STANDARD RMF MODEL

I. Vector densities

spin degeneracy factor

$$\boxed{\rho = \frac{2}{(2\pi)^3}\int_0^{K_F} d^3K.} = \frac{K_F^3}{3\pi^2} \qquad (1)$$

$$\Rightarrow \rho_p = \frac{K_F^{p\,3}}{3\pi^2} \qquad \rho_n = \frac{K_F^{n\,3}}{3\pi^2} \qquad \rho_e = \frac{K_F^{e\,3}}{3\pi^2} \qquad \rho_\mu = \frac{K_F^{\mu\,3}}{3\pi^2}$$

Vector density $\approx$ number density

II. Scalar densities

$$\boxed{\rho^S = \frac{2}{(2\pi)^3}\int_0^{K_F} \frac{m^*}{\sqrt{K^2+m^{*2}}}\, d^3K} \qquad (2)$$

$$\rho_p^S = \frac{m_p^*}{2\pi^2}\left\{K_F^p\sqrt{K_F^{p\,2}+m_p^{*2}} - m_p^{*2}\, Ln\left[\frac{K_F^p+\sqrt{K_F^{p\,2}+m_p^{*2}}}{m_p^*}\right]\right\}$$

$$\rho_n^S = \frac{m_n^*}{2\pi^2}\left\{K_F^n\sqrt{K_F^{n\,2}+m_n^{*2}} - m_n^{*2}\, Ln\left[\frac{K_F^n+\sqrt{K_F^{n\,2}+m_n^{*2}}}{m_n^*}\right]\right\}$$

In general.

$$m_p^* = m + g_\sigma\sigma + g_\delta\delta$$

$$m_n^* = m + g_\sigma\sigma - g_\delta\delta.$$

Therefore, when $g_\delta = 0$, $m_n^* = m_p^*$.

III. Energy density

Within mean field approximation, for nuclear matter, the meson field obtained from following equations:

1) $m_\sigma^2 \sigma + b_2 \sigma^2 + b_3 \sigma^3 - d_2 V_0^2 - d_3 \sigma V_0^2$
$- f_2 b_0^2 - g_3 \sigma b_0^2 + g_\sigma (\rho_p^s + \rho_n^s) = 0$

2) $m_\omega^2 V_0 - g_\omega (\rho_p + \rho_n) + 2 d_2 \sigma V_0 + d_3 \sigma^2 V_0$
$+ c_1 V_0^3 = 0$

3) $m_\rho^2 b_0 - \frac{1}{2} g_\rho (\rho_p - \rho_n) + 2 f_2 \sigma b_0 = 0$

4) $m_\delta^2 \delta + g_\delta (\rho_p^s - \rho_n^s) = 0$

With meson fields in hand, the energy density can be calculated. The result is:

$$\epsilon = \sum_{i=n,p,e,\mu} \epsilon_i + g_\omega (\rho_p + \rho_n) + \frac{1}{2} g_\rho (\rho_p - \rho_n)$$
$$+ U$$

With

$$U = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\delta^2 \delta^2 - \frac{1}{2} m_\omega^2 V_0^2 + \frac{1}{2} m_\rho^2 b_0^2$$

$$+ \frac{1}{3} b_2 \sigma^3 + \frac{1}{4} b_3 \sigma^4 - \frac{1}{4} C_1 V_0^4$$

$$- d_2 \sigma V_0^2 - f_2 \sigma b_0^2 - \frac{1}{2} d_3 \sigma^2 V_0^2 - \frac{1}{2} g_\sigma \sigma^2 b_0^2$$

while

$$\boxed{\epsilon_i = \frac{2}{(2\pi)^3} \int_0^{K_F} \sqrt{k^2 + m^{*2}}\, d^3K} \qquad (3)$$

The integration results are.

$$\epsilon_p = \frac{1}{8\pi^2} \left\{ K_F^p \sqrt{K_F^{p2} + m_p^{*2}} \left(2K_F^{p2} + m_p^{*2} \right) - m_p^{*4} \ln \left[\frac{K_F^p + \sqrt{K_F^{p2} + m_p^{*2}}}{m_p^*} \right] \right\}$$

$$\epsilon_n = \frac{1}{8\pi^2} \left\{ K_F^n \sqrt{K_F^{n2} + m_n^{*2}} \left(2K_F^{n2} + m_n^{*2} \right) - m_n^{*2} \ln \left[\frac{K_F^n + \sqrt{K_F^{n2} + m_n^{*2}}}{m_n^*} \right] \right\}$$

$$\epsilon_e = \frac{1}{8\pi^2}\left\{ K_F^e \sqrt{K_F^{e^2} + m_e^2}\,\left(2K_F^{e^2} + m_e^2\right) - m_e^4 \ln\left[\frac{K_F^e + \sqrt{K_F^{e^2} + m_e^2}}{m_e}\right]\right\}$$

$$\epsilon_\mu = \frac{1}{8\pi^2}\left\{ K_F^\mu \sqrt{K_F^{\mu^2} + m_\mu^2}\,\left(2K_F^{\mu^2} + m_\mu^2\right) - m_\mu^4 \ln\left[\frac{K_F^\mu + \sqrt{K_F^{\mu^2} + m_\mu^2}}{m_\mu}\right]\right\}$$

IV Pressure (Isotropic case)

