SOLUTIONS: Communication Systems

Professor A. Manikas
Chair of Communications and Array Processing
Department of Electrical & Electronic Engineering
Imperial College London

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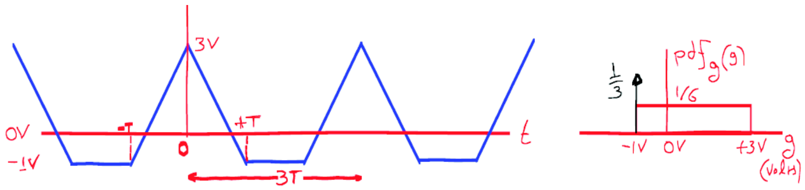
1 Topic: Introductory Concepts

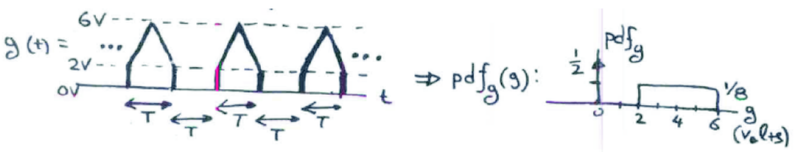
1. Solution

(a)



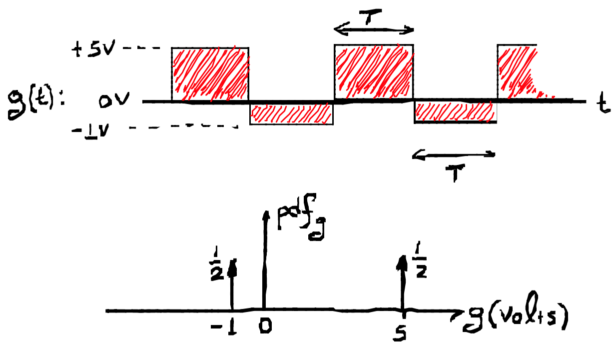
$$\text{i.e.pdf}_g(g) = \frac{1}{3}\delta(g+1) + \frac{1}{6}\text{rect}\left\{\frac{g-1}{4}\right\}$$

(b)



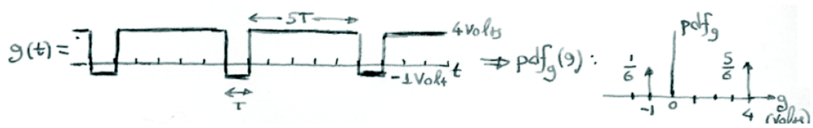
$$\text{i.e.pdf}_g(g) = \frac{1}{2}\delta(g) + \frac{1}{8}\text{rect}\left\{\frac{g-4}{4}\right\}$$

(c)



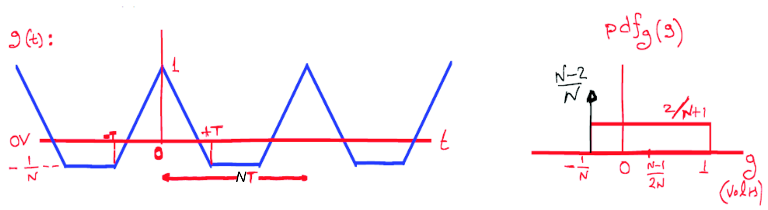
$$\text{pdf}_g(g) = \frac{1}{2}\delta(g+1) + \frac{1}{2}\delta(g-5)$$

(d)



$$\text{pdf}_g(g) = \frac{1}{6}\delta(g+1) + \frac{5}{6}\delta(g-4)$$

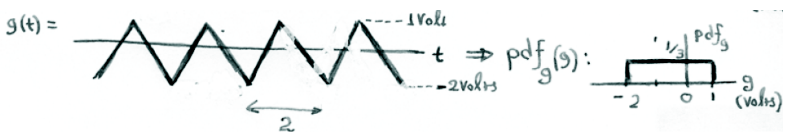
(e)



i.e.

$$\begin{aligned}
\text{pdf}_g(g) &= \frac{N-2}{N}\delta(g+\frac{1}{N}) + \frac{2}{N+1}\text{rect}\left\{\frac{g-\frac{N-1}{2N}}{\frac{N+1}{N}}\right\} \\
&= \frac{N-2}{N}\delta(g+\frac{1}{N}) + \frac{2}{N+1}\text{rect}\left\{\frac{2Ng-N+1}{2(N+1)}\right\}
\end{aligned}$$

(f)



$$\text{pdf}_g(g) = \frac{1}{3}\textbf{rect}\left\{\frac{g-0.5}{3}\right\}$$

2. Solution

(a)

$$\int_{-\infty}^{\infty} (t^4 - 3t + 1).\delta(t-2)\ .dt = (t^4 - 3t + 1)|_{t=2} = 2^4 - 3\times 2 + 1 = 11$$

(b) $\int_{-\infty}^{\infty} \left(\cos(4\pi t) * \delta(t+\frac{1}{4})\right).\delta(t-\frac{1}{8}).dt = \int_{-\infty}^{\infty} \cos\left(4\pi(t+\frac{1}{4})\right).\delta(t-\frac{1}{8}).dt =$

$= \cos\left(4\pi(t+\frac{1}{4})\right)|_{t=\frac{1}{8}} = \cos\left(4\pi(\frac{1}{8}+\frac{1}{4})\right) = \cos(\frac{3}{2}\pi) = 0$

(c) $\displaystyle\int_{-\infty}^{\infty} (t^3 - 3t^2 - 11).\delta(t-1)\ .dt = (t^3 - 3t^2 - 11)|_{t=1} = 1^3 - 3\times 1^2 - 11 = -13$

(d) $\displaystyle\int_{-\infty}^{\infty} \left\{(\sin(4\pi t) * \delta(t+\frac{1}{4})\right\}.\delta(t-\frac{1}{4}).dt = \int_{-\infty}^{\infty} \left(\sin(4\pi\left(t+\frac{1}{4}\right)\right).\delta(t-\frac{1}{4}).dt =$

$\sin\left(4\pi\left(t+\frac{1}{4}\right)\right)|_{t=\frac{1}{4}} = \sin\left(4\pi\left(\frac{1}{4}+\frac{1}{4}\right)\right) = \sin(2\pi) = 0$

(e) $\int_{-\infty}^{\infty} (t^3 - 2t^2 + 1).\delta(t-2)\ .dt = (t^3 - 2t^2 + 1)|_{t=2} = 2^3 - 2\times 2^2 + 1 = 1$

(f) $\int\limits_{-\infty}^{\infty}\left(\cos(2\pi t)*\delta(t-\frac{1}{4})\right).\delta(t-\frac{1}{12}).dt=\int\limits_{-\infty}^{\infty}\cos\left(2\pi(t-\frac{1}{4})\right).\delta(t-\frac{1}{12}).dt=$
$=\cos\left(2\pi(t-\frac{1}{4})\right)|_{t=\frac{1}{12}}=\cos\left(2\pi(\frac{1}{12}-\frac{1}{4})\right)=\cos(-\frac{1}{3}\pi)=\frac{1}{2}$

(g) $h(3)$ where $h(t)=\left(t.\text{rect}\left\{\frac{t}{8}\right\}\right)*\delta(t+3)$ (5%)
$h(t)=\left(t.\text{rect}\left\{\frac{t}{8}\right\}\right)*\delta(t+3)=(t+3)\,.\text{rect}\{\frac{t+3}{8}\}$
$\Rightarrow h(3)=0$

(h) $h(3)$ where $h(t)=\left(t.\text{rect}\left\{\frac{1}{8T}\right\}\right)*\delta(t-2)$ (10%)
$h(t)=\left(t.\text{rect}\left\{\frac{t}{8T}\right\}\right)*\delta(t-2)=(t-2)\,.\text{rect}\{\frac{t-2}{8T}\}$
$h(3)=(3-2).\text{rect}\{\frac{3-2}{8T}\}$
$$\Rightarrow h(t)=\begin{cases}t-2 & \text{if } -0.5<\frac{t-2}{8T}<0.5\\ 0 & \text{otherwise}\end{cases}$$
$$\Rightarrow h(3)=\begin{cases}1 & \text{if } -0.5<\frac{t-2}{8T}<0.5\\ 0 & \text{otherwise}\end{cases}=\begin{cases}1 & \text{if } T>\frac{1}{4}\\ 0 & \text{if } T\le\frac{1}{4}\end{cases}$$

(i) $h(3.5)$ where $h(t)=\left(t.\text{rect}\left\{\frac{1}{8T}\right\}\right)*\delta(t-3)$ (10%)

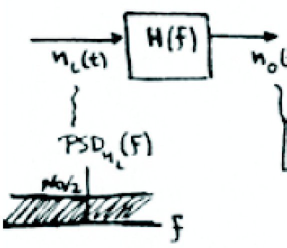
i.e. $h(t)=(t-3)\,\text{rect}\,\frac{t-3}{8T}\Rightarrow h(3.5)=(3.5-3)\,\text{rect}\,\frac{3.5-3}{8T}\Rightarrow$
$h(3.5)=0.5\times\text{rect}\,\frac{0.5}{8T}$
$\frac{0.5}{8T}<\frac{1}{2}\Rightarrow T>\frac{1}{8}$
i.e. $h(3.5)=\begin{cases}0.5 & \text{if } T>\frac{1}{8}\\ 0 & \text{if } T<\frac{1}{8}\end{cases}$

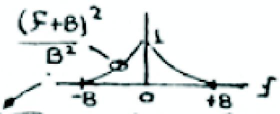
3. Solution

(a) $R_{bb}(\tau)=\frac{N+1}{N}\mathbf{rep}_{NT_c}\left\{\Lambda\left(\frac{\tau}{T_c}\right)\right\}-\frac{1}{N}$

(b) PSD(f) =FT$\{R_{bb}(\tau)\}=\frac{N+1}{N^2}\text{comb}_{\frac{1}{NT_c}}\left\{\text{sinc}^2(fT_c)\right\}-\frac{1}{N}\delta(f)$

4. Solution





$\frac{N_0}{2}=\frac{3}{2}\times10^{-6}\Rightarrow\boxed{N_0=3\times10^{-6}}$

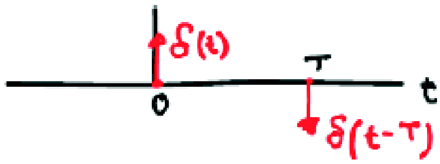
$\boxed{B=10^6}$

$PSD_{n_o}(f)=\underbrace{PSD_{n_i}(f)}_{\frac{N_0}{2}}.\underbrace{|H(f)|^2}$

$P_{n_o}=\int_{-\infty}^{+\infty}PSD_{n_i}(f).|H(f)|^2.df$
$=\frac{N_0}{2}\int_{-B}^{+B}|H(f)|^2df=\frac{N_0}{2}2\int_{-B}^{0}|H(f)|^2df$
$=N_0\int_{-B}^{0}\frac{(f+B)^2}{B^2}df=\frac{N_0}{B^2}\frac{(f+B)^3}{3}\Big]_{-B}^{0}$
$\Rightarrow\boxed{P_{n_o}=\frac{N_0B}{3}}\boxed{=\frac{3\times10^{-6}\times10^6}{3}=1}$

