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Revisiting rolling and sliding in two-dimensional discrete element models

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ABSTRACT

It has long been recognized that the rotation of single particles plays a very important role in simulations of granular flow using the discrete element method (DEM). Many researchers have also pointed out that the effect of rolling resistance at the contact points should be taken into account in DEM simulations. However, even for the simplest case involving two-dimensional circular particles, there is no agreement on the best way to define rolling and sliding, and different definitions and calculations of rolling and sliding have been proposed. It has even been suggested that a unique rolling and sliding definition is not possible. In this paper we assess results from previous studies on rolling and sliding in discrete element models and find that some researchers have overlooked the effect of particles of different sizes. After considering the particle radius in the derivation of rolling velocity, all results reach the same outcome: a unique solution. We also present a clear and simple derivation and validate our result using cases of rolling. Such a decomposition of relative motion is *objective*, or independent of the reference frame in which the relative motion is measured.

Introduction

Granular materials consist of a large number of particles, each having translational and rotational motion depending on the total force and torque applied. Particles interact via contact areas and move relative to neighboring particles. The relative motion between touching particles includes motion in the normal direction, sliding in the tangential direction, and rolling over one another. Therefore, the macroscopic behavior of granular assemblies can be very complex. It has long been recognized that particle rotation and rolling play a key role in the mechanical behavior of granular materials, especially for those composed of circular or spherical particles. This has been pointed out since the pioneering work on rolling resistance by Oda, Konishi, and Nemat-Nasser (1982), who defined the rolling velocity between circular particles, and observed that inter-particle rolling dominates the microscale deformation of granular media.

The discrete element method (DEM) has emerged as an ideal tool to investigate the behavior of granular materials (Cundall & Strack, 1979). Different researchers reported that in DEM simulations of granular flow, both single-particle rotation and rolling resistance need to be incorporated into the model, otherwise unrealistic results are generated. Iwashita and Oda (1998, 2000) noted that the conventional DEM could not reproduce the large voids and high rotational gradients observed in shear band experiments. They recognized that rolling resistance causes an arching action at the contact points, and permits the easy formation of voids in physical tests. Therefore they proposed a modified model of the conventional discrete element method that takes the rolling resistance into account. Bardet and Proubet (1991) and Bardet (1994) examined the structure of shear bands in granular materials by simulating idealized granular media numerically. They showed that particle rotations concentrate inside shear bands and found that rotations have a significant effect on the shear strength of granular materials.

Tordesillas et al. (Tordesillas, Peters, & Muthuswamy, 2005; Tordesillas & Walsh, 2002) incorporated rolling resistance in the DEM and examined the influence of particle rotation and rolling resistance in the rigid flat-punch problem. They found that extensive particle rotations occur near the edges of the punch where

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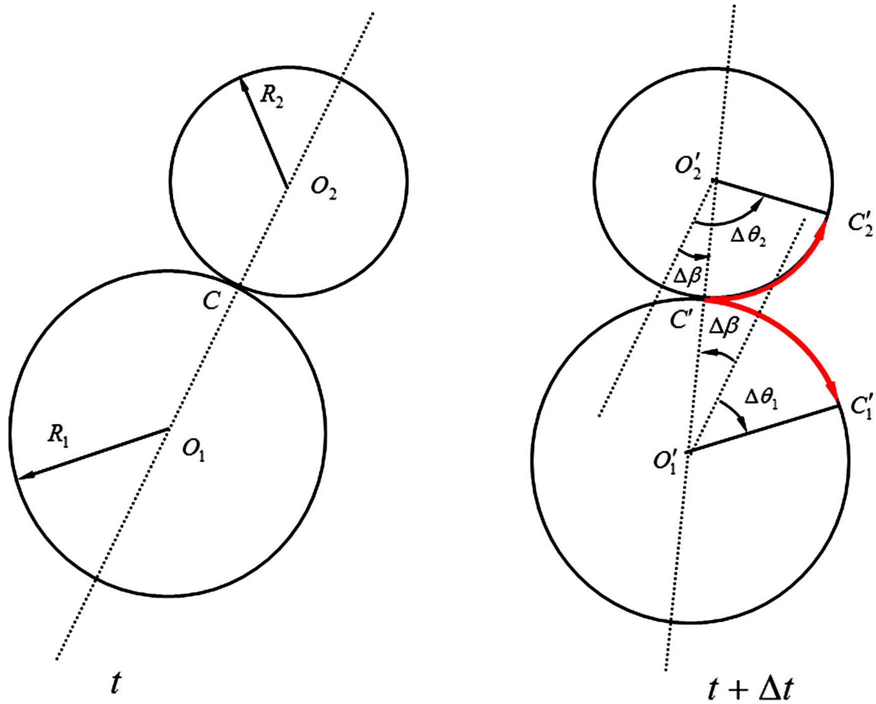