It can be calculated.

$$\underline{P} = \sum_{i=n,p,e,\mu} \frac{1}{3} P_i - U$$

with

$$\boxed{P = \frac{2}{(2\pi)^3}\int_0^{K_F} \frac{K^2}{\sqrt{K^2 + m^{*2}}}\, d^3K .} \qquad (4)$$

the integration results are

$$P_p = \frac{1}{8\pi^2}\left\{K_F^p\sqrt{K_F^p + m_p^{*2}}\left(2K_F^{p^2} - 3m_p^{*}\right) + 3m_p^{*4}\ln\left[\frac{K_F^p + \sqrt{K_F^{p^2} + m_p^{*2}}}{m_p^*}\right]\right\}$$

$$P_n = \frac{1}{8\pi^2}\left\{K_F^n\sqrt{K_F^{n^2} + m_n^{*2}}\left(2K_F^{n^2} - 3m_n^{*2}\right) + 3m_n^{*4}\ln\left[\frac{K_F^n + \sqrt{K_F^{n^2} + m_n^{*2}}}{m_n^*}\right]\right\}$$

$$P_e = \frac{1}{8\pi^2}\left\{K_F^e\sqrt{K_F^{e^2} + m_e^2}\left(2K_F^e - 3m_e^2\right) + 3m_e^4\ln\left[\frac{K_F^e + \sqrt{K_F^e + m_e^2}}{m_e}\right]\right\}.$$

$$P_\mu = \frac{1}{8\pi^2}\left\{K_F^\mu\sqrt{K_F^{\mu^2} + m_\mu^2}\left(2K_F^\mu - 3m_\mu^2\right) + 3m_\mu^4\ln\left[\frac{K_F^\mu + \sqrt{K_F^{\mu^2} + m_\mu^2}}{m_\mu}\right]\right\}.$$

5) K_F^i is dertermined by using β Stability.

1) neutrality.

$$\boxed{\rho^p = \rho^e + \rho^\mu} \quad (5)$$

2) chemical potential equilibrium. #

$$\boxed{\mu^\mu = \mu^n - \mu_p + g_\rho b_0}$$

$$\sqrt{K_F^{\mu 2} + m_\mu^2} = \sqrt{K_F^{n2} + m_n^{*2}} - \sqrt{K_F^{p2} + m_p^{*2}} + g_\rho b_0$$

$$\boxed{\mu^e = \mu^\mu} \quad (6)$$

$$\sqrt{K_F^{\mu 2} + m_\mu^2} = \sqrt{K_F^{e2} + m_e^2}.$$

note that

$$\boxed{\mu_i = \frac{\partial \epsilon}{\partial \rho_i} = \frac{\partial \epsilon}{\partial K}\frac{\partial K^i}{\partial \rho_i}} \quad (7)$$

$$\Rightarrow Y_p^{2/3} = \left(\frac{1}{3\pi^2}\right)^{2/3}\left\{\left[-\frac{g_\rho b_0}{\rho^{1/3}} + \sqrt{[3\pi(1-Y_p)]^{2/3} + \frac{m_n^{*2}}{\rho^{2/3}}} - \sqrt{[3\pi Y_p]^{2/3} + \frac{m_p^{*2}}{\rho^{2/3}}}\,\right]^2 - \frac{m_e}{\rho^{2/3}}\right\}^{3/2} + \left(\frac{1}{3\pi^2}\right)^{2/3}$$

$$\times\left\{-\frac{g_\rho b_0}{\rho^{1/3}} + \sqrt{[3\pi(1-Y_p)]^{2/3} + \frac{m_n^{*2}}{\rho^{2/3}}} - \sqrt{[3\pi Y_p]^{2/3} + \frac{m_p^{*2}}{\rho^{2/3}}} - \frac{m_\mu}{\rho^{2/3}}\right\}^{3/2}. \quad (6)$$

Minimal Length RMF Model.

Eq. (1) becomes.

$$\bar{\rho} = \frac{2}{(2\pi)^3} \int_0^{K_F} \frac{d^3K}{[1+\beta K^2]^2}$$

Eq. (2) becomes.

$$\bar{\rho}_s = \frac{2}{(2\pi)^3} \int_0^{K_F} \frac{d^3K}{[1+\beta K^2]^2} \frac{m^*}{\left[\frac{1}{\beta}\left(\tan^{-1}\sqrt{\beta}\,K\right)^2 + m^{*2}\right]^{1/2}}$$

Eq (3) becomes.

$$\bar{\epsilon} = \frac{2}{(2\pi)^3} \int_0^{K_F} \frac{\left[\frac{1}{\beta}\left(\tan^{-1}\sqrt{\beta}\,K\right)^2 + m^{*2}\right]^{1/2} d^3K}{(1+\beta K^2)^2}$$

Eq. (4) becomes.

