EE303: Communication Systems

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An Overview of Fundamentals: Digital Modulators and Line Codes

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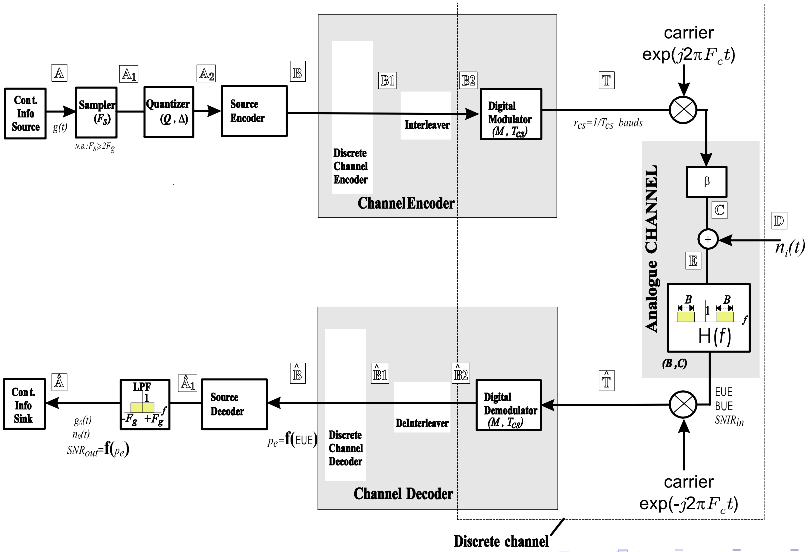
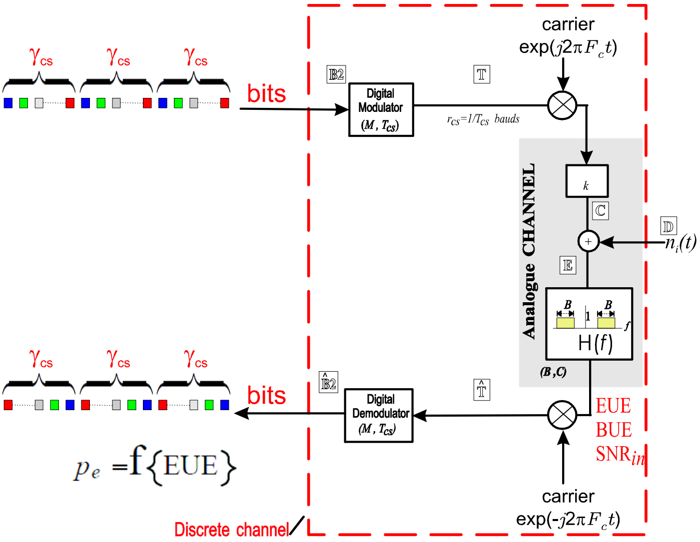
Introduction

General Block Structure of a Digital Communication System



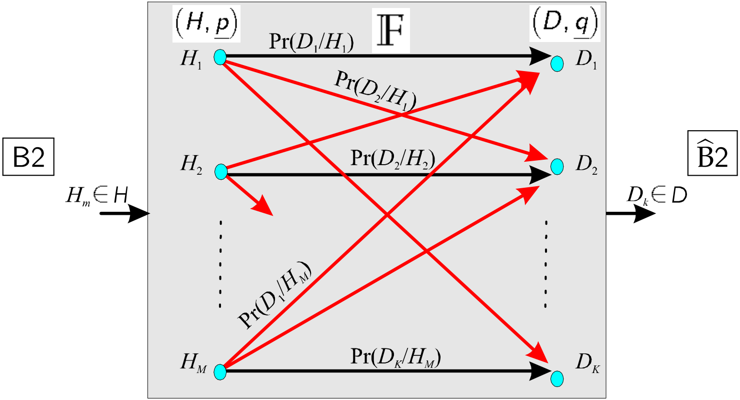
Let us focus on the "discrete channel"



With reference the previous figures:

- at point $\boxed{\text{T}}$: $s(t)$ **waveform**.
  - The digital modulator
    - ★ takes γ_{cs}-bits at a time at some uniform rate $r_{cs} = \frac{1}{T_{cs}}$ and
    - ★ transmits one of $M = 2^{\gamma_{cs}}$ distinct waveforms $s_1(t), \ldots, s_M(t)$ i.e. we have an M-ary communication system.
    - ★ If $\gamma_{cs} = 1$ we have one bit at a time $\begin{cases} 0 \longmapsto s_1 \\ 1 \longmapsto s_2 \end{cases}$ i.e. a binary comm. system
  - A new waveform (corresponding to a new γ_{cs}-bit-sequence) is transmitted every T_{cs} seconds
- at point $\boxed{\widehat{\text{T}}}$: **noisy waveform** $r(t) = \beta s(t) + n(t)$.
  - The transmitted waveform $s(t)$, affected by the channel, is received at point $\boxed{\widehat{\text{T}}}$
- at point $\boxed{\widehat{\text{B}}2}$: **a binary sequence.**
  - based on the received signal $r(t)$ the digital demodulator has to decide which of the M waveforms $s_i(t)$ has been transmitted in any given time interval T_{cs}

- If $M = 2 \Rightarrow$ Binary Digital Modulator $\Rightarrow$ Binary Comm. System
- If $M > 2 \Rightarrow$ M-ary Digital Modulator $\Rightarrow$ M-ary Comm. System



$$\mathbb{F} = \begin{bmatrix} \Pr(D_1|H_1), & \Pr(D_1|H_2), & \ldots, & \Pr(D_1|H_M) \\ \Pr(D_2|H_1), & \Pr(D_2|H_2), & \ldots, & \Pr(D_2|H_M) \\ \vdots & \vdots & \ldots, & \vdots \\ \Pr(D_K|H_1), & \Pr(D_K|H_2), & \ldots, & \Pr(D_K|H_M) \end{bmatrix}. \quad (1)$$

- **Binary** Comm Systems: use $M = 2$ possible waveforms

$$\{\overset{T_{cs}}{\overleftrightarrow{s_0(t)}}, \overset{T_{cs}}{\overleftrightarrow{s_1(t)}}\}; \; T_{cs} = T_b \tag{2}$$

M-**ary** Comm Systems: use M possible waveforms

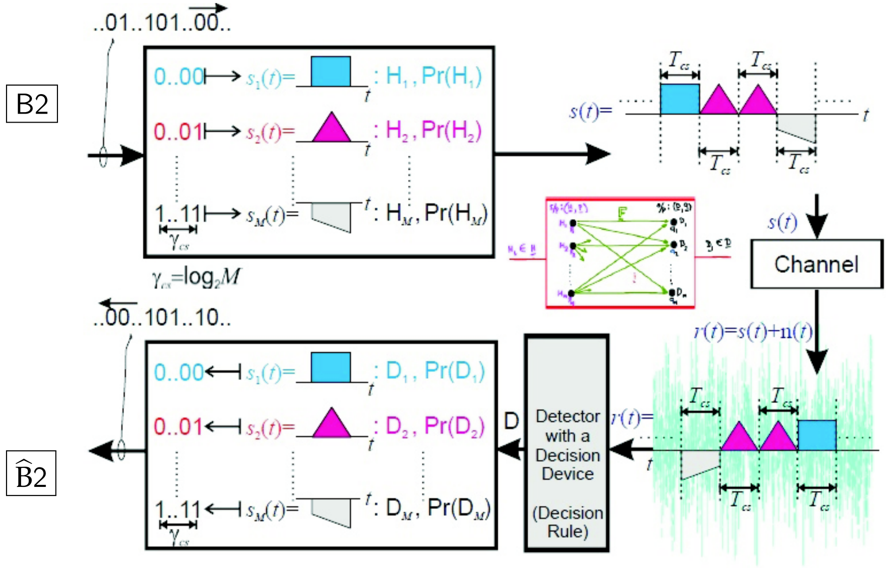
$$\{\underset{T_{cs}}{\underleftrightarrow{s_1(t)}}, \underset{T_{cs}}{\underleftrightarrow{s_2(t)}}, \ldots, \underset{T_{cs}}{\underleftrightarrow{s_M(t)}}\}; \quad T_{cs} = \gamma_{cs} T_b \tag{3}$$

