# EE401: Advanced Communication Theory

Professor A. Manikas
Chair of Communications and Array Processing

Imperial College London

## Multi-Antenna Wireless Communications

Part-A: An Introductory Overview

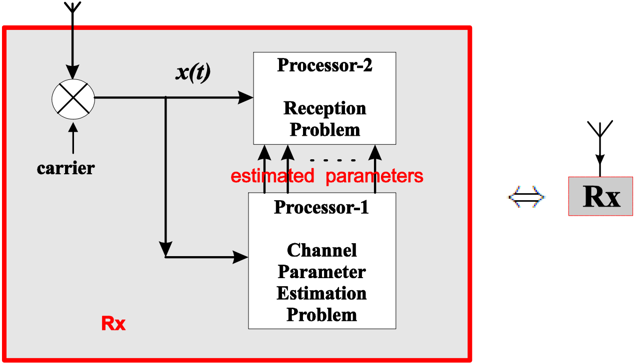
# Table of Contents

# SISO Wireless Channel Tx & Rx

- A wireless system can be partitioned into 3 main parts:
  1. **Tx** (a "**source** " that sends/transmits some information using wave propagation)
  2. **Wireless Channel** (the **physical propagation paths** )
  3. **Rx** ( a "**sink** " that receives the transmitted waves)

  and the objective in general is
  - to increase the **communication speed** (which is known as channel capacity)
    without sacrificing the **quality of service** (for a given energy + bandwidth)

# Generic Rx Architecture



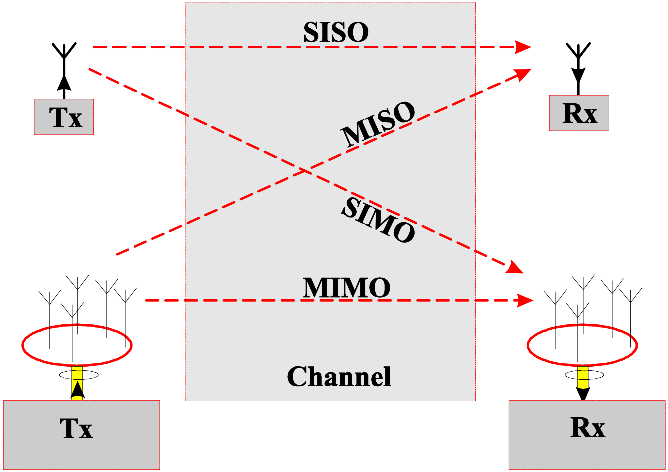

- The **quality of the receiver (Rx)** is a function of the **quality of the estimated channel parameters**
- Note that the receiver is **continuously designed** (based on these estimates) from **time frame** to **time frame.**

# Wireless Systems Classification

- There are many classifications. For instance:
  1. according to the **bandwidth/carrier**: **narrowband** or **wideband**
  2. according to the **spreading capabilities**: **conventional** or **spread spectrum**
  3. according to the **number of carriers**: **single carrier** or **multicarrier**
  4. according to the **"generation"**: 1G, 2G, **3G** , **3G+**
  5. according to the **"access"**: **TDMA,FDMA** , **CDMA** ,
- The **overall aims:**
  - **speed** $= \uparrow$,
  - **but maintaining reliability** (quality of service) & **spectral efficiency** (EUE,BUE)
- The current speed is expected to increase by the utilisation of the new technology of multiple antennas (MIMO) and this gives rise to **a new classification which super-sets** all the above.

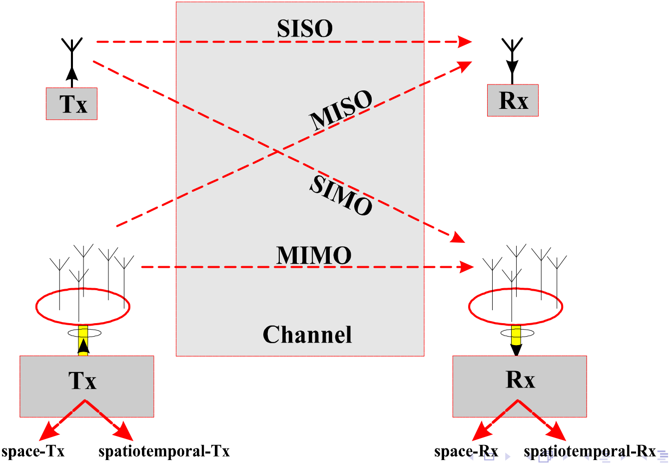
# New Wireless Systems Classification

- This **new** classification is according to the **number of antennas** used in both Tx and Rx



# New Wireless Systems Classification

- This **new** classification is also according to the **number of antennas** and the space-only or spatiotemporal signal processing used in both Tx and Rx (this is the main focus of this topic)