$$\bar{P} = \frac{2}{(2\pi)^3} \int_0^{K_F} \frac{K^2 \quad d^3K}{\left[\frac{1}{\beta}\left(\tan^{-1}\sqrt{\beta}\,K\right)^2 + m^{*2}\right]^{1/2} (1+\beta K^2)^2}$$

or

$$\bar{\rho} = \frac{2}{2\pi^2} \int_0^{K_F} \frac{K^2 dK}{[1+\beta K^2]^2}$$

$$\bar{\rho}_s = \frac{2}{2\pi^2} \int_0^{K_F} \frac{K^2 dK}{[1+\beta K^2]^2} \frac{m^*}{\left[\frac{1}{\beta}(\tan^{-1}\sqrt{\beta} K)^2 + m^{*2}\right]^{1/2}}$$

$$\bar{\epsilon} = \frac{2}{2\pi^2} \int_0^{K_F} \frac{K^2 dK}{[1+\beta K^2]^2} \left[\frac{1}{\beta}(\tan^{-1}\sqrt{\beta} K)^2 + m^{*2}\right]^{1/2}$$

$$\tilde{P} = \frac{2}{2\pi^2} \int_0^{K_F} \frac{K^4 dK}{[1+\beta K^2]^2} \frac{1}{\left[\frac{1}{\beta}(\tan^{-1}\sqrt{\beta} K)^2 + m^{*2}\right]^{1/2}}$$

Note on thermodynamics

① Z. Roupas, Class Quant Grav 32, 11 9051 (2015) & references therein

Theorem 1: For static, spherical symmetric perfect fluids in GR, thermal equilibrium requires, the TOV equation to hold along with Tolman's relations:

1) $T(r)\sqrt{g_{tt}} = T_0 \equiv$ Constant

2) $\mu(r)\sqrt{g_{tt}} = \mu_0 \equiv$ Constant.

Theorem 1 generelized in [T. Jacobson, PRL 75, 1260 (1995)] for stasionary case.

$\Rightarrow$ at least in the case under study, thermal equilibrium implies dynamical equilibrium, while the inverse is not necessarily true, since EFE donot imply eq. 1) & 2)

② According to 1st Law Thermodynamics (see Glendening book for example)

$$3)\ \underline{P} := \rho^2 \left[\frac{\partial \left(\frac{\epsilon}{\rho}\right)}{\partial \rho} \right] \quad \text{and} \quad \mu = \frac{d\epsilon}{d\rho}$$

⑨

2) Please see

$$P = \rho^2 \frac{1}{\rho} \underbrace{\frac{d\epsilon}{d\rho}}_{\mu} - \epsilon \rho^2 \frac{1}{\rho^2}$$

$$\Rightarrow P + \epsilon = \rho \mu$$

or

$$\boxed{\mu = \frac{P+\epsilon}{\rho}}$$

can be used to calc consisten[cy] with 1st TDL

constant

From 2) $\frac{(\epsilon + P)}{\rho} \sqrt{g_{tt}} = C$

If $\bar{\epsilon} = \frac{\epsilon}{\rho}$, $\bar{P} = \frac{P}{\rho}$

$$\Rightarrow (\bar{\epsilon} + \bar{P}) = \frac{c}{g_{tt}^{1/2}}$$

$$\bar{\epsilon} + \rho \frac{d\bar{\epsilon}}{d\rho} = \frac{c}{g_{tt}^{1/2}}$$

$$\frac{d}{d\rho}(\bar{\epsilon}\rho) = \frac{c}{g_{tt}^{1/2}}$$

From 1st TD: $\boxed{\bar{P} = \rho \frac{\partial \bar{\epsilon}}{\partial \rho}}$ ①

$$\Rightarrow \boxed{\bar{\epsilon} + \bar{P} = \frac{d}{d\rho} \epsilon} \quad \boxed{P = -\epsilon + \rho \frac{d\epsilon}{d\rho}} \text{ ②}$$

$\Rightarrow$ ① & ② Consistent !!

Cek : Polytropic !!

$$\boxed{\begin{aligned} \epsilon &= a\rho + b\rho^{\Gamma} \\ P &= (\Gamma - 1)\, b\rho^{\Gamma} \end{aligned}}$$

$$\rho \frac{\partial \epsilon}{\partial \rho} = a\rho + \Gamma b \rho^{\Gamma}$$

$$P = -\epsilon + \rho \frac{\partial \epsilon}{\partial \rho} = -\cancel{a\rho} - b\rho^{\Gamma} + \cancel{a\rho} + \Gamma b \rho^{\Gamma}$$

$$\boxed{P = (\Gamma - 1)\, b\rho^{\Gamma}}$$ ✓ ok.

Approximation up to first order of β $\quad \frac{1}{\beta}\left(\tan^{-1}\sqrt{\beta}\,K\right)^2 \approx K^2 - \frac{2}{3}\beta K^4$

$$(1+\beta K^2)^{-2} \approx 1 - 2\beta K^2$$

$$\left[\frac{1}{\beta}\left(\tan^{-1}\sqrt{\beta}\,K\right)^2 + m^{*2}\right]^{1/2} =$$

$$\left[K^2 + m^{*2} - \frac{2}{3}\beta K^4\right]^{\pm 1/2} = \left[K^2 + m^{*2}\right]^{\pm\frac{1}{2}}$$

$$\left(1 - \frac{2}{3}\frac{\beta K^4}{(K^2+m^{*2})}\right)^{\pm\frac{1}{2}}$$

$$= (K^2+m^{*2})^{\pm\frac{1}{2}}\left[1 \mp \frac{1}{3}\frac{\beta K^4}{(K^2+m^{*2})}\right]$$

$$\bar{\rho} = \frac{\gamma}{2\pi^2}\int_0^{K_F}\left[1 - 2\beta K^2\right]K^2\,dK$$

