# Insufficient or Excessive Investment Under Sovereign Default Risk*

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**Abstract**

Private agents do not internalize the impact of their investment decisions on the sovereign's bond prices and default risk. Therefore, a standard externality argument implies that investment is insufficient and that a subsidy can improve welfare, if financed by nondistortionary means. We contrast this logic with a countervailing force. When the sovereign is impatient relative to households, plausibly due to political economy factors, it finds laissez-faire capital accumulation excessive and might prefer instead to tax it. We embed both mechanisms in a sovereign default model with decentralized capital investment, long-term public debt, and stochastic trend growth, calibrated to salient features of the Spanish economy. We find that the impatience channel dominates quantitatively, to such an extent that laissez-faire is preferable to the government's ideal fiscal policy, based on households' welfare.

Keywords: sovereign default, capital accumulation, decentralized investment
JEL classification: E22, F34, F43

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This paper explores private capital accumulation in economies where the government implements policy under discretion and can default on outstanding public debt, in the tradition of Eaton and Gersovitz (1981). We find that, as expected from the prior work of Park (2017) and Gordon and Guerron-Quintana (2018), higher investment generally lowers default risk. In turn, this lower risk translates into higher sovereign bond prices now, more favorable borrowing terms and expanded fiscal space for the sovereign. Our analysis concerns the interplay of two mechanisms which determine the fiscal treatment of investment and its desirability.

First, as the private sector does not internalize the impact of its investment choices on public bond prices and default policies, a standard externality argument implies that an investment subsidy would improve welfare, if funded by non-distortionary means. Seoane and Yurdagul (2023) and Esquivel (2024) are two recent papers touching upon this argument. Second, governments in emerging markets and other countries at risk of default seem plagued by political economy dysfunctions, incentives that render fiscal authorities effectively more impatient than their constituents. Several works highlight this feature and its role in the quantitative success of sovereign default models, including Alesina and Tabellini (1990), Chatterjee and Eyigungor (2019), Aguiar, Amador, and Fourakis (2020), Azzimonti and Mitra (2023a, 2023b), Acharya, Rajan, and Shim (2024), and Cotoc, Johri, and Sosa-Padilla (2025). We do not take a stand on the exact nature of these frictions but instead emphasize their consequences for capital accumulation. Generally, such a government finds laissez-faire private sector investment excessive, and would like to tax rather than subsidize it, to induce higher consumption immediately.[1]

We embed these two forces and evaluate their interaction in a quantitative model with private investment, random trend growth, and long-term, defaultable public debt. We calibrate our model to Spain, a prototypical European periphery economy, and use it to disentangle the two forces we study, as well as evaluate the welfare implications of decentralized investment. Our model reproduces well the business cycle properties in the data, together with bond yield spreads dynamics.

We compare our baseline economy, in which capital accumulation is decided by the private sector and the government is as impatient as called for by the data, to three counterfactual economies. First, we contrast with an otherwise identical economy in which the government perfectly controls investment. This isolates the externality impinging on the sovereign's bond prices and default policies. We then consider a version of our

1. The government being more impatient than domestic households is distinct from the finding in most quantitative work that sovereigns are more impatient than international lenders, a known driver of procyclical fiscal policy (Cuadra, Sanchez, and Sapriza 2010).

model in which the government is as patient as households, to highlight the impact of this disconnect on the tax treatment of investment in the baseline. Finally, we evaluate a scenario where the government is not impatient and has control over investment.

We find that giving the government access to an appropriately-funded subsidy on investment can indeed improve outcomes, but critically only if the government's discount factor is close enough to that of domestic households. This is not the case in the baseline calibration featuring non-trivial sovereign impatience, to such an extent that, if given the opportunity, our sovereign would want to tax investment, to induce higher consumption immediately. Such a policy significantly worsens household welfare, by about 5.25% in consumption equivalent measure. In contrast, we find much smaller, albeit positive, welfare gains from introducing the investment subsidy, of about 0.001%, if the sovereign shares its households' discount factor. Remarkably, these welfare gains and losses are almost exclusively attributable to changes in default propensity, from capital deepening and sovereign discounting, with little to no change in business cycle statistics and comovements.

Our results connect to several themes explored in the literature. With a first group of papers, we share the focus on interactions between capital accumulation and default, including Bai and Zhang (2012) on financial liberalization, Bocola (2016) on the pass-through of sovereign risk, Arellano, Bai, and Mihalache (2018), Esquivel (2024), and Deng and Liu (2024) on sectoral misallocation, Alessandria, Bai, and Deng (2020) on production with migrant flows, and Asonuma and Joo (2023) on public capital. Our paper is closest to Gordon and Guerron-Quintana (2018), including their discussion of decentralization. Also noteworthy, Park (2017) emphasizes the non-monotonic relationship between the capital stock and default incentives in a setting similar to ours, and this pattern's role in defaults during "good times."

Several papers have explored ways in which private decision-making constrains discretionary fiscal policy, for example Na et al. (2018) and Bianchi, Ottonello, and Presno (2023) on downward rigid wages and exchange rate regimes, Arellano, Bai, and Mihalache (2020) on pricing frictions and monetary policy rules, and Liu and Shen (2022) on frictional labor markets. For us, private sector capital accumulation plays this role.

Finally, our focus on decentralized physical capital investment is distinct but related to the question of decentralized international borrowing, as explored by Jeske (2006), Kim and Zhang (2012), and Seoane and Yurdagul (2023), among others. We maintain the standard assumption that asset markets are segmented, and that the sovereign alone can tap international markets.

# 1 Model

Our model focuses on the interactions between a sovereign fiscal authority, the domestic private sector, and foreign lenders. The model closest to our work is Gordon and Guerron-Quintana (2018), from which we mainly differ by adopting a random trend productivity process, informed by the evidence in Aguiar and Gopinath (2007). We discuss our agents' problems in turn and relegate the fairly standard definition of equilibria for these economies to the online Supplemental Material. We consider alternative assumptions about the ability of fiscal authorities to shape private investment behavior, as well as its degree of impatience. We start with the laissez-faire case, where the sovereign lacks fiscal instruments which impact investment directly, to then consider the polar opposite case of perfect public control over capital investment.

**The private sector.** The private sector consists of a representative household and a representative firm. The household owns shares in the firm and receives dividend payments. In turn, the firm's investment decisions reflect the household' marginal rate of substitution. Since there are no financial frictions between households and firms, it is convenient to think of them as a consolidated entity, the private sector.