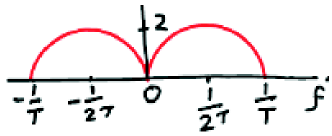
5. Solution

(a) $h(t) = \delta(t) - \delta(t - T)$



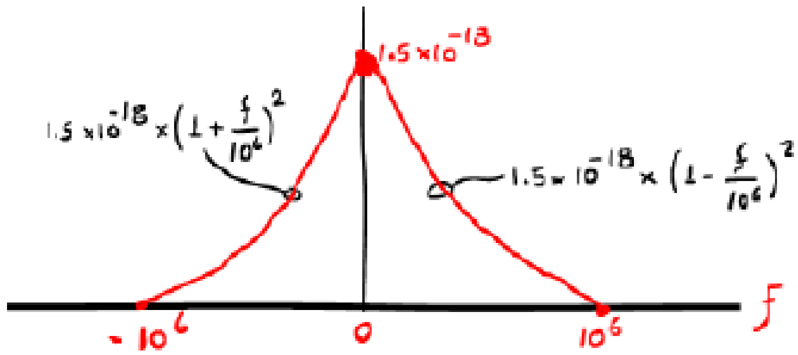
(b) $H(f) = \mathrm{FT}\{h(t)\} = 1 - \exp(-j2\pi fT)$
$= \{\exp(j\pi fT) - \exp(-j\pi fT)\} \exp(-j\pi fT)$
$= 2\sin(\pi fT)\exp(-j\pi fT)$

$\Rightarrow |H(f)| = 2\,|\sin(\pi fT)| =$



6. Solution

(a) $H(f) = \mathrm{FT}\{h(t)\} = \frac{1}{10^6}\Lambda\left(\frac{f}{10^6}\right)\exp\left(-j2\pi f \times 3\right)$
$\Rightarrow \mathrm{PSD}_n(f) = \mathrm{PSD}_{n_i}(f).\left|H(f)\right|^2 =$
$1.5 \times 10^{-6}\left(\frac{1}{10^6}\right)^2 \Lambda^2\left(\frac{f}{10^6}\right) = 1.5 \times 10^{-18}\Lambda^2\left(\frac{f}{10^6}\right)$



(b) $P_n = \int\limits_{-10^6}^{10^6} \mathrm{PSD}_n(f).df = 2\int\limits_{0}^{10^6} \mathrm{PSD}_n(f).df =$

$= 2\int\limits_{0}^{10^6}\left(1.5 \times 10^{-18} \times \left(1 - \frac{f}{10^6}\right)^2\right).df =$

$= 2 \times 1.5 \times 10^{-18} \times \int\limits_{0}^{10^6}\left(1 - \frac{f}{10^6}\right)^2 .df =$

$= 3 \times 10^{-18} \times \int\limits_{0}^{10^6}\left(1 - 2\frac{f}{10^6} + \frac{f^2}{10^{12}}\right).df$

$= 3 \times 10^{-18} \times \left(10^6 - 2\frac{10^{12}}{2\times 10^6} + \frac{10^{18}}{3\times 10^{12}}\right)$

$= 3 \times 10^{-18} \times \left(10^6 - 10^6 + \frac{1}{3}10^6\right)$

$= 10^{-12}$

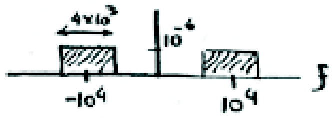
7. Solution

$$H(f) = FT\{h(t)\} = 8\times10^{3}\ FT\{\mathrm{sinc}(4\times10^{3}t)\cos(2\pi10^{4}t)\}$$

$$= 8\times10^{3}\ \underbrace{FT\{\mathrm{sinc}(4\times10^{3}t)\}}_{\frac{1}{4\times10^{3}}\mathrm{rect}\frac{f}{4\times10^{3}}} \underset{\text{convolution}}{\circledast} \left[\tfrac{1}{2}\delta(f-10^{4})+\tfrac{1}{2}\delta(f+10^{4})\right]$$

$$= \mathrm{rect}\,\frac{f-10^{4}}{4\times10^{3}} + \mathrm{rect}\,\frac{f+10^{4}}{4\times10^{3}}$$

$$PSD_{n}(f) = PSD_{n_i}(f)\cdot|H(f)|^{2} = 10^{-6}\,\mathrm{rect}\,\frac{f-10^{4}}{4\times10^{3}} + 10^{-6}\,\mathrm{rect}\,\frac{f+10^{4}}{4\times10^{3}}$$

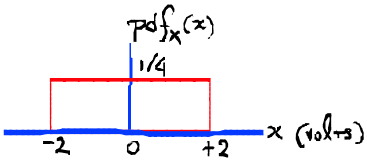


$$P_{n} = 4\times10^{3}\times10^{-6}\times2 = 8\times10^{-3}\,\mathrm{W} = \underline{\underline{8\,\mathrm{mW}}}$$

2 Topic: Information Sources

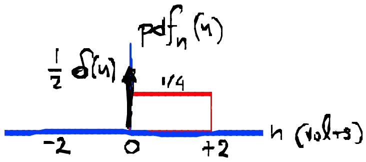
8. Solution

(a) pdfx(x)=$\frac{1}{4}$rect$\{\frac{x}{4}\}$



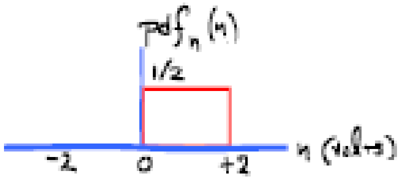
mean: $\mathcal{E}\{x(t)\} = \int_{-\infty}^{+\infty} x.\text{pdf}_\text{x}(x).dx = \int_{-2}^{+2} x.\frac{1}{4}.dx = 0$ Volts
power:$P_x = \mathcal{E}\{x^2(t)\} = \int_{-\infty}^{+\infty} x^2.\text{pdf}_\text{x}(x).dx = \int_{-2}^{+2} x^2.\frac{1}{4}.dx = \frac{4}{3}$
rms=$\sqrt{P_x} = \frac{2}{\sqrt{3}} = 1.1547$

(b) $\text{pdf}_n(n) = \frac{1}{2}\delta(n) + \frac{1}{4}\text{rect}\{\frac{n-1}{2}\}$



mean: $\mathcal{E}\{n(t)\} = \int_{-\infty}^{+\infty} n.\text{pdf}_\text{n}(n).dn = \int_{0}^{+2} n.\frac{1}{4}.dn = \frac{1}{2}$ Volts
power:$P_n = \mathcal{E}\{n^2(t)\} = \int_{-\infty}^{+\infty} n^2.\text{pdf}_\text{n}(n).dn = \int_{0}^{+2} n^2.\frac{1}{4}.dx = \frac{2}{3}$
rms=$\sqrt{P_x} = \sqrt{\frac{2}{3}} = 0.81650$

(c) $\text{pdf}_n(n) = \frac{1}{2}\text{rect}\{\frac{n-1}{2}\}$



mean: $\mathcal{E}\{n(t)\} = \int_{-\infty}^{+\infty} n.\text{pdf}_\text{n}(n).dn = \int_{0}^{+2} n.\frac{1}{2}.dn = 1$ Volts
power:$P_n = \mathcal{E}\{n^2(t)\} = \int_{-\infty}^{+\infty} n^2.\text{pdf}_\text{n}(n).dn = \int_{0}^{+2} n^2.\frac{1}{2}.dx = \frac{4}{3}$
rms=$\sqrt{P_x} = \frac{2}{\sqrt{3}} = 1.1547$

9. Solution

(a) average power (source with uniform pdf):
$P_x = \mathcal{E}\{x^2(t)\} = \int_{-\infty}^{+\infty} x^2.\text{pdf}_\text{x}(x).dx = \int_{-3}^{+3} x^2.\frac{1}{6}.dx = \frac{9}{3} = 3$

(b) $H_x = -\int_{-\infty}^{+\infty}\text{pdf}_\text{x}(x).\log_2(\text{pdf}_\text{x}(x))\,dx = -\int_{-3}^{+3}\frac{1}{6}.\log_2\left(\frac{1}{6}\right)dx =$
$= -\frac{1}{6}.\log_2\left(\frac{1}{6}\right)\int_{-3}^{+3} dx = \log_2 6 = 2.585$

(c) If $y(t)$=Gaussian with mean=μ and rms=$\sqrt{P_x}$ then
$H_y = \log_2\sqrt{2\pi e P_x} = \log_2\sqrt{2\pi e 3} = 2.8396$
$H_y - H_x = 2.8396 - 2.585 = 0.2546$

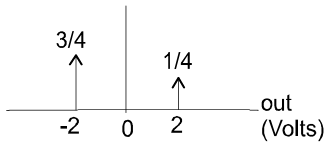
(d) $N_x = \frac{1}{2\pi e} 2^{2H_x} = \frac{1}{2\pi e} 2^{2\times 2.585} = 2.107\,9 = 4.2158$

10. Solution

(a) $r_x = 2 \times F_g = 8k \frac{levels}{\text{sec}}$

(b) $\Pr(-2V) = \frac{3}{4}; \Pr(+2V) = \frac{1}{4} \Rightarrow \underline{p} = [\frac{3}{4}, \frac{1}{4}]^T$

pdf:



(c) rms$=\sqrt{(-2)^2 \Pr(-2V) + 2^2 \Pr(+2V)} = \sqrt{(-2)^2 \frac{3}{4} + 2^2 \frac{1}{4}} = 2V$

(d) $H_X = -\frac{3}{4}\log_2\left(\frac{3}{4}\right) - \frac{1}{4}\log_2\left(\frac{1}{4}\right) = 0.81128 \frac{bits}{\text{level}}$

(e) $(X \times X, \mathbb{J}) = \left\{ \begin{array}{cc} \left(x_1 x_1, \frac{9}{16}\right) & \left(x_2 x_1, \frac{3}{16}\right) \\ \left(x_1 x_2, \frac{3}{16}\right) & \left(x_2 x_2, \frac{1}{16}\right) \end{array} \right\}$

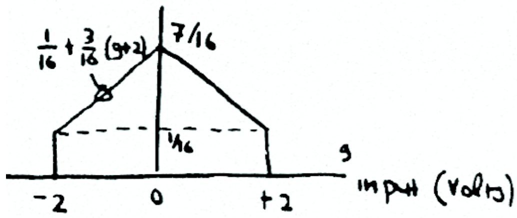
$H_{X\times X} = -\frac{9}{16}\log_2\left(\frac{9}{16}\right) - \frac{3}{16}\log_2\left(\frac{3}{16}\right) - \frac{3}{16}\log_2\left(\frac{3}{16}\right) - \frac{1}{16}\log_2\left(\frac{1}{16}\right) = 1.6226 \frac{bits}{\text{double level}}$

11. Solution

(a) Quantiser:

"end"-points	Quantisation levels
$b_0 = -2V$ (or $-\infty$)	
$b_1 = -1V$	$x_1 = -1.5V$
$b_2 = 0V$	$x_2 = -0.5V$
$b_3 = 1V$	$x_3 = 0.5V$
$b_4 = 2V$ (or $+\infty$)	$x_4 = 1.5V$

input pdf:

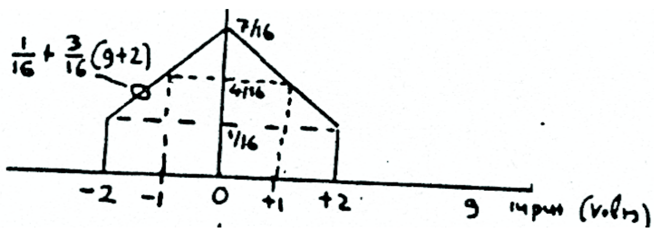


Therefore, power of $g(t) = P_g = \int_{-\infty}^{+\infty} g^2.\text{pdf}_g(g).dg$

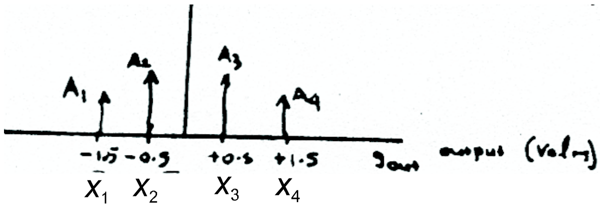
$= 2\int_{-2}^{0} g^2.\left(\frac{1}{16} + \frac{3}{16}(g+2)\right).dg = 2\int_{-2}^{0}\left(\frac{1}{16}g^2 + \frac{3}{16}g^3 + \frac{6}{16}g^2\right).dg = 2\int_{-2}^{0}\left(\frac{7}{16}g^2 + \frac{3}{16}g^3\right).dg$

$= \frac{2\times 7}{16}\frac{g^3}{3}]_{-2}^{0} + \frac{2\times 3}{16}\frac{g^4}{4}]_{-2}^{0} = \frac{5}{6} = 0.8333$

input pdf:



output pdf:



(b) rms:

$A_1 = Pr(g_{out} = -1.5V) = Pr(\underbrace{-2 < g < -1}_{\text{area}=A_1})$

$$= \int_{-2}^{-1} \left(\frac{1}{16} + \frac{3}{16}(g+2)\right) dg =$$

$$= \frac{1}{16} g + \frac{3}{16}\frac{(g+2)^2}{2} \Big]_{-2}^{-1} =$$

$$-\frac{1}{16} + \frac{3}{32} + \frac{2}{16} - 0 = \frac{5}{32} = 0.1563$$

$A_2 = Pr(g_{out} = -0.5V) = Pr(\underbrace{-1 < g < 0}_{\text{area}=A_2})$

$$= \int_{-1}^{0} \left(\frac{1}{16} + \frac{3}{16}(g+2)\right) dg =$$

$$= \frac{1}{16} g + \frac{3}{16}\frac{(g+2)^2}{2} \Big]_{-1}^{0} =$$

$$\frac{6}{16} + \frac{1}{16} - \frac{3}{32} = \frac{11}{32} = 0.3438$$

i.e. $A_3 = A_2 = \frac{11}{32}$ and $A_4 = A_1 = \frac{5}{32}$

$P_{g_{out}} = 2 \times \left((-1.5)^2 \times 0.1563 + (-0.5)^2 \times 0.3438\right) = 0.8752 \Rightarrow rms = \sqrt{0.8752} = 0.9355$

(c) $(X, \underline{p}) = \left\{\left(-1.5V, \frac{5}{32}\right), \left(-0.5V, \frac{11}{32}\right), \left(+0.5V, \frac{11}{32}\right), \left(+1.5V, \frac{5}{32}\right)\right\}$

$$(X \times X, \mathbb{J}) = \left\{ \begin{array}{cccc} \left(x_1x_1, \frac{25}{1024}\right) & \left(x_2x_1, \frac{55}{1024}\right) & \left(x_3x_1, \frac{55}{1024}\right) & \left(x_4x_1, \frac{25}{1024}\right) \\ \left(x_1x_2, \frac{55}{1024}\right) & \left(x_2x_2, \frac{121}{1024}\right) & \left(x_3x_2, \frac{121}{1024}\right) & \left(x_4x_2, \frac{55}{1024}\right) \\ \left(x_1x_3, \frac{55}{1024}\right) & \left(x_2x_3, \frac{121}{1024}\right) & \left(x_3x_3, \frac{121}{1024}\right) & \left(x_4x_3, \frac{55}{1024}\right) \\ \left(x_1x_4, \frac{25}{1024}\right) & \left(x_2x_4, \frac{55}{1024}\right) & \left(x_3x_4, \frac{55}{1024}\right) & \left(x_4x_4, \frac{25}{1024}\right) \end{array} \right\}$$

(d) Entropy:

$H_{X \times X} = \underline{1}_4^T \left(\mathbb{J} \odot \log_2 \mathbb{J}\right) \underline{1}_4 = 3.7921 \ \frac{bits}{\text{double level}}$

3 Topic: Communication Channels

12. Solution

- The matrix $\mathbb{F}$ is:

$$\mathbb{F} = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$$

- $\therefore$ the overall matrix $\mathbb{F}_{csc}$ is as follows:

$$\begin{aligned}
\mathbb{F}_{casc} &= \mathbb{F}.\mathbb{F} = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix} \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix} \\
&= \begin{bmatrix} 0.7 \times 0.7 + 0.3 \times 0.3, & 0.7 \times 0.3 + 0.3 \times 0.7 \\ 0.3 \times 0.7 + 0.7 \times 0.3, & 0.7 \times 0.7 + 0.3 \times 0.3 \end{bmatrix} \\
&= \begin{bmatrix} 0.58 & 0.42 \\ 0.42 & 0.58 \end{bmatrix} = \begin{bmatrix} \Pr(z_0|x_0), & \Pr(z_0|x_1) \\ \Pr(z_1|x_0) & \Pr(z_1|x_1) \end{bmatrix}
\end{aligned}$$

$\therefore$

$$\begin{aligned}
p_e &= \Pr(z_1|x_0)\Pr(x_0) + \Pr(z_0|x_1)\Pr(x_1) \\
&= 0.42 \times 0.4 + 0.42 \times 0.6 = 0.42
\end{aligned}$$

13. Solution

$C = B \log 2(1 + \underbrace{SNR_{in}}_{=30/2}) = B \log_2(1 + 15) = 4B \quad (1)$

However, BUE$= \frac{B}{r_b} \Rightarrow B = \underbrace{\text{BUE}}_{=2} \times \underbrace{r_b}_{=100} = 200 \quad (2)$

Therefore: $(1) \wedge (2) \Rightarrow C = 4 \times 200 = 800 \frac{bits}{\text{sec}}$

14. Solution

(a)

$$\left.\begin{array}{l} EUE = 30 \\ N_0/2 = 0.5 \times 10^{-6} \Rightarrow N_0 = 10^{-6} \\ C = 16 \frac{kbits}{sec} \\ B = 4KHz \end{array}\right\} \Rightarrow \begin{array}{l} C = B \log_2(1 + SNR_{in}) \quad (1) \\ BUE = \frac{B}{r_b} \quad (2) \end{array}$$

$$(1) \Rightarrow SNR_{in} = 2^{C/B} - 1 \Rightarrow \frac{EUE}{BUE} = 2^{C/B} - 1 \Rightarrow$$

$$\Rightarrow BUE = \frac{EUE}{2^{C/B} - 1} \Rightarrow \frac{B}{r_b} = \frac{EUE}{2^{C/B} - 1} \Rightarrow$$

$$\Rightarrow r_b = \frac{(2^{C/B} - 1)B}{EUE} = 2k \frac{bits}{sec}$$

(b) $P_n = N_0 B = 10^{-6} \times 4k = 4mW$

15. Solution

(a) $p_e = \Pr(H_2, D_1) + \Pr(H_1, D_2) = \Pr(D_1|H_2)\Pr(H_2) + \Pr(D_2|H_1)\Pr(H_1)$
$= 0.04 \times \frac{2}{3} + 0.018 \times \frac{1}{3} = \boxed{0.032667}$

(b) any expression of mutual information H_{mut} can be used.

For instance: $\mathrm{H}_{mut} = \sum_{m=1}^{2}\sum_{k=1}^{2} J_{km}\log_2\left(\frac{p_m q_m}{J_{km}}\right)$

where

$$\underline{q} = \mathbb{F}.\underline{p} \Rightarrow \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0.982 & 0.04 \\ 0.018 & 0.96 \end{bmatrix}}_{\mathbb{F}} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$

$p_1 = \Pr(H_1) = \frac{1}{3}$,

$p_2 = \Pr(H_2) = \frac{2}{3}$

$J_{11} = \Pr(H_1, D_1) = \Pr(D_1|H_1)\Pr(H_1) = 0.982 \times \frac{1}{3} = -0.3273$

$J_{12} = \Pr(H_2, D_1) = \Pr(D_1|H_2)\Pr(H_2) = 0.04 \times \frac{2}{3} = -0.0267$

$J_{21} = \Pr(H_1, D_2) = \Pr(D_2|H_1)\Pr(H_1) = 0.018 \times \frac{1}{3} = -0.006$

$J_{22} = \Pr(H_2, D_2) = \Pr(D_2|H_2)\Pr(H_2) = 0.96 \times \frac{2}{3} = -0.64$

$$or\ \mathbb{J} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} = \mathbb{F}diag\left(\underline{p}\right) = \underbrace{\begin{bmatrix} 0.982 & 0.04 \\ 0.018 & 0.96 \end{bmatrix}}_{\mathbb{F}} . \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{2}{3} \end{bmatrix}$$

$\Rightarrow \boxed{\mathrm{H}_{mut} = 0.7326}$

16. Solution

(a) $B = \frac{r_{cs}}{2} = \frac{1}{2T_{cs}} = \frac{1}{2\times 20\times 10^{-6}} = \boxed{25kHz}$

(b) $r_{cs} = \frac{1}{T_{cs}} = r_b = \boxed{50kbits/\sec}$

(c) EUE$=\frac{E_b}{N_0}$,

$$E_1 = \int_{-10\mu}^{10\mu} 4\Lambda^2\left\{\frac{t}{10\mu}\right\}dt = 2\int_{-10\mu}^{0} 4\left(\frac{t+10\mu}{10\mu}\right)^2 dt$$

$$= \frac{8}{(10\mu)^2}\int_{-10\mu}^{0}\left(t^2 + 20\mu t + (10\mu)^2\right)dt = \frac{1}{3}80\mu = E_2 = E_b$$

i.e. $E_b = \frac{80}{3}\times 10^{-6} = 26.667\times 10^{-6}$

$\frac{N_0}{2} = 10^{-6} \Rightarrow N_0 = 2\times 10^{-6}$

Thus $\boxed{\text{EUE}=\frac{E_b}{N_0} = \frac{26.667\times 10^{-6}}{2\times 10^{-6}} = 13.334}$

(d) $C = B\log_2(1 + SNR_{in}) = B\log_2\left(1 + \frac{EUE}{BUE}\right) =$

$$25\times 10^3 \times \log_2\left(1 + \frac{13.334}{\frac{25\times 10^3}{50\times 10^3}}\right) = 25\times 10^3 \times \log_2(1 + 26.668)$$

