E303: Communication Systems

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An Overview of Fundamentals: PN-codes/signals & Spread Spectrum
(Part B)

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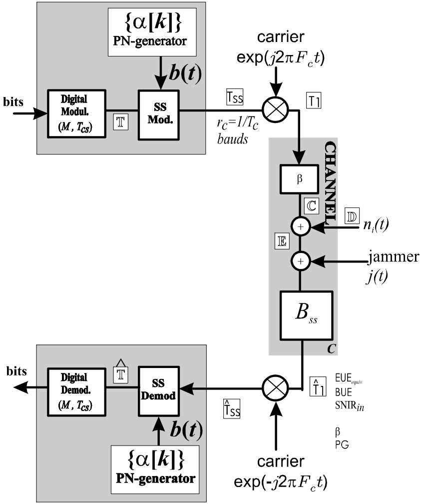
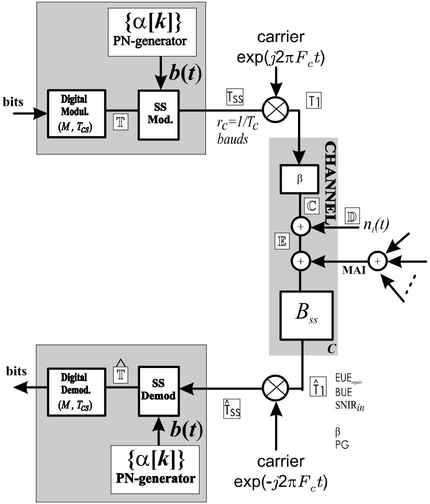
Introduction

(a) SSS (jammers):



(b) CDMA (K users):



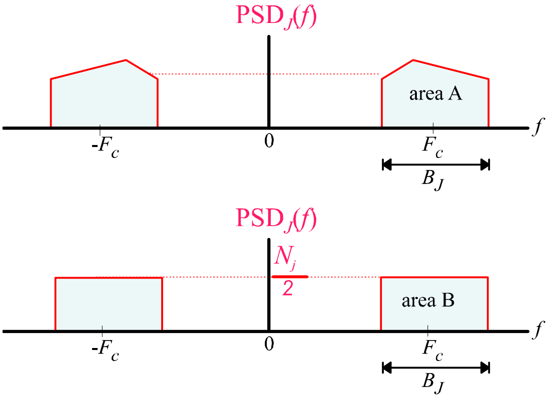
Modelling of PN-signals in SSS

- Consider
  – $\begin{cases} \{\alpha[n]\} = \text{a sequence of } \pm 1\prime s \\ c(t) = \text{an energy signal of duration } T_c, \text{ e.g. } c(t) = \text{rect}\left\{\frac{t}{T_c}\right\} \end{cases}$

  We have seen that the PN signal $b(t)$ can be modelled as follows:
  - DS-SSS (Examples: DS-BPSK, DS-QPSK):
  $$b(t) = \sum_n \alpha[n].c(t - nT_c) \tag{1}$$
  - FH-SSS (Examples: FH-FSK)
  $$b(t) = \sum_n \exp\{j(2\pi \mathrm{k}[n]F_1 t + \phi[n])\}.c(t - nT_c) \tag{2}$$
  where $\{\mathrm{k}[n]\}$ is a sequence of integers such that
  $$\{\alpha[n]\} \mapsto \{\mathrm{k}[n]\} \tag{3}$$
  and with $\phi[n] = \text{random: } \text{pdf}_{\phi[n]} = \frac{1}{2\pi}\text{rect}\left\{\frac{\phi}{2\pi}\right\}$

Equivalent Energy Utilisation Efficiency (EUE)

- Remember:
  - ★ Jamming source, or, simply Jammer = intentional interference
  - ★ Interfering source = unintentional interference



  - ★ With $\boxed{\text{area-B} = \text{area-A}}$ we can find N_j
  - ★ $P_j = 2\times\ \underline{\text{area}\mathbf{A}} = 2\times\ \underline{\text{area}\mathbf{B}} = N_j B_j \Rightarrow N_j = \frac{P_j}{B_j}$

- if

$$B_J = qB_{ss};\ 0 < q \leq 1 \tag{4}$$

then

$$\mathrm{EUE}_J = \frac{E_b}{N_J} = \frac{P_s.B_J}{P_J.r_b} = \frac{P_s.q.B_{ss}}{P_J.B} = \mathrm{PG} \times \mathrm{SJR}_{in} \times q \tag{5}$$

$$\mathrm{EUE}_{equ} = \frac{E_b}{N_0 + N_J} \tag{6}$$

$$= \underbrace{\mathrm{PG} \times \mathrm{SJR}_{in} \times q}_{\mathrm{EUE}_j} \times \left(\frac{N_0}{N_j} + 1\right)^{-1} \tag{7}$$

where

$$\mathrm{SJR}_{in} \triangleq \frac{P_s}{P_J} \tag{8}$$

Comments

- EUE_{equ} (or EUE_J): very important since bit error probabilities are defined as function of EUE_{equ} (or of EUE_J)
- For a specified performance

$$- \begin{cases} \text{the smaller the } \text{SIR}_{in} \Rightarrow \text{the better for the signal} \\ \text{the larger the } \text{SIR}_{in} \Rightarrow \text{the better for the jammer} \end{cases}$$

- Jammer limits the performance of the communication system
  i.e. effects of channel noise can be ignored
  i.e. Jammer Power$\gg P_n \Rightarrow \text{EUE}_{equ} = \frac{E_b}{N_0+N_J} \simeq \frac{E_b}{N_J} = \text{EUE}_J$

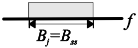
  N.B. exception to this:

  i) non-uniform fading channels

  ii) multiple access channels
- $\exists\ \infty$ number of possible jamming waveforms
- There is no single jamming waveform that is worst for all SSSs
- There is no single SSS that is best against all jamming waveforms.

Classification of Jammers in SSS

BROADBAND NOISE JAMMER



Spreads Gaussian Noise of total power P_j evently over the total frequency range of the spread spectrum bandwidth B_{ss}

$$EUE_J = \frac{E_b}{N_j}$$

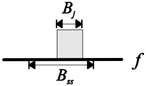
The only knowldge available to (and exploited by) the jammer is the bandwidth B_{ss} of the SSS

p_e: same as that with additive white Gaussian noise of $PSD_n(f) = N_j/2$

The performance with this type of jammer is known as

BASELINE PERFORMANCE

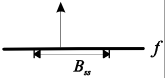
PARTIAL NOISE JAMMER



Spreads noise of total power P_j evently over some frequency range B_j with $B_{ss} \ni B_j$

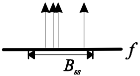
$$\rho = \frac{B_j}{B_{ss}} \leq 1$$

CW JAMMER



$$j(t) = \sqrt{2P_j}\cos(2\pi F_1 t + \vartheta)$$

MULTITONE JAMMER



$$j(t) = \sum_l \sqrt{\frac{2P_j}{N_j}}\cos(2\pi F_l t + \vartheta_l)$$

PULSE JAMMER

The jammer transmits with power $P_{j,peak} = \frac{P_j}{\rho}$ for a fraction ρ of the time and nothing for the ramaining $1-\rho$ of the time

REPEAT-BACK JAMMER or frequ.following jammer

The jammer first estimates some parameters of the SSSignal and then transmits jamming signals which use this information.

