E303: Communication Systems

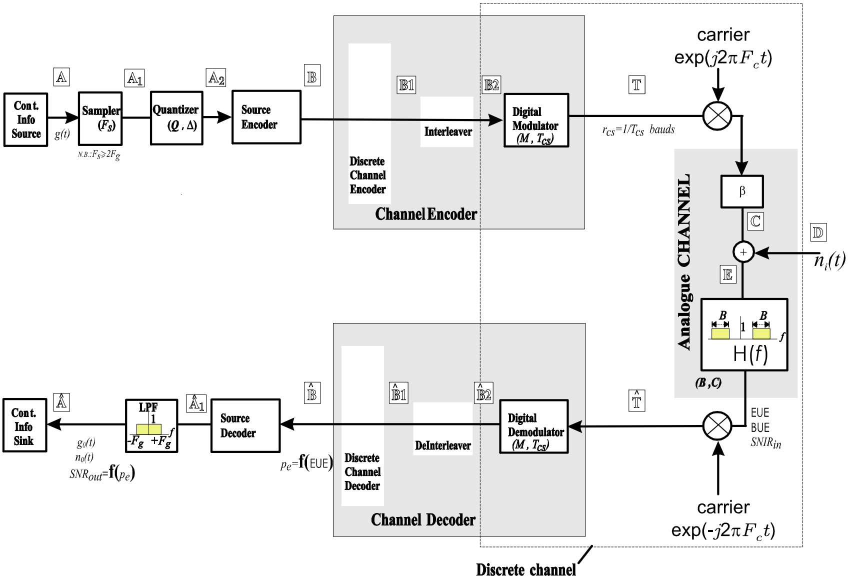
Professor A. Manikas
Chair of Communications and Array Processing

Imperial College London

Principles of PCM

Table of Contents

Introduction



- **PCM** $=$ **sampled quantized values** of an analogue signal are transmitted via a **sequence of codewords.**

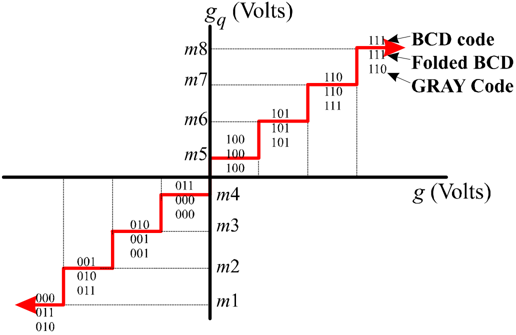
- i.e. after sampling & quantization, a Source Encoder is used to map the quantized levels (i.e. o/p of quantizer) to codewords of γ bits

  i.e. $\boxed{\textbf{quantized level}} \mapsto \boxed{\textbf{codeword of } \gamma \textbf{ bits}}$

  and, then a digital modulator is used to transmit the bits, i.e. PCM system

- There are three popular PCM source encoders (or, in other words, Quantization-levels Encoders).

  - **Binary Coded Decimal** (BCD) source encoder
  - **Folded BCD** source encoder
  - **Gray Code** (GC) source encoder



$$g(\text{input}) \mapsto g_q(\text{output})$$

$$g_q\text{: occurs at a rate } F_s \ \frac{\text{samples}}{\text{sec}}$$

$$(\text{N.B: } F_s \geq 2 \cdot F_g)$$

$$Q = \text{quantizer levels;}$$

$$\gamma = \log_2(Q) \ \frac{\text{bits}}{\text{level}}$$

- Note:

$$\boxed{\text{codeword rate (point B)}} = \boxed{\text{quant. levels rate}} = \boxed{\text{sampling rate}}$$

$$\underset{\frac{\gamma-\text{bit codewords}}{\text{sec}}}{\uparrow} \qquad \underset{\frac{\text{levels}}{\text{sec}}}{\uparrow} \qquad \underset{\frac{\text{samples}}{\text{sec}}}{\uparrow}$$

$$= F_s = 2F_g \qquad (1)$$

- bit rate: $\underset{\frac{\text{bits}}{\text{sec}}}{r_b} = \underset{\frac{\text{bits}}{\text{level}}}{\underset{\uparrow}{\gamma}} \; \underset{\frac{\text{levels}}{\text{sec}}}{\underset{\uparrow}{F_s}}$ e.g. for $Q = 16$ levels then $r_b = \underset{\gamma}{\underset{\uparrow}{4}} \; F_s$

  $$\text{(e.g. transmitted sequ.} = \overbrace{1010}^{\gamma=4}\underbrace{1100}_{\gamma=4}\overbrace{1101}^{\gamma=4}\ldots)$$

- versions of PCM:
  - Differential PCM (DPCM)$\triangleq$PCM with differential Quant.
  - Delta Modulation (DM): PCM with diff. quants having 2 levels i.e. $\underset{\substack{\text{are encoded using} \\ \text{a single binary digit}}}{\underset{\uparrow}{\underline{+\Delta \text{ or } -\Delta}}}$
  - Note: DM$\in$DPCM
  - Others

PCM: Bandwidth & Bandwidth Expansion Factor

- we transmit several digits for each quantizer's o/p level $\Rightarrow B_{PCM} > F_g$

$$\text{where} \begin{cases} B_{PCM} & \text{denotes the channel bandwidth} \\ F_g & \text{represents the message bandwidth} \end{cases}$$

- PCM Bandwidth
  baseband bandwidth:

$$\boxed{B_{PCM} \geq \frac{\text{channel symbol rate}}{2} \text{ Hz}} \tag{2}$$

  bandpass bandwidth:

$$\boxed{B_{PCM} \geq \frac{\text{channel symbol rate}}{2} \times 2 \text{ Hz}} \tag{3}$$