- **My Terminology**

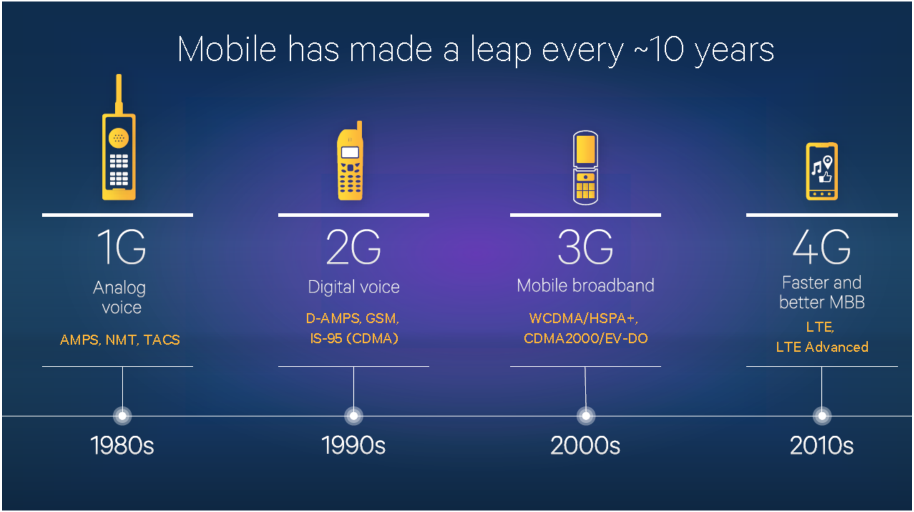
| **Terminology-1** (More Representative) | | |
|---|---|---|
| **1** | SISO: | Scalar-Input-Scalar-Output Channel |
| **2** | SIVO: | Scalar-Input-Vector-Output Channel |
| **3** | VISO: | Vector-Input-Scalar-Output Channel |
| **4** | VIVO: | Vector-Input-Vector-Output Channel |

- **Alternative Terminology**

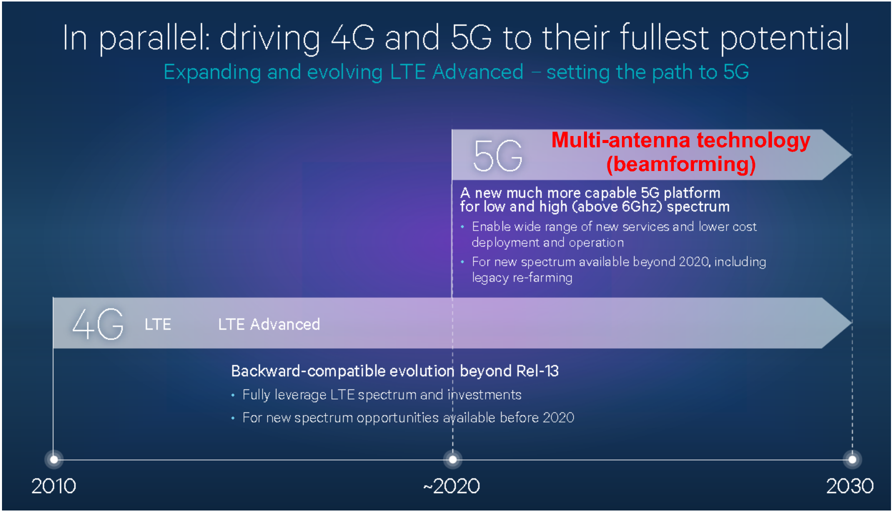
| **Terminology-2** (Initial) | | |
|---|---|---|
| **1** | SESE: | from Single-Element (SE) Tx to Single-Element (SE) Rx |
| **2** | SEME: | from Single-Element (SE) Tx to Multiple-Element (ME) Rx |
| **3** | MESE: | from Multiple-Element (ME) Tx to Single-Element (SE) Rx |
| **4** | MEME: | from Multiple-Element (ME) Tx to Multiple-Element (ME) Rx |

| **Terminology-3** (More Popular) | | |
|---|---|---|
| **1** | SISO: | Single-Input-Single-Output |
| **2** | SIMO: | Single-Input-Multiple-Output |
| **3** | MISO: | Multiple-Input-Single-Output |
| **4** | MIMO: | Multiple-Input-Multiple-Output |

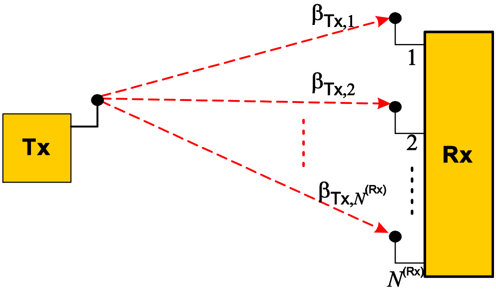
# Mobile Evolution - Motivation



# Mobile Evolution - Motivation (cont.)



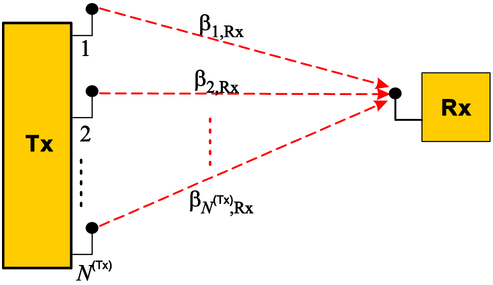
# SIMO Wireless Systems (non-parametric)




Single-Input Multiple-Output (SIMO)

