EE303: Communication Systems

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An Overview of Fundamentals: Channels, Criteria and Limits

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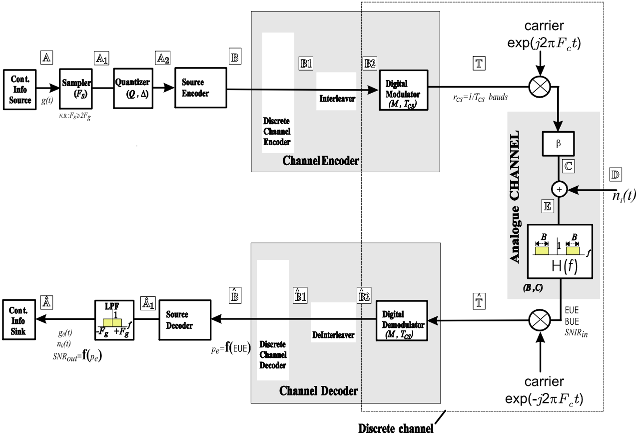
Introduction

- With reference to the following block structure of a Dig. Comm. System (DCS), this topic is concerned with the basics of both continuous and discrete communication channels.

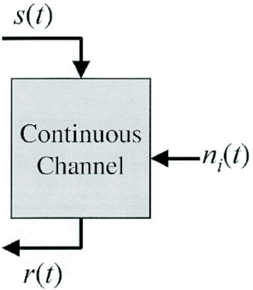


Block structure of a DCS

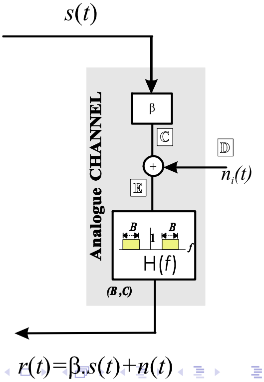
- Just as with sources, communication channels are either
  - discrete channels, or
  - continuous channels
    1. wireless channels (in this case the whole DCS is known as a Wireless DCS)
    2. wireline channels (in this case the whole DCS is known as a Wireline DCS)
- Note that a continuous channel is converted into (becomes) a discrete channel when a **digital modulator** is used to feed the channel and a **digital demodulator** provides the channel output.
- Examples of channels - with reference to DCS shown in previous page,
  - discrete channels:
    - ★ input: $\mathrm{A2}$ - output: $\widehat{\mathrm{A}}2$ (alphabet: levels of quantiser - Volts)
    - ★ input: $\mathrm{B2}$ - output: $\widehat{\mathrm{B}}2$ (alphabet: binary digits or binary codewords)
  - continuous channels:
    - ★ input: $\mathrm{A1}$ - output: $\widehat{\mathrm{A}}1$, (Volts) - continuous channel (baseband)
    - ★ input: T, - output: $\widehat{\mathrm{T}}$ (Volts) - continuous channel (baseband),
    - ★ input: $\mathrm{T1}$ - output: $\widehat{\mathrm{T}}1$ (Volts) - continuous channel (bandpass).

Continuous Channels

- A continuous communication channel (which can be regarded as an analogue channel) is described by
  - an input ensemble $(s(t), \text{pdf}_s(s))$ and $\text{PSD}_s(f)$
  - an output ensemble, $(r(t), \text{pdf}_r(r))$
  - the channel noise (AWGN) $n_i(t)$ and β,
  - the channel bandwidth B and channel capacity C.

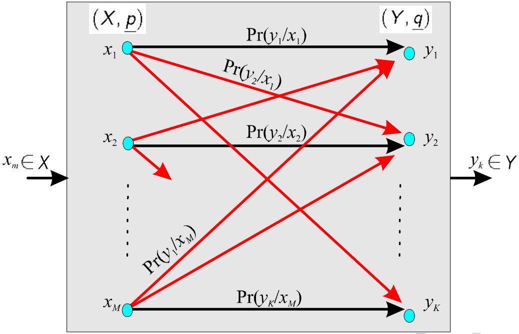


$\Longleftrightarrow$



Discrete Channels

- A discrete communication channel has a discrete input and a discrete output where
  - the symbols applied to the channel input for transmission are drawn from a finite alphabet, described by an input ensemble $(X, \underline{p})$ while
  - the symbols appearing at the channel output are also drawn from a finite alphabet, which is described by an output ensemble $(Y, \underline{q})$
  - the channel transition probability matrix $\mathbb{F}$.



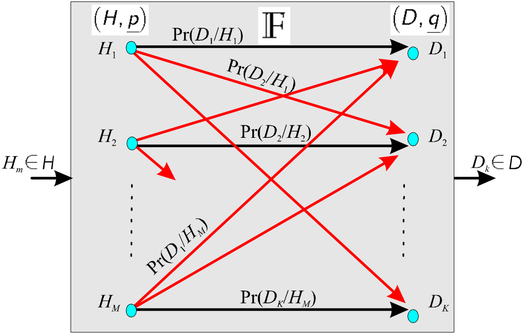
- In many situations the input and output alphabets X and Y are identical but in the general case these are different. Instead of using X and Y, it is common practice to use the symbols H and D and thus define the two alphabets and the associated probabilities as

$$\text{input:} \quad H = \{H_1, H_2, ..., H_M\} \quad \underline{p} = [\overbrace{\Pr(H_1)}^{\triangleq p_1}, \overbrace{\Pr(H_2)}^{\triangleq p_2}, ..., \overbrace{\Pr(H_M)}^{\triangleq p_M}]^T$$

$$\text{output:} \quad D = \{D_1, D_2, ..., D_M\} \quad \underline{q} = [\overbrace{\Pr(D_1)}^{\triangleq q_1}, \overbrace{\Pr(D_2)}^{\triangleq q_2}, ..., \overbrace{\Pr(D_K)}^{\triangleq q_K}]^T$$

where p_m abbreviates the probability $\Pr(H_m)$ that the symbol H_m may appear at the input while q_k abbreviates the probability $\Pr(D_k)$ that the symbol D_k may appear at the output of the channel.