$$\bar{\rho}_s = \frac{\gamma}{2\pi^2}\int_0^{K_F} dK\,K^2\left[1 - 2\beta K^2\right] m^* (K^2+m^{*2})^{-1/2}$$

$$\times\left[1 + \frac{1}{3}\frac{\beta K^4}{(K^2+m^{*2})}\right]$$

$$\bar{\epsilon} \simeq \frac{2}{2\pi^2} \int_0^{k_F} dk\, k^2 \left[1 - 2\beta k^2\right] \left(k^2 + m^{*2}\right)^{1/2}$$

$$\left[1 - \frac{1}{3} \frac{\beta k^4}{(k^2 + m^{*2})}\right]$$

$$\bar{P} \simeq \frac{2}{2\pi^2} \int_0^{k_F} dk\, k^4 \left[1 - 2\beta k^2\right] \left(k^2 + m^{*2}\right)^{-1/2}$$

$$\times \left[1 + \frac{1}{3} \frac{\beta k^4}{(k^2 + m^{*2})}\right]$$

$$\bar{\rho} = \rho + \beta \Delta \rho$$

$$\boxed{\Delta \rho \simeq -\frac{4}{2\pi^2} \int_0^{k_F} k^4\, dk}$$

$$\bar{\rho}_s = \rho_s + \beta \Delta \rho_s$$

$$\boxed{\Delta \rho_s \simeq -\frac{4}{2\pi^2} \int_0^{k_F} k^4 \frac{m^*}{(k^2 + m^{*2})^{1/2}}\, dk + \frac{2}{2\pi^2} \frac{1}{3} \int_0^{k_F} k^6 \frac{m^*}{(k^2 + m^{*2})^{3/2}}\, dk}$$

$$\bar{\epsilon} = \epsilon + \beta \Delta \epsilon$$

$$\Delta \epsilon = -\frac{4}{2\pi^2} \int_0^{K_F} K^4 (K^2 + m^{*2})^{1/2} \, dK - \frac{2}{2\pi^2} \frac{1}{3} \int_0^{K_F} K^6 \frac{1}{(K^2 + m^{*2})^{1/2}} \, dK$$

$$\bar{P} = P + \beta \Delta P$$

$$\Delta P = -\frac{4}{2\pi^2} \int_0^{K_F} K^6 (K^2 + m^{*2})^{-1/2} \, dK + \frac{2}{2\pi^2} \frac{1}{3} \int_0^{K_F} K^8 \frac{1}{(K^2 + m^{*2})^{3/2}} \, dK$$

$$\Delta \rho = -\frac{2K_F^5}{5\pi^2}$$

$$\Delta \rho_s = -\frac{m^*}{4\pi^2}\Big[K_F\,(2K_F^2 - 3m^{*2})\sqrt{K_F^2+m^{*2}}$$

$$+ 3\,m^{*4}\,\mathrm{Ln}\left[\frac{K_F+\sqrt{K_F^2+m^{*2}}}{m^*}\right]$$

$$+\frac{m}{24\pi^2}\Big[2K_F^5 - 5K_F^3 m^{*2} + 15\,m^4$$

$$\times\sqrt{K_F^2+m^{*2}}\;\mathrm{Ln}\left[\frac{K_F+\sqrt{K_F^2+m^{*2}}}{m^*}\right]$$

$$-15\,K_F\,m^{*4}\Big]\left[K_F^2+m^{*2}\right]^{1/2}.$$

$$\Delta \epsilon = -\frac{1}{24\pi^2}\Big[K_F\sqrt{K_F+m^{*2}}\,(8K_F^4+2K_F^2m^{*2}-3$$

$$+3m^{*6}\,\mathrm{Ln}\left[\frac{K_F+\sqrt{K_F^2+m^{*2}}}{m^*}\right]\Big]$$

$$+\frac{1}{144\pi^2}\Big[K_F\sqrt{K_F+m^{*2}}\,(8K_F^4-10K_F^2m^{*2}+15$$

$$-15\,m^{*6}\,\mathrm{Ln}\left[\frac{K_F+\sqrt{K_F^2+m^{*2}}}{m^*}\right]\Big]$$

1) $\Delta \rho = -\dfrac{2K_F^5}{5\pi^2}$.

2) $$\Delta \rho_s = \frac{1}{24\pi^2} \left(K_F^2 + m^{*2}\right)^{-1/2} m \left\{ -10K_F^5 + K_F^3 m^{*2} + 3K_F m^{*4} - 3m^{*4}\left(K_F^2 + m^{*2}\right)^{1/2} \ln\left[\frac{K_F + \sqrt{K_F^2 + m^{*2}}}{m^*}\right]\right\}$$

3) $$\Delta \epsilon = \frac{1}{144\pi^2} \left\{ K_F \left(K_F^2 + m^{*2}\right)^{1/2} \left(-40K_F^4 - 22K_F^2 m^{*2} + 33m^{*4}\right) - 33m^{*6} \ln\left[\frac{K_F + \sqrt{K_F^2 + m^{*2}}}{m^*}\right]\right\}$$