- The M signals (or channel symbols) are characterized by their energy E_i

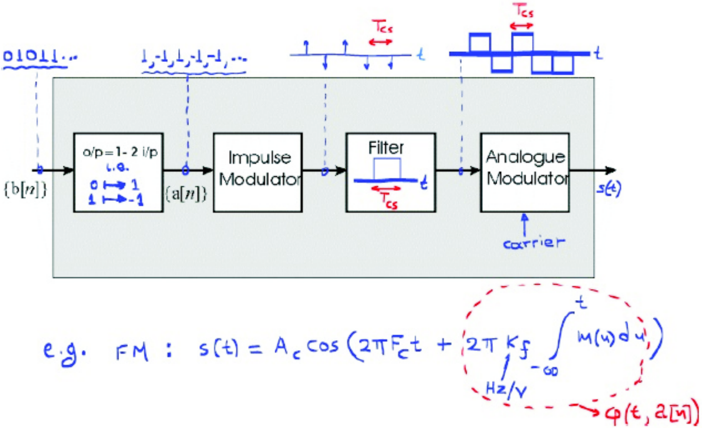
$$E_i = \int_0^{T_{cs}} s_i^2(t) \cdot dt; \quad \forall i \tag{4}$$

Furthermore their similarity (or dissimilarity) is characterized by their cross-correlation

$$\rho_{ij} = \frac{1}{\sqrt{E_i E_j}} \int_0^{T_{cs}} s_i(t) \cdot s_j^*(t) \cdot dt \tag{5}$$


..01..101..00..
B2
0..00 ⟼ $s_1(t)=$: H_1, $\Pr(H_1)$
0..01 ⟼ $s_2(t)=$: H_2, $\Pr(H_2)$
1..11 ⟼ $s_M(t)=$: H_M, $\Pr(H_M)$
γ_{cs}
$\gamma_{cs}=\log_2 M$
$s(t)=$
T_{cs}
$s(t)$
Channel
$r(t)=s(t)+n(t)$
$r(t)=$
Detector with a Decision Device (Decision Rule)
D
..00..101..10..
$\widehat{B}2$
0..00 ⟻ $s_1(t)=$: D_1, $\Pr(D_1)$
0..01 ⟻ $s_2(t)=$: D_2, $\Pr(D_2)$
1..11 ⟻ $s_M(t)=$: D_M, $\Pr(D_M)$

First Modelling of Digital Modulators



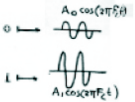
- Note: Transforming a sequence of 0's and 1's to a sequence of ± 1's

$$\boxed{\mathrm{o/p} = 1 - 2\times \mathrm{i/p}} \tag{6}$$

Examples of Binary Modulators

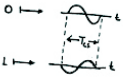
- A binary digital modulator maps 0's and 1's onto two analogue symbol-waveforms $s_0(t)$ and $s_1(t)$, that is $\begin{cases} 0 \mapsto s_o(t) \\ 1 \mapsto s_1(t) \end{cases}$ $0 \leq t \leq T_{cs}$
- The three basic binary digital modulators
  - **ASK** (Amplitude Shift-Keyed)

$$\begin{cases} 0 \mapsto s_o(t) = A_0 \cdot \cos(2\pi F_c t) \\ 1 \mapsto s_1(t) = A_1 \cdot \cos(2\pi F_c t) \\ \text{for } 0 \leq t \leq T_{cs} \end{cases}$$



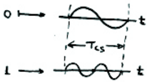
  - **PSK** (Phase Shift-Keyed)

$$\begin{cases} 0 \mapsto s_o(t) = A_c \cdot \cos(2\pi F_c t) \\ 1 \mapsto s_1(t) = A_c \cdot \cos(2\pi F_c t + 180^\circ) \\ \text{for } 0 \leq t \leq T_{cs} \end{cases}$$



  - **FSK** (Frequency Shift-Keyed)

$$\begin{cases} 0 \mapsto s_o(t) = A_c \cdot \cos(2\pi F_0 t) \\ 1 \mapsto s_1(t) = A_c \cdot \cos(2\pi F_1 t) \\ \text{for } 0 \leq t \leq T_{cs} \end{cases}$$



- The above binary digital modulators using complex representation:
  - **ASK** (Amplitude Shift-Keyed) $\begin{cases} 0 \mapsto s_o(t) = A_0 \cdot \exp(j2\pi F_c t) \\ 1 \mapsto s_1(t) = A_1 \cdot \exp(j2\pi F_c t) \\ \text{for } 0 \leq t \leq T_{cs} \end{cases}$
  - **PSK** (Phase Shift-Keyed)

$$\begin{cases} 0 \mapsto s_o(t) = \overbrace{A_c \exp(j0^\circ)}^{=+A_c} \cdot \exp(j2\pi F_c t) \\ 1 \mapsto s_1(t) = \underbrace{A_c \exp(j180^\circ)}_{=-A_c} \cdot \exp(j2\pi F_c t) \\ \text{for } 0 \leq t \leq T_{cs} \end{cases}$$

  - **FSK** (Frequency Shift-Keyed) $\begin{cases} 0 \mapsto s_o(t) = A_c \cdot \exp(j2\pi F_0 t) \\ 1 \mapsto s_1(t) = A_c \cdot \exp(j2\pi F_1 t) \\ \text{for } 0 \leq t \leq T_{cs} \end{cases}$

- Communication Systems Classification:
  1. Coherent (if demodulator is coherent, i.e. it uses a copy of the carrier)
  2. Non-coherent (if demodulator is non-coherent, e.g. it does not use the carrier, e.g. envelope detector)
- Note: Optimum demodulators are coherent.

Second Modelling of Digital Modulators

Signal Constellation

- We may represent the signal $s_i(t)$ by a **point** $\underline{w}_{s_i}$ in a D-dimensional Euclidean space with $D \leq M$.
- The set of points (vectors) specified by the columns of the matrix

$$\mathbb{W} = \left[\underline{w}_{s_1}, \quad \underline{w}_{s_2}, \quad \ldots, \quad \underline{w}_{s_M}\right] \tag{7}$$

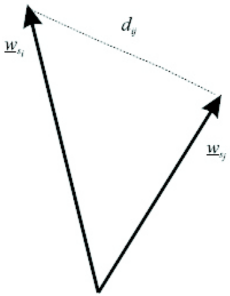
is known as "signal constellation".

$$\boxed{E_i = \underline{w}_{s_i}^H \underline{w}_{s_i} = \left\|\underline{w}_{s_i}\right\|^2} \tag{8}$$

$$\boxed{\rho_{ij} = \frac{1}{\sqrt{E_i E_j}} \underline{w}_{s_i}^H \underline{w}_{s_j} = \frac{\underline{w}_{s_i}^H \underline{w}_{s_j}}{\left\|\underline{w}_{s_i}\right\| \left\|\underline{w}_{s_j}\right\|}} \tag{9}$$

Distance Between two M-ary Signals

- The **distance** between two signals $s_i(t)$ and $s_j(t)$ is the **Euclidean distance** between their associate vectors $\underline{w}_{s_i}$ and $\underline{w}_{s_j}$



$$
\begin{aligned}
\text{i.e. } d_{ij} &= \left\| \underline{w}_{s_i} - \underline{w}_{s_j} \right\| \\
&= \sqrt{(\underline{w}_{s_i} - \underline{w}_{s_j})^H (\underline{w}_{s_i} - \underline{w}_{s_j})} \\
&= \sqrt{\underline{w}_{s_i}^H \underline{w}_{s_i} + \underline{w}_{s_j}^H \underline{w}_{s_j} - 2\underline{w}_{s_i}^H \underline{w}_{s_j}} \\
&= \sqrt{E_i + E_j - 2\rho_{ij}\sqrt{E_i E_j}} \\
&\Rightarrow \boxed{d_{ij}^2 = E_i + E_j - 2\rho_{ij}\sqrt{E_i E_j}}
\end{aligned}
$$

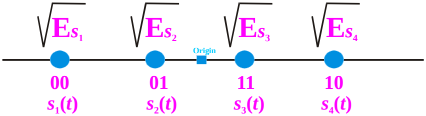
- It is clear from the above that the **Euclidean distance** d_{ij} associated with two signals $s_i(t)$ and $s_j(t)$ indicates, like the cross-correlation coefficient, the **similarity or dissimilarity** of the signals.

