Structural Identifiability in Julia: A Tutorial

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Outline

- Introduction
- `StructuralIdentifiability.jl`
- `SIAN.jl`
- Examples
- Conclusions

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- *Global* identifiability means we can determine the value of parameter(s) uniquely
- *Local* identifiability means we can determine up to multiple values

Example

$$\begin{cases} \dot{x} = A^2 x + b, \\ y = x \end{cases} \tag{1}$$

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Solution:

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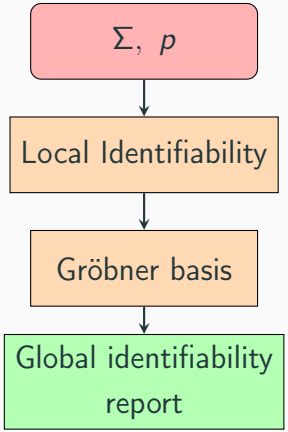
Solution:

- $x(0), b$ are globally identifiable, A is identifiable only locally according to `SIAN.jl`.
- b is globally identifiable, A is locally identifiable according to `StructuralIdentifiability.jl`. Note that there is no information about x.

SIAN.jl[1]

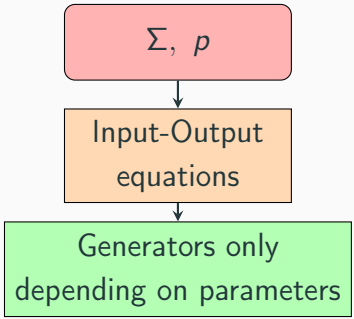


- SIAN is a Monte-Carlo algorithm.
- Input: ODE model with outputs Σ, probability of correctness p.
- Local identifiability is determined via identifiability rank condition.
- Global identifiability is analyzed via Gröbner basis.

[1] H. Hong et al. "Global identifiability of differential models". In: *Communications on Pure and Applied Mathematics* 73.9 (2020), pp. 1831–1879.

StructuralIdentifiability.jl[2]

- Input: ODE model with outputs Σ, probability of correctness p.
- Local identifiability is based on Sedoglavic's algorithm [3].
- Global identifiability is determined via field of coefficients of input-output equations
- Can differentiate Single- or Multi-experiment identifiability



[2] R. Dong et al. *Differential elimination for dynamical models via projections with applications to structural identifiability*. 2022.

Examples

Conclusions and Summary

- Parameter identifiability can be solved in Julia Language from 2 different perspectives
- No one-size-fits-all solution
- Future work includes mutliple enhancements to ODE preprocessing, Gröbner basis computation, integration with more SciML packages

References

[1] R. Dong et al. *Differential elimination for dynamical models via projections with applications to structural identifiability*. 2022.

[2] H. Hong et al. "Global identifiability of differential models". In: *Communications on Pure and Applied Mathematics* 73.9 (2020), pp. 1831–1879.

[3] A. Sedoglavic. "A probabilistic algorithm to test local algebraic observability in polynomial time". In: *Journal of Symbolic Computation* 33.5 (2002), pp. 735–755.