Fig. 1. Kinematic scheme of two disks in contact at times t and $t+\Delta t$.

high stress concentrations exist. These rotations lead to dilatation in the region adjacent to the sides of the punch. Wang and Mora (2008) showed that when only normal forces are transmitted, or when rolling resistance is absent, laboratory tests of wing-crack extension cannot be reproduced. The effect of rolling friction on granular flows has also been reported on recently by Balevicius, Sielamowicz, Mroz, and Kacianauskas (2012) and Goniva, Kloss, Deen, Kuipers, and Pirker (2012).

A quantitative investigation of the effects of rolling and sliding using the DEM demands a clear and unambiguous definition and calculation of rolling and sliding deformation. In principle, the relative motion between two particles in contact can be decomposed into several independent components: relative motion in the normal, tangential directions or sliding, relative rolling; and in the three-dimensional (3D) case, relative torsion. However, even for the simplest two-dimensional (2D) case involving circular particles, surprisingly, there is no agreement on the best way to define rolling and sliding. Different definitions and calculations of rolling and sliding have been proposed (Ai, Chen, Rotter, & Ooi, 2011; Alonso-Marroquin, Vardoulakis, Herrmann, Weatherley, & Mora, 2006; Bagi & Kuhn, 2004; Bardet, 1994; Bardet & Proubet, 1991; Iwashita & Oda, 1998; Jiang, Yu, & Harris, 2005; Kuhn & Bagi, 2004a,b; Luding, 2008; Mohamed & Gutierrez, 2010; Tordesillas et al., 2005; Tordesillas & Walsh, 2002). Some sources contradict other sources and this has led not only to confusion in the DEM field, but also to some researchers suggesting that there is no unique way to define the rolling displacement (Bagi & Kuhn, 2004).

The objective of our work is to answer three questions: Is there a unique way to define rolling and sliding deformation? If there is, how are rolling and sliding best determined in general cases? How can the different definitions of rolling resistance be consolidated in a unique formula? In this paper, we focus on the kinematics of two particles only. A thorough investigation of rolling resistance models is beyond the scope of this paper, but can be found in the literature (Ai et al., 2011; Mohamed & Gutierrez, 2010).

Problem statement

Fig. 1 shows the kinematic scheme of two discs in contact. During a time step from t to $t+\Delta t$, two particles 1 and 2, with radii R_1 and R_2, respectively, remain in contact. At time t, let O_1, O_2, and C denote the centers of the two particles and the contact point, respectively. At time $t+\Delta t$, the current centers of the two particles and the contact point are O_1', O_2', and C', respectively. The original contact point at C now appears at C_1' on particle 1 and at C_2' on particle 2. $\Delta\theta_1$ is the angle between O_1O_2 and $O_1'C_1'$, and $\Delta\theta_2$ is the angle between O_1O_2 and $O_2'C_2'$. By taking counter-clockwise rotation as positive, these two angles represent the incremental rotations of the two particles during the time step from t to $t+\Delta t$. $\Delta\beta$ is the angle between O_1O_2 and $O_1'O_2'$, representing an incremental change in the angle of contact direction between the particles. The arcs $C'C_1'$ and $C'C_2'$, which represent the displacement of the contact point C on particles 1 and 2, are denoted by Δa and Δb, respectively, and are positive when measured in a counter-clockwise direction. Generally, Δa and Δb have a rolling and sliding component, denoted by ΔU_r and ΔU_s, respectively. Now the question is how to determine and calculate ΔU_r and ΔU_s. To answer this question, we first need to define clearly what pure rolling and sliding are.