Constellation Diagram of Main Modems

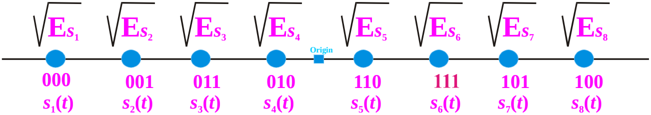
Consider an M-ary System having the following signals

$$\{s_1(t), s_2(t), \ldots, s_M(t)\} \text{ with } 0 \leq t \leq T_{cs}$$

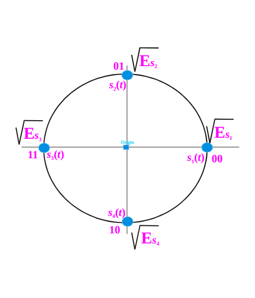
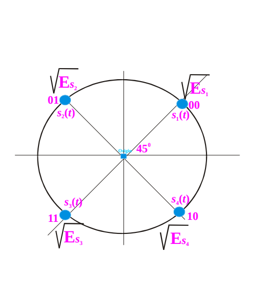
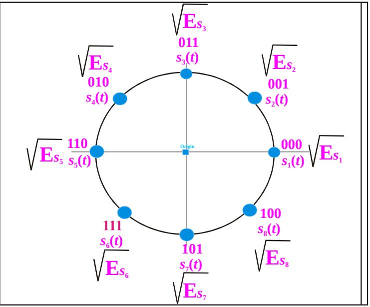
- M-ary ASK
  - channel symbols: $s_i(t) = A_i \cdot \cos(2\pi F_c t)$ where A_i =given= $2i - 1 - M$ (say)
  - dimensionality of signal space $= D = 1$
  - if $M = 4$ then



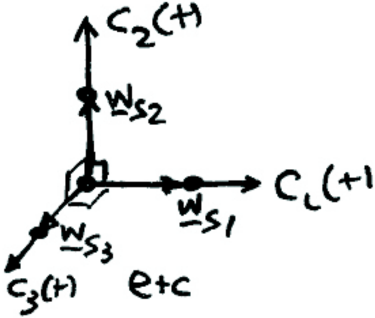
  - if $M = 8$ then



- M-ary PSK
  - channel symbols: $s_i(t) = A.\cos\left(2\pi F_c t + \frac{2\pi}{M}.(i-1) + \phi\right)$ for $i = 1, 2, ..,$
  - dimensionality of signal-space $= D = 2$

$M = 4$ & $\phi = 0°$	$M = 4$ & $\phi = 45°$	$M = 8$ & $\phi = 0°$
		

- M-ary FSK:
  very difficult to be represented using constellation diagram



with

$$f_i - f_j = n \, \frac{1}{2T_{cs}}$$

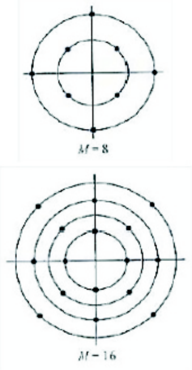
$$f_i + f_j = m \, \frac{1}{2T_{cs}}$$

where
$n, m =$ integers

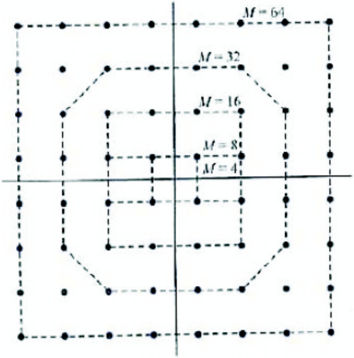
- **M-ary Amplitude & Phase - M-ary QAM:**
  1. channel symbols: $s_i(t) = A_i \cdot \cos(2\pi F_c t + \varphi_i + \phi) \quad i = 1, 2, \ldots, M$

     or $s_{nm}(t) = A_n \cdot \cos(2\pi F_c t + \varphi_m + \phi) \begin{cases} n = 1, 2, \ldots, M_1 \\ m = 1, 2, \ldots, M_2 \\ M = M_1 \times M_2 \end{cases}$
  2. dimensionality of signal space $= D = 2$

PAM-PSK

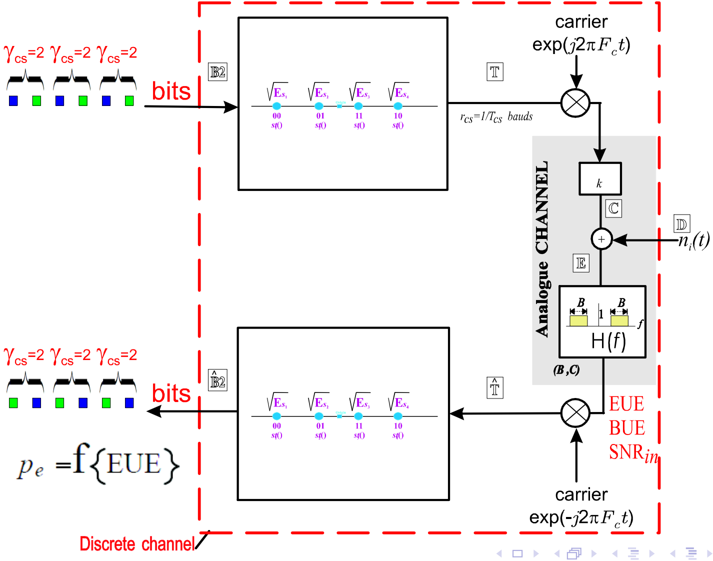


QAM



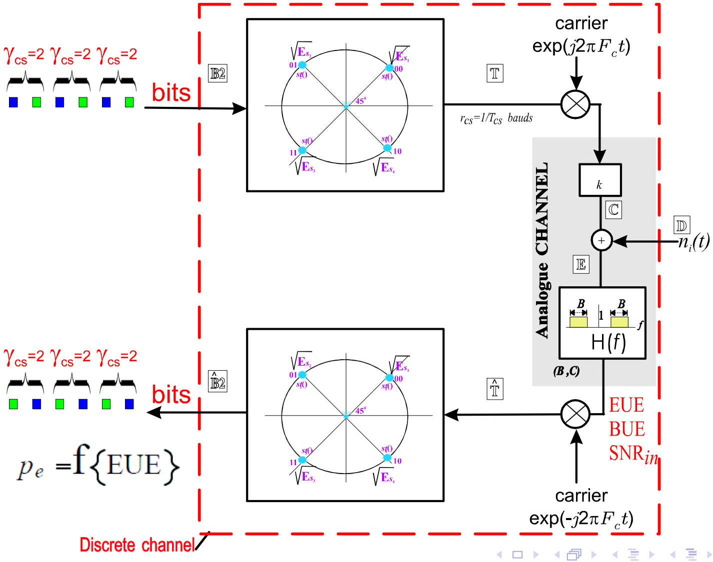
4ASK and QPSK Block Structures

An ASK (M=4) Communication System



$$p_e = f\{EUE\}$$

A QPSK Communication System



$$p_e = f\{EUE\}$$

QPSK: Comments

- A QPSK modulator has four "channel-symbols" which are described by the following equation:

$$s_i(t) = A\cos(2\pi F_c t + \frac{2\pi}{M}(i-1) + \phi) \quad \text{for } i = 1, 2, 3, 4 \tag{10}$$

with

$$M = 4 \text{ and } 0 \leq t \leq T_{cs}$$

and the modulation process is described by the so called "constellation diagram".

- The previous figure (page 20) shows the constellation for $\phi = 45^\circ$.
- From this figure it is clear that the constellation diagram shows the mapping of binary digits to QPSK channel symbols (constellation points) as well as the square root of the energy $\sqrt{E_s}$ of the channel symbols. The diagram also indicates a Gray code mapping from binary digits to channel symbols (constellation points).

