Problem Sheets: Communication Systems

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1 Topic: Introductory Concepts

1. Sketch and mathematicaly represent the pdfs of the following signals:

   (a) $4\ \mathrm{rep}_{3T}\left\{\Lambda\left(\frac{t}{T}\right)\right\}-1$ (10%)

   (b) $\mathrm{rep}_{2T}\left\{2\mathrm{rect}\left(\frac{t}{T}\right)+4\Lambda\left(\frac{2t}{T}\right)\right\}$ (10%)

   (c) $\mathrm{rep}_{2T}\left\{5\ \mathrm{rect}\left(\frac{t}{T}\right)-\ \mathrm{rect}\left(\frac{t-T}{T}\right)\right\}$ (10%)

   (d) $\mathrm{rep}_{6T}\left\{4\ \mathrm{rect}\left(\frac{t}{5T}\right)-\ \mathrm{rect}\left(\frac{t-3T}{T}\right)\right\}$ (5%)

   (e) $\frac{N+1}{N}\ \mathrm{rep}_{NT}\left\{\Lambda\left(\frac{t}{T}\right)\right\}-\frac{1}{N}$ with $N\in Z^{+}>2$ (15%)

   (f) $3\mathrm{rep}_{2}\left\{\Lambda\left(t\right)\right\}-2$ (5%)

2. Evaluate:

   (a) $\int\limits_{-\infty}^{\infty}(t^4-3t+1).\delta(t-2)\ .dt$ (10%)

   (b) $\int\limits_{-\infty}^{\infty}\left(\cos(4\pi t)*\delta(t+\frac{1}{4})\right).\delta(t-\frac{1}{8}).dt$ (10%)

   (c) $\int\limits_{-\infty}^{\infty}(t^3-3t^2-11).\delta(t-1)\ .dt$ (5%)

   (d) $\int\limits_{-\infty}^{\infty}\left\{(\sin(4\pi t)*\delta(t+\frac{1}{4})\right\}.\delta(t-\frac{1}{4}).dt$ (5%)

   (e) $\int\limits_{-\infty}^{\infty}(t^3-2t^2+1).\delta(t-2)\ .dt$ (5%)

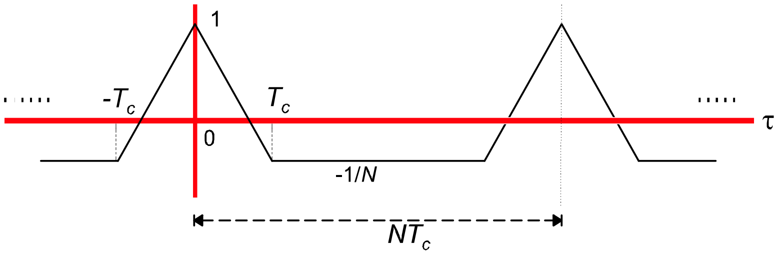
   (f) $\int\limits_{-\infty}^{\infty}\left\{(\cos(2\pi t)*\delta(t-\frac{1}{4})\right\}.\delta(t-\frac{1}{12}).dt$ (5%)

   (g) $h(3)$ where $h(t)=\left(t.\mathrm{rect}\left\{\frac{t}{8}\right\}\right)*\delta(t+3)$ (5%)

   (h) $h(3)$ where $h(t)=\left(t.\mathrm{rect}\left\{\frac{1}{8T}\right\}\right)*\delta(t-2)$ (10%)

   (i) $h(3.5)$ where $h(t)=\left(t.\mathrm{rect}\left\{\frac{1}{8T}\right\}\right)*\delta(t-3)$ (10%)

3. The waveform below shows the autocorrelation function $R_{bb}(\tau)$ of what is called in communications a pseudo-random (PN) signal $b(t)$.



   (a) Write a mathematical expression, using Woodward's notation, to describe the above autocorrelation function. (15%)

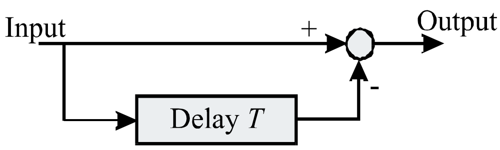
   (b) Find the power spectral density $\mathrm{PSD}_b(f)$ of $b(t)$. (20%)

4. At the input of a filter there is white Gaussian noise of power spectral density $\mathrm{PSD}_{n_i}(f) = \frac{3}{2}10^{-6}$. If the transfer function of the filter is

$$H(f) = \Lambda\left\{\frac{f}{10^6}\right\}\exp(-j\phi(f)$$

   calculate the power of the signal at the output of the filter. (10%)

5. For the following differential circuit



   find:

   (a) the impulse response and (5%)

   (b) frequency response (5%)

6. Consider the filter with impulse response

$$h(t) = \mathrm{sinc}^2\left\{10^6(t-3)\right\}$$

   and assume that the input signal $n_i(t)$ is white Gaussian noise with double-sided power spectral density $\mathrm{PSD}_{n_i}(f) = 1.5 \times 10^{-6}$ W/Hz.

   For the signal $n(t)$ at the output of the filter

   (a) find and plot its power spectral density $\mathrm{PSD}_n(f)$; (10%)

   (b) calculate its power P_n (5%)

7. Consider a bandpass filter with impulse response

$$h(t) = 8 \times 10^3 \mathrm{sinc}\left\{4 \times 10^3 t\right\}.\cos(2\pi 10^4 t)$$

   and assume that at the input of this filter there is white Gaussian noise $n_i(t)$ of power spectral density $\mathrm{PSD}_{n_i}(f) = 10^{-6}$.