- Note that, by default, the Lower bound of the 'baseband' bandwidth is assumed and used in this course
- Bandwidth expansion factor β :

$$\beta \triangleq \frac{\text{channel bandwidth}}{\text{message bandwidth}} \tag{4}$$

- Example - Binary PCM
  - Bandwidth:

$$B_{PCM} = \frac{\text{channel symbol rate}}{2} = \frac{\text{bit rate}}{2} = \frac{\gamma F_s}{2} = \underset{\substack{\uparrow \\ \log_2 Q}}{\gamma} \; F_g \, Hz$$

$$\Rightarrow \boxed{B_{PCM} = \gamma F_g} \quad (5)$$

  - Bandwidth Expansion Factor:

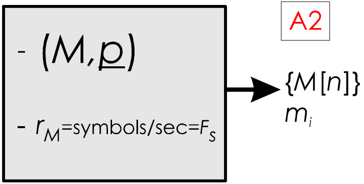
$$B_{PCM} = \gamma F_g \Rightarrow \frac{B_{PCM}}{F_g} = \gamma$$

$$\Rightarrow \boxed{\beta = \gamma} \quad (6)$$

The Quantization Process (output point-A2)

- at point $\boxed{\text{A2}}$:
  a signal discrete in amplitude and discrete in time.

  The blocks up to the point A2, combined, can be considered as a discrete information source where a discrete message at its output is a "level" selected from the output levels of the quantizer.

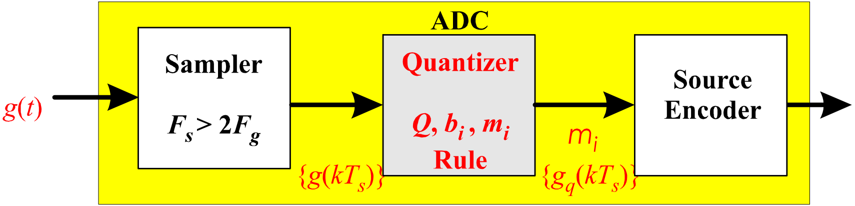


- $\boxed{\text{analogue samples}} \mapsto \boxed{\text{finite set of levels}}$

where the symbol $\mapsto$ denotes a "map"

In our case this **mapping** is called **quantizing**

i.e.



- quantizer parameters:

$$\begin{cases} Q : \text{number of levels} \\ b_i : \text{input levels of the quantizer, with } i = 0, 1, \dots, Q \\ \quad (b_0 = \text{lowest level}): \text{ known as quantizer's } \textbf{end-points} \\ m_i : \text{outputs levels of the quantizer} \\ \quad \text{(sampled values after quantization)} \\ \quad \text{with } i = 1, \dots, Q; \text{ known as } \textbf{output-levels} \\ \textit{rule:} \text{ connects the input of the quantizer to } m_i \end{cases}$$

RULE:

the sampled values $g(kT_s)$ of an analogue signal $g(t)$ are converted to one of Q allowable output-levels $m_1, m_2, \dots, m_Q$ according to the rule:

$$g(kT_s) \mapsto m_i \quad \text{(or equivalently } g_q(kT_s) = m_i)$$
$$\textbf{iff } b_{i-1} \leq g(kT_s) \leq b_i \quad \text{with } b_0 = -\infty, \; b_Q = +\infty$$

- quantization noise at each sample instance:

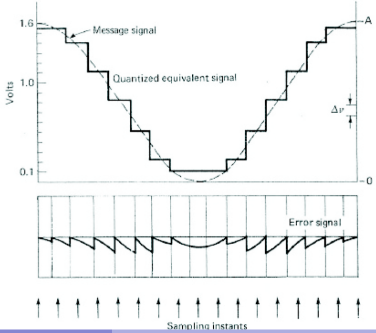
$$n_q(kT_s) = g_q(kT_s) - g_s(kT_s) \tag{7}$$

<u>**If**</u> the power of the quantization noise is small, i.e. $P_{n_q} = \mathcal{E}\left\{n_q^2(kT_s)\right\} =$ small,
<u>**then**</u> the quantized signal (i.e. signal at the output of the quantizer) is a good approximation of the original signal.

- **quality of approximation** may be improved by the careful choice of b_i's and m_i's and such as a measure of performance is optimized.

e.g. measure of performance: Signal to quantization Noise power Ratio (SNR_q)

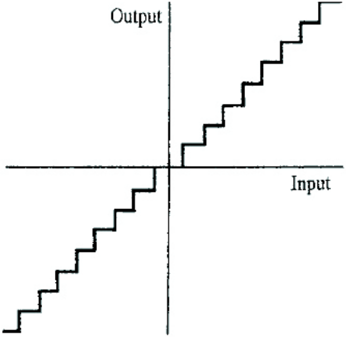
$$\text{SNR}_\text{q} = \frac{\text{signal power}}{\text{quant. noise power}} = \frac{P_g}{P_{nq}}$$



- Types of quantization: $\begin{cases} \text{uniform} \\ \text{non-uniform} \\ \text{differential} = \begin{cases} \text{uniform, or non-uniform} \\ \text{plus a differential circuit} \end{cases} \end{cases}$