- The probabilistic relationship between input symbols H and output symbols D is described by the so-called channel transition probability matrix $\mathbb{F}$, which is defined as follows:



$$\mathbb{F}=\begin{bmatrix}\Pr(D_1|H_1), & \Pr(D_1|H_2), & ..., & \Pr(D_1|H_M)\\ \Pr(D_2|H_1), & \Pr(D_2|H_2), & ..., & \Pr(D_2|H_M)\\ ..., & ..., & ..., & ...\\ \Pr(D_K|H_1), & \Pr(D_K|H_2), & ..., & \Pr(D_K|H_M)\end{bmatrix} \quad (1)$$

- $\Pr(D_k|H_m)$ denotes the probability that symbol $D_k \in D$ will appear at the channel output, given that $H_m \in H$ was applied to the input.
- The input ensemble $(H, \underline{p})$, the output ensemble $(D, \underline{q})$ and the matrix $\mathbb{F}$ fully describe the functional properties of the channel.
- The following expression describes the relationship between $\underline{q}$ and $\underline{p}$

$$\underline{q} = \mathbb{F}.\underline{p} \tag{2}$$

- Note that in a **noiseless channel**

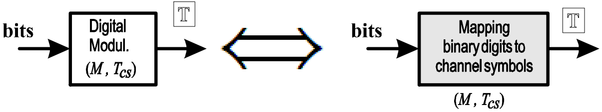
$$\begin{aligned} D &= H \\ \underline{q} &= \underline{p} \end{aligned} \tag{3}$$

i.e the matrix $\mathbb{F}$ is an identity matrix

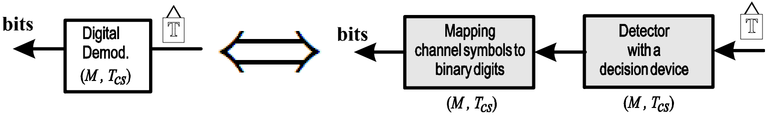
$$\mathbb{F} = \mathbb{I}_M \tag{4}$$

Converting a Continuous to a Discrete Channel

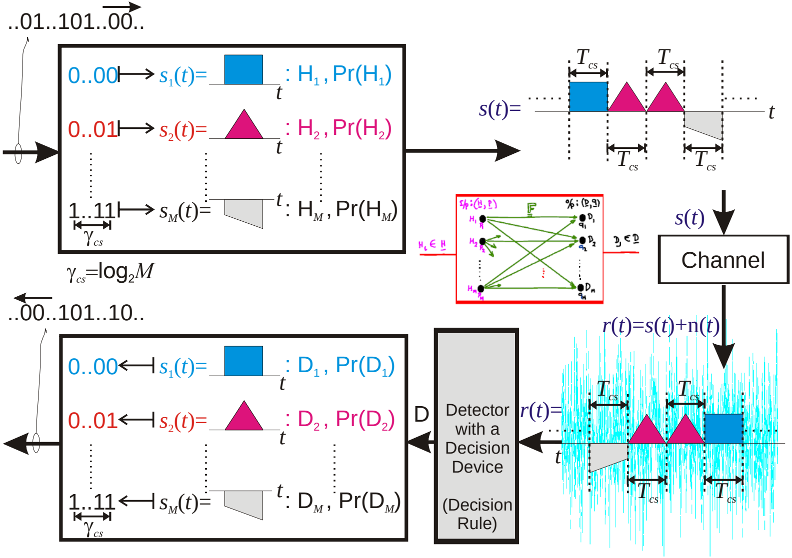
- A continuous channel is converted into (becomes) a discrete channel when a **digital modulator** is used to feed the channel and a **digital demodulator** provides the channel output.
- A digital modulator is described by M **different channel symbols** . These channel symbols are **ENERGY SIGNALS** of duration T_{cs}.
- Digital Modulator:



Digital Demodulator:



- If $M = 2$ $\Rightarrow$**Binary** Digital Modulator $\Rightarrow$**Binary** Comm. System
- If $M > 2$ $\Rightarrow$**M-ary** Digital Modulator $\Rightarrow$**M-ary** Comm. System


..01..101..00..
0..00 ⟼ $s_1(t)$= : H_1, $\Pr(H_1)$
0..01 ⟼ $s_2(t)$= : H_2, $\Pr(H_2)$
1..11 ⟼ $s_M(t)$= : H_M, $\Pr(H_M)$
γ_{cs}
$\gamma_{cs}=\log_2 M$
$s(t)$=
T_{cs}
t
$s(t)$
Channel
$r(t)=s(t)+n(t)$
$r(t)$=
Detector with a Decision Device
(Decision Rule)
D
..00..101..10..
0..00 ⟻ $s_1(t)$= : D_1, $\Pr(D_1)$
0..01 ⟻ $s_2(t)$= : D_2, $\Pr(D_2)$
1..11 ⟻ $s_M(t)$= : D_M, $\Pr(D_M)$