   For the signal $n(t)$ at the output of the filter

   (a) find and plot its power spectral density $\mathrm{PSD}_n(f)$; (10%)

   (b) calculate its power P_n (5%)

2 Topic: Information Sources

8. The signal at the output of an analogue information source $x(t)$ having a uniform pdf between ±2Volts, is passed through a half-wave and a full-wave rectifier circuits. Sketch and mathematically represent the pdfs of:

   (a) the original analogue information source, 5%

   (b) the output from the half-wave rectifier, 10%

   (c) the output from the full-wave rectifier. 10%

   (d) Determine

   - the mean value, and 15%
   - the rms value 15%

   of the signals in cases (a),(b) and (c) above.

   N.B.: Assume ideal diodes

9. Consider an analogue signal source $x(t)$ having a uniform amplitude probability density function

$$\text{pdf}_x(x) = \frac{1}{6}\text{rect}\left\{\frac{x}{6}\right\}$$

   (a) Estimate the average power P_x of the signal $x(t)$. 10%

   (b) Find the differential entropy H_x of the signal source $x(t)$ 10%

   (c) Find $H_y - H_x$ 10%

   where H_y denotes the differential entropy of an analogue signal source $y(t)$ having a Gaussian amplitude probability density function with mean μ_y and $\sigma_y = \sqrt{P_x}$

   (d) What is the entropy power of the signal $x(t)$. 10%

10. A signal $g(t)$ having the pdf shown in Fig.1 is bandlimited to 4 kHz. The signal is sampled at the Nyquist rate and is fed through a 2-level quantizer. The transfer function of the quantizer is shown in Fig.2.

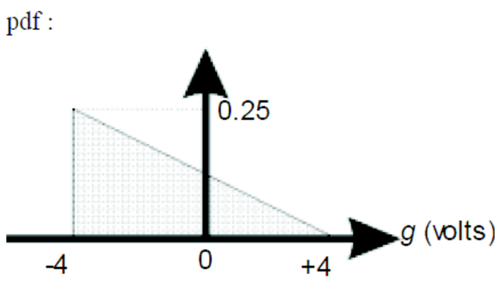


Fig. 1

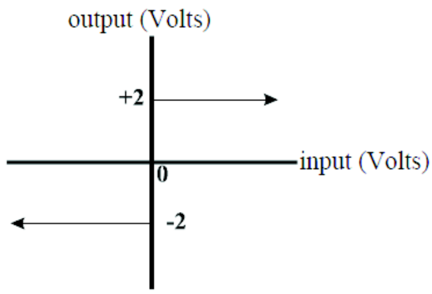


Fig. 2

Consider the output of the quantizer as the output of a discrete information source $(X, \underline{p})$. Calculate:

   (a) the symbol rate r_X of the source $(X, \underline{p})$. 10%

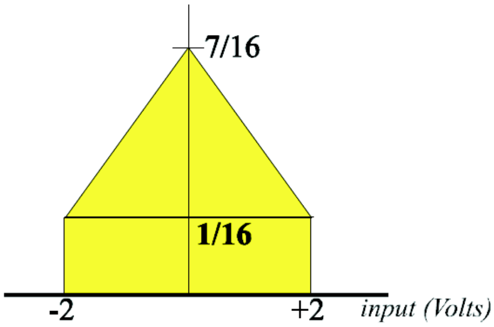
   (b) the amplitude pdf of the signal at the quantizer's output. Sketch this pdf. 10%

   (c) the rms value of the signal at the output of the quantizer. 10%

   (d) the entropy H_X 10%

   (e) the entropy of the source $(X \times X, \mathbb{J})$ (10%)

11. A signal $g(t)$ having the probability density function (pdf) shown below is sampled and fed through an 4-level quantizer.



Consider the output of the quantizer as the output of a discrete information source $(X, \underline{p})$.

(a) Calculate and sketch the pdf of the signal at the output of the quantizer. 10%

(b) Calculate the rms value of the signal at the output of the quantizer. 10%

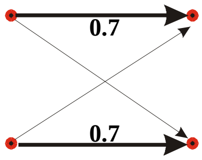
(c) What is the ensemble of the source $(X \times X, \mathbb{J})$? 10%

(d) Calculate the entropy $H_{X \times X}$ 10%

3 Topic: Communication Channels

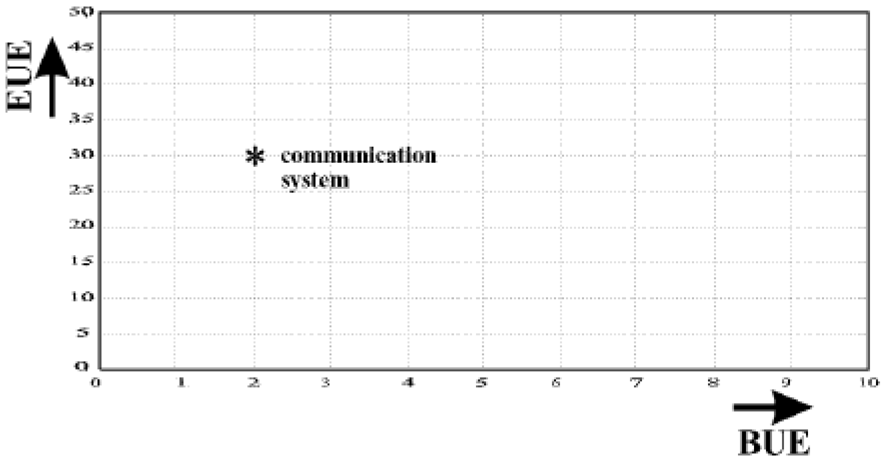
12. If one binary source and two binary channels are connected in cascade as shown below

where both channels have the following forward transition probability diagram



find the bit-error-rate p_e at the output of the second channel. 10%

13. A digital communication system, operating at 100 bits/sec in the presence of additive white Gaussian noise of power spectral density $\text{PSD}_n(f) = \frac{N_0}{2}$, is represented in the energy utilization efficiency (EUE) - bandwidth utilization efficiency (BUE) plane, as follows:



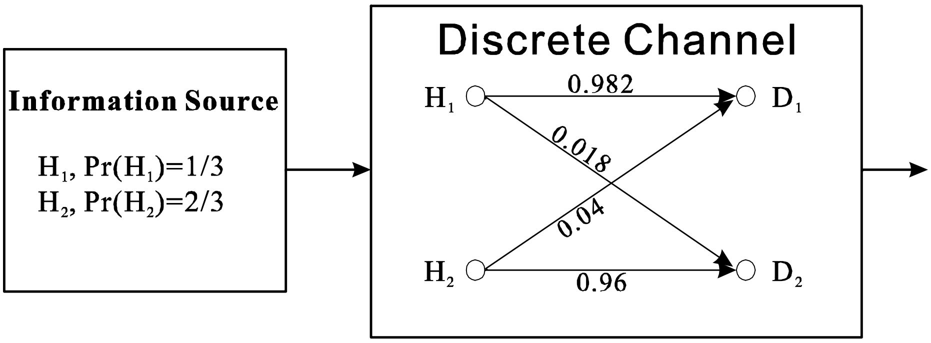
What is the capacity C of the channel in bits/sec? 20%

14. A digital communication system having an energy utilisation efficiency (EUE) equal to 30 operates in the presence of additive white Gaussian noise of double-sided power spectral density $\text{PSD}_n(f) = 0.5 \times 10^{-6}$ W/Hz. If the channel capacity C is 16 kbits/s and the channel bandwidth B is 4 kHz, estimate

    (a) the bit rate r_b 10%

    (b) the noise power at the channel output 10%

15. A discrete channel is modelled as follows:



Estimate:

(a) The probability of error at the output of the channel 5%

(b) The amount of information delivered at the output of the channel 10%

16. Consider a binary Communication System that uses the following two equally probable energy signals:

$$\begin{aligned} 0 &\mapsto s_0(t) = 2\Lambda\left\{\frac{t}{10\mu s}\right\} \\ 1 &\mapsto s_1(t) = -2\Lambda\left\{\frac{t}{10\mu s}\right\} \end{aligned}$$

The channel is assumed additive white Gaussian noise of double-sided power spectral density $\mathrm{PSD}_n(f) = 10^{-6}\,\mathrm{W/Hz}$. Find:

(a) the bandwith B of the channel; 5%

(b) the channel symbol rate r_{cs} (baud rate) & data bit rate; 5%

(c) the Energy Utilisation Efficiency (EUE); 10%

(d) the channel capacity C in bits/sec. 10%

17. Consider a binary Communication System that operates with a bit rate 100kbits/sec and uses the following two equally probable energy signals:

$$\begin{aligned} 0 &\mapsto s_0(t) = 3\left(\Lambda\left\{\frac{t}{5\mu s}\right\} + \mathrm{rect}\left\{\frac{t}{10\mu s}\right\}\right) \\ 1 &\mapsto s_1(t) = -3\left(\Lambda\left\{\frac{t}{5\mu s}\right\} + \mathrm{rect}\left\{\frac{t}{10\mu s}\right\}\right) \end{aligned}$$

The channel is assumed additive white Gaussian noise of double-sided power spectral density $\mathrm{PSD}_n(f) = 0.5 \times 10^{-6}\,\mathrm{W/Hz}$. Find:

(a) the bandwith B of the channel; 5%

(b) the channel symbol rate r_{cs} (baud rate); 5%

(c) the Energy Utilisation Efficiency (EUE); 20%

(d) the channel capacity C in bits/sec. 15%

18. Consider a binary digital communication system in which a binary sequence is transmitted as a signal $s(t)$ with a one being sent as $6\Lambda\left\{\frac{t}{T_{cs}/2}\right\}$ and a zero being sent as $-6\Lambda\left\{\frac{t}{T_{cs}/2}\right\}$. The source at the input to the system provides a binary sequence of ones and zeros, with the number of ones being twice the number of zeros. The transmitted signal is corrupted by channel noise $n(t)$ of bandwidth B and has an amplitude probability density function described by the following expression:

$$\mathrm{pdf_n}(n) = \frac{1}{6}.\mathrm{rect}\left\{\frac{n}{6}\right\}$$

Find a bound on the ratio C/B 20%

where C denotes the capacity of the channel in bits/s.

