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The Stability Problem of Broadcast Packet Switching Computer Networks

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Summary. Certain computer networks have been implemented using a radio broadcast frequency over which a large set of terminals are allowed to transmit packets of bits; one such example is the ALOHA computer network [1]. In such systems, a basic problem is that of the blocking of terminals whose transmission of a packet has overlapped in time with transmission by some other terminal. In this paper we consider a "slotted ALOHA" packet transmission scheme with an infinite set of terminals each transmitting at an infinitesimally small rate. We present a probabilistic model of such a system to show that the slotted ALOHA system with an infinite population is inherently unstable. This paper confirms the simulations and results of Kleinrock and Lam [8].

I. Introduction

Due to the inefficiency of allocating a large number of low capacity channels to a large set of user pairs transmitting data to each other, compared to the sharing of a high speed channel between the ensemble of users [2], various forms of packet switching schemes have been suggested [3, 4] and implemented in the ARPA [5], CYCLADES [10] and ALOHA computer networks [1].

We are concerned with networks using radio channels for packet switching similar to the ALOHA network in which *each* packet whose transmission overlaps in time the use of the channel by some other packet is lost. We can think of the shared radio channel being implemented by using a geostationary satellite which acts as a transponder, reflecting packets it receives back to earth. The information being transmitted can be heard by every user when it is reflected back; thus a terminal having transmitted apacket will know (by listening to the "echo") whether its packet has "collided" with another one and therefore is lost or whether the transmission has been sucessful. A terminal whose transmission is lost is *blocked*; it repeats the transmission of the same packet until a sucessful transmission occurs. A terminal which is not blocked is *active.* No collisions would occur if the transmission could be scheduled; the difficulty with this solution is that the only means of communication between the terminals is the channel itself. The two following methods for controlling transmissions over the channel have been suggested. The first allows the terminal to transmit a packet at any instant of time. The second method is known as the "slotted ALOHA" system, which is the subject of this paper. Here time is divided into equal slots whose duration is the transmission time of a packet (packet length is constant). Packet transmission is synchronized so as to be initiated at the beginning of a slot, and this apprach has been shown to increase the channel throughput over the first method [6]. Other control schemes are mentioned elsewhere [9].

Kleinrock and Lam [8] have discussed the stability problem of the slotted ALOHA channel. They give a qualitative argument, indicating that this channel will become saturated if the population of users is infinite independently of the value of the arrival rate of packets to the channel. Here, saturation is construed as the phenomenon, whereby the number of blocked terminals becomes arbitrarily large after a long period of operation. They also indicate that saturation has been observed during simulation runs, and they compute average first exit times into the set of channel states which lead to saturation from an initially empty channel for an infinite population model.

In this paper we concern ourselves with an infinite population system which is a worst case representation of reality. We present a simple mathematical model of the broadcast channel, allowing us to give a characterization which is different from [9] of the channel behavior. We then give a proof of the instability of the infinite population slotted broadcast channel. Our result confirms the discussion based on "fluid approximation" and the simulations of Kleinrock and Lam [8].

II. The Model

Consider the broadcast channel at the sequence of instants 0, 1, 2, ... when a slot begins. It is assumed that the slot, and packet data transmission time is of unit length. Denote by $N(k)$ the number blocked terminals at instant k.

Packets arrive from the non-blocked terminals according to a Poisson process of rate λ; the instants of "arrival" of packets (which are Poisson) are distinct from the instants of transmission of packets which are regular (as indicated above). We assume that the time between the completion of transmission of a packet and the time of retransmission of the same packet (if in the meanwhile the terminal becomes blocked) is exponentially distributed with expected value $1/\gamma$. This would probably not be so in a realistic situation, where a fixed time elapses (during which the terminal is silent) before the terminal knows that it is blocked. The exponential assumption is equivalent to an assumption of Kleinrock and Lam (Eq. (1) of [9]) for which they report that, based on simulation runs, it is seen that a good degree of accuracy is obtained with this representation. In addition to these assumptions, each blocked terminal behaves as an independent source, retransmitting the packet untill the transmission is successful.