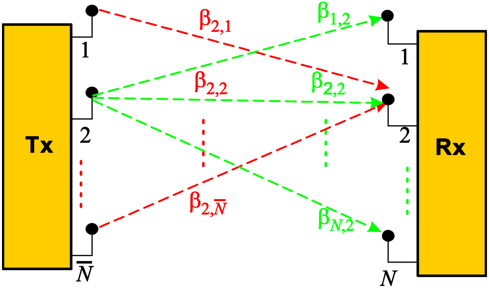
- Remember: SISO - one complex number $\beta$ per path

# MISO Wireless Systems (non-parametric)




Multiple-Input Single-Output (MISO)

# MIMO Wireless Systems (non-parametric)




Multiple-Input Multiple-Output (MIMO)

$$\begin{bmatrix} \beta_{1,1} & \beta_{1,2} & \cdots & \beta_{1,N^{(\mathrm{Tx})}} \\ \beta_{2,1} & \beta_{2,2} & \cdots & \beta_{2,N^{(\mathrm{Tx})}} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{N^{(\mathrm{Rx})},1} & \beta_{N^{(\mathrm{Rx})},2} & \cdots & \beta_{N^{(\mathrm{Rx})},N^{(\mathrm{Tx})}} \end{bmatrix}$$

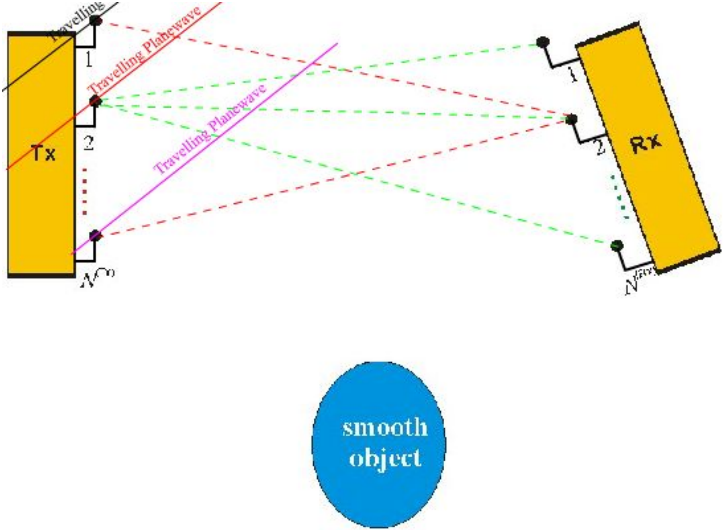
# Parametric Approaches

## Introduction

- The above modelling will result into a statistical approach (used in Wiener's estimation theory and Shannon's communication theory). e.g. many MIMO books, papers and tutorials: non parametric

- Although this approach is suitable for single antenna systems (i.e. SISO), it does not properly fit multiple antennas since it
  - ignores the **Cartesian coordinates** and orientations of Tx and Rx (i.e. ignoring the geometry/location of the multiple antennas),
  - ignores the **directions** of the signals,
  - ignores **propagation models** (planewaves or spherical waves),
  - etc.

# Revisiting Multiple-Input Multiple-Output (MIMO)



# Summary Table of SISO, MISO, SIMO and MIMO

| | Non-Parametric | Parametric (Array Processing) |
|---|---|---|
| **SISO:** | $\beta$ | |
| **SIMO:** | $\begin{bmatrix} \beta_{Tx,1} \\ \beta_{Tx,2} \\ \cdots \\ \beta_{Tx,N^{(\mathrm{Rx})}} \end{bmatrix}$ | $= \beta \underline{\mathrm{a}}^{(\mathrm{Rx})}$ |
| **MISO:** | $\begin{bmatrix} \beta_{1,\mathrm{Rx}} \\ \beta_{2,\mathrm{Rx}} \\ \cdots\cdot \\ \beta_{N^{(\mathrm{Tx})},\mathrm{Rx}} \end{bmatrix}$ | $= \beta \underline{\mathrm{a}}^{(\mathrm{Tx})}$ |
| **MIMO:** | $\begin{bmatrix} \beta_{1,1} & \beta_{1,2} & \cdots & \beta_{1,N^{(\mathrm{Tx})}} \\ \beta_{2,1} & \beta_{2,2} & \cdots & \beta_{2,N^{(\mathrm{Tx})}} \\ \cdots & \cdots & \cdots & \cdots \\ \beta_{N^{(\mathrm{Rx})},1} & \beta_{N^{(\mathrm{Rx})},2} & \cdots & \beta_{N^{(\mathrm{Rx})},N^{(\mathrm{Tx})}} \end{bmatrix}$ | $= \beta \underline{\mathrm{a}}^{(\mathrm{Rx})} \underline{\mathrm{a}}^{(\mathrm{Tx})^H}$ <br> $\Leftrightarrow \beta \underline{\mathrm{a}}^{(virtual)}$ |

# The Structure of the Array Response Vector

- From now on in this presentation the vector $\underline{a}$ will represent all multiple antenna wireless systems, i.e.

$$\underline{a} \triangleq \left\{ \begin{array}{ll} \underline{a}^{(\mathrm{Rx})} & \text{SIMO} \\ \underline{a}^{(\mathrm{Tx})} & \text{MISO} \\ \underline{a}^{(\mathrm{virtual})} = \underline{a}^{(\mathrm{Tx})} \otimes \underline{a}^{(\mathrm{Rx})} & \text{MIMO} \end{array} \right\}$$

- The vector $\underline{a}$ is known as
  - Array Manifold Vector or
  - Array Response Vector (alternative symbol $\underline{S}$)

- The vector $\underline{a}$ has a **profound mathematical structure** and is a function of a number of parameters such as Directions, carrier,etc

$$\underline{a}(\theta, \phi, F_c, c, \underline{r}_1, \underline{r}_2, \underline{r}_3, \ldots, \underline{r}_N)$$

# Note

- we can also add more wireless parameters from the Tx and Rx.