19. Consider a binary digital communication system in which the transmitted signal is corrupted by channel noise of bandwidth B having an amplitude probability density function described by the following expression:

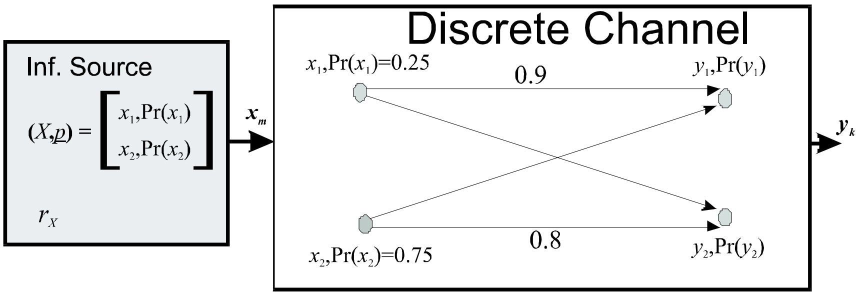
$$\mathrm{pdf_n}(n) = \frac{1}{6}.\mathrm{rect}\left\{\frac{n}{6}\right\}$$

If the power of the received signal is 12W then

(a) find the entropy power of the noise; 10%

(b) find an upper and a lower bound on the ratio C/B where C denotes the capacity of the communication channel. 10%

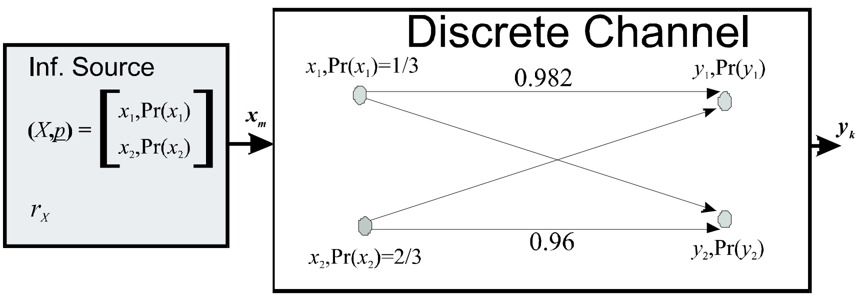
20. A discrete channel is modelled as follows:Estimate:



(a) The probability of error at the output of the channel 5%

(b) The amount of information delivered at the output of the channel 15%

21. A discrete channel is modelled as follows:



Estimate:

(a) The probability of error at the output of the channel 5%

(b) The amount of information delivered at the output of the channel 15%

22. A signal $g(t)$ bandlimited to 4kHz is sampled at the Nyquist rate and is fed through a 2-level quantizer. A Huffman encoder is used to encode triples of successive output quantization levels as follows:

symbols	probs	Huffman
$m_1m_1m_1$	27/64	1
$m_1m_1m_2$	9/64	001
$m_1m_2m_1$	9/64	010
$m_2m_1m_1$	9/64	011
$m_1m_2m_2$	3/64	00000
$m_2m_1m_2$	3/64	00001
$m_2m_2m_1$	3/64	00010
$m_2m_2m_2$	1/64	00011

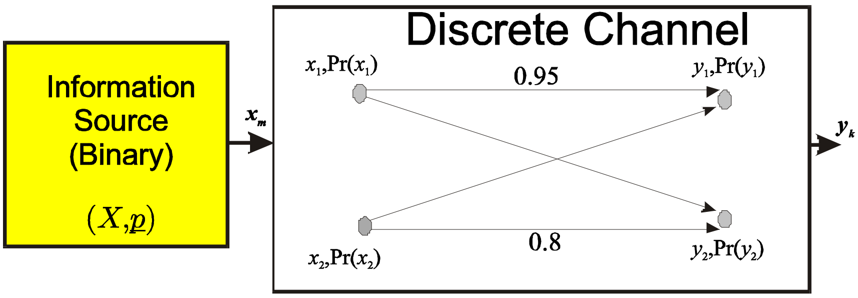
while the binary sequence at the output of the Huffman encoder is fed to a Binary on-off Keyed Communication System which employs the following two energy signals of duration T_{cs}

$$\begin{aligned} s_0(t) &= 0 \\ s_1(t) &= \sqrt{\frac{3}{8}}\mathbf{\Lambda}\left(\frac{t}{0.5\,T_{cs}}\right) \end{aligned}$$

The transmitted signals are corrupted by additive white Gaussian channel noise having a double-sided power spectral density of 10^{-3} W/Hz. The figure below shows a modelling of the whole system where the output of the Huffman encoder is modelled as the output of a binary discrete information source $(X, \underline{p})$ with

$$\begin{aligned} X &= \{x_1 = 1, x_2 = 0\}, \\ \underline{p} &= [\Pr(x_1), \Pr(x_2)]^T \end{aligned}$$

while the binary on-off Keyed system is modelled as a discrete channel as shown below.



(a) Find the entropy of the information source $(X, \underline{p})$, the information rate and the bit data rate (symbol rate) at the channel input. 15%

(b) Estimate the bit-error probability of the system. 10%

(c) Estimate the energy utilization efficiency (EUE) and bandwidth utilization efficiency (BUE) using the bit data rate as well as the information rate. 15%

(d) Represent the communication system, as a point on the (EUE,BUE) parameter plane. In this plane show also the locus of the system properly labelled. 10%

(e) Is the system a 'realizable' communication system? 5%

(f) What is the signal-to-noise ratio, SNR_{in}, at the receiver's input? 5%

23. A signal $g(t)$ having the pdf shown in Figure 1 is bandlimited to 4 kHz. The signal is sampled at the Nyquist rate and fed through a 2-level quantizer. The transfer function of the quantizer is shown in Figure 2.