Effective against FH-SSS with slow hop-rate enough for the jammer to respond within the hop duration.

Can be neutralized by increasing hop-rate.

ARBITRARY JAMMER POWER DISTRIBUTION

Spread Gaussian noise with arbitrary $PSD(f)$ of total power P_j over the total frequency range (or some frequency range) of the SSSignal bandwidth B_{ss}

Direct Sequence SSS

Introductory Concepts & Mathematical Modelling

- If BPSK digital modulator is employed then the BPSK-signal can be modelled as:

$$\boxed{\textbf{BPSK}} \boxed{s(t) = A_c \cdot \sin(2\pi F_c t + m(t) \cdot \tfrac{\pi}{2})} \tag{9}$$

where the data waveform can be modelled as follows:

$$m(t) \equiv \sum_n \mathrm{a}[n] \cdot \underbrace{c_1(t - n \cdot T_{cs})}_{rect\left\{\frac{t-n\cdot T_{cs}}{T_{cs}}\right\}}; \quad nT_{cs} \leq t < (n+1) \cdot T_{cs} \tag{10}$$

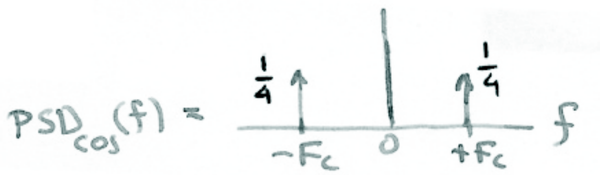
with $\{\mathrm{a}[n]\}$ = sequ. of independent data (message) bits (±1's)

- Equation (9) can be rewritten as follows:

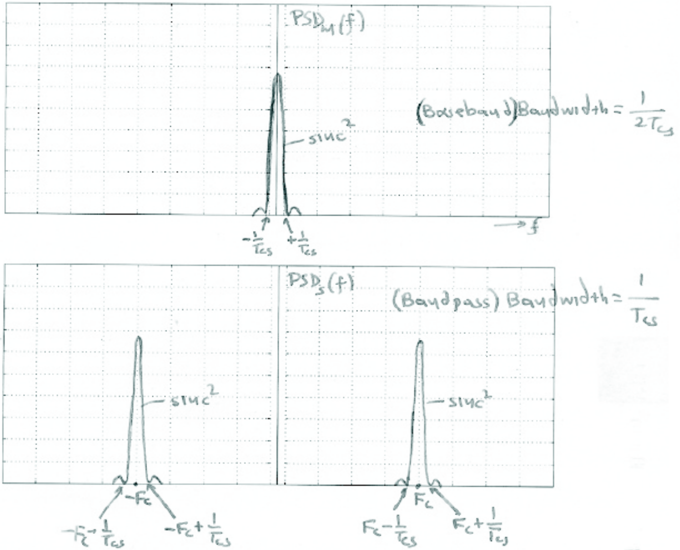
BPSK	$s(t) = A_c \cdot m(t) \cdot \cos(2\pi F_c t)$

$$\therefore \text{BPSK can be considered as } \begin{cases} \boxed{\text{PM}} \\ \boxed{\text{AM}} \end{cases} \tag{11}$$

- remember:



- The PSD(f)'s of $m(t)$ and $s(t)$ are shown below



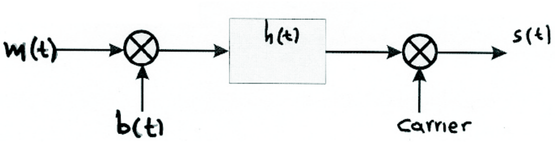
- If a DS/BPSK modulator is employed, then the SS-transmitted signal is

$$\begin{aligned}
\text{DS/BPSK}: s(t) &= A_c \sin\left(2\pi F_c t + \overbrace{m(t)b(t)}^{\pm 1}\frac{\pi}{2}\right) &(12)\\
&= A_c m(t) b(t) \cos(2\pi F_c t) &(13)
\end{aligned}$$

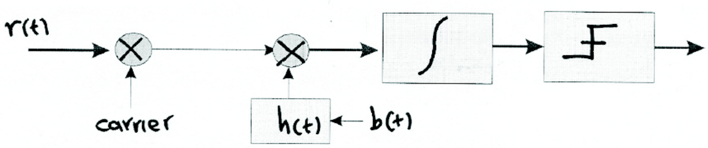
$$\text{where}\left\{\begin{array}{ll}
m(t) \equiv \sum_n \mathrm{a}[n] \cdot \underbrace{c_1(t - nT_{cs})}_{\substack{\uparrow \\ \mathrm{rect}\left\{\frac{t-nT_{cs}}{T_{cs}}\right\}}}; & nT_{cs} \leq t < (n+1)T_{cs} \\
b(t) = \sum_k \alpha[k] \cdot \underbrace{c_2(t - kT_{cs})}_{\substack{\uparrow \\ \mathrm{rect}\left\{\frac{t-kT_c}{T_c}\right\}}}; & kT_c \leq t < (k+1)\overset{\substack{\text{chip-duration} \\ \downarrow}}{T_c}
\end{array}\right| \quad (14)$$

BPSK DS/CDMA Transmitter & Receiver

- TX (see also Appendix-B):



- Rx (see also Appendix-B):



- The PN-sequence $\{\alpha[l]\}$ (whose elements have values ± 1)
- is M times faster than the data sequence $\{\mathrm{a}[n]\}$.
- i.e.