The household maximizes its expected utility $\mathbb{E}_0 \sum_t \beta^t u(c_t)$ by choosing shareholdings $a_{t+1}$ subject to a sequence of budget constraints $c_t + P_t a_{t+1} = w_t + (\text{div}_t + P_t)\, a_t + T_t$, where $c_t$ is the household's consumption, $P_t$ is the stock price, $a_t$ are the shares held at the start of the period, $\text{div}_t$ are dividend payments, and $T_t$ is a lump-sum transfer received from the government (a lump-sum tax, if negative). We assume, for simplicity, that a unit of labor is supplied inelastically, so that the wage income is simply $w_t$.[2] With a representative agent, in equilibrium $P_t$ is at a level such that $a_{t+1} = 1$.[3]

The firm hires labor $\ell_t$ and chooses investment $i_t$ each period, to maximize the present value of dividend payments $\mathbb{E}_0 \sum_t \Lambda_t \text{div}_t$, discounted using the household's pricing kernel $\Lambda_t = \beta^t u'(c_t)/u'(c_0)$, subject to a sequence of constraints and laws of motion,

$$\text{div}_t = z_t k_t^{\alpha} \left(\Gamma_t \ell_t\right)^{1-\alpha} - w_t \ell_t - i_t - \Theta(k_t, k_{t+1}), \quad \text{and} \quad k_{t+1} = (1-\delta)k_t + i_t,$$

where $\Theta(k_t, k_{t+1})$ is a quadratic adjustment cost, a standard feature of small open economy models (Schmitt-Grohé and Uribe 2003). Production is constant return to scale, subject to a

2. Our results extend naturally to the typical case of Greenwood, Hercowitz, and Huffman (1988) preferences, without wealth effects on labor supply, which are widely used in studies of small open economies. We abstract from elastic labor supply in order to isolate one private sector decision margin alone, investment.

3. Note that we implicitly rule out foreign ownership of shares. We expand on our model's market segmentation aspects when discussing the sovereign's problem.

stationary productivity shock $z_t$, and a random trend $\Gamma_t$. We assume that the stationary "cycle" shock $z_t$ is AR(1), with

$$\log z_{t+1} = \rho_z \log z_t + \varepsilon_{z,t}, \quad \varepsilon_{z,t} \sim \text{ iid } N(0, \sigma_z^2)$$

and the trend accumulates random gross growth rate shocks, $\Gamma_{t+1} = \Gamma_t g_t$, with

$$\log g_{t+1} = (1 - \rho_g) \log \mu_g + \rho_g \log g_t + \varepsilon_{g,t},\ \varepsilon_{g,t} \sim \text{ iid } N(0, \sigma_g^2).$$

Following Aguiar and Gopinath (2006), we set the adjustment cost function to $\Theta(k_t, k_{t+1}) = \frac{\theta}{2}\left(\frac{k_{t+1}}{k_t} - \mu_g\right)^2 k_t$, so that investment is costly to the extent to which the growth rate of the capital stock deviates from $\mu_g$, the long-run average growth rate of this economy.

**The fiscal authority.** Financial markets are segmented. The government alone operates in international markets by trading a long-term bond, modeled as a decaying perpetuity (Hatchondo and Martinez 2009). It has the option to default on this obligation at its discretion. It uses the net proceeds from these external operations to fund a level of public spending $G_t$ and a lump-sum transfer to the domestic households $T_t$, with $T_t < 0$ corresponding to lump-sum taxation. Its budget constraint is given by

$$(1 - D_t)\left[q_t(B_{t+1} - (1 - \phi)B_t) - \kappa B_t\right] = G_t + T_t.$$

$B_t$ are the outstanding bond units, $B_{t+1}$ is the bond level chosen for the next period, and the market price of bonds is $q_t$. $\kappa$ is the debt service payment called for by each of the $B_t$ units of outstanding debt, while the $\phi$ parameter controls the maturity structure of the debt. If the sovereign chooses to default today, and sets $D_t = 1$, no debt service payments are made and the primary deficit $G_t + T_t$ must be zero.

We make the simplifying assumption that $G_t$ is at all times proportional to the productivity trend $\Gamma_{t-1}$, rather than a choice variable for the sovereign. This frees us from having to take a stand on the social desirability of public spending levels, while broadly matching the data.

Default is followed by a spell of international financial market exclusion, which ends with probability $\lambda$. Eventually, the sovereign regains access to international markets without any outstanding debt.[4] The sovereign's choice to default causes disruptions in domestic production, captured in reduced form by a penalty function applied to the cycle

4. We assume full repudiation of the defaulted debt. Yue (2010) and Dvorkin et al. (2021) provide reference models with haircuts and recovery, using cooperative and non-cooperative bargaining, respectively.

component of productivity, $h(z_t)$, which proxies for the mechanisms explored by Mendoza and Yue (2012) and Arellano, Bai, and Bocola (2025), among others, of trade or banking disruptions induced by sovereign default.

We assume that the government values consumption the same as households, except possibly with a higher degree of impatience. Its objective is given by $\mathbb{E}_0 \sum_t \beta_g^t u(c_t)$, with $\beta_g \leq \beta$. This assumption captures in reduced form a long tradition of studying political economy frictions in sovereign borrowing.

**International lenders.** Foreign buyers of the government's bonds are competitive and risk-neutral.[5] Their opportunity cost of funds is given by a constant short-term risk-free rate $r^f$. These assumptions deliver the standard long-term bond pricing condition that ensures lenders break even in expectation,

$$q_t = \frac{1}{1+r^f} \mathbb{E}_t (1 - D_{t+1})(\kappa + (1-\phi) q_{t+1}),$$

consistent with our assumption of full debt repudiation upon default.

With this maturity structure, the yield-to-maturity of the bond is $r_t = \frac{\kappa}{q_t} - \phi$, the spread is $r_t - r^f$, and the risk-free Macaulay duration is $\frac{1+r^f}{\phi + r^f} \approx \phi^{-1}$.

## 1.1 A small $k$, big $K$ recursive formulation of laissez-faire

We introduce a recursive formulation of our environment via a standard "small $k$, big $K$" approach, so that firms do not internalize the impact of their individual investment decisions ($k'$) for the aggregate capital stock next period ($K'$) and thus allocations more broadly. To economize on notation, we group state variables into two sets: first, exogenous and stationary shocks $s = \langle z, g \rangle$, and second, the endogenous capital and debt stocks, together with the level of the trend in the previous period, $S = \langle K, B, \Gamma_{-1} \rangle$.[6]

The value function for the private sector, the household and firm consolidated, under normal international market access for the sovereign, given that the government will have

5. By restricting attention to the traditional case of risk-neutrality, we rule out any role for risk premia (Lizarazo 2013) or global financial shocks (Morelli, Ottonello, and Perez 2022) in driving yields in our model.

6. We reserve the details of the detrending and computation of our models for the online Supplemental Material. Detrending is enabled by the homogeneity of value and bond price functions, together with our functional form assumptions (Aguiar et al. 2016).

$B'$ outstanding debt next period, satisfies

$$
\begin{aligned}
v^r(k,s,S,B') = \max_{k'} \; & u(c) + \beta \mathbb{E}_{s'|s} v(k',s',S') \\
\text{s.t. } & c + k' + \Theta(k,k') = z k^\alpha \Gamma^{1-\alpha} + (1-\delta)k + T(s,S,B') \\
& K' = \mathcal{K}(s,S,B'),
\end{aligned} \tag{1}
$$

where $\mathcal{K}$ is the perceived law of motion for aggregate capital. In equilibrium, a consistency condition ensures that the capital investment choice of the representative household coincides with the aggregate variable, $k' = K' = \mathcal{K}(s,S,B')$.