$\Rightarrow C = \boxed{119.75\times 10^3}$

17. Solution

(a) $T_{cs} = 10\mu s \Rightarrow B = \frac{1}{2T_{cs}} = \frac{1}{2\times 10\mu s} = \frac{10^5}{2} = 50kHz$

(b) $r_{cs} = \frac{1}{T_{cs}} = 10^5$ symbols/sec (=bit rate r_b)

(c) $N_0 = 2\times 0.5\times 10^{-6} \Rightarrow$EUE$= \frac{E_b}{N_0} = \frac{E_b}{10^{-6}}$

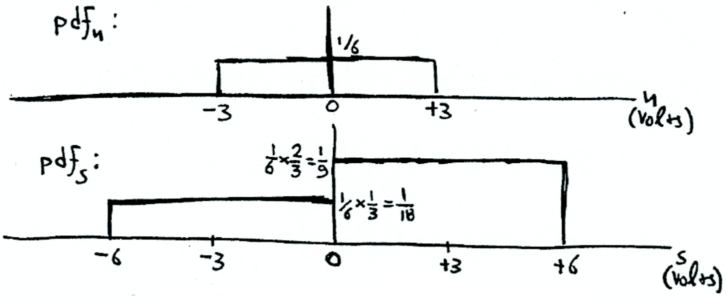
However,

$$\begin{aligned}
E_b &= \int_{-5\mu}^{5\mu} 9\left(\Lambda\left\{\frac{t}{5\mu}\right\}+\text{rect}\left\{\frac{t}{10\mu}\right\}\right)^2 dt \\
&= 2\times 9\int_{0}^{5\mu}\left(\Lambda\left\{\frac{t}{5\mu}\right\}+\text{rect}\left\{\frac{t}{10\mu}\right\}\right)^2 dt \\
&\quad \text{note that } \Lambda\left\{\frac{t}{5\mu}\right\}|_0^{5\mu} \text{ is equal to } \frac{-t+5\mu}{5\mu} \\
&\quad \text{and } \Lambda\left\{\frac{t}{5\mu}\right\}+\text{rect}\left\{\frac{t}{10\mu}\right\} = 1+\frac{-t+5\mu}{5\mu} = 2-2\times 10^5 t \\
&\quad \text{and } \left(\Lambda\left\{\frac{2t}{T_{cs}}\right\}+\text{rect}\left\{\frac{t}{T_{cs}}\right\}\right)^2 = 4+4\times 10^{10}t^2-8\times 10^5 t \\
&= 2\times 9\int_{0}^{5\times 10^{-6}}\left(4+4\times 10^{10}t^2-8\times 10^5 t\right)dt \\
&= 18\left(\int_0^{5\mu}4dt+\int_0^{5\mu}4\times 10^{10}t^2dt-\int_0^{5\mu}8\times 10^5 t dt\right) \\
&= 18\times\left(20+\frac{4}{3}\times 10^{-2}\times 125-4\times 10^{-1}\times 25\right)\times 10^{-6} \\
&= 210\times 10^{-6}
\end{aligned}$$

EUE= $\frac{210\times 10^{-6}}{10^{-6}} = 210$

(d) EUE=210; BUE= $\frac{B}{r_b} = \frac{50\times 10^3}{10^5} = 0.5$
$C = 50\times 10^3\log_2(1+\frac{210}{0.5}) = 435.88\ kbits/s$

18. Solution



$$P_s = \mathcal{E}\left\{s(t)^2\right\} = \int_{-\infty}^{+\infty}s^2\text{pdf}_s(s)ds = \int_{-6}^{0}s^2\tfrac{1}{18}ds+\int_0^6 s^2\tfrac{1}{9}ds = \tfrac{1}{18}\tfrac{s^3}{3}\Big|_{-6}^{0}+\tfrac{1}{9}\tfrac{s^3}{3}\Big|_0^6 = 4+8 = 12W$$

Since noise≠white Gaussian⇒ $B\log_2\left(\frac{P_s+N_n}{N_n}\right)\le C\le B\log_2\left(\frac{P_s+P_n}{N_n}\right)$

However, $P_n = \int_{-3}^{+3}n^2\frac{1}{6}dn = \frac{1}{6}\frac{n^3}{3}\Big|_{-3}^{+3} = \frac{1}{18}(27+27) = 3W$

N_n =entropy power=$\frac{1}{2\pi e}2^{2H}$

where $H = -\int_{-\infty}^{+\infty} \text{pdf}_n(n) \log_2 (\text{pdf}_n(n))\, dn = -\frac{1}{6}\log_2 \frac{1}{6}\int_{-3}^{+3} dn = -\log_2 \frac{1}{6} = \log_2 6 = 2.5850$

Thus, $N_n = \frac{1}{2\pi e} 2^{2\times 2.5850} = \frac{36.002}{2\pi e} = 2.1079$

$\log_2\left(\frac{12+2.1079}{2.1079}\right) \leq \frac{C}{B} \leq \log_2\left(\frac{12+3}{2.1079}\right)$

$2.7426 \leq \frac{C}{B} \leq 2.8311$

19. Solution

(a)

$$\begin{aligned}
P_n &= \mathcal{E}\{n(t)^2\} = \int_{-\infty}^{\infty} n.\text{pdf}_n(n).dn \\
&= \int_{-3}^{3} n.\frac{1}{6}.dn = 3 \\
N_n &= (\text{entropy power}) = \frac{1}{2\pi e} 2^{2H} \\
\text{where } H &= -\int_{-\infty}^{\infty} \text{pdf}_n(n) \log_2 (\text{pdf}_n(n))\,.dn \\
&= -\frac{1}{6}\log_2\left(\frac{1}{6}\right)\int_{-3}^{3} dn \\
&= -\log_2\left(\frac{1}{6}\right) = \log_2(6) = 2.585
\end{aligned}$$

$$\begin{aligned}
\therefore N_n = (\text{entropy power}) &= \frac{1}{2\pi e} 2^{2H} \\
&= \frac{36}{2\pi e} = 2.1078
\end{aligned}$$

(b)

$$\begin{aligned}
B\log_2\left(\frac{P_s+N_n}{N_n}\right) &\leq C \leq B\log_2\left(\frac{P_s+P_n}{N_n}\right) \ \frac{\text{bits}}{\text{sec}} \\
\log_2\left(\frac{P_s+N_n}{N_n}\right) &\leq \frac{C}{B} \leq \log_2\left(\frac{P_s+P_n}{N_n}\right) \\
\log_2\left(\frac{12+2.1078}{2.1078}\right) &\leq \frac{C}{B} \leq \log_2\left(\frac{12+3}{2.1078}\right) \\
2.7427 &\leq \frac{C}{B} \leq 2.8312
\end{aligned}$$

20. Solution

(a) $p_e = \underbrace{\Pr(y_2|x_1).\Pr(x_1)}_{\Pr(y_2,x_1)} + \underbrace{\Pr(y_1|x_2).\Pr(x_2)}_{\Pr(y_1,x_2)} = 0.1\times 0.25 + 0.2\times 0.75 = 0.175$

(b) $\mathbb{F} = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}$

$\underline{p} = \begin{bmatrix} 0.25 & 0.75 \end{bmatrix}^T$

$\mathbb{J} = \mathbb{F}.\text{diag}(p) = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix} diag\left(\begin{bmatrix} 0.25 \\ 0.25 \end{bmatrix}\right) = \begin{bmatrix} 0.225 & 0.15 \\ 0.025 & 0.6 \end{bmatrix}$

$\underline{q} = \mathbb{F}\underline{p} = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}\begin{bmatrix} 0.25 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 0.375 \\ 0.625 \end{bmatrix}$

$$H_R = \underline{q}^T . \log_2 \left(\underline{q}\right) = 0.9544$$
$$H_{R|M} = -1_2^T \ (\mathbb{J} \odot \log_2(\mathbb{F}))\, 1_2 = 0.658695$$
$$H_{mut} = H_R - H_{R|M} = 0.9544 - 0658695 = 0.295705 \quad \text{bits/symbol}$$

21. Solution

(a)

$$\begin{aligned} p_e &= \Pr(y_1, x_2) + \Pr(y_2, x_1) \\ &= \Pr(y_1|x_2). \Pr(x_2) + \Pr(y_2|x_1). \Pr(x_1) \\ &= 0.04 \times \frac{2}{3} + 0.018 \times \frac{1}{3} = 0.032667 \end{aligned}$$

(b)

$$\mathbb{J} = \begin{bmatrix} J_{11} = \Pr(x_1, y_1) = 0.3267 & J_{12} = \Pr(x_1, y_2) = 0.006 \\ J_{21} = \Pr(x_2, y_1) = 0.02667 & J_{22} = \Pr(x_2, y_2) = 0.64 \end{bmatrix}^T$$

$$\begin{aligned} \underline{p} &= \begin{bmatrix} p_1 = \Pr(x_1) = \frac{1}{3} \\ p_2 = \Pr(x_2) = \frac{2}{3} \end{bmatrix}, \mathbb{F} = \begin{bmatrix} F_{11} = 0.982 & F_{12} = 0.04 \\ F_{21} = 0.018 & F_{22} = 0.96 \end{bmatrix} \\ \underline{q} &= \mathbb{F}\underline{p} = \begin{bmatrix} q_1 = \Pr(y_1) = 0.982 \times \frac{1}{3} + 0.04 \times \frac{2}{3} = 0.354 \\ q_2 = \Pr(y_2) = 0.018 \times \frac{1}{3} + 0.96 \times \frac{2}{3} = 0.646 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} H_{mut} \triangleq H_{mut}(\underline{p}, \mathbb{F}) &= \text{(using any of the following expression)} \\ &= -\sum_{m=1}^{M} \sum_{k=1}^{K} F_{km}.p_m \log_2 \left(\frac{q_k}{F_{km}} \right) \\ &= -\sum_{m=1}^{M} \sum_{k=1}^{K} J_{km} \log_2 \left(\frac{p_m.q_k}{J_{km}} \right) \\ &= -\underline{1}_K^T \left(\underbrace{\mathbb{J} \odot \log_2 \left[\left(\overbrace{\mathbb{F}.\underline{p}}^{=\underline{q}} .\underline{p}^T \right) \oslash \mathbb{J} \right]}_{K \times M \quad matrix} \right) \underline{1}_M \ \frac{\text{bits}}{\text{symbol}} \\ &= 0.7327 \end{aligned}$$

22. Solution

a>
$F_g = 4\times10^3\,Hz$

$F_s = 2\times F_g = 8\times10^3\,Hz$

$\bar{\ell}_3 = 1\times\frac{27}{64} + \left(3\times\frac{9}{64}\right)\times3 + \left(5\times\frac{3}{64}\right)\times3 + 5\times\frac{1}{64} = 2.4685\ \frac{bits}{trip}$

$= 2.4685\ \frac{bits}{triple\text{-}level}$

Alphabet: $\underline{x} = \begin{pmatrix} x_1=1 \\ x_2=0 \end{pmatrix}$ $\boxed{M=2 \text{ channel symbols}}$

