Matching

- Finding Corresponding points between two images representing the same scene viewed in different imaging conditions.
- Matching for finding tranformation

Image transformation Matching patches

- Extracting and matching patches

Image transformation Matching patches

- Extracting and matching patches

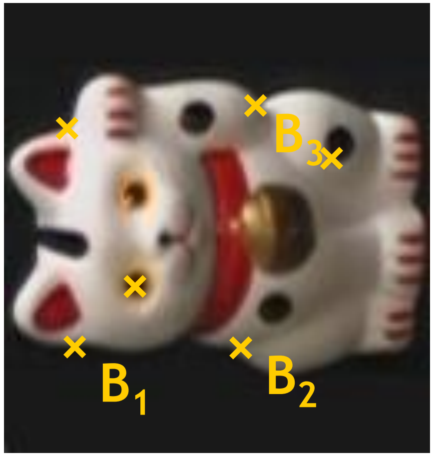


Image transformation
Matching patches

- Extracting and matching patches

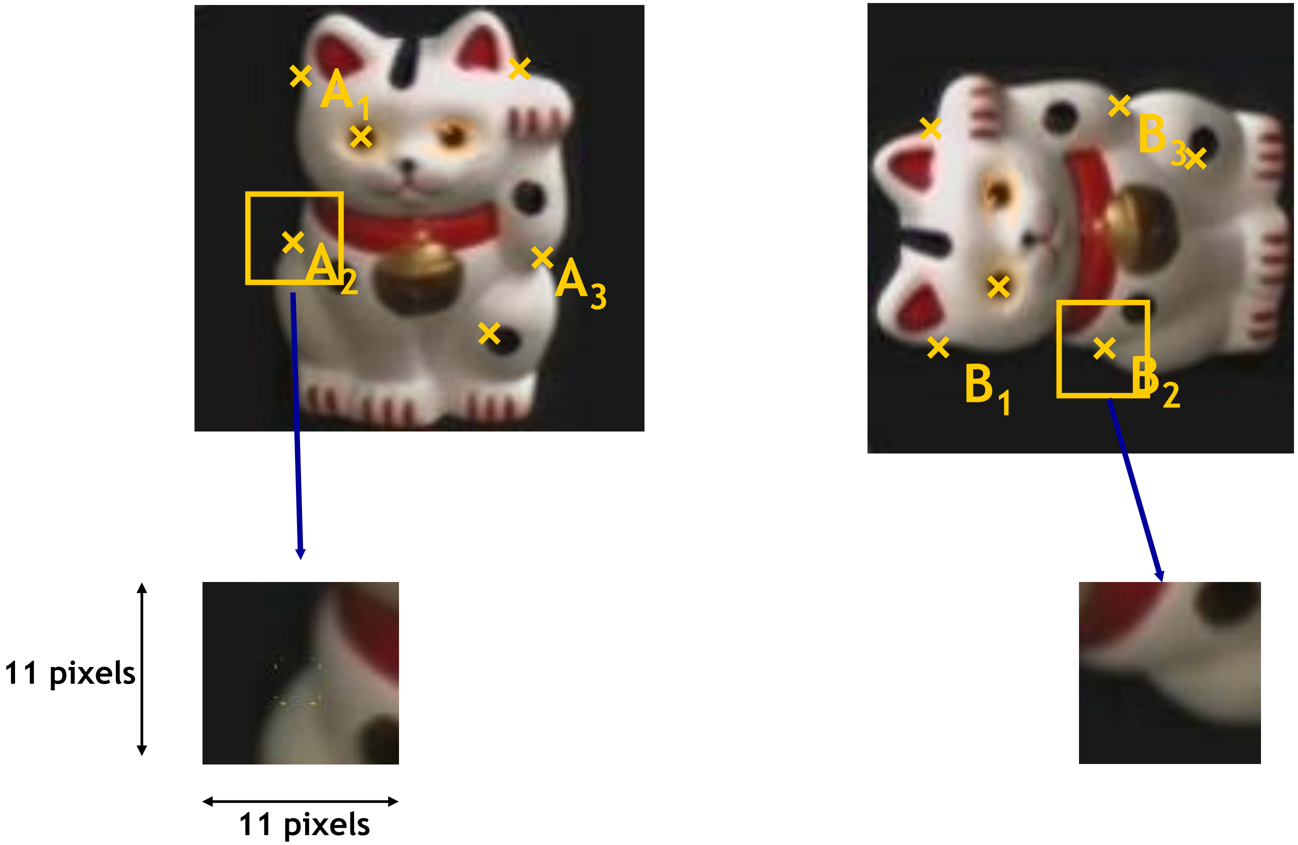


Image transformation
Matching patches

- Extracting and matching patches

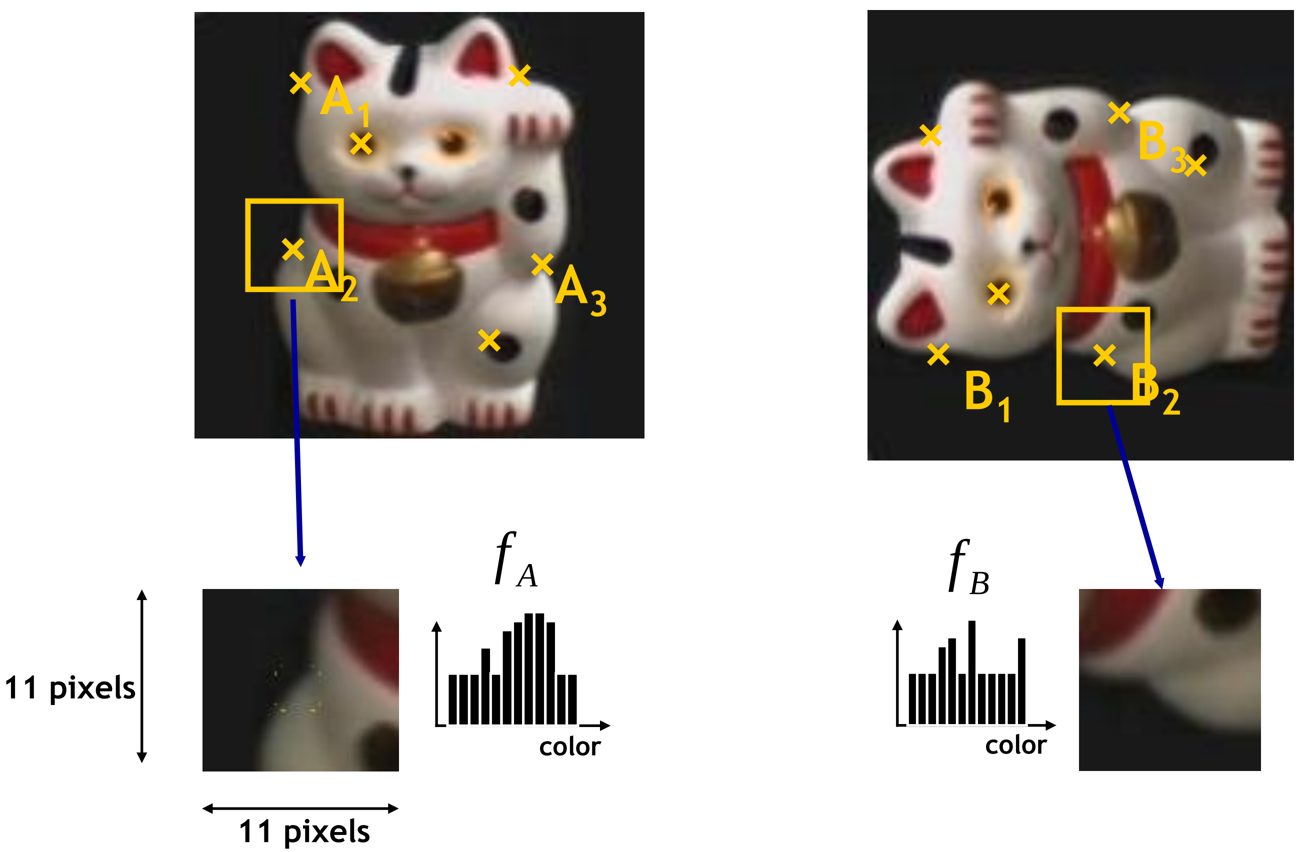


Image transformation
Matching patches

- Extracting and matching patches