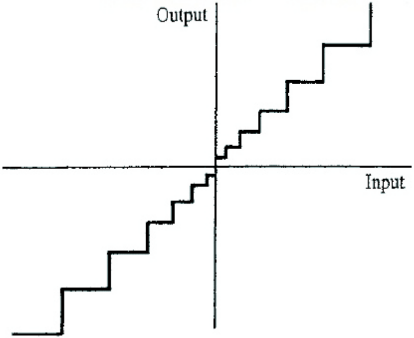
- Transfer Function:

uniform quantizer



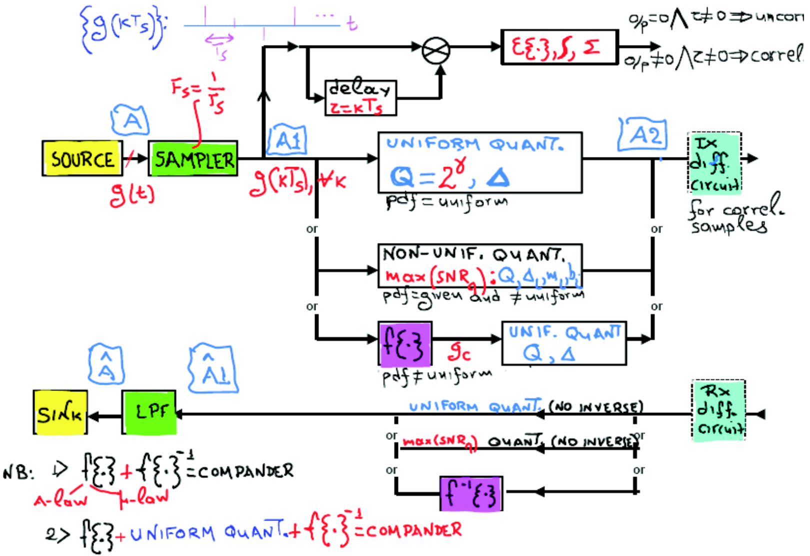
for signals with CF = small

non-uniform quantizer



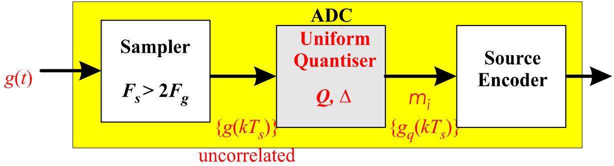
for signals with CF = large

The following figure illustrates the main characteristics of different types of quantizers



Uniform Quantizers

- Uniform quantizers are appropriate for uncorrelated samples



- let us change our notation: $g_q(kT_s)$ to g_q and $g(kT_s)$ to g
- the range of the continuous random variable g is divided into Q intervals of equal length Δ
- (value of g) $\mapsto$ (midpoint of the quantizing interval in which the value of g falls)

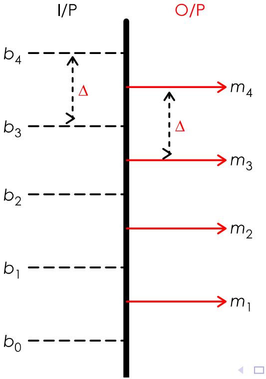
$$\text{or equivalently } m_i = \frac{b_{i-1} + b_i}{2} \text{ for } i = 1, 2, \ldots, Q \qquad (8)$$

- step size Δ:

$$\Delta = \frac{b_Q - b_0}{Q} \tag{9}$$

- rule:

$$\text{rule: } g_q = m_i \quad \text{iff} \quad b_{i-1} < g \le b_i \text{ where} \begin{cases} b_i = b_0 + i \cdot \Delta \\ m_i = \frac{b_{i-1}+b_i}{2} \end{cases} \tag{10}$$
$$\text{for } i = 1, 2, \ldots, Q$$



Comments on Uniform Quantiser

- Since, in general, $Q = \text{large} \Rightarrow P_{g_q} \simeq P_g \equiv \mathcal{E}\left\{g^2\right\}$
- Furthermore, large Q implies that Fidelity of Quantizer $= \uparrow$

$$g_q \simeq q$$

- $Q = 8 - 16$ are just sufficient for good intelligibility of speech;

  (but quantizing noise can be easily heard at the background)
  voice telephony: minimum <u>128 levels</u>; (i.e. $\text{SNR}_\text{q} \simeq 42\text{dB}$)

  N.B.: 128 levels $\Rightarrow$ 7-bits to represent each level
  $\Rightarrow$ transmission bandwidth $= \uparrow$

- if $\left\{ \begin{array}{l} \boxed{\text{Quantizer}} = \boxed{\text{UNIFORM}} \\ \boxed{\text{pdf of the input signal}} = \boxed{\text{UNIFORM}} \end{array} \right.$
  then
$$\boxed{\mathrm{SNR_q} = Q^2 = 2^{2\gamma}} \tag{11}$$
- Quantisation Noise Power P_{n_q}:
$$\text{Quantization Noise Power: } \boxed{P_{n_q} = \frac{\Delta^2}{12}} \tag{12}$$
- rms value of Quant. Noise:
$$\text{rms value of Quant. Noise} = \text{fixed} = \frac{\Delta}{\sqrt{12}} \neq \mathrm{f}\{g\} \tag{13}$$

$\therefore$ if $g(t)$ = small for extended period of time
$$\Rightarrow \qquad \boxed{\mathrm{SNR_q} < \text{the design value}} \tag{14}$$
$\uparrow$

this phenomenon is obvious

if the signal waveform has a large **CREST FACTOR**

- SNRq as a function of the Crest Factor

$$SNR_q = \frac{P_{g_q}}{P_{n_q}} = \frac{\sigma_{g_q}^2}{\Delta^2/12} = \left\{ \begin{array}{l} * \; \left. \begin{array}{c} +\hat{v} \\ \\ -\hat{v} \end{array} \right\} = \text{dynamic range} \\ * \; Q = \text{No. of quant. levels} \end{array} \right\} = \left\{ \begin{array}{l} * \; \Delta = \frac{2\hat{v}}{Q}, \quad Q = 2^{\gamma} \\ * \; \gamma = \text{No. of bits per quant. level} \end{array} \right\} =$$

$$= \frac{\sigma_{g_q}^2}{\frac{4\hat{v}^2}{12Q^2}} = 3Q^2 \frac{\sigma_{g_q}^2}{\hat{v}^2} = 3 \times 2^{2\gamma} \times CF^{-2} \Rightarrow$$

$$\boxed{SNR_q = 4.77 + 6\gamma - 20\log_{10} CF} \text{ in } \underline{dB}$$

- Remember:

$$\textbf{CREST FACTOR} \equiv \frac{\text{peak}}{\text{rms}} \tag{15}$$

- By using $\underbrace{\textbf{variable spacing}}$ $\Rightarrow$ CREST FACTOR effects $= \downarrow$