The value under default, in international market exclusion, is

$$
\begin{aligned}
v^d(k,s,S) = \max_{k'} \; & u(c) + \beta \mathbb{E}_{s'|s} \Big( \lambda v(k',s',S') + (1-\lambda) v^d(k',s',S') \Big) \\
\text{s.t. } & c + k' + \Theta(k,k') = h(z) k^\alpha \Gamma^{1-\alpha} + (1-\delta)k + T^d(s,S) \\
& K' = \mathcal{K}^d(s,S), \quad B' = 0,
\end{aligned} \tag{2}
$$

and the start-of-period value, prior to the sovereign's default decision, satisfies

$$
v(k,s,S) = \mathcal{D}(s,S) v^d(k,s,S) + (1-\mathcal{D}(s,S)) v^r(k,s,S,\mathcal{B}(s,S)), \tag{3}
$$

where $\mathcal{D}(s,S)$ is the sovereign's equilibrium default policy and $\mathcal{B}(s,S)$ is its choice of bonds for next period. The private sector is impacted by the sovereign's choices through the levels of the transfers $T$ and $T^d$, as well as the default productivity penalty $h$.

These private sector value functions induce policy functions under repayment and default, respectively: the consumption policies $\mathcal{C}(s,S,B')$ and $\mathcal{C}^d(s,S)$, and the capital policies $\mathcal{K}(s,S,B')$ and $\mathcal{K}^d(s,S)$. These act as further constraints on the government's choices, since the sovereign understands that, for example, following a counterfactually higher $B'$ the resulting capital stock $K'$ will respond as governed by the $\mathcal{K}$ policy.

The sovereign's value under repayment is

$$
\begin{aligned}
V^r(s,S) = \max_{B'} \; & u(\mathcal{C}(s,S,B')) + \beta_g \mathbb{E}_{s'|s} V(s',S') \\
\text{s.t. } & T(s,S,B') = q(s,S')(B' - (1-\phi)B) - \kappa B - G(S) \\
& K' = \mathcal{K}(s,S,B'),
\end{aligned} \tag{4}
$$

with associated bond issuance policy $\mathcal{B}(s, S)$, while the default value satisfies

$$V^d(s,S) = u(\mathcal{C}^d(s,S)) + \beta_g \mathbb{E}_{s'|s} \Big(\lambda V(s',S') + (1-\lambda)V^d(s',S')\Big) \tag{5}$$
$$\text{with } T^d(s,S) = -G(S)$$
$$K' = \mathcal{K}^d(s,S), \quad B' = 0.$$

The start of period value inclusive of the option to default is given by

$$V(s,S) = \max_{D\in\{0,1\}} D\, V^d(s,S) + (1-D)V^r(s,S) \tag{6}$$

and we encode the resulting equilibrium default policy in $\mathcal{D}(s, S)$.

Finally, the bond price schedule in recursive notation satisfies

$$q(s,S') = \frac{1}{1+r^f}\, \mathbb{E}_{s'|s}(1-\mathcal{D}(s',S'))\left[\kappa + (1-\phi)q(s',S'')\right] \tag{7}$$
$$\text{with } S'' = \langle \mathcal{K}(s',S',\mathcal{B}(s',S')), \mathcal{B}(s',S'), \Gamma g\rangle.$$

Due to long-term bonds, pricing reflects all future equilibrium default, borrowing, and investment policies, at all horizons.

## 1.2 A recursive formulation of centralized investment

We will contrast the values and policies from the previous section, under laissez-faire, to outcomes under centralized investment, where we allow the government to choose capital investment directly. We discuss in the following section how this can be decentralized using a linear investment tax or subsidy.

In the centralized case, denoted throughout by a $c$ subscript, the value of the government under repayment is

$$V_c^r(s,S) = \max_{K',B'} u(c) + \beta_g \mathbb{E}_{s'|s} V_c(s',S') \tag{8}$$
$$\text{s.t. } c + K' + \Theta(K,K') + G(S) = zK^\alpha\Gamma^{1-\alpha} + (1-\delta)K + q_c(s,S')(B' - (1-\phi)B) - \kappa B,$$

and the resulting policies are $K' = \mathcal{K}_c(s, S)$ and $B' = \mathcal{B}_c(s, S)$. In default, the choice of capital satisfies

$$V_c^d(s,S) = \max_{K'} u(c) + \beta_g \mathbb{E}_{s'|s} \Big( \lambda V_c(s',S') + (1-\lambda) V_c^d(s',S') \Big) \tag{9}$$
$$\text{s.t. } c + K' + \Theta(K,K') + G(S) = h(z) K^\alpha \Gamma^{1-\alpha} + (1-\delta)K$$
$$B' = 0,$$

with associated policy function $K' = \mathcal{K}_c^d(s,S)$. Finally, the default decision is

$$V_c(s,S) = \max_{D\in\{0,1\}} D\, V_c^d(s,S) + (1-D) V_c^r(s,S), \tag{10}$$

and the policy is encoded in $\mathcal{D}_c(s,S)$.

The lenders' bond pricing condition is analogous to the one under the laissez-faire case, except for the fact that now the relevant policy functions are $\mathcal{D}_c$, $\mathcal{K}_c$, and $\mathcal{B}_c$, respectively:

$$q_c(s,S') = \frac{1}{1+r^f} \mathbb{E}_{s'|s} (1 - \mathcal{D}_c(s',S')) \left[ \kappa + (1-\phi) q_c(s',S'') \right] \tag{11}$$
$$\text{with } S'' = \langle \mathcal{K}_c(s',S'), \mathcal{B}_c(s',S'), \Gamma g \rangle.$$

To evaluate welfare, we compute values for the private sector too, noting that it makes no decision. For example, under market access,

$$v_c^r(s,S) = u(c) + \beta \mathbb{E}_{s'|s} v_c(s',S')$$
$$\text{where } c = z K^\alpha \Gamma^{1-\alpha} + (1-\delta)K - K' - \Theta(K,K') - G(S) + q_c(s,S')(B' - (1-\phi)B) - \kappa B$$
$$K' = \mathcal{K}_c(s,S), \quad B' = \mathcal{B}_c(s,S).$$

The online Supplemental Material includes definitions for the Markov Perfect equilibria of the laissez-faire and centralized investment economies, respectively.

## 1.3 A first-order condition characterization

We start our analysis by comparing the first-order optimality conditions for investment, under laissez-faire and centralized investment, to highlight the forces shaping the optimal size of the capital stock. We focus on investment when the sovereign is choosing to service the debt, although analogous conditions can be derived for default. We set $\theta = 0$ in this section alone, for ease of illustration, and revert temporarily to the more compact sequential notation.