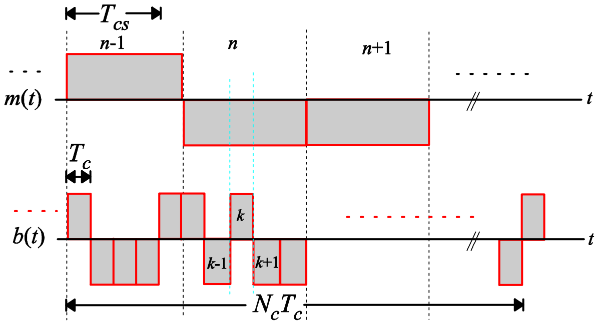
$$T_{cs} = M \cdot T_c \tag{15}$$

i.e.

$$\boxed{\text{PN-signal Bandwidth}} \gg \boxed{\text{data-Bandwidth}} \tag{16}$$

- Systems which have **coincident data** and **SS code clocks**

  are often said to have a **"data privacy feature"**
  - such systems are easy to build and can be combined in single units



$$T_{cs} = \underset{\overset{\shortparallel}{5}}{M} T_c$$

- Note: chip $= T_c =$ smallest time increment

- If the above **"data privacy feature"** is taken into account

then

$$
\begin{cases}
m(t) \equiv \sum_n \mathrm{a}[n].c_1(t - n \cdot T_{cs}) & nT_{cs} < t < (n+1)\,T_{cs} \\
b(t) = \sum_k \alpha[k].c_2(t - kT_c) & kT_c < t < (k+1)\,T_c \\
\text{with} \quad T_{cs} = MT_c; \;\; \left\lfloor \frac{k}{M} \right\rfloor = n;
\end{cases}
\tag{17}
$$

where

$$
\begin{aligned}
n \cdot T_{cs} + k' \cdot T_c \quad &\leq \quad t < n \cdot T_{cs} + (k'+1) \cdot T_c \\
\forall k' \quad &= \quad 0, 1, \ldots, M-1 = k \bmod M
\end{aligned}
$$

Conclusion

- $$\boxed{\text{DS/BPSK}}\ \underline{\text{similar to}}\ \boxed{\text{BPSK}}$$
  $$\uparrow$$
  except that the apparent data rate is M **times faster**
  $$\Downarrow$$
  signal spectrum is M **times wider**

- Therefore
$$\boxed{\mathrm{PG} = \frac{B_{ss}}{B} = M} \tag{18}$$

- Note:
  1. message cannot be recovered without knowledge of PN-sequence i.e. PRIVACY
  2. typical:
     - ★ PN-chip-rate $\longrightarrow$ several **M bits/sec**
     - ★ data rate $\longrightarrow$ few **bits/sec**

PSD of DS/BPSK/SS Transmitted Signal

- Tx signal

$$
\begin{aligned}
s(t) &= m(t)\cdot b(t)\cdot A_c\cdot\cos(2\pi F_c t)\\
&\Downarrow\\
\mathrm{PSD}_s(f) &= \mathrm{PSD}_m(f) * \mathrm{PSD}_b(f) * \mathrm{PSD}_{A_c\cos(2\pi F_c t)}(f)\\
&\Downarrow\\
\mathrm{PSD}_s(f) &= \mathrm{PSD}_m(f) * \mathrm{PSD}_b(f) * \mathrm{PSD}_{A_c\cos(2\pi F_c t)}(f)
\end{aligned}
$$

- **remember**

$$
\mathrm{PSD}_{A_c\cos(2\pi F_c t)}(f) = \frac{A_c^2}{4}\cdot\left(\delta\left(f - F_c\right) + \delta\left(f + F_c\right)\right)
$$

- $m(t) \equiv \sum_n \mathrm{a}[n].c(t - n \cdot T_{cs}) \Rightarrow$ (see "line-codes")

$$\boxed{\mathrm{PSD}_m(f) = \frac{|\mathrm{FT}(c(t))|^2}{T_{cs}} \cdot \left[R_{\mathrm{aa}}[0] + \sum_{\substack{k=-\infty \\ k\neq 0}}^{+\infty} R_{\mathrm{aa}}[k] \exp(-j2\pi f k T_{cs}) \right]} \tag{19}$$

- Note that if statistical indep. we have:

$$R_{\mathrm{aa}}[k] = \begin{cases} \mathcal{E}\left\{\mathrm{a}_n^2\right\} \text{ for } k = 0 \\ \mathcal{E}\left\{\mathrm{a}_n\right\} \cdot \mathcal{E}\left\{\mathrm{a}_{n+k}\right\} \text{ for } k \neq 0 \end{cases}$$

i.e. $R_{\mathrm{aa}}[k] = \begin{cases} \mu_{\mathrm{a}}^2 + \sigma_{\mathrm{a}}^2 \text{ for } k = 0 \text{ where } \mu_{\mathrm{a}} = \text{mean and } \sigma_{\mathrm{a}} = \text{std} \\ \mu_{\mathrm{a}}^2 \text{ for } k \neq 0 \end{cases}$

$\boxed{\text{then}}$

$$\boxed{\mathrm{PSD}_m(f) = \underbrace{\sigma_{\mathrm{a}}^2 \frac{|\mathrm{FT}(c(t))|^2}{T_{cs}}}_{\text{Continuous Spectrum}} + \underbrace{\frac{\mu_{\mathrm{a}}^2}{T_{cs}^2} \cdot \mathrm{comb}_{\frac{1}{T_{cs}}}\left(|\mathrm{FT}(c(t))|^2\right)}_{\text{Discrete Spectrum}}}$$

PSD(f) of a Random Pulse Signal

- For a random pulse signal $m(t) \equiv \sum_n \mathrm{a}[n].c(t - n \cdot T_{cs})$ (i.e. a sequence of pulses where there is an invariant average time of separation T_{cs} between pulses) **with all pulses of the same form but with**
  - random amplitudes $\mathrm{a}[n]$ with mean$=\mu_{\mathrm{a}} = \mathcal{E}\{\mathrm{a}[n]\}$ and std$=\sigma_{\mathrm{a}}^2 = \mathcal{E}\{(\mathrm{a}[n] - \mu_{\mathrm{a}})^2\}$
  - **statistical independent random time of occurrence** ,

Then:

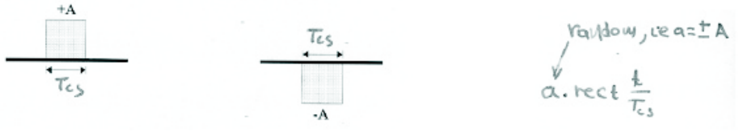
$$\mathrm{PSD}_m(f) = \underbrace{\sigma_{\mathrm{a}}^2 \frac{|\mathrm{FT}(c(t))|^2}{T_{cs}}}_{\text{Continuous Spectrum}} + \underbrace{\frac{\mu_{\mathrm{a}}^2}{T_{cs}^2} \cdot \mathrm{comb}_{\frac{1}{T_{cs}}}\left(|\mathrm{FT}(c(t))|^2\right)}_{\text{Discrete Spectrum}}$$

Note that if $\mu_{\mathrm{a}} = 0$ (**this is a very common case**) then

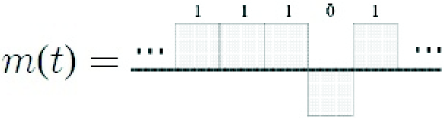
$$\boxed{\mathrm{PSD}_m(f) = \mathcal{E}\{\mathrm{a}[n]^2\} \frac{|\mathrm{FT}(\text{single pulse})|^2}{T_{cs}}}$$

Example: PSD of a random BINARY signal m(t)

- Consider a random binary sequence of 0's and 1's. This binary sequence is transmitted as random signal $m(t)$ with 1's and 0's being sent using the pulses shown below.