- Overall, using complex number representation, it is clear from the constellation diagram that

$$
\begin{array}{|c|l|}
\hline
00 \mapsto & s_1(t) = \underbrace{\sqrt{E_s}\exp(j45^\circ)}_{=m_1}\exp(j2\pi F_c t) \\
\hline
01 \mapsto & s_2(t) = \underbrace{\sqrt{E_s}\exp(j135^\circ)}_{=m_2}\exp(j2\pi F_c t) \\
\hline
11 \mapsto & s_3(t) = \underbrace{\sqrt{E_s}\exp(j225^\circ)}_{=m_3}\exp(j2\pi F_c t) \\
\hline
10 \mapsto & s_4(t) = \underbrace{\sqrt{E_s}\exp(j315^\circ)}_{=m_4}\exp(j2\pi F_c t) \\
\hline
\end{array}
\tag{11}
$$

- In other words,

$$s_i(t) = m_i \exp(j2\pi F_c t) \quad \text{for } i = 1, 2, 3 \text{ and } 4 \tag{12}$$

- It is important to point out that the four symbols m_1, m_2, m_3 and m_4 are known as **baseband QPSK "channel symbols"** and are used by the "QPSK constellation symbol mapping" block shown in the previous figure.
- For instance, if the bit-pair at the input of the QPSK modulator is "01" then the output is the baseband channel symbol m_2.
- With reference to the figure given in page 20, the term $\exp(j2\pi F_c t)$ (see Equations 11 and 12) is shown at the Transmitter as the up-conversion from baseband to bandpass.
- In a similar fashion the down-conversion from baseband to bandpass is shown at the receiver's front-end using the complex conjugate of the transmitter's carrier, i.e. using $\exp(-j2\pi F_c t)$.
- Thus, overall, we have $\exp(j2\pi F_c t)\exp(-j2\pi F_c t) = 1$.

- Based on the above discussion it is clear that the presence of the carrier does not affect the analysis of the system.

  Therefore, it is common practice to ignore the carrier when analyzing communication systems, by working on the baseband.

- For the rest of this explanatory note the carrier term will be ignored from both Tx and Rx.

Summarising

- a QPSK modulator/demodulator is represented by its constellation diagram and the QPSK symbol mapper transforms the binary sequence to a sequence of QPSK complex channel symbols m_i, forming the baseband QPSK message signal $m(t)$ of bandwidth B i.e.

$$m(t) = \sum_n \overbrace{a[n]}^{m_1, m_2, m_3, m_4} \cdot c(t - n \cdot T_{cs});\; nT_{cs} \leq t < (n+1) \cdot T_{cs}$$

where

$$\begin{cases} c(t) = rect\left\{\frac{t}{T_{cs}}\right\} \\ \{a[n]\} = \text{sequ. of independent data symbols } (m_i) \\ B = \frac{r_{cs}}{2} = \frac{1}{2T_{cs}} \end{cases}$$

- Thus with reference to the binary sequence of bits "001001" (message), by looking at the QPSK constellation diagram it is clear that we have the following mapping

00	$m_1 = A\exp(j45^\circ)$
10	$m_4 = A\exp(j315^\circ)$
01	$m_2 = A\exp(j135^\circ)$

where

$$A = \sqrt{E_s}$$

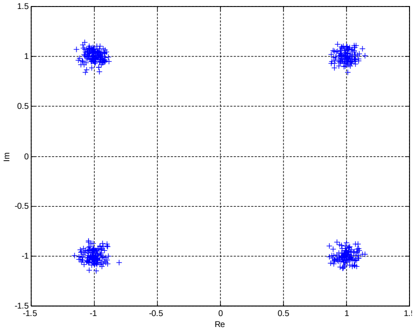
- Note that, for a binary system

$$m(t) \equiv \sum_n \overbrace{a[n]}^{\pm 1} \cdot c(t - n\cdot T_{cs});\;\; nT_{cs} \leq t < (n+1)\cdot T_{cs}$$

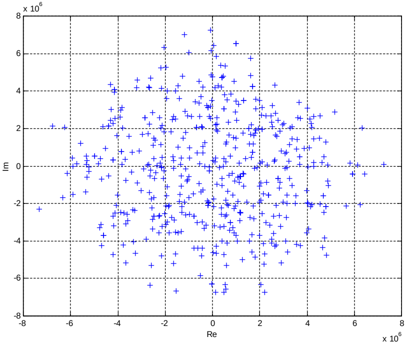
where

$$\begin{cases} c(t) = \text{rect}\left\{\frac{t}{T_{cs}}\right\} \\ \{a[n]\} = \text{sequ. of independent data bits } (\pm 1\text{s}) \\ B = \frac{r_{cs}}{2} = \frac{1}{2T_{cs}} \end{cases}$$

Examples of Plots of QPSK- Receiver's Constellation Diagram



"good"



"bad"

Performance Evaluation Criteria

- In general the quality of a digital communication system is expressed in terms of the accuracy with which the binary digits delivered at the output of the detector represent the binary digits that were fed into the digital modulator.

- It is generally taken that it is **the fraction of the binary digits that are delivered back in error** that is a measure of the quality of the communication system.

- This fraction, or rate, is referred to as the bit error probability $\mathbf{p_e}$ **, or, Bit-Error-Rate BER.**

- The performance of M-ary communication systems is evaluated by means of the average **probability of symbol error p $_{e,cs}$** , which, for $M > 2$, is different than the average **probability of bit error (or Bit-Error-Rate BER),** p_e.

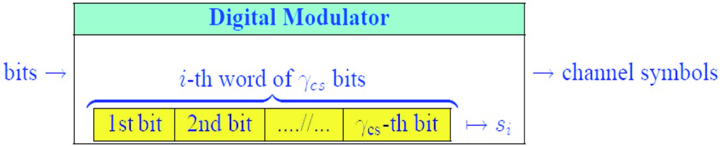
  That is
  $$\begin{cases} p_{e,cs} \neq p_e & \text{for } M > 2 \\ p_{e,cs} = p_e & \text{for } M = 2 \text{ (i.e. Binary Communication Systems)} \end{cases}$$

  However because we transmit binary data, the probability of bit error ρ_e is a more natural parameter for performance evaluation than $\rho_{e,cs}$.

- Although, these two probabilities are related i.e.

$$p_e = \mathsf{f}\{p_{e,cs}\}$$

their relationship depends on the encoding approach which is employed by the digital modulator for mapping binary digits to M-ary signals (channel symbols)



where

$$\gamma_{cs} = \log_2 M$$

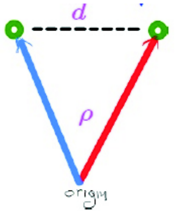
Performance of Binary (equiprobable) Digital Modulators/Demodulators

- Consider a Binary Communication system

$$\begin{cases} 0 \mapsto s_1 \\ 1 \mapsto s_2 \end{cases} \quad \text{or a more popular notation:} \quad \begin{cases} 0 \mapsto s_0 \\ 1 \mapsto s_1 \end{cases}$$

Constellation diagram:

0	1
s_0	s_1
$\sqrt{E_0}$	$\sqrt{E_1}$



$$p_e = \mathrm{T}\left\{\frac{d}{\sqrt{2N_0}}\right\} = \ldots = \mathrm{T}\left\{\sqrt{(1-\rho)\mathrm{EUE}}\right\}$$

- Thus, at the output of an optimum digital demodulator the probability of error can be calculated by using the following expression:

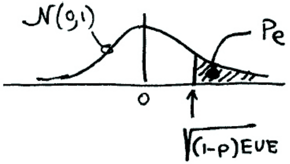
$$p_e = \mathrm{T}\left\{\sqrt{(1-\rho)\mathrm{EUE}}\right\} \tag{13}$$

where EUE$= \frac{E_b}{N_0}$ and $\mathrm{PSD}_{n_i}(f) = \frac{N_0}{2}$

$$\text{with}\begin{cases} E_b = \frac{1}{2}\cdot\int_0^{T_{cs}}(s_0(t)^2 + s_1(t)^2)dt \\ \quad = \text{average signal energy} \\ \\ \rho = \frac{1}{E_b}\cdot\int_0^{T_{cs}} s_0(t)s_1(t)dt \\ \quad = \text{the time cross-correlation between signals} \end{cases}$$

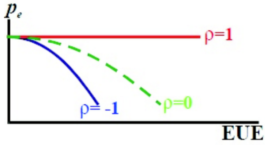
N.B.:

- $(1-\rho)\frac{E_b}{N_0} = \uparrow \Longrightarrow p_e = \downarrow$



- if $\frac{E_b}{N_0}$ = fixed **then** the optimum system is that for which the correlation coeff is -1

i.e. $\boxed{\rho = -1 \Longrightarrow s_0(t) = -s_1(t)}$
This is known as **optimum, or ideal binary Communication System**



Examples

Baseband MODEMS:

- Antipodal$\rightarrow \begin{cases} 0 \mapsto s_0(t) = -A_c \\ 1 \mapsto s_1(t) = A_c \end{cases} \qquad 0 \leq t \leq T_{cs}$
- $s(t) = A_c m(t)$ ($\{a[n]\}$ = sequ. of independent data bits ($\pm$1s))
- $p_e = \mathrm{T}\left\{\frac{A_c}{\sigma}\right\}$

