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Digital Image Procesing

Introduction to Image Enhancement
Histogram Processing

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Image Enhancement

- The goal is to process an image so that the resulting image is:
  - more suitable than the original image for a *specific* application
  - of better quality in terms of some quantitative metric
  - visually better

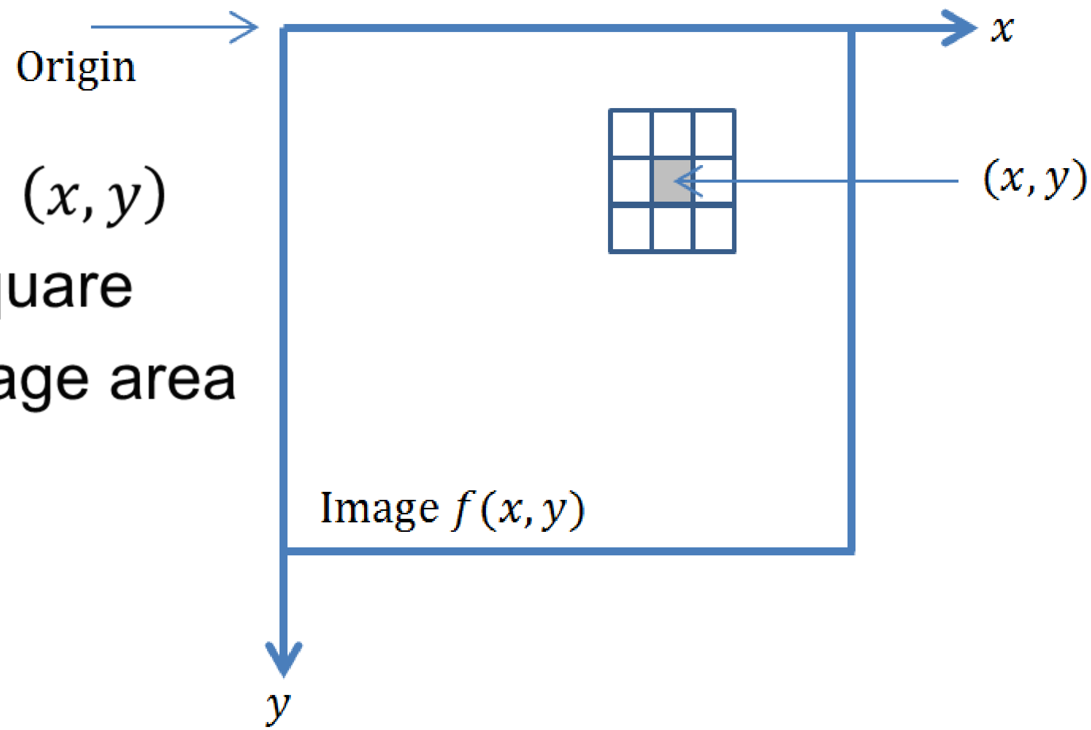
- Spatial domain methods

- Frequency domain methods.

Spatial Domain Methods: Local neighborhood processing

- Procedures that operate directly on the local aggregate of pixels composing an new image $g(x, y) = T[f(x, y).]$

- A neighborhood around (x, y) is defined by using a square (or rectangular) sub-image area centered at (x, y).



Spatial Domain Methods: Point processing

- When the neighborhood is 1 x 1 then $g(x,y)$ depends only on the value of $f(x,y)$ at (x,y) and T becomes a grey-level transformation (or mapping) function:

  $$s = T(r)$$

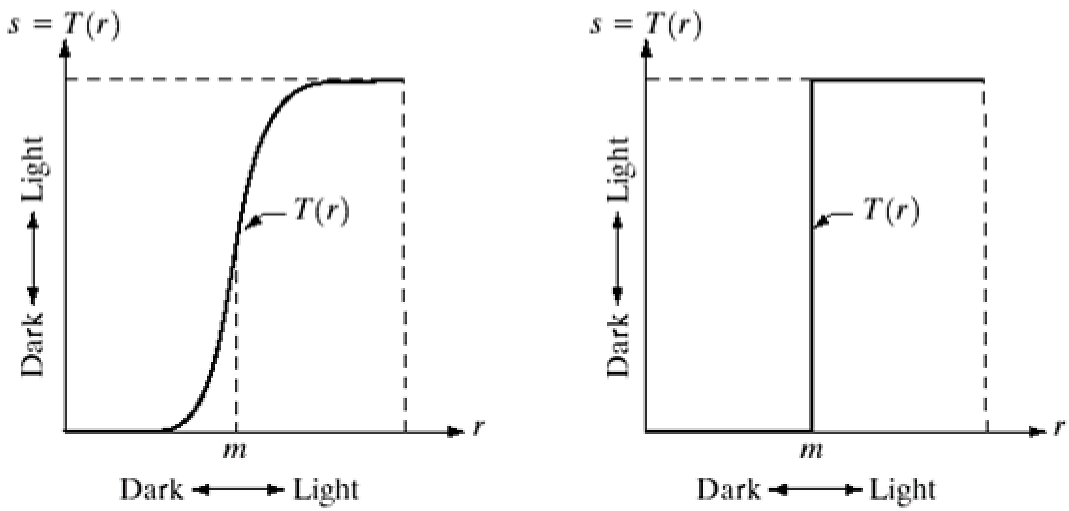
  r, s are the grey levels of $f(x,y)$ and $g(x,y)$ at (x,y).

- These techniques are called point processing techniques.
  - Histogram processing
  - Thresholding
  - Contrast stretching
  - Many others

Point Processing: contrast enhancement

- In the figures below you can see examples of two different intensity transformations.

- The figure on the right shows the process of binarization of the image.



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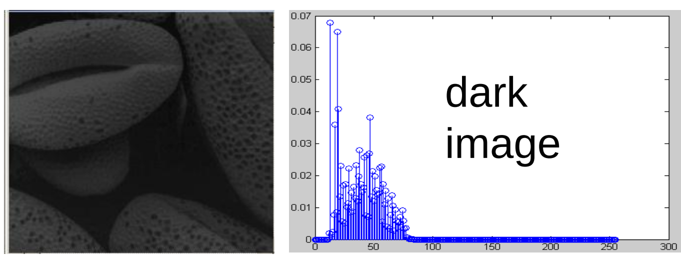
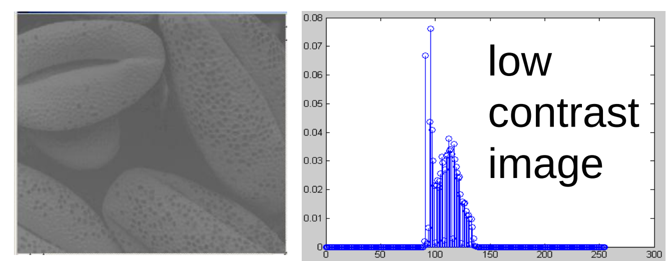
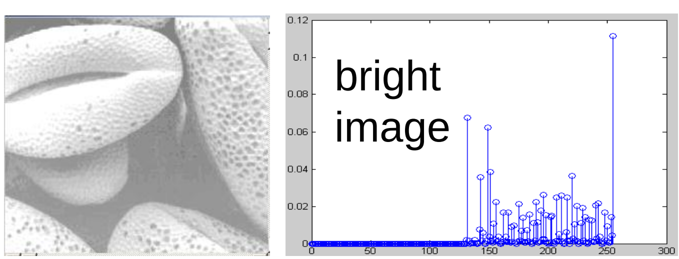
Histogram Processing: definition of image histogram

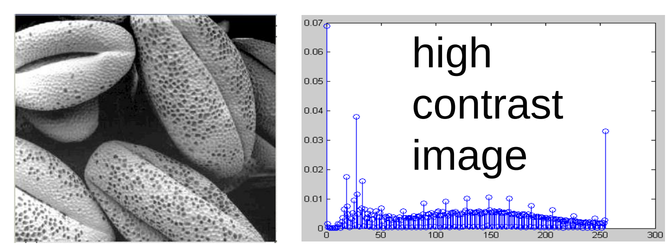
- Take an image with intensities $r_k, k \in [0, L-1]$ and size MN.
- The number of pixels with intensity r_k is n_k.
- The histogram of the image is the function $h(r_k) = n_k$.
- The normalized histogram is the function is the function