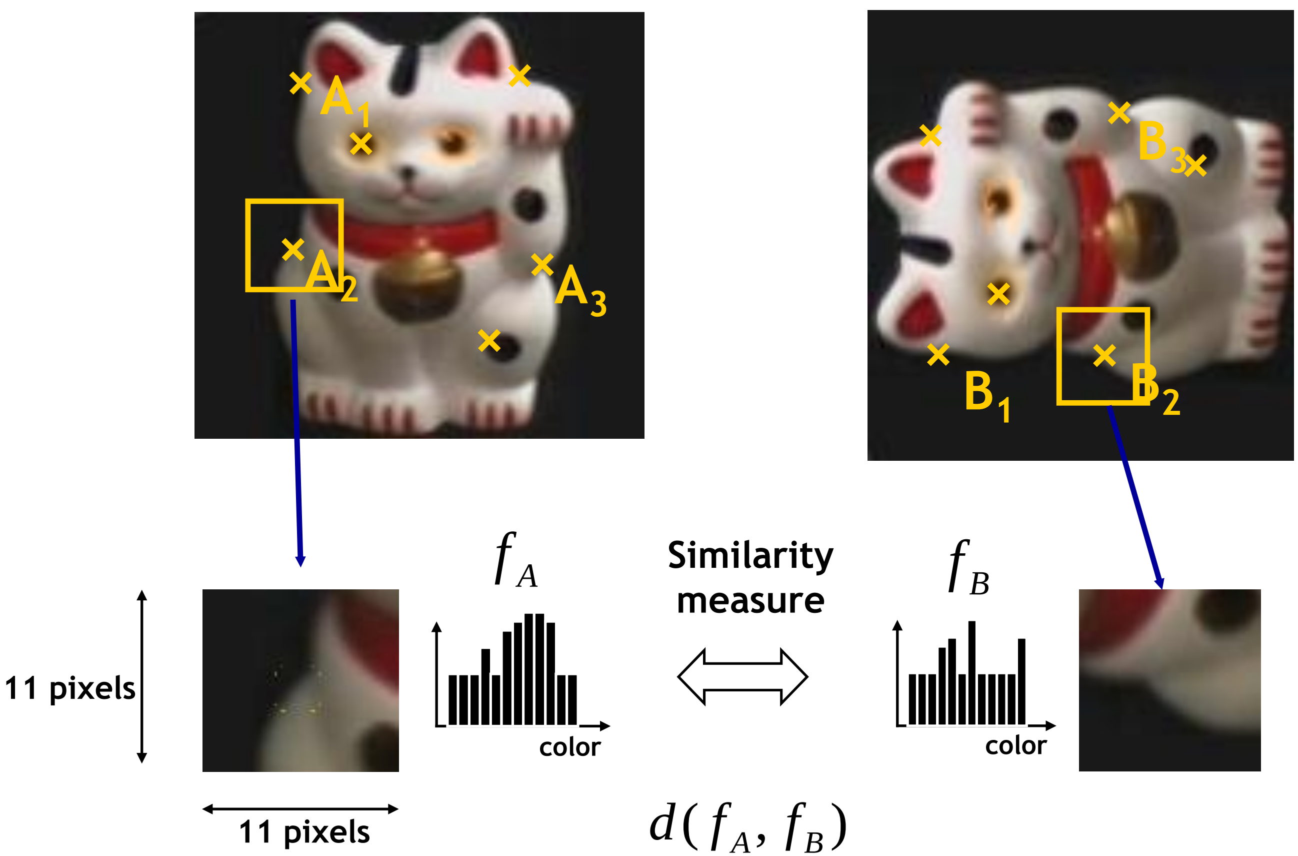


Image transformation
Matching patches

- Extracting and matching patches

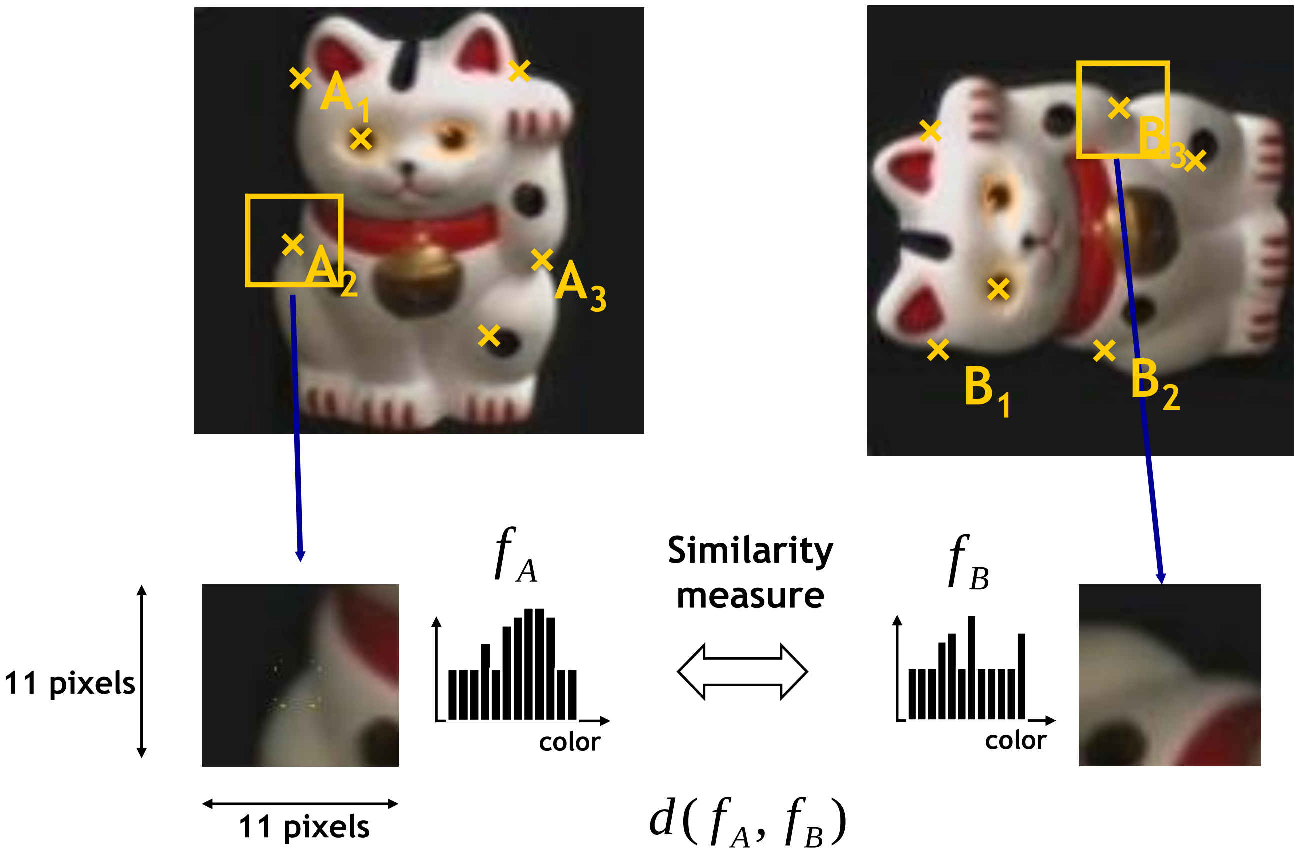


Image transformation
Matching patches

- Extracting and matching patches

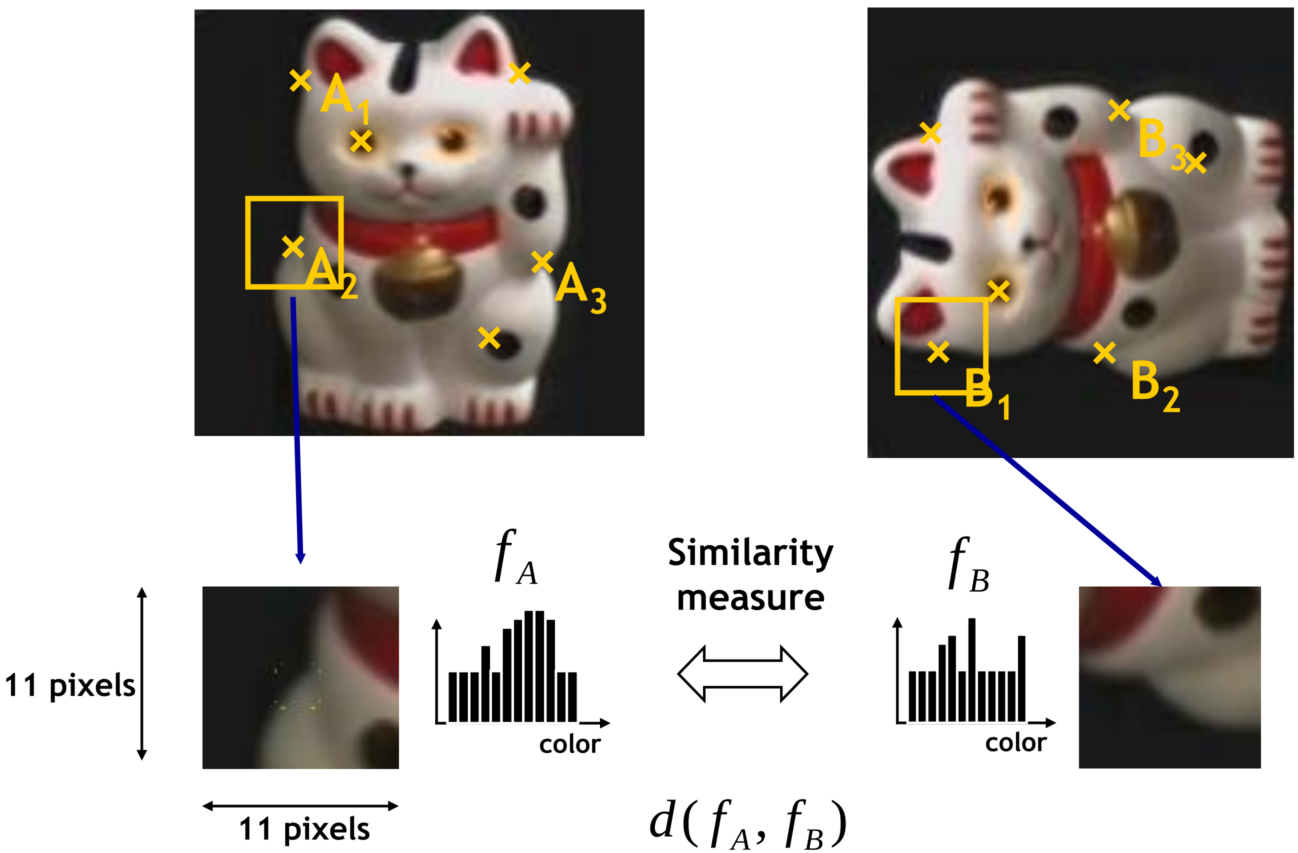
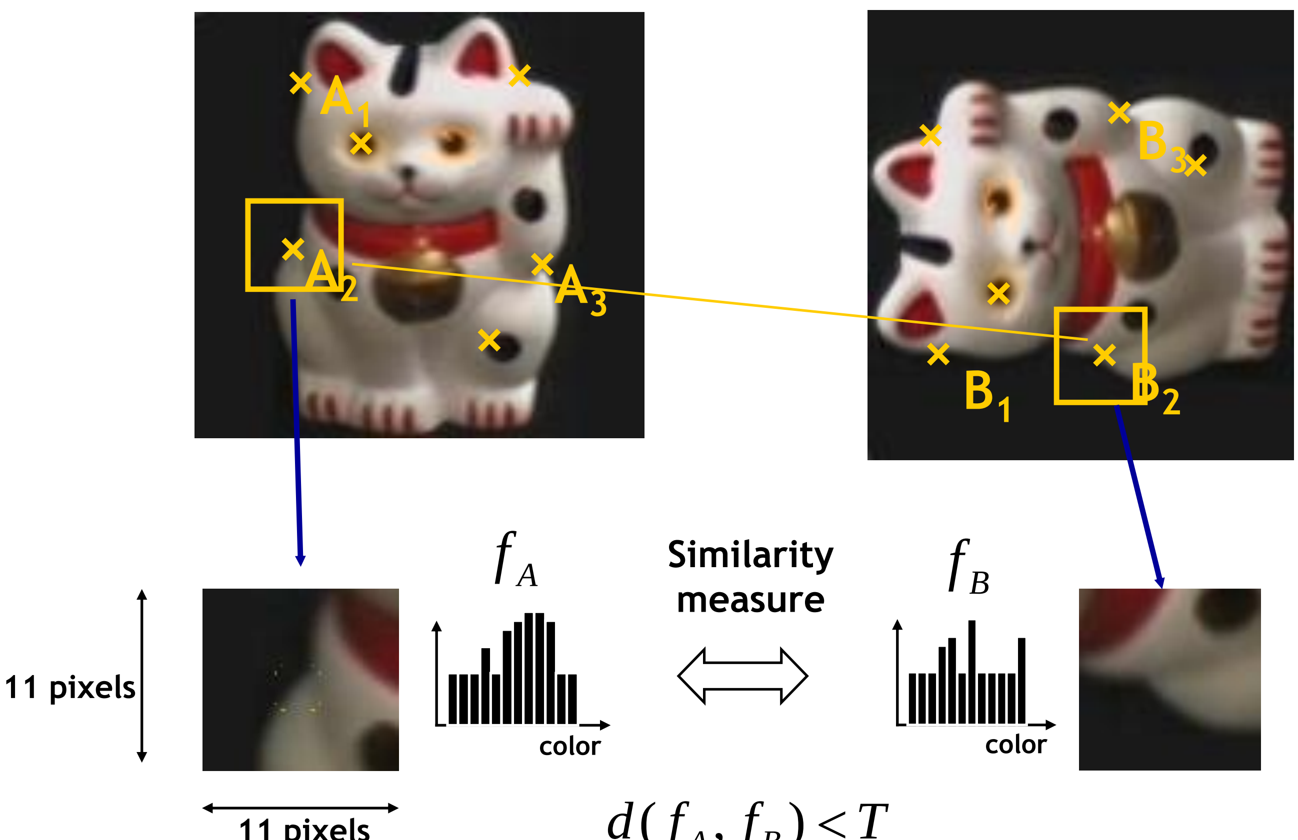


