EE303: Communication Systems

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Introductory Concepts

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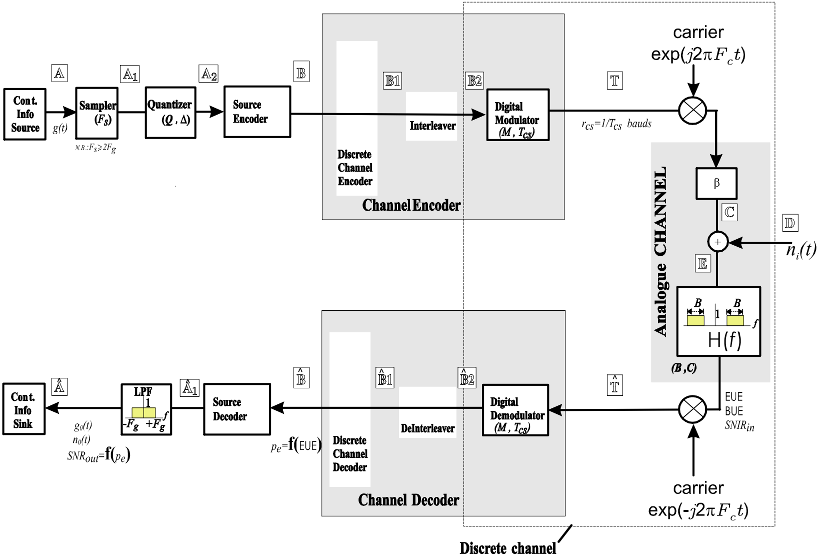
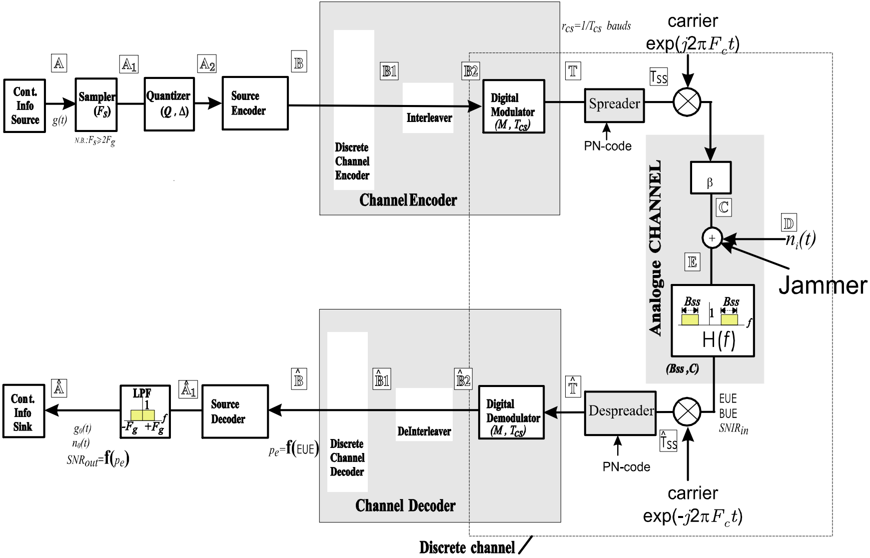
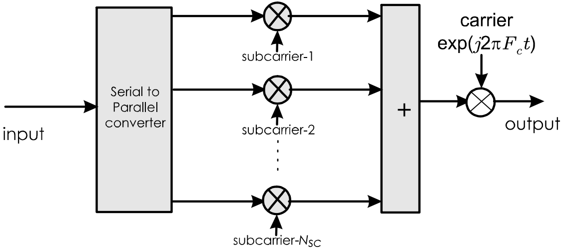
Block Structure of a Digital Comm System

