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Rotational Deformation of Neutron Stars *

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The rotational deformations of two kinds of neutron stars are calculated by using Hartle's slow-rotation formulism. The results show that only the faster rotating neutron star gives an obvious deformation. For the slow rotating neutron star with a period larger than hundreds of millisecond, the rotating deformation is very weak.

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Nowadays, neutron star is generally used to refer to a star with a mass on the order of $1.5\,M_\odot$, a radius about 12 km, a rapid spin even as fast as several hundred rotations per second and a central density as high as several times the nuclear saturation densities. It is widely believed that a neutron star is created in the aftermath of the gravitational collapse of the core of a massive star (more than $8\,M_\odot$) at the end of its life and is one of the densest forms of matter in the observable universe. Although all neutron stars are very far away from us, there are already some known credible observational properties, such as pulse period, masses (of neutron star in binary system), thermal emission, glitches, etc.[1,2] The enormous mass and the small figure make a very strong gravitational field in the neutron star and therefore provide a high-pressure environment in which numerous subatomic particles exist. One main aim of the study of neutron star is to understand the dense matters.[3]

Due to the strongly gravitational field, one must employ the general relativity to investigate the structure of the neutron star. To spherically symmetric non-rotating neutron star, if the matters can be looked on as perfect fluid, one can use the Tolman–Oppenheimer–Volkoff (TOV) equation[4,5] to calculate its structure. In this case, the system is highly symmetric, and then the dynamical equations (Einstein field equations) become very simple, one can obtain an exact analytical solution. For a real neutron star, it must be rotating and the star will be deformed by the rotation. In order to fully and accurately investigate the properties of neutron stars, such as the total masses, electromagnetic radiation mechanism, and the gravitational radiation, etc, one must consider the effect of rotation, especially the rotational deformation. For the rotating neutron star, it is only azimuthally symmetric, the Einstein field equations become very complicated, and it is too hard to obtain an exact analytical solution. In the research on the properties of rotating neutron stars, one must use numerical approximate methods. Several numerical approximate methods have been developed.[6,7] In this work, the numerical result of the rotational deformation of neutron star will be calculated by employing the method treated by Hartle and Thorne.[8,9] Here we adopt the metric signature $-+++$, $G=c=1$.

In Newtonian gravitational theory, the rotation of a massive star will not affect the inertial frame, but in general relativity, this rotation will drag the local inertial frame along the direction of the star rotation.[10,11] In general relativity, the metric of a rotating star can be written as

$$ds^2=-e^{2\nu}dt^2+e^{2\lambda}dr^2+e^{2\psi}(d\phi-\omega dt)^2+e^{2\mu}d\theta^2, \quad (1)$$

where ω denotes the angular velocity of the local inertial frames, which is proportional to the star angular velocity Ω with respect to the distant observer.[12]

Expanding the metric functions in Eq. (1), there are only even powers of Ω in the expansion. The expansions of the metric functions are given by[9]

$$e^{2\nu}=e^{2\varphi}[1+2(h_0+h_2P_2)], \quad (2)$$

$$e^{2\lambda}=\Big[1+\frac{2}{r}(m_0+m_2P_2)\Big(1-\frac{2M_0(r)}{r}\Big)^{-1}\Big]\cdot\Big(1-\frac{2M_0(r)}{r}\Big)^{-1}, \quad (3)$$

$$e^{2\psi}=r^2\sin^2\theta[1+2(v_2-h_2)P_2], \quad (4)$$

$$e^{2\mu}=r^2[1+2(v_2-h_2)P_2], \quad (5)$$

where $e^{2\varphi}$ and $M_0(r)$ denote the metric function and the mass of the non-rotating neutron star with the same central density, respectively; p_2 is the Legendre polynomial of order 2; the perturbation functions m_0, m_2, h_0, h_2, v_2 are proportional to Ω^2 and are to be calculated from Einstein field equations

$$R^\nu_\mu-\frac{1}{2}g^\nu_\mu R=8\pi T^\nu_\mu. \quad (6)$$

From the (t,t) and (r,r) components of Einstein field equations, one can obtain two coupled ordinary