Under laissez-faire, the first-order condition for the private sector's capital choice is

$$u'(c_t) = \beta \mathbb{E}_t \Bigg\{ D_{t+1} u'(c^d_{t+1}) \left[ \alpha h(z_{t+1}) \left( \frac{\Gamma_{t+1}}{K_{t+1}} \right)^{1-\alpha} + 1 - \delta \right] \\ + (1 - D_{t+1}) u'(c^r_{t+1}) \left[ \alpha z_{t+1} \left( \frac{\Gamma_{t+1}}{K_{t+1}} \right)^{1-\alpha} + 1 - \delta \right] \Bigg\} \tag{12}$$

while under centralized investment, when the government has perfect control over capital accumulation, the condition reads

$$\left[ 1 - \frac{\partial q_t}{\partial K_{t+1}} (B_{t+1} - (1-\phi) B_t) \right] u'(c_t) = \beta_g \mathbb{E}_t \Bigg\{ \\ D_{t+1} u'(c^d_{t+1}) \left[ \alpha h(z_{t+1}) \left( \frac{\Gamma_{t+1}}{K_{t+1}} \right)^{1-\alpha} + 1 - \delta \right] \\ + (1 - D_{t+1}) u'(c^r_{t+1}) \left[ \alpha z_{t+1} \left( \frac{\Gamma_{t+1}}{K_{t+1}} \right)^{1-\alpha} + 1 - \delta \right] \Bigg\} \tag{13}$$

Two differences are salient. First, the marginal benefit of investment, on the right-hand-side, is evaluated using $\beta_g$ rather than $\beta$. All else equal, future consumption is weakly less valuable today. This discourages investment. Second, the private sector does not internalize the impact of its investment on the bond price received by the sovereign today. This is reflected in the term in square brackets on the left-hand-side, scaling the marginal cost of investment.

Across the state space, the relation between capital levels and default propensity is non-monotonic (Park 2017). Still, for most of the states in the ergodic distribution, more capital means weaker incentives to default, and so the benefit of capital accumulation is underestimated by the private investment policy. This social desirability of higher capital is reflected by the $\frac{\partial q_t}{\partial K_{t+1}} > 0$ term. A more subtle implication of long-term debt is that when the government is buying back, retiring outstanding bonds in secondary markets, $B_{t+1} < (1-\phi)B_t$, the private sector could invest excessively, in that its high $K_{t+1}$ choice supports a higher bond price, a price the sovereign now pays rather than receives.

Our quantitative analysis in the following section aims to establish which of these differences in investment behavior dominates and how the answer depends on the nature of the frictions impinging on this economy.

**The decentralization tax or subsidy.** We laid out two polar opposite cases: laissez-faire, under which the private sector decides on investment and the government issues bonds, and a centralized economy, where the government controls both assets. We now sketch how this centralized outcome can be implemented by means of a state-contingent subsidy to capital accumulation, mirroring the analysis of Gordon and Guerron-Quintana (2018, Appendix A.2). We introduce a subsidy $\tau_t$ on capital accumulation in the firm's problem. The definition of dividends becomes

$$\mathrm{div}_t = z_t k_t^{\alpha} \left(\Gamma_t \ell_t\right)^{1-\alpha} - w_t \ell_t + (1-\delta) k_t - (1-\tau_t) k_{t+1} - \Theta(k_t, k_{t+1}),$$

and the problems facing private sector agents are otherwise unaltered. The subsidy also enters the primary deficit expression on the right-hand-side of the government's budget constraint,

$$(1-D_t)\left[q_t(B_{t+1}-(1-\phi)B_t) - \kappa B_t\right] = G_t + T_t + \tau_t K_{t+1}.$$

The private sector's first order condition for $K_{t+1}$ becomes

$$\begin{aligned}(1-\tau_t)u'(c_t) = \beta \mathbb{E}_t \Bigg\{ & D_{t+1} u'(c^d_{t+1}) \left[\alpha h(z_{t+1}) \left(\frac{\Gamma_{t+1}}{K_{t+1}}\right)^{1-\alpha} + 1 - \delta\right] \\ & + (1-D_{t+1}) u'(c^r_{t+1}) \left[\alpha z_{t+1} \left(\frac{\Gamma_{t+1}}{K_{t+1}}\right)^{1-\alpha} + 1 - \delta\right] \Bigg\}\end{aligned} \tag{14}$$

so that the only difference from (12) is the scaling of the marginal cost of investment on the left-hand side by the $1-\tau_t$ subsidy term. A comparison with the centralized investment condition (13) delivers a state-contingent subsidy rate which supports the centralized investment policies,

$$\tau_t = 1 - \left[1 - \frac{\partial q_t}{\partial K_{t+1}}(B_{t+1} - (1-\phi)B_t)\right] \frac{\beta}{\beta_g}. \tag{15}$$

Whether $\tau_t$ is a subsidy ($\tau_t > 0$) or a tax ($\tau_t < 0$) depends on three factors: first, the strength and sign of the investment externality, $\frac{\partial q_t}{\partial K_{t+1}}$, second, whether the sovereign is issuing or buying back bonds, $B_{t+1} \lesseqgtr (1-\phi)B_t$, and finally, the degree of sovereign impatience, $\beta_g \leq \beta$.

# 2 Quantitative Analysis

To evaluate our model's quantitative properties, we make further functional form assumptions. We use $u(c) = 1 - c^{-1}$ for the flow utility function, and thus employ a conventional value of 2 for the coefficient of relative risk aversion. Our default penalty follows Chatterjee and Eyigungor (2012), $h(z) = z - \max(0, \iota_0 z + \iota_1 z^2)$, with $\iota_0 < 0 < \iota_1$. The concavity of $h(z)$ is in line with the observation of Arellano (2008) that a convex penalty is key for countercyclical spreads and elevated default risk during recessions.

## 2.1 Calibration and model fit

We pursue a quarterly calibration of the model for the case of decentralized investment using data from Spain, a representative European periphery economy, over the 1995Q1 to 2019Q4 sample.[7] Moreover, Spain is part of a monetary union, the Euro area, which alleviates concerns about the role of monetary policy for our question.

We group parameters in two sets. First, those that can be set directly, based on estimates from the data and values common in the literature. Table 1 compiles their values and interpretation. A second set of parameters are specific to our model. They are set in order to match seven moments in the data. Table 2 lists these parameters and the model fit.

Our productivity process features a stationary cycle component $z_t$ and a random trend $\Gamma_t$ driven by potentially persistent growth rates $g_t$. Estimating such a process on our short sample is challenging. We rely on the insight of Aguiar and Gopinath (2007), that the volatility of consumption reflects the relative magnitude of trend versus cycle shocks. We find that the Spanish data is best described by growth rate shocks which are uncorrelated over time, yet volatile.