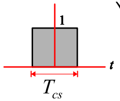
- For instance a random binary sequence/waveform could be



- If 1's and 0's are statistically independent with $\Pr(0) = \Pr(1) = 0.5$, the PSD of the transmitted signal can be estimated as follows:

- Solution:

$$\text{PSD}_m(f) = \mathcal{E}\left\{\text{a}[n]^2\right\}.\frac{\left\|\text{FT}\left(\quad\right)\right\|^2}{T_{cs}}.$$



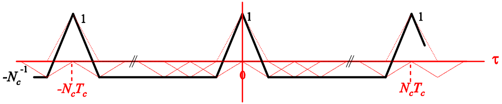
$$\begin{aligned}
&= \mathcal{E}\left\{\text{a}[n]^2\right\}.\frac{T_{cs}^2\cdot\text{sinc}^2(fT_{cs})}{T_{cs}} \\
&= \underbrace{\mathcal{E}\left\{\text{a}[n]^2\right\}}_{=(-A^2)\frac{1}{2}+(+A^2)\frac{1}{2}}\quad T_{cs}\cdot\text{sinc}^2\left\{fT_{cs}\right\} \\
&= A^2\ T_{cs}\text{sinc}^2\left\{fT_{cs}\right\}
\end{aligned}$$

- $R_{mm}(\tau) =$
$\text{FT}^{-1}\left\{\text{PSD}_m(f)\right\} = \text{FT}^{-1}\left\{A^2\ T_{cs}\text{sinc}^2\left\{fT_{cs}\right\}\right\} = A^2\ \Lambda\left\{\frac{\tau}{T_{cs}}\right\}$

PSD(f) of a PN-Signal b(t) in DS-SSSs

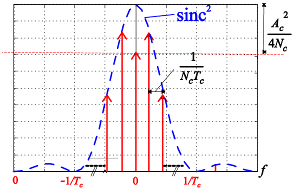
- Autocorrelation function: $R_{bb}(\tau)$

$$\boxed{R_{bb}(\tau) = \frac{N_c+1}{N_c}\mathrm{rep}_{N_c T_c}\left\{\Lambda\left(\frac{\tau}{T_c}\right)\right\} - \frac{1}{N_c}} \tag{20}$$



- Using the FT tables the $\mathrm{PSD}_b(f)$= $\mathrm{FT}\{R_{bb}(\tau)\}$ of the signal $b(t)$ is:

$$\boxed{\mathrm{PSD}_b(f) = \frac{N_c+1}{N_c^2}\mathrm{comb}_{\frac{1}{N_c T_c}}\left\{\mathrm{sinc}^2\left\{f \cdot T_c\right\}\right\} - \frac{1}{N_c}\delta(f)} \tag{21}$$



PSD(f) of DS/BPSK Spread Spectrum Tx Signal s(t)

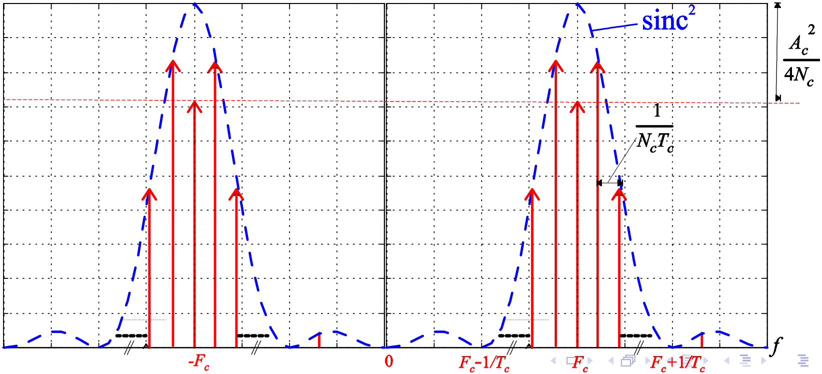
$$\begin{aligned} s(t) &= m(t)\cdot b(t)\cdot A_c\cdot\cos(2\pi F_c t) \\ \Rightarrow \mathrm{PSD}_s(f) &= \mathrm{PSD}_m(f) * \underbrace{\mathrm{PSD}_b(f) * \tfrac{A_c^2}{4}\left(\delta(f-F_c)+\delta(f+F_c)\right)}_{\text{term 1}} \end{aligned}$$

- **Ignore** (for the time being) the effects of $m(t)$

$$\begin{aligned} \mathrm{PSD}_{\mathrm{term1}}(f) &= \tfrac{A_c^2}{4}\cdot\mathrm{PSD}_b(f) * \left(\delta(f-F_c)+\delta(f+F_c)\right) \\ &= \tfrac{A_c^2}{4}\cdot\left\{\tfrac{N_c+1}{N_c^2}\cdot\mathrm{comb}_{\frac{1}{N_cT_c}}\left\{\mathrm{sinc}^2(f\cdot T_c)\right\} - \frac{1}{N_c}\cdot\delta(f)\right\} \\ &\quad * \left(\delta(f-F_c)+\delta(f+F_c)\right) \\ &= \tfrac{A_c^2}{4}\cdot\tfrac{N_c+1}{N_c^2}\cdot\Big(\mathrm{comb}_{\frac{1}{N_cT_c}}\left\{\mathrm{sinc}^2\left\{(f-F_c)T_c\right\}\right\} \\ &\qquad + \mathrm{comb}_{\frac{1}{N_cT_c}}\left\{\mathrm{sinc}^2\left\{(f+F_c)T_c\right\}\right\}\Big) \\ &\quad -\tfrac{A_c^2}{4}\tfrac{1}{N_c}\left(\delta(f-F_c)+\delta(f+F_c)\right) \end{aligned}$$

- i.e.

$$\begin{aligned}\mathrm{PSD}_{\mathrm{term1}}(f) &= \frac{A_c^2}{4}\cdot\frac{N_c+1}{N_c^2}\cdot\Big(\mathrm{comb}_{\frac{1}{N_cT_c}}\left\{\mathrm{sinc}^2\left\{(f-F_c)T_c\right\}\right\} \\ &\qquad + \mathrm{comb}_{\frac{1}{N_cT_c}}\left\{\mathrm{sinc}^2\left\{(f+F_c)T_c\right\}\right\}\Big) \\ &\quad -\frac{A_c^2}{4}\frac{1}{N_c}\left(\delta(f-F_c)+\delta(f+F_c)\right) \qquad (22)\end{aligned}$$