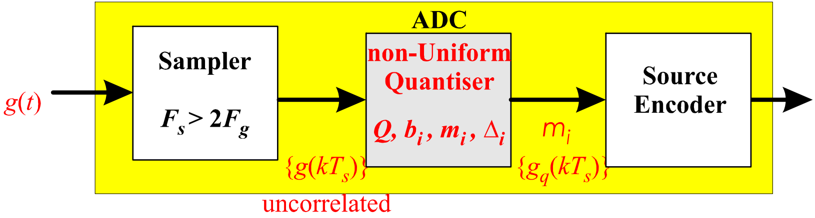
$\uparrow$

small spacing near 0 and
large spacing at the extremes

  - $\Longrightarrow$this leads to **NON-UNIFORM QUANTIZERS**

Non-Uniform Quantizers

- Non-Uniform quantizers are (like unif. quants) appropriate for **uncorrelated samples**



- step size $=$ variable $= \Delta_i$

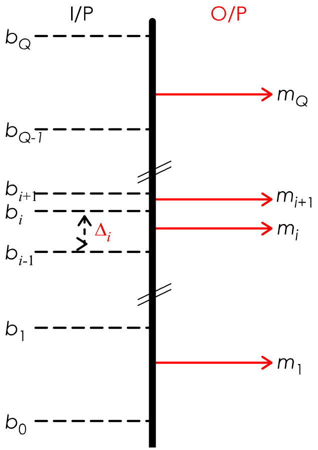
- <u>**if**</u> $\text{pdf}_{\text{i/p}} \neq$ uniform
  <u>**then**</u> non-uniform quants yield higher SNR_q than uniform quants

- rms value of n_q is not constant but depends on the sampled value $g(kT_s)$ of $g(t)$

- **rule:** $g_q = m_i \quad \text{iff} \quad b_{i-1} < g \leq b_i$

  $$\text{where } b_0 = -\infty,\ b_Q = +\infty \quad \Delta_i = b_i - b_{i-1} = \text{variable}$$

- example:



max(SNR) Non-Uniform Quantisers

- b_i, m_i are chosen to maximize SNR_q as follows:
  - since $Q = \text{large} \Rightarrow P_{g_q} \simeq P_g \equiv \mathcal{E}\left\{g^2\right\} \Rightarrow \text{SNR}_\text{q} = \max$ if $P_{n_q} = \min$ where
  $$P_{n_q} = \sum_{i=1}^{Q} \int_{b_{i-1}}^{b_i} (g - m_i)^2 \cdot \text{pdf}_g \cdot dg \tag{16}$$
  - Therefore:

$$\boxed{\min_{m_i, b_i} P_{n_q}} \tag{17}$$

$$(17) \iff \begin{cases} \frac{dP_{n_q}}{db_j} = 0 \\ \frac{dP_{n_q}}{dm_j} = 0 \end{cases} \tag{18}$$

$$\Rightarrow \begin{cases} (b_j - m_j)^2 \cdot \text{pdf}_g(b_j) - (b_j - m_{j+1})^2 \cdot \text{pdf}_g(b_j) = 0 & \text{for } j = 1, 2, \ldots, Q \\ -2 \cdot \int_{b_{j-1}}^{b_j} (g - m_j) \cdot \text{pdf}_g(g) \cdot dg = 0 & \text{for } j = 1, 2, \ldots, Q \end{cases} \tag{19}$$

Note:

- the above set of equations (i.e. (19)) cannot be solved in **closed form** for a general pdf. Therefore for a specific pdf an appropriate method is given below in a step-form:

METHOD:

1. choose a m_1
2. calculate b_i's, m_i's
3. check if m_Q is the mean of the interval $[b_{Q-1}, b_Q = \infty]$
   if yes $\longrightarrow$ STOP
   else $\longrightarrow$ choose a new m_1 and then goto step-2

A SPECIAL CASE

max(SNR) Non-Uniform Quantizer of a Gaussian Input Signal

- if the input signal has a Gaussian amplitude pdf, that is $\text{pdf}_\text{q} = \mathbb{N}(0, \sigma_g)$ then it can be proved that:

$$\boxed{P_{n_q} = 2.2\sigma_g^2 Q^{-1.96}} \tag{12}$$

$$\uparrow$$

not easy to derive

- In this case the Signal-to-quantization Noise Ratio becomes:

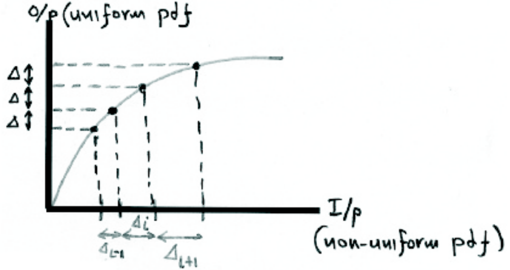
$$\boxed{\text{SNR}_\text{q} = \frac{P_{gq}}{P_{nq}} = \frac{\sigma_g^2}{2.2\sigma_g^2 Q^{-1.96}} = 0.45 Q^{1.96}} \tag{13}$$

Companders (non-Uniform Quantizers)