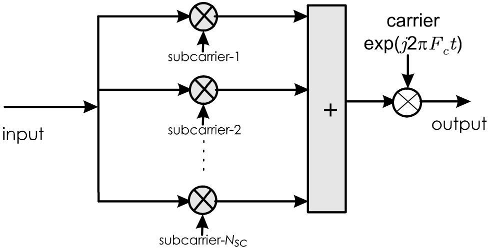


Block Structure of a Spread Spectrum Comm System



Multi-carrier Comm Systems

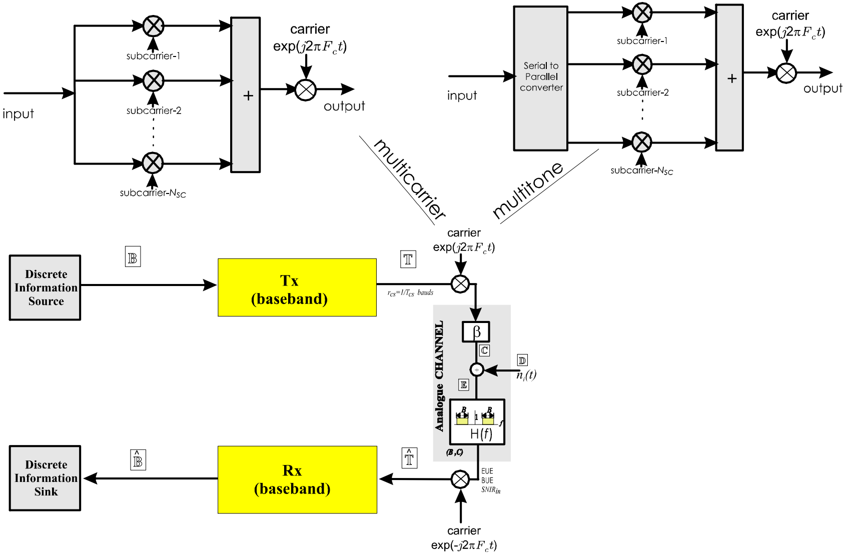




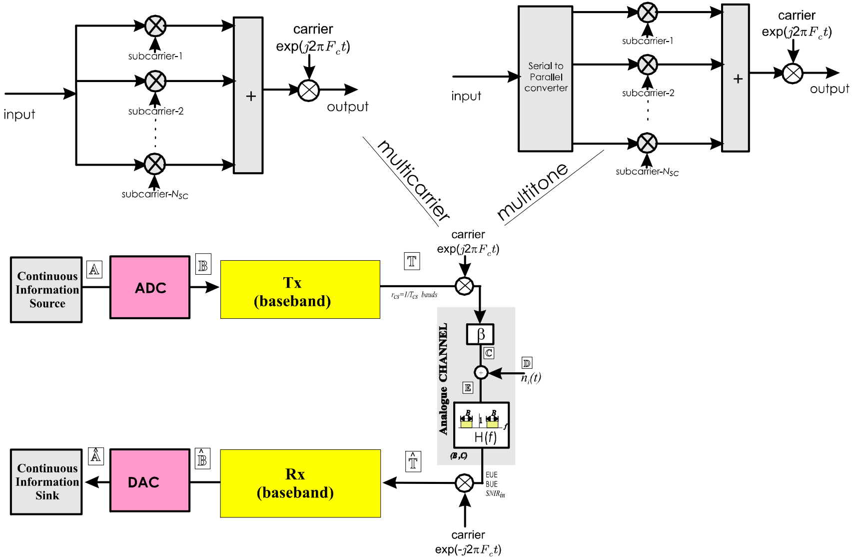
Simplified Block Diagrams

- A simplified and general block structure of a Digital Communication System is shown in the following page.
- it is common practice its **quality** to be expressed in terms of the accuracy with which the binary digits delivered at the output of the detector/Rx (point $\widehat{\mathrm{B}}$) represent the binary digits that were fed into the digital modulator/Tx (point B).
- It is generally taken that it is the fraction of the binary digits that are delivered back in error that is a measure of the quality of the communication system. This fraction, or rate, is referred to as the probability of a bit in error, or, **Bit-Error-Rate BER** (point $\widehat{\mathrm{B}}$).

Digital Communication



Digital Transmission of Analogue Signals



cont.

- N.B.: Quality is measured as
  - the Signal-to-Noise (SNR) at point $\widehat{\mathrm{T}}$ - known as **input SNR** (analogue signals degrade as noise level increases)
  - the SNR at point $\widehat{\mathrm{A}}$ - known as **output SNR**
  - **Bit-Error-Rate** at point $\widehat{\mathrm{B}}$
- Note that like Wireline and fiber communications wireless communications are also fully digital.
- It is clear from the previous discussion that signals (representing bits) propagate through the networks.
- Therefore the following sections are concerned with the main properties and parameters of communication signals.

Communication Signals

TRANSFORMATION

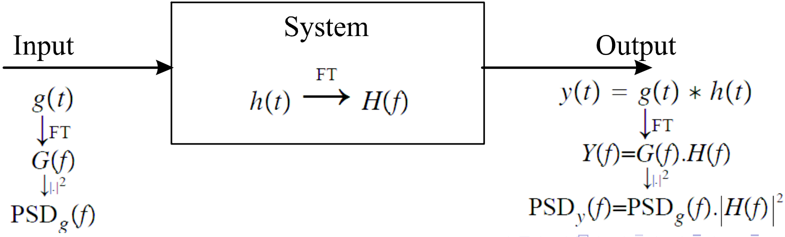
- Time Domain (TD) → Frequency Domain (FD), z-Domain, s-Domain, etc