COHERENT MODEMS:

1. Amplitude Shift-Keyed (ASK) or On-Off Keying (OOK)
   - (ASK or OOK) $\rightarrow \begin{cases} 0 \mapsto s_0(t) = 0 \\ 1 \mapsto s_1(t) = A_c \cos(2\pi F_c t) \end{cases} \qquad 0 \leq t \leq T_{cs}$
   - $s(t) = A_c \cdot m(t) \cdot \cos(2\pi F_c t)$
   - $p_e = \mathrm{T}\left\{\sqrt{\frac{E_{s_1}}{2N_0}}\right\}$

2. Biphase Shift-Keyed:

- general $\rightarrow \begin{cases} 0 \mapsto s_0(t) = A_c \cos(2\pi F_c t - \Delta\theta) \\ 1 \mapsto s_1(t) = A_c \cos(2\pi F_c t + \Delta\theta) \end{cases} \qquad 0 \leq t \leq T_{cs}$

  $$p_e = \mathrm{T}\left\{\sqrt{2 \cdot \mathrm{EUE} \cdot \sin^2(\Delta\theta)}\right\}$$

- Phase-Reversal Keying (RSK)

  $$(\mathrm{RSK}) \rightarrow \begin{cases} 0 \mapsto s_0(t) = -A_c \sin(2\pi F_c t) \\ 1 \mapsto s_1(t) = A_c \sin(2\pi F_c t) \end{cases} \qquad 0 \leq t \leq T_{cs}$$

  $$p_e = \mathrm{T}\left\{\sqrt{2 \cdot \mathrm{EUE}}\right\}$$

- **N.B.:** for $\Delta\theta = \pm\frac{\pi}{2}$ then general = RSK and it is called BPSK

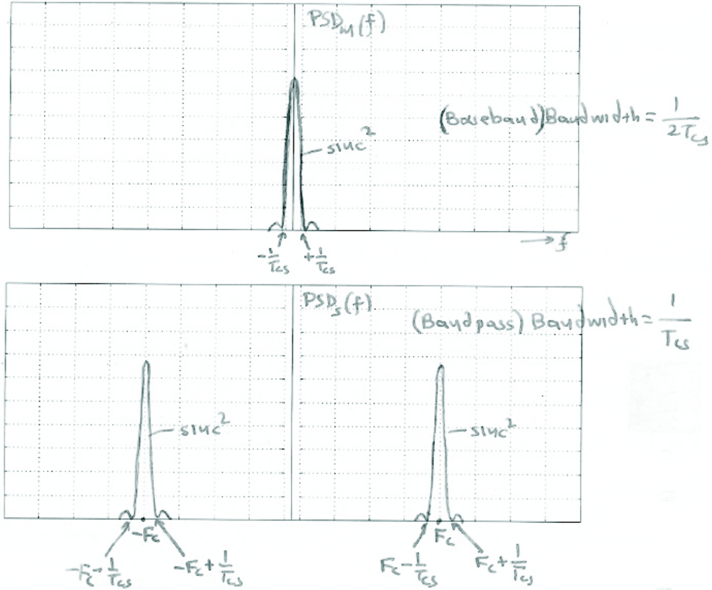
$$\boxed{\textbf{BPSK} \mid s(t) = A_c \cdot \sin(2\pi F_c t + m(t) \cdot \frac{\pi}{2})} \tag{14}$$

Equation 14 can be written as follows:

$$\boxed{\textbf{BPSK} \mid s(t) = A_c \cdot m(t) \cdot \cos(2\pi F_c t)} \tag{15}$$

$\therefore$ BPSK can be considered as $\left\{ \begin{array}{l} \text{PM} \\ \text{AM} \end{array} \right.$

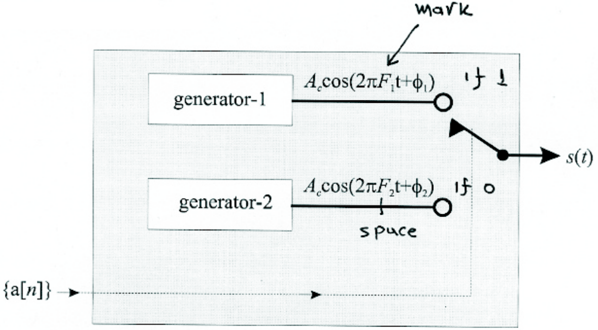
The PSD(f)'s of $m(t)$ and $s(t)$ are shown below:


$PSD_m(f)$
(Baseband) Bandwidth = $\frac{1}{2T_{cs}}$
sinc^2
$-\frac{1}{T_{cs}}$
$+\frac{1}{T_{cs}}$
f
$PSD_s(f)$
(Bandpass) Bandwidth = $\frac{1}{T_{cs}}$
sinc^2
sinc^2
$-F_c$
$-F_c-\frac{1}{T_{cs}}$
$-F_c+\frac{1}{T_{cs}}$
F_c
$F_c-\frac{1}{T_{cs}}$
$F_c+\frac{1}{T_{cs}}$

3. Frequency Shift-Keyed (FSK)

$$(\text{FSK}): \begin{cases} 0 \mapsto s_0(t) = A_c \cos(2\pi F_c t) \\ 1 \mapsto s_1(t) = A_c \cos(2\pi (F_c + \Delta f) t) \end{cases}$$
$$0 \leq t \leq T_{cs},\ \Delta f = \frac{m}{2T_{cs}}$$

- $\overbrace{\text{CFSK}}^{\text{coherent}}$: $p_e = \text{T}\left\{\sqrt{\text{EUE}}\right\}$
- discontinuous FSC



- continuous FSC

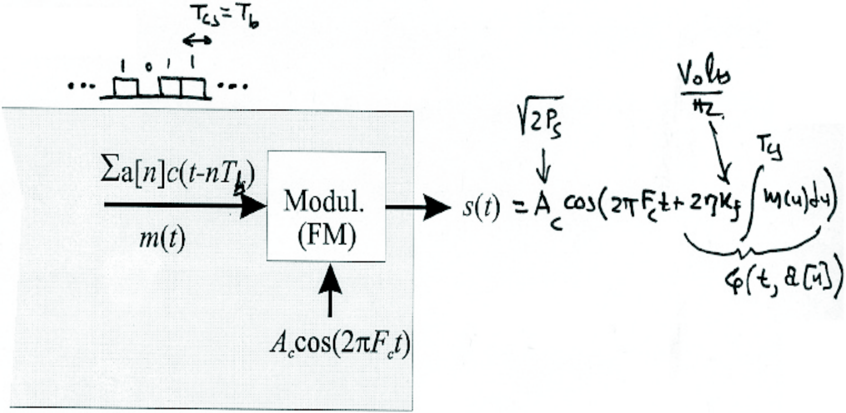


Table of BERs

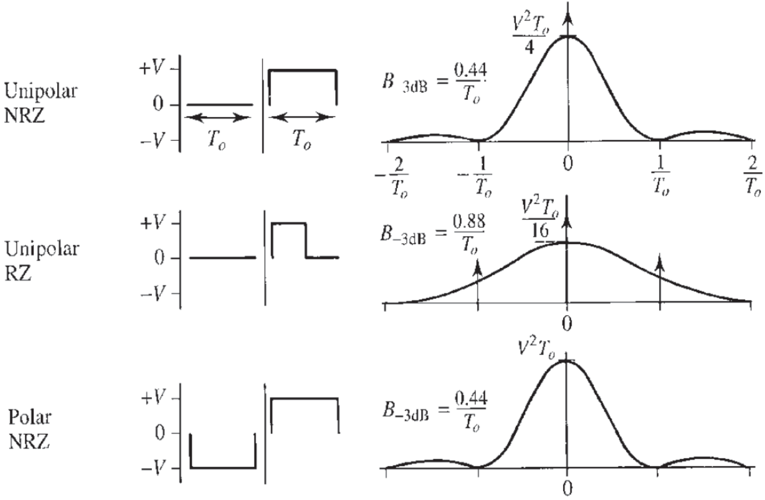
1	ASK	$p_e = \mathrm{T}\left\{\sqrt{\frac{1}{2}\mathrm{EUE}}\right\}$
2	non-coherent ASK	$p_e = 0.5\exp(-\frac{E_{s_1}}{4N_0}) + 0.5\mathrm{T}\left\{\sqrt{\frac{E_{s_1}}{2N_0}}\right\}$
3	FSK	$p_e = \mathrm{T}\left\{\sqrt{\mathrm{EUE}}\right\}$
4	FSK (non-coherent)	$p_e = \frac{1}{2}\exp\left\{-\frac{1}{2}\mathrm{EUE}\right\}$
5	BPSK	$p_e = \mathrm{T}\left\{\sqrt{2\mathrm{EUE}}\right\}$
6	BPSK (differential)	$p_e = \frac{1}{2}\exp\left\{-\mathrm{EUE}\right\}$
7	MSK	$p_e = \mathrm{T}\left\{\sqrt{1.7\mathrm{EUE}}\right\}$
8	Gaussian MSK	$p_e \approx \mathrm{T}\left\{\sqrt{1.36\mathrm{EUE}}\right\}$
9	M-ary PSK (coherent)	$p_e \approx 2\mathrm{T}\left\{\sqrt{4\mathrm{EUE}}\sin(\frac{\pi}{2M})\right\}$
10	M-ary QAM	$p_e \approx 4(1 - \frac{1}{\sqrt{M}})\mathrm{T}\left\{\sqrt{\frac{3}{M-1}\mathrm{EUE}}\right\}$