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differential equations of m_0 and h_0 as[9]

$$\frac{dm_0}{dr} = 4\pi r^2 \frac{d(p+\rho)}{dp}(\rho+p)p_0^* + \frac{1}{12}j^2 r^4\Big(\frac{d\bar{\omega}}{dr}\Big)^2 - \frac{1}{3}r^3\frac{d(j^2)}{dr}\bar{\omega}^2, \tag{7}$$

$$\frac{dp_0^*}{dr} = -\frac{m_0(1+8\pi r^2 p)}{[r-2M_0(r)]^2} - \frac{4\pi r^2(p+\rho)}{r-2M_0(r)}p_0^* + \frac{1}{12}\frac{r^4 j^2}{r-2M_0(r)}\Big(\frac{d\bar{\omega}}{dr}\Big)^2 + \frac{1}{3}\frac{d}{dr}\Big[\frac{r^3 j^2\bar{\omega}^2}{r-2M_0(r)}\Big], \tag{8}$$

where $p_0^* = -h_0 + \frac{1}{3}r^2 e^{-2\nu}\bar{\omega}^2 + C$, here C is a constant determined by the demand that h_0 be continuous across the star surface; $\bar{\omega} = \Omega - \omega$ denotes the angular velocity of the fluid relative to the local inertial frame, which can be numerical calculated by the differential equation[9]

$$\frac{1}{r^4}\frac{d}{dr}\Big(r^4 j\frac{d\bar{\omega}}{dr}\Big) + \frac{4}{r}\frac{dj}{dr}\bar{\omega} = 0, \tag{9}$$

in which $j(r) = e^{-\varphi}[1-2M_0(r)/r]^{\frac{1}{2}}$, with boundary conditions as $\bar{\omega} = \bar{\omega}_c$ at the center, $d\bar{\omega}/dr|_c = 0$, where $\bar{\omega}_c$ is chosen arbitrarily.

Integrating outward Eqs. (7)–(8) under the boundary conditions that both m_0 and p_0^* vanish at the origin, one can numerical calculate the perturbation functions both m_0 and p_0^*. At the same central density, the difference between the mass of the rotating star and the non-rotating star is

$$\delta M = m_0(R_0) + \frac{J^2}{R_0^3}, \tag{10}$$

where R_0 is the surface radius of the non-rotating neutron star. The difference of the mean radius is

$$\delta r = -p_0^*(\rho+p)\frac{dp}{dr}. \tag{11}$$

From the $(\theta,\theta), (\phi,\phi), (r,\theta)$ and (r,r) components of Einstein field equations, one can obtain equations of perturbation functions v_2, h_2, m_2 as[9]

$$\frac{dv_2}{dr} = -2\frac{d\varphi}{dr}h_2 + \Big(\frac{1}{r} + \frac{d\varphi}{dr}\Big)\Big[-\frac{1}{3}r^3\frac{d(j^2)}{dr}\bar{\omega}^2 + \frac{1}{6}j^2 r^4\Big(\frac{d\bar{\omega}}{dr}\Big)^2\Big], \tag{12}$$

$$\begin{aligned}\frac{dh_2}{dr} = &\Big\{-2\frac{d\varphi}{dr} + \frac{r}{r-2M_0(r)}\Big(2\frac{d\varphi}{dr}\Big)^{-1} \cdot \Big[8\pi(\rho+p) - \frac{4M_0(r)}{r^3}\Big]\Big\}h_2 \\ &- \frac{4v_2}{r[r-2M_0(r)]}\Big(2\frac{d\varphi}{dr}\Big)^{-1} + \frac{1}{6}\Big[\frac{d\varphi}{dr}r - \frac{1}{r-2M_0(r)}\Big(2\frac{d\varphi}{dr}\Big)^{-1}\Big]r^3 j^2\Big(\frac{d\bar{\omega}}{dr}\Big)^2 \\ &- \frac{1}{3}\Big[\frac{d\varphi}{dr}r + \frac{1}{r-2M_0(r)}\Big(2\frac{d\varphi}{dr}\Big)^{-1}\Big]r^2\frac{d(j)^2}{dr}\bar{\omega}^2,\end{aligned} \tag{13}$$