Image transformation
Matching patches

- Extracting and matching patches

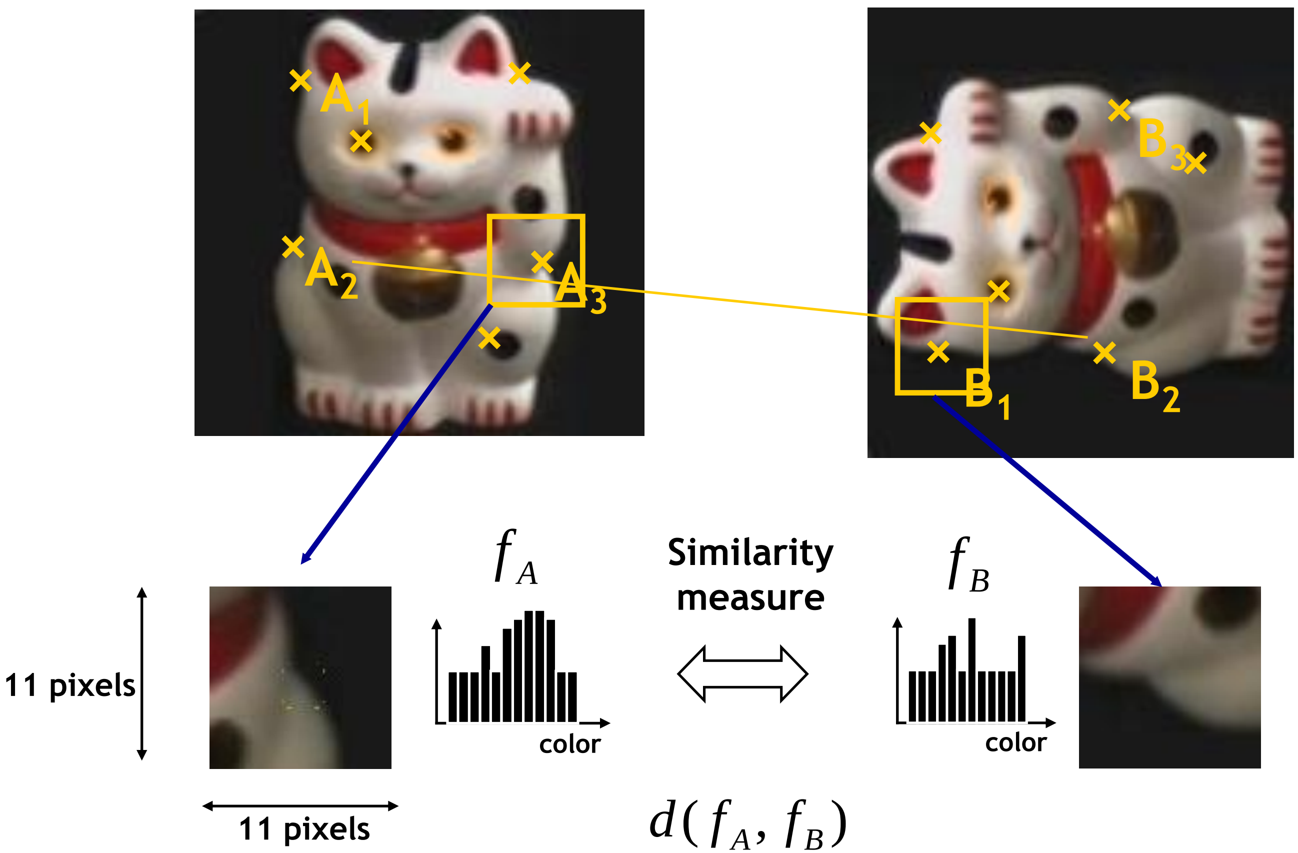


$$\mathbf{x}_{A_2} \leftrightarrow \mathbf{x}_{B_2}$$

$$d(f_A, f_B) < T$$

Image transformation
Matching patches

