Eur. Phys. J. C manuscript No.
(will be inserted by the editor)

Mass correction and deformation of slowly rotating anisotropic neutron stars based on Hartle-Thorne formalism

M. L. Pattersons[a,1], **A. Sulaksono**[b,1]

[1]Department of Physics, University of Indonesia, Kampus UI Depok, Indonesia 16424

Received: date / Accepted: date

Abstract Due to their compactness, neutron stars are the best objects to simultaneously study matter in high density and strong-field gravity. Hartle and Thorne have proposed a good approximation or perturbation procedure within general relativity for slowly rotating relativistic stars by assuming the matter inside the stars is an ideal isotropic fluid. In this study, we extend the analytical Hartle-Thorne formalism for slowly rotating, including the possibility that the neutron star pressure can be anisotropic. We study the impact of neutron stars anisotropy pressure on the mass correction and deformation numerically. For the anisotropic model, we use the Bowers-Liang model. For the equation of state of neutron stars, we use a relativistic mean-field BSP parameter set with the hyperons and crust equation of state are taken into account. We have found that the mass of neutron stars increases by increasing λ_{BL} value, and the EOS becomes stiffer when λ_{BL} is relatively large, and leads to a condition which NS is getting harder to deformed when the λ_{BL} increased.

1 Introduction

It has been understood that the evolution of the massive stars whose mass in the range $8-25\ M_\odot$ is ended when neutron stars (NSs) formed. The corresponding formation is due to the stars' gravitational collapse during Type-II, Ib, or Ic supernova explosion phenomena [1]. NSs have some convincing observational signatures, such as pulse period, masses (of NS in a binary system), thermal emission, glitches [2]. The corresponding pieces of information through X-rays, γ-rays, and neutrinos sources can be used to extract the NS properties[1]. NSs as the densest stars are 10^4 times denser than the earth. Therefore, NSs are the best avenues to study the matter at high density and strong-field gravity simultaneously. However, up to now, NS's matter composition and gravity are not entirely understood [3,4]. We need to note that accurate measurements of massive pulsars have been reproted in Refs [5–7]. These measurements provide maximum mass limit of NS around 2.0 $M_\odot$. Several constraints of NSs radii from X-ray sources also reported up to the recent years (see Ref. [8] for the discussions). The simultaniously mass and radius constraints from NICER using PSR J0030+0451 pulsar [9–11]. The tidal deformability gravitational waves (GW) observations of coalescence NSs with LIGO and VIGO can also provide NS mass and radius constraints [12–14]. It is also need to note that the LIGO/Virgo collaboration recently reported the GW190814 observation of the merger of a black hole (BH) of mass (22.2-24.3) $M_\odot$ and a secondary massive compact object with mass (2.50-2.67)$M_\odot$ [15]. If the secondary object is NS, then the mass is significantly larger than the known maximum mass constraint.

The equation of state (EOS) of NS as crucial information to know the radius and the maximum mass of NS is determined from NS matter composition[16]. Many authors have proposed phenomenological models or the ones based on many-body approaches to calculating EOS. The Phenomenological approaches, either relativistic or non-relativistic, are based on effective interactions developed to reproduce the properties of finite nuclei. The Skyrme Hartree-Fock (SHF) and relativistic mean-field (RMF) models belong to this approach. The recent discussions of the application status of RMF models in NS EOS can be consulted to Refs. [17,18]. On the other hand, many-body microscopic models are based on two- and three-body forces

[a]e-mail: muhammad.lawrence@ui.ac.id
[b]e-mail: anto.sulaksono@sci.ui.ac.id

that describe scattering data in free space and the properties of the deuteron [1]. To this end, we need to emphasize that NS's properties such as mass, radius, and spin are sensitively influenced by the nature of the NS EOS [19]. Please see the recent review of NS EOS in Refs. [20,21].

Due to the strong gravity of NS, one must use general relativity to describe the NS structure. Up to now, it is common that NS is modeled as a static, spherically symmetric, and non-rotating object while the matter is described by isotropic perfect fluid [2]. In this way, one can use Tolman-Oppenheimer-Volkoff (TOV) equation [22] to calculate NSs structures. The situation becomes relatively more complicated when we assume that NS can rotate and becomes deformed due to the rotation. Hartle [23], and then Hartle and Thorne [24] have proposed a magnificent approximation to handle this problem, i.e. approximation of slowly rotating relativistic stars. Once one employs the Hartle-Thorne (HT) formalism, deformation and mass correction of the relativistic stars can be calculated. The pioneering work of Hartle and Thorne has been investigated in many studies. According to Ref. [25], HT approach is relative better than those of Manko [26], and Cook-Shapiro-Teukolsky (CST) [27].

In many works, the NS matter is usually modeled as an isotropic fluid. However, it is natural to expect that there is a possibility of the appearance of anisotropic pressure, i.e., the unequal value between radian and tangential pressures inside NS matter. For details, please see Refs. [3,19,28–34] and the references therein. We need to note that the anisotropic pressure might also exist in other kinds of compact stars, such as strange quark stars [35–37]. Many factors can cause anisotropic pressure in compact objects due to matter properties which are linking to each other such as boson condensations, the existence of solid core, different kinds of phase transition, as well as the presence of strong magnetic and electric field,[28] and or due to the impact of modified geometry [38–41]. Studies on static anisotropic NSs to obtain their mass correction and deformation have been studied; for example, it is discussed in Refs. [19, 29–31]. In Refs. [19,29,30], the perturbation in metric appears due to NS's magnetic field is modeled as the source of anisotropic pressure of NS. Note that the formalism is analog to that used in perturbation matric due to HT formalism for rotation. However, the impact of rotation is still neglected in their works. It means that their works are not too realistic since celestial objects are normally rotating due to the law of angular momentum conservation [42]. It is worthed noting that many authors have investigated the numerical calculation of rotating NSs by considering NSs as stars with isotropic pressure within HT formalism. Please see Ref. [2] as an example. However, it is evident from many studies that the anisotropic pressure is more realistic to describe NSs. [3,19,28–30,32–36,43–50].

By Considering these factors and the fact that the HT model is better than the Manko and CST model, we argue that it is essential to extend HT formalism. Therefore, we can approach perturbations attributed to NS as a complete form that contains both anisotropic pressure and rotation. In this study, we reformulated the formalism of mass correction and deformation of slowly rotating anisotropic NS based on HT formalism, so we still used HT metric and followed Einstein tensor calculations done by Hartle and Thorne [23,24]. We added anisotropic pressure in the energy-momentum tensor and followed Hartle and Thorne's procedure in calculating Einstein field equations. We then applied the formalism to calculate mass correction and deformation of NS. We have obtained the results by employing numerical calculations. Here, we use Bowers-Liang (BL) model for anisotropic pressure of NS [51]. The EOS of the core of NS is calculated using the RMF model with the BSP parameter set [52,53] under which the standard SU(6) prescription and hyperon potential depths [54] are utilized to determine the hyperon coupling constants. For the inner and outer crusts, we use the crust EOS obtained from Miyatsu *et al.* [55].