Definition of pure rolling and sliding

We start by defining rigid-body rotation (RBR). RBR occurs when two particles rotate together as a single rigid body. The distance between any arbitrary chosen points on the two particles remains constant during the motion. In this case (see Fig. 2), we have:

$$\Delta\beta = \Delta\theta_1 = \Delta\theta_2 \neq 0, \tag{1a}$$

$$\Delta a = \Delta b = 0, \tag{1b}$$

$$\Delta U_r = \Delta U_s = 0. \tag{1c}$$

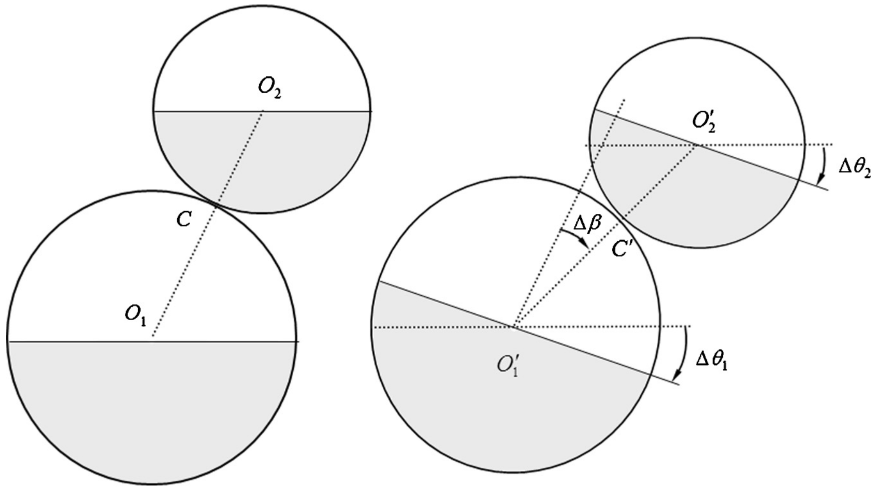

Fig. 2. Rigid-body rotation: two particles rotate together as if they were a single rigid body.

Pure rolling (PR) occurs when two particles rotate over each other in a gear-like fashion such that the contact point changes and no relative transverse displacement (or velocity) occurs at the contact point (Fig. 3, left). The arcs $C'C_1'$ and $C'C_2'$ have the same length but opposite signs, and are independent of the particle radii. The particle rotations have opposite signs:

$$\Delta a = -\Delta b \neq 0, \tag{2a}$$

$$\Delta \beta = 0, \tag{2b}$$

$$\Delta U_r = R_1 \Delta\theta_1 = -R_2 \Delta\theta_2. \tag{2c}$$

PR combined with RBR (PR + RBR, Fig. 3, right) is also defined:

$$\Delta a = -\Delta b \neq 0, \tag{3a}$$

$$\Delta \beta \neq 0, \tag{3b}$$

$$\Delta \theta_1 = 0, \tag{3c}$$

$$\Delta U_r = -R_1 \Delta\beta = -R_2(\Delta\theta_2 - \Delta\beta). \tag{3d}$$

PR + RBR can be obtained either from PR followed by a RBR of angle $\Delta\beta$, or directly from the initial state (Fig. 2, left) by rolling particle 2 over stationary particle 1.

