\sigma\omega$ - model)

Minimal length $\Longrightarrow$ Komutasi field operator harus bersifat non-lokal

$$\delta(\vec{x}-\vec{y}) \longrightarrow \tilde{\delta}(\vec{x}-\vec{y}) = \frac{1}{\pi(\vec{x}-\vec{y})} \sin\left[\frac{\pi(\vec{x}-\vec{y})}{2\hbar\sqrt{\beta}}\right]$$

referensi utama:

1) Kh. Nouicer, J. Phys. A: Math. Gen 38 (2005)
2) Frassino - Panella, PRD 85 (2012), 045030
3) Panella, PRD 76, 045012 (2007)

(1) Momentum modified to (to all orders in β):

$$p_i \longrightarrow \bar{p}_i(p) = \frac{1}{\sqrt{\beta}\, p}\left[\tan^{-1}(\sqrt{\beta} p)\right] p_i \quad \text{--- (1)}$$

(2) GUP ⊕ Dirac equation (in momentum rep):

$$\left(\gamma^{\mu}\bar{p}_{\mu} - m^{*}\right)\varphi(\vec{p}) = 0 \quad \text{--- (2)}$$

(3) State "plane-wave" = Maximized localized state:

$$\psi(\vec{x},t) = A\, e^{-i(\omega_p t - \vec{\bar{p}}\cdot\vec{x})} \quad \text{--- (3)}$$

(4) Dispersion relation

from (2) one will get:

$$(\bar{p}_0)^2 - \bar{p}^2 - m^{*2} = 0$$

$$\omega_p^2 = \bar{p}^2 + m^{*2}$$

$$\omega_p^2 = \frac{1}{\beta}\left[\tan^{-1}\sqrt{\beta} p\right]^2 + m^{*2} \quad \text{--- (4)}$$

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(5) Normalisasi Condition Yang disunakan

$$\sum_{\alpha=1}^{2} U^{\dagger}(\vec{p},\alpha)\, U(\vec{p},\alpha) = 2.$$

$$\sum_{\alpha=1}^{2} \bar{U}(\vec{p},\alpha)\, U(\vec{p},\alpha) = \frac{2m^{*}}{\omega_p} \qquad \text{---(5)}$$

(6) Field operator (2'nd quantization with GUP)

expand in maximilally localized state

$$\psi(\vec{x},t) = \sum_{\alpha} \int \frac{d^3p}{(1+\beta p^2)(2\pi)^3} N_p \Big[b(\vec{p},\alpha)\, e^{i(-\vec{p}\cdot\vec{x}+\omega_p t)}\, U(\vec{p},\alpha) + d^{\dagger}(\vec{p},\alpha)\, e^{-i(-\vec{p}\cdot\vec{x}+\omega_p t)}\, V(\vec{p},\alpha) \Big]$$

demanding the commutation (ref- Greiner & Ryder):

$$\{\psi_a(\vec{x},t), \psi_b^{\dagger}(\vec{y},t)\} = \delta_{ab}\, \tilde{\delta}(\vec{x}-\vec{y})$$

With Commutation of agnihilation & creation operator:

$$\boxed{\{ b(\vec{p},\alpha), b^{\dagger}(\vec{q},\tilde{\alpha})\} = (2\pi)^3 \delta_{\alpha\tilde{\alpha}}\, \delta(\vec{p}-\vec{q}) = \{d(\vec{p},\alpha), d^{\dagger}(\vec{q},\tilde{\alpha})\}}$$

left-hand side:

$$\{\psi_a(\vec{x},t), \psi_b^{\dagger}(\vec{y},t)\} = \sum_{\alpha\beta} \int \frac{d^3p\, d^3q\, N_p N_q}{(2\pi)^6 (1+\beta p^2)(1+\beta q^2)} \Big[(2\pi)^3 \delta_{\alpha\beta}\, \delta(\vec{p}-\vec{q})\, U_a(\vec{p},\alpha) U_b^{\dagger}(\vec{q},\beta)\, e^{i(\vec{q}\cdot\vec{y}-\vec{p}\cdot\vec{x})} e^{i(\omega_p-\omega_q)t}$$
$$+ (2\pi)^3 \delta_{\alpha\beta}\, V_a(\vec{p},\alpha) V_b^{\dagger}(\vec{q},\beta)\, \delta(\vec{p}-\vec{q})\, e^{i(\vec{q}\cdot\vec{x}-\vec{p}\cdot\vec{y})} e^{i(\omega_q-\omega_p)t} \Big]$$

$$\{\psi_a(\vec{x},t), \psi_b^\dagger(\vec{y},t)\} = \sum_\alpha \int \frac{d^3p \; N_p^2}{(2\pi)^3 (1+\beta p^2)^2} \Big[U_a(\vec{p},\alpha) U_b^\dagger(\vec{p},\alpha)\, e^{i\vec{p}\cdot(\vec{x}-\vec{y})} + \mathcal{V}_a(\vec{p},\alpha) \mathcal{V}_b^\dagger(\vec{p},\alpha)\, e^{i\vec{p}\cdot(\vec{x}} \Big]$$