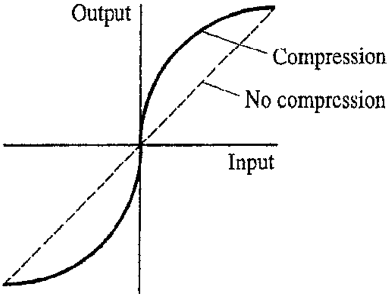
- Their performance independent of CF
- Non-unif. Quant $=$ $\boxed{\text{SAMPLE COMPRESSION}}$ $+$ $\boxed{\text{UNIFORM QUANTIZER}}$ $+$ $\boxed{\text{SAMPLE EXPANDER}}$
- Compressor $+$ Expander $\equiv$ **Compander**

$$g \overset{f}{\longmapsto} g_c \; i.e. \; \boxed{g_c = f\{g\} \quad \underset{\text{means "such that"}}{\underset{\uparrow}{:}} \quad \text{pdf}_{g_c} = \text{uniform}} \overset{f^{-1}}{\longmapsto} g_c$$

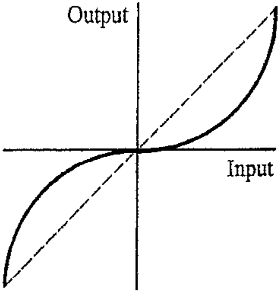
$\Delta = \frac{2}{Q}$



- Popular companders: use **log** compression

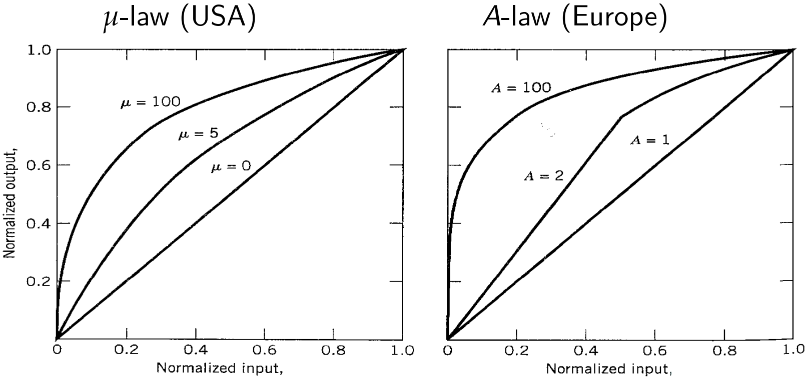


(a) Compression characteristic



(b) Expansion characteristic

- Two compression rules (A-law and μ-law) which are used in PSTN and provide a SNR_q **independent of signal statistics** **are given below:**



- In practice $\begin{cases} A \simeq 87.6 \\ \mu \simeq 100 \end{cases}$

Compression-Rules (PCM systems)

- The μ and A laws

μ-law	A-law
$g_c = \frac{\ln\left(1+\mu\cdot\left\vert\frac{g}{g_{\max}}\right\vert\right)}{\ln(1+\mu)} g_{\max}$	$g_c = \begin{cases} \frac{A\cdot\left\vert\frac{g}{g_{\max}}\right\vert}{1+\ln(A)}\cdot g_{\max} & 0 \leq \left\Vert\frac{g}{g_{\max}}\right\Vert < \frac{1}{A} \\ \frac{1+\ln\left(A\cdot\left\vert\frac{g}{g_{\max}}\right\vert\right)}{1+\ln(A)} g_{\max} & \frac{1}{A} \leq \left\Vert\frac{g}{g_{\max}}\right\Vert < 1 \end{cases}$

- where

$$\begin{aligned} g_c &= \text{compressor's output signal} \\ &\quad \text{(i.e. input to uniform quantiser)} \\ g &= \text{compressor's input signal} \\ g_{\max} &= \text{maximum value of the signal } g \end{aligned}$$

The 6dB LAW

- uniform quantizer:

$$\mathrm{SNR_q} = 4.77 + 6\gamma - 20\log(\mathrm{CF}) \quad dB \tag{20}$$

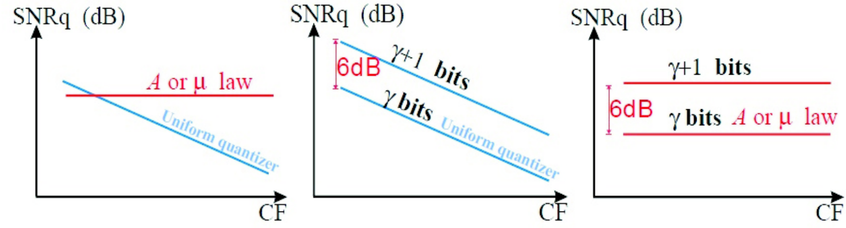
$$\text{remember } \mathrm{CF} = \frac{\text{peak}}{\text{rms}}$$

- μ-law:

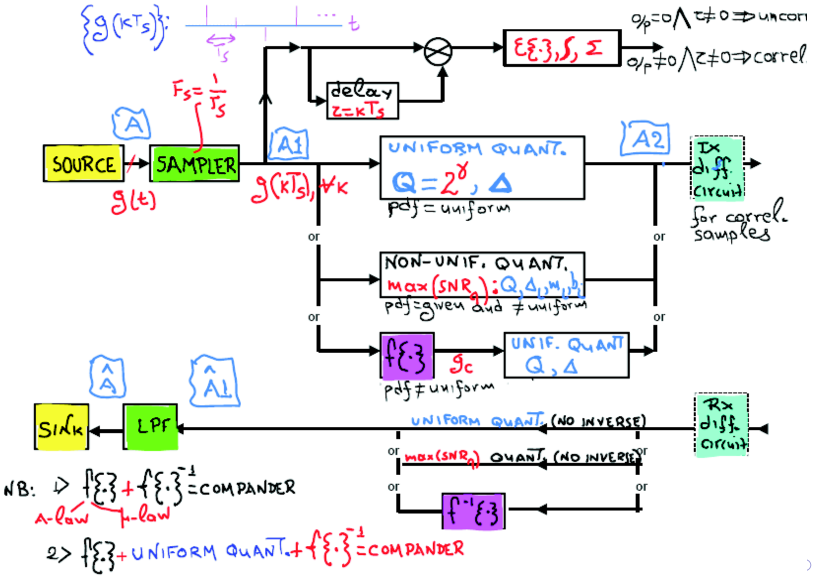
$$\mathrm{SNR_q} = 4.77 + 6\gamma - 20\log(\ln(1+\mu)) \quad dB \tag{21}$$