4) $$\Delta P = -\frac{1}{144\pi^2} \left(K_F^2 + m^{*2}\right)^{-1/2} \left\{ 40K_F^7 + 2K_F^5 m^{*2} - 5K_F^3 m^{*4} - 15K_F m^{*6} + 15m^{*6}\sqrt{K_F^2 + m^{*2}} \ln\left[\frac{K_F + \sqrt{K_F^2 + m^{*2}}}{m^*}\right]\right\}$$

chemical potential for kinetic term

$$\bar{\mu}_i^{\kappa} = \frac{\partial \bar{\epsilon}}{\partial \bar{\rho}_i} = \frac{\partial \epsilon}{\partial \bar{\rho}_i} + \beta \frac{\partial \Delta \epsilon}{\partial \bar{\rho}_i}$$

$$= \frac{1}{\left(\frac{\partial \bar{\rho}_i}{\partial \epsilon}\right)} + \beta \frac{1}{\left(\frac{\partial \bar{\rho}_i}{\partial \Delta \epsilon}\right)}$$

$$= \frac{1}{\left[\frac{\partial \rho_i}{\partial \epsilon} + \beta \frac{\partial \Delta \rho_i}{\partial \epsilon}\right]} + \beta \frac{1}{\left[\frac{\partial \rho_i}{\partial \Delta \epsilon} + \beta \right.}$$

$$\approx \frac{1}{\frac{\partial \rho_i}{\partial \epsilon}\left[1 + \beta \frac{\partial \epsilon}{\partial \rho_i} \frac{\partial \Delta \rho_i}{\partial \epsilon}\right]}$$

$$+ \frac{\beta}{\frac{\partial \rho_i}{\partial \Delta \epsilon}}$$

$$= \frac{\partial \epsilon}{\partial \rho_i}\left(1 - \beta \frac{\partial \Delta \rho_i}{\partial \rho_i}\right) + \beta \frac{\partial (\Delta \epsilon)}{\partial \rho_i}$$

$$\boxed{\bar{\mu}_i^{\kappa} = \frac{\partial \epsilon}{\partial \rho_i} + \beta \left[\frac{\partial \Delta \epsilon}{\partial \rho_i} - \frac{\partial \epsilon}{\partial \rho_i} \frac{\partial \Delta \rho_i}{\partial \rho_i}\right]}$$

$$\bar{\mu}_i^{\kappa} = \mu_i^{\kappa} + \beta \left[\frac{\partial \Delta \epsilon}{\partial k} \frac{\partial \kappa}{\partial \rho_i} - \mu_i \frac{\partial \Delta \rho_i}{\partial \kappa} \frac{\partial \kappa}{\partial \rho_i} \right]$$

<u>note</u> to remember

$$\mu_n = g_\omega V_0 - \frac{1}{2} g_\rho b_0 + \sqrt{K_n^2 + m_n^{*2}}$$

$$\mu_p = g_\omega V_0 + \frac{1}{2} g_\rho b_0 + \sqrt{K_p^2 + m_p^{*2}}$$

$$\mu_e = \sqrt{K_e^2 + m_e^2}$$

$$\mu_\mu = \sqrt{K_\mu^2 + m_\mu^2}$$

$$\frac{\partial \Delta \epsilon}{\partial K} = -\frac{2}{\pi^2} \left[K^4 (K^2 + m^{*2})^{1/2} \right]$$

$$- \frac{2}{3\pi^2} \left[K^6 (K^2 + m^{*2})^{-1/2} \right]$$

$$\frac{\partial K}{\partial \rho} = \frac{\pi^2}{K^2}$$

$$\frac{\partial \Delta \rho_i}{\partial K} = -\frac{2}{\pi^2} K^4$$

$$\Rightarrow \quad \frac{\partial \Delta \epsilon}{\partial K} \frac{\partial K}{\partial \rho} =$$

$$\left[- \left\{ \frac{2}{\pi^2} K^4 (K^2 + m^{*2})^{1/2} + \frac{2}{3\pi^2} K^6 (K^2 + m^{*2})^{-1/2} \right\} \right.$$

$$\left. \times \frac{\pi^2}{K^2} \right]$$

$$= - \left[2K^2 (K^2 + m^{*2})^{1/2} + \frac{2}{3} \frac{K^4}{(K^2 + m^{*2})^{1/2}} \right]$$

$$\Rightarrow \mu_i \frac{\partial \Delta \rho_i}{\partial K} \frac{\partial K}{\partial \rho_i} =$$

$$\left[(K^2 + m^{*2})^{1/2} \left(-\frac{2}{\pi^2} K^4 \right) \frac{\pi^2}{K^2} \right]$$

$$= -(K^2 + m^{*2})^{1/2} \, 2K^2$$

Therefore

$$\boxed{\bar{\mu}^{k} = \sqrt{k^2+m^{*2}} - \beta\,\frac{2}{3}\,\frac{k^4}{(k^2+m^{*2})^{1/2}}}$$

$$\therefore \quad \mu_n = g_\omega V_0 - \frac{1}{2} g_\rho b_0$$

$$+ \sqrt{k_n^2 + m_n^{*2}} - \frac{2}{3}\beta \frac{k_n^4}{(k_n^2+m_n^{*2})^{1/2}}$$

$$\mu_p = g_\omega V_0 + \frac{1}{2} g_\rho b_0$$

$$+ \sqrt{k_p^2 + m_p^{*2}} - \frac{2}{3}\beta \frac{k_p^4}{(k_p^2+m_p^{*2})^{1/2}}$$

$$\mu_e = \sqrt{k_e^2 + m_e^2} - \frac{2}{3}\beta \frac{k_e^4}{(k_e^2+m_e^2)^{1/2}}$$

$$\mu_\mu = \sqrt{k_\mu^2 + m_\mu^2} - \frac{2}{3}\beta \frac{k_\mu^4}{(k_\mu^2+m_\mu^2)^{1/2}}.$$

Therefore β stability becomes.