$$p(r_k) = \frac{n_k}{MN} \text{ for } k = 0,1,2,\dots,L-1$$

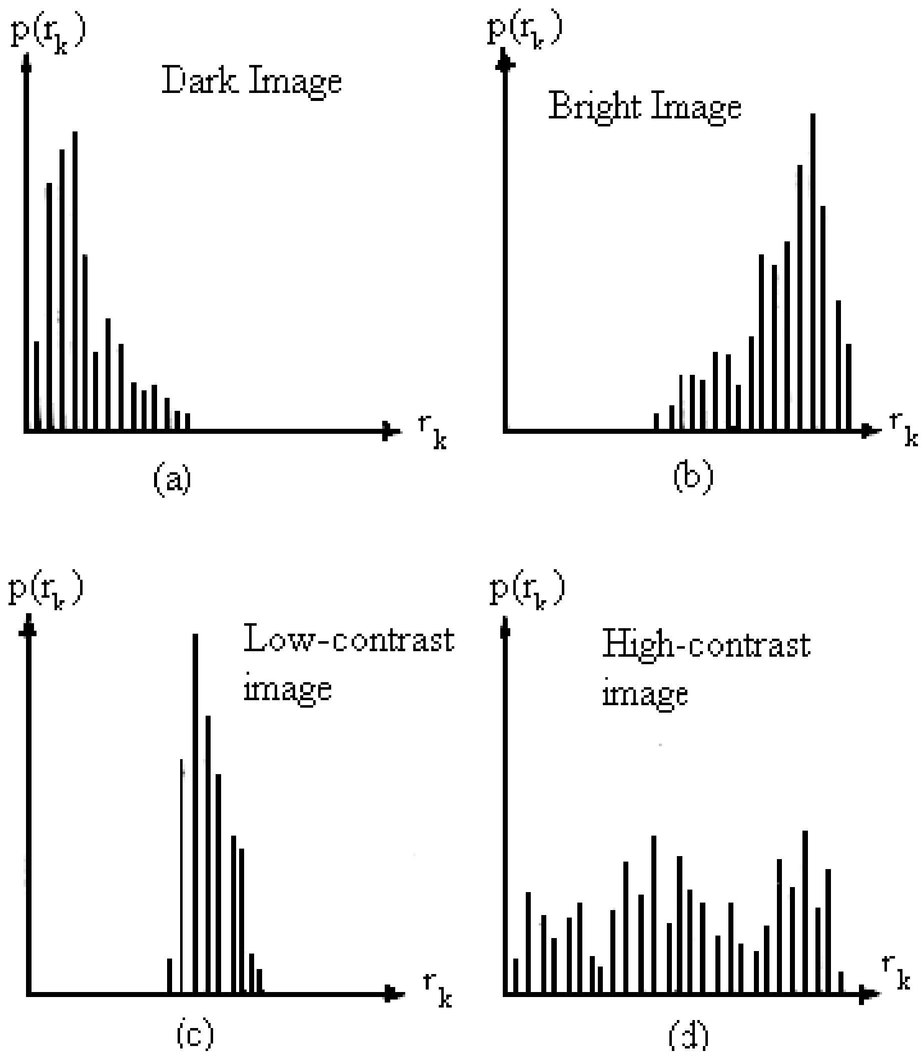








Generic figures of histograms



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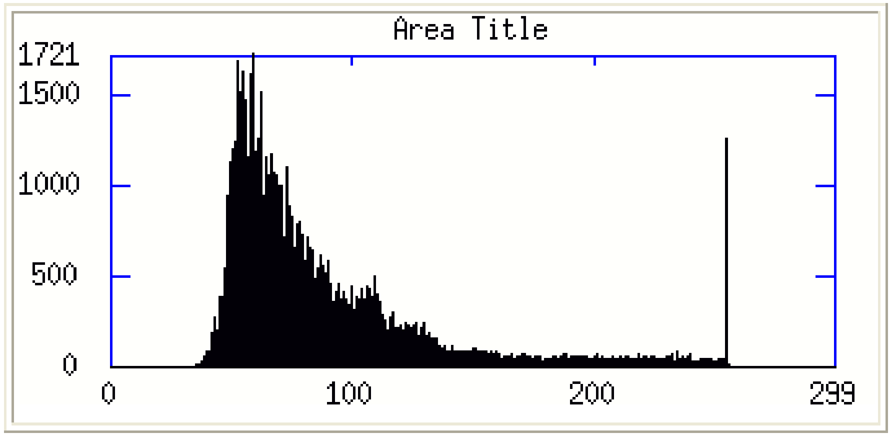
Two different images with the same histogram.

a) b)



Histogram Processing: definition of intensity transformation

- Consider for the moment **continuous** intensity values.
- The continuous intensity of an image is $r \in [0, L-1]$.
- $r = 0$ represents black and $L-1$ represent white.
- We are looking for intensity transformations of the form:

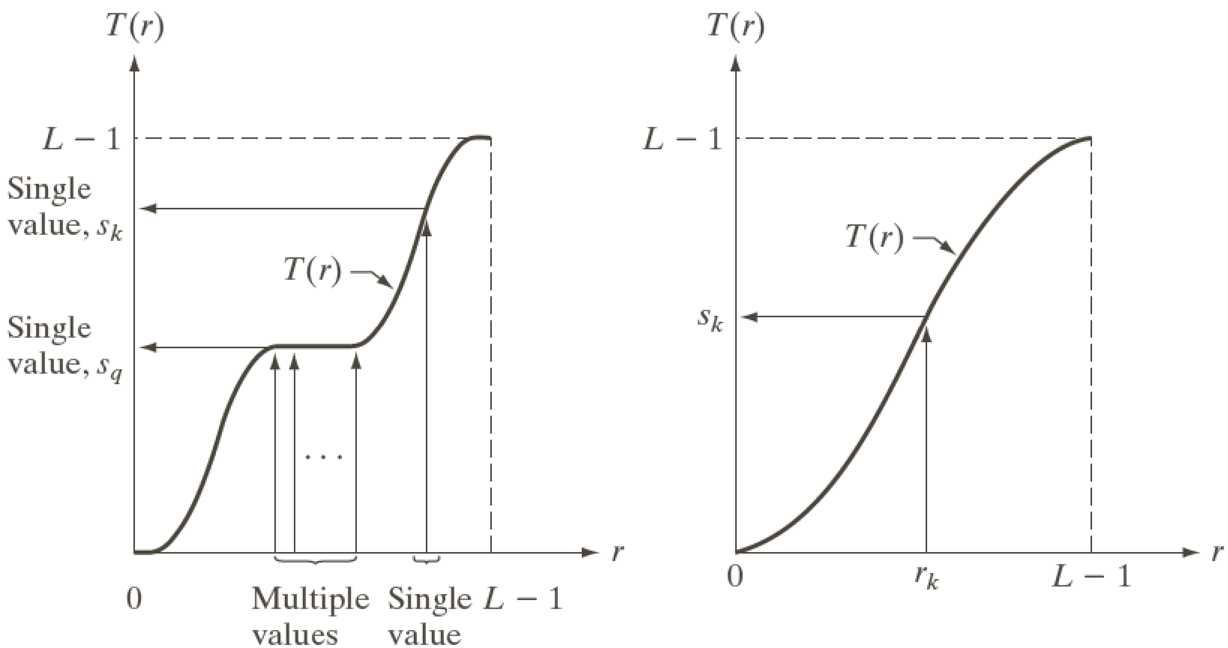
$$s = T(r), 0 \leq r \leq L-1$$

- Conditions on $T(r)$
  - $T(r)$ is monotonically increasing in $0 \leq r \leq L-1$ OR strictly monotonically increasing in $0 \leq r \leq L-1$
  - $0 \leq T(r) \leq L-1$ for $0 \leq r \leq L-1$

Histogram Processing: definition of intensity transformation

- The condition for $T(r)$ to be monotonically increasing guarantees that ordering of the output intensity values will follow the ordering of the input intensity values (avoids reversal of intensities).
- If $T(r)$ is strictly monotonically increasing then the mapping from s back to r will be 1-1.
- The second condition ($T(r)$ in $[0,1]$) guarantees that the range of the output will be the same as the range of the input.

Monotonicity versus strict monotonicity



a) We cannot perform inverse mapping (from s to r).

b) Inverse mapping is possible.

Modelling intensities as continuous variables

- We can view intensities r and s as random variables and their histograms as probability density functions $p_r(r)$ and $p_s(s)$.

- If $s = T(r)$ and $T(r)$ is continuous, differentiable and monotonically increasing, it is proven that:

$$p_s(s) = p_r(r)\frac{1}{\left|\frac{ds}{dr}\right|} = p_r(r)\left|\frac{dr}{ds}\right|$$

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Histogram Equalization: continuous form

- The pdf of the output is determined by the pdf of the input and the transformation.
- This means that we can determine the histogram of the output image.
- A transformation of particular importance in image processing is the cumulative distribution function (CDF) of a random variable:

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

Histogram Equalization: continuous form

- It satisfies the first condition as the area under the curve increases as r increases.
- It satisfies the second condition as for $r = L - 1$ we have $s = L - 1$.
- To find $p_s(s)$ we have to compute:

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = (L-1)\frac{d}{dr}\int_0^r p_r(w)dw = (L-1)p_r(r)$$

- Knowing that $p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$ we get

$$p_s(s) = \frac{1}{L-1},\ s = 1, \dots, L-1$$

- Therefore $p_s(s)$ is a uniform pdf!

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Histogram Equalization: discrete form

- The formula for histogram equalisation in the discrete case is given by

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j$$

Where:

r_k: input intensity

s_k: processed intensity

n_j: the frequency of intensity j

MN: number of image pixels

A histogram equalization example in discrete form

- A 3-bit 64x64 image with 8 intensities is described in the table.
- Discrete histogram equalised intensity levels are obtained through

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

- After applying histogram equalization

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7p_r(r_0) = 1.33$$

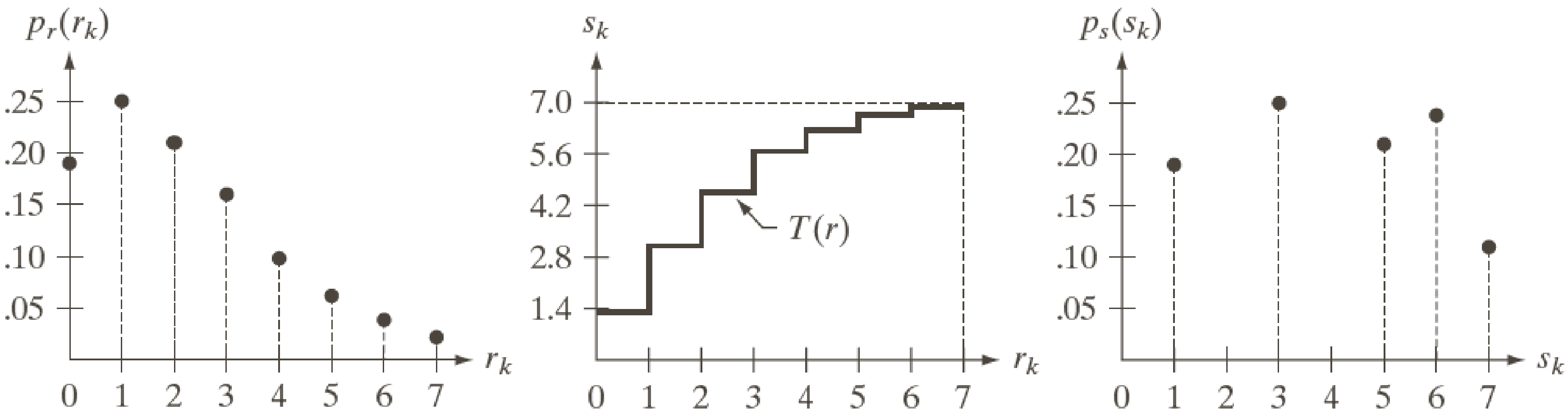
$$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7p_r(r_0) + 7p_r(r_1) = 3.08$$

and so on…

A histogram equalization example in discrete form

By rounding to the nearest integer we get:

$$s_0 = 1.33 \rightarrow 1 \quad s_1 = 3.08 \rightarrow 3 \quad s_2 = 4.55 \rightarrow 5 \quad s_3 = 5.67 \rightarrow 6$$
$$s_4 = 6.23 \rightarrow 6 \quad s_5 = 6.65 \rightarrow 7 \quad s_6 = 6.86 \rightarrow 7 \quad s_7 = 7 \rightarrow 7$$



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

A histogram equalization example in discrete form

Notice that due to discretization, the resulting histogram will rarely be perfectly flat. However, it will more “extended” compared to the original histogram.