Probabilities: $\underline{p} = \begin{bmatrix} p_1 = Pr(H_1) = 0.6344 \\ p_2 = Pr(H_0) = 0.3656 \end{bmatrix}$ ← to be proven

$H_x = -\sum_{m=1}^{2} p_m \log_2(p_m) = -\underline{p}^T.\log_2(\underline{p}) = 0.9473\ \frac{bits}{symbol}$

$r_{inf} = H_x \cdot r_{cs}$

symbol rate $= r_b = F_s\frac{1}{3}\bar{\ell}_3 = 6583.3\ \frac{symbols}{sec}$

i.e. $r_{inf} = 0.9473\times6583.3 = 6236.4\ \frac{bits}{symbol}$

$r_d = \ell \cdot r_{cs} = 6583.3\ \frac{bits}{sec}$

ℓ: 1 bit; r_{cs}: 6583.3

b> $P_e = 0.6344\times0.05 + 0.3656\times0.2 = 0.1048$

c> $1 = x_1 \longmapsto A_1 \cdot \Lambda\left(\frac{t}{0.5T_{cs}}\right)$ of Energy $= E_1 = ?$ ($A_1 = \sqrt{\frac{3}{8}}$)

$0 = x_2 \longmapsto 0$ Volts i.e. Energy $= E_2 = 0$

$E_1 = 2\int_0^{T_{cs}/2} A_1^2 \Lambda^2\left(\frac{t}{0.5T_{cs}}\right)dt$ (where $A_1 = \sqrt{\frac{3}{8}}$)

$= 2\int_0^{0.5T_{cs}} A_1^2\left(\frac{-t+0.5T_{cs}}{0.5T_{cs}}\right)^2 dt$

$= \ldots = \frac{1}{3}A_1^2 T_{cs} = \frac{1}{3}\frac{3}{8}T_{cs} = \frac{1}{8}T_{cs}$

$$E_b = E_1 \cdot P_1 + E_2 P_2 = \frac{1}{8} T_{cs} P_1 + 0 = 1.2046 \times 10^{-5}$$

$$EUE = \frac{E_b}{\underbrace{N_0}_{2\times 10^{-3}}} = 6.0228 \times 10^{-3} \quad \text{(data EUE)}$$

$$BUE = \frac{B}{r_b} = \frac{B}{r_{cs}} = \frac{B}{2B} = \frac{1}{2}$$

$$\text{Note: } B = \frac{r_{cs}}{2} \Rightarrow r_{cs} = 2B = r_b$$

$$\text{data point} = (EUE, BUE) = \left(6.0228\times 10^{-3}, \frac{1}{2}\right)$$

$$CS = \text{inf. point} = (EUE_{inf}, BUE_{inf}) = (\text{data point}) \times \frac{\log_2 M}{H_{mut}}$$

therefore the mutual entropy H_{mut} of the channel should be estimated

$$\text{ie} \quad H_{mut} = H_Y - H_{Y|X} \quad (\text{or } H_{mut} = H_X - H_{X|Y})$$

as follows:

$$* \quad \underline{p} = \begin{bmatrix} 0.6344 \\ 0.3656 \end{bmatrix}; \quad \mathbb{F} = \begin{bmatrix} 0.95, & 0.2 \\ 0.05, & 0.8 \end{bmatrix}; \quad \underline{q} = \mathbb{F} \cdot \underline{p} = \begin{bmatrix} 0.6758 \\ 0.3242 \end{bmatrix}$$

$$\mathbb{J} = \mathbb{F} \cdot \text{diag}(\underline{p}) = \begin{bmatrix} 0.6027, & 0.0731 \\ 0.0317, & 0.2925 \end{bmatrix}$$

$$* \quad H_Y = -\underline{q}^T \cdot \log_2(\underline{q}) = 0.9089 \ \frac{\text{bits}}{\text{symbol}}$$

$$* \quad H_{Y|X} = -\sum_{m=1}^{2}\sum_{k=1}^{2} J_{km} \log_2\left(\frac{J_{km}}{p_m}\right) = -\left\| \mathbb{J} \odot \log_2(\mathbb{F}) \right\|_{1*}$$

$$= 0.4456 \ \frac{\text{bits}}{\text{symbol}}$$

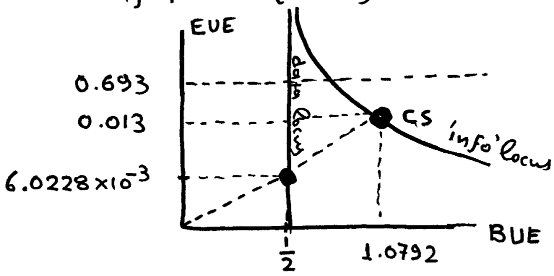
$$* \quad H_{mut} = H_Y - H_{Y|X} = 0.4633 \ \frac{\text{bits}}{\text{symbol}}$$

Note: a different approach is to use the following expression

$$H_{mut} = -\left\| \mathbb{J} \odot \log_2\left(\oslash \frac{\mathbb{F}\cdot \underline{p} \cdot \underline{p}^T}{\mathbb{J}} \right) \right\|_{1*} \ \frac{\text{bits}}{\text{symbol}} = 0.4633$$

where $\|\text{matrix}\|_{1*}$ = sum of the elements of the matrix-argument.

d> $\therefore$ CS=inf. point= (0.013, 1.0792)



e> $\therefore$ CS = inf · point = (0.013, 1.0792) $\Rightarrow$

CS is not realizable (since $EUE_{inf} = 0.013 < \underbrace{0.693}_{\text{theoretical limit}}$)

f> $SNR_{in} = \frac{EUE_{inf}}{BUE_{inf}} = \frac{EUE_d}{BUE_d} = 0.012 \Rightarrow SNR_{in} = -19.2082\,dB$

23. Solution

a) $F_g = 4 \times 10^3$ Hz; $F_s = 2 \times F_g = 8 \times 10^3$ Hz; $Q = 2$

From Figures 1 and 2 we get:
$\Pr(-2\text{V}) = 3/4$
$\Pr(+2\text{V}) = 1/4$

$N_0 = 2 \times 10^{-3}$

symbols	probabilities	Huffman	l_i (bits)
$x_1x_1x_1$	$27/64$	1	1
$x_1x_1x_2$	$9/64$	001	3
$x_1x_2x_1$	$9/64$	010	3
$x_2x_1x_1$	$9/64$	011	3
$x_1x_2x_2$	$3/64$	00000	5
$x_2x_1x_2$	$3/64$	00001	5
$x_2x_2x_1$	$3/64$	00010	5
$x_2x_2x_2$	$1/64$	00011	5

$$\bar{l} = 1 \times 27/64 + 3 \times 9/64 + 3 \times 9/64 + 3 \times 9/64 + 5 \times 3/64 + 5 \times 3/64 + 5 \times 3/64 + 5 \times 1/64 = 2.46875 \text{ bits/}\underbrace{\text{triple-level}}_{\text{(or 3-samples)}}$$

Alphabet: $\underline{X} = \begin{Bmatrix} x_1 = 1 \\ x_2 = 0 \end{Bmatrix}$ (since $\Pr(x_1) > \Pr(x_2)$)

with probabilities: $\underline{p} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} \Pr(x_1) \\ \Pr(x_2) \end{bmatrix} = \begin{bmatrix} 0.6344 \\ 0.3656 \end{bmatrix}$

Note:

$$\Pr(x_2) = \frac{2}{3} \times \frac{9}{64} + \frac{2}{3} \times \frac{9}{64} + \frac{1}{3} \times \frac{9}{64} + \frac{5}{5} \times \frac{3}{64} + \frac{4}{5} \times \frac{3}{64} + \frac{4}{5} \times \frac{3}{64} + \frac{3}{5} \times \frac{1}{64} = 0.3656$$

$$\begin{aligned} p_e &= \Pr(y_2, x_1) + \Pr(y_1, x_2) \\ &= \Pr(y_2|x_1)\Pr(x_1) + \Pr(y_1|x_2)\Pr(x_2) \\ &= 0.05 \times 0.6344 + 0.2 \times 0.3656 \\ &= 0.1048 \end{aligned}$$

b) $\mathrm{H}_x = -\sum_{m=1}^{2} p_m.\log_2 p_m = -\underline{p}^T.\log_2 \underline{p} = 0.9473 \frac{\text{bits}}{\text{symbol}}$

data rate:

$r_{\text{data}} = r_b = F_s \frac{1}{3} \bar{l} = 6583.3$ bits/sec

information rate:

$r_{\text{inf}} = r_b \times \mathrm{H}_x = r_b \times 0.9473 = 6236.4$ bits/sec

c) $M = 2$ i.e. binary CS

Therefore: $T_{cs} = \frac{1}{r_{cs}} = 1.5190 \times 10^{-4}$ sec

$E_b = \frac{0.5^2}{2} T_{cs} \times \Pr(x_1) = 1.2046 \times 10^{-5}$

$\text{EUE} = \frac{E_b}{N_0} = 6.0228 \times 10^{-3}$ (data EUE)

$\text{BUE} = \frac{B}{r_{cs}} = \frac{B}{2B \times \log_2(M)} = \frac{1}{2}$ (data BUE with B denoting the baseband bandwidth)

data point $= (\text{EUE}, \text{BUE}) = (6.0228 \times 10^{-3}, \frac{1}{2})$

d) CS = inf.point $= (\text{EUE}_{inf}, \text{BUE}_{inf}) =$ (data point) $\times \frac{\log_2(M)}{\mathbf{H}_{\text{mut}}}$

Therefore we have to estimate the mutual information $\mathbf{H}_{\text{mut}}$

$\mathbf{H}_{\text{mut}} = \mathbf{H}_Y - \mathbf{H}_{Y|X}$ or $\left(\mathbf{H}_{\text{mut}} = \mathbf{H}_X - \mathbf{H}_{X|Y}\right)$

i.e.