① $\rho^{p} = \rho^{e} + \rho^{\mu}$

② a) $\mu_{\mu} = \sqrt{K_F^{n^2} + m_n^{*2}} - \sqrt{K_F^{p^2} + m_p^{*2}}$

$$+ g_{\rho} b_0 + \frac{2}{3}\beta \left[\frac{K_F^{p\,4}}{\sqrt{K_F^{p^2} + m_p^{*2}}} - \frac{K_F^{n\,4}}{\sqrt{K_F^{n^2} + m_n^{*2}}} \right]$$

b) $\mu_e = \mu_{\mu}$

③ c) $\mu_e = \sqrt{K_F^{e^2} + m_e^2} - \frac{2}{3}\beta \frac{K_F^{e\,4}}{\sqrt{K_F^{e^2} + m_e^2}}$

d) $\mu_{\mu} = \sqrt{K_F^{\mu^2} + m_{\mu}^2} - \frac{2}{3}\beta \frac{K_F^{\mu\,2}}{\sqrt{K_F^{\mu 2} + m_{\mu}^2}}$

Note:

$$\rho = \frac{K_F^{3}}{3\pi^2} - \frac{2\beta}{5\pi^2} K_F^{5}$$

$$K_F :\simeq K_F^{(0)} + \beta K_F^{(1)}$$

$$\rho = \frac{K_F^{(0)3}}{3\pi^2} + \beta \underbrace{\left(\frac{K_F^{(0)2} K_F^{(1)}}{\pi^2} - \frac{2 K_F^{(0)5}}{5\pi^2} \right)}_{\equiv 0} + O(\beta^2)$$

$$K_F^{(1)} = \frac{\cancel{\pi^2}}{\cancel{K_F^{(0)(2)}}} \frac{2}{5} \frac{K_F^{(0)\cancel{5}\,3}}{5\cancel{\pi^2}}$$

$$K_F \simeq K_F^{(0)} + \frac{2\beta}{5} K_F^{(0)3}$$

$$K_F^{(0)} = (3\pi^2 \rho)^{1/3}$$

$$\boxed{K_F = (3\pi^2 \rho)^{1/3} + \frac{2\beta}{5} (3\pi^2 \rho)}$$

$$\sqrt{K_F^2+m^2} = \sqrt{\left(K_F^{(0)} + \frac{2\beta}{5} K_F^{(0)3}\right)^2 + m^2}$$

$$= \left(K_F^{(0)2}\left(1+\frac{2\beta}{5}K_F^{(0)2}\right)^2 + m^2 \right)^{1/2}$$

$$= \left[K_F^{(0)2} + m^2 + \frac{4\beta}{5} K_F^{(0)4}\right]^{1/2}$$

$$= \left(K_F^{(0)2} + m^2\right)^{1/2}\left(1 + \frac{\frac{2\beta}{5} K_F^{(0)4}}{(K_F^{(0)} + m^2)}\right)^{1/2}$$

$$= \left(K_F^{(0)} + m^2\right)^{1/2} + \frac{2\beta}{5}\frac{K_F^{(0)4}}{(K_F^0 + m^2)^{1/2}}$$

(2) $\Longrightarrow$

$$\mu_e = \mu_\mu = \sqrt{K_F^{on2} + m_n^{*2}} - \sqrt{K_F^{on2} + m_p^{*2}} + g_\rho b_0$$

$$+ \frac{4}{15}\beta\left[\frac{K_F^{op4}}{\sqrt{K_F^{op2} + m_p^{*2}}} - \frac{K_F^{on4}}{\sqrt{K_F^{on2} + m_n^{*2}}}\right]$$

$$= 0$$

and (3)

$$\mu_e = \sqrt{K_F^{0e2} + m_e^2} - \frac{4}{15}\beta \frac{K_F^{e0\,4}}{\sqrt{K_F^{0e2} + m_e^2}}$$

$$= \sqrt{(3\pi^2 \rho_e^0)^{2/3} + m_e^2} - \frac{4}{15}\beta \frac{(3\pi^2 \rho_e^0)^{4/3}}{\sqrt{(3\pi \rho_e^0)^{2/3} + m_e^2}}$$

$$\boxed{\mu_e^2 = (3\pi^2 \rho_e^0)^{2/3} + m_e^2 - 2\frac{4}{15}\beta\,(3\pi^2 \rho_e^0)^{4/3}}$$

$$(\mu_e^2 - m_e^2) = (3\pi^2 \rho_e)^{2/3} - 2\frac{4}{15}\beta(3\pi^2\rho_e)^{4/3}$$

$$(3\pi^2 \rho_e^0)^{2/3} = (\mu_e^2 - m_e^2)\left[1 + \frac{8}{15}\beta \frac{(3\pi \rho_e^0)^{4/3}}{(\mu_e^2 - m_e^2)}\right.$$