$$m_2 = [r-2M_0(r)]\Big[-h_2 - \frac{1}{3}r^3\frac{d(j)^2}{dr}\bar{\omega}^2 + \frac{1}{6}r^4 j^2 \cdot \Big(\frac{d\bar{\omega}}{dr}\Big)^2\Big]. \tag{14}$$

Equations (12) and (13) are the two coupled ordinary differential equations about v_2 and h_2, its boundary conditions can be written as

$$v_2(0) = h_2(0) = v_2(\infty) = h_2(\infty) = 0. \tag{15}$$

To a given set of parameters: equation of state $p = p(\rho)$, central density ρ_c and the central angular velocity $\bar{\omega}_c$ with respect to the local inertial frame, according to the boundary conditions and the calculated function $\bar{\omega}(r)$, one can numerical calculate the perturbation functions v_2 and h_2.

The rotational deformation of the neutron star relative to the corresponding non-rotating star is[10]

$$\delta r = \xi_0(r) + \xi_2(r)P_2(\cos\theta), \tag{16}$$

where $\xi_0(r) = -p_0^*(\rho+p)\frac{dp}{dr}$, $\xi_2(r) = -p_2^*(\rho+p)\frac{dp}{dr}$ and $p_2^* = -h_2 - \frac{1}{3}r^2 e^{-2\nu}\bar{\omega}^2$. It is clear that $\delta r(R_0)$ expresses the max deformation. According to Eq. (16), the surface eccentricity of the rotating neutron star is

$$e = \Big[1 - \Big(\frac{R_p}{R_e}\Big)^2\Big]^{\frac{1}{2}}, \tag{17}$$

where

$$R_p = R_0 + \xi_0(R_0) + \xi_2(R_0) \tag{18}$$

is the surface radius at the pole, which is the radius along the rotating axis direction; and

$$R_e = R_0 + \xi_0(R_0) - \frac{1}{2}\xi_2(R_0) \tag{19}$$

is the radius at the equator.

From the above results, one can see that to the perturbation functions of the metric, m_0 and h_0 determine the change of the mass and the mean radius of the rotating neutron star; m_2, h_2 and v_2 determine its deformation.

In this work, the relativistic $\sigma - \omega$ model will be adopted to deal with the EOS of the neutron stars, here two kinds of neutron stars will be considered: the traditional neutron stars (TNS), in which n, p, e are the main elements; and hyperon stars (HS), in which $n, p, e, \mu, \Sigma, \Lambda, \Xi, \Delta$ are the main elements[13–15] (in fact, the compositions of the neutron star may be much more complex than this). Figure 1 shows the EOS of TNS and HS, it is clear that the EOS of TNS is stiffer than the EOS of HS.

The rotating effect on the figuration and the total mass of the neutron star are showed in Table 1. As we know, only a few masses have been determined in observation from more than thousand neutron stars, such as the mass of Hulse–Taylor radio pulsar PSR $1913+16$,[18] given by $1.444 \pm 0.003 M_\odot$, the mass of X-ray binaries Cygnus X-2,[17] give by $1.8 \pm 0.04 M_\odot$, etc. In order to compare numerical results with observation, the mass of the neutron star in our calculation lets to be with a value as one of the above two observational values. To TNS, as the EOS is stiffer, which can give a bigger mass neutron star, so the mass of rotating TNS is adopted to be about $1.80M_\odot$; since the EOS of HS is softer, the mass of rotating HS is adopted to be about $1.44M_\odot$. From Table 1, one can see that only the fast rotating neutron star can give a effectual deformation, even TNS rotates as a period not smaller than 2.4 ms and HS rotates as a period not smaller than 2.7 ms, their eccentricities will be smaller than the eccentricity of our earth, with a value of about 0.082. To the rotating neutron star with a period larger than 1.2 ms, our previous work[18] shows that the rotating deformation of the star is very weak, the mean deformation $\delta R/R$ is small than 0.02, so we believe that Hartle's approximation method is rational to deal with the rotating neutron star with a period bigger than 1.2 ms.