- A-law:

$$\mathrm{SNR_q} = 4.77 + 6\gamma - 20\log(1+\ln A) \quad dB \tag{22}$$



- REMEMBER the following figure (illustrates the main characteristics of different types of quantizers)



COMMENTS

- uniform & non-uniform quantizers:

  use them when samples are uncorrelated with each other (i.e. the sequence is quantized independently of the values of the preceding samples)

- practical situation:

  the sequence $\{g(kT_s)\}$ consists of samples which are correlated with each other. In such a case use **differential quantizer.**

Examples

- **PSTN**

  $F_s = 8\text{kHz},\ Q = 2^8\ (\text{A} = 87.6 \text{ or } \mu = 100),\ \gamma = 8 \text{ bits/level}$

  i.e. bit rate: $r_b = F_s \times \gamma = 8\text{k} \times 8 = 64 \text{ kbits/sec}$

- **Mobile-GSM**

  $F_s = 8\text{kHz},\ Q = 2^{13} \text{ uniform} \Rightarrow \gamma = 13 \text{ bits/level},$

  i.e. bit rate: $r_b = F_s \times \gamma = 8\text{k} \times 13 = 104 \text{ kbits/sec}$

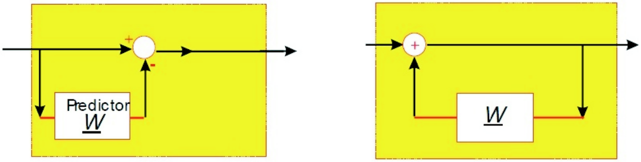
  which, with a differential circuit, is reduced to $r_b = 13 \text{ kbits/sec}$

Differential Quantizers

- $\boxed{\text{Differential quantizers are appropriate for } \textbf{correlated samples}}$ namely they take into account the sample to sample correlation in the quantizing process;
- e.g.

input (current message symbol)

Transmitter (Tx) Receiver (Rx)

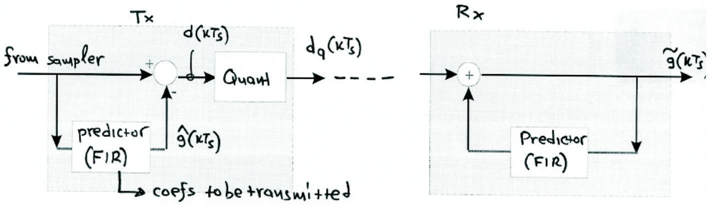


- The weights $\underline{w}$ are estimated based on autocorr. function of the input
- **The Tx & Rx predictors should be identical.**
  - Therefore, the Tx transmits also its weights to the Rx (i.e. weights $\underline{w}$ are transmitted together with the data)

- In practice, the variable being quantized is not $g(kT_s)$ but the variable $d(kT_s)$

where $$\boxed{d(kT_s) = g(kT_s) - \hat{g}(kT_s)} \qquad (14)$$

i.e.

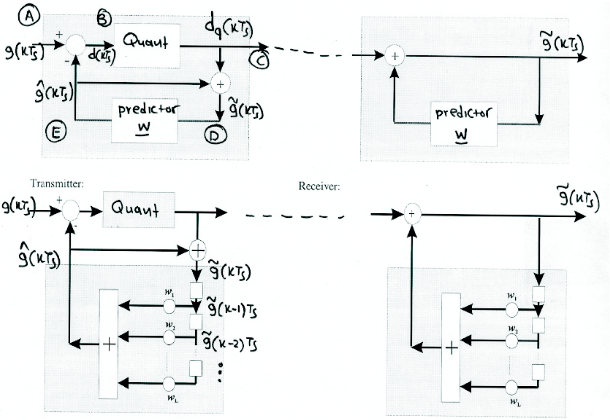


- Because $d(kT_s)$ has small variations, to achieve a certain level of performance, fewer bits are required. This implies that DPCM can achieve PCM performance levels with lower bit rates.
- 6dB law:

$$\boxed{\begin{array}{l} \text{SNR}_\text{q} = 4.77 + 6\gamma - \mathbf{a} \quad \textbf{in dB} \\ \hline \text{where} \quad -10\text{dB} < \mathbf{a} < 7.77\text{dB} \end{array}} \qquad (15)$$

A Better Differential Quantiser: mse Diff. Quant.

- the largest error reduction occurs when the differential quantizer operates on the differences between $g(kT_s)$ and the minimum mean square error (min-mse) estimator $\hat{g}(kT_s)$ of $g(kT_s)$ - (N.B.: but more hardware)



$$\hat{g}(kT_s) = \underline{w}^T \underline{\tilde{g}}$$

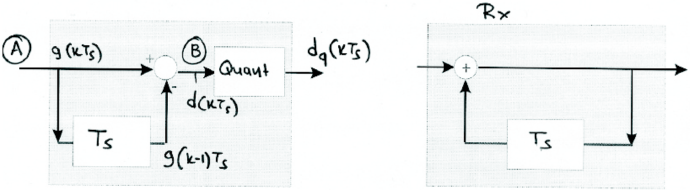
where

$$\begin{cases} \underline{\tilde{g}} = [\tilde{g}((k-1)T_s), \tilde{g}((k-2)T_s), \ldots, \tilde{g}((k-L)T_s)]^T \\ \underline{w} = [w_1, w_2, \ldots, w_L]^T \end{cases}$$

rule:

$$\begin{cases} \text{choose } \underline{w} \text{ to minimize } \mathcal{E}\left\{(g(kT_s) - \hat{g}(kT_s))^2\right\} \ \ldots \text{ for the Transmitter} \\ \text{choose } \underline{w} \text{ to minimize } \mathcal{E}\left\{(d_q(kT_s) + \hat{g}(kT_s))^2\right\} \ \ldots \text{ for the Receiver} \end{cases}$$