More on Discrete Channels

Backward transition Matrix

- There are also occassions where we **get/observe the output** of a channel and then, based on this knowledge, **we refer to the input** .
- In this case we may use the concept of an imaginary "**backward** " channel and its associated transition matrix, known as **backward transition matrix** defined as follows:

$$\mathbb{B} = \begin{bmatrix} \Pr(H_1|D_1), & \Pr(H_1|D_2), & ..., & \Pr(H_1|D_K) \\ \Pr(H_2|D_1), & \Pr(H_2|D_2), & ..., & \Pr(H_2|D_K) \\ ..., & ..., & ..., & ... \\ \Pr(H_M|D_1), & \Pr(H_M|D_2), & ..., & \Pr(H_M|D_K) \end{bmatrix}^T \quad (5)$$

Joint transition Probability Matrix

- The joint probabilistic relationship between input channel symbols $H = \{H_1, H_2, ..., H_M\}$ and output channel symbols $D = \{D_1, D_2, ..., D_M\}$,
is described by the so-called joint-probability matrix,

$$\mathbb{J} \triangleq \begin{bmatrix} \Pr(H_1, D_1), & \Pr(H_1, D_2), & ..., & \Pr(H_1, D_K) \\ \Pr(H_2, D_1), & \Pr(H_2, D_2), & ..., & \Pr(H_2, D_K) \\ ..., & ..., & ..., & ... \\ \Pr(H_M, D_1), & \Pr(H_M, D_2), & ..., & \Pr(H_M, D_K) \end{bmatrix}^T \quad (6)$$

- $\mathbb{J}$ is related to the forward transition probabilities of a channel with the following expression (compact form of Bayes' Theorem):

$$\mathbb{J} = \mathbb{F}. \underbrace{\begin{bmatrix} p_1 & 0 & ... & 0 \\ 0 & p_2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & p_M \end{bmatrix}}_{\triangleq \text{diag}(\underline{p})} = \mathbb{F}.\text{diag}(\underline{p}) \quad (7)$$

- Note: This is equivalent to a new (joint) source having alphabet

$$\left\{ \left(H_1, D1 \right), \left(H_1, D_2 \right), ..., \left(H_M, D_K \right) \right\}$$

and ensemble (joint ensemble) defined as follows

$$(H \times D, \mathbb{J}) = \left\{ \begin{array}{c} \left(H_1, D_1 \right), \Pr\left(H_1, D_1 \right) \\ \left(H_1, D_2 \right), \Pr\left(H_1, D_2 \right) \\ ... \\ \left(H_m, D_k \right), \Pr\left(H_m, D_k \right) \\ ... \\ \left(H_M, D_K \right), \Pr\left(H_M, D_K \right) \end{array} \right\} \tag{8}$$

$$= \left\{ \left(\left(H_m, D_k \right), \underbrace{\Pr\left(H_m, D_k \right)}_{=J_{km}} \right), \forall mk : 1 \leq m \leq M, 1 \leq k \leq K \right\}$$

Measure of Information at the Output of a Channel

- In general three measures of information are of main interest:

  1. the **Entropy of a Source** - in (info) bits per source symbol
  2. the **Mutual Entropy** (or Multual Information) **of a Channel**, in (info) bits per channel symbol
  3. the Discrimination of a Sink

- Next we will focus on the Mutual Information of a Channel H_{mut}

Mutual Information of a Channel

- The mutual information measures the **amount of information that the output of the channel** (i.e. received message) **gives about the input to the channel** (transmitted message).
- That is, when symbols or signals are transmitted over a noisy communication channel, information is received. **The amount of information received** is given by the **mutual** information,

$$H_{mut} \geqq 0 \tag{9}$$

$$H_{mut} \triangleq H_{mut}(\underline{p}, \mathbb{F}) = -\sum_{m=1}^{M}\sum_{k=1}^{K} F_{km}.p_m \log_2\left(\frac{q_k}{F_{km}}\right) \quad (10)$$

$$= -\sum_{m=1}^{M}\sum_{k=1}^{K} J_{km} \log_2\left(\frac{p_m.q_k}{J_{km}}\right) \quad (11)$$

$$= -\underline{1}_K^T \left(\underbrace{\mathbb{J} \odot \log_2\left[\left(\mathbb{F}.\underline{p}.\underline{p}^T\right) \oslash \mathbb{J}\right]}_{K \times M \quad matrix}\right) \underline{1}_M \ \frac{\text{bits}}{\text{symbol}} \quad (12)$$

where

$$\underline{1}_M = \text{a column vector of } M \text{ ones}$$
$$\odot, \oslash = \text{Hadamard operators (mult. and div.)}$$

Note that

$$\underline{1}^T \mathbb{A} \underline{1} = \text{adds all elements of } \mathbb{A} \quad (13)$$

Equivocation & Mutual Information of a Discrete Channel

- Consider a discrete source $(H, \underline{p})$ followed by a discrete channel, as shown below
- The **average amount of information gained** (or uncertainty removed) about the H source (channel input) by observing the outcome of the D source (channel output), is given by the conditional entropy $H_{H|D}$ which is defined as follows:

$$H_{H|D} \triangleq H_{H|D}(\mathbb{J}) = -\sum_{m=1}^{M}\sum_{k=1}^{K} J_{km}.\log_2\left(\frac{J_{km}}{q_k}\right) \quad (14)$$

$$= -\underline{1}_K^T \left(\underbrace{\mathbb{J} \odot \log_2\left(\overbrace{\text{diag}\left(\underline{q}\right)^{-1}}^{\mathbb{B}} \mathbb{J} \right)}_{K \times M \ \ matrix} \right) \underline{1}_M \ \frac{\text{bits}}{\text{symbol}} \quad (15)$$