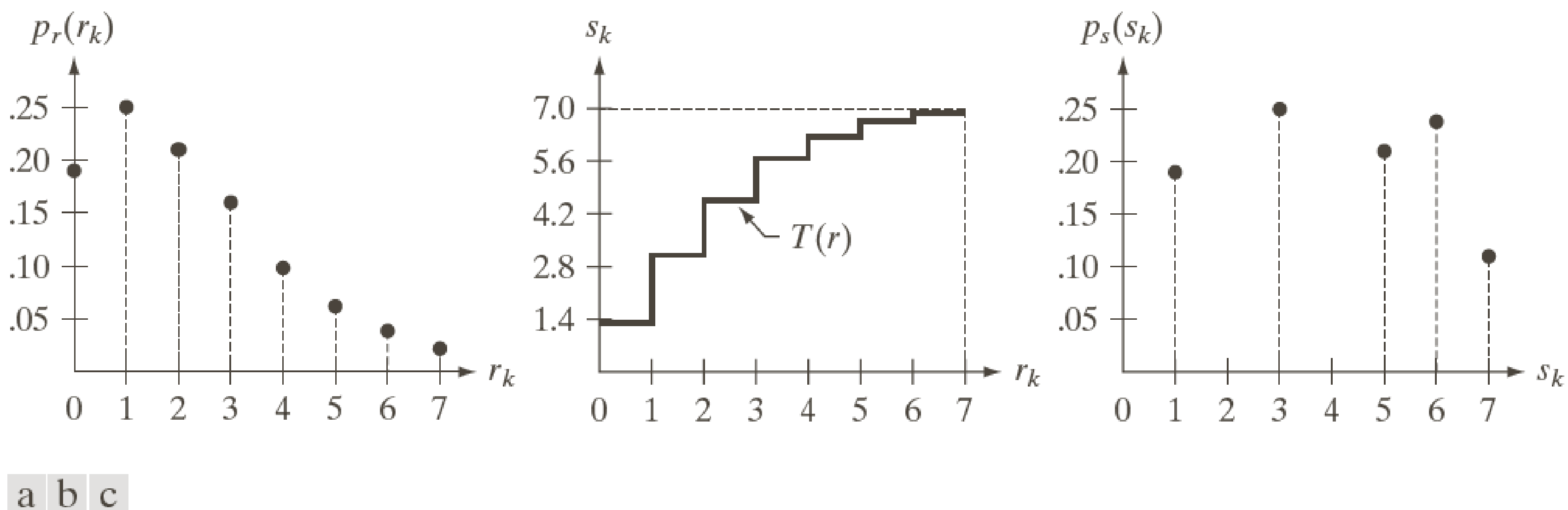


FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

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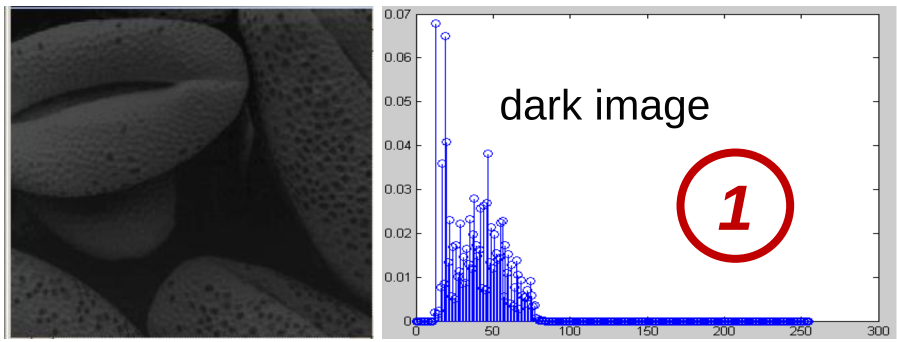
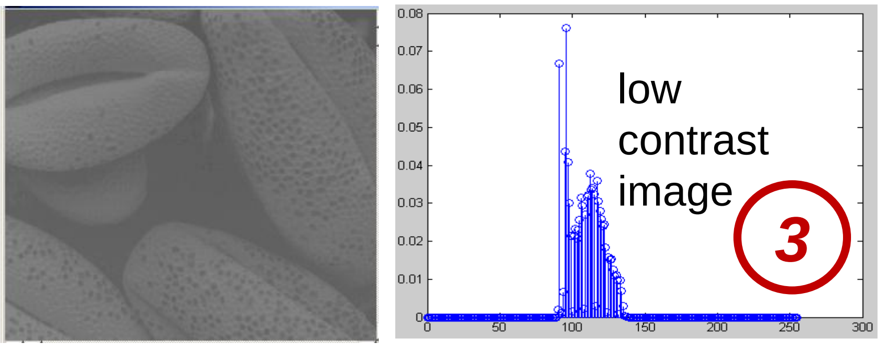
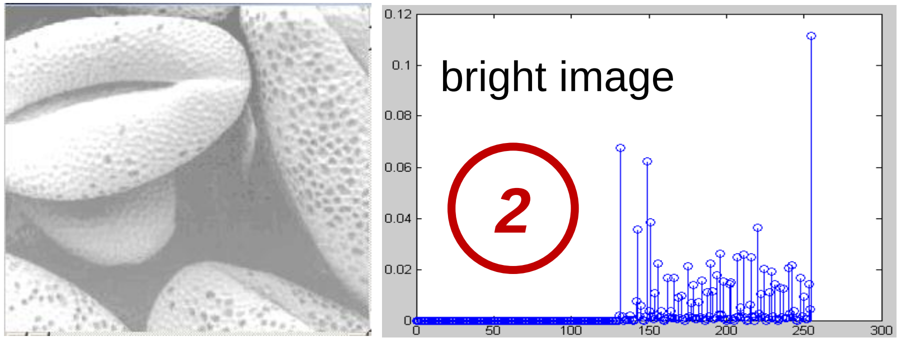
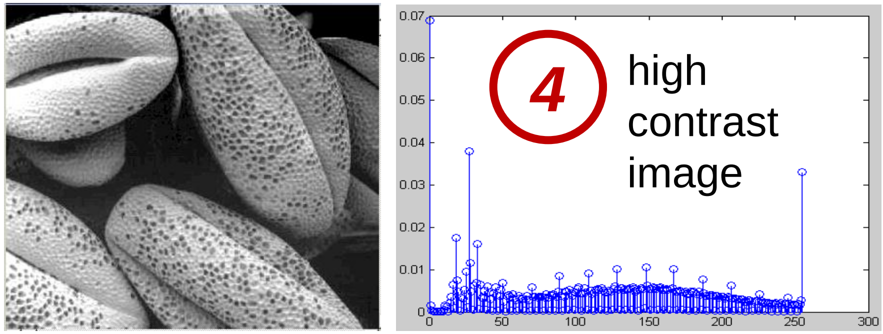
A set of images with same content but different histograms

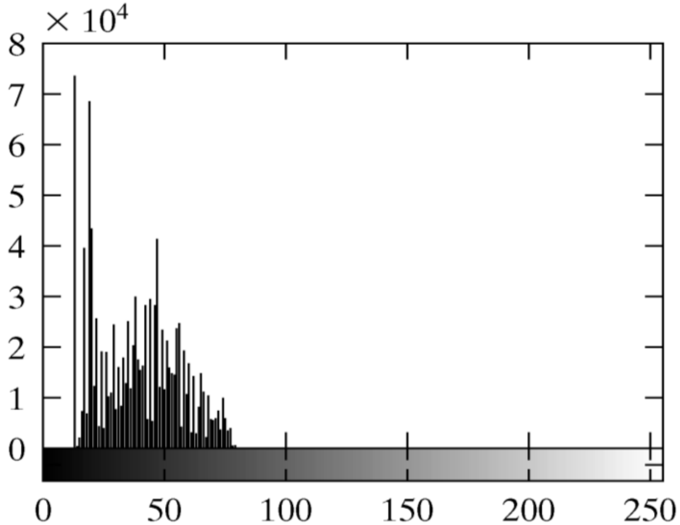


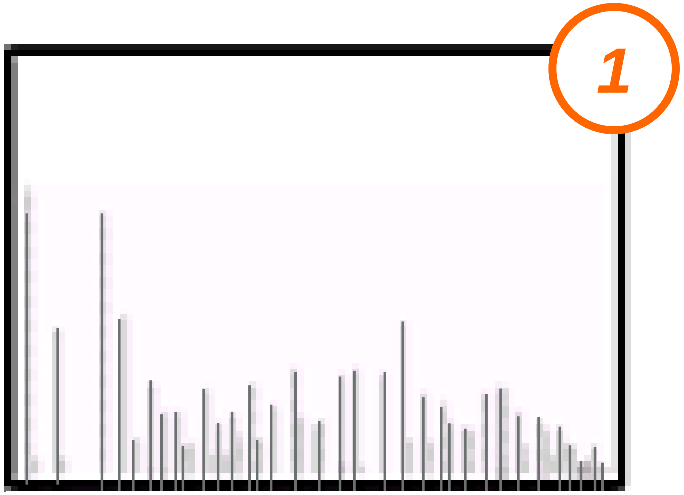


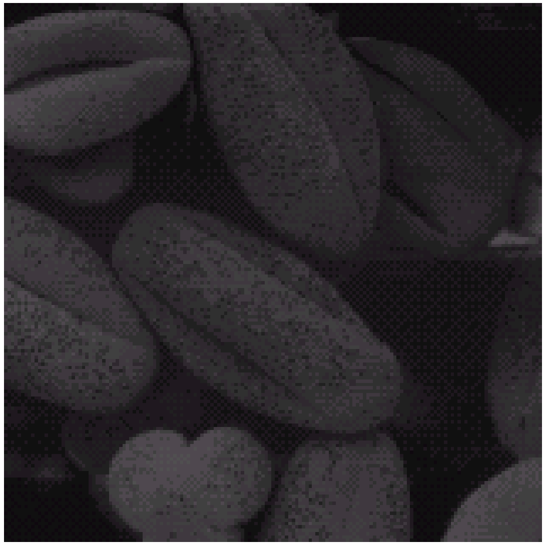




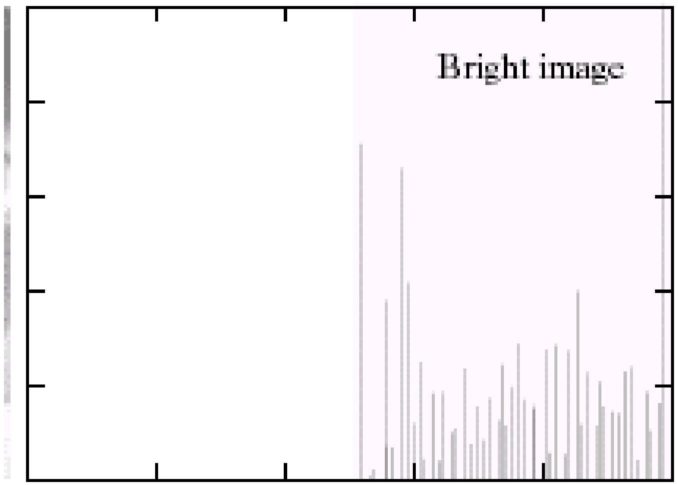
Histogram equalization applied to the dark image