Introduction: Lines Codes (Wireline Digital Comms)

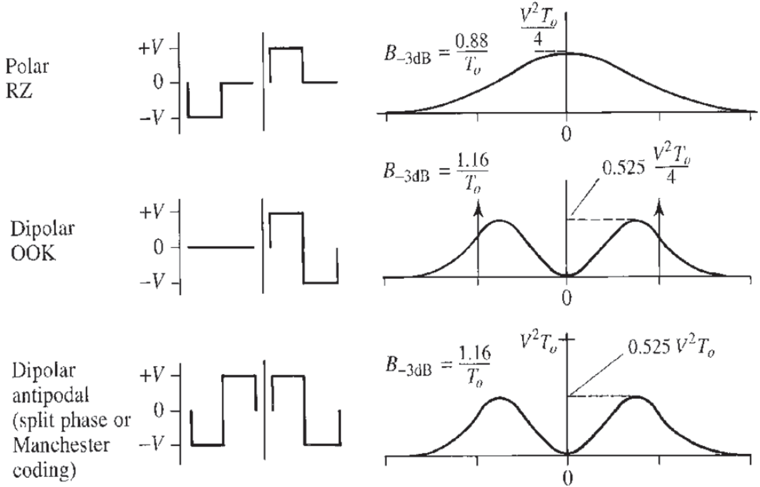
- Line codes are used for data transmission in a metallic wire, twisted pair, cable.
- Line coding is **baseband signal transmission** having:
  - a **Tx** (Digital Modulator[1] without carrier)
  - a **cable** (communication channel)
  - a **Rx** (Digital Demodulator[1] without carrier).
- Appropriate coding of the transmitted symbol pulse shape can minimise the probability of error by ensuring that the spectral characteristics of the digital signal are well matched to the transmission channel.
- The line code must also permit the receiver to extract accurate PCM bit and word timing signals directly from the received data, for proper detection and interpretation of the digital pulses.
- for more info see Chapter 6 in [*Ian Glover and Peter Grant, "Digital Communications", 3rd edition,Pearson Education Limited*, 2010]

[1] Commonly used names: Tx="Line Encoder" and Rx="Line Decoder".

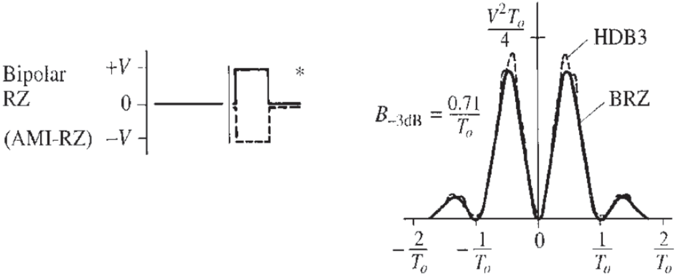
Main Types of Line Codes and their PSD

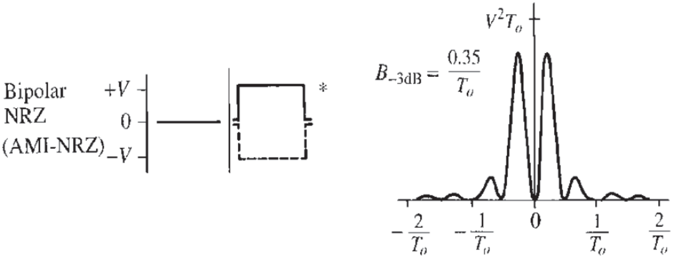


Main Types of Line Codes and their PSD (cont.)

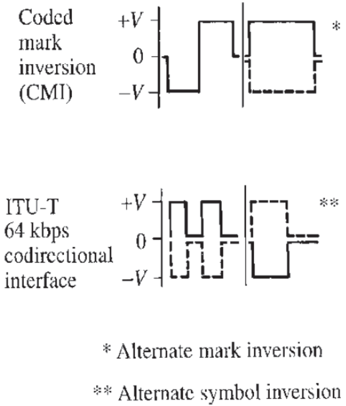


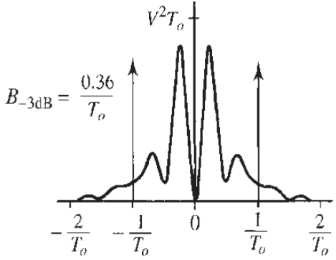
Main Types of Line Codes and their PSD (cont.)



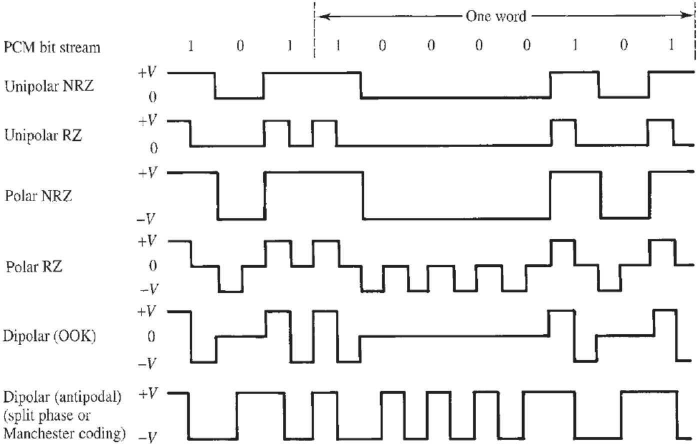


Main Types of Line Codes and their PSD (cont.)

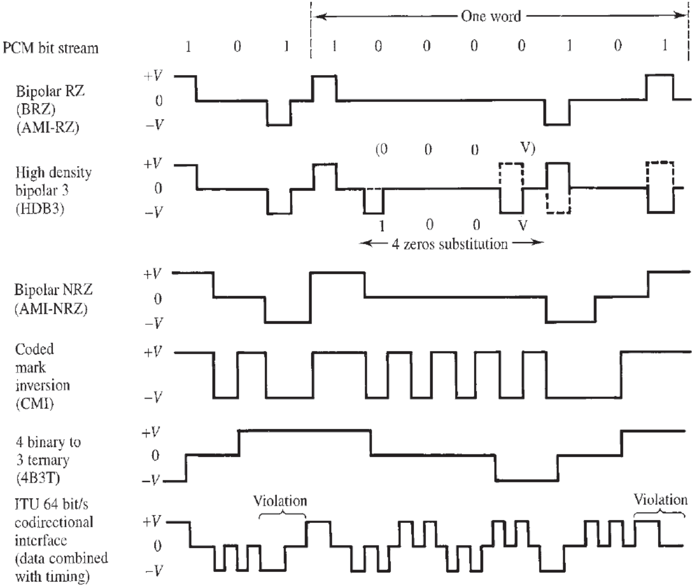




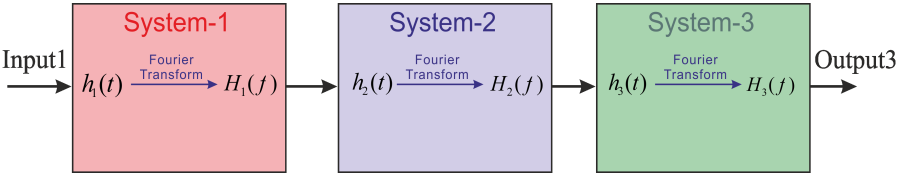
Popular Line Codes



Popular Line Codes (cont.)