- For instance

  $\underline{a}(\theta, \phi, F_c, c, \underline{r}_x, \underline{r}_y, \underline{r}_z,$
  pseudorandom sequ, delay, polarisation parameters,
  No.of subcarriers/carriers, bandwidth, Doppler frequency).

- Various forms of $\underline{a}$ have different dimensions but always a **profound mathematical structure**
  This leads to Differential Geometry which complements the statistical signal processing and Shannon's communication theory in array processing problems and wireless systems.

# Differential Geometry

- **Differential geometry** is a branch of mathematics that is concerned with the application of differential calculus for the investigation of the properties of geometric **curves**, **surfaces** and **other objects** known as **'manifolds'**.

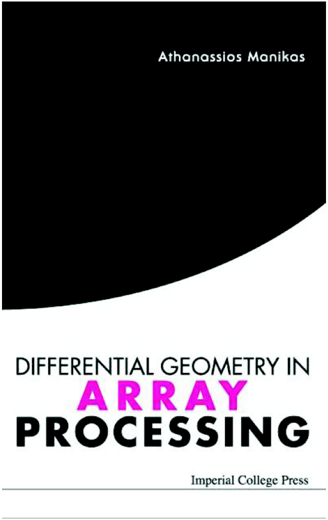
- Manifolds have a **deep** and **profound mathematical structure** and have been **an area of intense pure mathematical analysis**.

$$\underline{p} \longmapsto \text{mathematical object}$$

- In Physics, Albert Einstein (Nobel 1921) used differential geometry to express his **general theory of relativity**
  - where the universe is a **smooth manifold** equipped with pseudo-Riemannian metric (described the curvature of space-time).

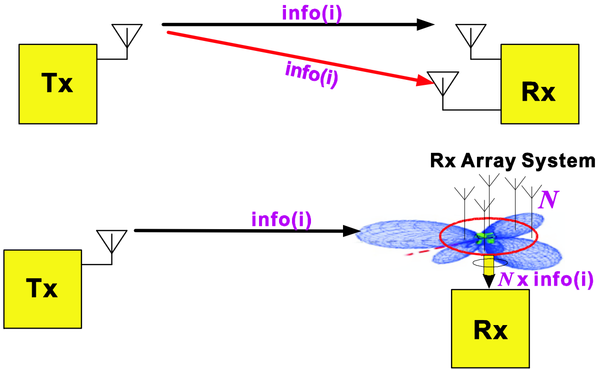
# Fundamental Questions

- Diff. Geom. helps **answering some fundamental questions** such as:
  - Q1 Is it possible to **express** a wireless system as a space curve or a surface (or a manifold - in general)?
  - Q2 Is it possible to **analyse** a wireless system **by analysing** a curve or a surface?
  - Q3 Is it possible to **design** a wireless system **by designing** a curve or a surface?
  - Q4 What do we **stand to gain** by expressing wireless systems as mathematical objects such as curves or surfaces?



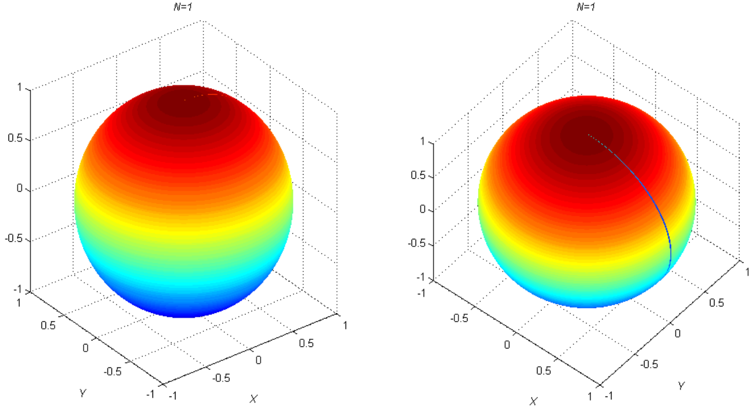
# Antenna Array Space Response

- **SIMO Example: Rx-Array Diversity**



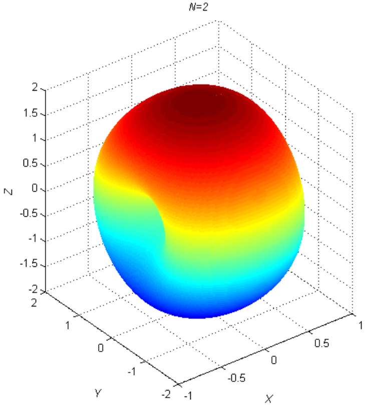

- Array systems (smart antennas) and techniques can be seen as the most sophisticated and advanced space diversity systems/techniques. (This type of systems/techniques will be considered in this course.)