- Above the effects of $m(t)$ have not been taken into account.
  - If $m(t)$ is used,
    then each discrete frequency in Equation (22) becomes a sinc[2] function.
  - There are two cases:

- **CASE-1:**

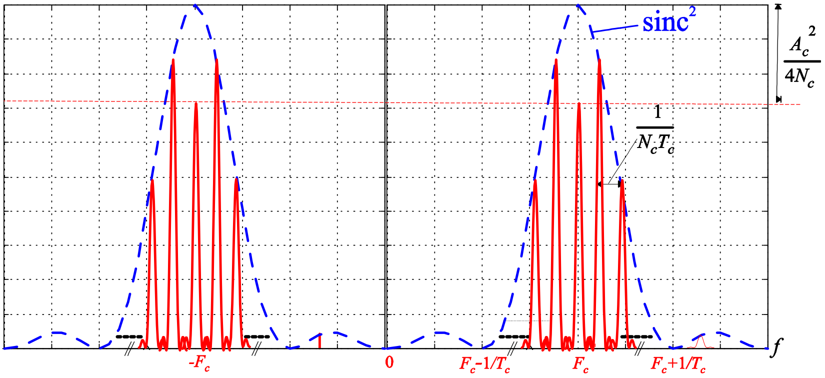
$$\frac{1}{T_{cs}} < \frac{1}{2N_c T_c}$$

- **CASE-2:**

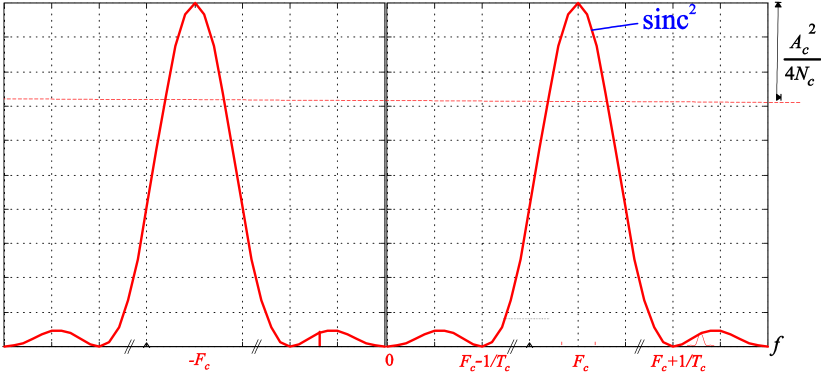
$$\frac{1}{T_{cs}} \geq \frac{1}{N_c T_c}$$

the peaks will merge into a continuous smooth spectrum

- **CASE-1:**



- **CASE-2:**

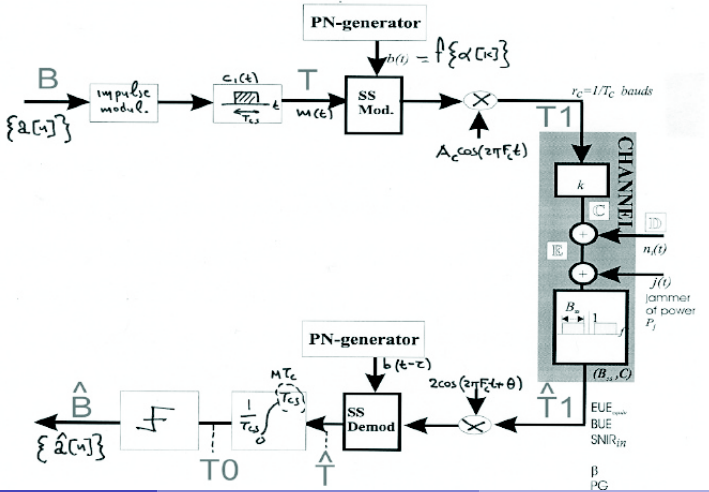


- **N.B.:**
  - If
    $$b(t) = \text{random}$$
    then
    $$s(t) = A_c m(t) b(t) \cos(2\pi F_c t)$$
  - If the effects of $m(t)$ **are ignored** then
    $$\begin{aligned} \text{PSD}_s(f) &= A_c^2 T_c^2 \text{sinc}\{fT_c\} * \frac{1}{4}\left(\delta(f - F_c) + \delta(f + F_c)\right) \\ &\Downarrow \\ \text{PSD}_s(f) &= \frac{T_c}{4} A_c^2 \left(\text{sinc}^2\{(f - F_c)T_c\} + \text{sinc}^2\{(f + F_c)T_c\}\right) \end{aligned}$$
    i.e. PSD is similar to **'CASE-2'** above

DS-BPSK Spread Spectrum:

Output SNIR

- Consider the block diagram of a SS-Communication System which employs a BPSK digital modulator:



- N.B.: $B_{ss} = \frac{1}{T_c}$; $B = \frac{1}{T_{cs}}$; $\text{PG} = \frac{B_{ss}}{B} = \frac{T_{cs}}{T_c}$

- Then, at point $\boxed{\text{T1}}$, we have

$$s(t) = A_c \cdot m(t) \cdot b(t) \cdot \cos(2\pi F_c t) \tag{23}$$

and at point $\boxed{\overset{\wedge}{\text{T1}}}$:

$$s(t) + n(t) + j(t) \quad (\text{for } k = 1) \tag{24}$$

- At the input of the receiver the Signal-to-Noise-plus-Interference Ratio (SNIR_{in}) is:

$$\text{SNIR}_{\text{in}} = \frac{\text{EUE}}{\text{PG} \cdot (1 + \text{JNR}_{\text{in}})} = \frac{\text{EUE}_{\text{equ}}}{\text{PG}} \tag{25}$$

- at point $\boxed{\overset{\wedge}{\mathrm{T}}}$:

$$(s(t)+n(t)+j(t))\cdot b(t-\tau)\cdot 2\cos(2\pi F_c t+\theta) \tag{26}$$

- at point $\boxed{\mathrm{T0}}$:

$$P_{\text{unwanted}} = P_{n_{\text{out}}} + P_{\text{code-noise}} + P_{j_{\text{out}}} \tag{27}$$