The $G$ parameter matches the average share of public spending in GDP. For the factor shares parameter $\alpha$ and the depreciation rate $\delta$ we rely on common values in the literature, 0.36 and 2%, respectively. The risk-free rate is 1% quarterly, based on a longer sample of German bond yields, a value comparable to rates in the literature, albeit somewhat higher than recent levels. The Macaulay duration of the debt is 5 years, the value from Chatterjee and Eyigungor (2012), somewhat lower than the values reported by Bocola and Dovis (2019) for Italy. $\lambda$ is pinned down by the average length of market exclusion following default, roughly five years (Tomz and Wright 2013). The debt service parameter $\kappa$ is normalized to $\phi + r^f$, so that the risk-free long-term bond price is one, with the desirable

7. The online Supplemental Material reports data from five European countries, collectively and informally known as the PIIGS, often studied in relation to the European debt crisis of 2009–2010, to establish the degree to which Spain is representative for the group as a whole.

| | Value | Description |
|---|---|---|
| *Fiscal policy and international markets* | | |
| $G$ | 0.18 | Mean public spending to GDP |
| $r^f$ | 0.01 | International risk-free rate |
| $\phi$ | 0.05 | Macaulay duration of debt |
| $\lambda$ | 0.05 | International market exclusion |
| $\kappa$ | $\phi + r^f$ | Normalization, $q^f = 1$ |
| *Production and productivity* | | |
| $\alpha$ | 0.36 | Income shares |
| $\delta$ | 0.02 | Capital depreciation |
| $\rho_z$ | 0.95 | Persistence of cycle |
| $\rho_g$ | 0.0 | Persistence of growth rates |
| $\mu_g$ | 1.005 | Trend growth rate |

Table 1: Parameters Set Externally

property of expressing $B$ in units of GDP. Finally, we use $\mu_g$ to reproduce the trend growth rate of GDP on our model's Balanced Growth Path.

Seven parameters allow us to target seven key moments. Nearly all parameters impact most statistics to at least some degree, but some are more influential than others, and therefore we align them in Table 2 with the moment that is most sensitive. The adjustment cost $\theta$ is set to match the volatility of investment relative to that of GDP, roughly 2.5 times more volatile. The default penalty parameters $\iota_0$ and $\iota_1$ drive the mean and volatility of yield spreads, in the data about 1.2% and 1.3%, respectively. As standard in growth models, the private sector discount factor $\beta$ controls the capital to income ratio, equivalently the ratio of investment to GDP, about 22%. In turn, the sovereign's discount factor $\beta_g$ is most important for the debt to GDP ratio. We find that $\beta_g$ is lower than $\beta$ by about 0.008 in our quarterly calibration, a nontrivial degree of relative impatience for the fiscal authority. The volatility of shock innovations $\sigma_z$ and $\sigma_g$ shape the volatility of detrended GDP and the relative volatility of consumption to GDP. Overall, the model is largely successful in this moment matching exercise, with one exception. It has difficulty in delivering the volatility of spreads, with a standard deviation of about 1.1% in the model, versus 1.3% in the data.

## 2.2 Findings

Table 3 compiles our results. It reports key statistics from the data, our model calibration, and model-based counterfactuals aimed at decomposing the relative contribution of the frictions we study. Our baseline model is the "Impatient ($\beta_g < \beta$)" + "Laissez-faire" case.

| | Value | Target | Data | Model |
|---|---|---|---|---|
| $\beta$ | 0.987 | Investment to GDP | 0.22 | 0.22 |
| $\theta$ | 30.0 | Relative volatility of investment | 2.56 | 2.76 |
| $\sigma_z$ | 0.003 | Volatility of GDP | 1.72 | 1.94 |
| $\sigma_g$ | 0.005 | Relative volatility of consumption | 1.12 | 1.27 |
| $\beta_g$ | 0.979 | Debt to GDP | 0.27 | 0.27 |
| $\iota_0$ | $-3.03$ | Bond yield spread | 1.17 | 1.15 |
| $\iota_1$ | 3.12 | Standard deviation of spread | 1.31 | 1.09 |

Table 2: Parameters Set Internally

The model exhibits weakly countercyclical spreads, an acyclical trade balance, together with business cycle volatilities and a debt to GDP ratio comparable to the data.

The third column of Table 3 allows us to explore the consequences of endowing the sovereign with direct control over the capital accumulation decision or, equivalently, under a richer set of fiscal instruments. This internalizes the externality induced by investment on the bond price schedule. On the other hand, all policies in this economy are shaped by the government's discount factor $\beta_g$ and the private sector's $\beta$ is only relevant for assessing welfare. The net result is a reduction in the average level of the capital stock, by about 19%, a higher default risk, and an increase in average spreads of about 67bps. Spread volatility increases too, by about a third, while other moments are mostly unaffected.

We employ a consumption equivalent measure[8] for welfare, to assess the value to the household of centralizing investment. We find that households are *worse off*, with roughly a 5.25% drop in consumption equivalent welfare. Households would strictly prefer not to delegate investment decisions to the sovereign, even though it would internalize the externality, because of the government's impatience.

The last two columns of Table 3 report results under the assumption that public decision-making mirrors the household's discount factor, $\beta_g = \beta$, e.g., the absence of political economy frictions, while keeping all other parameters unaltered. From the fourth column, we learn that eliminating the sovereign's impatience lowers spreads and default risk. The household is *better off* with such a patient sovereign, and consumption equivalent welfare increases by 0.2%. Even though both regimes suffer from the investment externality, allocations are improved due to the reduced willingness of the government to risk costly default episodes.

8. We report welfare when shocks are at their mean, with debt and the capital stock at the mean of the baseline model's ergodic distribution. Results are similar elsewhere in the state space. We use the household's value function for this calculation, $c_{\text{eq}} = u^{-1}((1-\beta)v)$, and report the percent change in this implied consumption measure across models.

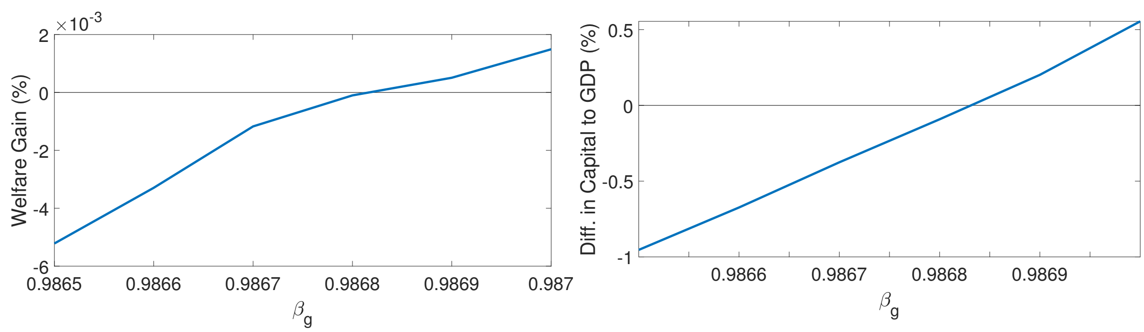
| Moment | Data | Impatient ($\beta_g < \beta$) Laissez-faire | Impatient ($\beta_g < \beta$) Centralized | Patient ($\beta_g = \beta$) Laissez-faire | Patient ($\beta_g = \beta$) Centralized |
|---|---|---|---|---|---|
| *Ratios to GDP* | | | | | |
| Debt | 0.27 | 0.27 | 0.29 | 0.24 | 0.24 |
| Capital stock | — | 2.72 | 2.21 | 2.72 | 2.73 |
| Consumption | 0.60 | 0.59 | 0.61 | 0.59 | 0.59 |
| Investment | 0.22 | 0.22 | 0.18 | 0.22 | 0.22 |
| *Yield spread and default (%)* | | | | | |
| Mean spread | 1.17 | 1.15 | 1.82 | 0.31 | 0.30 |
| St dev spread | 1.31 | 1.09 | 1.47 | 0.44 | 0.43 |
| Default risk | — | 0.97 | 1.51 | 0.30 | 0.28 |
| *Standard deviations, relative to GDP* | | | | | |
| Consumption | 1.12 | 1.27 | 1.41 | 1.23 | 1.23 |
| Investment | 2.56 | 2.76 | 3.17 | 2.45 | 2.44 |
| *Correlations with GDP* | | | | | |
| Consumption | 0.94 | 0.88 | 0.84 | 0.91 | 0.91 |
| Investment | 0.85 | 0.68 | 0.63 | 0.70 | 0.70 |
| Trade balance | −0.06 | −0.10 | −0.10 | −0.07 | −0.07 |
| Spread | −0.22 | −0.28 | −0.30 | −0.26 | −0.26 |
| Household welfare (%) | | | | | |
| vs laissez-faire | | | −5.25 | | +0.001 |
| vs impatient | | | | +0.20 | |