- Extracting and matching patches



$$\mathbf{x}_{A_2} \leftrightarrow \mathbf{x}_{B_2}$$

Image transformation
Matching patches

- Extracting and matching patches

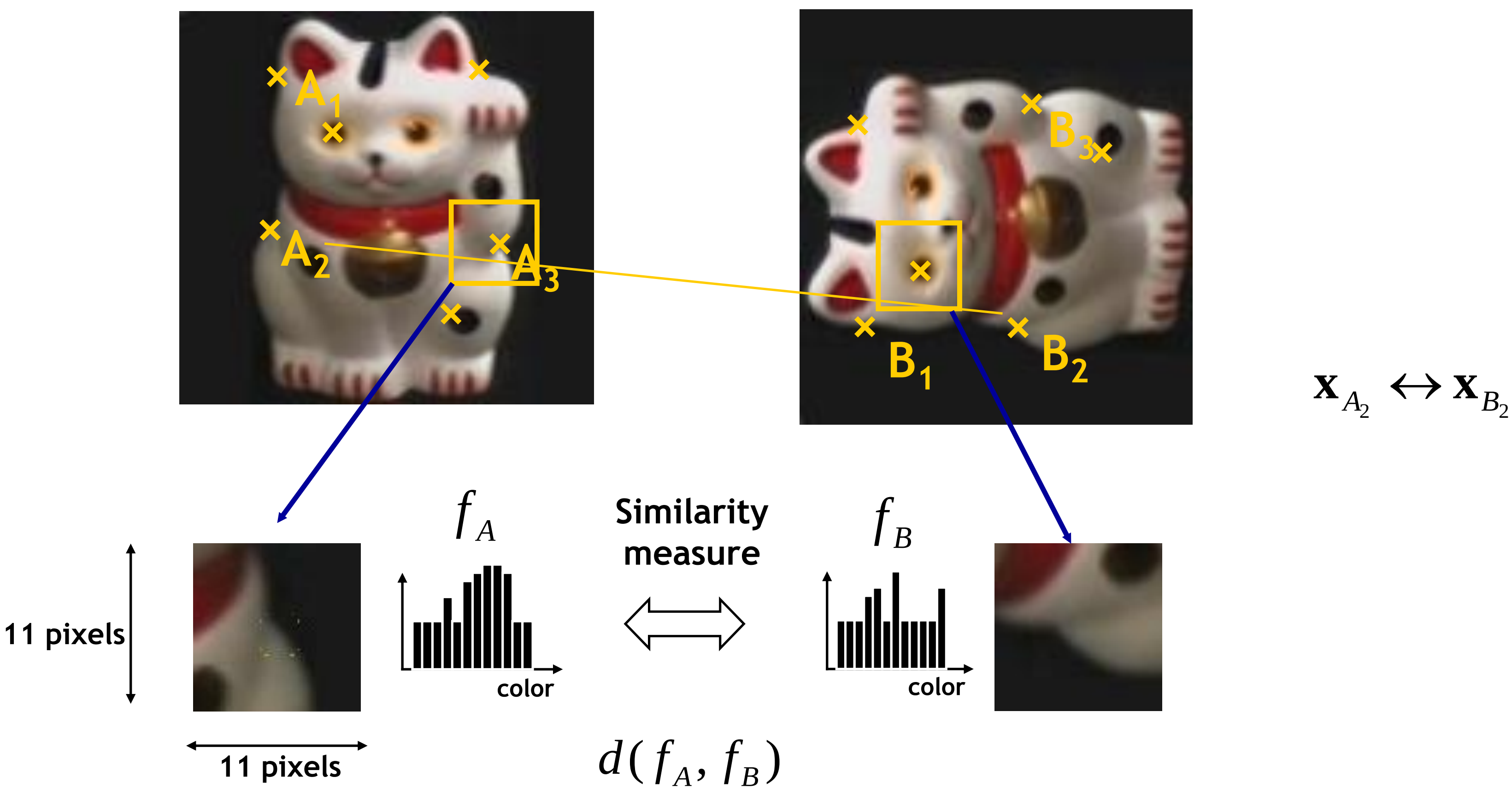
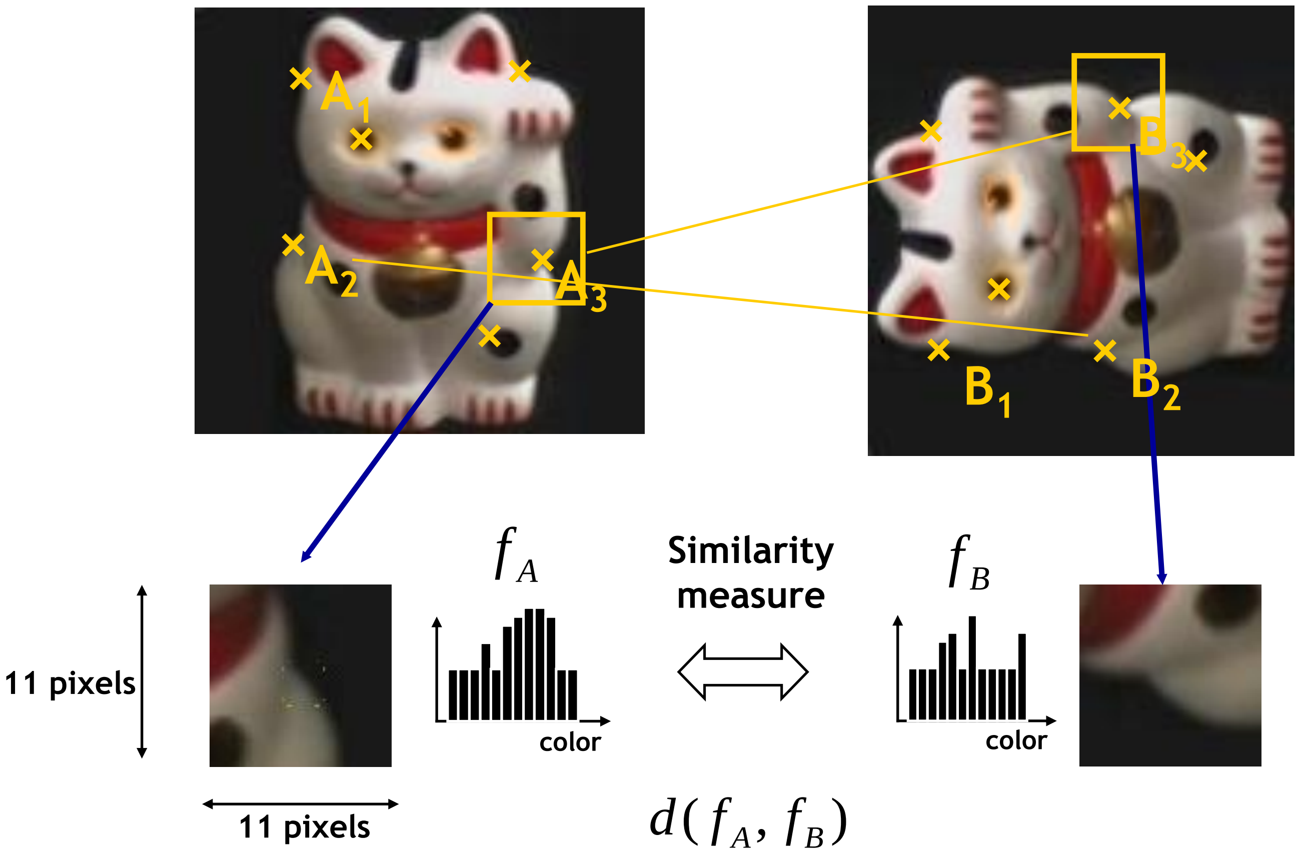