$$\underline{p} = \begin{bmatrix} 0.6344 \\ 0.3656 \end{bmatrix} \qquad \mathbb{F} = \begin{bmatrix} 0.95, & 0.2 \\ 0.05, & 0.8 \end{bmatrix} \quad \underline{q} = \mathbb{F}.\underline{p} = \begin{bmatrix} 0.6758 \\ 0.3242 \end{bmatrix}$$

$$\mathbb{B} = \text{diag}(\underline{q})^{-1}.\mathbb{F}.\text{diag}(\underline{p}) = \begin{bmatrix} 0.8918, & 0.1082 \\ 0.0978, & 0.9022 \end{bmatrix}$$

$$\mathbb{J} = \mathbb{F}.\text{diag}(\underline{p}) = \text{diag}(\underline{q}).\mathbb{B} = \begin{bmatrix} 0.6027, & 0.0731 \\ 0.0317 & 0.2925 \end{bmatrix}$$

$$\mathbf{H}_{\boldsymbol{X}} = -\sum_{m=1}^{2} p_m.\log_2\left(p_m\right) = -\underline{p}^T \log_2\left(\underline{p}\right) = 0.9473 \ \frac{\text{bits}}{\text{symbol}}$$

$$\mathbf{H}_{\boldsymbol{Y}} = -\sum_{k=1}^{2} p_k.\log_2\left(p_k\right) = -\underline{q}^T \log_2\left(\underline{q}\right) = 0.9089 \ \frac{\text{bits}}{\text{symbol}}$$

$$\mathbf{H}_{X\times\boldsymbol{Y}} = -\sum_{m=1}^{2}\sum_{k=1}^{2} J_{km}.\log_2\left(J_{km}\right) = -\left\|\mathbb{J} \odot \log_2\left(\mathbb{J}\right)\right\|_{1*} = 1.3929 \ \frac{\text{bits}}{\text{symbol}}$$

$$\mathbf{H}_{\boldsymbol{Y}|\boldsymbol{X}} = \mathbf{H}_{\boldsymbol{Y}|\boldsymbol{X}}(\mathbb{J}) \equiv -\sum_{m=1}^{2}\sum_{k=1}^{2} J_{km}.\log_2\left(\frac{J_{km}}{p_m}\right)$$

$$= -\left\|\mathbb{J} \odot \log_2\Big(\underbrace{\mathbb{J}.\text{diag}(\underline{p})^{-1}}_{\mathbb{F}}\Big)\right\|_{1*} = 0.4456 \ \frac{\text{bits}}{\text{symbol}}$$

$$\Rightarrow \boxed{\mathbf{H}_{\text{mut}} = \mathbf{H}_Y - \mathbf{H}_{Y|X} = 0.4633}$$

Therefore $\boxed{\text{CS} = \text{inf.point} = (0.013, 1.0792)}$

e)

$\Rightarrow$ $\boxed{\text{CS is not realizable}}$ (since $\text{EUE}_{inf} = 0.013 < 0.693$)

f)

$\text{SNR}_{\text{in}} = \frac{\text{EUE}}{\text{BUE}} = 0.012 \Rightarrow \text{SNR}_{\text{in}} = -19.2082\text{dB}$

4 Topic:Wireless Channels

24. Solution

$T_{spread} = \frac{30}{3\times10^8} = 10^{-7}\sec = T_{c,\max}$

In this case $L = \left\lfloor \frac{T_{spread}}{T_c} \right\rfloor + 1 = 1 + 1 = 2$

chip rate$=\frac{1}{T_c} = \frac{1}{10^{-7}} = 10Mchips/\sec$

25. Solution

(a) $Tc = 61ns;$ $\quad Bcoh = 3MHz$ $\quad T_c = 61ns < T_{spread} = \frac{1}{B_{coh}} = \frac{1}{3\times10^6} = 333ns$ and $T_c = 61ns < T_{coh} = \frac{1}{B_D} = \frac{1}{8\times10^6} = 125ns$

(b) $T_c = 61ns;$ $\quad Bcoh = 100MHz$ $\quad T_c = 61ns > T_{spread} = \frac{1}{B_{coh}} = \frac{1}{100\times10^6} = 10ns$ and $T_c = 61ns < T_{coh} = \frac{1}{B_D} = \frac{1}{8\times10^6} = 125ns$

(c) $Tc = 244ns \Rightarrow Bcoh = 3MHz$ $\quad T_c = 244ns < T_{spread} = \frac{1}{B_{coh}} = \frac{1}{3\times10^6} = 333ns$ and $T_c = 244ns > T_{coh} = \frac{1}{B_D} = \frac{1}{8\times10^6} = 125ns$

(d) $Tc = 244ns \Rightarrow Bcoh = 100MHz$ $\quad T_c = 244ns > T_{spread} = \frac{1}{B_{coh}} = \frac{1}{100\times10^6} = 10ns$ and $T_c = 244ns > T_{coh} = \frac{1}{B_D} = \frac{1}{8\times10^6} = 125ns$

(e) None of the above.

Thus, the answer is (c)

26. Solution

$T_{spread} = \frac{30}{3\times10^8} = 10^{-7}\sec = T_{c,\max}$

In this case $L = \left\lfloor \frac{T_{spread}}{T_c} \right\rfloor + 1 = 1 + 1 = 2$

chip rate$=\frac{1}{T_c} = \frac{1}{10^{-7}} = 10Mchips/\sec$

That is, the answer is (a)

5 Topic: Digital Modulators & Line Codes

27. Solution

    (a) $d_{ij}^2 = E_{s_i} + E_{sj} - 2\rho_{ij}\sqrt{E_{s_i}E_{sj}}$ with $E_{s_i} = E_{sj} = E$
    $\rho_{ij} = -1$
    $10^2 = 2E + 2E = 4E \Rightarrow E = 25\backslash$
    That is, the correct answer is (a)

28. Solution
    $A = 3mV$
    $T_{cs} = 1ms$

    single pulse $= \alpha.\text{rect}\frac{t}{T_{cs}}$ with α = random $\begin{cases} \alpha = +A & \text{with prob. } 0.5 \\ \alpha = -A & \text{with prob } 0.5 \end{cases}$

    $$\text{PSD}(f) = \frac{1}{T_{cs}}\mathcal{E}\left\{|\text{FT}(\text{single pulse})|^2\right\} =$$
    $$= \frac{1}{T_{cs}}\mathcal{E}\left\{\left|\text{FT}\left(\alpha\,\text{rect}\frac{t}{T_{cs}}\right)\right|^2\right\} =$$
    $$= \frac{1}{T_{cs}}\mathcal{E}\left\{|\alpha T_{cs}\,\text{sinc}(fT_{cs})|^2\right\} =$$
    $$= \frac{1}{T_{cs}}T_{cs}^2\,\text{sinc}^2(fT_{cs}).\underbrace{\mathcal{E}\{\alpha^2\}}_{(-A)^2\times 0.5 + (+A)^2\times 0.5 = A^2}$$
    $$= T_{cs}A^2\,\text{sinc}^2(fT_{cs})$$

    $= 9 \times 10^{-9}\,\text{sinc}^2\left(f10^{-3}\right)$

29. Solution

    $\frac{N_o}{2} = 0.5\times 10^{-6} \Rightarrow N_o = 10^{-6}$
    $r_b = 220$ kbits/sec
    $10^{-5} = \mathrm{T}\left(\sqrt{2\frac{E_b}{N_o}}\right) \Rightarrow \underbrace{\sqrt{2\frac{E_b}{N_o}}}_{\text{from 'Tau' graph}} = 4.2$
    $P_s = E_b.r_b \Rightarrow E_b = \frac{P_s}{r_b}$
    $\Rightarrow \sqrt{2\frac{P_s}{r_b N_o}} = 4.2$
    $\Rightarrow P_s \simeq 2W$
    (1.9869)

30. Solution

    (a) EUE$=\frac{Eb}{N_0} = \frac{\frac{3^2}{2}4\times 10^{-9}}{10^{-9}} = 18$

    (b) Initially you have to prove that $p_e = \mathrm{T}\left\{\sqrt{2\mathrm{EUE}\sin^2(30^0)}\right\}$

    $p_e = \mathrm{T}\left\{\sqrt{2\times 18\sin^2(30^0)}\right\} = \mathrm{T}\left\{\sqrt{2\times 18\times\frac{1}{4}}\right\} = \mathrm{T}\{3\} \simeq 10^{-3}$

31. Solution

    The correct answer is (c)

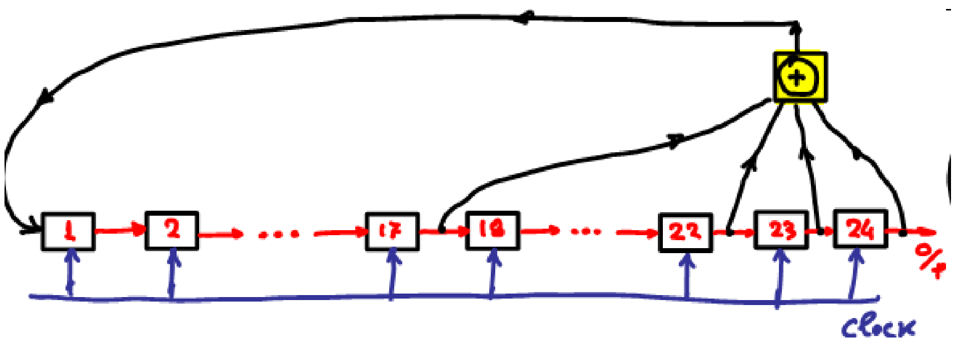
6 Topic: SSS and PN-Codes

32. Solution

    (a) $N_c = 31 \Rightarrow N_c = 2^m - 1$
    $\Rightarrow m = \log_2(N_c + 1)$
    $\Rightarrow m = \log_2(31 + 1) = \boxed{5}$

    (b) $T_c = 10^{-8}$
    $\Delta f = \frac{1}{N_c T_c} = \frac{1}{31 \times 10^{-8}} = \boxed{3.2258 \times 10^6}$

33. Solution



$N = 2^m - 1$

$m = 24$

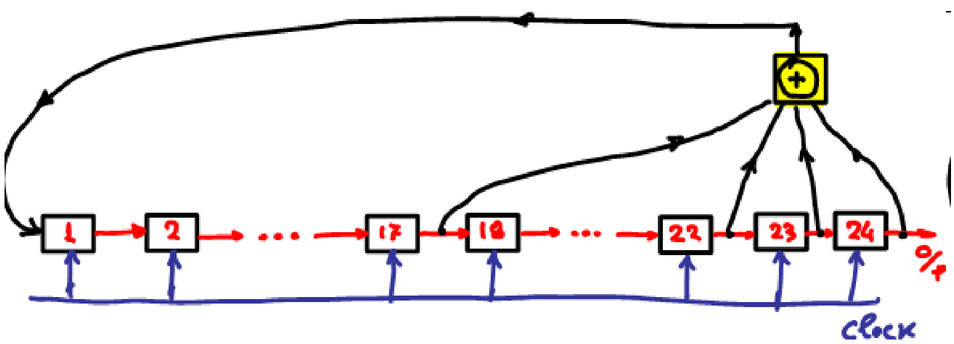
Thus $N = 2^{24} - 1 = 16777215$

$T_c = \frac{1}{2.7} \frac{s}{chip} = 0.37037 \frac{s}{chip}$

$NT_c = \left(2^{24} - 1\right) \times 0.37037 = 6.21377712 \times 10^6 s$

$= \frac{6.21377712 \times 10^6}{60} \min = 1.0356 \times 10^5 \min$

34. Solution



$$\left.\begin{array}{r} N = 2^m - 1 \\ m = 24 \end{array}\right\} \Rightarrow N = 2^{24} - 1 == 16.777 \times 10^6 (16777215)$$