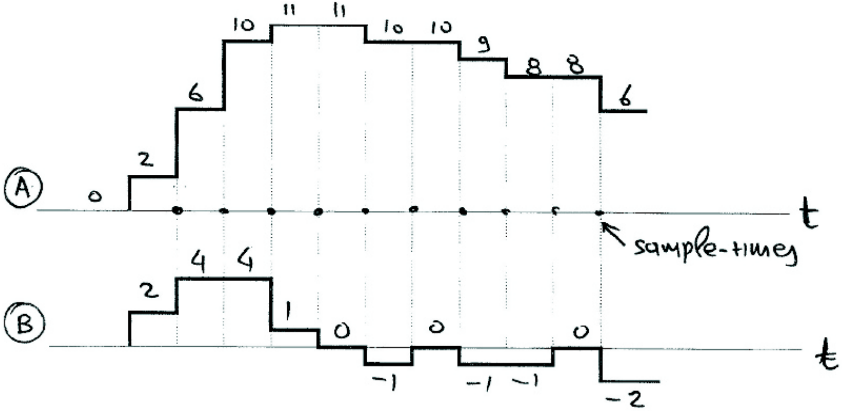
Differential Quantisers: Examples



- The power of $d(kT_s)$ can be found as follows:

$$
\begin{aligned}
\sigma_d^2 &= \mathcal{E}\left\{d^2\right\} \\
&= \underbrace{\mathcal{E}\left\{g^2(kT_s)\right\}}_{=\sigma_g^2} + \underbrace{\mathcal{E}\left\{g^2((k-1)T_s)\right\}}_{=\sigma_g^2} - 2\underbrace{\mathcal{E}\left\{g(kT_s)g((k-1)T_s)\right\}}_{2\cdot R_{gg}(T_s)} \\
&\Downarrow \\
\sigma_d^2 &= 2\cdot\sigma_g^2 - 2\cdot R_{gg}(T_s) = 2\cdot\sigma_g^2\cdot\left(1-\frac{R_{gg}(T_s)}{\sigma_g^2}\right) \qquad (23)
\end{aligned}
$$

- e.g.

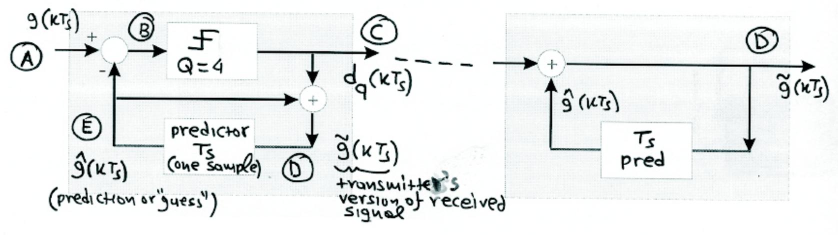


- disadvantages: unrecoverable degradation is introduced by the quantization process.
  - (Designer's task is to keep this to a subjective acceptable level)

Remember

1. $\sigma_g^2 = R_{gg}(0)$
2. $\frac{R_{gg}(\tau)}{\sigma_g^2}$ = is known as the *normalized* autocorrelation function
3. DPCM with the same No of bits/sample $\longrightarrow$ generally gives better results than PCM with the same number of bits.

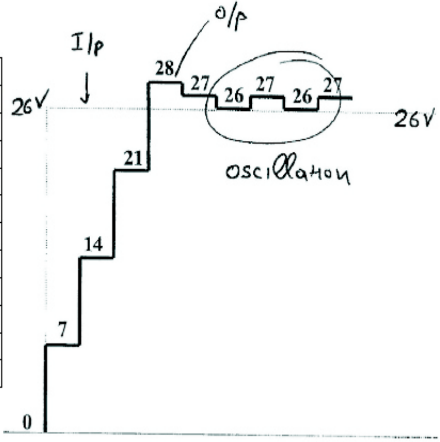
Example of mse DPCM



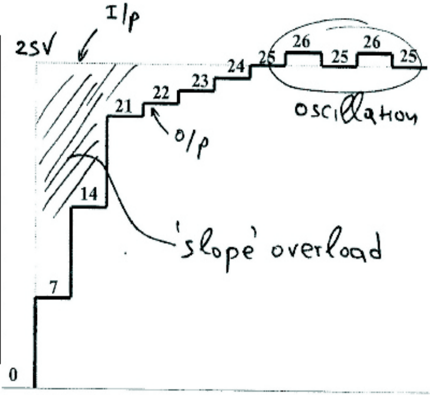
- assume a 4-level quantizer:

I/P	O/P
$+5 \leq$ input $\leq +255$	$+7$
$0 \leq$ input $\leq +4$	$+1$
$-4 \leq$ input ≤ -1	-1
$-255 \leq$ input ≤ -5	-7

INPUT step from 0V to 26V				
A_n	$E_n=D_{n-1}$	$B_n=A_n-E_n$	C_n	$D_n=C_n+E_n$
i/p	prediction	error	quant. error	o/p
26	0	26	+7	7
26	7	19	+7	14
26	14	12	+7	21
26	21	5	+7	28
26	28	−2	−1	27
26	27	−1	−1	26
26	26	0	+1	27
26				26
26				27



INPUT step from 0V to 25V				
A_n	$E_n=D_{n-1}$	$B_n=A_n-E_n$	C_n	$D_n=C_n+E_n$
i/p	prediction	error	quant. error	o/p
25	0	25	+7	7
25	7	18	+7	14
25	14	11	+7	21
25	21	4	+1	22
25	22	3	+1	23
25	23	2	+1	24
25	24	1	+1	25
25	25	0		26
25				25



