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Efek Anisotropi Teori Generalized Uncertainity Principle pada Persamaan Keadaan Bintang Netron*

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Abstract

kita sudah menurunkan transformasi ruang fase GUP pada besaran-besaran termodinamika di bintang netron. Sejauh ini apa yang kita bisa simpulkan bahwa dari perbandingan hasil perhitungan dengan data Symmetric Nuclear Matter (SNM) dan Pure Neutron Matter (PNM). Kita mendapati bahwa data tersebut mengkonstrain parameter GUP hingga $\beta \leq 10^{-8}$. Dari situ kami menghitung materi bintang netron dengan harga parameter GUP $\beta = 10^{-8}$, serta melihat efek anisotropinya. Kami mendapati bahwa efek anisotropinya tidak berpengaruh signifikan.

I. INTRODUCTION

Heisenberg Uncertainty Principles (HUP) is one of the most fundamental principle in Quantum Mechanics. Generalized Uncertainty Principles (GUP) try to include gravity to quantum mechanics by modify the commutation relation of HUP. So the observable in quantum mechanics in Planck scale is also modified. In this study we want to see the impact in the neutron star

II. METHODS

Number Density

$$\begin{aligned}
\rho_0 &= \frac{2}{(2\pi)^3}\int_0^{k_f} \frac{d^3k}{(1+\beta_k k_k^2)^3} \\
&= \frac{2}{(2\pi)^2}\int_0^{k_f} dk \int_0^{\pi} d\theta \frac{k^2 \sin\theta}{[1+k^2\beta_r(1+\alpha\cos^2\theta)]^3} \\
&\approx \frac{k_f^3}{3\pi^2} - \frac{k_f^5(3+\alpha)\beta}{5\pi^2}
\end{aligned} \tag{1}$$

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* A footnote to the article title

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Scalar Density

$$
\begin{aligned}
\rho_s &= \frac{2}{(2\pi)^3}\int_0^{k_f} \frac{d^3k}{(1+\beta_k k_k^2)^3}\frac{m^*}{\sqrt{k^2+m^{*2}+2\beta_r(1+\alpha\cos^2\theta)k^4}} \\
&= \frac{2}{(2\pi)^2}\int_0^{k_f} dk \int_0^{\pi} d\theta \frac{k^2 m^*}{\sqrt{k^2+m^{*2}+2\beta_r(1+\alpha\cos^2\theta)k^4}}\ \frac{\sin\theta}{[1+k^2\beta_r(1+\alpha\cos^2\theta)]^3} \\
&= \rho_s^{(0)} + \Delta\rho_s
\end{aligned}
\tag{2}
$$

where

$$
\rho_s^{(0)} = \frac{1}{2\pi}\left[k_f m^* \sqrt{k_f^2+m^{*2}} - m^{*3}\ln\left(\frac{k_f+\sqrt{k_f^2+m^{*2}}}{m^*}\right)\right] \tag{3}
$$

$$
\Delta\rho_s = -\frac{m(3+\alpha)\beta}{3\pi^2}\left[\frac{k_f(k_f^4-k_f^2m^{*2}-3m^{*4})}{\sqrt{k_f^2+m^{*2}}} + 3m^{*4}\ln\left(\frac{k_f+\sqrt{k_f^2+m^{*2}}}{m^*}\right)\right] \tag{4}
$$

Energy Density

$$
\begin{aligned}
\varepsilon_{kin} &= \frac{2}{(2\pi)^3}\int_0^{k_f} \frac{d^3k}{(1+\beta_k k_k^2)^3}\sqrt{k^2+m^{*2}+2\beta_r(1+\alpha\cos^2\theta)k^4} \\
&= \frac{2}{(2\pi)^2}\int_0^{k_f} dk \int_0^{\pi} d\theta\ k^2\sqrt{k^2+m^{*2}+2\beta_r(1+\alpha\cos^2\theta)k^4}\ \frac{\sin\theta}{[1+k^2\beta_r(1+\alpha\cos^2\theta)]^3} \\
&= \varepsilon^{(0)} + \Delta\varepsilon
\end{aligned}
\tag{5}
$$