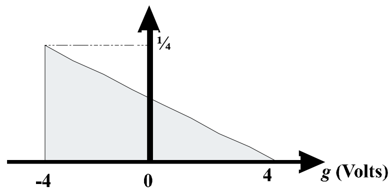


Figure-1

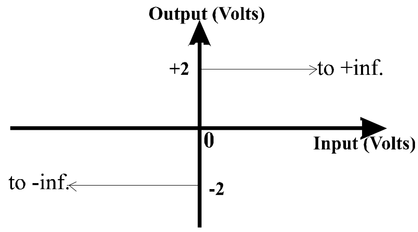
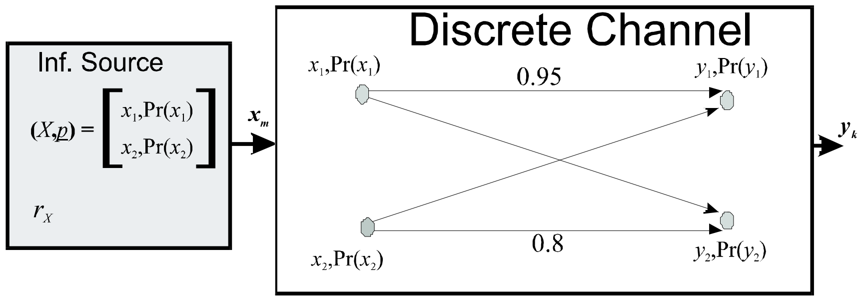


Figure-2

A Huffman encoder is used to encode triples of successive output quantization levels while the binary sequence at the output of the Huffman encoder is fed to a Binary on-off Keyed Communication System which employs the following two energy signals

$$s_1(t) = 0; s_2(t) = 0.5 \cos\left(2\pi \frac{5}{T_{cs}} t\right); \text{with } 0 < t < T_{cs}$$

The whole system is modelled as follows



where the binary information source represents the system up to the output of the Huffman encoder. The discrete channel models the binary on-off keyed Transmitter/Receiver (with $x_1 = 1$ and $x_2 = 0$) and the additive white Gaussian noisy channel with noise having a double-sided power spectral density of 10^{-3} W/Hz.

(a) Estimate the bit-error probability of the system. 5%

(b) Find the information rate and the bit data rate (symbol rate) at the channel input. 10%

(c) Estimate the data point (EUE,BUE), where EUE denotes the energy utilization efficiency and BUE represents the bandwidth utilization efficiency of the system. 10%

(d) Estimate the information point (EUE,BUE), where EUE denotes the information energy utilization efficiency and BUE represents the information bandwidth utilization efficiency of the system. 15%

(e) Is the system a 'realizable' communication system? 5%

(f) What is the signal-to-noise ratio SNR, at the receiver's input? 5%

4 Topic: Wireless Channels

24. Find the minimum channel symbol rate needed by a digital communication system to resolve a multipath, with an additional path length of 30m compared to the direct path. 10%

25. If $B_D = 8MHz$ denotes the Doppler spread, B_{coh} represents the coherent bandwidth and T_{cs} is the channel symbol period, then in a frequency selective fast fading channel which of the following is correct? 10%

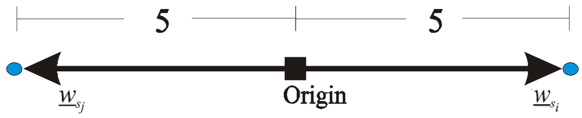
    (a) $T_c = 61n\sec$ and $B_{\mathrm{coh}} = 3\text{MHz}$.
    (b) $T_c = 61n\sec$ and $B_{\mathrm{coh}} = 100\text{MHz}$.
    (c) $T_c = 244n\sec$ and $B_{\mathrm{coh}} = 3\text{MHz}$.
    (d) $T_c = 244n\sec$ and $B_{\mathrm{coh}} = 100\text{MHz}$.
    (e) None of the above.

26. The minimum chip rate needed by a DS-BPSK spread spectrum system to resolve a multipath, with an additional path length of 30m compared to the direct path, is

    (a) 10 Mchips/second
    (b) 20 Mchips/second
    (c) 40 Mchips/second
    (d) 60 Mchips/second
    (e) none of the above.

5 Topic: Digital Modulators & Line Codes

27. The next figure illustrates the signal constellation points of two M-ary signals $s_i(t)$ and $s_j(t)$ of equal energy.



The energy of each of these two signals is

(a) 25,
(b) 50,
(c) 75,
(d) 100,
(e) none of the above.

28. Consider a random binary sequence of 0's and 1's. This binary sequence is transmitted as a random signal with 1's and 0's being sent using the pulses $s_1(t)$ and $s_0(t)$ described below:

$$0 \longmapsto s_0(t) = 3\text{rect}\left\{\frac{t}{1ms}\right\} \text{ mV}$$

and

$$1 \longmapsto s_1(t) = -3\text{rect}\left\{\frac{t}{1ms}\right\} \text{ mV}$$

If 1's and 0's are statistically independent with $\Pr(1) = \Pr(0) = 0.5$, find the Power Spectral Density of the transmitted signal.