$T_c = \frac{1}{10^6} s/bit = 10^{-6} s/bit$

$NT_c = (2^{24} - 1) 10^{-6}/60 = 0.27962 \text{minutes}$

35. Solution

    1 1 1 1

    0 1 1 1

    0 0 1 1

    0 0 0 1

    1 0 0 0

    0 1 0 0

    0 0 1 0

    1 0 0 1

    1 1 0 0

    0 1 1 0

    1 0 1 1

    0 1 0 1

    1 0 1 0

    1 1 0 1

    1 1 1 0

    1 1 1 1

    o/p = 1 1 1 1 0 0 0 1 0 0 1 1 0 1 0

7 Topic: Direct Sequence and Frequency Hopping

36. Solution

    $r_b = 9.6$ kbits$\Rightarrow T_{cs} = \frac{1}{9.6\times10^3}$

    PG $= N = \frac{T_{cs}}{T_c}$

    $B_{ss} \leq 25$ MHz$\Rightarrow \frac{1}{T_c} \leq 25\times10^6 \Rightarrow T_c \geq \frac{1}{25\times10^6}$

    $\Rightarrow \frac{T_{cs}}{N} \geq \frac{1}{25\times10^6} \Rightarrow N \leq 25\times10^6 T_{cs}$

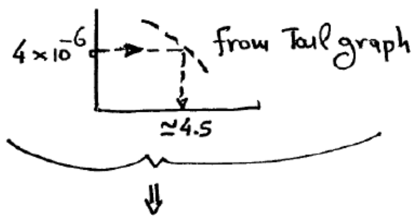
    $\Rightarrow N \leq \frac{25\times10^6}{9.6\times10^3} = 2604.2$

    However, $N = 2^m - 1 \Rightarrow N = 2^{11} - 1 = 2047 \leq 2604.2$

    That is, the correct answer is (d)

37. Solution

    (a) $r_{cs} = 8\times10^3 bits/\sec$
    $\frac{N_0}{2} = 10^{-12} \Rightarrow N_0 = 2\times10^{-12}$
    $P_j = 1W$;PG$=10^5$; $p_e = 4\times10^{-6}$; $p_{e,PR} = 4\times10^{-2}$
    PG$=\frac{T_{cs}}{T_c} \Rightarrow B_{ss} = \frac{1}{T_c} =PG\times \underbrace{r_{cs}}_{=\frac{1}{T_{cs}}} = 10^5\times8\times10^3 = 800\times10^6 Hz$
    Baseline Performance: $B_j = B_{ss}$
    $\Rightarrow p_e = \mathrm{T}\left\{\sqrt{(1-\rho)\mathrm{EUE}_{equ}}\right\}; \rho = -1$
    $\Rightarrow 4\times10^{-6} = \mathrm{T}\left\{\sqrt{2\mathrm{EUE}_{equ}}\right\}$

    $\Rightarrow \sqrt{2\mathrm{EUE}_{equ}} = 4.5 \Rightarrow \mathrm{EUE}_{equ} = \frac{4.5^2}{2} = 10.125$
    $\Rightarrow \frac{E_b}{N_0+\frac{P_j}{B_{ss}}} = 10.125 \Rightarrow E_b = 10.125\times\left(N_0 + \frac{P_j}{B_{ss}}\right) = 10.125\times\left(2\times10^{-12} + \frac{1}{800\times10^6}\right) =$
    12.677×10^{-9}
    $Ps = \frac{E_b}{T_{cs}} = E_b r_{cs} = 12.677\times10^{-9}\times8\times10^3 = 101.42\times10^{-6}$
    $A = \sqrt{2Ps} = \sqrt{2\times101.42\times10^{-6}} \Rightarrow \boxed{A = 14.242mV}$

    (b) pulse jammer with $q = 0.4$ "on"
    $$p_e = \underbrace{(1-q)\mathrm{T}\left\{\sqrt{2\frac{E_b}{N_0}}\right\}}_{jammer="off"} + \underbrace{q\mathrm{T}\left\{\sqrt{2\frac{E_b}{N_0+\frac{P_J}{qB_{ss}}}}\right\}}_{jammer="on"}$$
    $p_e = 0 + 0.4\times\mathrm{T}\left\{\sqrt{2\times4.05}\right\} = 0.4\times\mathrm{T}\{2.8474\} = 0.4\times2.2\times10^{-3}$
    i.e. $\boxed{p_e = 8.8\times10^{-4}}$

    (c) $p_{e,PR} = q\,\mathrm{T}\left\{\sqrt{2q\mathrm{EUE}_{PR}}\right\} \Rightarrow 4\times10^{-2} = 0.4\,\mathrm{T}\left\{\sqrt{2\times0.4\times\mathrm{EUE}_{PR}}\right\}$
    $\Rightarrow 10^{-1} = \mathrm{T}\left\{\sqrt{0.8\times\mathrm{EUE}_{PR}}\right\}$
    $\Rightarrow \sqrt{0.8\times\mathrm{EUE}_{PR}} = 1.25 \Rightarrow \mathrm{EUE}_{PR} = \frac{1.25^2}{0.8} = 1.9531$
    AJM$= 10\log_{10}\mathrm{EUE}_{equ} - 10\log_{10}\mathrm{EUE}_{pr} = \boxed{7.1467}$
    (or AJM $= 10\log_{10}(10.125) - 10\log_{10}(0.7813) = 11.126$)

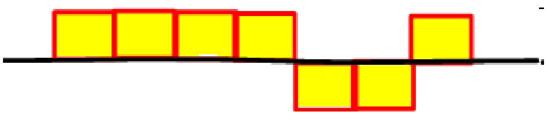
38. Solution

    (a) fully synchronised system $\Rightarrow$ $\boxed{\text{code noise} = \text{zero}}$

    (b) $F_s = 2 \times 2 \times 4k = 16kHz$ $\Rightarrow$bit-rate$= r_b = r_{cs} = 8 \times 16k = 128k$
    $\Rightarrow T_{cs} = \frac{1}{r_{cs}} = \frac{1}{128 \times 10^3} = 7.8125 \times 10^{-6}$
    $N_0 = 2 \times 0.5 \times 10^{-12} = 10^{-12}$
    $P_{n,out} = \frac{N_0}{T_{cs}} = \frac{10^{-12}}{7.8125 \times 10^{-6}} = \boxed{1.28 \times 10^{-7}}$

    (c) $T_c = \frac{1}{B_{ss}} = \frac{1}{32 \times 10^6} = 3.125 \times 10^{-8}\,\text{sec}$
    PG$= \frac{T_{cs}}{T_c} = \frac{7.8125 \times 10^{-6}}{3.125 \times 10^{-8}} = 250$
    $P_{j,out} = \frac{P_j}{\text{PG}} = \frac{1.6}{25} = \boxed{0.0064}$

39. Solution



(or, other valid codes - for various delays)

40. Solution

$r_b = 28kbits/\text{sec} \Rightarrow T_{cs} = \frac{1}{28 \times 10^3}$

Note: PG$=N = \frac{T_{cs}}{T_c}$

$B_{ss} \leqq 25MHz \Rightarrow \frac{1}{T_c} \leqq 25 \times 10^6 \Rightarrow T_c \geqq \frac{1}{25 \times 10^6} = 40 \times 10^{-9}\,\text{sec}$

$\Rightarrow \frac{T_{cs}}{N} \geqq 40 \times 10^{-9}\,\text{sec} \Rightarrow N \leqq 25 \times 10^6 T_{cs} \Rightarrow N \leqq \frac{25 \times 10^6}{28 \times 10^3} = 892$

However,

$$\boxed{N = 2^m - 1 \leqq 892} \tag{1}$$

This implies that $m = 9 \Rightarrow N = 2^9 - 1 = 511$ that satisfies Equation 1 given above.

41. Solution

$$\left.\begin{array}{r}\frac{1}{T_c}=10M\Rightarrow T_c=10^{-7}\,\mathrm{sec}\\ \frac{1}{NT_c}=39.2k\Rightarrow NT_c=25.5\mu\,\mathrm{sec}\end{array}\right\}\Rightarrow\quad N=255$$
$$N=2^m-1\Rightarrow\qquad m=\log_2(N+1)=8$$

42. Solution

$$\left.\begin{array}{l}\gamma=2\\ F_g=4\mathrm{KHz}\\ PG=10^8\\ \frac{N_0}{2}=10^{-12}\Rightarrow N_0=2\times10^{-12}\\ EUE=40\\ q=50\%=0.5\end{array}\right\}\Rightarrow P_e=3\times10^{-5}\Rightarrow$$

$$T\left(\sqrt{\frac{2E_b}{N_0+N_J}}\right)=3\times10^{-5}\Rightarrow$$

$$\sqrt{\frac{2E_b}{N_0+\underset{\uparrow\ \frac{P_J}{qB_{ss}}}{N_J}}}=4\Rightarrow\boxed{P_J=\underset{0.5}{q}\,\underset{?}{B_{ss}}\,\underset{2\times10^{-12}}{N_0}\left(\frac{2\overset{40}{EUE}}{16}-1\right)}\ (1)$$

However, $r_{cs}=\frac{1}{T_{cs}}=\underset{2}{\gamma}\,2\underset{4\mathrm{KHz}}{F_g}\Longrightarrow r_{cs}=16k\,\frac{\mathrm{bits}}{\mathrm{sec}}$

$$B_{ss}=\frac{1}{T_c}$$

$$PG=\frac{T_{cs}}{T_c}=\frac{B_{ss}}{r_{cs}}=10^8\Rightarrow\boxed{B_{ss}=10^8\times16k}\ (2)$$

$$(1)\wedge(2)\Rightarrow\boxed{P_J=6.4\,\mathrm{W}}$$

N.B.: $\mathrm{EUE}_{equ}=8;T_c=\frac{1}{16}\times10^{-11};T_{cs}=\frac{1}{16}\times10^{-3}$

43. Solution

$r_b=\frac{1}{Tcs}=\frac{1}{MT_c}=\frac{1}{100\times4\times10^{-6}}=2.5\mathrm{kbits}$

$$\text{bandwidth}=B_{ss}=c\times L\times F_1=\begin{pmatrix}c=8\\ L=2^{10}\\ F_1=250k\end{pmatrix}=8\times2^{10}\times250\times10^3=2.048\times10^9\mathrm{Hz}$$

$\frac{bandwidth}{r_b}=\frac{c\times L\times F_1}{r_b}=\frac{2.048\times10^9}{2.5\times10^3}=819200=59.13dB$

8 Topic: DS-CDMA

44. Solution

   (a) $F_s = 2 \times 4K = 8KHz$
   $\gamma = \log_2 Q = \log_2 128 = 7$
   $r_b = \gamma F_s = 7 \times 8K = 56Kbits/s$
   $T_{cs} = 2T_b = 2\frac{1}{r_b} = 2\frac{1}{56K} = 3.5714 \times 10^{-5}$
   PG$= 20dB \Rightarrow 10\log_{10}\frac{T_{cs}}{T_c} = 20 \Rightarrow 100 = \frac{T_{cs}}{T_c} \Rightarrow T_c = \frac{T_{cs}}{100} = 0.35714 \times 10^{-6}$
   i.e. chip rate$= r_c = \frac{1}{T_c} = \frac{1}{0.35714\times10^{-6}} = 2.8$Mchips/s

