Progress Report

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Anisotropic GUP

GUP versi Kovarian

$$[\hat{x}^{\mu}, \hat{p}_{\nu}] = i\delta^{\mu}_{\nu}(1 + \beta \hat{p}^{\alpha}\hat{p}_{\alpha})$$

Dimana bentuk operator yang memenuhinya adalah

$$\hat{p}_{\mu} = p_{\mu}$$

$$\hat{x}_{\mu} = i(1 + \beta p^{\alpha} p_{\alpha})\frac{\partial}{\partial p_{\mu}}$$

Agar kerja operator tetap simetri, Integral seluruh ruang fase bertransformasi menjadi

$$\int_{\infty}^{\infty} d^3p \; d^3x \rightarrow \int_{\infty}^{\infty} \frac{d^3p}{(1 + \beta_k p_k^2)^3} d^3x$$

dimana $k = 1, 2, 3$

Misalkan panjang minimum bersifat anisotropi, ada perbedaan ketidakpastian untuk arah yang berbeda.
Asumsikan anisotropi memiliki simetri silindris dimana $\beta_x = \beta_y = \beta_r$, $p_x^2 + p_y^2 = p_r^2$ sehingga

$$\begin{aligned}
\beta_k p_k^2 &= \beta_x p_x^2 + \beta_y p_y^2 + \beta_z p_z^2 \\
&= \beta_r p_r^2 + \beta_z p_z^2 \\
&= \beta_r p^2 \sin^2\theta + \beta_z p^2 \cos^2\theta \\
&= \beta_r p^2(\sin^2\theta + \cos^2\theta) + (\beta_z - \beta_r)p^2\cos^2\theta \\
&= \beta_r p^2\left[1 + \alpha\cos^2\theta\right]
\end{aligned}$$

dimana $\alpha \equiv \frac{\beta_z - \beta_r}{\beta_z}$ maka

$$\int_{\infty}^{\infty} d^3p\ d^3x \to \int_{\infty}^{\infty} \frac{d^3p}{(1 + \beta_r p^2\left[1 + \alpha\cos^2\theta\right])^3} d^3x \tag{1}$$

Pada teori QHD standar, EOS didapatkan dengan menyelesaikan

$$\phi_0 = \frac{g_s}{m_s^2}\rho_s \tag{2}$$

$$V_0 = \frac{g_v}{m_\omega^2}\rho_0 \tag{3}$$

$$m^* = M - \frac{g_s^2}{m_s^2}\rho_s \tag{4}$$

$$\varepsilon = \varepsilon_{kin} + \frac{1}{2}m_s^2\phi_0^2 + \frac{1}{2}m_\omega^2 V_0^2 \tag{5}$$

$$P = P_{kin} - \frac{1}{2}m_s^2\phi_0^2 + \frac{1}{2}m_\omega^2 V_0^2 \tag{6}$$

dimana

$$\rho_0 = \frac{2}{(2\pi)^3} \int_0^{k_f} d^3k \tag{7}$$

$$\rho_s = \frac{2}{(2\pi)^3} \int_0^{k_f} d^3k \frac{m^*}{\sqrt{k^2 + m^{*2}}} \tag{8}$$

$$\varepsilon_{kin} = \frac{2}{(2\pi)^3} \int_0^{k_f} d^3k \sqrt{k^2 + m^{*2}} \tag{9}$$

$$P_{kin} = \frac{1}{3} \left(\frac{2}{(2\pi)^3} \int_0^{k_f} d^3k \frac{k^2}{\sqrt{k^2 + m^{*2}}} \right) \tag{10}$$

Besaran-besaran tersebut perlu ditransformasi. Makasetelah ditransformasi akan menjadi

Number Density

$$\begin{aligned}\rho_0 &= \frac{2}{(2\pi)^3}\int_0^{k_f}\frac{d^3k}{(1+\beta_k k_k^2)^3} \\ &= \frac{2}{(2\pi)^2}\int_0^{k_f} dk \int_0^{\pi} d\theta \frac{k^2\sin\theta}{[1+k^2\beta_r(1+\alpha\cos^2\theta)]^3} \\ &\approx \frac{k_f^3}{3\pi^2} - \frac{k_f^5(3+\alpha)\beta}{5\pi^2} \end{aligned} \tag{11}$$