- Frequency Domain (spectrum) is very important in Communications

$$\text{TD} \xrightarrow{\textbf{Fourier Transform}} \text{FD}$$

signal: $g(t)$ → $G(f)$

system: $h(t)$=impulse response → $H(f)$=transfer function

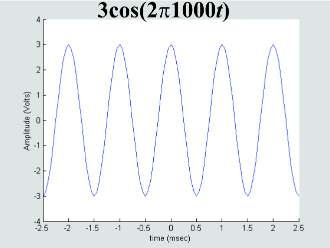


Classifications of Signals

According to their **description**

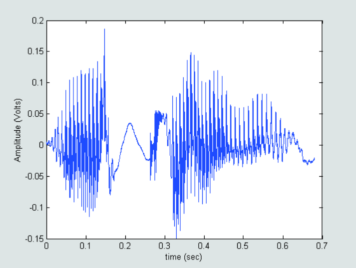
Deterministic Signals

* describable by mathematical functions



Random

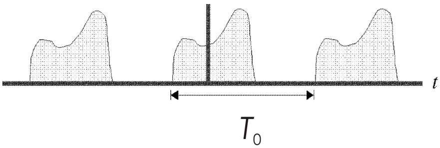
* these are unpredictable
* cannot be expressed as a function
* can be expressed probabilistically



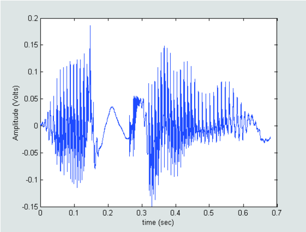
Classifications of Signals

According to their **periodicity**

Periodic



Non-Periodic



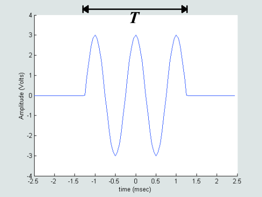
- N.B.:according to Fourier Series Theorem any periodic waveform can be represented by a sum of sinusoidals having frequencies $F_0 = \frac{1}{T_0}$, $2F_0$, $3F_0$, etc.

Classifications of Signals

According to their **Energy**

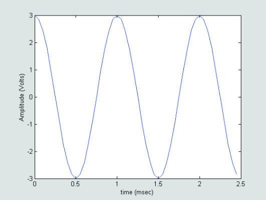
Energy Signals

(Energy$<\infty$)



Power Signals

(Energy$=\infty$)

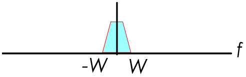


- N.B.:
  - Energy$= \int_{\infty}^{\infty} g(t)^2 dt$
  - Signals of finite duration are Energy Signals
  - Periodic Signals are Power Signals

Classifications of Signals

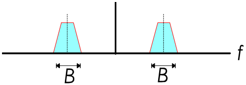
According to their **Spectrum**

Low Pass Signals



Bandwidth$=W$
W=max Freq.

Band Pass Signals



Bandwidth$=B$

Other types

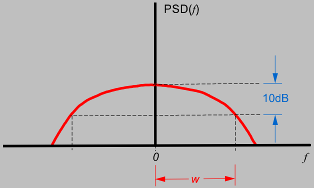
e.g all-pass, high-pass

Classifications of Signals (cont.)

According to their **Spectrum (Power Spectral Density, PSD(f))**

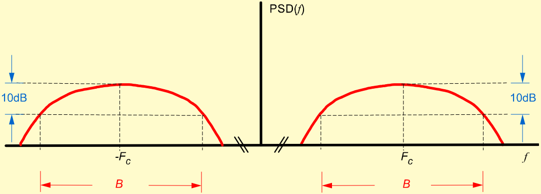
Low Pass Signals

(Bandwidth$=W$)



Band Pass Signals

Bandwidth$=B$



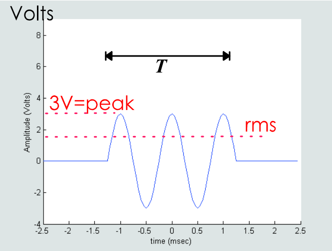
Fractional Bandwidth$=\frac{B}{F_c}$

Narrow Band	Wide Band	Ultra Wide Band
$0 < B_{fr} < 0.01$	$0.01 < B_{fr} < 0.25$	$0.25 < B_{fr} < 2.00$

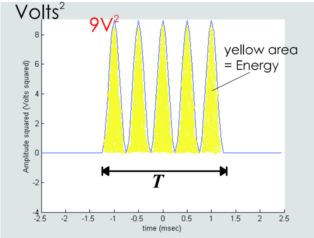
Some Signal Parameters

The figures below show the following parameters:

- peak (Volts) or peak-to-peak
- Energy (J) (or Power (W)) - over 1Ω resistor
- rms (Volts)
- Crest Factor: $\mathrm{CF} = \frac{peak}{rms}$