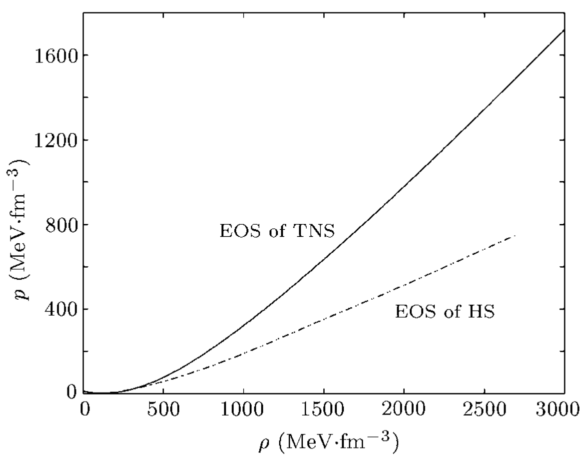

Fig. 1. Equations of state of traditional neutron stars (TNS) and hyperon stars (HS).

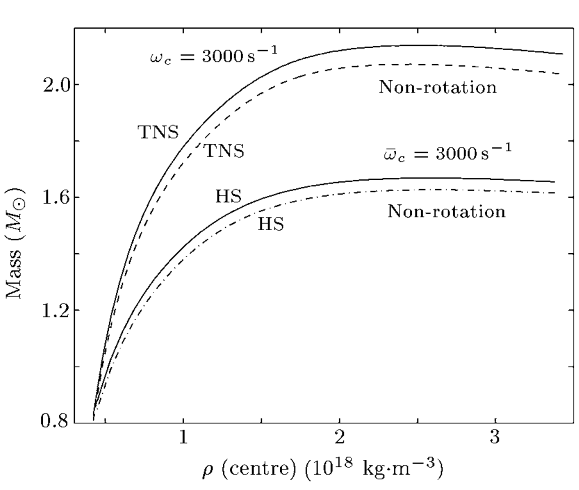

Fig. 2. Mass of the non-rotating and rotating neutron stars (at $\bar{\omega}_c = 3000\,\mathrm{s}^{-1}$) as a function of central densities.

Figure 2 presents the masses of non-rotating and rotating neutron stars as a function of central densities. It is clear that the stiffer the EOS is, the larger the total mass of the corresponding neutron star will be; when the central densities are larger than $2\times10^{18}\,\mathrm{kg{\cdot}m^{-3}}$, the change of the mass will be very weak as the central density increases. On the other hand, Figs. 3 and 4 show that as the central densities are larger than $2\times10^{18}\,\mathrm{kg{\cdot}m^{-3}}$, the radius will diminish as the central density increases. As we know, only in the region where the mass increases while the radius decrease, neutron star is stabile, so to both TNS and HS, a stabile neutron star are more likely with a central density in the region of $0.7\sim2\times10^{18}\,\mathrm{kg{\cdot}m^{-3}}$.

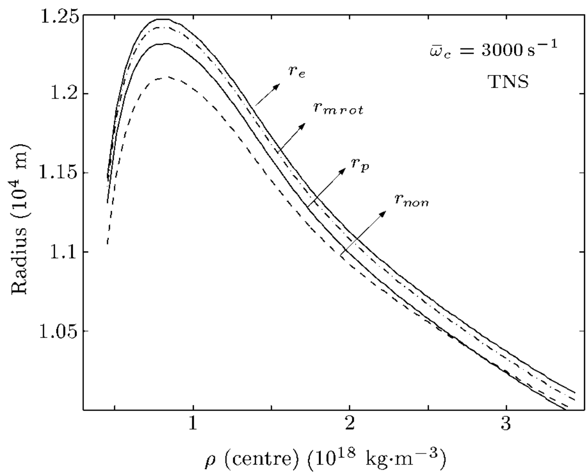

Fig. 3. The deformation of the rotating TNS (at $\bar{\omega}_c = 3000\,\mathrm{s}^{-1}$) as a function of central densities, where r_p denotes the radius at pole, r_e denotes the radius at equator, r_{mrot} denotes the mean radius of the rotating star, r_{non} denotes the radius of the non-rotating star.