Table 3: Model Statistics

NOTES: The "Data" column reports moments for Spain, 1995Q1–2019Q4. Our calibrated model corresponds to the "Impatient ($\beta_g < \beta$)" + "Laissez-faire" column. Model moments are computed based on long simulations, $10^6$ periods, excluding default spells and the first five years following return to market. The welfare measure is described in footnote 8.

Finally, we can isolate the value of internalizing the externality by comparing the fourth and fifth columns. In both of these cases the sovereign is patient and the only difference is that in the fifth column the government controls investment directly. Capital levels are modestly higher and spreads fall on average. By internalizing the externality, fiscal policy makes households *better off*, welfare increases by 0.001%.

**The role of $\beta_g$.** The sovereign's impatience weighs heavily on our results. We explore how much by varying $\beta_g$ while keeping $\beta$ fixed. Figure 1 plots the welfare gain from centralizing investment as a function of $\beta_g$ in panel (a), and the change in the capital to GDP ratio in panel (b). With low $\beta_g$, households are robustly worse off, by up to 5.5% of their consumption equivalent measure. This reflects depressed investment and a lower capital stock. As $\beta_g$ crosses a critical threshold at 0.9869, the welfare loss turns into a gain. Households thereafter are better off with centralized investment due to the desirable effects of internalizing the externality on the bond price $q$. In the limit, we recover the welfare gain from centralizing investment with a patient sovereign, from Table 3. We conclude that even a subtle degree of impatience can be critical for the desirability of additional fiscal instruments.




(a) Households' welfare gain

(b) Difference in Capital to GDP

Figure 1: Comparison across sovereign's impatience

NOTES: Panel (a) plots the consumption equivalent welfare gain to the private sector from centralizing the investment decision, as a function of the government's discount factor $\beta_g$. Panel (b) plots the change in the average level of the capital to GDP ratio associated with centralizing investment, also as a function of $\beta_g$.

**Implied taxes and subsidies.** Table 4 compiles key statistics for the implied taxes or subsidies needed to align private and public incentives. We report positive values for taxes and negative values for subsidies, i.e. $-\tau$, since our baseline result features taxes rather than subsidies.

On average, an impatient sovereign would impose a 0.75% tax on investment, to stimulate immediate consumption, with some variation captured by the 0.13% standard deviation. The tax is roughly acyclical, but positively correlated with domestic absorption and negatively with the trade balance and the yield spread. In sum, the tax is higher in "good times" and lower in "bad times," when the sovereign must service the debt under elevated spreads. In contrast, the patient government ($\beta_g = \beta$) employs a modest 0.02% investment subsidy, also acyclical, to support capital accumulation and address the externality from $\frac{\partial q_t}{\partial K_{t+1}}$.

| | Impatient ($\beta_g < \beta$) | Patient ($\beta_g = \beta$) |
|---|---|---|
| *Implied tax, market access (%)* | | |
| Mean | 0.75 | $-0.02$ |
| St dev | 0.13 | 0.13 |
| *Implied tax, default (%)* | | |
| Mean | 0.75 | $-0.01$ |
| St dev | 0.09 | 0.09 |
| *Implied tax, 3 years prior to default (%)* | | |
| Mean | 0.72 | $-0.03$ |
| St dev | 0.16 | 0.14 |
| *Correlation with tax, market access* | | |
| GDP | 0.08 | 0.05 |
| Consumption | 0.23 | 0.11 |
| Investment | 0.29 | 0.10 |
| Trade balance | $-0.32$ | $-0.13$ |
| Yield spread | $-0.23$ | $-0.03$ |

Table 4: Implied Investment Tax Rates

NOTES: Positive values correspond to taxes, that is $-\tau_t$ in our model notation, while negative values denote subsidies. Implied tax rates are computed using equation (14), state by state, given equilibrium allocations.

Turning to the fiscal treatment of investment during default, Table 4 documents that the tax is on average the same as during market access, albeit with a lower volatility. Interestingly, the tax is lower during the 3 years preceding defaults. This reflects the shifting priorities of the government, away from immediate consumption and towards avoiding the crisis, whenever conditions involve a high enough degree of risk.

# 3 Discussion

Our results imply that the potential benefit from internalizing the investment externality via an appropriately designed and funded subsidy is outweighed by the sovereign's preference for immediate consumption. This is due to its lower discount factor, a classic proxy for political economy frictions. What is to be done? Aguiar, Amador, and Fourakis (2020) consider the gains from outright banning international market access for the sovereign and find them to be potentially large, in an endowment model. Less radically, Hatchondo, Martinez, and Roch (2022) propose fiscal rules, institutional limits on the sovereign's ability to borrow into unfavorable bond prices, by means of either a debt or spread break. Azzimonti and Mitra (2023a, 2023b) tackle political constraints and their fiscal consequences directly, in a microfounded model of legislative bargaining over tax and spending outcomes, as well as sovereign default. This broader research agenda points to a need to better understand the exact nature and causes of sovereigns' shortsightedness.

In sum, our results call into question the standard prescription according to which households can be better off from the introduction of targeted incentives for capital accumulation, without also addressing fiscal policy-making distortions. One noteworthy caveat to this takeaway is that we studied a setting with a modest risk of default, of about 1% annualized, a relatively developed economy part of the European Union. This is far from the typical emerging market environment considered in the default literature.

# References

Acharya, Viral V, Raghuram G Rajan, and Jack B Shim. 2024. "Sovereign debt and economic growth when government is myopic and self-interested." *Journal of International Economics* 150:103906.

Aguiar, Mark, Manuel Amador, and Stelios Fourakis. 2020. "On the Welfare Losses from External Sovereign Borrowing." *IMF Economic Review* 68 (1): 163–194.

Aguiar, Mark, Satyajit Chatterjee, Harold Cole, and Zachary Stangebye. 2016. "Quantitative models of sovereign debt crises." In *Handbook of macroeconomics,* 2:1697–1755. Elsevier.

Aguiar, Mark, and Gita Gopinath. 2006. "Defaultable debt, interest rates and the current account." *Journal of International Economics* 69 (1): 64–83.