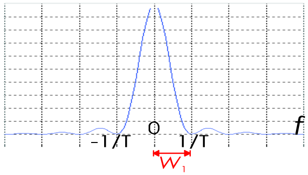
$\longrightarrow \xrightarrow{(.)^2}$

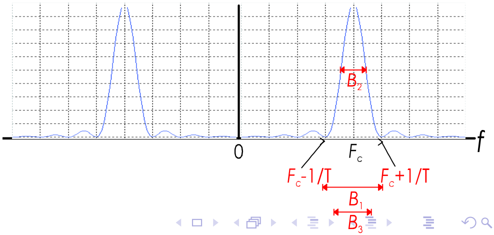


$$Power = \frac{Energy}{T}$$

Signal Bandwidth

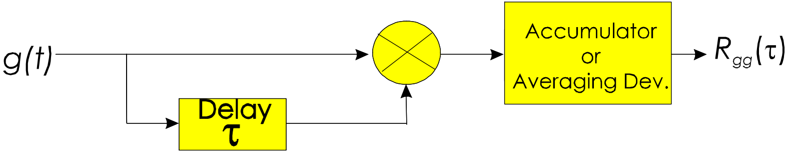
- Bandwidth of a signal: is the range of the significant frequency components in a signal waveform
- Examples of message signals (baseband signals) and their bandwidth:
  - television signal bandwidth 5.5MHz
  - speech signal bandwidth 4KHz
  - audio signal bandwidth 8kHz to 20kHz
- Note that there are various definitions of bandwidth, e.g.
  - 3dB bandwidth, (see B_2) or 10dB bandwidth
  - null-to-null bandwidth, (see $B_1 = \frac{2}{T}$,where T =signal duration).
  - Nyquist (minimum) bandwidth (see $B_3 = \frac{1}{T}$)





Redundancy - Autocorrelation

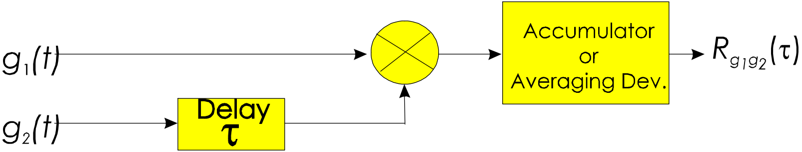
- The degree of **Redundancy** in a signal is provided by its **autocorrelation function** .
- For instance the autocorrelation function of a signal is $R_{gg}(\tau)$



- NB:
  - if $\tau =$ *fixed* then $R_{gg}(\tau) =$ a number
  - if $\tau =$ *variable* (i.e. $\forall\tau$) then $R_{gg}(\tau) =$ a function of τ

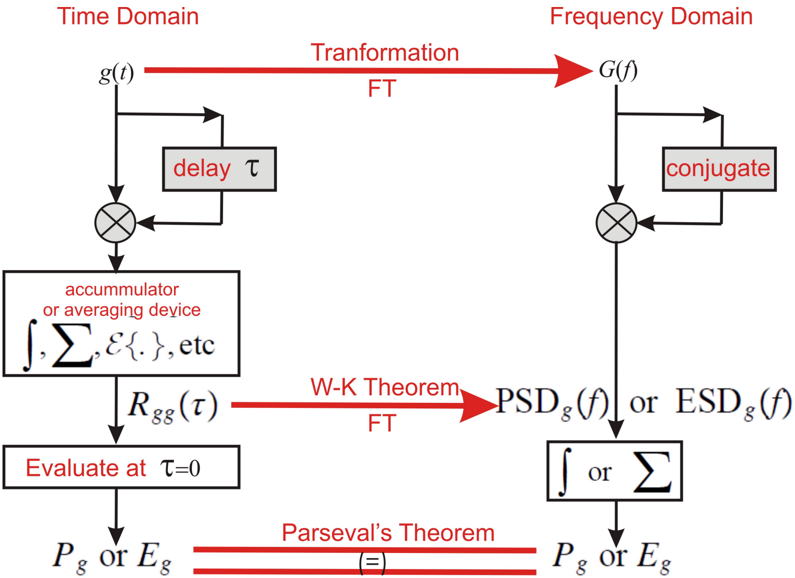
Similarity - Cross Correlation

- The degree of **Similarity** between two signals is given by their **cross-correlation function** .
- For instance the cross-correlation function between two signals $g_1(t)$ and $g_2(t)$ is $R_{g_1 g_2}(\tau)$



- NB:
  - if $\tau = \textit{fixed}$ then $R_{g_1 g_2}(\tau) =$ a number
  - if $\tau = \textit{variable}$ (i.e. $\forall\tau$) then $R_{g_1 g_2}(\tau) =$ a function of τ

Parseval's and Wiener-Khinchin(W-K) Theorems



"Accumulators" and "Averaging" Devices

- Definitions

Accumulators:	$\int_{t_1}^{t_2}$	$\sum_{i=1}^{M}$	
Averaging:	$\frac{1}{t_2-t_1}\int_{t_1}^{t_2}$	$\frac{1}{M}\sum_{i=1}^{M}$	$\mathcal{E}\{.\}$