$$\approx (\mu_e^2 - m_e^2)\left[1 + \frac{8}{15}\beta\,(\mu_e^2 - m_e^2)\right]$$

$$\boxed{Y_e = \frac{1}{3\pi^2}\frac{1}{\rho_B}\left[(\mu_e^2 - m_e^2)\left\{1 + \frac{8}{15}\beta\,(\mu_e^2 - m_e^2)\right.\right.}$$

$$\boxed{\rho_B = \rho_p + \rho_n.}$$

where ② $\Rightarrow$

$$\boxed{\mu_\mu = \mu_e = \left\{ \left[\sqrt{(3\pi^2[1-Y_p])\frac{m_n^{*2}}{\rho_B^{2/3}}} - \sqrt{(3\pi^2 Y_p)^{2/3} + \frac{m_p^{*2}}{\rho_B^{2/3}}} \right. \right. + \frac{\frac{1}{2} g_\rho^2 [1 - 2Y_p] \rho_B^{2/3}}{m_\rho^{*2}} + \frac{4}{15}\beta \left[\frac{(3\pi^2 Y_p)^{4/3} \rho_B^{2/3}}{\sqrt{(3\pi^2 Y_p)^{2/3} + \frac{m_p^{*2}}{\rho_B^{2/3}}}} \left. \left. - \frac{(3\pi^2[1-Y_p]^{4/3} \rho_B^{2/3}}{\sqrt{(3\pi^2[1-Y_p]^{2/3} + \frac{m_n^{*2}}{\rho_B^{2/3}}}} \right] \right\} \rho_B^{1/3}}$$

$$\boxed{\Rightarrow \quad Y_p = \frac{1}{3\pi^2 \rho_B} \left[\left(\mu_e^2 - m_e^2\right)^{3/2} \left\{ 1 + \frac{8}{15}\beta\left(\mu_e^2 - m_e^2\right) \right\}^{3/2} + \left(\mu_\mu^2 - m_\mu^2\right)^{3/2} \left\{ 1 + \frac{8}{15}\beta\left(\mu_\mu^2 - m_\mu^2\right) \right\}^{3/2} \right]}$$

PNM, SNM

SNM : $Y_e = Y_\mu = 0$, $Y_p = Y_n = \frac{1}{2}$ $;x^2$

PNM : $Y_e = Y_\mu = 0$, $Y_p = 0$, $Y_n = 1$

Properties SNM in saturation:

1) $E \rightarrow$ Binding energy $\rightarrow$ subrotine FED

$$E = \frac{B}{A} = \left(\frac{\epsilon}{\rho} \times (\hbar c)^3 \right)_{\rho = \rho_0}, \quad \epsilon \text{ energy density}$$

$\rho = \rho_0$ ← "saturation density of SNM"

2) Pressure → can be expressed analitically

Can be obtained from derivative of Binding energy

$$P = \left(\rho^2 \frac{d}{d\rho} E \right)_{\rho_0}$$

3) Incompresibility

$$K_0 = 9 \rho_0^2 \frac{d^2 E}{d\rho^2} \Big|_{\rho_0}$$

4) Slope in compresibility

$$J_0 = 27 \rho_0^3 \frac{d^3 E}{d\rho^3} \Big|_{\rho_0}$$

5) Symmetry Energy.

$$E_{sym}(\rho) = \frac{1}{2} \frac{\partial^2 E(\rho,\delta)}{\partial \delta^2}\Big|_{\delta=0}$$

$\delta = 0$, SNM, $\delta = 1$, PNM

In standard code, It calculated analytically

However we can approximated for relatitle complicated case by

$$E_{sym}(\rho) \approx E_{PNM}^{(\rho)} - E_{SNM}(\rho)$$

In saturation ~~E_{sym}~~ $J = E_{sym}(\rho_0)$

6. Slope of symmetry energy

$$L = 3\rho_0 \frac{dJ}{d\rho}\Big|_{\rho_0}$$

and

7. $$K_{sym} = 9\rho_0^2 \frac{d^2 J}{d\rho^2}\Big|_{\rho_0}$$

8. $$K_{asy} = K_{sym} - 6L$$

$$K_{sat2} = K_{asy} - \frac{J_0}{K_0} L$$

Cal Consistency of 1^{st} law of TD.

Hugenholtz-Van Hove theorem state:

In MBP of system Fermion

$$E_F = \underbrace{E}_{\text{binding energy}} + \frac{P}{\rho}$$

$$E = \frac{\epsilon}{\rho}$$

for $T = 0$ $\quad \mu = E_F$

1^{st} law of TD

$$\boxed{\mu = \frac{\epsilon + P}{\rho}}$$

Now we will check for the simplest GUP EOS

→ EOS of non interacting Fermion within GUP

$$\bar{\mu} = \mu + \beta \Delta \mu$$

$$\bar{\epsilon} = \epsilon + \beta \Delta \epsilon$$

$$\bar{P} = P + \beta \Delta P$$

$$\bar{\rho} = \rho + \beta \Delta \rho$$

$$\bar{\mu}\bar{\rho} = (\mu + \beta \Delta \mu)(\rho + \beta \Delta \rho)$$

$$= \mu\rho + \beta(\Delta\mu\,\rho + \mu\,\Delta\rho)$$

$$\bar{\epsilon} + \bar{P} = \epsilon + P + \beta(\Delta\epsilon + \Delta P)$$

$$\bar{\mu}\bar{\rho} - (\bar{\epsilon} + \bar{P}) = \overset{\,=0}{\mu\rho - (\epsilon + P)} \qquad * \times \ln\left[\frac{K_F + \sqrt{K_F^2 + m^2}}{m}\right]$$