The structure of this paper is, in Sec. 2, we discuss the formalism used. In Sec. 3, we discuss the obtained numerical results, and we conclude in Sec. 4.

2 Formalism

To make the discussion self-contained, in sub-section 2.1, we briefly review the standard HT analitical formalism for isotropic stars, while the the one for anisotropic is given in sub-section 2.2.

2.1 HT formalism for mass correction and deformation of slowly rotating relativistic stars

Here we review briefly the HT formalism for isotropic stars. In this paper we use geometrized units $G = c = 1$. The HT metric can be expressed as [2]

$$ds^2 = -e^{2\nu}dt^2 + e^{2\lambda}dr^2 + e^{2\psi}(d\phi - \omega dt)^2 + e^{2\mu}d\theta^2, \quad (1)$$

where ω denotes the angular velocity of local inertial frame, and it is proportional to the angular velocity of the star Ω.

The exponential factors in each terms of the right-hand side of Eq. (1) are expanded as

$$e^{2\nu} = e^{2\varphi}[1 + 2(h_0 + h_2 P_2(\cos\theta))], \quad (2)$$

$$e^{2\lambda} = \left[1 + \tfrac{2}{r}\left(m_0 + m_2 P_2(\cos\theta)\right)\left(1 - \tfrac{2m(r)}{r}\right)^{-1}\right], \quad (3)$$

$$e^{2\psi} = r^2 \sin^2\theta[1 + 2(v_2 - h_2)P_2(\cos\theta)], \quad (4)$$

$$e^{2\mu} = r^2[1 + 2(v_2 - h_2)P_2(\cos\theta)], \quad (5)$$

where h_0, h_2, m_0, m_2, and v_2 are perturbation functions; $P_2(\cos\theta)$ is the second order of Legendre polynomial; $e^{2\varphi}$ and $m(r)$ are functions which are bound by constraints [23]

$$\frac{d\varphi}{dr} = \frac{m(r) + 4\pi r^3 p(r)}{r(r - 2m(r))}, \quad (6)$$

and

$$\frac{dm}{dr} = 4\pi r^2 \epsilon(p). \quad (7)$$

where ϵ denotes energy density, p denotes pressure of the star. TOV equation [22] can be obtained from Eq. (6). It is written as

$$\frac{dp}{dr} = -\frac{(\epsilon + p)(m(r) + 4\pi r^3 p(r))}{r(r - 2m(r))}. \quad (8)$$

In order to obtain the perturbation functions h_0, h_2, m_0, and m_2, one has to solve the Einstein equations as

$$R^\mu_\nu - \frac{1}{2}\delta^\mu_\nu R = 8\pi T^\mu_\nu. \quad (9)$$

By calculating (t, t) and (r, r) components of Einstein equations, one can obtain

$$\frac{dm_0}{dr} = 4\pi r^2 \frac{d\epsilon}{dp}(\epsilon + p)\, p_0^* + \frac{1}{12} j^2 r^4 \left(\frac{d\bar{\omega}}{dr}\right)^2 - \frac{1}{3} r^3 \frac{dj^2}{dr}\bar{\omega}^2, \quad (10)$$

$$\frac{dp_0^*}{dr} = -\frac{m_0(1 + 8\pi r^2 p)}{(r - 2m)^2} - \frac{4\pi(\epsilon + p)\, r^2}{(r - 2m)} p_0^* + \frac{1}{12}\frac{r^4 j^2}{(r - 2m)}\left(\frac{d\bar{\omega}}{dr}\right)^2 + \frac{1}{3}\frac{d}{dr}\left(\frac{r^3 j^2 \bar{\omega}^2}{r - 2m}\right), \quad (11)$$

where $p_0^* = -h_0 + \frac{1}{3} r^2 e^{-2\varphi}\bar{\omega} + h$, and h is a constant that is needed so that h_0 is continuous across the surface of the star; $\bar{\omega} = \Omega - \omega$, i.e. angular velocity of the star relative to the local inertial frame; and $j = e^{-\varphi}\left(1 - \frac{2m(r)}{r}\right)^{1/2}$. Boundary conditions for both m_0 and p_0^* are zero at $r = 0$ since they vanish at the origin. In this calculation concatenation, one has to find $\bar{\omega}$ by solving a differential equation, i.e.

$$\frac{1}{r^4}\frac{d}{dr}\left(r^4 j \frac{d\bar{\omega}}{dr}\right) + \frac{4}{r}\frac{dj}{dr}\bar{\omega} = 0. \quad (12)$$

Boundary conditions at $r = 0$ that satisfies Eq. (12) are $\bar{\omega} = \omega_c$ and $\frac{d\bar{\omega}}{dr} = 0$, where ω_c can be chosen arbitrarily.

The mass correction can be written as

$$\delta M = m_0(R) + \frac{J^2}{R^3}, \quad (13)$$

where R is the radius of the star without deformation, and J denotes moment of inertia of the star.

From the (θ, θ), (ϕ, ϕ), (θ, r), and (r, r) components of Einstein equations, one can obtains

$$\frac{dv_2}{dr} = -2\frac{d\varphi}{dr}h_2 + \left(\frac{1}{r} + \frac{d\varphi}{dr}\right) \times \left[-\frac{1}{3} r^3 \frac{dj^2}{dr}\bar{\omega}^2 + \frac{1}{6} j^2 r^4 \left(\frac{d\bar{\omega}}{dr}\right)^2\right], \quad (14)$$

$$\begin{aligned}\frac{dh_2}{dr} = &-2\frac{d\varphi}{dr}h_2 + \frac{r}{r - 2m}\left(2\frac{d\varphi}{dr}\right)^{-1} \\ &\times \left\{8\pi(\epsilon + p) - \frac{4m}{r^3}\right\} h_2 - \frac{4v_2}{r(r - 2m)} \\ &\times \left(2\frac{d\varphi}{dr}\right)^{-1} + \frac{1}{6}\left[\frac{d\varphi}{dr}r - \frac{1}{r - 2m}\left(2\frac{d\varphi}{dr}\right)^{-1}\right] \\ &\times r^3 j^2 \left(\frac{d\bar{\omega}}{dr}\right)^2 - \frac{1}{3}\left[\frac{d\varphi}{dr}r + \frac{1}{r - 2m}\left(2\frac{d\varphi}{dr}\right)^{-1}\right] \\ &\times r^2 \frac{dj^2}{dr}\bar{\omega}^2. \end{aligned} \quad (15)$$