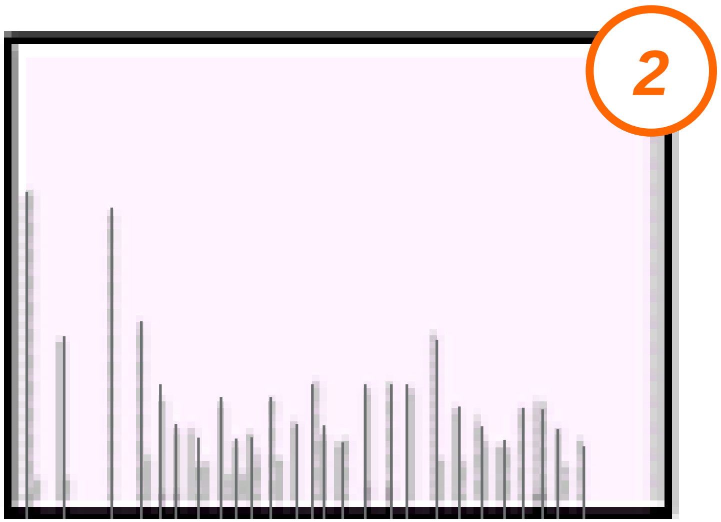




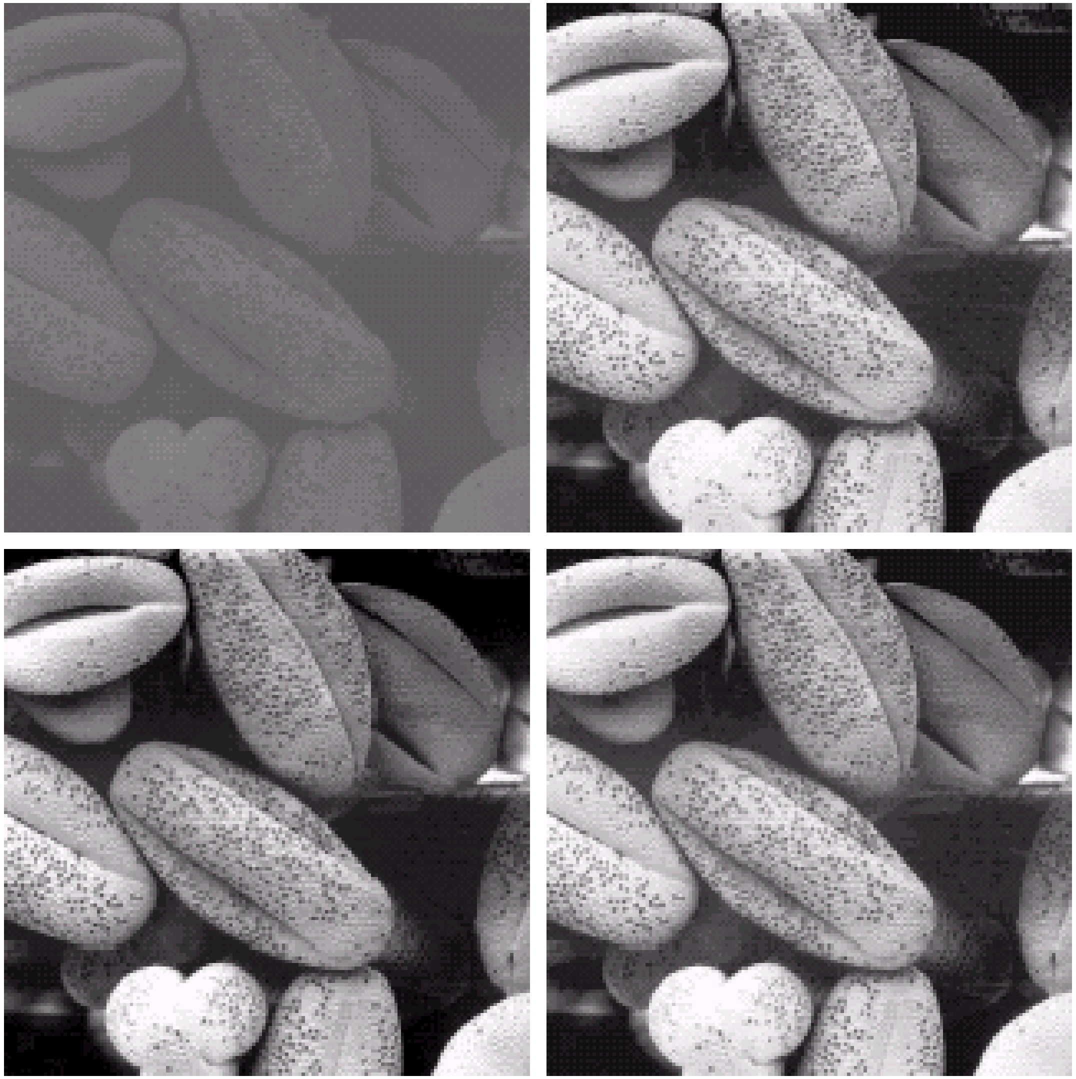


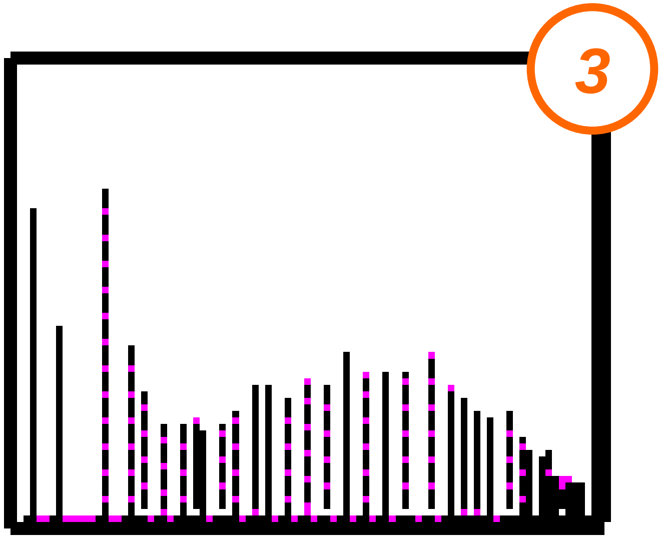
Histogram equalization applied to the bright image

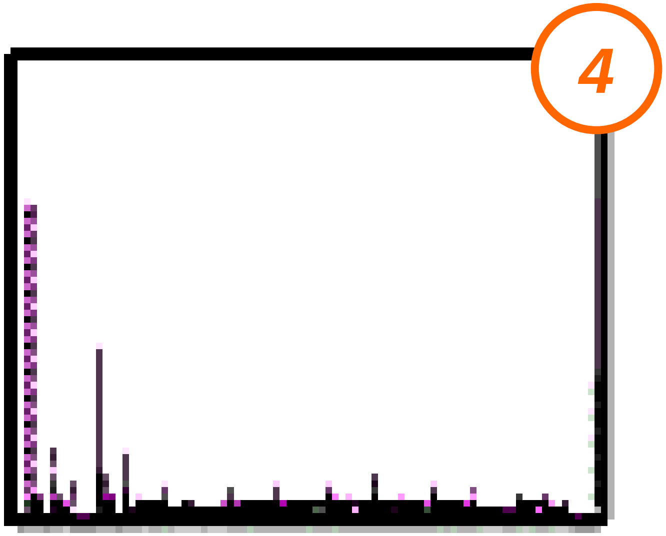




Histogram equalization applied to the low and high contrast images







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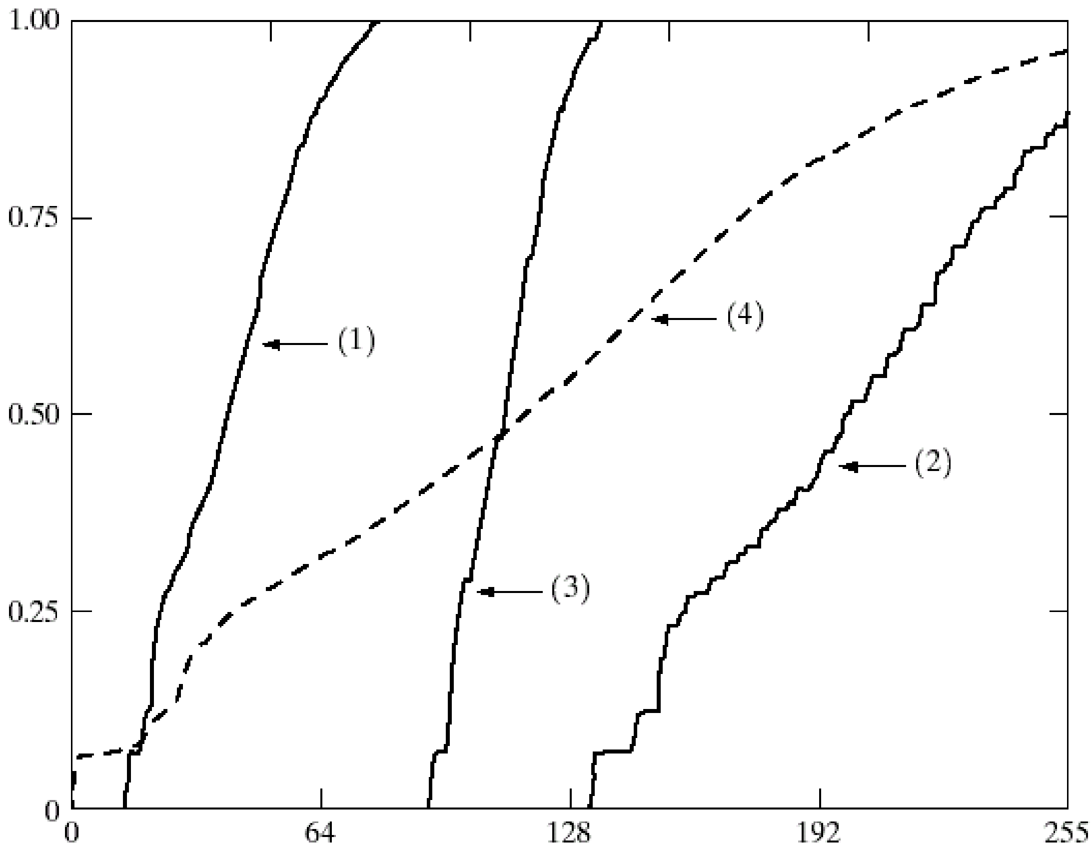
Transformation functions for histogram equalization for the previous example