- From the last 2 figures we can see that
  small variation to the i/p signal (25V $\Longleftrightarrow$ 26V)

$$\Downarrow$$

large variations to o/p waveforms

Noise Effects in a Binary PCM

- It can be proved that the Signal-to-Noise Ratio at the output of a binary Pulse Code Modulation (PCM) system, which employs a BCD encoder/decoder and operates in the presence of noise, is given by the following expression

$$\mathrm{SNR}_{\mathrm{out}} = \frac{\mathcal{E}\left\{g_0(t)^2\right\}}{\mathcal{E}\left\{n_0(t)^2\right\} + \mathcal{E}\left\{n_{q0}(t)^2\right\}} = \frac{2^{2\gamma}}{1 + 4 \cdot p_e \cdot 2^{2\gamma}} \tag{24}$$

where

$$\begin{aligned} p_e &= \text{f(type of digital modulator)} \\ p_e &= \mathrm{T}\left\{\sqrt{(1-\rho)\cdot \mathrm{EUE}}\right\} \end{aligned}$$

e.g. if the digital modulator is a PSK-mod. then

$$p_e = \mathrm{T}\left\{\sqrt{2 \cdot \mathrm{EUE}}\right\}$$

Threshold Effects in a Binary PCM

- We have seen that: $\text{SNR}_{\text{out}} = \frac{2^{2\gamma}}{1+4\cdot p_e \cdot 2^{2\gamma}}$
- Let us examine the following two cases: $\text{SNR}_{\text{in}} = \text{high}$ and $\text{SNR}_{\text{in}} = \text{low}$

i) SNR $_{\text{in}}$ = HIGH

$\text{SNR}_{\text{in}} = \text{high} \Rightarrow p_e = \text{small}$

$\Rightarrow 1 + 4 \cdot p_e \cdot 2^{2\gamma} \simeq 1$

$\Rightarrow \text{SNR}_{\text{out}} = 2^{2\gamma}$

$\Rightarrow \boxed{\text{SNR}_{\text{out}} \simeq 6\gamma \text{ dB}}$

ii) SNR $_{\text{in}}$ = LOW

$\text{SNR}_{\text{in}} = \text{low} \Rightarrow p_e = \text{large}$

$\Rightarrow 1 + 4 \cdot p_e \cdot 2^{2\gamma} \simeq 4 \cdot p_e \cdot 2^{2\gamma}$

$\Rightarrow \boxed{\text{SNR}_{\text{out}} \simeq \frac{1}{4\cdot p_e}}$

Threshold Point - Definition

- Threshold point is arbitrarily defined as the $\mathrm{SNR_{in}}$ at which the $\mathrm{SNR_{out}}$, i.e.

$$\mathrm{SNR_{out}} = \frac{2^{2\gamma}}{1 + 4 \cdot p_e \cdot 2^{2\gamma}}$$

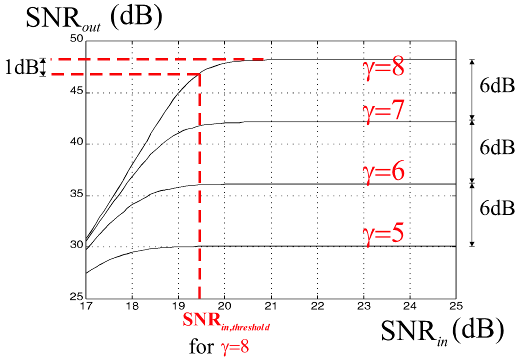
falls 1dB below the maximum $\mathrm{SNR_{out}}$
(i.e. 1dB below the value $2^{2\gamma}$).

- By using the above definition it can be shown (...for you...) that the threshold point occurs when

$$\boxed{p_e = \frac{1}{16 \cdot 2^{2\gamma}}}$$

where γ is the number of bits per level.

Threshold Effects



Comments

- The onset of threshold in PCM will result in a sudden $\uparrow$ in the output noise power.
- $P_{signal} = \uparrow \Rightarrow \text{SNR}_{\text{in}} = \uparrow \Rightarrow \text{SNR}_{\text{out}}$ reaches 6γ dB and becomes independent of P_{signal}
  $\therefore$ above threshold: increasing signal power $\Longrightarrow$ no further improvement in SNR_{out}
- The limiting value of SNR_{out} depends only on the number of bits γ per quantization levels

CCITT Standards: Differential PCM (DPCM)

- DPCM = PCM which employs a differential quantizer

  i.e. DPCM reduces the correlation that often exists between successive PCM samples

The CCITT standards $32\frac{\text{kbits}}{\text{sec}}$ DPCM	The CCITT standards $64\frac{\text{kbits}}{\text{sec}}$ DPCM
speech signal - $F_g = 3.2\text{kHz}$	audio signal - $F_g = 7\text{kHz}$
$F_s = 8\frac{\text{ksamples}}{\text{sec}}$	$F_s = 16\frac{\text{ksamples}}{\text{sec}}$
$Q = 16$ levels (i.e. $\gamma = 4\frac{\text{bits}}{\text{level}}$)	$Q = 16$ levels (i.e. $\gamma = 4\frac{\text{bits}}{\text{level}}$)

Problems of DPCM:

1. **slope overload noise:**
   occurs when outer quantization level is too small for large input transitions and has to be used repeatedly

2. **"Oscillation" or granular noise:**
   occurs when the smallest Q-level is not zero. Then, for constant input, the coder output oscillates with amplitude equal to the smallest Q-level.

3. **"Edge Busyness" noise:**
   occurs when repetitive edge waveform is contaminated by noise which causes it to be coded by different sequences of Q-levels.

Core and Access Networks