$$= \int \frac{d^3p \; N_p^2 \; e^{i\vec{p}\cdot\Delta\vec{x}}}{(2\pi)^3 (1+\beta p^2)^2} \Big[\sum_\alpha \Big(U_a(\vec{p},\alpha) U_b^\dagger(\vec{p},\alpha) + \mathcal{V}_a(\vec{p},\alpha) \mathcal{V}_b^\dagger(\vec{p},\alpha) \Big) \Big]$$

$$= \int \frac{d^3p \; N_p^2}{(2\pi)^3 (1+\beta p^2)^2} \Big[2\delta_{ab} \Big] e^{i\vec{p}\cdot(\vec{x}-\vec{y})}$$

$$= 2\int \frac{d^3p \; N_p^2}{(2\pi)^3 (1+\beta p^2)^2} e^{i\vec{p}\cdot(\vec{x}-\vec{y})}$$

Since r.h.s :

$$\{\psi_a(\vec{x},t), \psi_b^\dagger(\vec{y},t)\} = \delta_{ab}\, \tilde{\delta}(\vec{x}-\vec{y})$$

$$= \delta_{ab} \int \frac{d^3p}{(2\pi)^3 (1+\beta p^2)^2} e^{i\vec{p}\cdot(\vec{x}-\vec{y})}$$

So we have $2N_p^2 = 1$ or $N_p = \frac{1}{\sqrt{2}}$

accordingly:

$$\psi(\vec{x},t) = \sum_\alpha \int \frac{d^3p}{(1+\beta p^2)(2\pi)^3 \sqrt{2}} \Big[b(\vec{p},\alpha)\, e^{ip^\mu x_\mu}\, U(\vec{p},\alpha) + d^\dagger(\vec{p},\alpha)\, e^{-ip^\mu x_\mu}\, \mathcal{V}(\vec{p},\alpha) \Big]$$

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7.) $\sigma\omega$ Lagrangian

$$\mathcal{L} = \bar{\psi}\left[i\gamma^{\mu}(\partial_{\mu} + i g_{\omega}\omega_{\mu}) - \overbrace{(m - g_{\sigma}\sigma)}^{m^*}\right]\psi + \frac{1}{2}\left[(\partial_{\mu}\sigma)(\partial^{\mu}\sigma) - m_{\sigma}^2\sigma^2\right] - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_{\omega}^2\omega_{\mu}\omega^{\mu}$$

in the mean relativistic scheme :

7.1) Dirac equation for nucleon

$$\left[\gamma_{\mu}(i\partial^{\mu} - g_{\omega}\omega^{\mu}) - (m - g_{\sigma}\sigma)\right]\psi(x) = 0 \quad \text{---}(7.1)$$

dengan $\psi(x)$ spinor expand in maximally localized state

7.2) Condition for meson $\sigma\omega$ (under MRF scheme):

$$m_{\sigma}^2\langle\sigma\rangle = g_{\sigma}\langle\bar{\psi}\psi\rangle$$

$$m_{\omega}^2\langle\omega_0\rangle = g_{\omega}\langle\psi^{\dagger}\psi\rangle \quad \text{---}(7.2)$$

$$m_{\omega}^2\langle\omega_k\rangle = g_{\omega}\langle\bar{\psi}\gamma_k\psi\rangle = 0$$

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8.) Equation of State

8.1) Scalar density $\rho_s = \langle \bar{\psi} \psi \rangle$

$$\langle \bar{\psi}\psi \rangle = \sum_{\alpha\beta} \int \frac{d^3p \, d^3q}{(1+\beta p^2)(1+\beta q^2)(2\pi)^3 2} \bar{U}(\vec{p},\alpha) U(\vec{q},\beta) e^{i x^n (\bar{p}_n - \bar{q}_n)} \delta(\vec{p}-\vec{q}) \delta_{\alpha\beta}$$

$$= \int \frac{d^3 p}{2(2\pi)^3 (1+\beta p^2)^2} \underbrace{\sum_{\alpha} \bar{U}(\vec{p},\alpha) U(\vec{p},\alpha)}_{\frac{2M^*}{\omega_p}}$$

$$= \int \frac{d^3 p}{(2\pi)^3 (1+\beta p^2)^2} \frac{M^*}{\omega_p}$$

$$= \int \frac{d^3 p}{(2\pi)^3 (1+\beta p^2)^2} \frac{M - g_\sigma \sigma}{\sqrt{\bar{p}^2 + M^{*2}}}$$

$$= \int \frac{d^3 p}{(2\pi)^3 (1+\beta p^2)^2} \frac{M - g_\sigma \sigma}{\left[\frac{1}{\beta}\left(\tan^{-1}\sqrt{\beta} p\right)^2 + (M - g_\sigma \sigma)^2\right]^{1/2}}$$

$$\boxed{\rho_s = \langle \bar{\psi}\psi \rangle = \frac{1}{2\pi^2} \int_0^{p_F} \frac{p^2 \, dp}{(1+\beta p^2)^2} \frac{M - g_\sigma \sigma}{\left[\frac{1}{\beta}\left(\tan^{-1}\sqrt{\beta} p\right)^2 + (M - g_\sigma \sigma)^2\right]^{1/2}} \quad \cdots (8.1)}$$