In this framework $N(k)$ is a random variable. Let

$$p_n(k) = \text{Prob}\{N(k) = n\}. \tag{1}$$

We may then write the equation[1]:

$$\begin{aligned} p_n(k+1) = \sum_{j=2}^{n} p_{n-j}(k) \frac{\lambda^j}{j!} e^{-\lambda} + p_{n+1}(k)\, \pi_1(n+1)\, e^{-\lambda} \\ + p_n(k)\,[1-\pi_1(n)]\, e^{-\lambda} + p_n(k)\, \pi_0(n)\, \lambda e^{-\lambda} \\ + p_{n-1}(k)\,[1-\pi_0(n-1)]\, \lambda e^{-\lambda} \end{aligned} \tag{2}$$

1 Eq. (2) remains valid for all non-negative integer values of n if we use the rule that $p_i(k) = 0$ for $i < 0$. This rule is necessary for $n = 0, 1$.

where $\pi_i(n)$, $i=0, 1, 2, \ldots$, is the probability that, during one slot, i blocked terminals transmit a packet conditional on there being n blocked terminals at the beginning of the slot.

We have

$$\pi_0(n) = e^{-\gamma n} \tag{3}$$

and

$$\pi_1(n) = n e^{-\gamma(n-1)}(1-e^{-\gamma}). \tag{4}$$

On the right-hand side of (2), the first term represents the fact that, if in a slot two or more "external arrivals" occur (i.e., those which do not originate from blocked terminals), then these tend to increase the number of blocked terminals independently of the number of packets generated by the blocked terminals. The second term covers the case where no external arrival occurs, and exactly one blocked terminal emits a packet, thus becoming unblocked. The fourth term covers the case where no blocked terminal retransmits a packet, and one external arrival occurs. In the fifth term, one external arrival and at least one transmission from a blocked terminal take place. Eq. (2) is the transition equation of a discrete time Markov chain which is aperiodic and irreducible.

Consider the invariant probability measure $\{p_n\}$ satisfying (2); that is

$$\begin{aligned} p_n = \sum_{j=2}^{n} p_{n-j} \frac{\lambda^j}{j!} e^{-\lambda} + p_{n+1}\pi_1(n+1)e^{-\lambda} + p_n[1-\pi_1(n)]e^{-\lambda} \\ + p_n \pi_0(n)\lambda e^{-\lambda} + p_{n-1}[1-\pi_0(n-1)]\lambda e^{-\lambda} \end{aligned} \tag{6}$$

and

$$\sum_{n=0}^{\infty} p_n = 1. \tag{7}$$

Definition. We shall say that the slotted broadcast channel is ***unstable*** if an invariant probability measure, solution to (6) and (7), does not exist, such that $p_n > 0$ for all non-negative integer n.

In other terms, if the Markov chain given by (2) is not ergodic, then we shall say that the channel is unstable. In physical terms, this implies that after a sufficiently long time the number of blocked terminals becomes arbitrarily large.

Theorem. The slotted broadcast channel is unstable.

To prove the theorem let us write (6) in slightly different form:

$$\begin{aligned} p_n = \sum_{j=0}^{n} p_{n-j} \frac{\lambda^j}{j!} e^{-\lambda} + p_{n+1}\pi_1(n+1)e^{-\lambda} + p_n e^{-\lambda}[\lambda\pi_0(n) - \pi_1(n)] \\ - p_{n-1}\pi_0(n-1)\lambda e^{-\lambda} \end{aligned} \tag{8}$$

and define

$$S_N = \sum_{j=0}^{N} p_j. \tag{9}$$

Then (8) yields

$$S_N = p_{N+1}\pi_1(N+1)e^{-\lambda} + p_N\pi_0(N)\lambda e^{-\lambda} + \sum_{n=0}^{N} S_{N-n}\frac{\lambda^n}{n!}e^{-\lambda}$$

or

$$S_N[1-e^{-\lambda}] = \sum_{n=1}^{N} S_{N-n} \frac{\lambda^n}{n!} e^{-\lambda} + p_{N+1}\pi_1(N+1)e^{-\lambda} + p_N\pi_0(N)\lambda e^{-\lambda}$$

leading to the inequality

$$S_N[1-e^{-\lambda}] \leqq S_{N-1}[1-e^{-\lambda}] + p_{N+1}\pi_1(N+1)e^{-\lambda} + p_N\pi_0(N)\lambda e^{-\lambda} \tag{10}$$

equivalently

$$p_N[1-e^{-\lambda}] \leqq p_{N+1}\pi_1(N+1)e^{-\lambda} + p_N\pi_0(N)\lambda e^{-\lambda}$$

or

$$\frac{p_{N+1}}{p_N} \geqq \frac{1-e^{-\lambda}-\pi_0(N)\lambda e^{-\lambda}}{\pi_1(N+1)e^{-\lambda}}. \tag{11}$$

But the right-hand side of (11) tends to infinity as $N\to\infty$. Therefore, the series $\sum_{N=0}^{\infty} p_N$ diverges unless $p_N=0$ for all N. Since (7) must be satisfied, we conclude that $p_n=0$ for all n (finite) and the channel is unstable, as was to be shown.