Pure sliding (PS) occurs when both particles rotate through the same angle λ (Fig. 4, left). In this case, there is a relative transverse displacement at the contact point, but no relative rotation between the two particles:

$$\Delta \beta = 0, \tag{4a}$$

$$\Delta\theta_1 = \Delta\theta_2 = \lambda, \tag{4b}$$

$$\Delta U_s = (R_1 + R_2)\lambda. \tag{4c}$$

It should be noted that in this case, the equality is not the arc length, but the angle. In other words, $\Delta a = \Delta b$ is only valid when $R_1 = R_2$, but not when $R_1 \neq R_2$. Instead the following equation (Eq. (5)) always holds in the case of PS regardless of particle size. As will be pointed out in Comparison with earlier derivations section, assuming that $\Delta a = \Delta b$ in PS has led to the incorrect calculation of rolling velocity.

$$\Delta\theta_1 = \Delta\theta_2 = \frac{\Delta a}{R_1} = \frac{\Delta b}{R_2}. \tag{5}$$

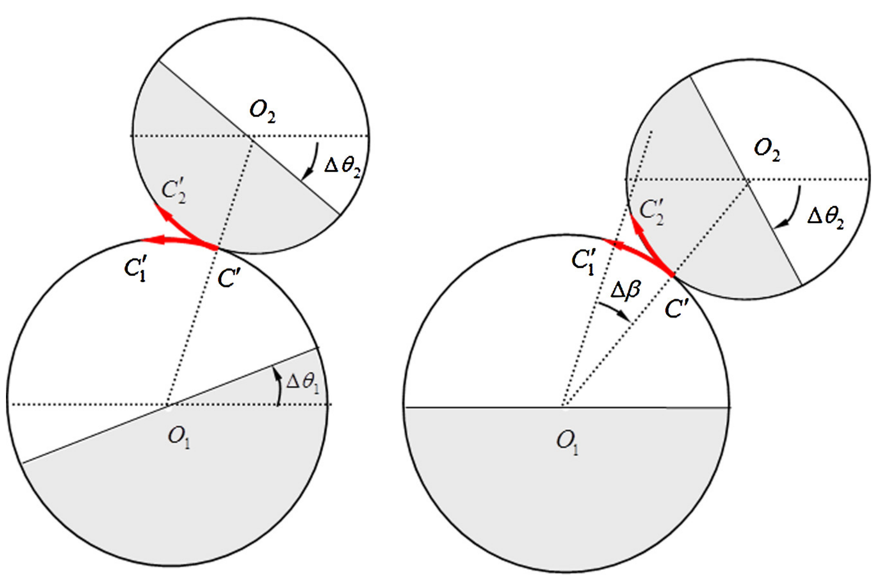

Fig. 3. Pure rolling (left) and pure rolling combined with rigid-body rotation (right).

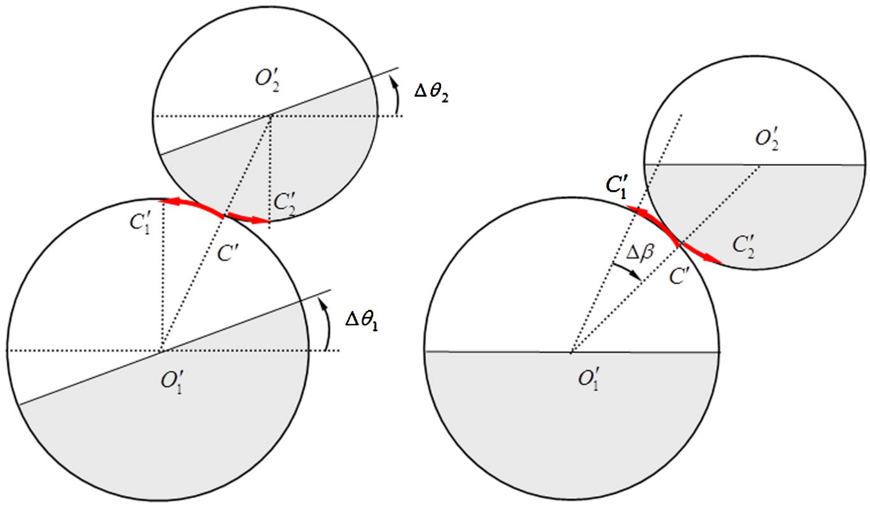

Fig. 4. Pure sliding (left) and pure sliding combined with rigid-body rotation (right).