with the boundary conditions

$$v_2(0) = h_2(0) = v_2(\infty) = h_2(\infty) = 0. \quad (16)$$

There is a formula [2,23]

$$p_2^* = -h_2 - \frac{1}{3} r^2 e^{-2\varphi}\bar{\omega}^2. \quad (17)$$

The deformation is expressed as

$$\delta r = \xi_0(R) + \xi_2(R) P_2(\cos\theta), \quad (18)$$

where

$$\xi_0(r) = -p_0^*(\epsilon + p)\left(\frac{dp}{dr}\right)^{-1}, \quad (19)$$

$$\xi_2(r) = -p_2^*(\epsilon + p)\left(\frac{dp}{dr}\right)^{-1}. \quad (20)$$

From this point, we can calculate the radius of the pole R_p and the radius of the equator R_e,

$$R_p = R + \xi_0 + \xi_2, \quad (21)$$

$$R_e = R + \xi_0 - \tfrac{1}{2}\xi_2. \quad (22)$$

We can also find the eccentricity of the star, i.e.

$$e = \sqrt{1 - \frac{R_p}{R_e}}. \quad (23)$$

The star is spherically symmetric if the value of e is equal to 0, and its shape is getting more oblate as the value of e is getting close to 1.

2.2 Modified HT formalism due to anisotropic pressure of NS

The energy-momentum tensor of anisotropic NS [28,33] can be written as

$$T^{\mu}_{\nu} = \epsilon u^{\mu} u_{\nu} + q\delta^{\mu}_{\nu} + \sigma k^{\mu} k_{\nu}, \tag{24}$$

where $\sigma = p - q$, and k_{ν} is a unit radial vector that satisfies $u^{\mu} k_{\nu} = 0$.

In BL model, σ is expressed as [3,51]

$$\sigma = -\lambda_{BL} \frac{G\epsilon r^2}{3} \left(1 + \frac{3p}{\epsilon}\right) \frac{(1 + \frac{p}{\epsilon})}{1 - \frac{2Gm(r)}{r}}, \tag{25}$$

where λ_{BL} is defined as the strength of anisotropic terms of the model, and G is the universal constant of gravity.

TOV equation for anisotropic NS [28] is expressed by

$$\frac{dp}{dr} = -\frac{(\epsilon + p)(m(r) + 4\pi r^3 p(r))}{r(r - 2m(r))} - \frac{2\sigma}{r}. \tag{26}$$

By reformulating HT model which can be compatible for slowly rotating anisotropic NS, several eqs. are modified. We followed calculation procedure for Einstein equations. which is written in Ref. [23]. The multipole expansion terms are limited to $l = 2$. One can obtain the expansion terms of Einstein tensor and Ricci tensor. Those are written as

$$\begin{aligned}\left(\delta G^t_t\right)_{l=0} = \frac{j}{6r^2} &\left[8r^3 \frac{dj}{dr}\, \omega(\Omega - \omega) + jr^4 \left(\frac{d\omega}{dr}\right)^2\right] \\ &- \frac{dm_0}{dr} \frac{2}{r^2},\end{aligned} \tag{27}$$

$$\begin{aligned}\left(\delta G^t_t\right)_{l=2} = \frac{j}{6r^2} &\left[8r^3 \frac{dj}{dr}\, \omega(\Omega - \omega) + jr^4 \left(\frac{d\omega}{dr}\right)^2\right] \\ &- \frac{dm_2}{dr} \frac{2}{r^2} + \left(1 - \frac{2m}{r}\right) \\ &\times \left[2\left(\frac{d^2 v_2}{dr^2} - \frac{d^2 h_2}{dr^2}\right) + \frac{6}{r}\left(\frac{dv_2}{dr} - \frac{dh_2}{dr}\right)\right] \\ &- 2\left(\frac{dv_2}{dr} - \frac{dh_2}{dr}\right) \frac{d}{dr}\left(\frac{m}{r}\right) \\ &- \frac{6m_2}{r^2(r - 2m)} - \frac{4(v_2 - h_2)}{r^2},\end{aligned} \tag{28}$$

$$\begin{aligned}\left(\delta G^r_r\right)_{l=0} = \frac{1}{6} r^2 j^2 &\left(\frac{d\omega}{dr}\right)^2 - \frac{2m_0}{r^2}\left(2\frac{d\varphi}{dr} + \frac{1}{r}\right) \\ &+ \left(1 - \frac{2m}{r}\right) \frac{2}{r} \frac{dh_0}{dr},\end{aligned} \tag{29}$$

$$\begin{aligned}\left(\delta G^r_r\right)_{l=2} = -\frac{1}{6} r^2 j^2 &\left(\frac{d\omega}{dr}\right)^2 - \frac{2m_2}{r^2}\left(2\frac{d\varphi}{dr} + \frac{1}{r}\right) \\ &+ \left(1 - \frac{2m}{r}\right) \frac{2}{r} \frac{dh_2}{dr} - \frac{6h_2}{r^2} \\ &+ \left(1 - \frac{2m}{r}\right)\left(\frac{dv_2}{dr} - \frac{dh_2}{dr}\right)\left(2\frac{d\varphi}{dr} + \frac{2}{r}\right) \\ &- \frac{4(v_2 - h_2)}{r},\end{aligned} \tag{30}$$

$$\left(\delta R_{r\theta}\right)_{l=0} = 0, \tag{31}$$

$$\begin{aligned}\left(\delta R_{r\theta}\right)_{l=2} = h_2 &\left(\frac{1}{r} - \frac{d\varphi}{dr}\right) - \frac{dv_2}{dr} + \frac{m_2}{r - 2m} \\ &\times \left(\frac{1}{r} + \frac{d\varphi}{dr}\right),\end{aligned} \tag{32}$$

$$\begin{aligned}\left(R^{\theta}_{\theta} - R^{\phi}_{\phi}\right) = \frac{1}{2}\sin^2\theta &\left[-\frac{3\left(h_2 + \frac{m_2}{r - 2m}\right)}{r^2}\right. \\ &+ \frac{j^2 r^2}{4}\left(\sin^2\theta \frac{d\omega}{dr}\right)^2 + \frac{1}{2} r\omega(\Omega - \omega) \\ &\times \left(\frac{dj^2}{dr}\right)\sin^2\theta.\end{aligned} \tag{33}$$