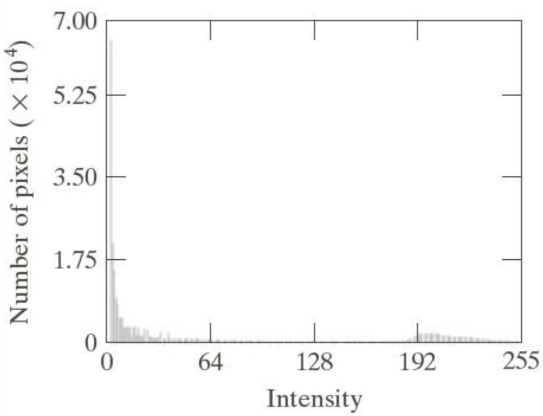
The functions $T(r)$ used to equalise the images in the previous examples (**try to explain their form!**)

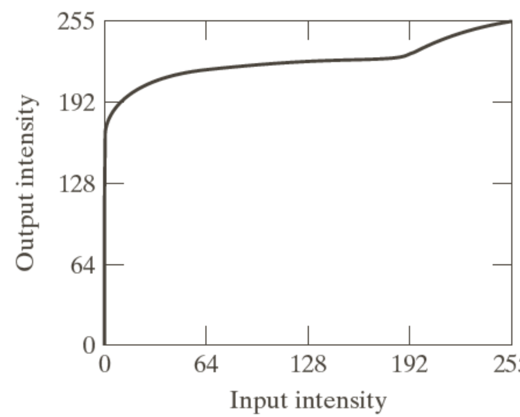
The numbering in the figure below is consistent with the previous numbering of the four images.

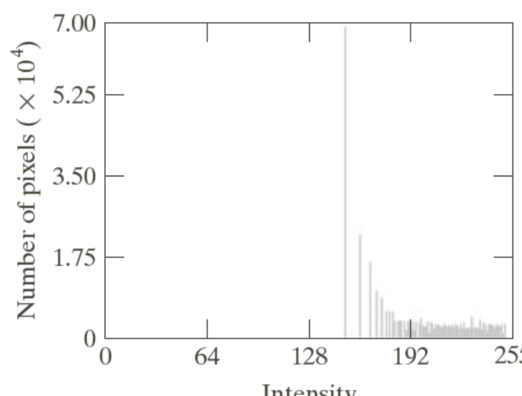


An example of an unfortunate histogram equalization

- Example of image of Phobos (Mars moon) and its histogram.
- Histogram equalization (bottom of right image) does not always provide the desirable results.







Histogram Specification

- We are looking for a technique which can provide an image with any pre-specified histogram.
- This is called histogram specification.
- We assume that the original image has a pdf $p_r(r)$.
- We are looking for a transformation $z = T(r)$ which provides an image with a specific pdf $p_z(z)$.
- This technique will use histogram equalization as an intermediate step.

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Histogram Specification

- We first equalize the given image
  $s = T(r) = (L-1)\int_0^r p_r(w)dw$
- If we had the desired image we could equalized it and obtain
  $$s = G(z) = (L-1)\int_0^z p_z(w)dw$$
- Based on the above we can assume that
  $G(z) = T(r) \Rightarrow z = G^{-1}(T(r))$
- In the case of continuous variables, if $p_r(r)$ and $p_z(z)$ are given we can get z after formulating the functions T, G and G^{-1}.

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Histogram Specification

- In the discrete case we first equalize the initial histogram of the image:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j$$

- The we equalize the target histogram:

$$s_k = G(z_q) = (L-1)\sum_{i=0}^{q} p_z(r_i)$$

- Obtain the inverse transform:
$z_q = G^{-1}(s_k) = G^{-1}\,(T(r_k))$
- Notice the difference in indices of z and s.

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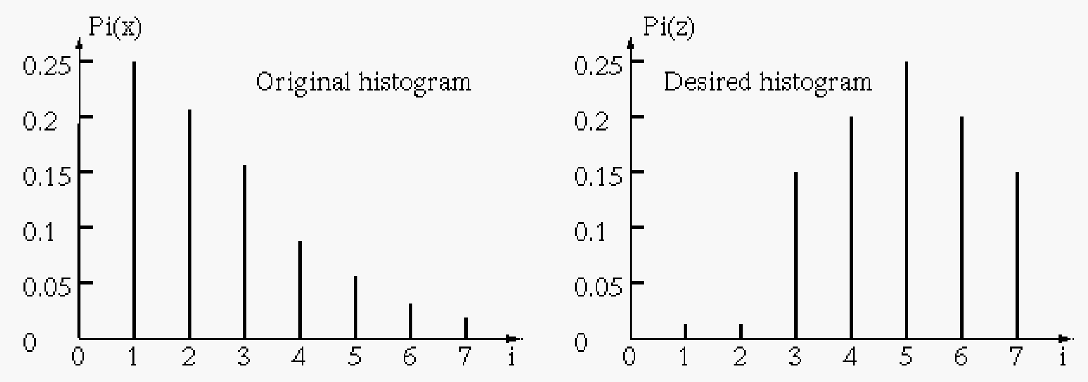
Histogram Specification: Example



original intensities	number of pixels	probability	cumulative probability CM	equalised intensities CM x 7	normalised equalised intensities
0	790	0.19	0.19	1.33	1
1	1023	0.25	0.44	3.08	3
2	850	0.21	0.65	4.55	5
3	656	0.16	0.81	5.67	6
4	329	0.08	0.89	6.23	6
5	245	0.06	0.95	6.65	7
6	122	0.03	0.98	6.86	7
7	81	0.02	1	7	7

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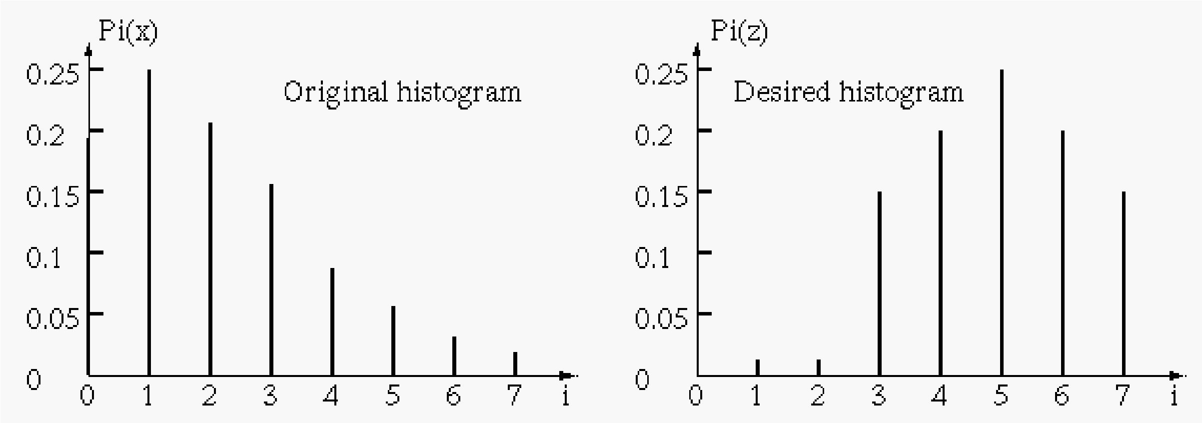
Histogram Specification: Example



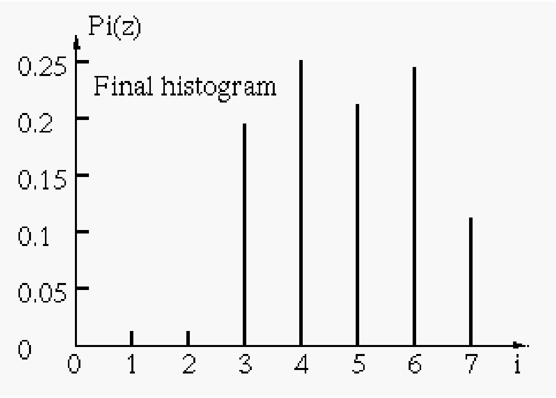
desired intensities	probability	cumulative probability CM	equalised intensities CM x 7	normalised equalised intensities
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0.15	0.15	1.05	1
4	0.2	0.35	2.45	2
5	0.3	0.65	4.55	5
6	0.2	0.85	5.95	6
7	0.15	1	7	7

Histogram Specification: Example

original intensities	equalised intensities (AVAILABLE)
0	1
1	3
2	5
3	6
4	6
5	7
6	7
7	7

desired intensities	equalised intensities (NOT AVAILABLE!!!)
0	0
1	0
2	0
3	1
4	2
5	5
6	6
7	7