- Examples

Accumulators corresponding to Energy	$R_{gg}(\tau) \triangleq \int_{t_1}^{t_2} g(t).g(t-\tau)dt$ $\text{ESD}_g(f) = \text{FT}\{R_{gg}(\tau)\}$
Averaging corresponding to Power	$R_{gg}(\tau) \triangleq \frac{1}{t_2-t_1}\int_{t_1}^{t_2=t_1+\tau} g(t).g(t-\tau)dt$ $R_{gg}(\tau) \triangleq \mathcal{E}\{g(t).g(t-\tau)\}$ $\text{PSD}_g(f) = \text{FT}\{R_{gg}(\tau)\}$

Woodwards's Notation and FT

- The evaluation of FT, that is

$$\mathrm{FT}\{g(t)\} = G(f) \triangleq \int_{-\infty}^{\infty} g(t).\exp(-j2\pi ft).dt \qquad (1)$$

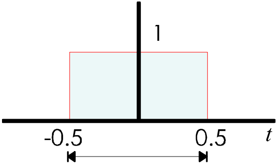
$$\mathrm{FT}^{-1}\{G(f)\} = g(t) \triangleq \int_{-\infty}^{\infty} G(f).\exp(+j2\pi ft).df \qquad (2)$$

  involves integrating the product of a function and a complex exponential - which can be difficult; so tables of useful transformations are frequently used.

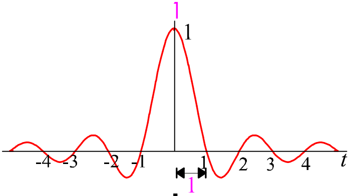
- However, the use of tables is greatly simplified by employing Woodward's notation for certain commonly occurring situations.
- Main advantage of using Woodward's notation: allows periodic time/frequency functions to be handled with FT rather than Fourier Series.

cont.

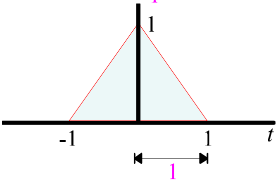
1. $\text{rect}\{t\} \triangleq \begin{cases} 1 & \text{if } |t| < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$



2. $\text{sinc}\{t\} \triangleq \dfrac{\sin(\pi t)}{\pi t}$

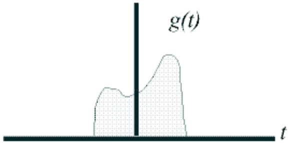


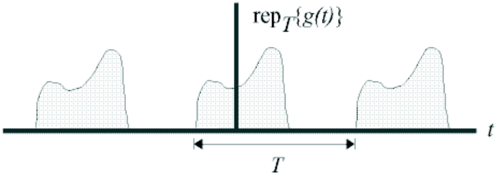
3. $\Lambda\{t\} \triangleq \begin{cases} 1 - t & \text{if } 0 \leq t \leq 1 \\ 1 + t & \text{if } -1 \leq t \leq 0 \end{cases}$



cont.

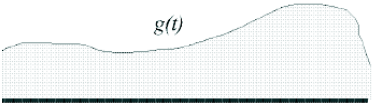
4. $\text{rep}_T\{g(t)\} \triangleq \sum_{n=-\infty}^{+\infty} g(t-nT) \qquad \forall n: .... -2, -2, 1, 0, 1, ...$

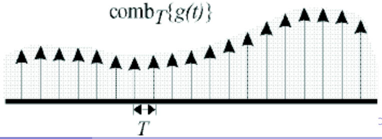




5. $\text{comb}_T\{g(t)\} \triangleq \sum_{n=-\infty}^{+\infty} g(nT).\delta(t-nT)$

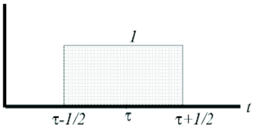
also known as **sampling function**





Examples

- we can generate any desired "rect" function by scaling and shifting - see for instance the following table

<table>
<tr><th>shifting</th><th>scaling</th><th>shifting+scaling</th></tr>
<tr>
<td>$g(t) = \text{rect}\{t-\tau\}$</td>
<td>$g(t) = \text{rect}\left\{\frac{t}{T}\right\}$</td>
<td>$g(t) = \text{rect}\left\{\frac{t-\tau}{T}\right\}$</td>
</tr>
<tr>
<td>
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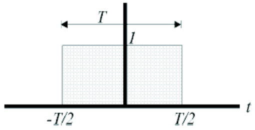
<td>
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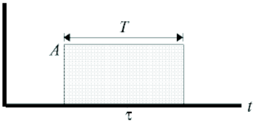
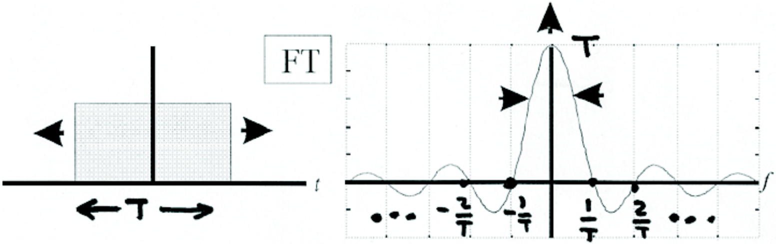
<td>
</td>
</tr>
<tr>
<td>$-\frac{1}{2} \leq t-\tau \leq \frac{1}{2}$
$\Downarrow$
$\tau-\frac{1}{2} \leq t \leq \tau+\frac{1}{2}$</td>
<td>$-\frac{1}{2} \leq \frac{t}{T} \leq \frac{1}{2}$
$\Downarrow$
$-\frac{T}{2} \leq t \leq \frac{T}{2}$</td>
<td>$-\frac{1}{2} \leq \frac{t-\tau}{T} \leq \frac{1}{2}$
$\Downarrow$
$\tau-\frac{T}{2} \leq t \leq \tau+\frac{T}{2}$</td>
</tr>
</table>