According to Ref. [23], correction of Einstein tensor can be written as

$$\Delta G^{\nu}_{\mu} = \delta G^{\nu}_{\mu} + \xi \frac{\partial}{\partial r}\left[G^{\nu(0)}_{\mu}\right], \tag{34}$$

where $G^{\nu(0)}_{\mu}$ is unperturbed Einstein tensor. Recall unperturbed Einstein equations $G^{\nu(0)}_{\mu} = 8\pi T^{\nu(0)}_{\mu}$, where $T^{\nu(0)}$ is energy-momentum tensor which is not perturbed by rotation, so one can obtain

$$\Delta G^{\nu}_{\mu} = \delta G^{\nu}_{\mu} + 8\pi\xi \frac{\partial}{\partial r}\left[T^{\nu(0)}_{\mu}\right]. \tag{35}$$

The explicit forms of correction of Einstein tensor are written as

$$\Delta G^t_t = \delta G^t_t - 8\pi\xi \frac{\partial \epsilon}{\partial r}, \tag{36}$$

$$\Delta G^r_r = \delta G^r_r + 8\pi\xi \frac{\partial p}{\partial r}, \tag{37}$$

$$\Delta G^{\theta}_{\theta} = \delta G^{\theta}_{\theta} + 8\pi\xi \frac{\partial q}{\partial r}, \tag{38}$$

$$\Delta G^{\phi}_{\phi} = \delta G^{\phi}_{\phi} + 8\pi\xi \frac{\partial q}{\partial r}, \tag{39}$$

$$\Delta G^{\theta}_{r} = \delta G^{\theta}_{r}. \tag{40}$$

The mathematical entity ξ is expanded as

$$\xi = \xi_0(r) + \xi_2(r) P_2(\cos\theta) + \ldots. \tag{41}$$

Here we limited the expansion terms of ξ to $l = 2$.

Considering Eq. (24), so we have new forms of p_0^* and p_2^*, i.e.

$$p_0^* = -\xi_0 \left(\frac{1}{\epsilon + q} \frac{dq}{dr} \right), \tag{42}$$

$$p_2^* = -\xi_2 \left(\frac{1}{\epsilon + q} \frac{dq}{dr} \right), \tag{43}$$

and ξ_0 and ξ_2 can be expressed as

$$\xi_0 = -p_0^* (\epsilon + p) \left(1 - \frac{\sigma}{\epsilon + p} \right) \left(\frac{dp}{dr} - \frac{d\sigma}{dr} \right)^{-1}, \tag{44}$$

$$\xi_2 = -p_2^* (\epsilon + p) \left(1 - \frac{\sigma}{\epsilon + p} \right) \left(\frac{dp}{dr} - \frac{d\sigma}{dr} \right)^{-1}. \tag{45}$$

Eq. (44) and Eq. (45) are anisotropic modified forms of Eq. (19) and Eq. (20).

While the correction of energy-momentum tensor for anisotropic NS can be written as

$$\Delta T_t^t = \delta T_t^t + \xi \frac{\partial \epsilon}{\partial r}, \tag{46}$$

$$\Delta T_r^r = \delta T_r^r + \xi \frac{\partial p}{\partial r}, \tag{47}$$

$$\Delta T_\theta^\theta = \delta T_\theta^\theta + \xi \frac{\partial q}{\partial r}, \tag{48}$$

$$\Delta T_\phi^\phi = \delta T_\phi^\phi + \xi \frac{\partial q}{\partial r}, \tag{49}$$

where

$$\delta T_\nu^\mu = (\Delta\epsilon + \Delta q) u^\mu u_\nu + \Delta q \delta_\nu^\mu + \Delta\sigma k^\mu k_\nu, \tag{50}$$

$$\Delta\epsilon = (\epsilon + q) \left(\frac{\partial \epsilon}{\partial q} \right) p^* = -\frac{\partial \epsilon}{\partial r}\, \xi, \tag{51}$$

$$\Delta q = (\epsilon + q)\, p^* = -\frac{\partial q}{\partial r}\, \xi, \tag{52}$$

$$\Delta\sigma = (\epsilon + q) \left(\frac{\partial \sigma}{\partial q} \right) p^* = -\frac{\partial \sigma}{\partial r}\, \xi. \tag{53}$$

It is easy to show that

$$(\Delta T_t^t)_{l=0} = -\tfrac{2}{3} (\epsilon + p)\, e^{-\nu} r^2 \bar{\omega}^2 \left(1 - \tfrac{\sigma}{\epsilon + p} \right), \tag{54}$$

$$(\Delta T_r^r)_{l=0} = 0. \tag{55}$$

We step further to Einstein equations for the corrections of the tensors

$$\Delta G_\nu^\mu = 8\pi \Delta T_\nu^\mu. \tag{56}$$

With a little algebra, for $\mu = \nu = t$, and $l = 0$, one can obtain

$$\begin{aligned} \frac{dm_0}{dr} &= 4\pi r^2 \frac{d\epsilon}{dp} \left(1 - \frac{d\sigma}{dp} \right)^{-1} (\epsilon + p) \left(1 - \frac{\sigma}{\epsilon + p} \right) p_0^* \\ &\quad + \frac{1}{12} j^2 r^4 \left(\frac{d\bar{\omega}}{dr} \right)^2 - \frac{1}{3} r^3 \left(\frac{dj^2}{dr} \right) \bar{\omega}^2 \\ &\quad \times \left(1 - \frac{\sigma}{\epsilon + p} \right), \end{aligned} \tag{57}$$

and for $\mu = \nu = r$, with $l = 0$, one can obtain

$$\begin{aligned} \frac{dp_0^*}{dr} &= -\frac{m_0 (1 + 8\pi r^2 p)}{(1 - 2m)^2} - \frac{4\pi r^2 (\epsilon + p)}{r - 2m} \\ &\quad \times \left(1 - \frac{\sigma}{\epsilon + p} \right) p_0^* + \frac{1}{12} \frac{r^4 j^2}{r - 2m} \left(\frac{d\bar{\omega}}{dr} \right)^2 \\ &\quad + \frac{1}{3} \frac{d}{dr} \left(\frac{r^3 j^2 \bar{\omega}^2}{r - 2m} \right). \end{aligned} \tag{58}$$

Eq. (57) and Eq. (58) are modified forms of Eq. (10) and Eq. (11), respectively. It is important to keep in mind that both m_0 and p_0^* vanish at the origin.