- if $\tau = 0$ and $\theta = 0$ (i.e. the system is synchronized) then:

$$P_{\text{code-noise}} = 0$$

and

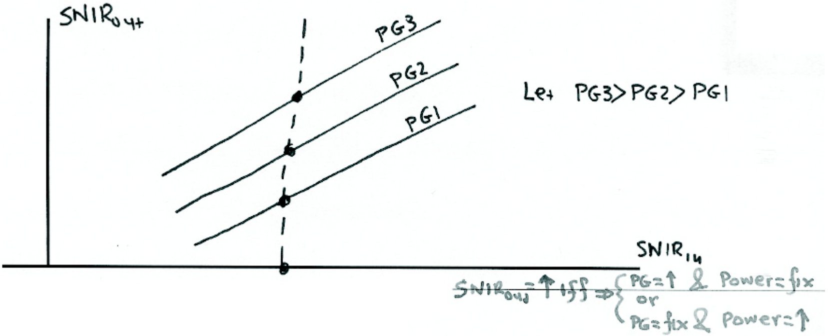
$$\textbf{SNIR}_{\textbf{out-max}} = \textbf{2EUE}_{\textbf{equ}} \qquad (28)$$

- However,

$$\text{SNIR}_{\text{in}} = \frac{\text{EUE}}{\text{PG} \cdot (1 + \text{JNR}_{\text{in}})} = \frac{\text{EUE}_{\text{equ}}}{\textbf{PG}} \tag{29}$$

- Therefore,

$$\boxed{\text{SNIR}_{\textbf{out-max}} = 2\text{EUE}_{\text{equ}} = 2.\text{PG}.\text{SNIR}_{\text{in}}} \tag{30}$$



Bit Error Probability with Jamming

A. CONSTANT POWER BROADBAND JAMMER:

- From the "Detection Theory" topic we know that the bit-error-probability p_e for a **Binary Phase-Shift Key (BPSK)** communication system is given by:

$$p_e = \mathrm{T}\left\{ \underbrace{\sqrt{2 \cdot \mathrm{EUE}}}_{\mathrm{SNR}_{\text{out, matched filter}}} \right\} \quad \text{where } \mathrm{EUE} = \frac{E_b}{N_0} \tag{31}$$

- Consider a DS/BPSK SSS which operates in the presence of a constant amplitude broadband jammer with double sided power spectral density

$$\mathrm{PSD}_j(f) = \frac{N_j}{2} \tag{32}$$

- Then,

$$p_e = T\left\{\sqrt{2 \cdot \mathrm{EUE}_{\mathrm{equ}}}\right\}$$

$$\text{where } \mathrm{EUE}_{\mathrm{equ}} = \frac{E_b}{N_0 + N_j} \quad \text{with } N_j = \frac{P_J}{B_{ss}}$$

- If we make the assumption that $N_j \gg N_0$ then

$$\boxed{p_e = \mathrm{T}\left\{\sqrt{2 \cdot \mathrm{EUE}_J}\right\}} \tag{33}$$

$$\text{where } \mathrm{EUE}_J = \frac{E_b}{N_j}$$

- This is known as the **BASELINE PERFORMANCE** of a DS/BPSK SSS

B. PULSE JAMMER:

- Consider a DS/BPSK SSS which operates in the presence of a jammer which transmits "broadband noise" with large power but only a fraction of the time.

- The double-sided power spectral density of the jammer is given by:

$$\mathrm{PSD}_j(f) = \frac{N_j}{2\rho} \tag{34}$$

- where
  - $\rho \equiv$ the fraction of time the jammer is **"on".**
  - P_j = average jamming power
  - $\frac{P_j}{\rho}$ = actual power during a jamming pulse duration

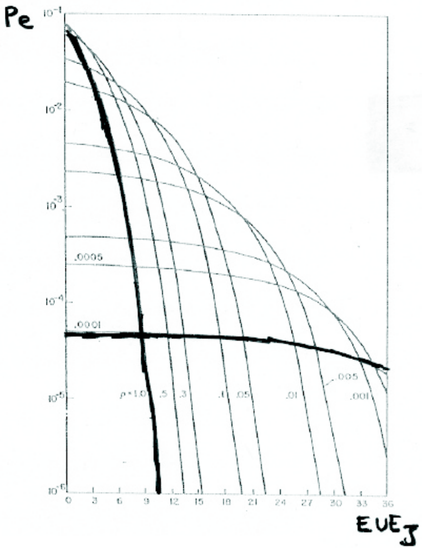
- Let the jammer pulse duration be greater than T_{cs} (data bit time).
  - Then $\begin{cases} \Pr(\text{jammer} = \text{"on"}) = \rho \\ \Pr(\text{jammer} = \text{"off"}) = 1-\rho \end{cases}$
    and the bit-error-probability is given by:

$$p_e = \underbrace{(1-\rho)\mathrm{T}\left\{\sqrt{2\frac{E_b}{N_0}}\right\}}_{\simeq 0 \text{ (very small)}} + \rho\mathrm{T}\left\{\sqrt{2\frac{E_b}{N_0 + \frac{N_j}{\rho}}}\right\} \tag{35}$$

which can be simplified to

$$\boxed{p_e = \rho \cdot \mathrm{T}\left\{\sqrt{2\rho \cdot \mathrm{EUE}_J}\right\}} \quad \text{where } \mathrm{EUE}_J = \frac{E_b}{N_j} \tag{36}$$

- By plotting the above equation for different values of ρ we get:



- Note that
  - the value of which maximizes p_e decreases with increasing values of EUE_j
  - there is a value of ρ which maximizes the probability of error p_e. This value can be found by differentiating Equation-36 with respect to ρ. That is

$$\frac{dp_e}{d\rho} = 0 \Rightarrow \rho^* = \begin{cases} \frac{0.709}{\text{EUE}_j} & \text{if } \text{EUE}_j > 0.709 \\ 1 & \text{if } \text{EUE}_j \leq 0.709 \end{cases} \tag{37}$$

  - Therefore

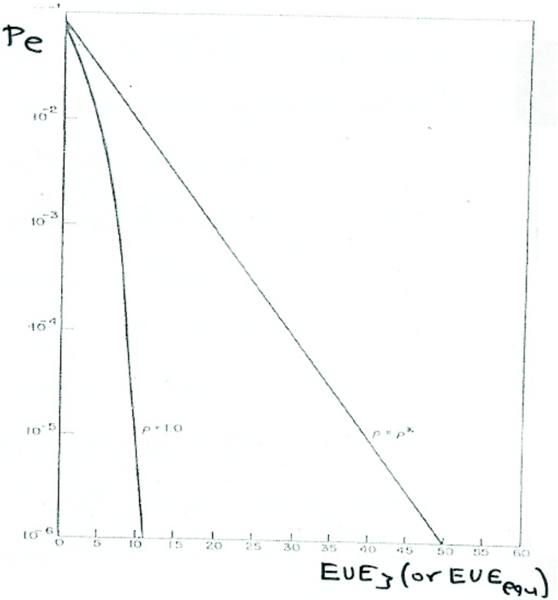
$$p_{e_{\max}} = \max_{\rho} \left\{ \rho \cdot \text{T}\left\{ \sqrt{2\rho \text{EUE}_j} \right\} \right\} \tag{38}$$

$$p_{e_{\max}} = \rho^* \cdot \text{T}\left\{ \sqrt{2\rho^* \text{EUE}_j} \right\}$$