III. Model Assumptions under which Instability Is Verified

In fact, the theorem we have proved remains valid under broader assumptions which we state as follows.

A1. Let X_k be the number of external arrivals during the k-th slot; $\{X_k\}_{0\leqq k<\infty}$ is a sequence of independent identically distributed random variables whose sample space is the set of non-negative integers and

$$\text{Prob}\{X_k=i\}=c_i, \quad i\geqq 0, \quad \sum_{i=0}^{\infty} c_i = 1.$$

A2. Let

$$Y_{ki} = \begin{cases} 1 & \text{if the } i\text{-th blocked terminal transmits during the } k\text{-th slot} \\ 0 & \text{otherwise.} \end{cases}$$

Y_{ki} is a random variable; it is independent of Y_{kj} for $i\neq j$, of Y_{li} for $l\neq k$, and of Y_{lj} for $l\neq k$, $i\neq j$. Furthermore, $\text{Prob}\{Y_{ki}=1\}=a>0$.

Eq. (2) is now replaced by

$$\begin{aligned} p_n(k+1) = \sum_{j=2}^{n} p_{n-j}(k)c_j + p_{n+1}(k)\pi_1(n+1)c_0 + p_n(k)[1-\pi_1(n)]c_0 \\ + p_n(k)\pi_0(n)c_1 + p_{n-1}(k)[1-\pi_0(n-1)]c_1 \end{aligned} \tag{12}$$

where, instead of (3) and (4), we have

$$\pi_0(n) = (1-a)^n, \tag{13}$$

$$\pi_1(n) = n(1-a)^{n-1}a \tag{14}$$

and (10) is replaced by

$$S_N[1-c_0] \leqq S_{N-1}[1-c_0] + p_{N+1}\pi_1(N+1)c_0 + p_N\pi_0(N)c_1 \tag{15}$$

so that

$$\frac{p_{N+1}}{p_N} \geq \frac{1 - c_0 - (1-a)^N c_1}{(N+1)a(1-a)^N c_0}. \tag{16}$$

The arguments used in the preceding section are then used to prove the theorem.

Note Added in Proof. Extensions of the results given here can be found in [11] where it is shown that:

The instability result can also be derived starting from a model in which the total number of terminals is a constant M, each active terminal transmitting independently with probability b in a slot, as we let $M \to \infty$, $b \to 0$ and $M.b \to d > 0$, d being constant.

The channel throughput in steady state is zero both for the model in the present paper and for the limiting case outlined above.

In [11] some simulation and analytical results are presented to illustrate the instability and to justify certain stabilizing control policies for the channel.

References

1. Abramson, N.: The ALOHA system-Another alternative for computer communications. AFIPS 1970 FJCC **37**. Montvale (N.J.): AFIPS Press 1970, p. 281–285
2. Kleinrock, L.: Resource allocation in computer systems and computer-communication networks. Proc. IFIP Congress 1974, Stockholm (to appear)
3. Baren, P., Boehm, S., Smith, P.: On distributed communications. Series of eleven reports available from Rand Corp., Santa Monica (Calif.), 1964
4. Davies, D. W.: The principles of a data communication networks fòr computers and remote peripherals. Proc. IFIP Congress 1968 (Hardware), Edinburgh
5. Roberts, L. G.: Multiple computer networks and inter-computer communications. ACM Symposium on Operating Systems, Gatlinburg (Tenn.), 1967
6. Abramson, N.: Packet switching with satellites. National Computer Conference, 1973. Montvale (N.J.): AFIPS Press 1973, p. 695–702
7. Roberts, L. G.: Dynamic allocation of satellite capacity through packet reservation. National Computer Conference, 1973. Montvale (N.J.): AFIPS Press 1973, p. 711–716
8. Kleinrock, L., Lam, S. S.: Packet switching in a slotted satellite channel. National Computer Conference, 1973. Montvale (N.J.): AFIPS Press 1973, p. 703–710
9. Kleinrock, L., Lam, S. S.: On stability of packet switching in a random multiaccess broadcast channel (to appear)
10. Pouzin, L.: Pressentation and major design aspects of the CYCLADES computer network. Submitted for presentation at the Third Data Communications Symposium, April 1973
11. Fayolle, G., Gelenbe, E., Labetoulle, J.: Stability and control of packet switching broadcast channels (to appear)

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