From (θ, θ) and (ϕ, ϕ) components, we can obtain

$$T_\theta^\theta - T_\phi^\phi = -(\epsilon + p) e^{-\nu} \bar{\omega}^2 r^2 \sin^2\theta \left(1 - \frac{\sigma}{\epsilon + p} \right). \tag{59}$$

Recall the Einstein equations for (θ, θ) and (ϕ, ϕ) components

$$8\pi \left(T_\theta^\theta - T_\phi^\phi \right) = R_\theta^\theta - R_\phi^\phi. \tag{60}$$

From Eq. (60), one can obtain

$$\begin{aligned} m_2 &= \left[-h_2 - \left(\frac{1}{3} - \frac{2\sigma}{\epsilon + p} \right) r^3 (j^2)_r\, \bar{\omega}^2 + \frac{1}{6} j^2 r^4 (\bar{\omega}_r)^2 \right] \\ &\quad \times r \left(1 - \frac{2m}{r} \right). \end{aligned} \tag{61}$$

It is clear that non-diagonal components of energy-momentum tensor is equal to zero, so

$$\left(\delta T_r^\theta \right)_{l=2} = (\delta T_{r\theta})_{l=2} = 0. \tag{62}$$

Recall the Einstein equation for (r, θ) component with $l = 2$

$$\begin{aligned} 8\pi\, (\delta T_{r\theta})_{l=2} &= (\delta R_{r\theta})_{l=2}\,, \\ 0 &= -\frac{dh_2}{dr} + h_2 \left(\frac{1}{r} - \frac{d\varphi}{dr} \right) \\ &\quad - \left(\frac{dv_2}{dr} - \frac{dh_2}{dr} \right) + \frac{m_2}{r - 2m} \\ &\quad \times \left(\frac{1}{r} + \frac{d\varphi}{dr} \right) \\ \frac{dv_2}{dr} &= \frac{m_2}{r - 2m} \left(\frac{1}{r} + \frac{d\varphi}{dr} \right) + h_2 \\ &\quad \times \left(\frac{1}{r} - \frac{d\varphi}{dr} \right). \end{aligned} \tag{63}$$

By inserting Eq. (61) into Eq. (63), we can obtain

$$\begin{aligned} \frac{dv_2}{dr} &= -2 \frac{d\varphi}{dr} h_2 + \left(\frac{1}{r} + \frac{d\varphi}{dr} \right) \\ &\quad \times \left[-\left(\frac{1}{3} - \frac{2\sigma}{\epsilon + p} \right) r^3 \frac{dj^2}{dr} \bar{\omega}^2 \right] \\ &\quad + \left(\frac{1}{r} + \frac{d\varphi}{dr} \right) \left[\frac{1}{6} j^2 r^4 \left(\frac{d\bar{\omega}}{dr} \right)^2 \right]. \end{aligned} \tag{64}$$

Eq. (64) is modified form of Eq. (14).

From Eq. (47) it is clear that

$$(\Delta T^r_r)_{l=2} = 0. \tag{65}$$

Recall the Einstein equation for (r, r) component, with $l = 2$

$$\begin{aligned}
8\pi\,(\Delta T^r_r)_{l=2} &= (\Delta G^r_r)_{l=2}\,, \\
0 &= -\frac{1}{6}r^2 j^2 \left(\frac{d\bar{\omega}}{dr}\right)^2 - \frac{2m_2}{r^2}\left(2\frac{d\varphi}{dr}+\frac{1}{r}\right) \\
&\quad + \left(1-\frac{2m}{r}\right)\frac{2}{r}\frac{dh_2}{dr} - \frac{6h_2}{r^2} \\
&\quad + \left(1-\frac{2m}{r}\right)\left(\frac{dv_2}{dr}-\frac{dh_2}{dr}\right) \\
&\quad \times\left(2\frac{d\varphi}{dr}+\frac{2}{r}\right) - \frac{4(v_2-h_2)}{r} \\
&\quad + 8\pi\xi_2\frac{dp}{dr}, \\
0 &= -\frac{1}{6}r^2 j^2 \left(\frac{d\bar{\omega}}{dr}\right)^2 - \frac{2m_2}{r^2}\left(2\frac{d\varphi}{dr}+\frac{1}{r}\right) \\
&\quad + \left(1-\frac{2m}{r}\right)\frac{2}{r}\frac{dh_2}{dr} - \frac{6h_2}{r^2} \\
&\quad + \left(1-\frac{2m}{r}\right)\left(\frac{dv_2}{dr}-\frac{dh_2}{dr}\right) \\
&\quad \times\left(2\frac{d\varphi}{dr}+\frac{2}{r}\right) - \frac{4(v_2-h_2)}{r} \\
&\quad - 8\pi(\epsilon+p)\left(1-\frac{\sigma}{\epsilon+p}\right) \\
&\quad \times\left(1-\frac{d\sigma}{dp}\right)p_2^*.
\end{aligned} \tag{66}$$

Recall the formula [23] $p_2^* = -h_2 - \frac{1}{3}e^{-\nu}r^2\bar{\omega}^2$, so we can write Eq. (66) as

$$\begin{aligned}
\frac{dh_2}{dr} &= -2h_2\frac{d\varphi}{dr} + \frac{r}{r-2m}\left(2\frac{d\varphi}{dr}\right)^{-1} h_2 \\
&\quad \times 8\pi(\epsilon+p)\left(1-\frac{\sigma}{\epsilon+p}\right)\left(1-\frac{d\sigma}{dp}\right)^{-1} \\
&\quad - \frac{r}{r-2m}\left(2\frac{d\varphi}{dr}\right)^{-1}\frac{4m}{r^3}h_2 \\
&\quad - \frac{4v_2}{r(r-2m)}\left(2\frac{d\varphi}{dr}\right)^{-1} + r^3 j^2\left(\frac{d\bar{\omega}}{dr}\right)^2 \\
&\quad \times\frac{1}{6}\left[\frac{d\varphi}{dr}r - \frac{1}{r-2m}\left(2\frac{d\varphi}{dr}\right)^{-1}\right] \\
&\quad - \frac{1}{3}\left(1-\frac{6\sigma}{\epsilon+p}\right)\frac{d\varphi}{dr}r^3\left(\frac{dj^2}{dr}\right)\bar{\omega}^2 \\
&\quad + \frac{r^2}{3}\left(\frac{dj^2}{dr}\right)\bar{\omega}^2\frac{1}{r-2m}\left(2\frac{d\varphi}{dr}\right)^{-1} \\
&\quad \times\left(1-\frac{\sigma}{\epsilon+p}\right)\left(1-\frac{d\sigma}{dp}\right)
\end{aligned} \tag{67}$$

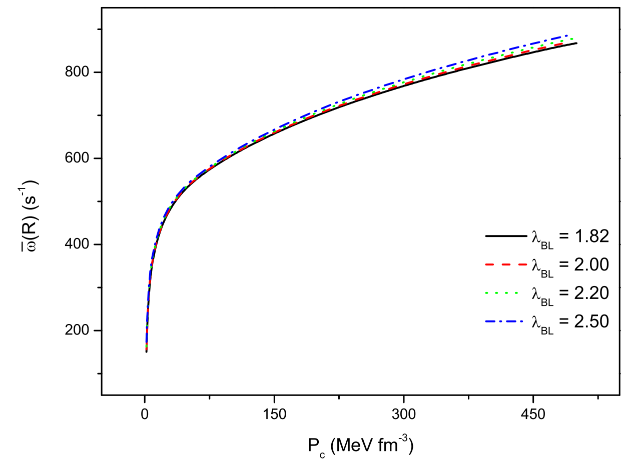


Fig. 1 Relation between pressure at the center of the star and angular velocity of the star relative to local inertial frame at the star's surface, i.e. $\bar{\omega}(R)$. We used four values of λ_{BL} which represent strength of anisotropic terms. The influence of anisotropic pressure of the star can be seen obviously after P_c has already exceeded 150 MeV fm^{-3}.