- A similar expression can be also given for the average information gained about the channel output D by observing the channel input H, i.e.

$$H_{D|H} \triangleq H_{D|H}(\mathbb{J}) = -\sum_{m=1}^{M}\sum_{k=1}^{K} J_{km}.\log_2\left(\frac{J_{km}}{p_m}\right) \quad (16)$$

$$= -\underline{1}_K^T\left(\underbrace{\mathbb{J}\odot\log_2\left(\overbrace{\mathbb{J}.\mathrm{diag}\left(\underline{p}\right)^{-1}}^{\mathbb{F}}\right)}_{K\times M \quad matrix}\right)\underline{1}_M \ \frac{\text{bits}}{\text{symbol}} \quad (17)$$

- The conditional entropy $H_{H|D}$ is also known as **equivocation** and **it is the entropy of the noise** or, otherwise, the uncertainty in the input of the channel from the receiver's point of view.

- Notes

1. for a **noiseless** channel:

$$H_{H|D} = 0 \tag{18}$$

2. For a discrete memoryless channel,

$$H_{mut} \triangleq H_{mut}(\underline{p}, \mathbb{F}) = H_H - H_{H|D} \tag{19}$$

$$= H_D - H_{D|H} \tag{20}$$

Capacity of a Channel

Shannon's Capacity Theorem

- There is a theoretical upper limit to the performance of a specified digital communication system with the upper limit depending on the actual system specified.
- However, in addition to the specific upper limit associated with each system, there is an overall upper limit to the performance which no digital communication system, and in fact no communication system at all, can exceed.
- This bound (limit) is important since it provides the performance level against which all other systems can be compared.
- The closer a system comes, performance wise, to the upper limit the better.

- The theoretical upper limit was given by Shannon (1948) as an upper bound to the maximum rate at which information can be transmitted over a communication channel.
- This rate is called channel capacity and is denoted by the symbol C. Shannon's capacity theorem states:

$$C \triangleq \max\left(H_{mut}\right) \quad \frac{\text{bits}}{\text{symbol}} \tag{21}$$

or

$$C \triangleq r_{cs} \times \max\left(H_{mut}\right) \quad \frac{\text{bits}}{\text{sec}} \tag{22}$$

where r_{cs} denotes the channel-symbol rate (in channel-symbols per sec) with

$$r_{cs} = \frac{1}{T_{cs}} \tag{23}$$

$$B \geq \frac{r_{cs}}{2} \tag{24}$$

- i.e. if $H_{mut}(\underline{p}, \mathbb{F})$ is maximised with respect to the input probabilities $\underline{p}$, then it becomes equal to C, the channel capacity (in bits/symbol)

Capacity of AWGN Channels

- In the case of a continuous channel corrupted by additive white Gaussian noise the capacity is given by

$$\begin{aligned} C &= \frac{1}{2}\log_2\left(1+\mathrm{SNR}_{in}\right) \quad \frac{\text{bits}}{\text{symbol}} && (25)\\ & \text{or} && (26)\\ C &= B\log_2\left(1+\mathrm{SNR}_{in}\right) \quad \frac{\text{bits}}{\text{sec}} && (27)\end{aligned}$$

where

$$\begin{aligned} B &= \text{baseband band width of channel}\\ \mathrm{SNR}_{in} &= \frac{P_s}{P_n}\\ P_s &= \text{Power of the signal at point } \widehat{\mathrm{T}}\\ P_n &= \text{Power of the noise at point } \widehat{\mathrm{T}} \; = N_0 B\end{aligned}$$

Capacity of non-Gaussian Channels

- If the pdf of the noise is arbitrary (non-Gaussian) then it is very difficult to estimate the capacity .
- However, it can be proved [Shannon 1948] that in this case the capacity is bounded as follows:

$$B \log_2 \left(\frac{P_s + N_n}{N_n} \right) \leq C \leq B \log_2 \left(\frac{P_s + P_n}{N_n} \right) \ \frac{\text{bits}}{\text{sec}} \tag{28}$$

- where

$$\begin{aligned} P_s &: \text{ is the average received signal power,} \\ N_n &: \text{ is the entropy power of the noise, and} \\ P_n &: \text{ is the power of the noise} \end{aligned}$$

- Equation 28 is important in that it can be used to provide bounds for any kind of channel.

Shannon's Channel Capacity Theorem based on Continuous Channel Parameters

- Consider a time-continuous channel which comprises of a linear time invariant filter with transfer function $H(f)$, the output of which is corrupted by an additive zero mean stationary noise $n(t)$ of $\mathrm{PSD}_n(f)$.
  - if the average power of the channel input signal is constraint to be P_s, then

$$P_s = \int_{-\infty}^{\infty} \max\left\{0, \theta - \frac{\mathrm{PSD_n}(f)}{|H(f)|^2}\right\}.df \tag{29}$$

$$\text{and } C \geq \int_{-\infty}^{\infty} \max\left\{0, \frac{1}{2}\log_2\left(\frac{\theta.|H(f)|^2}{\mathrm{PSD_n}(f)}\right)\right\}.df \tag{30}$$

    with the equality holding if the noise is Gaussian.
  - Further, if the channel noise is white Gaussian with $\mathrm{PSD}_n(f) = \frac{N_0}{2}$ then Equation 30 simplifies to the well known result

$$C = B\log_2(1+\mathrm{SNR}_{in}) \quad \frac{\text{bits}}{\text{sec}} \tag{31}$$