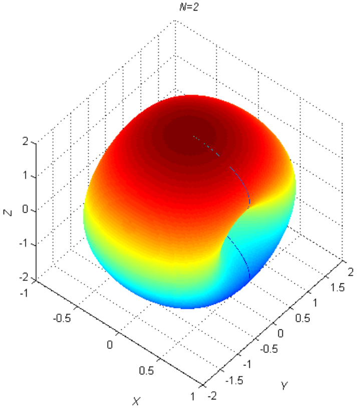
# Single Antenna (N=1): Space Response




(rotated by 90°)

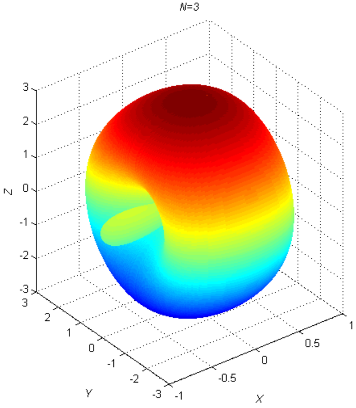
# Two Antennas (N=2): Space Response

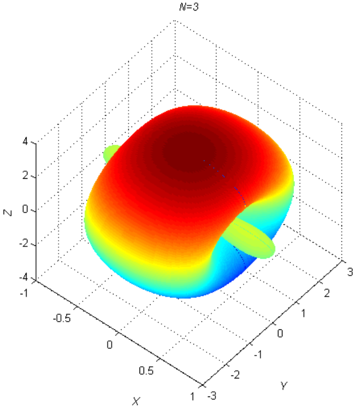



(rotated by 90°)

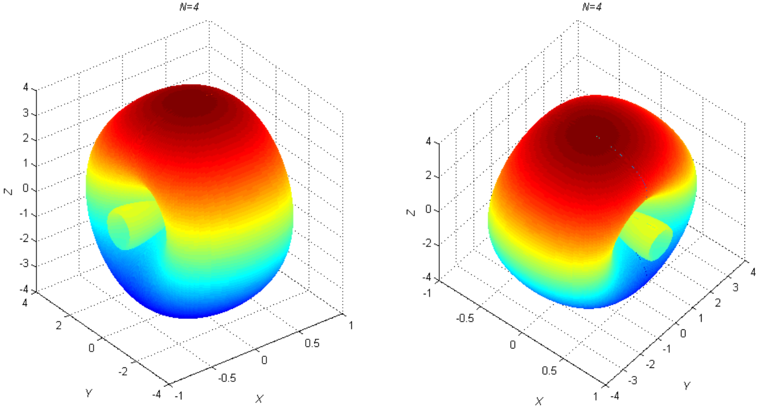
# Three Antennas (N=3): Space Response



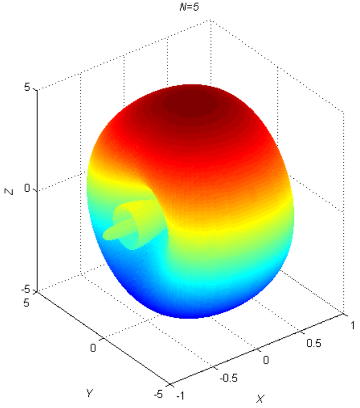

(rotated by 90°)

# Four Antennas (N=4): Space Response




(rotated by 90°)

# Five Antennas (N=5): Space Response



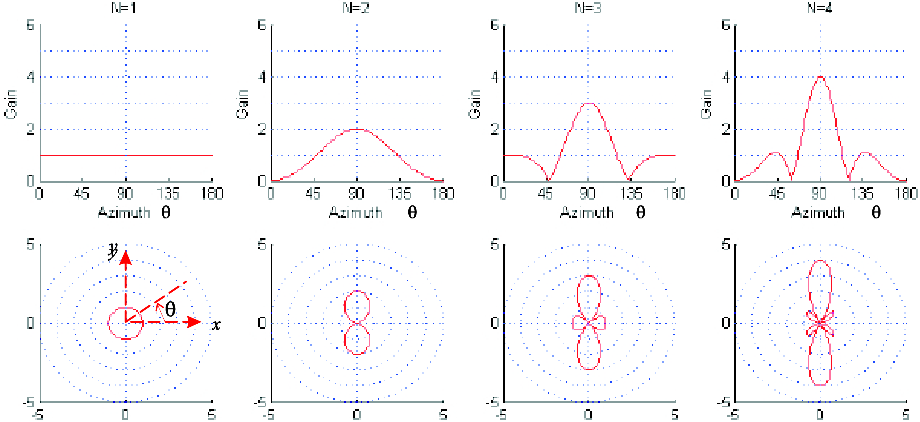

(rotated by 90°)