PS combined with RBR (PS + RBR, Fig. 4, right) is defined as follows:

$$\Delta\beta \neq 0, \tag{6a}$$

$$\Delta\theta_1 = \Delta\theta_2 = 0, \tag{6b}$$

$$\Delta U_s = -(R_1 + R_2)\Delta\beta. \tag{6c}$$

Similarly, PS + RBR can be obtained either from PS followed by a RBR of angle $\Delta\beta$, or directly from the initial state (Fig. 2, left) by moving particle 2 without rotation in the tangential direction over stationary particle 1.

PR and PS contacts are two extreme cases. Most instances of contact fall between the two extremes, and can be decomposed into PR and PS. For RBR, there is no relative translational or rotational motion between two particles, and the RBR should be excluded while decomposing the relative deformation at contact points.

Derivation of rolling and sliding in the general case

Instead of decomposing the arc lengths $C'C_1'$ and $C'C_2'$ into rolling and sliding distances ΔU_r and ΔU_s directly, we decompose the rotations of each particle into a PR and a PS part.

After excluding the RBR component, the particle rotations relative to contact direction $O_1'O_2'$ are $\Delta\theta_1 - \Delta\beta$ and $\Delta\theta_2 - \Delta\beta$. These rotations can be decomposed into two parts: a rotation of λ for both particles contributing to PS, and a rotation of α_1 and α_2 for particles 1 and 2, respectively, contributing to PR. Then the following equations can be used:

$$\alpha_1 + \lambda = \Delta\theta_1 - \Delta\beta, \tag{7a}$$

$$\alpha_2 + \lambda = \Delta\theta_2 - \Delta\beta, \tag{7b}$$

$$-R_2\alpha_2 = R_1\alpha_1. \tag{7c}$$

The solutions of Eqs. (7a)–(7c) are:

$$\alpha_1 = \frac{R_2}{R_1 + R_2}(\Delta\theta_1 - \Delta\theta_2) \tag{8a}$$

$$\alpha_2 = \frac{-R_1}{R_1 + R_2}(\Delta\theta_1 - \Delta\theta_2), \tag{8b}$$

$$\lambda = \frac{R_2\Delta\theta_2 + R_1\Delta\theta_1}{R_1 + R_2} - \Delta\beta \tag{8c}$$

The rolling and sliding distance can be determined by substituting Eqs. (8a)–(8c) into Eqs. (2a)–(2c) and (4a)–(4c):

$$\Delta U_r = \frac{R_1 R_2}{R_1 + R_2}(\Delta\theta_1 - \Delta\theta_2) \tag{9}$$

$$\Delta U_s = R_2\Delta\theta_2 + R_1\Delta\theta_1 - (R_1 + R_2)\Delta\beta. \tag{10}$$

Rolling and sliding velocity can be obtained from the time derivatives:

$$V_r = \frac{R_1 R_2}{R_1 + R_2}(\omega_1 - \omega_2), \tag{11}$$

$$V_s = R_1\omega_1 + R_2\omega_2 + V_t, \tag{12}$$

where ω_1 and ω_2 are the angular velocities of two particles measured in the space-fixed frame, and $V_t = \Delta X_t/\Delta t$ is the relative velocity in the tangential direction caused by the relative translational motion between the two particles. $\Delta X_t = -(R_1 + R_2)\Delta\beta$ is the tangential displacement caused by the relative translational motion during t and $t + \Delta t$. The vector form of V_t can be expressed as:

$$\mathbf{V}_t = [(\mathbf{V}_2 - \mathbf{V}_1)\cdot\hat{\mathbf{t}}]\hat{\mathbf{t}} \approx \mathbf{V}_{21} - \mathbf{X}_{21}\frac{\mathbf{V}_{21}\cdot\mathbf{X}_{21}}{(R_1 + R_2)^2}, \tag{13}$$