Eq. (67) is modified form of Eq. (67). Boundary conditions for both v_2 and h_2 are not changed, i.e. Eq. (16).

For anisotropic NS, The momen of inertia equation in Eq. (12) is modified to be following expression

$$\frac{1}{r^4}\frac{d}{dr}\left(r^4 j\frac{d\bar{\omega}}{dr}\right) + \frac{4}{r}\frac{dj}{dr}\left(1-\frac{\sigma}{\epsilon+p}\right)\bar{\omega} = 0. \tag{68}$$

These modified differential equations due to anisotropic pressure of h_0, h_2, m_0, m_2, and v_2 and the one of inertia moment are the main analytical results of this work.

3 Numerical results

In these numerical calculations, we used Runge-Kutta fourth-order method to solve every differential equation. For the EOS, we used the model from ref. [53], and the hyperons are taken into account. The same EOS has been used in several studies, e.g. [3,29,30]. The main differential equations in this work are Eq. (57), Eq. (58), Eq. (67), and Eq. (63), with boundary conditions $m_0(0) = 0$, $p_0^*(0) = 0$, $v_2(0) = 0$, and $h_2(0) = 0$. The values of $m_0(R)$, $p_0^*(R)$, $v_2(R)$, and $h_2(R)$ are obtained when calculations of differential equations stop, since those processes end when $r = R$. The value of $m_0(R)$ is used to find mass correction so that we can substitute it to Eq. (13), while the value of $p_0^*(R)$ is used to find $\xi_0(R)$ which is related to the star's deformation. Note that differential equations of p_0^* and m_0 are coupled to each other, so we have to solve both simultaneously. The value of $h_2(R)$ is needed to find $p_2^*(R)$ so that we

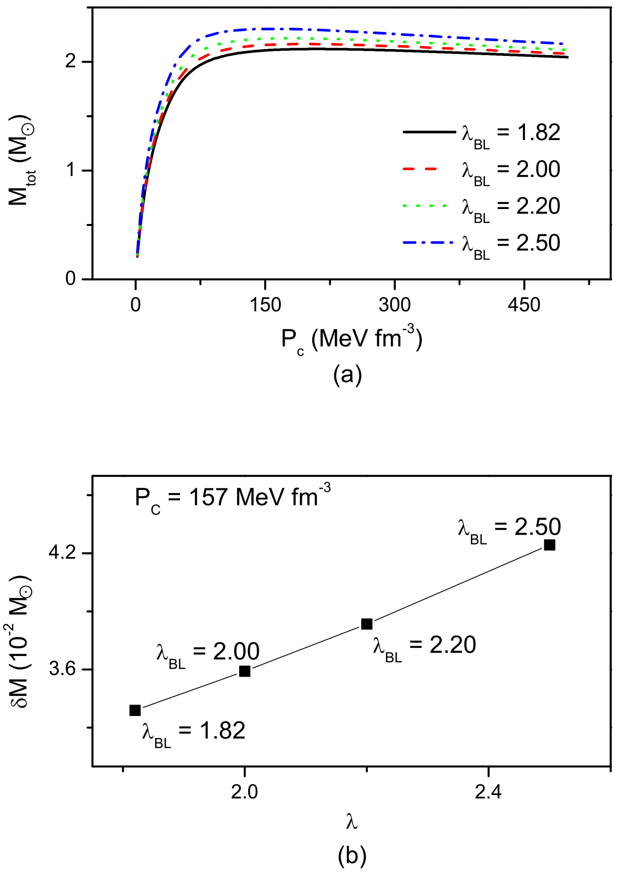


Fig. 2 Impact of anisotropic pressure to total mass and mass correction of NS. The λ are specified in the figures. (a) Relation between pressure at the center of the star and total mass. Maximum mass of NS with $\lambda_{BL} = 2.50$ (blue dot dash curve) exceeds 2.3 $M_\odot$. (b) Mass correction, i.e. δM for different values of λ_{BL} at P_c = 157 MeV fm^{-3}, which is the point where star with $\lambda_{BL} = 2.50$ reach the higher value of mass correction.

can find $\xi_2(R)$ which is related to the star's shape, see Eq. (17) and Eq. (45). The function $v_2(r)$ is needed to calculate to help calculating h_2, because h_2 and v_2 are coupled to each other, see Eq. (63) and Eq. (67). In order to solve those differential equations above, there are components within those equations that have to be obtained from other differential equations, i.e. $p(r)$, $\varphi(r)$, $m(r)$, and $\bar{\omega}(r)$, which are obtained by solving Eq. (26), Eq. (6), Eq. (7), and Eq. (68), respectively.

In solving differential equation of angular velocity, we used $\bar{\omega}_c = 0$ as the boundary condition of $\bar{\omega}$. Note that this value is much less than the value used in ref. [2], i.e. $\bar{\omega}$ = 3000 s^{-1}. Our initial value for $\bar{\omega}$ is more realistic for slowly rotating condition, considering that $\bar{\omega}(r)$ increases along with increment of r. Setting $\bar{\omega}_c$ = 3000 s^{-1} is so risky that we could lose the physical meaning of slowly rotating, considering that HT formalism basically is just an approximation of slowly rotating relativistic stars. We can see in Fig. 1 that $\bar{\omega}(R)$, i.e. the angular velocities relative to local inertial frame at the star's surface, significantly increase along with the increment of pressure at the center, but it only happens in range where the values of pressure at the center are relatively small, i.e. between 0 - 150 MeV fm^{-3}. In this range, anisotropic pressure does not obviously influence the relation between pressure at the center and $\bar{\omega}(R)$, because the curves tend to stick together, so in the result we cannot see the difference when we employ different values of λ_{BL}. The influence of the anisotropic pressure can be seen more obviously when the curves exceed P_c = 150 MeV fm^{-3}. From the curves, we also see that the stars with stronger anisotropic terms (which have larger λ_{BL} values) rotate faster. Note that according to Eq. (68), $\bar{\omega}$ is not only depended on σ as the correction element of anisotropic pressure, it is also depended on r and other parameters such as pressure and energy density. The fastest angular velocity we obtained does not exceed 1000 s^{-1}, which is even much less than the value of $\bar{\omega}_c$ used in Ref. [2]. This indicates that our parameter for $\bar{\omega}_c$ is more realistic to describe slowly rotating NS.