where

$$
\varepsilon^{(0)} = \frac{1}{8\pi}\left[k_f\sqrt{k_f^2+m^{*2}}(2k_f^2+m^{*2}) - m^{*4}\ln\left(\frac{k_f+\sqrt{k_f^2+m^{*2}}}{m^*}\right)\right] \tag{6}
$$

$$
\Delta\varepsilon = -\frac{(3+\alpha)\beta}{18\pi^2}\left[k_f\sqrt{k_f^2+m^{*2}}\ (2k_f^4+2k_f^2m^{*2}-3m^{*4}) + 3m^{*6}\ln\left(\frac{k_f+\sqrt{k_f^2+m^{*2}}}{m^*}\right)\right] \tag{7}
$$

Pressure(Isotrophy)

$$
\begin{aligned}
P_{kin} &= \frac{1}{3}\left(\frac{2}{(2\pi)^3}\int_0^{k_f}\frac{d^3k}{(1+\beta_k k_k^2)^3}\frac{k^2}{\sqrt{k^2+m^{*2}+2\beta_r(1+\alpha\cos^2\theta)k^4}}\right)\\
&= \frac{1}{3}\left(\frac{1}{2\pi^2}\int_0^{k_f}dk\int_0^{\pi}d\theta\ \frac{k^4}{\sqrt{k^2+m^{*2}+2\beta_r(1+\alpha\cos^2\theta)k^4}}\ \frac{\sin\theta}{[1+k^2\beta_r(1+\alpha\cos^2\theta)]^3}\right)\\
&= P^{(0)}+\Delta P
\end{aligned} \tag{8}
$$

where

$$
P^{(0)} = \frac{1}{3}\ \frac{1}{8\pi^2}\left[k_f\sqrt{k_f^2+m^{*2}}(2k_f^2-3m^{*2})+3m^{*4}\ln\left(\frac{k_f+\sqrt{k_f^2+m^{*2}}}{m^*}\right)\right] \tag{9}
$$

$$
\Delta P = -\frac{1}{3}\ \frac{(3+\alpha)\beta}{72\pi^2}\left[\frac{k_f(-16k_f^6+10k_f^4m^{*2}-25k_f^2m^{*4}-75m^{*6})}{\sqrt{k_f^2+m^{*2}}}+75m^{*6}\ln\left(\frac{k_f+\sqrt{k_f^2+m^{*2}}}{m^*}\right)\right] \tag{10}
$$

PNM AND SNM

KESEIMBANGAN POTENSIAL KIMIA

https://www.overleaf.com/project/61151e9a607f9fdfe4370730 Consider the system has beta stability and neutral in charge. Those condition are described in their chemical potential equilibrium

$$
\mu_i = B_i\mu_n - Q_i\mu_e \tag{11}
$$

$$
\mu_p = \mu_n - \mu_e \tag{12}
$$

$$
\mu_\mu = \mu_e \tag{13}
$$

$$\varepsilon = \sum_B \varepsilon_k^B + \sum_L \varepsilon_k^L + g_\omega(\rho^p + \rho^n)V_0 + \tfrac{1}{2}g_\rho(\rho^p - \rho^n)b_0 + U \tag{14}$$

$$\mu_i = \frac{\partial \varepsilon}{\partial \rho_i} = \frac{\partial \varepsilon}{\partial k_f}\frac{\partial k_f}{\partial \rho_i} \tag{15}$$

Dimana dengan transformasi GUP, terdapat koreksi yang tertuang pada suku terakhir untuk masing-masing potensial kimia

$$\mu_n = g_\omega V_0 - \tfrac{1}{2}g_\rho b_0 + \sqrt{k_n^2 + m_n^{*2}} + \frac{\beta(3+\alpha)}{3}\frac{k_n^4}{\sqrt{k_n^2 + m_n^{*2}}} \tag{16}$$

$$\mu_p = g_\omega V_0 + \tfrac{1}{2}g_\rho b_0 + \sqrt{k_p^2 + m_p^{*2}} + \frac{\beta(3+\alpha)}{3}\frac{k_p^4}{\sqrt{k_p^2 + m_p^{*2}}} \tag{17}$$