———. 2007. "Emerging market business cycles: The cycle is the trend." *Journal of Political Economy* 115 (1): 69–102.

Alesina, Alberto, and Guido Tabellini. 1990. "A Positive Theory of Fiscal Deficits and Government Debt." *The Review of Economic Studies* 57 (3): 403–414.

Alessandria, George, Yan Bai, and Minjie Deng. 2020. "Migration and sovereign default risk." *Journal of Monetary Economics* 113:1–22.

Arellano, Cristina. 2008. "Default risk and income fluctuations in emerging economies." *American Economic Review* 98, no. 3 (June): 690–712.

Arellano, Cristina, Yan Bai, and Luigi Bocola. 2025. "Sovereign default risk and firm heterogeneity." *Journal of the European Economic Association* 0 (0): 0.

Arellano, Cristina, Yan Bai, and Gabriel Mihalache. 2018. "Default risk, sectoral reallocation, and persistent recessions." *Journal of International Economics* 112:182–199.

———. 2020. "Monetary Policy and Sovereign Risk in Emerging Economies (NK-Default)." NBER Working Paper No. 26671, January.

Asonuma, Tamon, and Hyungseok Joo. 2023. "Sovereign Defaults and Public Investment (Capital)." University of Surrey Discussion Paper in Economics, June.

Azzimonti, Marina, and Nirvana Mitra. 2023a. "Political constraints and sovereign default." *Journal of International Money and Finance* 137:102895.

———. 2023b. *Sovereign Default and Tax-smoothing in the Shadow of Corruption and Institutional Weakness.* Technical report. National Bureau of Economic Research.

Bai, Yan, and Jing Zhang. 2012. "Financial integration and international risk sharing." *Journal of International Economics* 86 (1): 17–32.

Bianchi, Javier, Pablo Ottonello, and Ignacio Presno. 2023. "Fiscal stimulus under sovereign risk." *Journal of Political Economy* 131 (9): 2328–2369.

Bocola, Luigi. 2016. "The Pass-Through of Sovereign Risk." *Journal of Political Economy* 124 (4): 879–926.

Bocola, Luigi, and Alessandro Dovis. 2019. "Self-fulfilling debt crises: A quantitative analysis." *American Economic Review* 109 (12): 4343–4377.

Chatterjee, Satyajit, and Burcu Eyigungor. 2012. "Maturity, indebtedness, and default risk." *American Economic Review* 102 (6): 2674–2699.

———. 2019. "Endogenous political turnover and fluctuations in sovereign default risk." *Journal of International Economics* 117:37–50.

Cotoc, Johnny, Alok Johri, and César Sosa-Padilla. 2025. "Sovereign spreads and the political leaning of nations." *International Economic Review* 66 (2): 687–709.

Cuadra, Gabriel, Juan M Sanchez, and Horacio Sapriza. 2010. "Fiscal policy and default risk in emerging markets." *Review of Economic Dynamics* 13 (2): 452–469.

Deng, Minjie, and Chang Liu. 2024. "Sovereign risk and intangible investment." *Journal of International Economics,* 104009.

Dvorkin, Maximiliano, Juan M Sánchez, Horacio Sapriza, and Emircan Yurdagul. 2021. "Sovereign debt restructurings." *American Economic Journal: Macroeconomics* 13, no. 2 (April): 26–77.

Eaton, Jonathan, and Mark Gersovitz. 1981. "Debt with potential repudiation: Theoretical and empirical analysis." *The Review of Economic Studies* 48 (2): 289–309.

Esquivel, Carlos. 2024. "Underinvestment and capital misallocation under sovereign risk." *Journal of International Economics* 151:103973.

Gordon, Grey, and Pablo A. Guerron-Quintana. 2018. "Dynamics of investment, debt, and default." *Review of Economic Dynamics* 28:71–95.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W Huffman. 1988. "Investment, capacity utilization, and the real business cycle." *The American Economic Review,* 402–417.

Hatchondo, Juan Carlos, and Leonardo Martinez. 2009. "Long-duration bonds and sovereign defaults." *Journal of international Economics* 79 (1): 117–125.

Hatchondo, Juan Carlos, Leonardo Martinez, and Francisco Roch. 2022. "Fiscal rules and the sovereign default premium." *American Economic Journal: Macroeconomics* 14 (4): 244–273.

Hodrick, Robert J, and Edward C Prescott. 1997. "Postwar US business cycles: an empirical investigation." *Journal of Money, credit, and Banking,* 1–16.

Jeske, Karsten. 2006. "Private international debt with risk of repudiation." *Journal of political Economy* 114 (3): 576–593.

Kim, Yun Jung, and Jing Zhang. 2012. "Decentralized borrowing and centralized default." *Journal of International Economics* 88 (1): 121–133.

Liu, Siming, and Hewei Shen. 2022. "Sovereign Default and Labor Market Dynamics." *Available at SSRN 3785746.*

Lizarazo, Sandra Valentina. 2013. "Default risk and risk averse international investors." *Journal of International Economics* 89 (2): 317–330.

Mendoza, Enrique G., and Vivian Z. Yue. 2012. "A General Equilibrium Model of Sovereign Default and Business Cycles*." *The Quarterly Journal of Economics* 127, no. 2 (April): 889–946.

Mihalache, Gabriel. 2020. "Sovereign default resolution through maturity extension." *Journal of International Economics* 125c (July): 103326.

———. 2025. "Solving default models." In *Oxford Research Encyclopedia of Economics and Finance.*

Morelli, Juan M, Pablo Ottonello, and Diego J Perez. 2022. "Global banks and systemic debt crises." *Econometrica* 90 (2): 749–798.

Na, Seunghoon, Stephanie Schmitt-Grohé, Martín Uribe, and Vivian Yue. 2018. "The twin ds: Optimal default and devaluation." *American Economic Review* 108 (7): 1773–1819.

Ohio Supercomputer Center. 1987. *Ohio Supercomputer Center.*

Park, JungJae. 2017. "Sovereign default and capital accumulation." *Journal of International Economics* 106:119–133.

Schmitt-Grohé, Stephanie, and Martın Uribe. 2003. "Closing small open economy models." *Journal of international Economics* 61 (1): 163–185.

Seoane, Hernán D, and Emircan Yurdagul. 2023. "Business cycles and sovereign defaults: a decentralized approach." June.

Tomz, Michael, and Mark LJ Wright. 2013. "Empirical research on sovereign debt and default." *Annu. Rev. Econ.* 5 (1): 247–272.

Yue, Vivian Z. 2010. "Sovereign default and debt renegotiation." *Journal of international Economics* 80 (2): 176–187.

*Insufficient or Excessive Investment Under Sovereign Default Risk*
— Supplemental Material —
Ilhwan Song and Gabriel Mihalache

**Data Sources.** Our sample consists of the PIIGS countries (Portugal, Ireland, Italy, Greece, and Spain) over the 1995Q1 to 2019Q4 period. We obtain data on debt to GDP, yields and spreads from the European Central Bank (ECB) and National Accounts data from EuroStat. Table A.1 compiles data moments for each country, together with the source and cross-country averages.