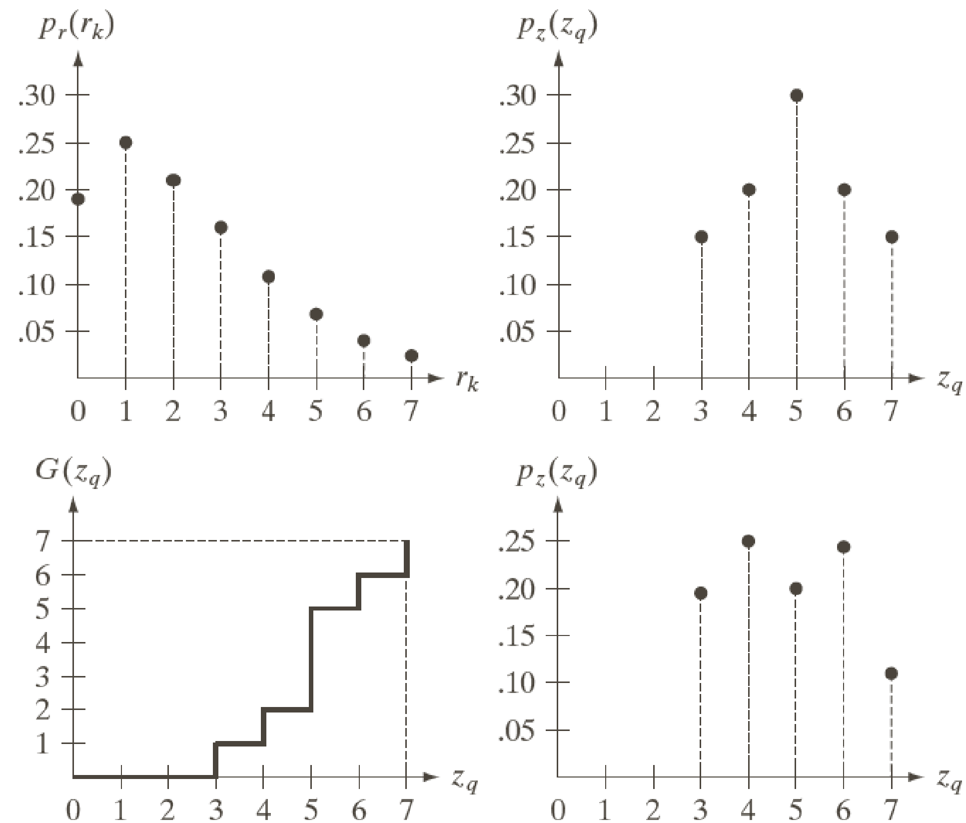
equalised intensities (available)	NEW intensities (available)
1	3
3	4
5	5
6	6
6	6
7	7
7	7
7	7



Histogram Specification: Example

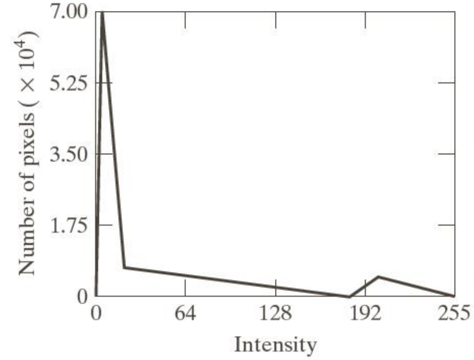
Notice that due to discretization, the resulting histogram will rarely be exactly the same as the desired histogram.

- Top left: original pdf
- Top right: desired pdf
- Bottom left: desired CDF
- Bottom right: resulting pdf

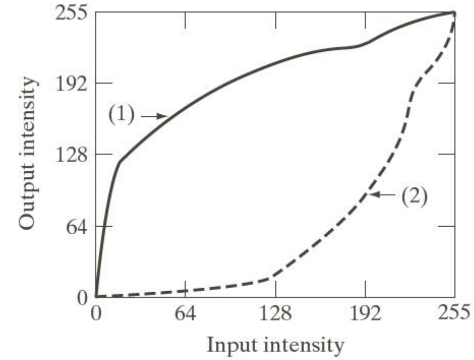


Histogram Specification: Example

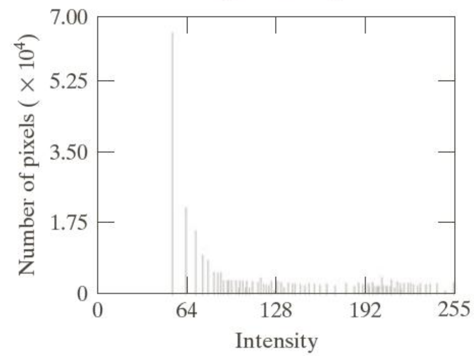
•Specified histogram.



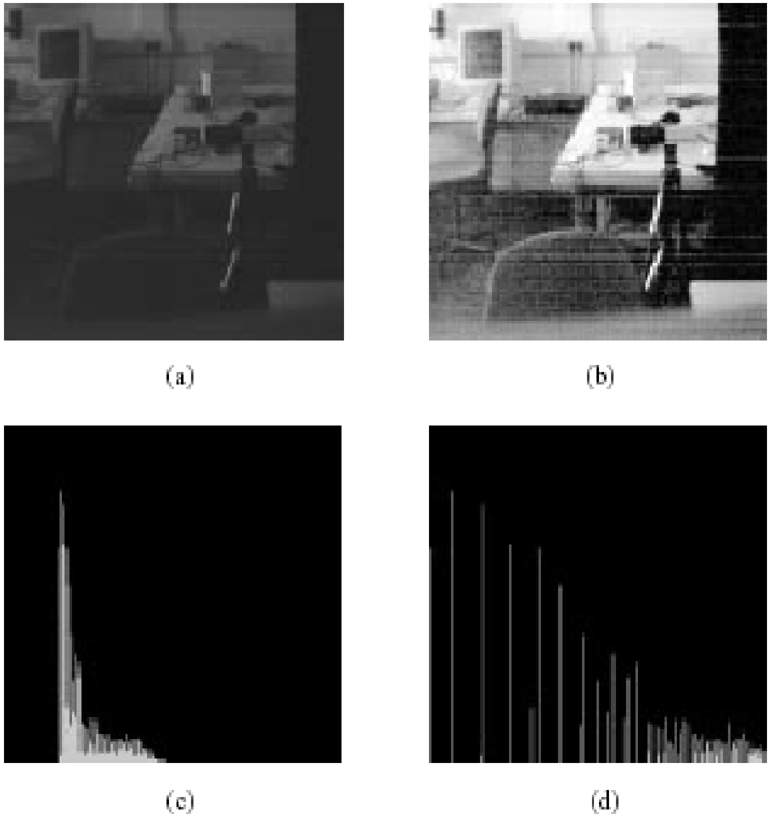
•Transformation function and its inverse.

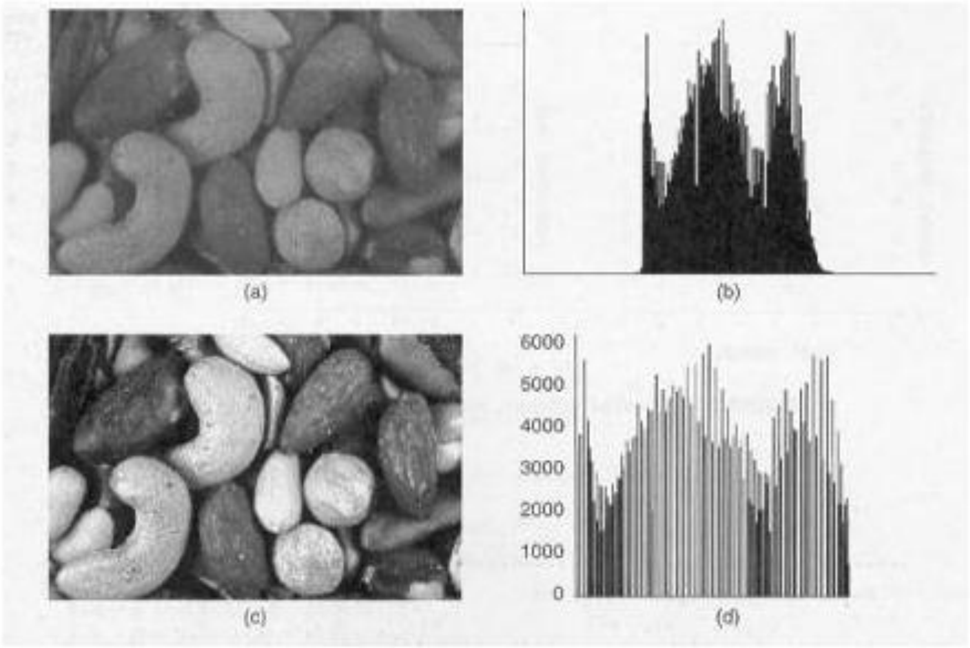


•Resulting histogram.