$$\mu_e = \sqrt{k_e^2 + m_e^{*2}} + \frac{\beta(3+\alpha)}{3}\frac{k_e^4}{\sqrt{k_e^2 + m_e^{*2}}} \tag{18}$$

$$\mu_\mu = \sqrt{k_\mu^2 + m_\mu^{*2}} + \frac{\beta(3+\alpha)}{3}\frac{k_\mu^4}{\sqrt{k_\mu^2 + m_\mu^{*2}}} \tag{19}$$

dimana *Number Density* memiliki bentuk

$$\rho_0 = \frac{k_f^3}{3\pi^2} - \frac{k_f^5}{5\pi^2}(3+\alpha)\beta \tag{20}$$

dengan persamaan k_f yang diaproksimasi adalah

$$\begin{gathered} k_f \approx k_f^{(0)} + \beta k_f^{(1)} \\ k_f \approx (3\pi^2\rho_0)^{1/3} + \beta\frac{3\pi^2}{5}(3+\alpha)\rho_0 \end{gathered} \tag{21}$$

dengan pendekatan ini kita dapat lebih mudah melakukan substitusi kedalam persaaan keseimbangan potensial kimia $\mu_p = \mu_n - \mu_e$, sehingga kita dapatkan fraksi elektron

$$Y_e = \frac{1}{3\pi^2}\left[\frac{\mu_e^2 - m_e^{*2}}{1+2\frac{\beta(3+\alpha)}{3}(\mu_e^2 - m_e^{*2})}\right]^{3/2}\frac{1}{\rho_B} \tag{22}$$

dimana

$$\begin{aligned}\mu_e = &\left(\sqrt{(3\pi^2[1-Y_p])^{2/3} + \frac{m_n^{*2}}{\rho_B^{2/3}}} - \sqrt{(3\pi^2 Y_p)^{2/3} + \frac{m_p^{*2}}{\rho_B^{2/3}}} + \frac{\frac{1}{2}g_\rho^2[1-2Y_p]\rho_B^{2/3}}{m_\rho^2 + 2f_2\sigma + g_3\sigma^2 + g_4\omega_0^2}\right.\\ &\left. + \frac{\beta(3+\alpha)}{3}\left[\frac{(3\pi^2[1-Y_p])^{4/3}\rho_B^{2/3}}{\sqrt{(3\pi^2[1-Y_p])^{2/3} + \frac{m_n^{*2}}{\rho_B^{2/3}}}} - \frac{(3\pi^2 Y_p)^{4/3}\rho_B^{2/3}}{\sqrt{(3\pi^2 Y_p)^{2/3} + \frac{m_p^{*2}}{\rho_B^{2/3}}}}\right]\right)\rho_B^{1/3}\end{aligned} \tag{23}$$

Sementara untuk mendapatkan fraksi proton, ditentukan dengan netralitas $Y_p = Y_e + Y_\mu$, karena $\mu_\mu = \mu_e$, maka persamaannya akan menjadi sebagai berikut

$$Y_p = \frac{1}{3\pi^2\rho_B}\left(\left[\frac{\mu_e^2 - m_e^{*2}}{1+2\frac{\beta(3+\alpha)}{3}(\mu_e^2 - m_e^{*2})}\right]^{3/2} + \left[\frac{\mu_\mu^2 - m_\mu^{*2}}{1+2\frac{\beta(3+\alpha)}{3}(\mu_\mu^2 - m_\mu^{*2})}\right]^{3/2}\right) \tag{24}$$

III. RESULT

CHECK ON PNM AND SNM

ACKNOWLEDGMENTS

We wish to acknowledge the support of the author community in using REVTEX, offering suggestions and encouragement, testing new versions, ...

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[1] A. Rahmansyah, A. Sulaksono, A. B. Wahidin and A. M. Setiawan, Eur. Phys. J. C **80**, no.8, 769 (2020).