Bandwidth and Channel Symbol Rate

- The following expression are given without any proof:

$$\text{Baseband Bandwidth} \geq \frac{\text{channel symbol rate}}{2} \quad (32)$$

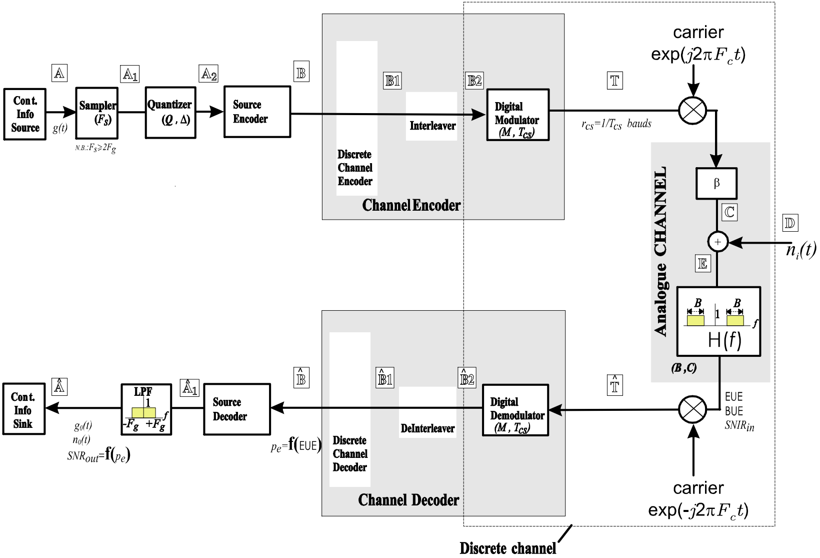
$$\text{Bandpass Bandwidth} \geq \frac{\text{channel symbol rate}}{2} \times 2 \quad (33)$$

- The equality is known as **Nyquist Bandwidth.**
- In this course, except if it is defined otherwise,
  - the word "bandwidth" will mean "Nyquist bandwidth"
  - the carrier will be ignored and thus "bandwidth" by default will refer to "baseband bandwdith"

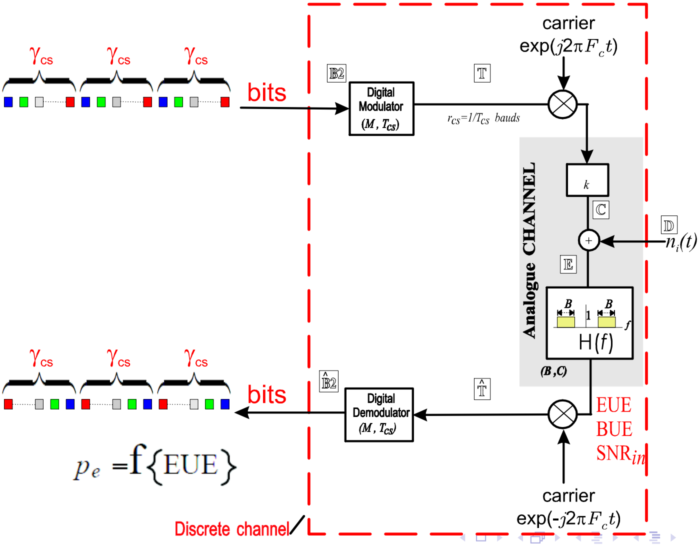
Criteria and Limits of DCS

Introduction

- Digital Communications provide excellent message-reproduction and greatest Energy (EUE) and Bandwidth (BUE) Utilization Efficiency through effective employment of two fundamental techniques:
  - **source compression coding** (to **reduce the transmission rate** for a given degree of fidelity)
  - **error control coding** and digital modulation (to **reduce the SNR** and **bandwidth** requirements)
- With reference to the general structure of a DCS given in the next page,
  - the **source compression coding** is implemented by the blocks "Source Encoder" and "Source Decoder"
  - the **error control coding** is implemended by the "Discrete Channel Endoder", "Interleaver", "DeInterleaver" and "Discrete Channel Decoder".


Cont. Info Source
g(t)
Sampler (F_s)
N.B.: $F_s \geq 2F_g$
Quantizer (Q, Δ)
Source Encoder
A
A1
A2
B
B1
B2
T
Interleaver
Discrete Channel Encoder
Channel Encoder
Digital Modulator (M, T_{cs})
$r_{cs}=1/T_{cs}$ bauds
carrier
exp($j2\pi F_c t$)
Analogue CHANNEL
β
C
D
E
$n_i(t)$
B
1
f
H(f)
(B,C)
EUE
BUE
$SNIR_{in}$
carrier
exp($-j2\pi F_c t$)
Digital Demodulator (M, T_{cs})
DeInterleaver
Discrete Channel Decoder
Channel Decoder
Discrete channel
p_e=f(EUE)
Source Decoder
LPF
$-F_g$ $+F_g$
$g_0(t)$
$n_0(t)$
SNR_{out}=f(p_e)
Cont. Info Sink

- Let us focus on the Discrete Channel: We have seen that a digital modulator is described by $M = 2^{\gamma_{cs}}$ different channel symbols which are ENERGY SIGNALS of duration T_{cs}.



Energy Utilisation Efficiency (EUE)

- The parameter EUE is a measure of how efficiently the system utilises the available energy in order to transmit information in the presence of additive white Gaussian noise of double-sided power spectral density $\text{PSD}_n(f) = N_0/2$ and it is defined as follows:

$$\text{EUE} \triangleq \frac{E_b}{N_0} \tag{34}$$

  Note that EUE is directly related to the received signal power. It will be appreciated of course that this is, in turn, directly related to the transmitted power by the attenuation factor introduced by the channel.