$$+ \beta\Big[\Delta\mu\,\rho + \mu\,\Delta\rho$$

$$- \Delta\epsilon - \Delta P\Big]$$

$$= \beta\left[\Delta\mu\,\rho + \mu\,\Delta\rho - (\Delta\epsilon + \Delta P)\right]$$

$$\rho = \frac{K_F^3}{3\pi^2} \qquad \Delta\rho = -\frac{2K_F^5}{5\pi^2}$$

$$\mu = \sqrt{K_F^2 + m^2} \qquad \Delta\mu = -\frac{2}{3}\frac{K_F^4}{(K_F^2 + m^2)^{1/2}}$$

$$\Delta\epsilon = \frac{1}{144\pi^2}\Big\{K_F (K_F^2 + m^2)^{1/2}\left[-40K_F^4 - 22K_F^2 m^2\right.$$

$$\left. + 33m^4\right) - 33m^6 \ln\left[\frac{K_F + \sqrt{K_F^2 + m^2}}{m}\right]\Big\}$$

$$\Delta P = -\frac{1}{144\pi^2}(K_F^2 + m^2)^{1/2}\Big\{40K_F^7 + 2K_F^5 m^2$$

(18) $$- 5K_F^3 m^4 - 15K_F m^6 + 15m^6\sqrt{K_F^2 + m^2} \; *$$

$$\bar{\mu}\bar{\rho} - (\bar{\epsilon} + \bar{P}) = 0$$

$$+ \frac{\beta}{45\pi^2}\Big[-3K_F^7 + 2K_F^2 m^2 - 15 K_F m^6$$

$$+ 15 m^6 \sqrt{K_F^2 + m^2} \, \ln\left(\frac{K_F + \sqrt{K_F^2 + m^2}}{m^2} \right) \Big] (K_F^2 + m^2)^{-1}$$

$$\neq 0$$

∴ Thermodynamic Consistentcy is broken!!

Alka, anisotropi ini bukan seperti yang Alka jelaskan di WA (karena distribusi partikel dinukleir matter yang tidak homogen) tapi karena faktor lain yang kita tidak tahu, bisa geometri atau yang lain yang secara fenomenologi informasinya ada di σ.

Misal kita ambil koordinat sedemikian hingga tekanan yang dihitung dengan mean field ada pada arah $r \Rightarrow P_r$.

Tekanan rata-rata secara umum

$$\underline{P} := \frac{1}{3}\left(P_r + 2P_t\right)$$

Untuk materi isotropik $P_t = P_r$

sehingga

$$\boxed{\underline{P} = P_r}$$

Karena 1 dan lain hal di materi bintang neutron $P_r \neq P_t$, maka, Jika

$P_t := P_r - \sigma$, maka

$$\boxed{\underline{P} = \frac{1}{3}(3P_r - 2\sigma) = P_r - \frac{2}{3}\sigma}$$

Disini σ general, tidak selalu dikarenakan distribusi partikel yang tidak homogen di materi, bisa juga karena faktor lain.

HK Termodinamika I

$$\boxed{\mu \rho - (\epsilon + \underline{P}) = 0}$$

untuk materi isotropik $\underline{P} = P_r$

$$\mu \rho - \epsilon + P_r = 0$$

untuk GUP, lihat Catatan GUP EOS.

diasumsikan

$$\bar{\mu}\bar{\rho} - \bar{\epsilon} + \bar{P} \equiv \bar{\mu}\bar{\rho} - (\bar{\epsilon} + \bar{P}_r) + \frac{2}{3}\bar{\sigma}$$

$$(2) \quad = 0$$

$$\bar{\mu}\bar{\rho} - (\bar{\epsilon} + \bar{P}_r) + \frac{2}{3}\bar{\sigma}$$

$$= \mu\rho - (\epsilon + P_r) + \frac{2}{3}\bar{\sigma}$$

$$+ \beta\left[\Delta\mu\, \rho + \mu\, \Delta\rho - (\Delta\epsilon + \Delta P)\right]$$

$$:= 0 \quad !!$$

Maka agar konsisten dengan TDI,

$$\boxed{\bar{\sigma} \equiv -\frac{2}{3}\left[\Delta\mu\, \rho + \mu\, \Delta\rho - (\Delta\epsilon + \Delta P)\right]}$$

untuk kasus non interakting

$$\bar{\sigma} = -\frac{3}{2}\frac{\beta}{45\pi^2}\left\{-3K_F^7 + 2K_F^2 m^2 - 15 K_F m^6 + 15\, m^6 \sqrt{K_F^2 + m^2}\ \mathrm{Ln}\left[\frac{K_F + \sqrt{K_F^2 + m^2}}{m^2}\right]\right\}$$

$$\times (K_F^2 + m^2)^{-1/2}$$

tampak $\beta \to 0$ $\bar{\sigma} \to 0$, $\therefore$ anisotropik karena GUP !!