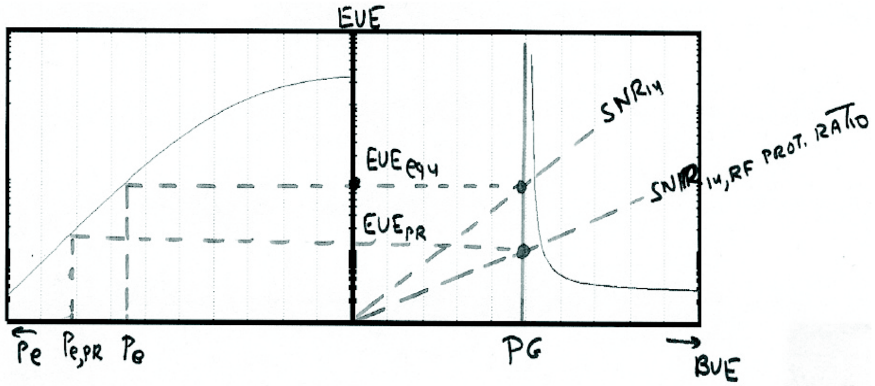
$$\Longrightarrow p_{e_{\max}} = \begin{cases} \frac{0.083}{\text{EUE}_j} & \text{if } \text{EUE}_j > 0.709 \\ \text{T}\left\{ \sqrt{2\text{EUE}_j} \right\} & \text{if } \text{EUE}_j \leq 0.709 \end{cases} \tag{39}$$

- N.B.:
  - when jammer pulse length is shorter than a data bit time T_{cs} then the above expression is not valid.
  - However, Equation-39 represents an UPPER BOUND on the bit-error-probability p_e.

- The next graph illustrates
  - the bit-error-prob. plotted against the EUE_j for a baseline jammer (i.e. $\rho = 1$) and
  - the worst case jammer (i.e. $\rho = \rho^*$) for a DS/BPSK SSS.
- Note the huge difference between the two curves.


Pe
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0 5 10 15 20 25 30 35 40 45 50 55 60
ρ = 1.0
ρ = ρ*
EUEJ (or EUEequ)

DS-SSS on the (p_e, EUE, BUE)-parameter plane



Anti-Jam Margin

- An important parameter of SSS is the ANTIJAM MARGIN which is defined as follows:

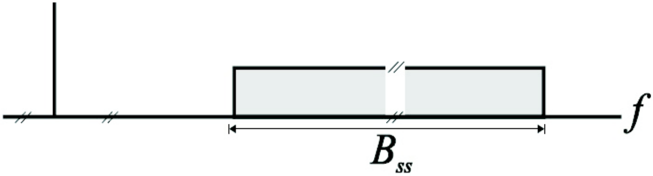
  $$\text{dB(AJM)} \equiv \text{dB}(\text{EUE}_{\text{equ}}) - \text{dB(EUE which corresponds to the } p_{e,PR})$$

  $$\Rightarrow \boxed{\text{dB(AJM)} \equiv 10 * \log(\text{EUE}_{\text{equ}}) - 10 * \log(\text{EUE}_{p_e,PR})}$$

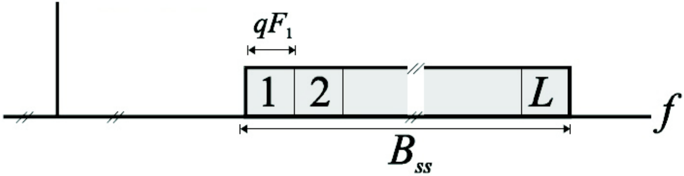
- AJM represents a safety margin against jammer (or against jammer plus noise).

Frequency Hopping SSSs

- Consider that the following part of the spectrum has been allocated to a FH/SSS:



- Let us partition the above spectrum onto L different frequency slots of bandwidth F_1 (or of bandwidth qF_1 where q is a constant). Then $B_{ss} = q \cdot F_1 \cdot L$



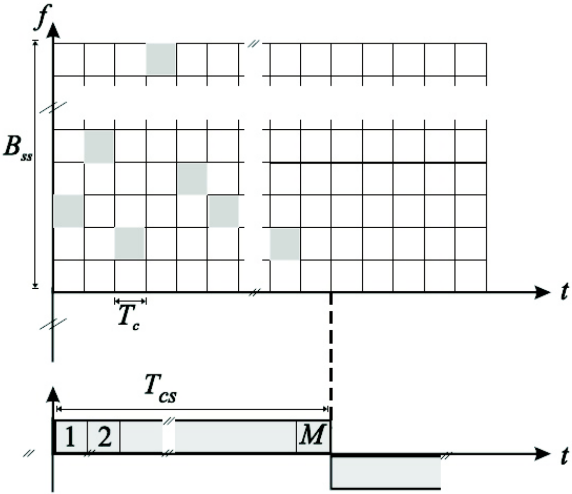
$L = 2^m$

- Define the following symbols:

$$T_c = \text{hop duration (i.e. hop rate } r_{hop} = \tfrac{1}{T_c})$$
$$T_{cs} = \text{message bit duration (i.e. bit rate } r_b = \tfrac{1}{T_{cs}})$$
$$M = \text{number of hops per message bit (i.e. } T_{cs} = M \cdot T_c)$$

- FH/SSS: $\begin{cases} \text{Fast hop} & \rightarrow r_{hop} > r_b \\ \text{slow hop} & \rightarrow r_{hop} < r_b \\ \text{balance hop} & \rightarrow r_{hop} = r_b \end{cases}$

- The frequency slot is constant in each time chip T_c, BUT changes from chip-to-chip.
  This can be represented by the following diagram:



- In general, the No. of different frequency slots L over which the signal may hop, is a power of 2.

- F_1 is, in general, equal to $\frac{1}{T_c}$,

  i.e. $F_1 = \frac{1}{T_c}$ (but this is not a necessary requirement).