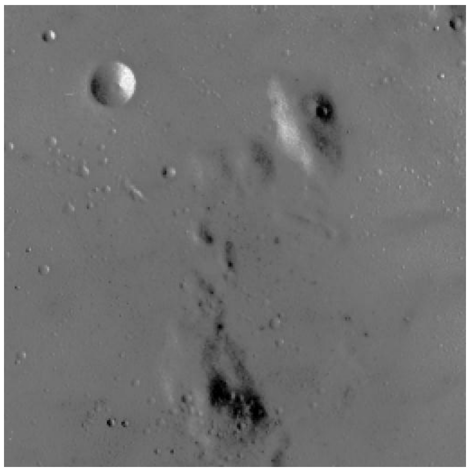


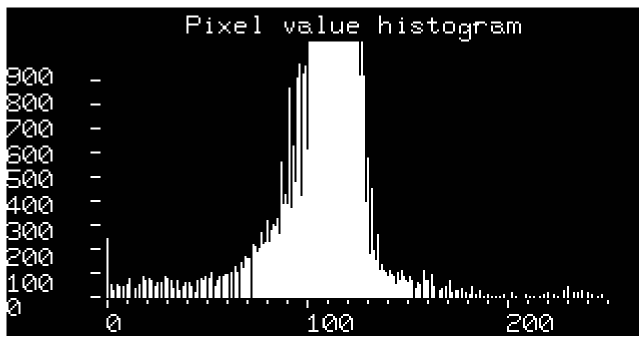
Histogram Equalization: Examples

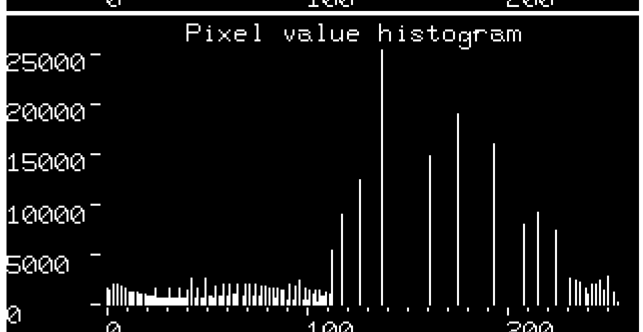


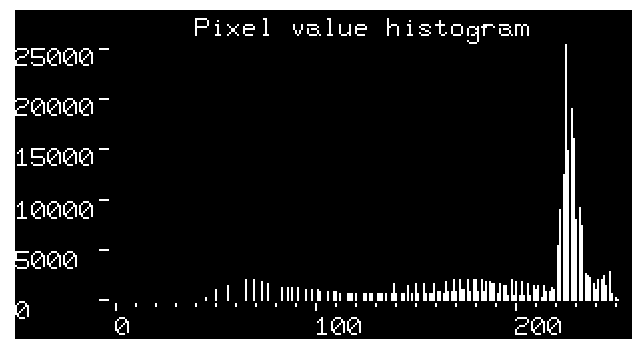


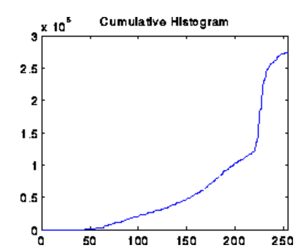
Histogram Equalization: Examples



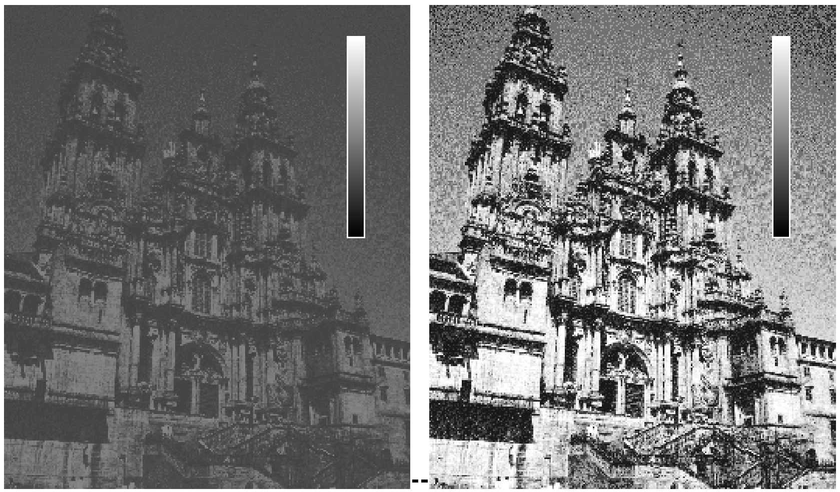




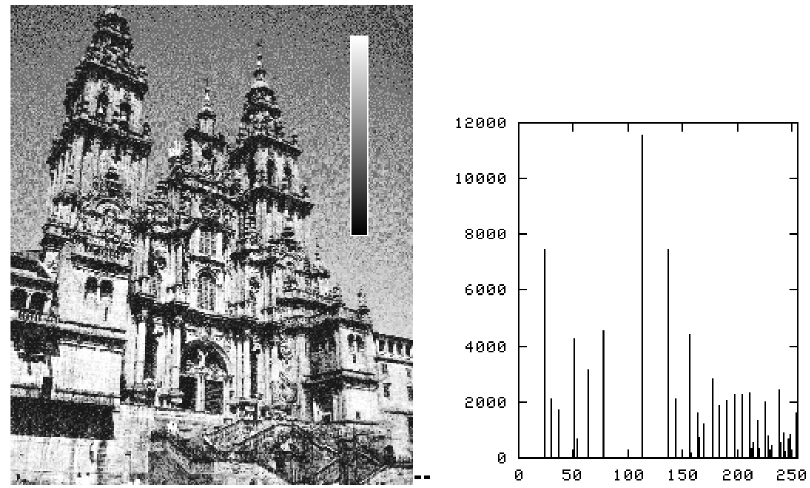




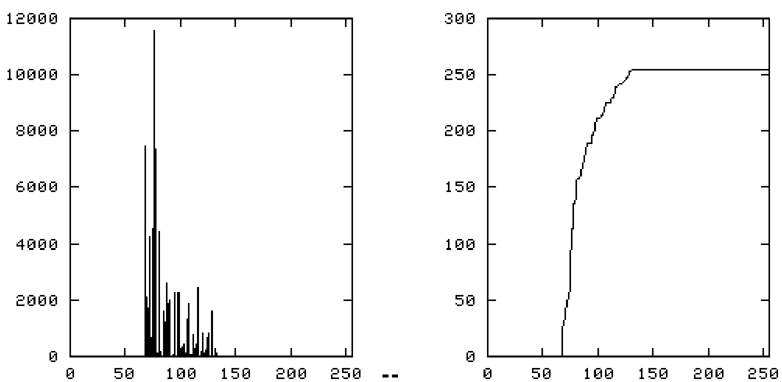
Histogram Equalization: Examples



Left: original image, Right: histogram equalized image

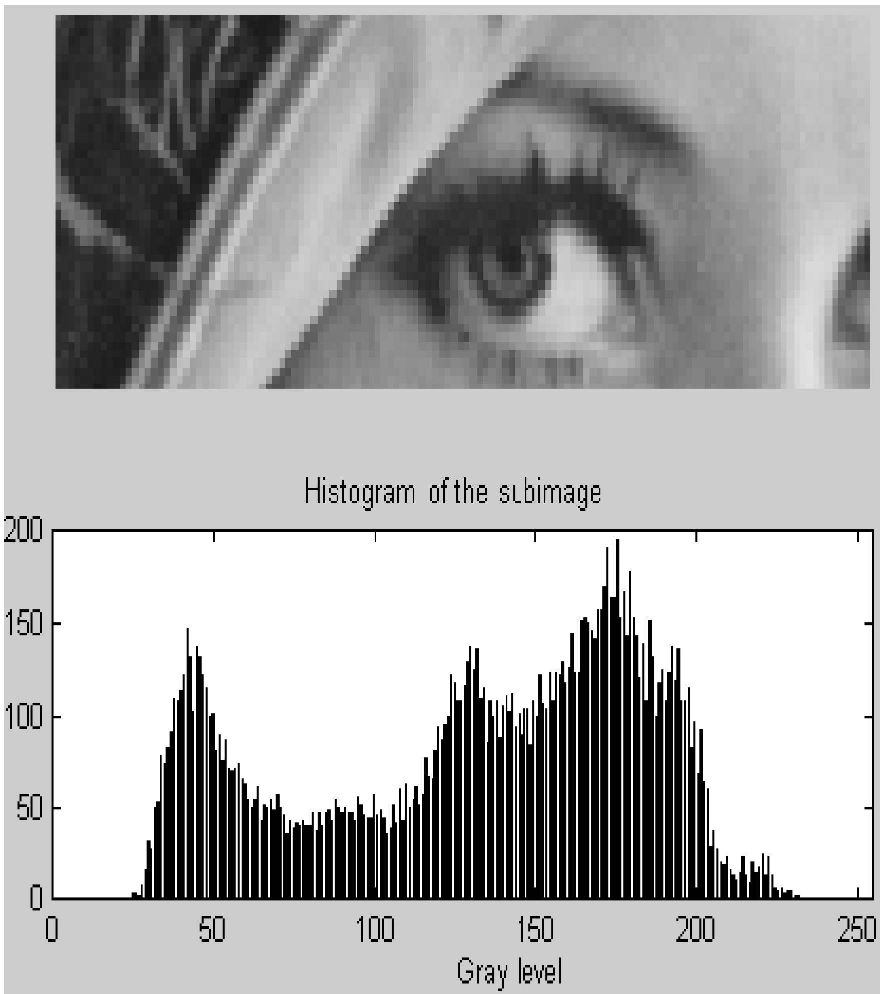


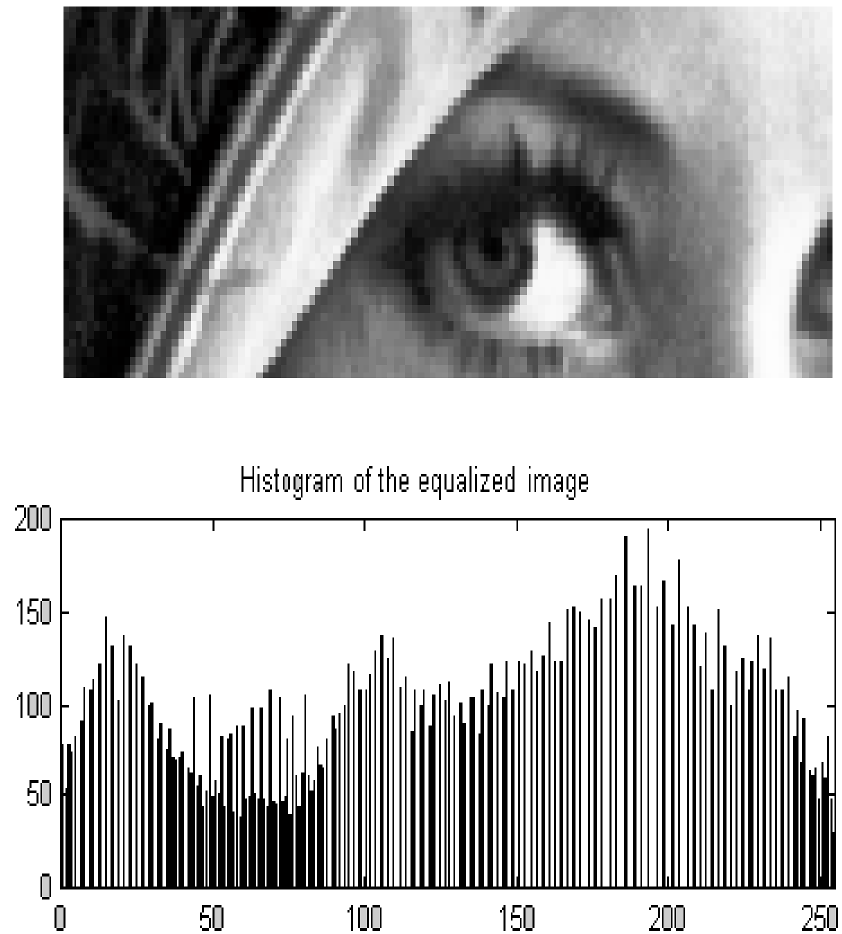
Left: histogram equalized image, Right: histogram of the histogram equalized image