Scalar Density

$$
\begin{aligned}
\rho_s &= \frac{2}{(2\pi)^3}\int_0^{k_f}\frac{d^3k}{(1+\beta_k k_k^2)^3}\frac{m^*}{\sqrt{k^2+m^{*2}+2\beta_r(1+\alpha\cos^2\theta)k^4}} \\
&= \frac{2}{(2\pi)^2}\int_0^{k_f}dk\int_0^{\pi}d\theta\frac{k^2m^*}{\sqrt{k^2+m^{*2}+2\beta_r(1+\alpha\cos^2\theta)k^4}}\frac{\sin\theta}{[1+k^2\beta_r(1+\alpha\cos^2\theta)]^3} \\
&= \rho_s^{(0)}+\Delta\rho_s \qquad (12)
\end{aligned}
$$

dimana

$$
\rho_s^{(0)}=\frac{1}{2\pi}\left[k_fm^*\sqrt{k_f^2+m^{*2}}-m^{*3}\ln\left(\frac{k_f+\sqrt{k_f^2+m^{*2}}}{m^*}\right)\right] \qquad (13)
$$

$$
\Delta\rho_s=-\frac{m(3+\alpha)\beta}{3\pi^2}\left[\frac{k_f(k_f^4-k_f^2m^{*2}-3m^{*4})}{\sqrt{k_f^2+m^{*2}}}+3m^{*4}\ln\left(\frac{k_f+\sqrt{k_f^2+m^{*2}}}{m^*}\right)\right] \qquad (14)
$$

Energy Density

$$
\begin{aligned}
\varepsilon_{kin} &= \frac{2}{(2\pi)^3}\int_0^{k_f}\frac{d^3k}{(1+\beta_k k_k^2)^3}\sqrt{k^2+m^{*2}+2\beta_r(1+\alpha\cos^2\theta)k^4} \\
&= \frac{2}{(2\pi)^2}\int_0^{k_f}dk\int_0^{\pi}d\theta\ k^2\sqrt{k^2+m^{*2}+2\beta_r(1+\alpha\cos^2\theta)k^4} \\
&\quad\times\frac{\sin\theta}{[1+k^2\beta_r(1+\alpha\cos^2\theta)]^3} \\
&= \varepsilon^{(0)}+\Delta\varepsilon \qquad (15)
\end{aligned}
$$

dimana

$$
\varepsilon^{(0)}=\frac{1}{8\pi}\left[k_f\sqrt{k_f^2+m^{*2}}(2k_f^2+m^{*2})-m^{*4}\ln\left(\frac{k_f+\sqrt{k_f^2+m^{*2}}}{m^*}\right)\right] \qquad (16)
$$

$$
\begin{aligned}
\Delta\varepsilon &= -\frac{(3+\alpha)\beta}{18\pi^2}\left[k_f\sqrt{k_f^2+m^{*2}}\,(2k_f^4+2k_f^2m^{*2}-3m^{*4})\right. \\
&\quad\left.+3m^{*6}\ln\left(\frac{k_f+\sqrt{k_f^2+m^{*2}}}{m^*}\right)\right] \qquad (17)
\end{aligned}
$$

Pressure(Isotrophy)

$$\begin{aligned} P_{kin} &= \frac{1}{3}\left(\frac{2}{(2\pi)^3}\int_0^{k_f}\frac{d^3k}{(1+\beta_k k_k^2)^3}\frac{k^2}{\sqrt{k^2+m^{*2}+2\beta_r(1+\alpha\cos^2\theta)k^4}}\right) \\ &= \frac{1}{3}\frac{1}{2\pi^2}\int_0^{k_f}dk\int_0^{\pi}d\theta\,\frac{k^4}{\sqrt{k^2+m^{*2}+2\beta_r(1+\alpha\cos^2\theta)k^4}} \\ &\quad\times\frac{\sin\theta}{[1+k^2\beta_r(1+\alpha\cos^2\theta)]^3} \\ &= P^{(0)}+\Delta P \end{aligned} \tag{18}$$