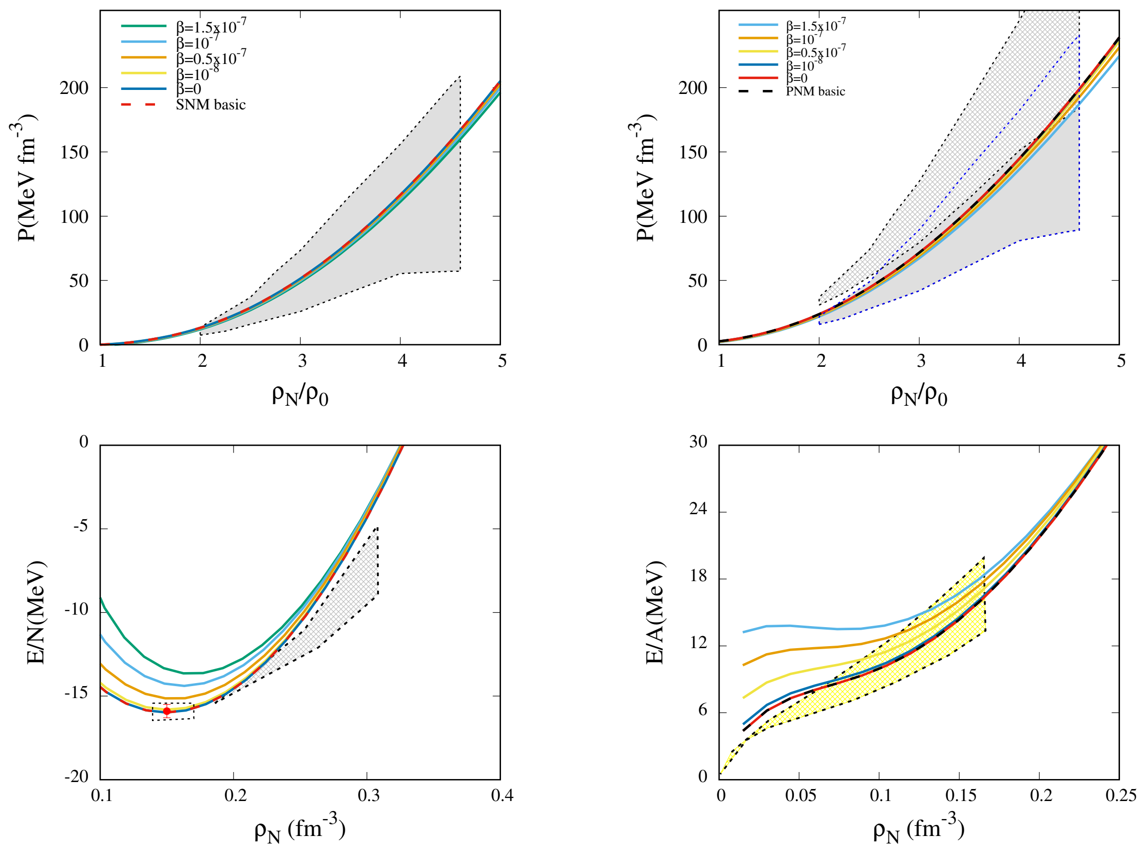

FIG. 1. Kasus SNM(kiri) dan PNM(kanan) masing-masing ditunjukan plot Tekanan P terhadap rapat jumlah dalam satuan kerapatan saturasi nuklir ρ_N/ρ_0 (atas) serta energi ikat E/N atau E/A terhadap rapat jumlah ρ_N(bawah).

TABLE I. In this table we show the comparison of the nuclear matter properties for several GUP parameter β at nuclear saturation density ($\rho/\rho_0 = 1$) for SNM case. We approximate the symmetric energy by $E_{sym} \approx E/N|_{PNM} - E/N|_{SNM}$. The bold number shows which values that lies outside the constrain from table II.