Image transformation
Matching patches

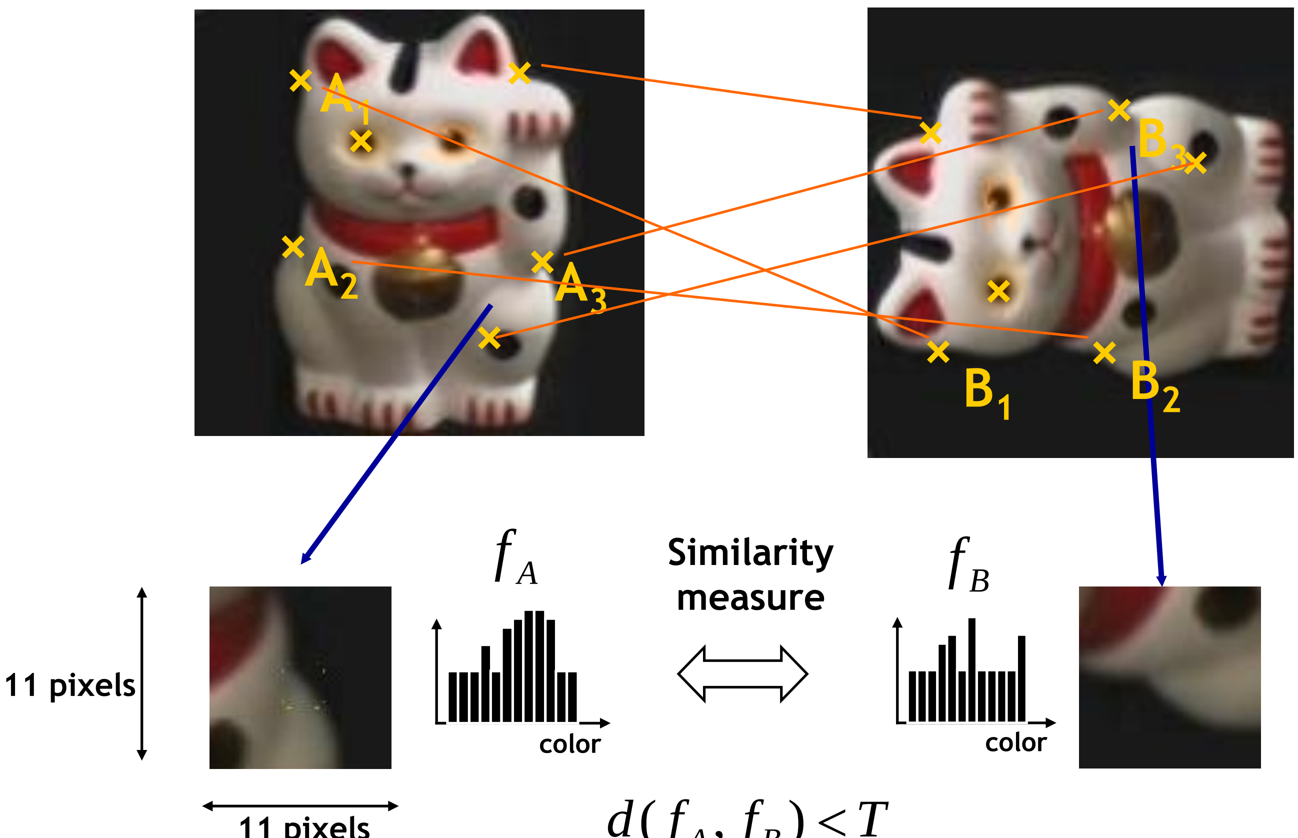
- Extracting and matching patches



$$\mathbf{x}_{A_2} \leftrightarrow \mathbf{x}_{B_2}$$
$$\mathbf{x}_{A_3} \leftrightarrow \mathbf{x}_{B_3}$$

Image transformation
Matching patches

- Extracting and matching patches



$$d(f_A, f_B) < T$$

$$\mathbf{x}_{A_2} \leftrightarrow \mathbf{x}_{B_2}$$
$$\mathbf{x}_{A_3} \leftrightarrow \mathbf{x}_{B_3}$$
$$\mathbf{x}_{A_1} \leftrightarrow \mathbf{x}_{B_1}$$
$$\vdots$$

Questions: goo.gl/K61te5

Image transformation Matching patches

- Matching algorithm
  1. Detecting local features in two images (e.g. corners)
  2. Computing descriptors (e.e. SIFT, extracting patches and computing a histogram for each one)
  3. Comparing one feature from image 1 to every feature in image 2 and selecting the pair which gives the minimum distance between histograms
  4. Repeating the above for each feature from image 1
  5. Using the list of best pairs to estimate the transformation between images

- Computing transformation
  - Each pair of corresponding points gives 4 parameters $(x,y),(x',y')$

Image transformation

translation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & h_{13} \\ 0 & 1 & h_{23} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

rotation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\det \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = 1$$

similarity

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\det \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = scale^2$$

affine

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

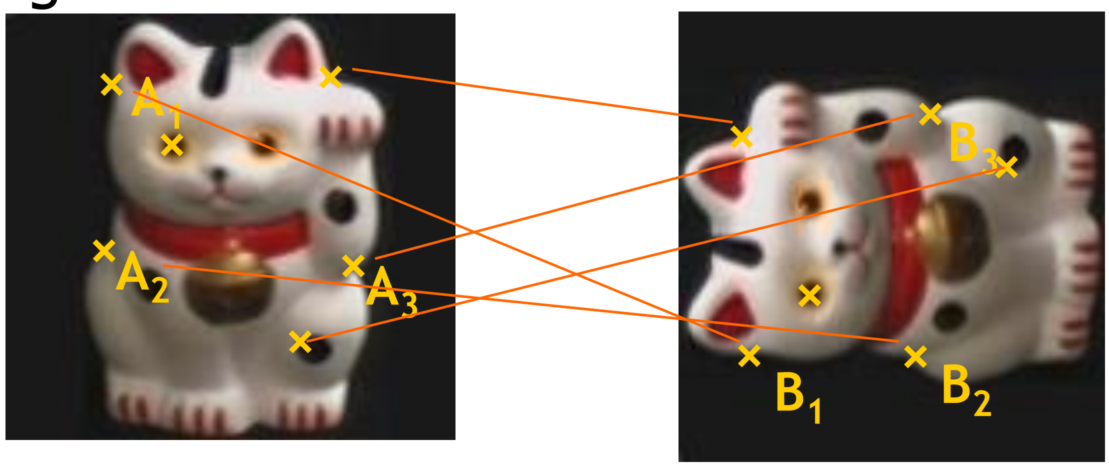
$$\det \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = scale_x scale_y$$

perspective

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Image transformation
Matching patches