dimana

$$P^{(0)}=\frac{1}{3}\,\frac{1}{8\pi^2}\left[k_f\sqrt{k_f^2+m^{*2}}(2k_f^2-3m^{*2})+3m^{*4}\ln\left(\frac{k_f+\sqrt{k_f^2+m^{*2}}}{m^*}\right)\right] \tag{19}$$

$$\begin{aligned} \Delta P &= -\frac{1}{3}\,\frac{(3+\alpha)\beta}{72\pi^2}\left[\frac{k_f(-16k_f^6+10k_f^4m^{*2}-25k_f^2m^{*4}-75m^{*6})}{\sqrt{k_f^2+m^{*2}}}\right. \\ &\quad\left.+75m^{*6}\ln\left(\frac{k_f+\sqrt{k_f^2+m^{*2}}}{m^*}\right)\right] \end{aligned} \tag{20}$$

Anisotrhophy Pressure (radial term)

$$
\begin{aligned}
P_r &= \frac{2}{(2\pi)^3}\int_0^{k_f}\frac{d^3k}{(1+\beta_k k_k^2)^3}\frac{\cos^2\theta\ k^2}{\sqrt{k^2+m^{*2}+2\beta_r(1+\alpha\cos^2\theta)k^4}} \\
&= \frac{1}{2\pi^2}\int_0^{k_f}dk\int_0^{\pi}d\theta\ \frac{\cos^2\theta\ k^4}{\sqrt{k^2+m^{*2}+2\beta_r(1+\alpha\cos^2\theta)k^4}}\ \frac{\sin\theta}{[1+k^2\beta_r(1+\alpha\cos^2\theta)]^3} \\
&= P^{(0)}+\Delta P_r \qquad (21)
\end{aligned}
$$

dimana

$$
P^{(0)}=\frac{1}{3}\ \frac{1}{8\pi^2}\left[k_f\sqrt{k_f^2+m^{*2}}(2k_f^2-3m^{*2})+3m^{*4}\ln\left(\frac{k_f+\sqrt{k_f^2+m^{*2}}}{m^*}\right)\right] \qquad (22)
$$

$$
\begin{aligned}
\Delta P_r &= \frac{(5+3\alpha)\beta}{360\pi^2}\left[\frac{k_f(-16k_f^6+10k_f^4m^{*2}-25k_f^2m^{*4}-75m^{*6})}{\sqrt{k_f^2+m^{*2}}}\right. \\
&\left.+75m^{*6}\ln\left(\frac{k_f+\sqrt{k_f^2+m^{*2}}}{m^*}\right)\right]
\end{aligned}
$$

(23)

Anisotrhophy Pressure (tangential term)

$$
\begin{aligned}
P_t &= \frac{2}{(2\pi)^3}\int_0^{k_f} \frac{d^3k}{(1+\beta_k k_k^2)^3}\frac{\frac{1}{2}(1-\cos^2\theta)k^2}{\sqrt{k^2+m^{*2}+2\beta_r(1+\alpha\cos^2\theta)k^4}} \\
&= \frac{1}{2\pi^2}\int_0^{k_f} dk\int_0^{\pi} d\theta\ \frac{\frac{1}{2}(1-\cos^2\theta)k^4}{\sqrt{k^2+m^{*2}+2\beta_r(1+\alpha\cos^2\theta)k^4}}\ \frac{\sin\theta}{[1+k^2\beta_r(1+\alpha\cos^2\theta)]^3} \\
&= P^{(0)}+\Delta P_t \qquad (24)
\end{aligned}
$$

dimana

$$
P^{(0)} = \frac{1}{3}\ \frac{1}{8\pi^2}\left[k_f\sqrt{k_f^2+m^{*2}}(2k_f^2-3m^{*2})+3m^{*4}\ln\left(\frac{k_f+\sqrt{k_f^2+m^{*2}}}{m^*}\right)\right] \qquad (25)
$$

$$
\begin{aligned}
\Delta P_t &= \frac{(5+\alpha)\beta}{360\pi^2}\left[\frac{k_f(-16k_f^6+10k_f^4m^{*2}-25k_f^2m^{*4}-75m^{*6})}{\sqrt{k_f^2+m^{*2}}}\right. \\
&\qquad \left. +75m^{*6}\ln\left(\frac{k_f+\sqrt{k_f^2+m^{*2}}}{m^*}\right)\right]
\end{aligned}
$$