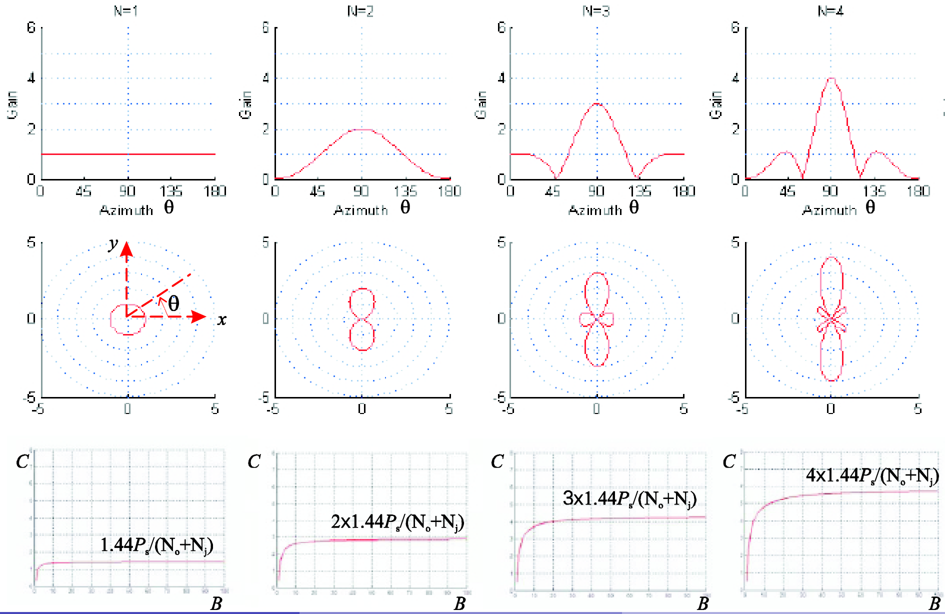
# Space-Only Example: Uniform Linear Array (ULA)

- Intersensor spacing$=\lambda/2$;
- $N$ = number of antennas (located on the x-axis).
- Channel Capacity (AWGN):

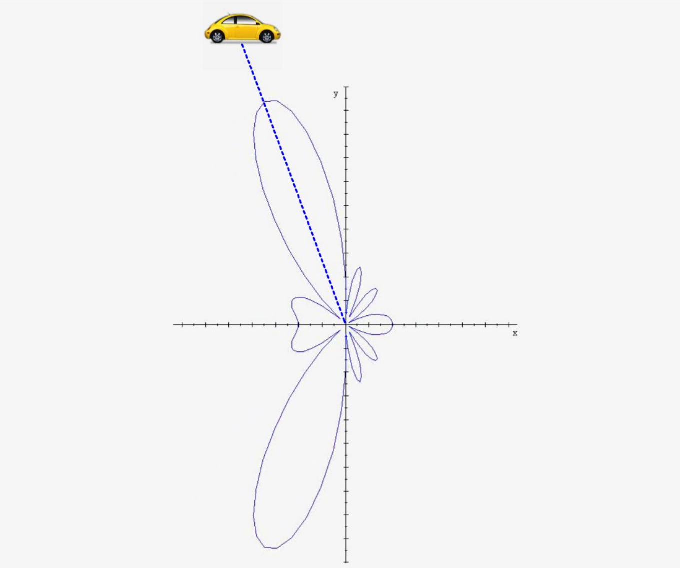
$$C = B\log_2(1 + N \times \text{SNIR}_{in}) \tag{1}$$

$$B \longrightarrow \infty \Rightarrow C \longrightarrow N \times 1.44\frac{P_s}{N_0 + N_j \downarrow} \tag{2}$$

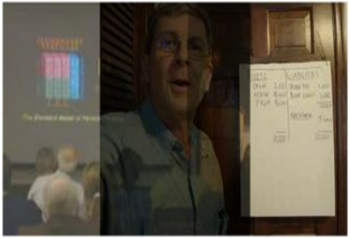

N=1
N=2
N=3
N=4
Gain
Azimuth θ
0
45
90
135
180
θ
x
y
5
0
-5


N=1
N=2
N=3
N=4
Gain
Azimuth θ
y
x
θ
C
B
1.44$P_s$/($N_o$+$N_j$)
2x1.44$P_s$/($N_o$+$N_j$)
3x1.44$P_s$/($N_o$+$N_j$)
4x1.44$P_s$/($N_o$+$N_j$)

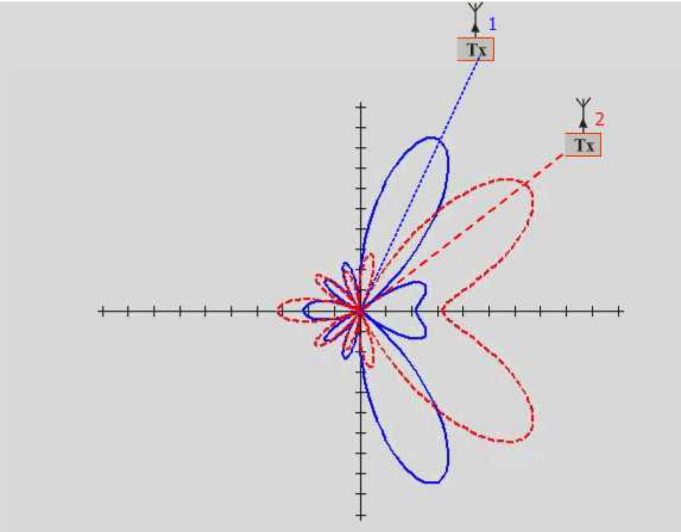
# SIMO Wireless Reception and Tracking (ULA, N=5)