- Computing transformation



$$\mathbf{x}_{A_1} \leftrightarrow \mathbf{x}_{B_1}$$
$$\mathbf{x}_{A_2} \leftrightarrow \mathbf{x}_{B_2}$$
$$\mathbf{x}_{A_3} \leftrightarrow \mathbf{x}_{B_3}$$
$$\vdots$$

Homogenous coordinates

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x_{A_1} = \frac{h_{11}\ x_{B_1} + h_{12} y_{B_1} + h_{13}}{h_{31}\ x_{B_1} + h_{32} y_{B_1} + 1}$$

Image coordinates

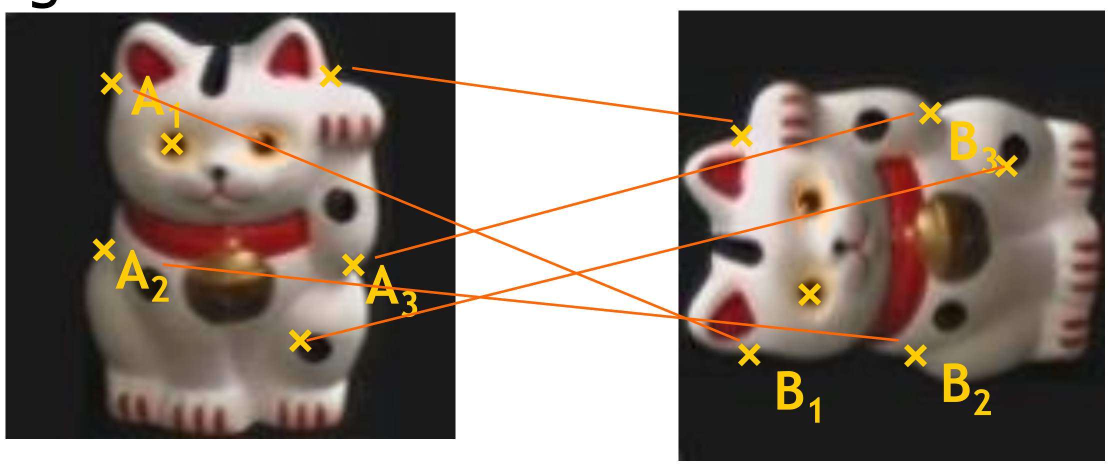
$$\begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = \boxed{\frac{1}{z'}} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

Matrix notation

$$\mathbf{x}' = \mathbf{H}\mathbf{x}$$
$$\mathbf{x} = \mathbf{H}^{-1}\mathbf{x}'$$

Image transformation
Matching patches

- Computing transformation



$$\mathbf{x}_{A_1} \leftrightarrow \mathbf{x}_{B_1}$$
$$\mathbf{x}_{A_2} \leftrightarrow \mathbf{x}_{B_2}$$
$$\mathbf{x}_{A_3} \leftrightarrow \mathbf{x}_{B_3}$$
$$\vdots$$

Homogenous coordinates

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Image coordinates

$$\begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = \frac{1}{z'} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

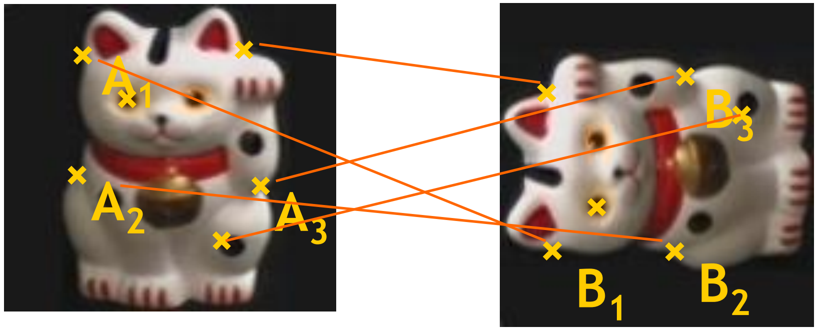
Matrix notation

$$\mathbf{x}' = \mathbf{H}\mathbf{x}$$
$$\mathbf{x} = \mathbf{H}^{-1}\mathbf{x}'$$

$$x_{A_1} = \frac{h_{11}\, x_{B_1} + h_{12} y_{B_1} + h_{13}}{h_{31}\, x_{B_1} + h_{32} y_{B_1} + 1} \qquad y_{A_1} = \frac{h_{21}\, x_{B_1} + h_{22} y_{B_1} + h_{23}}{h_{31}\, x_{B_1} + h_{32} y_{B_1} + 1}$$

Image transformation Matching patches

- Computing transformation



$$\mathbf{Ah} = 0$$

SVD

$$\mathbf{x}_{A_1} \leftrightarrow \mathbf{x}_{B_1}$$
$$\mathbf{x}_{A_2} \leftrightarrow \mathbf{x}_{B_2}$$
$$\mathbf{x}_{A_3} \leftrightarrow \mathbf{x}_{B_3}$$
$$\vdots$$

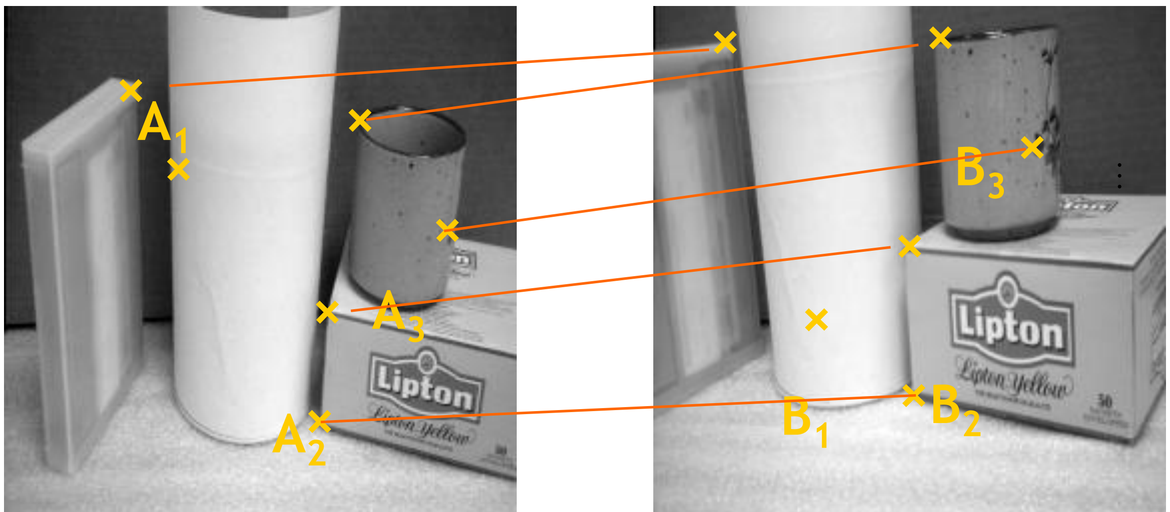
$$\mathbf{A}^T\mathbf{A} = \mathbf{UDV}^T = \mathbf{U}\begin{bmatrix} d_{11} & \cdots & d_{19} \\ \vdots & \ddots & \vdots \\ d_{91} & \cdots & d_{99} \end{bmatrix}\begin{bmatrix} v_{11} & \cdots & v_{19} \\ \vdots & \ddots & \vdots \\ v_{91} & \cdots & v_{99} \end{bmatrix}^T$$