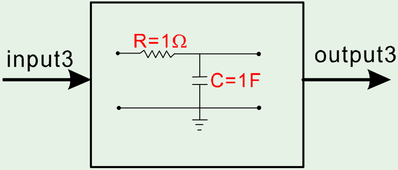


Example-1: Connecting Systems



Example (1)

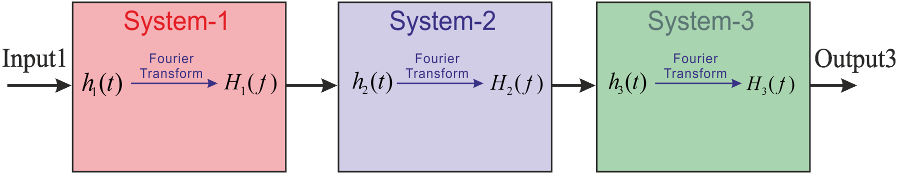
3rd system: $H_3(f)$



$$H_3(f) = \frac{1}{1 + j2\pi f}$$

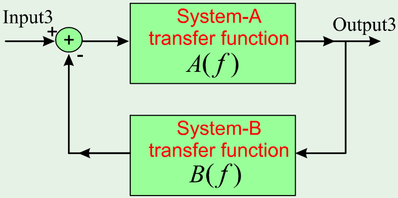
N.B.: This is a Low Pass Filter

Example-1: Connecting Systems (cont.)



Example (2)

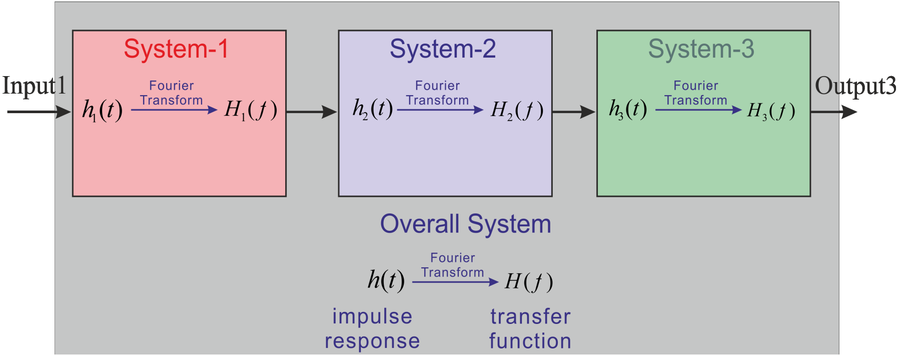
3rd system: $H_3(f)$



$$H_3(f) = \frac{A(f)}{1 + A(f).B(f)}$$

NB: This is a Control System

Example-1: Connecting Systems (cont.)

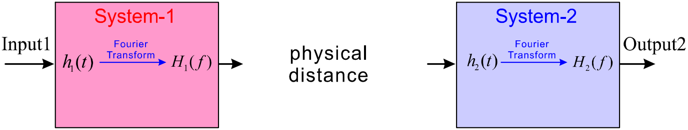


$$
\begin{aligned}
H(f) &= H_1(f).H_2(f).H_3(f) \\
h(t) &= h_1(t) \star h_2(t) \star h_3(t)
\end{aligned}
$$

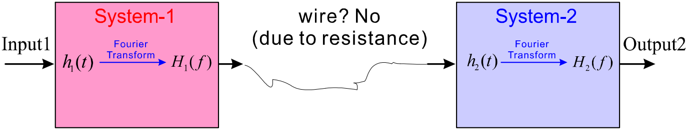
- the dot-symbol denotes "multiplication" and $\star$ denotes "convolution".
- These days we have "system-on-chip" (SOC)

Example-1: Connecting Systems

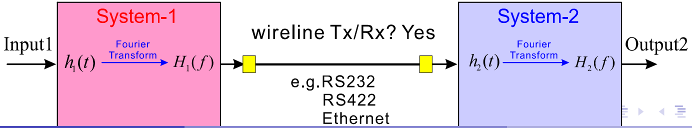
Connecting Systems - Far Apart



Wire Connection? **No** (due to effects equivalent to "Friction")



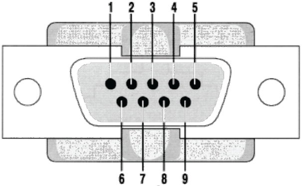
Wireline Comm Connection? **Yes**



Example-1: Connecting Systems (cont.)

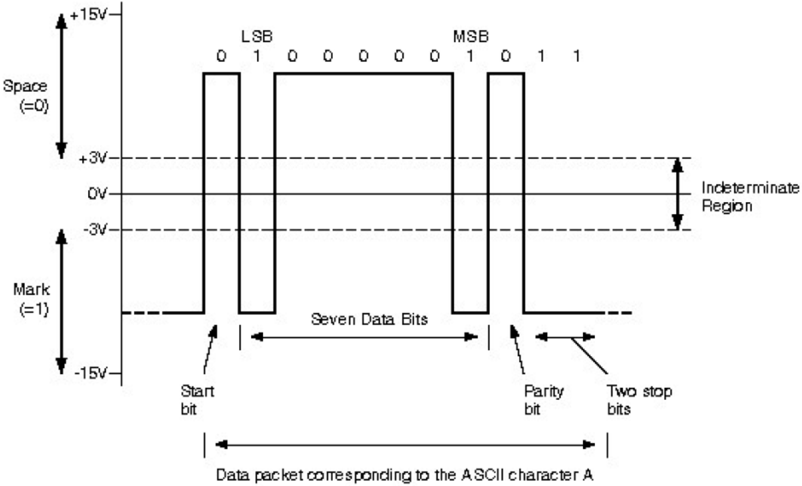
Connecting Systems with RS232 Data Port

- It establishes a two-way (i.e. "full-duplex") data communication channel using a single cable of length up to 15m. It uses:
  - the pin 2 to Tx data
  - the pin 3 to Rx data



Pin	Signal	Pin	Signal
1	Data Carrier Detect	6	Data Set Ready
2	Received Data	7	Request to Send
3	Transmitted Data	8	Clear to Send
4	Data Terminal Ready	9	Ring Indicator
5	Signal Ground		

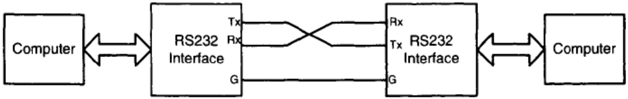
- Transmitted signal corresponding to the letter "A", using data transmission over a cable (RS232 data interface/port)



- N.B.:one byte of asynchronous transmission

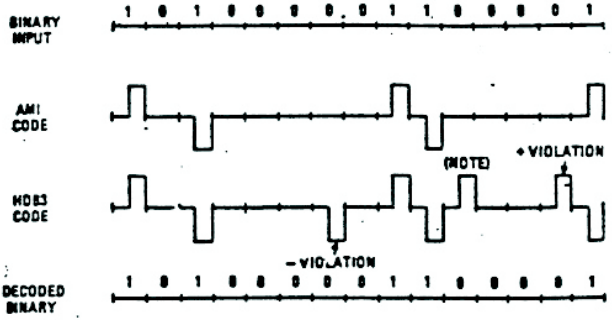
Example-1: Connecting Systems (cont.)

- Note: The RS232 is a wireline communication system connecting "smart" devices, e.g. Computers, CPU, PLC (Programmable Logic Controllers), etc.:



Example-2: PSTN and HDB3 Line Codes

- Example :



Note: Added mark in first *zero* position to ensure that consecutive violations are of opposite polarity

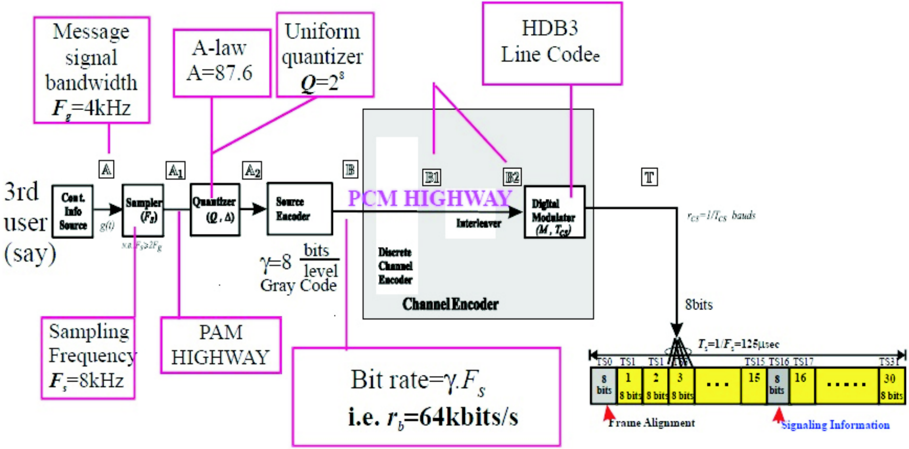
Example-2: PSTN and HDB3 Line Codes (cont.)