- Clearly, a question of major importance is **how large** EUE **needs to be in order to achieve communication at some specific bit error probability** p_e.
- Obviously **the smaller** EUE to achieve a specified error probability **the better**.

Bandwidth Utilisation Efficiency (BUE)

- The BUE measures how efficiently the system utilises the bandwidth B available to send information and it is defined as follows:

$$\text{BUE} \triangleq \frac{B}{r_b} \tag{35}$$

  where r_b denotes the bit rate.

- Specifically, the BUE **indicates how much bandwidth is being used per transmitted information bit** and hence, for a given level of performance, **the smaller** BUE **the better** since this means that less bandwidth is being used to achieve a given rate of data transmission.

- N.B.:

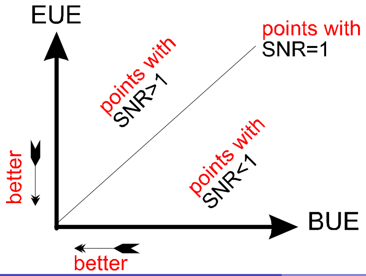
$$\text{signaling speed} \triangleq \frac{r_b}{B} = \text{BUE}^{-1} \tag{36}$$

Visual Comparison of Comm Systems

- By using EUE and BUE the SNR*in* can be expressed as follows

$$\mathrm{SNR}_{in} = \frac{P_s}{P_n} = \frac{\frac{E_b}{T_b}}{N_0 B} = \frac{E_b}{N_0 B T_b} = \frac{E_b}{N_0 B \frac{1}{rb}} = \frac{\frac{E_b}{N_0}}{\frac{B}{r_b}} = \frac{\mathrm{EUE}}{\mathrm{BUE}} \qquad (37)$$

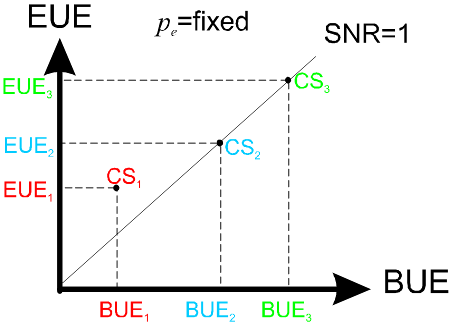
- By determining the EUE and BUE **of any particular system**, that system can be represented as a **point in the plane** (EUE,BUE).
- It is desirable for this point to be **as close to the origin as possible**



$$C = B \log_2 \left(1 + \frac{\mathrm{EUE}}{\mathrm{BUE}}\right) \frac{\text{bits}}{\text{sec}} \qquad (38)$$

$$C/B = \log_2 \left(1 + \frac{\mathrm{EUE}}{\mathrm{BUE}}\right) \frac{\frac{\text{bits}}{\text{sec}}}{\text{Hz}} \qquad (39)$$

- N.B.:
  - a line from origin represents those points (systems) in the plane for which the SNR_{in}**=constant**
  - By comparing points representing one system with those representing another $\Rightarrow$ **VISUAL COMPARISON !**



  - It can be observed that
    - **CS1** beter than **CS2** which is better than CS3
    - **CS2** and CS3 have the same SNR_{in}

Theoretical Limits

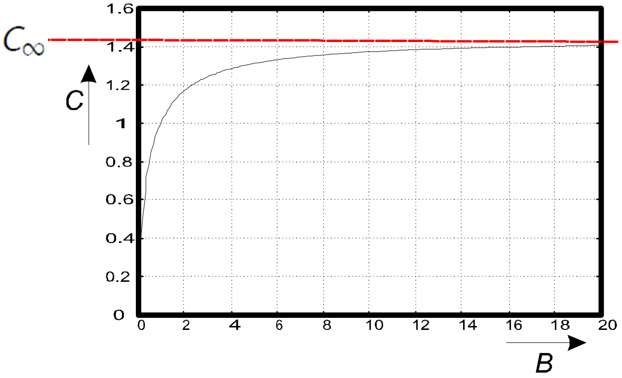
- We have seen that the capacity of a white Gaussian channel of bandwidth B is

$$C = B\log_2\left(1 + \mathrm{SNR}_{in}\right) \ \ \frac{\text{bits}}{\text{sec}} \tag{40}$$

  Please don't forget that the above equation refers to bandlimited white-noise channel with a constraint on the average transmitted power.

- **Question:** if $B = \uparrow$ (and in particluar if $B = \infty$) then $C = ?$
  Answer :
  - From the capacity-equation (Equ 40) it can be seen that $B \uparrow \Longrightarrow C \uparrow$
  - However, when B tends to ∞ then

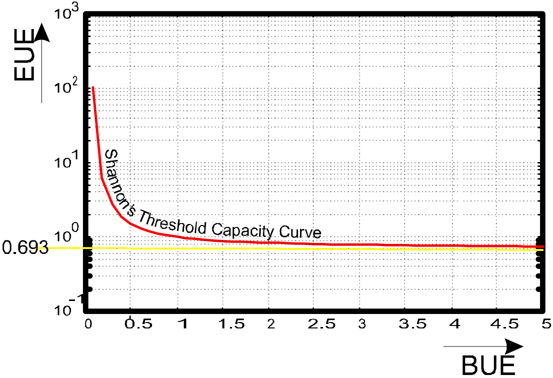
$$C_\infty = 1.44\frac{P_s}{N_0} \tag{41}$$