Fig. 2 describes impact of anisotropic pressure to total mass and mass correction of NS. Total mass is the the summation of mass of static star that is obtained from Eq. (7) and mass correction which is obtained from Eq. (13). Fig. 2(a) shows the relation between pressure at the center and the total mass of the star. We plotted curves for four different values of λ_{BL}. We can see that total mass increases for every larger value of λ_{BL}. As λ_{BL} decreases to 1.82, the total mass becomes lower. This is the result of Eq. (7), Eq. (57), Eq. (58), and Eq. (68), as the increment of total mass is influenced by factors within those equations. Fig. 2(b) shows mass correction for each value of λ_{BL}. The curves are plotted at certain pressure of center, i.e. P_c = 157 MeV fm^{-3}, which is the point where star with $\lambda_{BL} = 2.50$ reaches maximum mass. As we expect, the mass correction increment is not linear to λ_{BL}. This indicates that anisotropic pressure of NS has significant impact to mass correction, which in the next turn affects the total mass, although the maximum value of mass correction we obtained only in order of 10^{-2} $M_\odot$. However, without our formalism and any mass correction, the anisotropic pressure itself affects mass increment of the star, see and mass-radius relation of NS in Ref. [3].

Another interesting result on total mass that we have obtained is the maximum mass for $\lambda_{BL} = 2.50$ (i.e. the blue dot dash curve) which exceeds 2.25 $M_\odot$. We are optimistic that the formalism we have developed might be powerful for modeling stable NS with larger mass, since there are other EOS and anisotropic

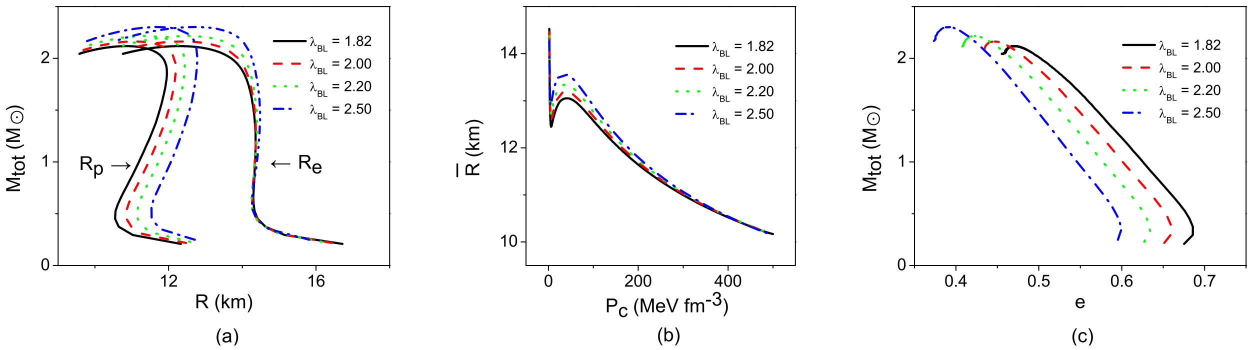
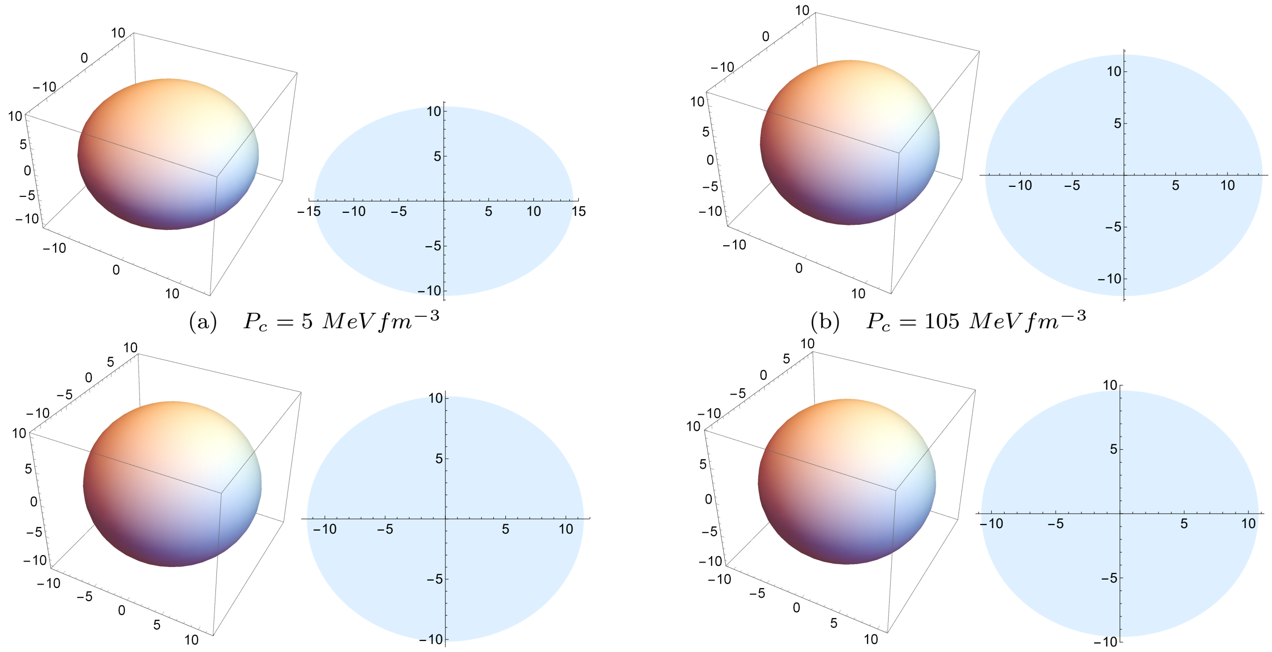


Fig. 3 Impact on anisotropic pressure to the star's radius. (a) Relation between radius and total mass. We label polar radius and equatorial radius as R_p and R_e, respectively. The appearance of R_p and R_e obviously shows that the star is deformed. The points where R_p and R_e overlap indicates no deformation at certain masses. (b) Relation between pressure at center and average radius of star. Here we can see that anisotropic pressure of NS affect obvious increment of average radius of NS, mainly in range (5 - 400) MeV fm^{-3}. (c) Relation between eccentricity and total mass. Star with larger value of λ_{BL} is harder to be deformed, since curves of larger λ_{BL} is closer to zero. Remember that while $e = 0$, the object's shape is spherical.



