Approximation EOS $\epsilon(P)$ from $P(\epsilon)$
In Vergara et al PRD 2019

Invertible form.

$$P := \frac{1}{3}(\epsilon - 4B) - \alpha(\epsilon - B)^{1/2}.$$

$$\Rightarrow \epsilon = \frac{3}{2}\left\{\sqrt{3}\,\alpha\left(3\alpha^2 + 4(B+P)\right)^{1/2} + 3\alpha^2 + 2(B+P)\right\}$$

$$+ B.$$

$$\boxed{\epsilon = 3(P+B) + \Delta\epsilon(\alpha) + B.} \quad (1)$$

with

$$\boxed{\Delta\epsilon(\alpha, P) = \frac{3\sqrt{3}}{2}\alpha\left(3\alpha^2 + 4(B+P)\right)^{1/2} + \frac{9}{2}\alpha^2} \quad (2)$$

our case invertible because [Eq. (1), Vergara et al]

$$\boxed{\begin{aligned} P &= \frac{1}{3}(\epsilon - 4B) - \alpha(\epsilon - B)^{1/2} \\ &+ \beta\left[1 + K \ln\left\{\gamma(\epsilon - B)^{1/2}\right\}\right] \end{aligned}} \quad (3)$$

Therefore, we approximate eq. (3) by subtitute eq. (1) into the last term of eq. (3).

$$P - \beta\left[1 + K \ln\left\{\gamma\left[3(P+B) + \Delta\epsilon\right]^{1/2}\right\}\right]$$

$$= \frac{1}{3}(\epsilon - 4B) - \alpha(\epsilon - B)^{1/2}$$

If we define

$$\boxed{\tilde{P} := P - \beta\left[1 + K \ln\left\{\gamma\left[3(P+B) + \Delta\epsilon\right]^{1/2}\right\}\right]} \quad (4a)$$

then we have inverted form as:

$$\boxed{\begin{aligned}\epsilon \approx \frac{3}{2}\Big[\!\Big[\sqrt{3}\,\alpha\left[3\alpha^2 + 4(B + \tilde{P})\right]^{1/2} \\ + 3\alpha^2 + 2(\tilde{P} + B)\Big]\!\Big] + B.\end{aligned}} \quad (4b)$$

Here:

$$\alpha := \frac{m_s^2}{3\pi}\frac{1}{\sqrt{a_4}} \qquad \gamma := \frac{8\pi}{3m_s^2}\frac{1}{\sqrt{a_4}}$$

$$\beta := \frac{m_s^4}{12\pi^2}\left(1 - \frac{1}{a_4}\right) \qquad K := \frac{3}{\left(1 - \frac{2}{a_4}\right)}.$$

eq. (4b) + eq (4) in Vergera et. al. we can solve TOV eq. (7) & (8) in Vergera et. al.