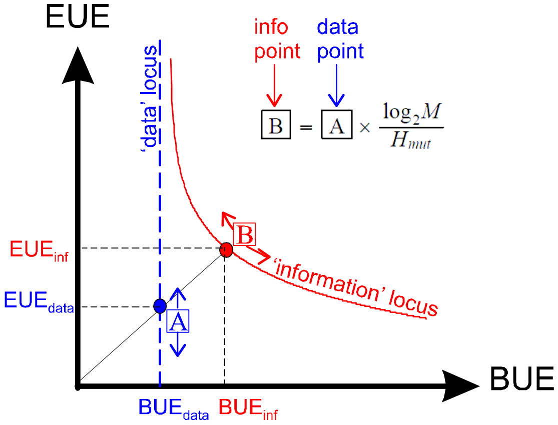
C_∞
C
B
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
14
16
18
20

- **LIMIT-1** : **limit on bit rate**
  - when binary information is transmitted in the channel, r_b should be limited as follows:
$$r_b \leq C \tag{42}$$
  - ideal case:
$$r_b = C \tag{43}$$
- **LIMIT-2** : **limit on EUE**
  - the best Energy Efficiency is EUE=0.693. This is the ultimate limit below which no physical channel can transmit without errors i.e
$$\text{EUE} \geq 0.693$$
- **LIMIT-3** : **Shannon's threshold channel capacity curve**
  - This is the curve EUE=$\mathrm{f}\{\text{BUE}\}$ for a bit rate r_b equal to its maximum value, i.e.
$$r_b = C \Rightarrow \text{EUE} = \frac{2^{\text{BUE}} - 1}{\text{BUE}^{-1}} \tag{44}$$

- Plot of Equation 44:



- **No physical realizable CS** could occupy a point in the plane (EUE,BUE) lying **below this theoretical channel capacity curve** .



$$\mathrm{SNR}_{in} = \frac{\mathrm{EUE}_{\mathrm{data}}}{\mathrm{BUE}_{\mathrm{data}}} = \frac{\mathrm{EUE}_{\mathrm{inf}}}{\mathrm{BUE}_{\mathrm{inf}}} \quad (45)$$

$$\text{data rate} : r_{b,\mathrm{data}} = r_{cs} \times \log_2 M \quad (46)$$

$$\text{info rate} : r_{b,\mathrm{info}} = r_{cs} \times H_{mut} \quad (47)$$

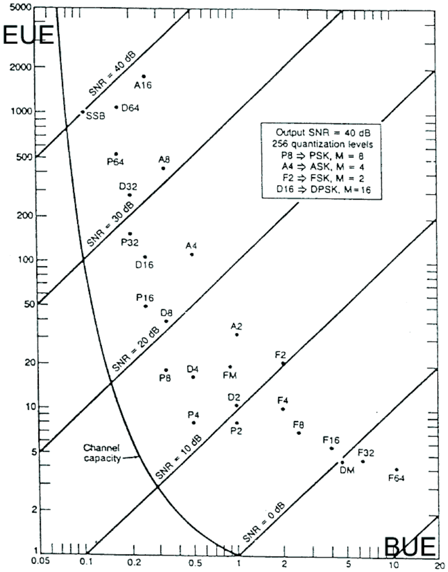
- Point $\boxed{\mathrm{B}}$ always should be above the Shannon's thr. capacity curve.
- Point $\boxed{\mathrm{A}}$ may be above or below the Shannon's thr. capacity curve.
- The following table gives the corresponding equivalent parameters between Analogue and Digital Comm Systems which can be used to place on the (EUE,BUE) not only digital but also analogue communication systems

Digital CS		Analogue CS
EUE$= \frac{E_b}{N_0} = \frac{P_s}{N_0 r_b}$	$\Leftrightarrow$	$\mathrm{SNR}_{in\text{-}mb} = \frac{P_s}{N_0 . F_g}$
r_b	$\Leftrightarrow$	F_g
BUE$= \frac{B}{r_b}$	$\Leftrightarrow$	$\beta \triangleq \frac{B}{F_g}$

where:

- F_g denotes the maximum frequency of the message signal $g(t)$, i.e. it represents the bandwidth of the message
- β is known as "bandwidth expansion factor", e.g. SSB: $\beta = 1$;AM: $\beta = 2$

- Comparison of various Digital and Analogue CS are shown below (for a fixed SNR_{out})



Other Comparison-Parameters

- SPECTRAL CHARACTERISTICS of transmitted signal (rate at which spectrum falls off).
- INTERFERENCE RESISTANCE
  - it may be necessary to increase (EUE,BUE) in order to increase interf. resistance
- FADING
  - Fading problem $p_e = \uparrow$
  - Note that, if $\left\{ \begin{array}{c} \text{BUE} = \uparrow \\ \text{EUE} = \downarrow \end{array} \right\}$ then Fading$=\downarrow$
- DELAY DISTORTION
  - Try to avoid this problem by selecting appropriate signals
- COST and COMPLEXITY

Appendix: SNR at the output of an Ideal Comm System

- In this section a so-called ideal system of communication will be considered and it will be shown that bandwidth can be exchanged for signal-to-noise performance.