(a) $P_c = 5\ MeV fm^{-3}$ (b) $P_c = 105\ MeV fm^{-3}$

(c) $P_c = 330\ MeV fm^{-3}$ (d) $P_c = 490\ MeV fm^{-3}$

Fig. 4 Illustration of 3- and 2-dimensional deformation of neutron stars in each particular pressure in center at $\lambda_{BL} = 1.82$. At lower pressure, the star's shape becomes very oblate. At $Pc = 5\ MeV\ fm^-3$, $R_p = 10.54$ km, and $R_e = 14.42$ km; at $Pc = 105\ MeV\ fm^-3$, $R_p = 11.66$ km, and $R_e = 13.38$ km; $Pc = 330\ MeV\ fm^-3$, $R_p = 10.19$ km, and $R_e = 11.49$ km; and $Pc = 490\ MeV\ fm^-3$, $R_p = 9.60$ km, and $R_e = 10.79$ km.

models of NS that we have not explored yet. We are motivated to use this modified HT formalism for further study to find larger maximum mass of NS, which involves other EOS and anisotropic models that we have not used so far. It possibly has great significance on GW research area. Recently LIGO/Virgo collaboration reported GW190814, with the secondary compact object has mass 2.6 $M_\odot$, see Ref. [15]. It is still not clear what is the secondary compact object, it might be either a BH or a NS. Abbot et al. [15] tend to argue that the secondary compact object is BH. If it was a BH, then it might be the lightest BH that has been observed. If it was a NS, then it might be a heaviest NS that is ever been observed, and the mass is significantly larger than the known maximum mass constraint. With the formalism we developed, there is an opportunity to obtain maximum mass of NS which reach 2.6 $M_\odot$. However, we leave it as our next project.

Fig. 3(a) shows the relation between radii, i.e. polar radius and equatorial radius, and total mass of the star. As a star rotates, the deformation occurs, this is like the deformation due to rotation of the earth. The rotation causes polar radius is less than equatorial radius. We labeled polar radius and equatorial radius as R_p and R_e, respectively. Wen et al. [2] have worked on rotational deformation of isotropic NS, but there is an issue, i.e. the rotation is weak for slowly rotating NS. Other previous studies [19,30] showed that the deformation occurs at static anisotropic stars, but both also have issues. In Ref. [30] there is no significant influence of varied anisotropic pressure to the star's deformation, although generally, the deformation is not weak; while in Ref. [19], the deformation appears, but it is weak enough compared to deformation obtained in ref. [30] (see radius-mass relation in both Ref. [19] and Ref. [30] to compare their results). In this work we obtained magnificent results that handled those issues so well. In our result, the deformation of slowly rotating anisotropic NS is not weak, and variation of anisotropic strength has obvious impact to the star's shape. In Fig. 4, we plotted 3- and 2-dimensional illustration of NS in each particular pressure of the center at $\lambda_{BL} = 1.82$. We can see that at lower mass, the star's shape tend to be more oblate. As the mass increase, the star is getting more spherical. This shows that radius-mass relation becomes stiffer as the mass increases. Note that at a certain mass for each curve, R_p and R_e overlap each other as we can see in Fig. 3(a). Such this condition indicates that there is no deformation of the mass.

From Fig. 3(b) we can see the relation between the pressure at the center and the average radius of the anisotropic star. Generally, anisotropic pressure affects star's average radius. The effect can be seen obviously at range (5 - 400) MeV fm^{-3}. Maximum average radius is different for each value of λ_{BL}, but it it clear that for every value of λ_{BL}, average radius decreases when pressure at center exceeds 100 MeV fm^{-3}. When pressure at the center exceeds 400 MeV fm^{-3}, the curves still decrease and tend to stick together, which indicate that anisotropic pressure no longer affects average radius.

Fig. 3(c) shows relation between eccentricity and total mass. We can see that larger eccentricity is associated with lower mass, so star is easier to become more oblate at lower mass. This fact is matched with what we can see in Fig. 3(a) and Fig. 4.

Another important insight we can get from Fig. 3(c) is fact that stars whose stronger anisotropic terms have less value of eccentricity, since curves of larger λ_{BL} are closer to zero. This condition is same to the result in Ref. [30], where the anisotropic pressure comes from strong magnetic field of NS, but in Ref. [30], the difference between each curve of eccentricity cannot be seen obviously (see radius-mass relation in Ref. [29]), which indicate different magnitudes of magnetic field do not affect the eccentricity significantly. An interesting highlight we can get from our result in Fig. 3(c) is the obvious effect of anisotropic pressure to the eccentricity. We can see that stars with weaker anisotropic terms tend to be more easy to deform. We have an explanation for this condition. From pressure-mass relation in Fig. 2(a) and radius-mass relation in Fig. 3(a), we can see that larger value of λ_{BL} produces larger mass. This condition analogous to stiff EOS, since larger maximum mass could be produced by stiff EOS [2]. It has been understood that EOS is related to compressiblity of the star, i.e. star's ability to be compressed. Stiff EOS causes low compressibility of the star [56–58]. Similar to condition we face, we can get larger maximum mass larger when we employed larger λ_{BL}, it is likely larger λ_{BL} makes EOS stiffer. To be deformed, a compression at star's poles has to be done, so when the EOS becomes stiffer, star is getting harder to be deformed. This leads to condition which star is not deviate too much from spherical shape. So the question that might arise on why the eccentricity of star with larger λ_{BL} is closer to zero can be answered.

4 Conclusion

This work extends the analytical HT formalism for slowly rotating stars for isotropic matter by including the anisotropic pressure correction in the energy-momentum tensor. The results apply to the study of anisotropic NSs. To describe the EOS of NS matter, we use the RMF model with the BSP parameter set [52,53] where the standard SU(6) prescription and hyperon potential depths [54] is used to determine the hyperon coupling constants. For the inner and outer crusts, we use the crust EOS obtained from Miyatsu *et al.* [55]. While for the anisotropic pressure model, we use the Bowers-Liang (BL) model for the anisotropic pressure of NS [51]. Note that the values of the anisotropic parameter of BL model λ_{BL} can be interpreted as the strength of the anisotropic term. We have found that the impact of increasing λ_{BL} value from $\lambda_{BL} = 1.82$ to $\lambda_{BL} = 2.50$ on angular velocity relative to local inertial frame at the surface, *i.e.* $\bar{\omega}(R)$ is the NS rotate faster. The corresponding increasing λ_{BL} value also implies incremement of mass and average radius of the NSs. It is also evident that for larger λ_{BL}, the EOS becomes stiffer, so NSs are getting harder to be deformed.

Acknowledgements MLP acknowledges Indonesian Endowment Fund for Education (LPDP), Ministry of Finance, Republic of Indonesia for the fully funded scholarship.

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