(26)

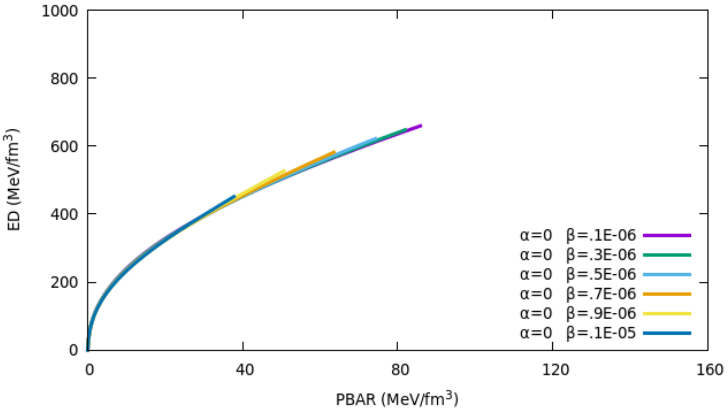


Figure: Plot *enegry density* ε dan tekanan rata-rata $P = \frac{1}{3}(P_r + 2P_t)$ dengan variasi parameter anisotropi $\alpha \equiv \frac{\beta_z - \beta_r}{\beta_z}$ dan parameter GUP radial β_r. Pada gambar ini, hanya divariasikan parameter GUP β, sementara $\alpha = 0$ di fix.

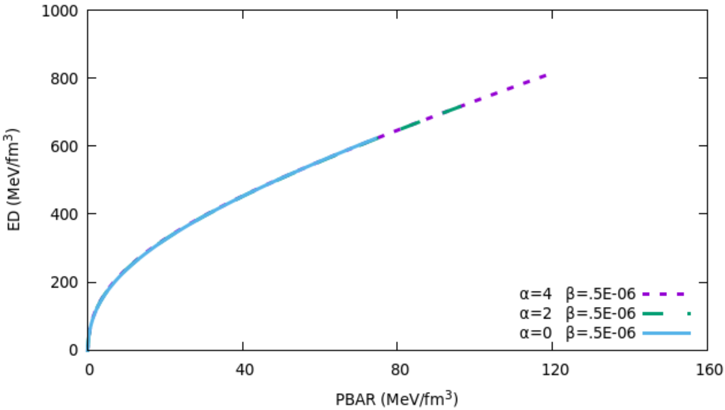
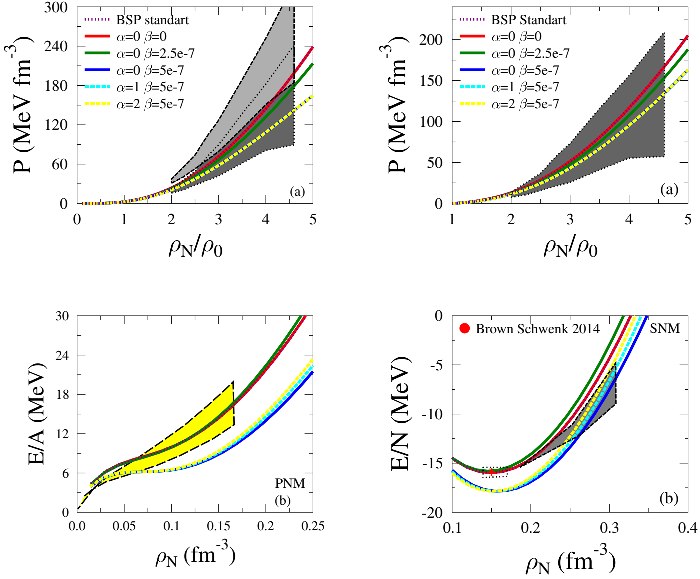


Figure: Pada gambar ini, hanya divariasikan parameter GUP α, sementara di fix $\beta = 5 \times 10^7\ \mathrm{MeV}^{-2}$.