Effects of Scaling



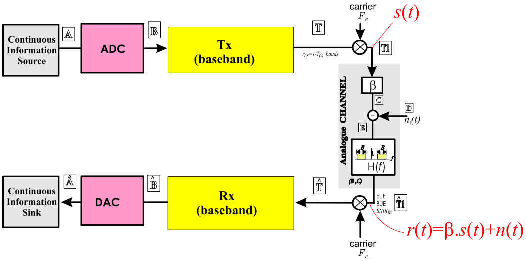
as $T \to \infty$ $\Rightarrow$ FT becomes **narrower** and amplitude **rises**

$\Rightarrow$ δ-function at 0 frequency when $T \to \infty$

Fourier Transform Tables

	Description	Function	Transformation
1	Definition	$g(t)$	$G(f)=\int_{-\infty}^{\infty} g(t).e^{-j2\pi ft}dt$
2	Scaling	$g\left(\frac{t}{T}\right)$	$\lvert T\rvert .G(fT)$
3	Time shift	$g(t-T)$	$G(f).e^{-j2\pi fT}$
4	Frequency shift	$g(t).e^{j2\pi Ft}$	$G(f-F)$
5	Complex conjugate	$g^*(t)$	$G^*(-f)$
6	Temporal derivative	$\frac{d^n}{dt^n}g(t)$	$(j2\pi f)^n.G(f)$
7	Spectral derivative	$(-j2\pi t)^n.g(t)$	$\frac{d^n}{df^n}G(f)$
8	Reciprocity	$G(t)$	$g(-f)$
9	Linearity	$A.g(t)+B.h(t)$	$A.G(f)+B.H(f)$
10	Multiplication	$g(t).h(t)$	$G(f)*H(f)$
11	Convolution	$g(t)*h(t)$	$G(f).H(f)$
12	Delta function	$\delta(t)$	1
13	Constant	1	$\delta(f)$
14	Rectangular function	$\mathbf{rect}\{t\} \triangleq \begin{cases} 1 & \text{if } \lvert t\rvert<\frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$	$\mathbf{sinc}\{f\} \triangleq \frac{\sin(\pi f)}{\pi f}$
15	Sinc function	$\mathbf{sinc}(t)$	$\mathbf{rect}\{f\}$
16	Unit step function	$u(t) \triangleq \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$	$\frac{1}{2}\delta(f)-\frac{j}{2\pi f}$
17	Signum function	$\mathrm{sgn}(t) \triangleq \begin{cases} 1 & t>0 \\ -1 & t<0 \end{cases}$	$-\frac{j}{\pi f}$
18	decaying exp (two-sided)	$e^{-\lvert t\rvert}$	$\frac{2}{1+(2\pi f)^2}$
19	decaying exp (one-sided)	$e^{-\lvert t\rvert}.u(t)$	$\frac{1-j2\pi f}{1+(2\pi f)^2}$
20	Gaussian function	$e^{-\pi t^2}$	$e^{-\pi f^2}$
21	Lambda function	$\Lambda\{t\} \triangleq \begin{cases} 1-t & \text{if } 0\le t\le 1 \\ 1+t & \text{if } -1\le t\le 0 \end{cases}$	$\mathbf{sinc}^2\{f\}$
22	Repeated function	$\mathbf{rep}_T\{g(t)\}=g(t)*\mathbf{rep}_T\{\delta(t)\}$	$\left\lvert\frac{1}{T}\right\rvert \mathbf{comb}_{\frac{1}{T}}\{G(f)\}$
23	Sampled function	$\mathbf{comb}_T\{g(t)\}=g(t).\mathbf{rep}_T\{\delta(t)\}$	$\left\lvert\frac{1}{T}\right\rvert \mathbf{rep}_{\frac{1}{T}}\{G(f)\}$