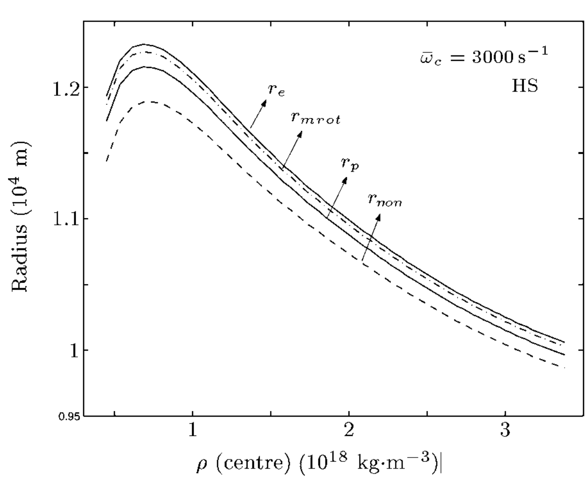

Fig. 4. Deformation of the rotating HS (at $\bar{\omega}_c = 3000\mathrm{s}^{-1}$) as a function of central densities.

Figures 3 and 4 show the rotating deformation of neutron stars as a function of central densities at a central angular velocity in the local inertial frame with $\bar{\omega}_c = 3000\,\mathrm{s}^{-1}$, which corresponds with a period near

Table 1. Deformation of the rotating neutron star. TNS and HS denote the traditional neutron stars and the hyperon stars respectively; $\bar{\omega}_c$ is the angular velocity relative to the local inertial frame at the center in units of $(10^3 s^{-1})$; ρ_c is the central density in units of $(10^{18}\,\mathrm{kg{\cdot}m^{-3}})$; P is the rotating period in units of ms; R_{non} denotes the radius of the non-rotating star; R_{mrot} denotes the mean radius of the rotating star; R_e and R_p denote the radius at the equator and at the pole respectively, the unit of the radius is km; e denotes the surface eccentricity; M_{non} and M_{rot} denote the masses of non-rotating and rotating stars, respectively, in units of the solar mass $(M_\odot)$.

	$\bar{\omega}_c$	ρ_c	p	R_{non}	R_{mrot}	R_e	R_p	e	M_{non}	M_{rot}
TNS	1.50	1.110	2.40	11.93	12.00	12.02	11.98	0.078	1.793	1.807
	3.00	1.022	1.30	12.02	12.31	12.36	12.21	0.155	1.738	1.795
HS	1.50	1.109	2.70	11.62	11.70	11.71	11.67	0.078	1.433	1.444
	3.00	1.065	1.30	11.66	11.99	12.03	11.89	0.155	1.413	1.456

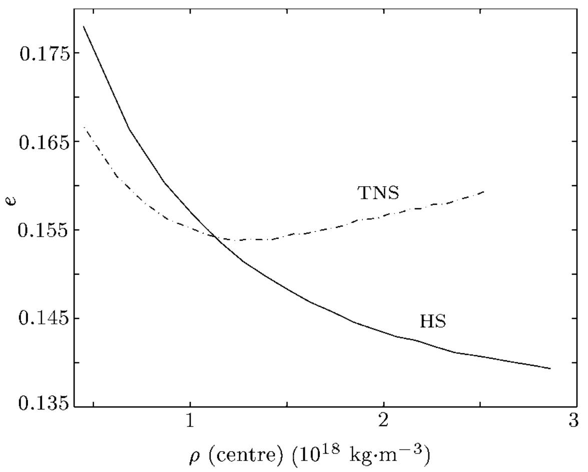

Fig. 5. Eccentricity of the rotating neutron stars (at $\bar{\omega}_c = 3000\,\mathrm{s}^{-1}$) as a function of central densities.