BSP standart
α=0 β=0
α=0 β=2.5e-7
α=0 β=5e-7
α=1 β=5e-7
α=2 β=5e-7
P (MeV fm^-3)
ρN/ρ0
(a)
BSP Standart
E/A (MeV)
ρN (fm^-3)
PNM
(b)
Brown Schwenk 2014
SNM
E/N (MeV)

Minimmal Length Update

Update Transformasi yang baru

$$\bar{\rho} = \frac{2}{(2\pi)^3} \int_0^{k_F} \frac{d^3k}{[1+\beta k^2]^2} \tag{27}$$

$$\bar{\rho_s} = \frac{2}{(2\pi)^3} \int_0^{k_F} \frac{d^3k}{[1+\beta k^2]^2} \frac{m^*}{[\frac{1}{\beta}(\tan^{-1}\sqrt{\beta}k)^2 + m^{*2}]^{1/2}} \tag{28}$$

$$\bar{\varepsilon} = \frac{2}{(2\pi)^3} \int_0^{k_F} \frac{d^3k}{[1+\beta k^2]^2} \left[\frac{1}{\beta}(\tan^{-1}\sqrt{\beta}k)^2 + m^{*2}\right]^{1/2} \tag{29}$$

$$\bar{P} = \frac{2}{(2\pi)^3} \int_0^{k_F} \frac{d^3k}{[1+\beta k^2]^2} \frac{k^2}{[\frac{1}{\beta}(\tan^{-1}\sqrt{\beta}k)^2 + m^{*2}]^{1/2}} \tag{30}$$

bandingkan dengan transformasi sebelumnya

$$\int_{\infty}^{\infty} d^3p \; d^3x \rightarrow \int_{\infty}^{\infty} \frac{d^3p}{(1+\beta_k p_k^2)^3} d^3x$$

Number Density

$$\rho = \frac{k_f^3}{3\pi^3}$$

$$\Delta\rho = -\frac{2k_f^5}{5\pi^3}$$

$$\bar{\rho} = \rho + \beta\Delta\rho$$

sedang untuk mencari k_F menjadi

$$k_F = (3\pi^2\rho)^{1/3} + \frac{2\beta}{5}(3\pi^2\rho)$$

Scalar Density

$$\rho_s = \frac{m^*}{2\pi^2}\left[k_F\sqrt{k_F^2 + m^{*2}} - m^{*2}\ln\left(\frac{k_F + \sqrt{k_F^2 + m^{*2}}}{m^{*2}}\right)\right]$$

$$\begin{aligned}\Delta\rho_s \quad = \quad & \frac{1}{24\pi^2}\left[\left(k_F^2 + m^{*2}\right)^{-1/2}\left\{k_F m^{*5} + k_F^3 m^{*3} - 10k_F^5 m^*\right\}\right. \\ & \left. -3m^{*5}\ln\left(\frac{k_F + \sqrt{k_F^2 + m^{*2}}}{m^{*2}}\right)\right]\end{aligned}$$

$$\bar{\rho_s} = \rho_s + \beta\Delta\rho_s$$

Energy density

$$\epsilon = \frac{1}{8\pi^2}\left[k_F\sqrt{k_F^2+m^{*2}}(2k_F^2+m^{*2}) - m^{*4}\ln\left(\frac{k_F+\sqrt{k_F^2+m^{*2}}}{m^{*2}}\right)\right]$$

$$\Delta\epsilon = \frac{1}{144\pi^2}\left[\sqrt{k_F^2+m^{*2}}\left\{3k_F m^{*4} - 2k_F^3 m^{*2} - 56k_F^5\right\} - 3m^{*6}\ln\left(\frac{k_F+\sqrt{k_F^2+m^{*2}}}{m^{*2}}\right)\right]$$

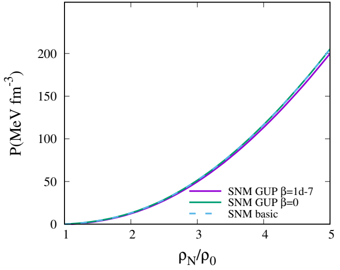
$$\bar{\epsilon} = \epsilon + \beta\Delta\epsilon$$