- HDB3 is used in Europe (and many other countries) in Public Switched Telephone Networks (PSTN) according to the 2nd CCITT recommendation (32 channel PCM)
- it is used to transmit a frame of duration $\frac{1}{F_s} = T_s = 125\mu s$, containing data from:
  - 30 users (one 8 bit word per user),
  - one Frame Alignment 8-bit word, and
  - one signalling info 8-bit word

  using an HDB3 line encoder at the Tx and HDB3 line decoder in the Rx.

Example-2: PSTN and HDB3 Line Codes (cont.)

Single-Channel Path of 2nd CCITT rec. (32-channels PCM)



PSD(f) of Line Code Signals

- Signals - Definition:

$$s(t) = \sum_n \mathrm{a}[n] c(t - nT_{cs}); \;\; nT_{cs} < t < (n+1) T_{cs} \tag{16}$$

where

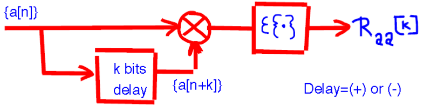
- $c(t)$ is an energy signal of duration T_{cs}. This implies that $c(t - nT_{cs})$ is defined in the interval $nT_{cs} < t < (n+1)T_{cs}$
- $\{\mathrm{a}[n]\}$ is a binary sequence

- Autocorrelation function of a binary sequence $\{\mathrm{a}[n]\}$

$$R_{\mathrm{aa}}[k] = \mathcal{E}\{\mathrm{a}[n]\mathrm{a}[n+k]\} \tag{17}$$

$$= \sum_{i=1}^{I} (\mathrm{a}[n]\mathrm{a}[n+k])_{i^{th}\text{-pair}} . \Pr(i^{th}\text{-pair}) \tag{18}$$

- where I denotes the total number of "pairs"



$$\boxed{\mathrm{PSD}_s(f) = \frac{|\mathrm{FT}(c(t))|^2}{T_{cs}} \cdot \left[R_{\mathrm{aa}}[0] + \sum_{\substack{k=-\infty \\ k\neq 0}}^{+\infty} R_{\mathrm{aa}}[k] \exp(-j2\pi f k T_{cs}) \right]} \quad (19)$$

- <u>Note that</u> if $R_{\mathrm{aa}}[k] = \begin{cases} \mathcal{E}\left\{\mathrm{a}_n^2\right\} \text{ for } k=0 \\ \mathcal{E}\left\{\mathrm{a}_n\right\} \cdot \mathcal{E}\left\{\mathrm{a}_{n+k}\right\} \text{ for } k \neq 0 \end{cases}$

  i.e. $R_{\mathrm{aa}}[k] = \begin{cases} \mu_{\mathrm{a}}^2 + \sigma_{\mathrm{a}}^2 \text{ for } k=0 \text{ where } \mu_{\mathrm{a}} = \text{mean and } \sigma_{\mathrm{a}} = \text{std} \\ \mu_{\mathrm{a}}^2 \text{ for } k \neq 0 \end{cases}$

  $\boxed{\text{then}}$

  $$(19) \Rightarrow \mathrm{PSD}_s(f) = \underbrace{\sigma_{\mathrm{a}}^2 \frac{|\mathrm{FT}(c(t))|^2}{T_{cs}}}_{\text{Continuous Spectrum}} + \underbrace{\frac{\mu_{\mathrm{a}}^2}{T_{cs}^2} \cdot \mathrm{comb}_{\frac{1}{T_{cs}}}\left(|\mathrm{FT}(c(t))|^2\right)}_{\text{Discrete Spectrum}}$$

EXAMPLE...FOR YOU...

- if $a_n = 0, 1$ with $\Pr\{a_n = 0\} = \Pr\{a_n = 1\} = \frac{1}{2}$ find the spectrum of "unipolar RZ" line-code signal.

Example: Autocorr. & PSD(f) of a Bipolar Line Code

1. Show that for a bipolar line code the autocorrelation function of the code sequence $\{\mathrm{a}[n]\}$ is as follows:

$$R_{\mathrm{aa}}[k] = \begin{cases} 1/2 & \text{if } k = 0 \\ -1/4 & \text{if } k = 1 \\ 0 & \text{if } k \geq 2 \end{cases} \tag{20}$$

2. If $\Pr(0) = \Pr(\pm 1) = 0.5$ and $c(t) = \mathrm{rect}\left\{\frac{t}{T_{cs}}\right\}$, derive an expression for the power spectral density $\mathrm{PSD}(f)$ for the bipolar line code waveform

$$m(t) = \sum_{n=-\infty}^{\infty} \mathrm{a}[n] \cdot c(t - nT_{cs}) \tag{21}$$

Solution: Autocorrelation R[0], R[1]

$$
\begin{aligned}
R_{aa}[0] &= \mathcal{E}\left\{\mathrm{a}[n]^2\right\} = \left\{ \begin{array}{l} 0\times 0 = 0 \rightarrow \frac{1}{2} \\ (\pm 1)\times(\pm 1) = +1 \rightarrow \frac{1}{2} \end{array} \right\} \\
&= 0\times\frac{1}{2} + 1\times\frac{1}{2} \\
&= \frac{1}{2}
\end{aligned} \tag{22}
$$

$$
R_{aa}[1] = \mathcal{E}\left\{\mathrm{a}[n]\mathrm{a}[n+1]\right\} =
$$

1^{st}	2^{nd}	$1^{st}\times 2^{rd}$	Pr
0,	0,	= 0	1/4
0,	± 1,	= 0	1/4
± 1,	0,	= 0	1/4
± 1,	∓ 1,	= −1	1/4

$$
\begin{aligned}
&= 0\times\frac{1}{4} + 0\times\frac{1}{4} + 0\times\frac{1}{4} + (-1)\times\frac{1}{4} \\
&= -\frac{1}{4}
\end{aligned} \tag{23}
$$

Solution: Autocorrelation R[k],k>1

$R_{aa}[k]$ with $k>1$ =

1^{st}	2^{nd}	3^{rd}	$1^{st}\times 3^{rd}$	Pr
$0,$	$0,$	0	$=0$	$1/8$
$0,$	$0,$	± 1	$=0$	$1/8$
$0,$	$\pm 1,$	0	$=0$	$1/8$
$0,$	$\pm 1,$	∓ 1	$=0$	$1/8$
$\pm 1,$	$0,$	0	$=0$	$1/8$
$\pm 1,$	$0,$	∓ 1	$=-1$	$1/8$
$\pm 1,$	$\mp 1,$	0	$=0$	$1/8$
$\pm 1,$	$\mp 1,$	± 1	$=+1$	$1/8$

$$
\begin{aligned}
&= \left(\text{ignoring the 2nd column and multiplying 1st with 3rd we have}\right)\\
&= 0\times\frac{6}{8}+1\times\frac{1}{8}+(-1)\times\frac{1}{8}\\
&= 0
\end{aligned}
\tag{24}
$$

Solution: PSD(f)

$$
\begin{aligned}
\text{PSD}(f) &= \frac{\left|\text{FT}\left(\text{rect}\left\{\frac{t}{T_{cs}}\right\}\right)\right|^2}{T_{cs}} \left\{R[0] + 2R[k]\cos(2\pi k T_{cs})\right\} \\
&= \frac{T_{cs}^2 \text{sinc}^2(fT_{cs})}{T_{cs}} \left\{\frac{1}{2} - 2 \times \frac{1}{4}\cos(2\pi T_{cs})\right\} \\
&= T_{cs}\text{sinc}^2(fT_{cs}) \left\{\frac{1}{2} - \frac{1}{2}\cos(2\pi T_{cs})\right\} \\
&= T_{cs}\text{sinc}^2(fT_{cs}) \cdot \sin^2\left(\frac{2\pi T_{cs}}{2}\right) \qquad (25)
\end{aligned}
$$