29. A binary PSK signal is decoded coherently in the presence of white noise having a double sided power spectral density 0.5×10^{-6} Watts/Hz. If $\Pr(1) = \Pr(0)$ and the bit rate is 220 kbits/sec, what is the average received signal power at which a probability of error of 10^{-5} can be achieved? (10%)

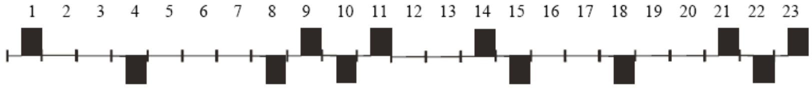
30. Consider a Biphase Shift-keyed digital modulator/demodulator operating the presence of additive white Gaussian noise with double-sided power spectral density $0.5 \times 10^{-9} W/Hz$. The digital modulator maps zeros and ones as follows:

$$\begin{aligned} 0 &\longmapsto s_0(t) = 3\cos(2\pi F_c t - 30^0) \\ 1 &\longmapsto s_1(t) = 3\cos(2\pi F_c t + 30^0) \\ \text{for } 0 &\leq t \leq T_{cs} \end{aligned}$$

where $\Pr(0) = \Pr(1)$, $T_{cs} = 4ns$ and $F_c = \frac{5}{T_{cs}}$. Find

(a) the Energy Utilisation Efficiency (EUE); [5 marks]
(b) the bit error rate p_e at the demodulator's output. [5 marks]

31. The following HDB3 encoded signal



represents the binary sequence:

(a) 1 0 0 1 0 0 0 0 1 1 1 0 0 0 1 0 0 0 0 0 1 1 1

(b) 1 0 0 1 0 0 0 1 1 1 1 0 0 1 1 0 0 1 0 0 1 1 1

(c) 1 0 0 1 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 1 1 1

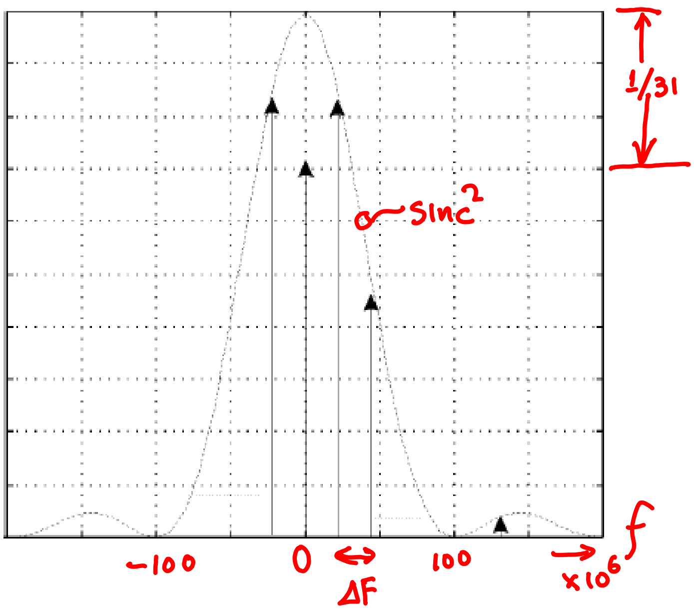
(d) 1 0 0 1 0 0 0 0 1 1 0 0 0 0 1 0 0 0 0 0 1 1 1

(e) None of the above.

6 Topic: SSS and PN-Codes

32. A pseudo random (PN) signal $b(t)$ is generated by using a maximal length shift register of m-stages and has the following double-sided Power Spectral Density.

$\mathrm{PSD}_b(f)$:



   (a) Find the number m of shift register stages. 5%

   (b) Find ΔF. 5%

33. Sketch the feedback shift register whose feedback connections are represented by the primitive polynomial $x^{24} + x^7 + x^2 + x + 1$ and find the length N of this sequence. [6 marks]

    If the clock rate is 2.7 chips/s, find the period of this sequence in minutes. [4 marks]

34. Sketch the feedback shift register whose feedback connections are represented by the primitive polynomial $D^{24} + D^7 + D^2 + D + 1$ and operates with a clock rate 1Mb/sec. 5%

    Find the period of the output sequence in minutes.

35. Consider a feedback shift register whose feedback connections are represented by the primitive polynomial $D^4 + D^1 + 1$. Give one period of its output sequence - starting with all 1's (initial condition). 15%

7 Topic: Direct Sequence and Frequency Hopping

36. A 'short-code' BPSK DS/SSS uses an m-sequence and a data rate 9.6 kbits/sec. If it is required that the spread spectrum signal will have bandwidth no larger than 25MHz, what is the largest period of the m-sequence that can be used?

    (a) 255
    (b) 511
    (c) 1023
    (d) 2047
    (e) None of the above

37. Consider a binary message signal of rate 8 kbits/s at the input of a fully synchronized BPSK direct sequence spread spectrum system (DS/SSS-BPSK). The system operates in the presence of both additive white noise, $n(t)$, and a broadband noise jammer, $j(t)$, of power 1 Watt. The double sided power spectral density of the noise is 10^{-12} Watts/Hz and the processing gain of the system is 10^5. The bit error probability at the output of the receiver is equal to 4×10^{-6} while the protection probability is equal to 4×10^{-2}.

    (a) What is the amplitude A of the sinewaves which are used by the binary PSK modulator? 15%
    (b) What is the bit error probability if the jammer switches to a "pulse jammer" mode, which is "on" for 40% and "off" for 60% of the time? 10%
    (c) What is the Anti-jam Margin, in dBs, when the jammer switches to the above-mentioned mode? 10%

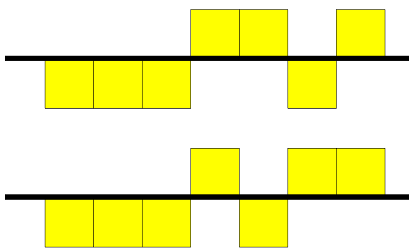
38. A speech signal having a maximum frequency of 4kHz is sampled at twice the Nyquist rate and then fed through an 8-bit uniform quantizer. The generated binary sequence is then fed through a binary PSK direct sequence spread spectrum system which operates in the presence of a broadband jammer of power 1.6 Watts and in the presence of additive white Gaussian noise with double-sided power spectral density 0.5×10^{-12} Watts/Hz. The amplitude of the BPSK signal is 0.5V.