Basic Performance Criteria

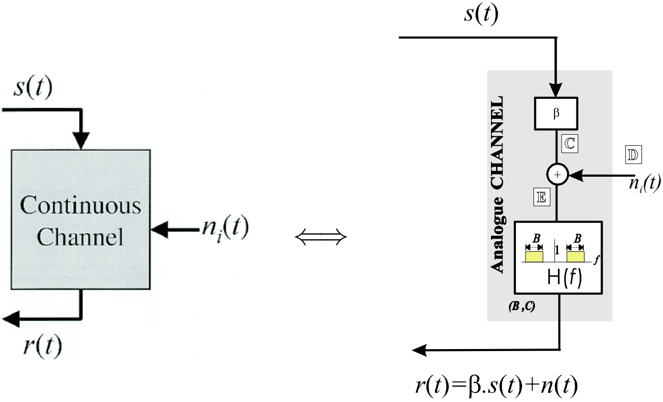


$$\mathrm{SNR}_{in} = \frac{\text{Power of signal at } \hat{\mathrm{T}}}{\text{Power of noise at } \hat{\mathrm{T}}} = \frac{\mathcal{E}\left\{(\beta s(t))^2\right\}}{\mathcal{E}\left\{n(t)^2\right\}} = \frac{\beta^2 P_s}{\underbrace{N_0 B}_{\triangleq P_n}} \quad (3)$$

$$p_e = \text{BER at point } \widehat{\mathrm{B}} \quad (4)$$

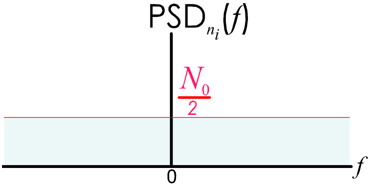
$$\mathrm{SNR}_{out} = \frac{\text{Power of signal at } \hat{\mathrm{A}}}{\text{Power of noise at } \hat{\mathrm{A}}} = \underbrace{\mathrm{f}\{p_e\}}_{\text{denotes: a function of } p_e} \quad (5)$$

Additive Noise



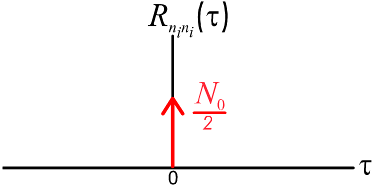
- types of channel signals
  - $s(t), r(t), n(t)$: bandpass
  - $n_i(t) = AWGN$: allpass

- $n_i(t)$
  - it is a random all-Pass signal
  - its Power Spectral Density is "White" i.e. "flat". That is,



$$\mathrm{PSD}_{n_i}(f) = \frac{N_0}{2} \tag{6}$$

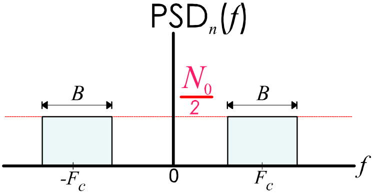
  - its amplitude probability density function is Gaussian
  - its Autocorrelation function $\left(\text{i.e. } \mathrm{FT}^{-1}\left\{\mathrm{PSD}(f)\right\}\right)$ is:



$$R_{n_i n_i}(\tau) = \frac{N_0}{2}\delta(\tau) \tag{7}$$

Bandlimited AWGN

- $n(t)$
  - it is a random Band-Pass signal of bandwidth B (equal to the channel bandwidth)
  - its Power Spectral Density is "bandmimited White". That is,



$$\mathrm{PSD}_{n_i}(f) = \frac{N_0}{2}\left(\mathrm{rect}\left\{\frac{f+F_c}{B}\right\} + \mathrm{rect}\left\{\frac{f-F_c}{B}\right\}\right) \tag{8}$$

  - Its power is:

$$\begin{aligned} P_n &= \sigma_n^2 = \int_{-\infty}^{\infty} PSD_{n_i}(f).df = \frac{N_0}{2} \times B \times 2 \\ \Rightarrow & \quad P_n = N_0 B \end{aligned} \tag{9}$$

- more on $n(t)$:
  - its amplitude probability density function is Gaussian

$$\mathrm{pdf}_n = \mathrm{N}(0, \sigma_n^2 = N_0 B) \tag{10}$$

  - It is also known as **bandlimited-AWGN**
  - It can be written as follows:

$$\begin{aligned} n(t) &= n_c(t)\cos(2\pi F_c t) - n_s(t)\sin(2\pi F_c t) && (11) \\ &= \underbrace{\sqrt{n_c^2(t) + n_s^2(t)}}_{\triangleq r_n(t)}\cos(2\pi F_c t + \phi_n(t)) && (12) \end{aligned}$$

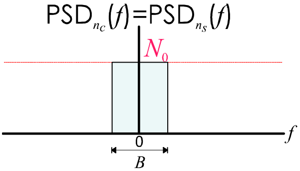
  where
    - ★ $n_c(t)$ and $n_s(t)$ are random signals - with pdf=Gaussian distribution
    - ★ $r_n(t)$ is a random signal - with pdf=Rayleigh distribution
    - ★ $\phi_n(t)$ is a random signal - with pdf=uniform distribution: $[0, 2\pi])$

  N.B.: all the above are low pass signals & appear at Rx's o/p

  - Equ. 11 is known as **Quadrature Noise Representation** .