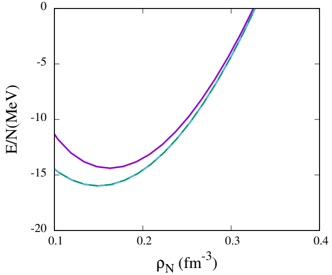
Pressure

$$P = \frac{1}{8\pi^2}\left[k_F\sqrt{k_F^2+m^{*2}}(2k_F^2-3m^{*2}) - 3m^{*4}\ln\left(\frac{k_F+\sqrt{k_F^2+m^{*2}}}{m^{*2}}\right)\right]$$

$$\begin{aligned}\Delta P \quad = \quad & -\frac{1}{144\pi^2}\left[\left(k_F^2+m^{*2}\right)^{-1/2}\left\{105k_F m^{*6} + 65k_F^3 m^{*4} - 26k_F^5 m^{*2} + 56k_F^7\right\}\right. \\ & \left. -105m^{*6}\ln\left(\frac{k_F+\sqrt{k_F^2+m^{*2}}}{m^{*2}}\right)\right]\end{aligned}$$

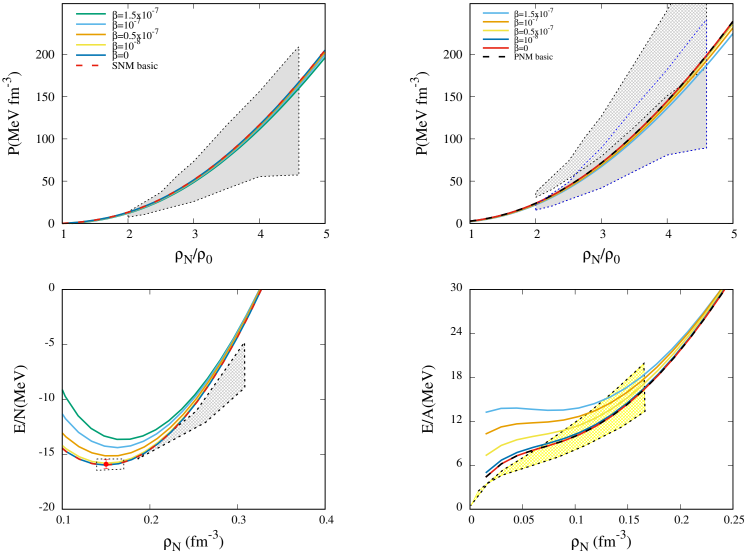
$$\bar{P} = P + \beta\Delta P$$





Pada gambar berikut, ditampilkan Tekanan terhadap ρ_N/ρ_0 (Atas) dan Energi ikat terhadap ρ_N (Bawah). Garis hijau dan garis bitu putus-putus berhimpit menandakan bahwa saat $\beta = 0$, ia kembali ke SNM basic. Terlihat pada kurva ungu, dengan adanya efek GUP menjadikan EOSnya menjadi lebih *soft* dan energi ikatnya berkurang.

Cek SNM dan PNM



Kasus SNM(kiri) dan PNM(kanan) masing-masing ditunjukan plot Tekanan P terhadap rapat jumlah dalam satuan kerapatan saturasi nuklir ρ_N/ρ_0 (atas) serta energi ikat E/N atau E/A terhadap rapat jumlah ρ_N(bawah).

Table: Perbandingan besaran-besaran pada saturasi nuklir (RHBO=1) untuk kasus SNM. Untuk Energi Simetri E_{sym} pada kasus GUP digunakan aproksimasi $E_{sym} \approx E/N|_{PNM} - E/N|_{SNM}$. Bagian yang diarsir merah adalah besaran yang memiliki nilai diluar nilai dari tabel 2.