the shortest observational period of pulsars, 1.6 ms. From these figures, one can see that to a rotating neutron star, the mean radius is always larger than the radius at the pole and smaller than the radius at the equator. This case is just like the earth. If we consider the rotating earth as a normal sphericity, we can obtain a mean radius, but when we consider the rotating deformation, because of the centrifugal force at the equator, the deformation must make the radius at the equator larger than the radius at the pole. For the neutron star, the matter can be considered as a perfect fluid, so its mechanism of deformation is more easy to understand. From these figures, one can see that in the central region with larger density, the radius at the pole of a fast rotating TNS is smaller than the radius of the corresponding non-rotating neutron star, but HS is not so. This case can be explained as follows. The EOS of HS is softer, which gives a bigger rotating effect; this gives a larger mean rotating radius. However, from Fig. 5 one can see that at the larger-density central region, the eccentricity of HS is smaller than that of TNS. To TNS, smaller mean rotating effect and larger eccentricity make its rotating radius at the pole smaller than the radius of the non-rotating star. To the rotating deformation, because up to now there has been no observation of them, and even no direct observation on the radius of pulsars, the calculated rotating deformation cannot be used as a criterion now, although it may be very useful in the research on gravitational radiation. Figure 5 presents the eccentricity of rotating neutron star at a central angular velocity relative to the local inertial frame with $\bar{\omega}_c = 3000\,\mathrm{s}^{-1}$. From this figure, one can see that the smaller central density correspond a larger eccentricity: that is, the deformation is greater in this case. This is because that the smaller central density gives a smaller mass but a relatively larger radius. Therefore, the gravitational field of this star is relatively weaker and the rotating deformation is relatively greater.

References

[1] Weber F http://xxx.lanl.gov/abs/astrop-ph//0407155
[2] Bai H and Peng Q H 2004 *Chin. Phys. Lett.* **21** 1402
[3] Mao G J, Kondratyev V N, Iwamoto A, Li Z X, Wu X Z, Greiner W and Mikhailov I N 2003 *Chin. Phys. Lett.* **20** 1234
[4] Tolman R C 1939 *Phys. Rev.* **55** 364
[5] Oppenheimer J R and Volkoff G M 1939 *Phys. Rev.* **55** 374
[6] Butterworth E M and Ipser J R 1976 *Astrophys. J.* **204** 200
[7] Komatsu H, Eriguchi Y and Hachisu I 1989 *Mon. Not. R. Astron. Soc.* **239** 153
[8] Hartle J B 1967 *Astrophys. J.* **150** 1005
[9] Hartle J B and Thorne K S 1968 *Astrophys. J.* **153** 807
[10] Brill D R and Cohen J M 1966 *Phys. Rev.* **143** 1011
[11] Kalogera V and Psaltis D 1999 *Phys. Rev.* D **61** 024009
[12] Weber F, Glendernning N K and Weigel M K 1991 *Astrophys. J.* **373** 579
[13] Serot B D and Walecka H D 1986 *Adv. Nucl. Phys.* **16** 1
[14] Liu L G, Bentz W and Aima A 1989 *Ann. Phys.* (N.Y.) **194** 387
[15] Chen W, Wen D H and Liu L G 2003 *Chin. Phys. Lett.* **20** 436
[16] Taylor J H and Weisberg J M 1989 *Astrophys. J.* **345** 434
[17] Orosz J A and Kuulkers E 1999 *Mon. Not. R. Astron. Soc.* **305** 1320
[18] Wen D H, Chen W, Wang X J, Ai B Q, Liu G T, Dong D Q and Liu L G 2004 *Eur. Phys. J.* A **21** 349