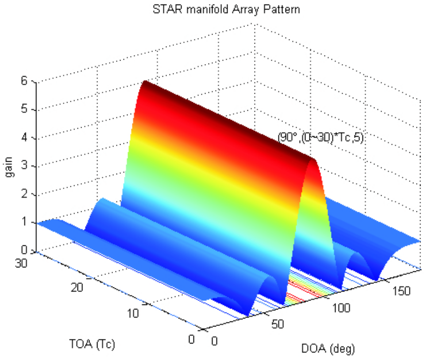
# Multiple Access Interference Cancellation (ULA, N=5)

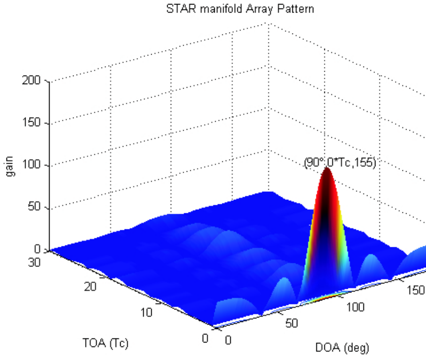


# Co-Channel Interference Cancellation with Motion (ULA, N=5)

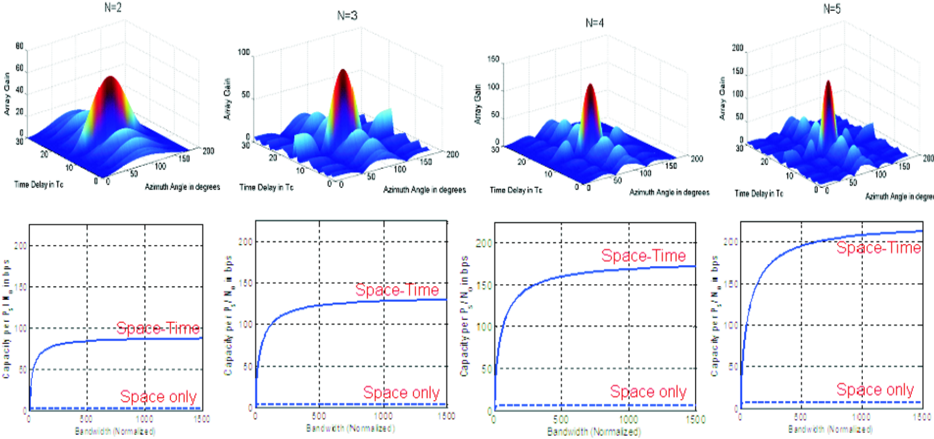


# Example: Space-only & Spatiotemporal Gain Patterns



**Spatio-Temporal Example:** PN-code of period 31; Uniform Linear Array (ULA) with intersensor spacing$=\lambda/2$; $N$ = number of antennas



- **SISO capacity** :

$$C = B\log_2(1+\mathrm{SNIR}_{out}) \quad \text{bits/sec} \tag{3}$$

- **MIMO Capacity** :

$$C = B\log_2\left(\frac{\det(\mathbb{R}_{xx})}{\det(\mathbb{R}_{nn})}\right) \quad \text{bits/sec} \tag{4}$$

- If **bandwidth B** $\longrightarrow \infty$ then $C = ?$

$$\text{SISO} \quad : \quad \lim_{B\to\infty} C = 1.44\frac{P_s}{N_0+N_J} \tag{5}$$

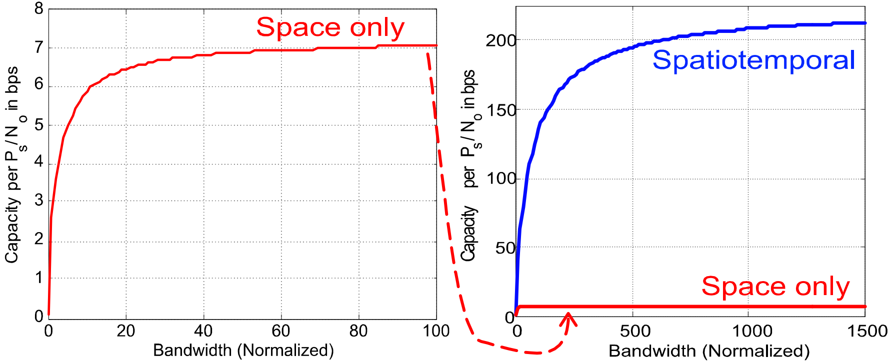
$$\text{space-only SIMO} \quad : \quad \lim_{B\to\infty} C = N \times 1.44\frac{P_s}{N_0+\underset{0}{\underset{\downarrow}{N_J}}} \tag{6}$$

$$\text{spatiotemporal-SIMO} \quad : \quad \lim_{B\to\infty} C = N \times N_{SP} \times 1.44\frac{P_s}{N_0+\underset{0}{\underset{\downarrow}{N_J}}} \tag{7}$$