   (b) $N_c T_c = 5\text{hours} \Rightarrow N_c \geqq \frac{5\text{hours}}{T_c} = \frac{5\times3600}{T_c} = \frac{5\times3600}{0.35714\times10^{-6}} = 0.0504 \times 10^{12}$
   $\Rightarrow N_c = 2^m - 1 \Rightarrow 2^m = N_c + 1 \Rightarrow m = \log_2(N_c+1) = \log_2(0.0504\times10^{12}+1) = 35.553$
   i.e. $m = 36$

45. Solution

   $P = 10^{-2}$
   SNIR$_{out} = 14$
   $r_{cs} = 25 \Rightarrow T_{cs} = \frac{1}{25k}$
   PG=400$\Rightarrow$PG$= \frac{B_{ss}}{B} = \frac{T_{cs}}{T_c} \Rightarrow T_c = \frac{T_{cs}}{\text{PG}} = 10^{-7}$
   $B_{ss} = \frac{1}{T_c} = 10\text{MHz}$
   SNIR$_{out} = 2\text{EUE}_{equ} = 2\frac{E_b}{N_0+N_j} = 2\frac{PT_{cs}}{N_0+(K-1)\frac{P}{B_{ss}}}$
   $\Rightarrow N_0 + (K-1)\frac{P}{B_{ss}} = \frac{2PT_{cs}}{\text{SNIR}_{out}}$
   $\Rightarrow K = \left(\frac{2PT_{cs}}{\text{SNIR}_{out}} - N_0\right)\frac{B_{ss}}{P} + 1 \simeq 58$ users

46. Solution

   (a) $P = 10mW$
   $r_b = 500\,\text{kbits/ sec} \Rightarrow T_{cs} = \frac{1}{500}$ msec

   (b) $K = 201$ users
   $N_0 = 2 \times 10^{-9}$
   $p_e = 3 \times 10^{-5}$
   $a = 0.375$
   $s = 1/3$
   $E_b = PT_{cs} = 10 \times 10^{-3} \times \frac{1}{500} \times 10^{-3} = 2 \times 10^{-8}$
   $p_e = \text{T}\left\{\sqrt{2\,\text{EUE}_{equ}}\right\} \Rightarrow 3 \times 10^{-5} = \text{T}\left\{\sqrt{2\,\text{EUE}_{equ}}\right\}$
   $\Rightarrow$ (using "tail graph" supplied)
   $4 = \sqrt{2\,\text{EUE}_{equ}} \Rightarrow \text{EUE}_{equ} = 8$

   (c) However, EUE$_{equ}$$=\frac{E_b}{N_0+N_j}$
   where $E_b = PT_{cs}$ and $N_j = \frac{(K-1).P.a.s}{B_{ss}} = \frac{(K-1).P.a.s}{\text{PG}/T_{cs}}$
   Therefore, EUE$_{equ} = \frac{PT_{cs}}{N_0+\frac{(K-1).P.a.s}{\text{PG}/T_{cs}}}$ $\Rightarrow$ ....$\Rightarrow$PG$= \frac{(K-1).P.a.s.T_{cs}}{\frac{PT_{cs}}{\text{EUE}_{equ}}-N_0}$
   $\Rightarrow$ ... $\Rightarrow$PG$= 1000$

47. Solution

(a) $K = 256$

AJM=30dB $\log_{10}\text{EUE}_{equ} - 10\log_{10}\text{EUE}_{PR} = 30$

$\Rightarrow \frac{\text{EUE}_{equ}}{\text{EUE}_{PR}} = 10^3$ (1)

$p_{e,PR}$=10^{-2}

$m = 21 \Rightarrow N_c = 2^m - 1 \Rightarrow N_c = 2^{21} - 1 = 2.0972 \times 10^6$

$P = 0.1915$

$N_0 = 10^{-6}$

$p_{e,PR} = \text{T}\{\sqrt{2\ \text{EUE}_{PR}}\} \Rightarrow 10^{-2} = \text{T}\{\sqrt{2\ \text{EUE}_{PR}}\}$

using tail function graph we have

$\sqrt{2\ \text{EUE}_{PR}} = 2.3(\text{or } 2.3263) \Rightarrow \text{EUE}_{PR} = 2.645$ (or 2.7058) (2)

$(1) \bigwedge (2) \Rightarrow \text{EUE}_{equ} = 2645$ (or 2705.8)

$\text{EUE}_{equ} = \frac{E_b}{N_0 + N_j} = 2645$ (or 2705.8)

$\Rightarrow\ E_b = \text{EUE}_{equ}(N_0 + \frac{(K-1).E_b}{N_c})$

$E_b\left(1 - \frac{\text{EUE}_{equ}(K-1)}{N_c}\right) = \text{EUE}_{equ}\ N_0$

$E_b = \frac{\text{EUE}_{equ}\ N_0}{1 - \frac{\text{EUE}_{equ}(K-1)}{N_c}} = \frac{2705.8\times10^{-6}}{1 - \frac{2705.8\times(256-1)}{2.0972\times10^6}} = 4.032\,5 \times 10^{-3}$

(b) $T_{cs} = \frac{E_b}{P} = \frac{4.032\,5\times10^{-3}}{0.1915} = 21.05\,7 \times 10^{-3} = 21.057ms$

$N_c = \frac{T_{cs}}{T_c} \Rightarrow T_c = \frac{T_{cs}}{N_c} = \frac{21.056\times10^{-3}}{2.0972\times10^6} = 10.04 \times 10^{-9} = 10.04\text{ns}$

PN-code-rate $= \frac{1}{T_c} = \frac{1}{10.04\times10^{-9}} = 99.602 \times 10^6 = \boxed{99.602\text{Mchips/s}}$

48. Solution

$P = 5mW$

$r_b = 25kbits/sec \Rightarrow T_{cs} = 2T_b = 2\frac{1}{r_b} = 2\frac{1}{25\times10^3} = \frac{1}{12500} = 8 \times 10^{-5}$

$\frac{N_0}{2} = 10^{-9} \Rightarrow N_0 = 2 \times 10^{-9}$

PG= $N_c = 400 \Rightarrow 400 = \frac{T_{cs}}{T_c} \Rightarrow T_c = \frac{8\times10^{-5}}{400} = 2 \times 10^{-7}$

$p_e = 3 \times 10^{-5}$

$a = 0.375$

$s = 3$

$p_e = \text{T}\{\sqrt{2\text{EUE}_{equ}}\} \Rightarrow 3\times10^{-5} = \text{T}\{\sqrt{2\text{EUE}_{equ}}\} \overset{\text{(using the tail function graph)}}{\Rightarrow} 4 = \sqrt{\text{EUE}_{equ}}$

$\Rightarrow \text{EUE}_{equ} = \frac{16}{2} = 8 \Rightarrow$

$$\left.\begin{array}{c} \frac{E_b}{N_0+N_j} = 8 \\ E_b = \frac{PT_{cs}}{2} = \frac{5\times10^{-3}\times8\times10^{-5}}{2} = 2\times10^{-7} \\ N_j = \frac{(K-1)Pas}{B_{ss}} \end{array}\right\} \Rightarrow \frac{E_b}{N_0 + (K-1)PasT_c} = 8$$

$\Rightarrow K = (\frac{PT_{cs}/2}{8} - N_0).\frac{1}{PasT_c} + 1$

$\Rightarrow K = (\frac{5\times10^{-3}\times8\times10^{-5}}{2\times8} - 2\times10^{-9})\frac{1}{5\times10^{-3}\times0.375\times1/3\times2\times10^{-7}} + 1 = 185$

9 Topic: PCM & PSTN

49. Solution

    $Q = 2^\gamma \Rightarrow r_b = 2F_g\gamma = 4 \times 10^3 \times 2 \times \log_2(256) = 64k$

    That is, the correct answer is (d)

50. Solution

    point 'D' data sequence: 23V, 46V, 40V, 41V, 42V

51. Solution

    PCM using a 256-level uniform quantizer:

    $Q = 256 \Rightarrow 2^\gamma = 256 \Rightarrow \gamma = 8\frac{\text{bits}}{\text{level}}$

    CF=crest factor of the signal= $\frac{\text{peak}}{\text{rms}}$

    peak= $\widehat{g} = 2V$

    rms= $\sigma_g = \sqrt{P_g} \Rightarrow P_g = 2\int_0^2 g^2 \ \text{pdf}_{\text{g}}(g)dg = 2\int_0^2 g^2 \ \frac{1}{2}\Lambda\left(\frac{g}{2}\right) dg$

    $= 2\int_0^2 g^2 \ \frac{1}{2}\frac{2-g}{2}dg = \int_0^2 \left(g^2 \ - \frac{1}{2}g^3\right) dg$

    $= \left(\frac{g^3}{3} \ - \frac{1}{2}\frac{g^4}{4}\right)]_0^2 = \frac{2}{3}$

    i.e. rms= $\sqrt{\frac{2}{3}}$

    $\text{SNR}_q = 4.77 + 6 \times \gamma - \underbrace{20\log_{10}\frac{2}{\sqrt{\frac{2}{3}}}}_{20\log_{10}(\sqrt{6})=7.7815} = 4.77 + 6 \times 8 - 7.7815 = 44.989\text{dB}$

52. Solution

    $\text{SNR}_q \geqslant 50 \ dB$;CR= $\frac{\hat{V}}{\sigma} = 4.4668$

$$\begin{aligned}\text{SNR}_q &= 4.77 + 6\gamma - \underbrace{20\log_{10}\frac{\hat{V}}{\sigma}}_{=13dB} \text{ dB} \\ \text{SNR}_q &= 4.77 + 6\gamma - 13\end{aligned}$$

    i.e.

$$\begin{aligned}&\Rightarrow \quad 4.77 + 6\gamma - 13 \geqslant 50 \\ &\Rightarrow \quad \gamma \geqslant \frac{50 + 13 - 4.77}{6} \\ &\Rightarrow \quad \gamma \geqslant 10 \Rightarrow \log_2 Q \geqslant 10 \Rightarrow Q \geqslant 2^{10} = 1024\end{aligned}$$

    Therefore,

$$r_b = \gamma F_s = \gamma 2F_g = 10 \times 2 \times 18k = 360kbits/\sec$$

53. Solution

    $B=\frac{\text{channel symbol rate}}{2} = \frac{\text{bit rate}}{2} = \frac{\gamma F_s}{2} = \gamma F_g \Rightarrow B = \gamma F_g \Rightarrow \frac{B}{F_g} = \gamma \Rightarrow \beta = \gamma$

54. Solution

    (a) $\frac{\text{g}}{\text{g}_{max}} = \frac{2.4}{10} = 0.24$

    $\text{g}_c = \frac{\ln(1+100\times 0.24)}{\ln(1+100)} \times g_{\max} = \frac{3.218}{4.615} \times 10 = 6.974$

    $\Rightarrow b_{13} < g_c < b_{14} \Rightarrow g_c = 6.875V = m_{14}$

(b) $g_{q,out} = \frac{1}{\mu}\left(\exp\left(\frac{m_{14}}{\mathrm{g}_{maz}} \times \ln(1+\mu)\right) - 1\right) \times 10 = 2.287$

(c) $n_{\mathrm{q}} = 2.4 - 2.28 = 0.12\,V (or\ -0.12V)$

55. Solution

    The correct answer is (a)

56. Solution

    The correct answer is (d)

END