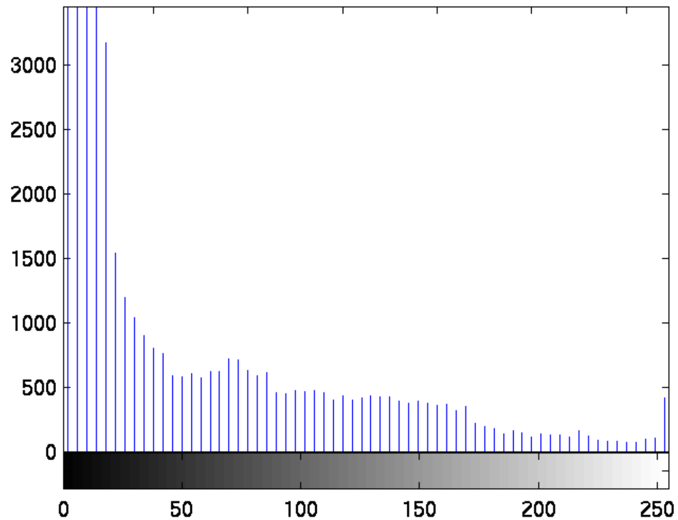
Left: histogram of the original image, Right: normalized cumulative histogram of the original image

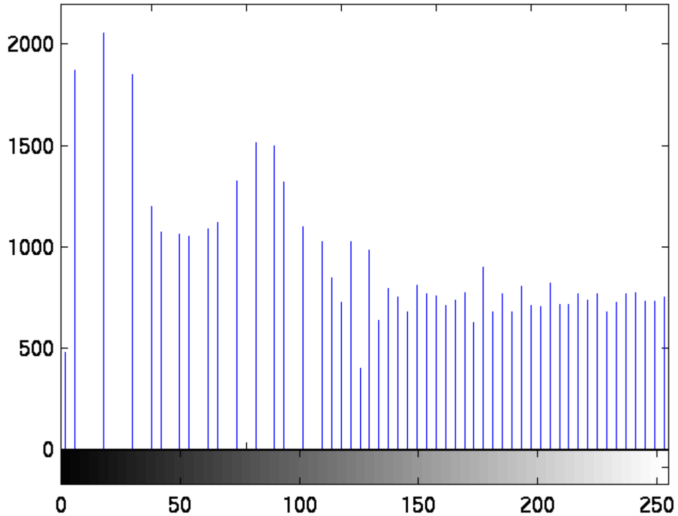
Histogram Equalization: Examples



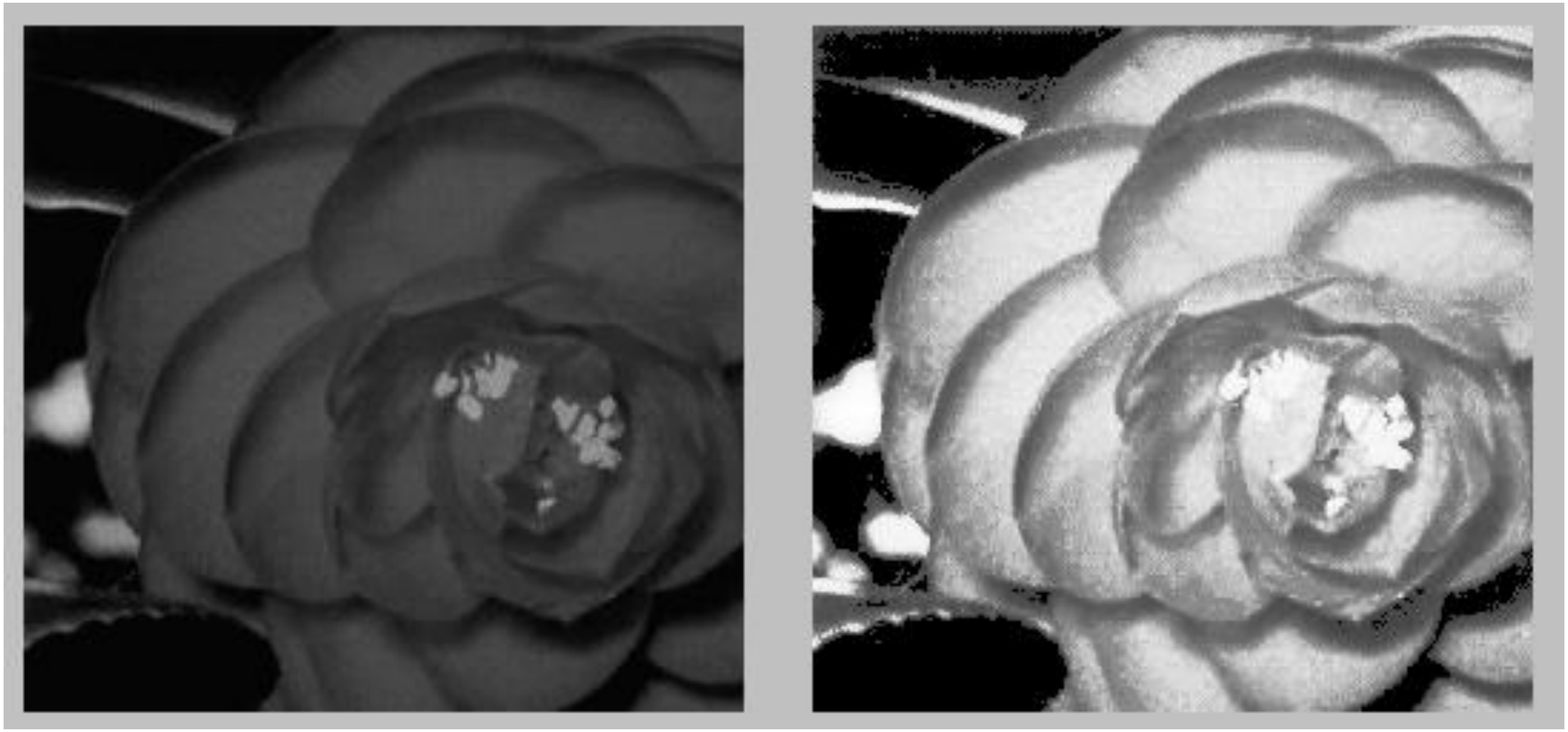


Histogram Equalization: Examples





Histogram Equalization: Examples



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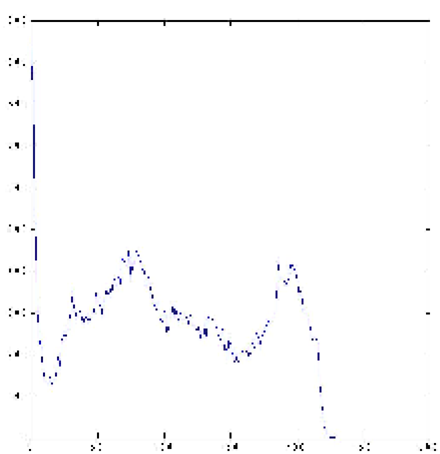
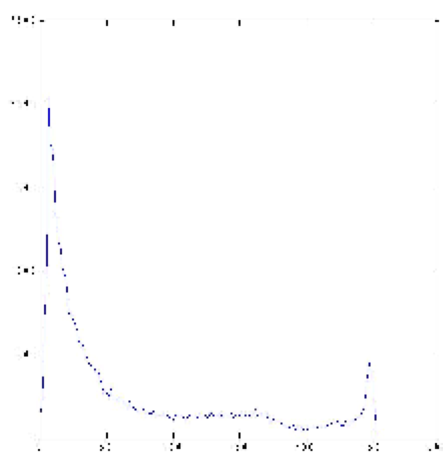
Histogram Specification: Examples

Original Image

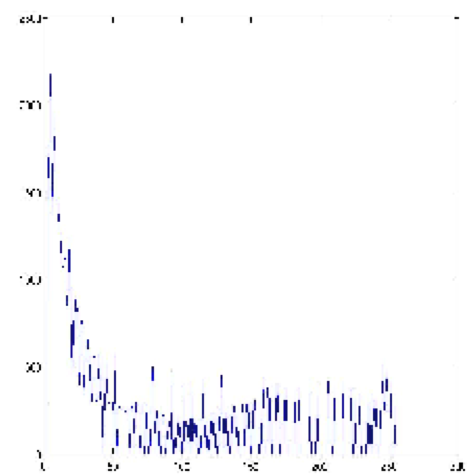
Image after histogram modification



Original Histogram



Specified Histogram

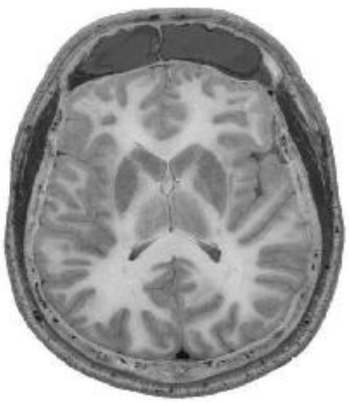


Histogram attained on modification

Local Histogram Specification

- The histogram processing methods discussed previously are global (transformation is based on the intensity distribution of the entire image).
- This global approach is suitable for overall enhancement.
- There are cases in which it is necessary to enhance details over small areas in an image.
- The number of pixels in these areas may have negligible influence on the computation of a global transformation.
- The solution is to devise transformation functions based on the intensity distribution in a neighbourhood around every pixel.

Local Histogram Specification: Examples



Orignal Image

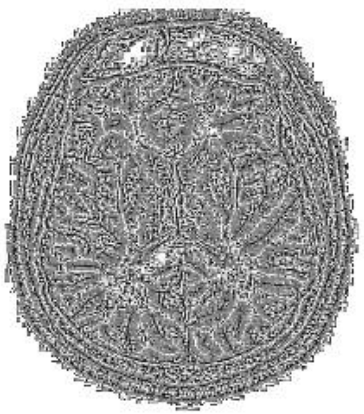
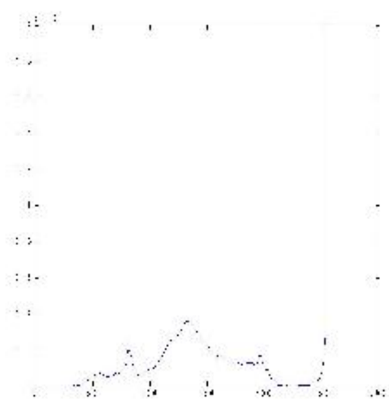
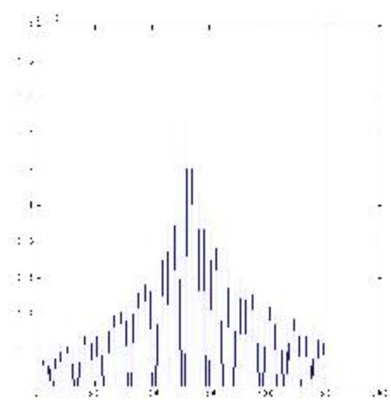


Image after Local Enhancement by 7*7 mask



Orignal Histogram



Histogram after local enhancement

Local Histogram Specification: Examples