	BSP	β				
		0	10^{-8}	5×10^{-8}	10^{-7}	1.5×10^{-7}
E/N (MeV)	-16.0	-16.0	-15.8	**-15.1**	**-14.2**	**-13.3**
K_0 (MeV)	230.97	230.97	233.66	249.20	**281.84**	**334.25**
E_{sym} (MeV)	28.86	30.28	30.29	30.25	30.10	29.82
L (MeV)	50.10	50.10	50.23	50.79	51.54	52.38
J_0 (MeV)	-341.38	-341.38	-354.34	-511.62	**-944.72**	**-1729.59**
K_{sym} (MeV)	9.34	9.34	9.06	7.72	5.46	2.33

TABLE II. Comparison with other works from Rahmansyah, et al [a]

	other works
E/N (MeV)	-15.9 ± 0.4
K_0 (MeV)	230 ± 40
E_{sym} (MeV)	31.7 ± 3.2
L (MeV)	58.7 ± 28.1
J_0 (MeV)	$-800 \leq J_0 \leq 400$
K_{sym} (MeV)	$-400 \leq K_{sym} \leq 100$

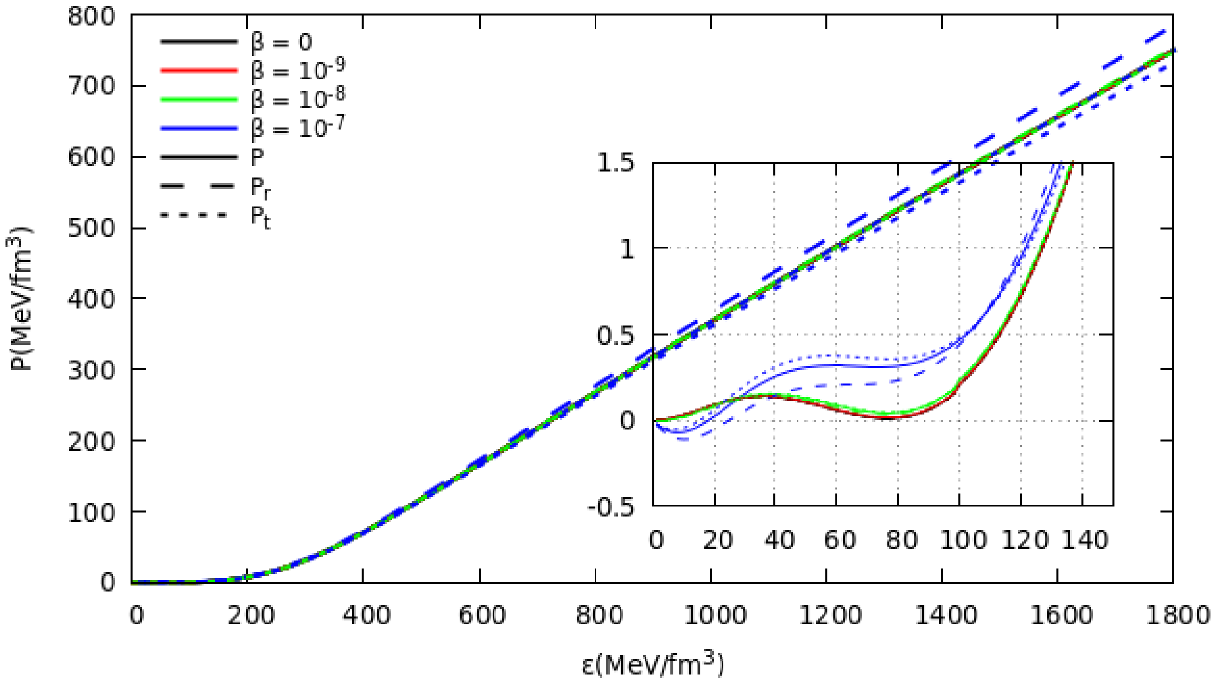

FIG. 2. Berikut adalah persamaan keadaan untuk materi bintang netron dengan variasi faktor anisotropi GUP. Gambar disajikan dalam 3 jenis garis : garis tegas (isotropik) dan 2 jenis garis putus-putus (anisotropik) yang masing-masing adalah tekanan arah radial P_r dan tangensial P_t. Sedangkan warna menunjukan variasi faktor GUP β. Untuk $\beta = 10^{-7}$ memiliki variasi efek anisotropi yang besar, namun tidak fisis karena memiliki tekanan negatif pada rapat energi rendah. sedangkan untuk kasus yang fisis ada pada $\beta < 10^{-8}$. Terlihat bahwa harganya dekat dengan $\beta = 0$ dan efek anisotropinya kecil sekali.