**Equilibrium definition.** We define the equilibria corresponding to the two regimes, with respect to the ability of the sovereign to impact private investment choices.
*MPE Laissez-Faire.* A recursive Markov Perfect Equilibrium of the laissez-faire economy consists of

- private sector value functions $v^r$, $v^d$, and $v$, and policies $\mathcal{C}$ and $\mathcal{K}$,
- government value functions $V^r$, $V^d$, and $V$, and policies $\mathcal{D}$ and $\mathcal{B}$, and
- the bond price schedule $q$,

which together satisfy the following conditions:

- given government policies, the private sector values and policies solve (1) and (2),
- given private sector policies and the bond price schedule, government values and policies solve (4) and (6),
- given policies, lenders break even and the bond price schedule (7) holds.

*MPE Centralized.* A recursive Markov Perfect Equilibrium of the centralized investment economy consists of

- government value functions $V_c^r$, $V_c^d$, and $V_c$,
- government policies $\mathcal{D}_c$, $\mathcal{B}_c$, and $\mathcal{K}_c$, and
- the bond price schedule $q_c$,

which together satisfy the following conditions:

- given the bond price schedule, policies and values solve (8), (9), and (10),
- given policies, lenders break even and the bond price schedule (11) holds.

| | Mean | Greece | Ireland | Italy | Portugal | Spain | Source |
|---|---|---|---|---|---|---|---|
| $b/y$ | 0.373 | n.a. | 0.337 | 0.409 | 0.510 | 0.266 | ECB |
| $c/y$ | 0.582 | 0.676 | 0.381 | 0.604 | 0.601 | 0.603 | EuroStat |
| $i/y$ | 0.215 | 0.187 | 0.265 | 0.199 | 0.201 | 0.217 | EuroStat |
| $g/y$ | 0.182 | 0.205 | 0.152 | 0.190 | 0.184 | 0.182 | EuroStat |
| spread | 2.390 | 5.137 | 1.662 | 0.658 | 2.443 | 1.173 | ECB |
| $\sigma_{\text{spread}}$ | 2.077 | 4.738 | 1.121 | 0.672 | 1.929 | 1.311 | ECB |
| $\sigma_c/\sigma_y$ | 1.004 | 1.079 | 0.705 | 0.997 | 1.117 | 1.121 | Eurostat |
| $\sigma_i/\sigma_y$ | 4.158 | 4.898 | 6.916 | 2.898 | 3.514 | 2.562 | Eurostat |
| $\sigma_{tb/y}/\sigma_y$ | 2.167 | 1.763 | 3.910 | 1.078 | 2.070 | 2.013 | Eurostat |
| $\rho_{y,c}$ | 0.798 | 0.888 | 0.463 | 0.801 | 0.900 | 0.937 | EuroStat |
| $\rho_{y,i}$ | 0.641 | 0.573 | 0.180 | 0.823 | 0.779 | 0.851 | EuroStat |
| $\rho_{y,tb/y}$ | -0.059 | -0.254 | 0.061 | 0.072 | -0.115 | -0.058 | EuroStat |
| $\rho_{y,\text{spread}}$ | -0.221 | -0.520 | -0.274 | 0.082 | -0.174 | -0.218 | EuroStat |
| $\sigma_y$ | 2.051 | 2.45 | 3.07 | 1.45 | 1.57 | 1.72 | EuroStat |
| $\sigma_{\Delta y}$ | 1.324 | 1.58 | 2.90 | 0.70 | 0.79 | 0.65 | EuroStat |
| $\rho_{y_t,y_{t-1}}$ | 0.815 | 0.88 | 0.60 | 0.81 | 0.87 | 0.91 | EuroStat |
| $\rho_{\Delta y_t,\Delta y_{t-1}}$ | 0.372 | 0.32 | -0.21 | 0.51 | 0.42 | 0.82 | EuroStat |

Table A.1: Data Moments by Country and Average

NOTES: "spread" is the difference in yield-to-maturity between each country's bond and that of Germany, as reported by the ECB. $\sigma$ denotes standard deviation, and $\rho$ correlation. Debt to GDP data is not available for Greece. When appearing in levels, and not as ratios or log differences ($\Delta$), variables are Hodrick and Prescott (1997) filtered with $\lambda = 1600$.

**Model Detrending.** Our models feature a random trend, encoded in the $\Gamma_{-1}$ state variables. Our detrending strategy is largely adapted from Aguiar and Gopinath (2007). We guess and verify that quantities are linear in $\Gamma_{-1}$ and values are proportional to the inverse of $\Gamma_{-1}$, under our assumption of a coefficient of risk-aversion of 2. We denote by $\hat{}$ the detrended variables and values. For any quantity $x$ (consumption, investment, etc.) we have therefore $x = \hat{x}\Gamma_{-1}$, and for any value function $v$ we write $v = \hat{v}/\Gamma_{-1}$. The bond price function is homogeneous of degree zero in $\Gamma_{-1}$, $B'$, and $K'$ and can be written as a function of $\hat{B}'$ and $\hat{K}'$ directly. Finally, our functional form assumption for the investment adjustment cost $\Theta$ is consistent with this detrending strategy.

Detrending the model produces small alterations to the resource and budget constraints, as all future stock variables are scaled by the current gross growth rate shock $g$. For example, the resource constraint for the sovereign, under centralized investment, becomes

$$\hat{c}_t + \hat{K}_{t+1}g_t + \hat{\Theta}_t + \hat{G} = z_t(\hat{K}_t)^\alpha(g_t)^{1-\alpha} + (1-\delta)\hat{K}_t + q_{c,t}(\hat{B}_{t+1}g_t - (1-\phi)\hat{B}_t) - \kappa\hat{B}_t.$$

The final necessary adjustment concerns discounting. The effective discount factor in the detrended model is now $\beta/g_t$ or $\beta_g/g_t$, respectively.

**Numerical Solution.** The computation of models with defaultable long-term debt is notoriously challenging (Chatterjee and Eyigungor 2012; Gordon and Guerron-Quintana 2018). We tackle this task in our model with capital investment by employing discrete choice methods. See Dvorkin et al. (2021) and Mihalache (2020) for early uses of these methods for default models, and Mihalache (2025) for a more pedagogical treatment.

Our method involves augmenting the default ($\mathcal{D}$) and borrowing ($\mathcal{B}$) decisions with Extreme Value Type I taste shocks. We set the variance of these shocks to the smallest value consistent with convergence. The capital investment choice ($\mathcal{K}$) is not perturbed. Instead, we rely on root-finding on the private sector's first-order condition to find the preferred investment level, given any arbitrary $B'$ borrowing level. In the centralized model, we again find the best $K'$ for each possible $B'$ using golden section search and linear interpolation. We solve all versions of the model using the same grids. We discretize the shock process using 13 points for $z$, 7 for $g$, 250 for $B$, and 150 points for $K$, and allow interpolation over the $K$ dimension. We experimented with grid sizes and location, to confirm the robustness of welfare measures and business cycle statistics.