"I" and "Q" Noise Components

- $n_c(t)$ (i.e. "I") and $n_s(t)$ (i.e. "Q")
  - their Power Spectral Densities are:



$$\mathrm{PSD}_{n_c}(f) = \mathrm{PSD}_{n_s}(f) = N_0 \mathrm{rect}\left\{\frac{f}{B}\right\} \tag{13}$$

  - their power are:

$$\begin{aligned} P_{n_c} &= \sigma_{n_c}^2 = \int_{-\infty}^{\infty} PSD_{n_c}(f).df = N_0 \times B \\ \Rightarrow & \quad P_{n_c} = P_{n_s} = P_n = N_0 B \end{aligned} \tag{14}$$

  - Amplitude probability density functions: Gaussian,

$$\mathrm{pdf}_{n_c} = \mathrm{pdf}_{n_s} = \mathrm{N}(0, N_0 B) \tag{15}$$

  - are uncorrelated i.e. $\mathcal{E}\{n_c(t).n_s(t)\} = 0$

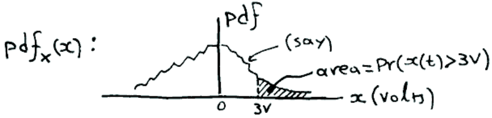
Tail function (or Q-function) for Gaussian Signals

Probablity and Probability-Density-Function (pdf)

- Consider a random signal $x(t)$ with a known amplitude probability density function $\text{pdf}_x(\text{x})$ - not necessarily Gaussian. Then the probability that the amplitude of $x(t)$ is greater than A Volts (say) is given as follows:

$$\Pr(x(t) > A) = \int_A^{\infty} \text{pdf}_x(\text{x}).dx \tag{16}$$

- e.g.
  if $A = 3V \Rightarrow \Pr(x(t) > 3V) = \int_3^{\infty} \text{pdf}_x(\text{x}).dx = \text{highlighted area}$



Gaussian pdf and Tail function

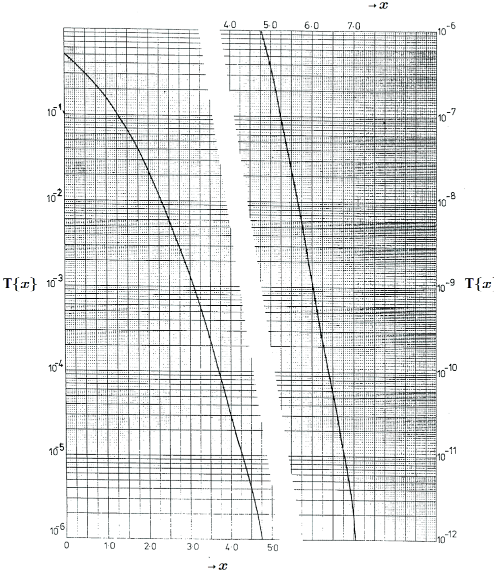
- If $\text{pdf}_x(\mathrm{x})$ = Gaussian of mean μ_x and standard deviation σ_x (notation used: $\text{pdf}_x(\mathrm{x}) = \mathrm{N}(\mu_x, \sigma_x^2)$), then the above area is defined as the Tail-function (or Q-function)

$$\Pr(x(t) > A) = \int_A^{\infty} \text{pdf}_x(\mathrm{x}).dx \triangleq \mathrm{T}\left\{\frac{|A-\mu_x|}{\sigma_x}\right\} \tag{17}$$

- e.g.
  - if $\text{pdf}_x(\mathrm{x}) = \mathrm{N}(1,4)$ - i.e. $\mu_x = 0, \sigma_x = 2$ - and $A = 3V$ then $\Pr(x(t) > 3V) = \int_3^{\infty} \text{pdf}_x(\mathrm{x}).dx \triangleq \mathrm{T}\left\{\frac{|3-1|}{2}\right\} = \mathrm{T}\left\{1\right\}$
  - if $\text{pdf}_x(\mathrm{x}) = \mathrm{N}(0,1)$ and $A = 3V$ then $\Pr(x(t) > 3V) = \int_3^{\infty} \text{pdf}_x(\mathrm{x}).dx \triangleq \mathrm{T}\left\{3\right\}$
- The Tail function graph is given in the next page

The graph below shows the Tail function $\mathbf{T}\{x\}$ which represents the area from x to ∞ of the Gaussian probability density function N(0,1), i.e.

$$\mathbf{T}\{x\} = \int_x^\infty \frac{1}{\sqrt{2\pi}}.\exp\left(-\frac{y^2}{2}\right)dy$$



Note that if $x > 6.5$ then $\mathbf{T}\{x\}$ may be approximated by $\mathbf{T}\{x\} \approx \frac{1}{\sqrt{2\pi}.x}.\exp\left\{-\frac{x^2}{2}\right\}$