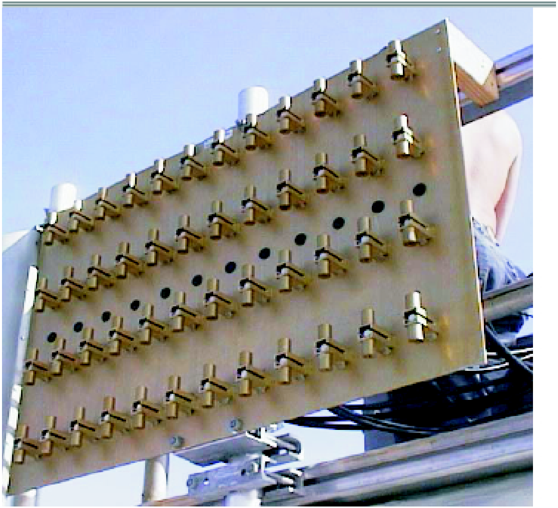
where $N$ denotes the number of array elements (antennas)

# Space and Spatiotemporal Capacity Curves

$N = 5$ antennas

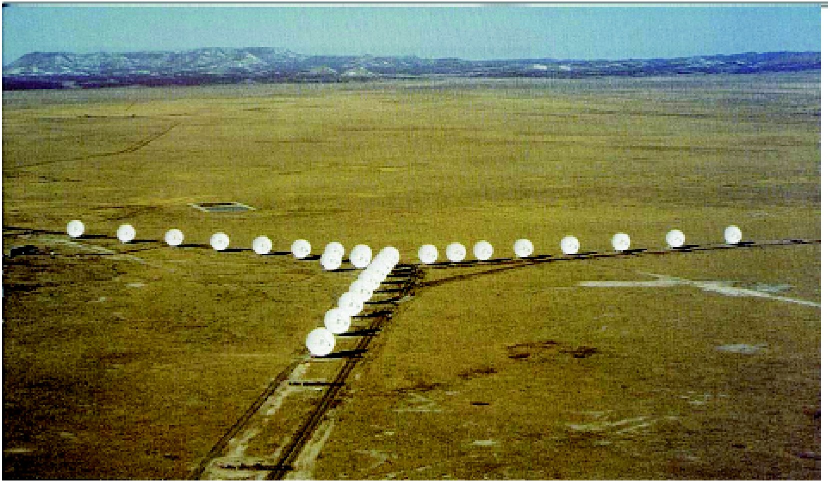


# A 2GHz Antenna Array of 48 Elements



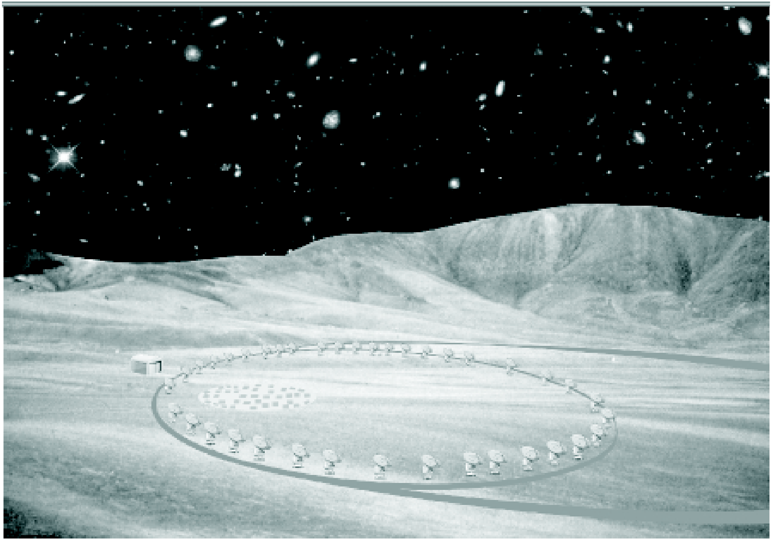
# Owens Valley Radio Observatory Array

# The New Mexico Very Large Array of 27 Elements

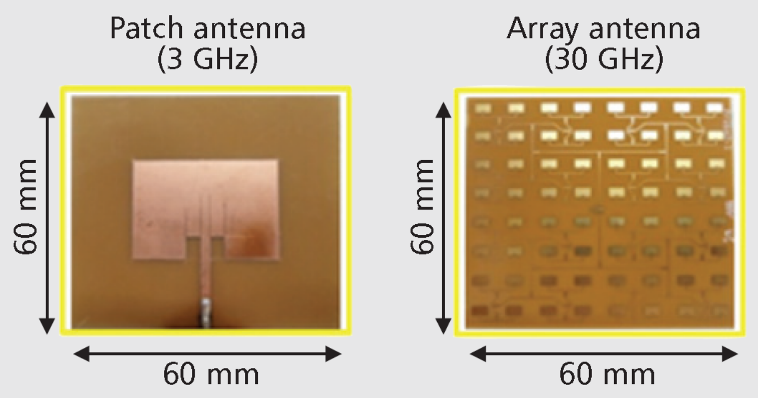


(along rail road tracks - 35km)

# A Large Circular Array



# Antenna Arrays for Modern Wireless Systems



# Antenna Arrays for Modern Wireless Systems (cont.)