Scalar density

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(8.2) Vector density $\rho_0 = \langle \psi^\dagger \psi \rangle$

$$\langle \psi^\dagger \psi \rangle = \sum_{\alpha\beta} \int \frac{d^3p\, d^3q}{2(2\pi)^3 (1+\beta p^2)(1+\beta q^2)} U^\dagger(\vec{p},\alpha)\, U(\vec{q},\beta)\, e^{ix^n(p_n - q_n)}\, \delta(\vec{p}-\vec{q})\, \delta_{\alpha\beta}$$

$$= \int \frac{d^3p}{2(2\pi)^3 (1+\beta p^2)^2} \underbrace{\sum_\alpha U^\dagger(\vec{p},\alpha)\, U(\vec{q},\alpha)}_{2}$$

$$= \int \frac{d^3p}{(2\pi)^3 (1+\beta p^2)^2}$$

$$= \frac{1}{2\pi^2} \int_0^{p_F} \frac{p^2\, dp}{(1+\beta p^2)^2}$$

$$= \frac{1}{2\pi^2} \left[\frac{\tan^{-1}[\sqrt{\beta}\, p_F] - \frac{\sqrt{\beta}\, p_F}{1+\beta p_F^2}}{2\beta^{3/2}} \right]$$

$$\boxed{\rho_0 = \langle \psi^\dagger \psi \rangle = \frac{1}{4\pi^2 (1+\beta p_F^2) \beta^{3/2}} \left[(1+\beta p_F^2) \tan^{-1}(\sqrt{\beta}\, p_F) - \sqrt{\beta}\, p_F \right] \quad \cdots (8.2)}$$

Vector density

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8.3) Energy density

menggunakan bentuk (4.156) pada Glendenning :

$$\epsilon = -\langle \mathcal{L} \rangle + \langle \bar{\Psi} \gamma_0 k_0 \Psi \rangle$$

with

$$k_0 = g_\omega \omega_0 + \omega_p = g_\omega \omega_0 + \sqrt{\bar{p}^2 + m^{*2}}$$

So:

(•) $\langle \bar{\Psi} \gamma_0 k_0 \Psi \rangle = \langle \Psi^\dagger k_0 \Psi \rangle$

$$= \frac{1}{2\pi^2} \int_0^{p_F} \frac{p^2\, dp}{(1+\beta p^2)^2} \left[g_\omega \omega_0 + \sqrt{\bar{p}^2 + m^{*2}} \right]$$

$$= g_\omega \omega_0 \langle \Psi^\dagger \Psi \rangle + \frac{1}{2\pi^2} \int_0^{p_F} p^2 \sqrt{\frac{1}{\beta} \left(\tan^{-1} \sqrt{\beta} p \right)^2 + m^{*2}}\, dp$$

(•) $\langle \mathcal{L} \rangle = -\frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2$

accordingly from (7-2):

$$m_\omega^2 \omega_0 = g_\omega \langle \Psi^\dagger \Psi \rangle$$

$$m_\omega^2 \omega_0^2 = g_\omega \omega_0 \langle \Psi^\dagger \Psi \rangle$$

So

$$\epsilon = -\langle \mathcal{L} \rangle + \langle \bar{\Psi} \gamma_0 k_0 \Psi \rangle$$

$$\boxed{\epsilon = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2\pi^2} \int_0^{p_F} p^2 \sqrt{\frac{1}{\beta} \left(\tan^{-1} \sqrt{\beta} p \right)^2 + (m - g_\sigma \sigma)^2}\, dp} \quad \text{--- (8.3)}$$

7 energy density

8.3) Pressure

$$P = \langle \mathcal{L} \rangle + \frac{1}{3} \langle \bar{\psi} \gamma_i k_i \psi \rangle$$

$$= -\frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{6\pi^2} \int_0^{p_F} \frac{p^2 dp}{(1+\beta p^2)^2} \frac{\bar{P}^2}{\sqrt{\bar{P}^2 + m^{*2}}}$$

$$P = -\frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{6\pi^2} \int_0^{p_F} \frac{p^2 dp}{(1+\beta p^2)^2} \frac{\frac{1}{\beta}(\tan^{-1}\sqrt{\beta}p)^2}{\left[\frac{1}{\beta}(\tan^{-1}\sqrt{\beta}p)^2 + m^{*2}\right]^{1/2}}$$

$$\boxed{P = -\frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{6\pi^2} \int_0^{p_F} \frac{p^2 (\tan^{-1}(\sqrt{\beta}p))^2}{\beta(1+\beta p^2)^2 \left[\frac{1}{\beta}(\tan^{-1}(\sqrt{\beta}p))^2 + (m - g\sigma)^2\right]^{1/2}}}$$

Pressure

d

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