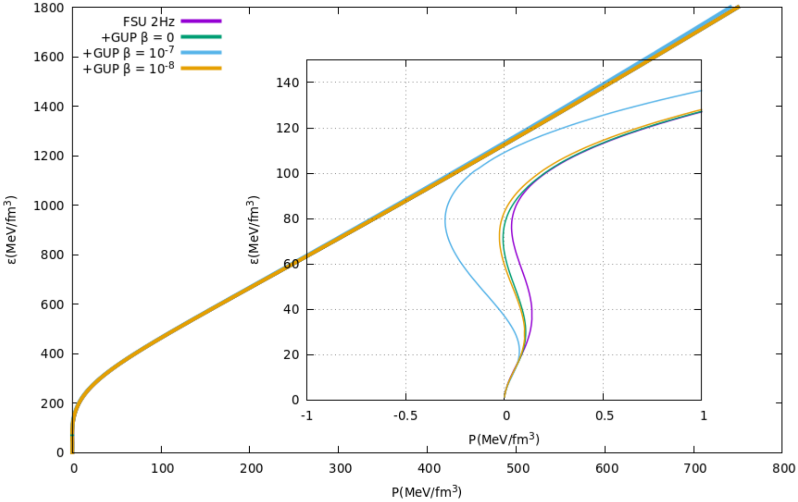
	BSP	β				
		0	10^{-8}	5×10^{-8}	10^{-7}	1.5×10^{-7}
E/N (MeV)	-16.0	-16.0	-15.8	-15.1	-14.2	-13.3
K_0 (MeV)	230.97	230.97	233.66	249.20	281.84	334.25
E_{sym} (MeV)	28.86	30.28	30.29	30.25	30.10	29.82
L (MeV)	50.10	50.10	50.23	50.79	51.54	52.38
J_0 (MeV)	-341.38	-341.38	-354.34	-511.62	-944.72	-1729.59
K_{sym} (MeV)	9.34	9.34	9.06	7.72	5.46	2.33

Table: Komparasi dengan hasil lainnya yang diambil dari paper Andri [1]

	other works
E/N (MeV)	-15.9 ± 0.4
K_0 (MeV)	230 ± 40
E_{sym} (MeV)	31.7 ± 3.2
L (MeV)	58.7 ± 28.1
J_0 (MeV)	$-800 \leq J_0 \leq 400$
K_{sym} (MeV)	$-400 \leq K_{sym} \leq 100$

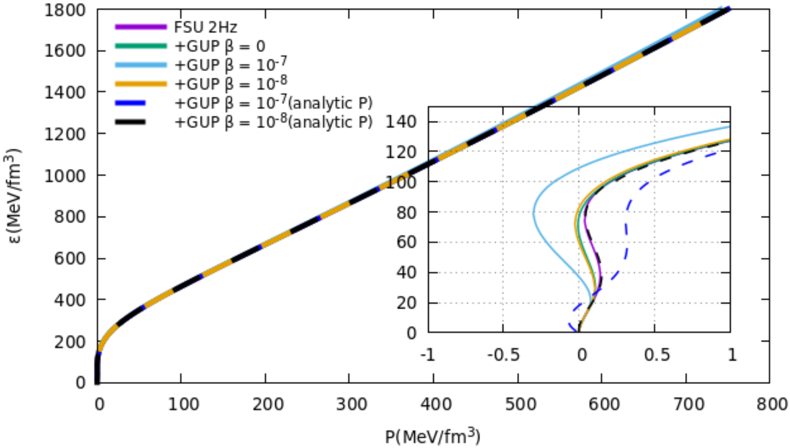
[1] A. Rahmansyah, A. Sulaksono, A. B. Wahidin and A. M. Setiawan, Eur. Phys. J. C **80**, no.8, 769 (2020)

Cek Neutron Star Matter (P Numerik)



Pada gambar ini saya menampilkan plot EOS "FSU 2 Hz" sebelum dan sesudah ditransformasi GUP. Parameter GUP (β) yang digunakan adalah 0, 10^{-8} dan 10^{-7}. Grafik yang lebih kecil menunjukan detail EOS pada nilai rapat energi yang lebih kecil. Grafik tersebut ditampilkan dalam grid untuk menunjukan daerah yang memiliki tekanan negatif.

Cek Neutron Star Matter (P Analitik)



Tekanan Negatif sudah teratasi pada $\beta = 10^{-8}$