$$\boxed{\mathbf{h} = \frac{[v_{19}, \cdots, v_{99}]}{v_{99}}}$$

- minimizes least square error

Image transformation
Matching patches

- Computing transformation



$$\mathbf{x}_{A_1} \leftrightarrow \mathbf{x}_{B_1}$$
$$\mathbf{x}_{A_2} \leftrightarrow \mathbf{x}_{B_2}$$
$$\mathbf{x}_{A_3} \leftrightarrow \mathbf{x}_{B_3}$$
$$\vdots$$

Homogenous coordinates

Image coordinates

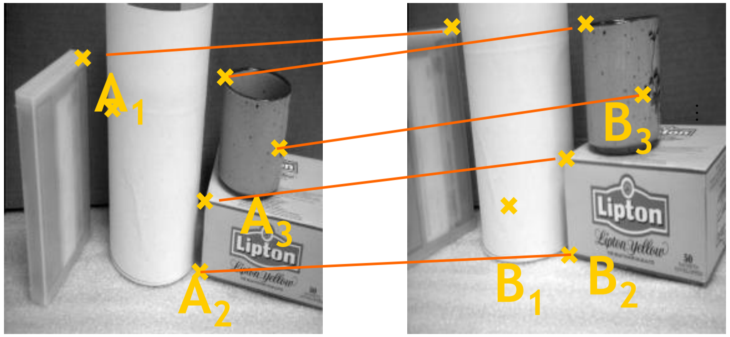
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \cdot \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

Matrix notation

$$\mathbf{x}'\mathbf{F}\mathbf{x} = 0$$

Image transformation
Matching patches

- Computing transformation



$$\mathbf{Af} = 0 \qquad \mathbf{x}'\mathbf{Fx} = 0$$

SVD

$$\mathbf{x}_{A_1} \leftrightarrow \mathbf{x}_{B_1}$$
$$\mathbf{x}_{A_2} \leftrightarrow \mathbf{x}_{B_2}$$
$$\mathbf{x}_{A_3} \leftrightarrow \mathbf{x}_{B_3}$$
$$\vdots$$

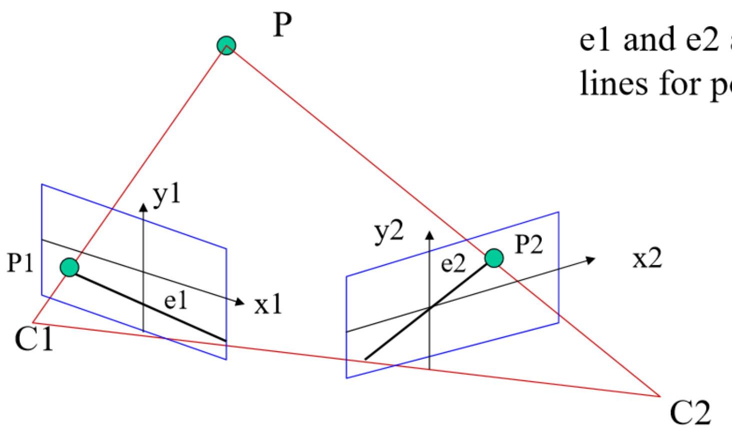
$$\mathbf{A}^T\mathbf{A} = \mathbf{UDV}^T = \mathbf{U}\begin{bmatrix} d_{11} & \cdots & d_{19} \\ \vdots & \ddots & \vdots \\ d_{91} & \cdots & d_{99} \end{bmatrix}\begin{bmatrix} v_{11} & \cdots & v_{19} \\ \vdots & \ddots & \vdots \\ v_{91} & \cdots & v_{99} \end{bmatrix}^T$$

$$\boxed{\mathbf{f} = \frac{[v_{19}, \cdots, v_{99}]}{v_{99}}}$$

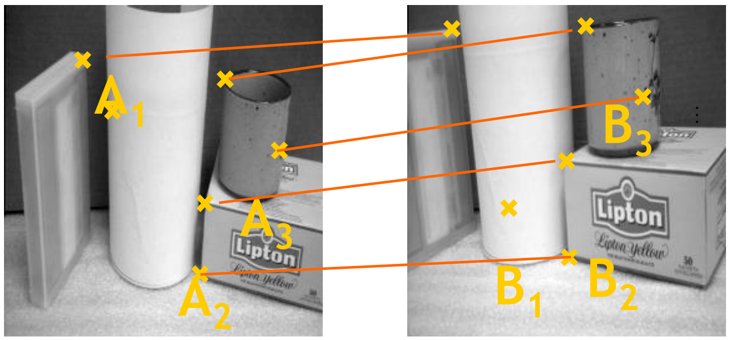
- minimizes least square error

Image transformation
Matching patches

- Computing transformation



e1 and e2 are the epipolar lines for point P



$$\mathbf{A}\mathbf{f} = 0$$

$$\mathbf{x}'\mathbf{F}\mathbf{x} = 0$$
$$\mathbf{x}\mathbf{F}^T\mathbf{x}' = 0$$

$$\mathbf{x}_{A_1} \leftrightarrow \mathbf{x}_{B_1}$$
$$\mathbf{x}_{A_2} \leftrightarrow \mathbf{x}_{B_2}$$
$$\mathbf{x}_{A_3} \leftrightarrow \mathbf{x}_{B_3}$$
$$\vdots$$

$$\mathbf{x}_{B_1}\mathbf{F}\mathbf{e}_A = 0$$

$\mathbf{e}_A$ - Intersection of all epipolar lines in image A, projection of camera centre from image B to A,

$\mathbf{F}\mathbf{e}_A = 0$ - null vector of F

$\mathbf{l}_{A_1}$ - epipolar line in image A, containing $\mathbf{x}_{A_1}$

$\mathbf{l}_{A_1}$ - Can be calculated from $\mathbf{x}_{A_1}$ and $\mathbf{e}_A$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Geometric transformation

- Estimation problems
  - Noise – inaccuracy in point positions (feature detectors)
  - Mismatches – incorrect matches (outliers)

- Solution
  - Use as many corresponding points as possible to minimize the error
  - Use Hough Transform or RANSAC