    For this system, the spread spectrum bandwidth B_{ss} is 32 MHz and the system is fully synchronised.

    Find:

    (a) the power of the code noise, 5%
    (b) the power of the noise at the output of the correlator, 5%
    (c) the power of the jammer at the output of the correlator. 10%

39. Two m-sequence PN-signals, generated by two 3-stage shift registers, are shown below.

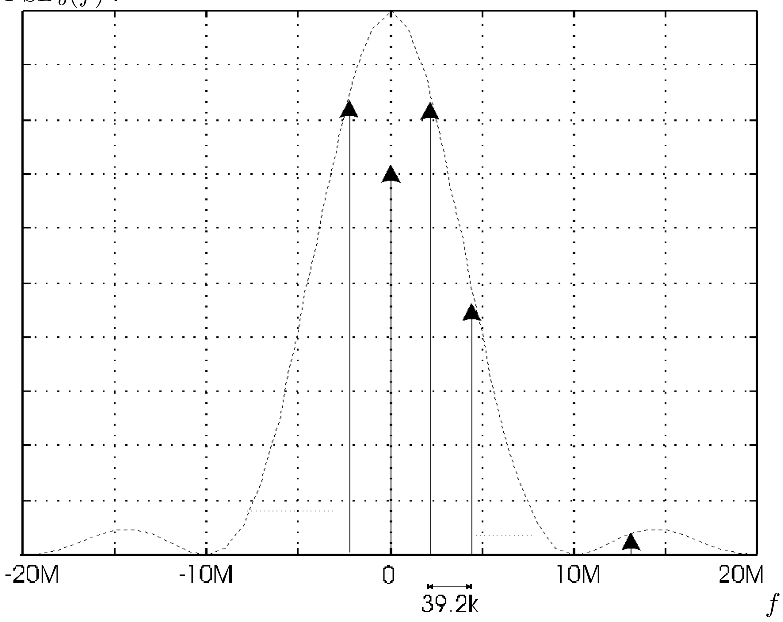


    Construct a Gold code signal from these two PN-signals. 10%

40. A DS/SSS uses an m-sequence for spreading the spectrum with a processing gain equal to one period of the m-sequence. If the data rate is 28 kbits/sec and it is required that the spread-spectrum signal has a bandwidth no larger than 25 MHz, what is the largest period of the m-sequence that can be used? 15%

41. A pseudo random (PN) signal $b(t)$ is generated by using a maximal length shift register of m-stages and has the following double-sided Power Spectral Density.

    $\mathrm{PSD}_b(f)$:



    Find the number m of shift register stages. 15%

42. An analogue message signal having a maximum frequency of 4kHz is sampled at the Nyquist rate and then is fed through a 4-level quantizer where each level is encoded using 2 bit codewords. The binary sequence is then fed through a fully synchronized Binary PSK Direct Sequence Spread Spectrum System (BPSK/DS-SSS) of processing gain 10^8. The system operates in the presence of white Gaussian noise having a double-sided power spectral density of 10^{-12} W/Hz and its Energy Utilization Efficiency is 40 (i.e. EUE= $\frac{E_b}{N_0}$ =40). What would be the power P_J of a jammer which, if it was distributed over 50% of the spread spectrum signal bandwidth, would provide a bit error probability p_e of 3×10^{-5} ? 25%

43. Consider a Frequency Hopping Spread Spectrum System (FH-SSS) in which there are 1024 frequency slots each of bandwidth 250kHz and 100 frequency hops for each message bit. Assuming that the hop-duration is 4μsec and a frequency multiplication of 8 is employed, calculate the ratio $\frac{\text{bandwidth}}{\text{bit rate}}$ of the system.

8 Topic: DS-CDMA

44. A recorded conversation is to be transmitted by a QPSK Direct Sequence Spread Spectrum System (DS/SSS). Assuming the spectrum of the speech waveform is bandlimited to 4 kHz, and that a 128-level quantizer is used:

    (a) find the chip rate required to obtain a processing gain of 20 dB, 10%

    (b) given that the sequence length is to be greater than 5 hours, find the number of shift register stages required. 10%

45. Consider a DS-BPSK CDMA systems where the received powers from all users are equal to 10^{-2} (a perfectly power controlled system). The system operates in the presence of additive white Gaussian noise of double sided power spectral density 0.5×10^{-11} while the processing gain of the system is 400. If the bit rate for each user is 25 kbits/sec and the Signal-to-Noise-plus-Interference ratio at the output of the j^{th} receiver is equal to 14, how many users are supported by the system? 50%

46. Consider a digital cellular DS-BPSK CDMA communication system which employs three directional antennas each having 120° beamwidth, thereby dividing each cell into 3 sectors. The system can support up to 201 users/subscribers and operates with a data bit-rate of 500 kbits/sec in the presence of additive white Gaussian noise of double-sided power spectral density 10^{-9}. With a bit-error-probability for each user of 3×10^{-5}, a power equal to 10 mWatts, and a voice activity factor $\alpha = 0.375$, find:

    (a) the average energy per bit E_b, 5%

    (b) the equivalent EUE (EUE$_{equ}$), 5%

    (c) the processing gain (PG) of the system. 10%

47. Consider a DS-BPSK CDMA system of 256 users where each user has a protection probability equal to 10^{-2} and an Anti-jam Margin of 30 dB. Each user employs a feedback shift register of 21 stages, whose feedback connections are described by a primitive polynomial. The system is perfectly power controlled and the received power from each user is equal to $P = 0.1915$W operating in the presence of additive white Gaussian noise of double sided power spectral density 0.5×10^{-6} Watts/Hz. Find:

    (a) the average energy per bit E_b and 20%

    (b) the PN-code rate. 10%

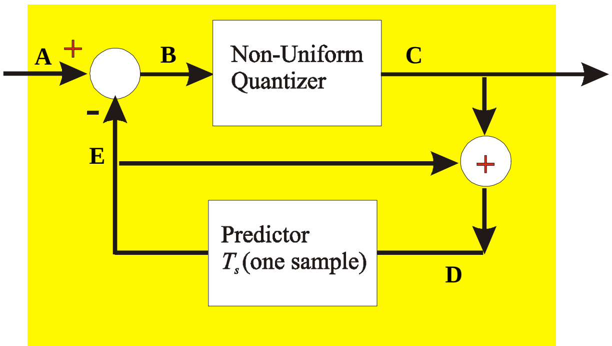
48. Consider a digital cellular DS-QPSK CDMA communication system with a Gray encoder/decode which employs three directional antennas each having 120° beamwidth, thereby dividing each cell into 3 sectors. The system operates with a data bit-rate 25 kbits/sec. in the presence of additive white Gaussian noise of double-sided power spectral density 10^{-9}, while the processing gain of the system is 400. With a desired bit-error-probability for each user 3×10^{-5}, a power equal to 5 mWatts, and a voice activity factor $\alpha = 0.375$, how many users/subscribers can be supported by the system? 30%

9 Topic: PCM & PSTN

49. For a speech signal of 4 kHz bandwidth transmitted using a uniform quantiser of 256 levels the bit rate at the output of the source encoder is

    (a) 8 kbits/s
    (b) 16 kbits/s
    (c) 32 kbits/s
    (d) 64 kbits/s
    (e) 128 kbits/s

50. Consider the *mse*-differential quantizer shown in the following figure



which employs a 6-level non-uniform quantizer, having the following input and output levels

I/P (volts)	O/P (volts)
$+8 \leq$input$\leq +255$	$+23$
$+4 \leq$input$\leq +7$	$+6$
$0 \leq$input$\leq +3$	$+1$
$-3 \leq$input≤ -1	-1
$-7 \leq$input≤ -4	-6
$-255 \leq$input≤ -8	-23

If the input is a step signal of amplitude 0V$\rightarrow$ 41V, and assuming "E"=0 as an initial value, then find the data sequence that is read from point 'D'

51. An analogue message signal $g(t)$ with amplitude probablity density function $0.5\Lambda\left(\frac{g}{2}\right)$ and a bandwidth of 10kHz, is applied to a 256-levels uniform PCM system (i.e. PCM system which employs a uniform quantizer of 256 levels). For this system calculate the Signal-to-Quantization-Noise ratio (SNR_q).

52. A high quality music signal with a Crest Factor of 4.4668, having a maximum frequency 18 kHz, is applied to a uniform PCM system (i.e. PCM with a uniform quantizer). If it is specified that the Signal-to-Quantisation-Noise ratio (SNR_q) should be better than 50dB, find the minimum data bit rate required [6 marks]

53. In a binary PCM communication system prove that bandwidth expansion factor β is equal to the average number of bits γ per quantization level, i.e. [4 marks]

$$\beta = \gamma$$

54. Consider a PCM system where its quantizer consists of a μ-law compander (with $\mu = 100$) followed by a uniform quantizer with "end points" b_i, and "output levels" m_i. The maximum value of the input signal is $10\,Volts$ and the input/output characteristics of the uniform quantizer are given in the following tables:

b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7
-10V	-8.75V	-7.5V	-6.25V	-5V	-3.75V	-2.5V	-1.25V

b_8	b_9	b_{10}	b_{11}	b_{12}	b_{13}	b_{14}	b_{15}	b_{16}
0V	1.25V	2.5V	3.75V	5V	6.25V	7.5V	8.75V	10V

m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8
-9.375V	-8.125V	-6.875V	-5.625V	-4.37V	-3.125V	-1.875V	-0.625V

m_9	m_{10}	m_{11}	m_{12}	m_{13}	m_{14}	m_{15}	m_{16}
0.625V	1.875V	3.125V	4.375V	5.625V	6.875V	8.125V	9.375V

Note that μ-law compression is defined as follows:

output= $\frac{\ln(1+\mu.\ |x|)}{\ln(1+\mu)}$.sign(x) where $x = \frac{\text{input value in } Volts}{\text{maximum input value in } Volts}$

(a) If the signal at the output of the sampler at time kT_s is equal to $2.4\,Volts$, what is the corresponding output level of the uniform quanitizer? [5 marks]

(b) Find the corresponding value of the signal at the output of the expander. [5 marks]

(c) Estimate the instantaneous quantisation noise $n_q(kT_s)$ [1 marks]

Note: T_s is the sampling period and k is an integer.

55. The CCITT standards 32kbits/second Differential PCM are

(a) for speech signals with bandwidth 3.2 kHz.

(b) for audio signals with bandwidth 7 kHz

(c) specifying a sampling frequency 16 ksamples/second

(d) specifying an 8 levels quantizer

(e) none of the above

56. The first TDMA multiplexing level of a 30-channel PCM Telephone system uses

(a) an AMI line code;

(b) a polar RZ line code;

(c) a Manchester line code;

(d) an HDB3 line code;

(e) none of the above.

END