  The ideal system forms a benchmark against which other communication systems can be compared.
- An ideal system has been defined as one that transmits data at a bit rate

$$r = C \tag{48}$$

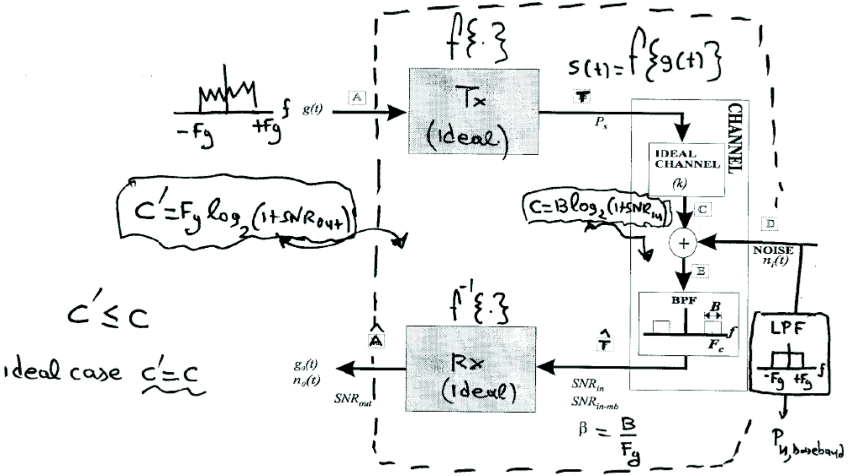
  where C is the channel capacity i.e. $C = B \log_2 (1 + \text{SNR}_{in})$ bits/sec

- Furthermore we have seen that for an ideal communication system:

$$\text{EUE} = \frac{\text{SNR}_{in}}{\log_2(1+\text{SNR}_{in})} \Rightarrow \lim_{\text{SNR}_{in}\to 0} \text{EUE} = 0.693 \tag{49}$$

$$\begin{aligned} \text{BUE} &= \frac{1}{\log_2(1+\frac{\text{EUE}}{\text{BUE}})} \qquad (50)\\ \Rightarrow \text{EUE} &= \frac{2^{\text{BUE}^{-1}}-1}{\text{BUE}^{-1}} \Rightarrow \lim_{\text{BUE}\to\infty} \text{EUE} = 0.693 \qquad (51) \end{aligned}$$

Block Diagram of an Ideal Communication System:



- The previous figure shows the elements of a basic ideal communication system.
- An input analogue message signal $g(t)$, which is of bandwidth F_g, is applied to a signal mapping unit which, in response to $g(t)$, produces an analogue signal $s(t)$ of bandwidth B and this signal is transmitted over an analogue channel having a similar bandwidth, B.
  - The channel is corrupted by additive white Gaussian noise of double sided power spectral density $N_0/2$ which is bandlimited to the channel bandwidth B.
  - Let the signal-to-noise ratio at the input of the receiver be SNR_{in}.
  - Assume further that the received signal, plus noise, is then fed to a detector having a bandwidth F_g, equal to the message bandwidth.
  - Let the signal-to-noise ratio at the output of the detector be SNRout.

- Now the capacity of the analogue transmission system (channel) is

$$C = B\log_2(1+\text{SNR}_{in}) \quad \text{bits/s} \tag{52}$$

- Also, the "mapping-unit/channel/detector" can be regarded as a channel having a signal-to-noise ratio SNRout and hence it too can be regarded as an AWGN channel and its capacity is given by

$$C' = F_g\log_2(1+\text{SNR}_{out}) \quad \text{bits/s} \tag{53}$$

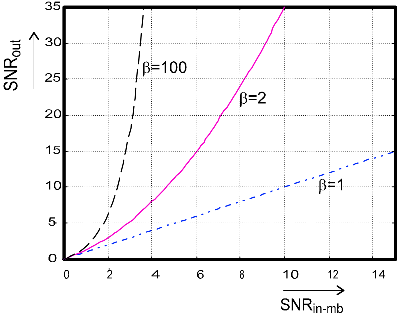
- If, in order to avoid information loss (ideal case), the capacities are set equal then it can be seen, after simple mathematical manipulation, that

$$C' = C \Rightarrow ... \Rightarrow \text{SNR}_{out,ideal} = (1+\text{SNR}_{in})^{\frac{B}{F_g}} - 1 \tag{54}$$

$$= \left(1+\frac{\text{SNR}_{in-mb}}{\beta}\right)^{\beta} - 1 \tag{55}$$

where β is the bandwidth expansion factor.

- The above expression is fundamentally important since it shows that the overall system performance, SNR_{out}, can be improved by using more channel bandwidth.
- The figure below shows, as a function of the bandwidth expansion factor β, typical curves of SNR_{out} versus SNR_{in-mb} for the ideal communication system.



- Note that all other known communication systems should be compared with this optimum performance provided by Equation .

- From the previous figure it can be seen that:
  - if SNR_{in-mb} is **small** then on increasing β the effect on the SNR_{out} is **small** (i.e. very little increase in the SNR_{out} is obtained).
  - If, however, SNR_{in-mb} is **large** then a small increase in the bandwidth expansion factor results in a **large** increase in the SNR_{out}.
  - A practical consequence of this is that if the SNR_{in-mb} is small then there is little to be gained from using more channel bandwidth.

	Case-1:$\text{SNR}_{in\text{-}mb}$=small	Case-2:$\text{SNR}_{in\text{-}mb}$=large=10
	$\text{SNR}_{in\text{-}mb}$=1	$\text{SNR}_{in\text{-}mb}$=10
$\beta = 1$	SNR_{out}=1	SNR_{out}=10
$\beta = 2$	SNR_{out}=1.25	SNR_{out}=35
$\beta = 100$	SNR_{out}=1.7048	SNR_{out}=13780