- several FH signals occupy a common RF channel

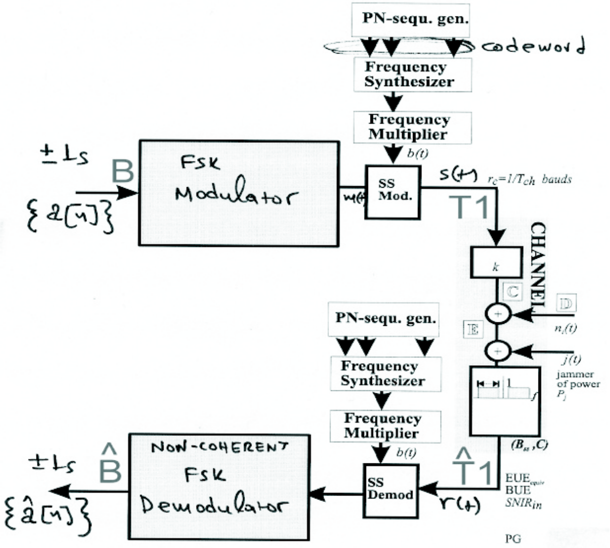
- FH model of $b(t)$ - (complex representation):

$$b(t) = \sum_n \exp\left\{j(2\pi.k[n].F_1 t + \phi_n)\right\} \cdot \text{rect}\left\{\frac{t - nT_c}{T_c}\right\}$$

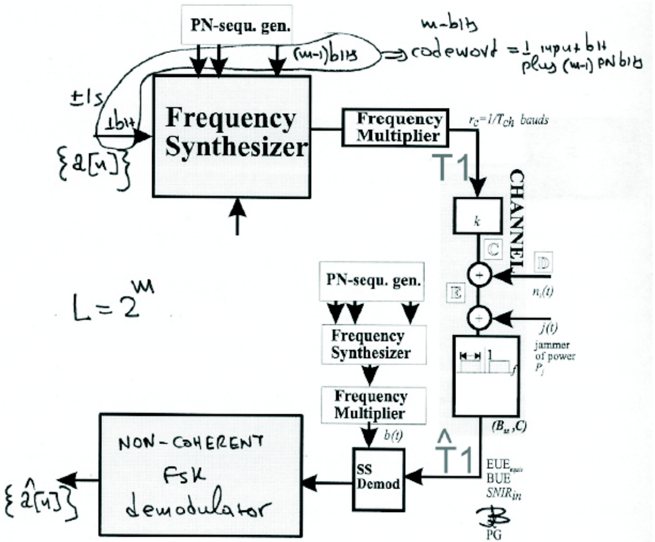
where

  - $k[n] = \mathsf{f}\{\text{PN-sequ } \{\alpha[n]\}\}$
  - $k[n]$ is an integer that is formed by a codeword which is formed by one or more m-sequences

- Transmitter-Receiver path:



A Different Implementation:

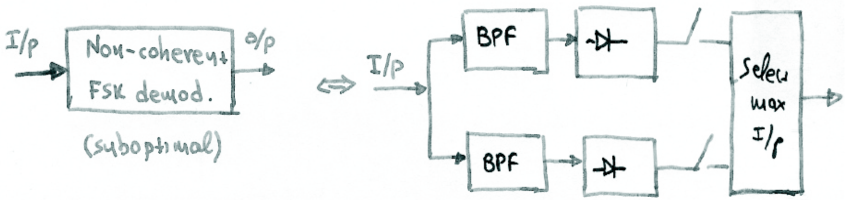


- L frequencies are produced by the digital Frequ. Synthesizer, separated by F_1

$$B_s \simeq q \cdot F_1 \cdot L \tag{40}$$

- Note:
  - if reception= $\boxed{\text{coherent}}$:
    - ★ more difficult to achieve
    - ★ places constraints on the transmitted signal and transmitted medium
  - if reception= $\boxed{\text{non-coherent}}$:
    - ★ PN-gen. can run at a considerably slower rate in this type of system than in a DS system

- FH: non-coherent $\Rightarrow$ poorer performance against thermal noise.



- Performance:
  - Coherent FSK (CFSK)

$$p_{e,CFSK} = T\left\{\sqrt{\text{EUE}}\right\}$$

  - Non-Coherent FSK (NFSK)

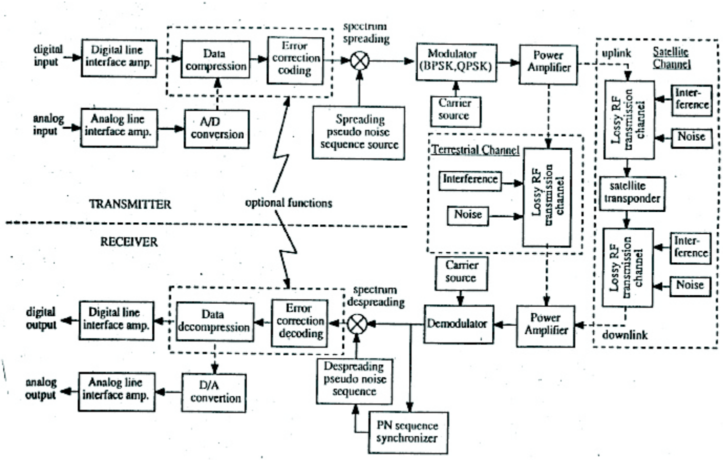
$$p_{e,NFSK} = \frac{1}{2}\exp\left(-\frac{\text{EUE}}{2}\right)$$

- A serious problem is the $\overbrace{\text{“near-far” problem}}^{\substack{\text{very strong signals at receiver} \\ \text{swapping out the effects} \\ \text{of weaker signal} \\ \Uparrow}}$:
  - DS: severe problem
  - FH: much more susceptible
- acquisition: much faster in FH than in DS
- $\text{PG} = \frac{B_{ss}}{B} =$ it is not very good criterion for FH

Appendices

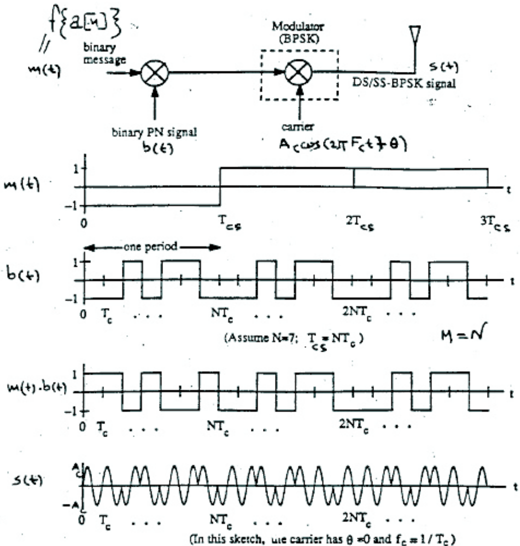
Appendix A. Block Diagram of a Typical SSS

(terrestrial & satellite comm. systems)



Appendix B. BPSK/DS/SS Transmitter and Receiver

BPSK/DS/SS Transmitter



BPSK/DS/SS Receiver