where $\mathbf{X}_{21} = \mathbf{X}_2 - \mathbf{X}_1$ and $\mathbf{V}_{21} = \mathbf{V}_2 - \mathbf{V}_1$. $\mathbf{X}_1$ and $\mathbf{X}_2$ are the position vectors, and $\mathbf{V}_1$ and $\mathbf{V}_2$ are the velocity vectors of the two particles. $\hat{\mathbf{t}} = \mathbf{X}_{21}/|\mathbf{X}_{21}| \times \hat{\mathbf{k}}$ is the unit tangent vector at contact point C, where $\hat{\mathbf{k}}$ is the unit vector in the z-direction. The bold letters in Eq. (13) (and in Eqs. (16), (23) and (24)) represent vectors.

It can be seen from Eq. (11) that the rolling velocity depends only on the angular velocities of the two particles. In Eqs. (10) and (12), the first and second terms are the contribution of particle rotations to sliding, while the third term is from the relative translational motion between the two particles. Eqs. (9)–(12) are *objective*, as any common rotation (RBR) of the two particles vanishes in the derivation process. By saying that they are *objective*, we indicate that the rolling and sliding deformations are independent of the reference frame in which the motion of the two particles is measured (Luding, 2008).

Further validations for special cases

Pure rolling and sliding

It can be easily tested whether PR and PS are special cases of Eqs. (9) and (10). For PR, applying $\Delta\beta = 0$ and $R_1\Delta\theta_1 = -R_2\Delta\theta_2$ to Eqs. (9) and (10) yields $\Delta U_s = 0$ and $\Delta U_r = R_1\Delta\theta_1$. For PS, applying $\Delta\beta = 0$ and $\Delta\theta_1 = \Delta\theta_2 = \lambda$ to Eqs. (9) and (10) yields $\Delta U_r = 0$ and $\Delta U_s = (R_1 + R_2)\lambda$. The cases of PR + RBR and PS + RBR can be tested in a similar way.

Disk rolling and sliding on a flat floor

A special case exists in which the radius of particle 1 becomes infinite. Despite the fact that $R_1 \to \infty$ and $\Delta\beta \to 0$, $\Delta X_t = -(R_1 + R_2)\Delta\beta$ still holds, and in this case $\Delta X_t = -(R_1 + R_2)\Delta\beta = -R_1\Delta\beta$ is a finite value. The PR and PS can be discussed on the basis of PS + RBR (Fig. 4, right) and PR + RBR (Fig. 3, right). For PR, according to Eq. (3), we have $\Delta U_r = -R_1\Delta\beta = -R_2(\Delta\theta_2 - \Delta\beta)$, thus:

$$\Delta U_r = -R_2\Delta\theta_2 = \Delta X_t, \quad V_2 = -R_2\omega_2. \tag{14}$$

For PS, $\Delta\theta_2 = 0$ and $\Delta\theta_1 = 0$, and Eq. (6) yields

$$\Delta U_s = -(R_1 + R_2)\Delta\beta = \Delta X_t = V_2\Delta t. \tag{15}$$

Disk rotating without linear displacement

Generally, both rolling and sliding occur, and the rolling and sliding distances are determined using Eqs. (9) and (10), with $\Delta U_r = -R_2\Delta\theta_2$ and $\Delta U_s = R_2\Delta\theta_2 + V_2\Delta t$. One special and interesting case is a disk rotating (clockwise for example, so that $\Delta\theta_2$ is negative) without any linear displacement ($V_2 = 0$). This can be decomposed as rolling a certain distance ($\Delta U_r = -R_2\Delta\theta_2$) and then sliding backwards exactly the same distance ($\Delta U_s = R_2\Delta\theta_2$).

Comparison with earlier derivations

Based on Iwashita and Oda's paper (1998), Bagi and Kuhn (2004) derived the following rolling velocity:

$$V_r = \frac{1}{2}\left[R_1\omega_1 - R_2\omega_2 - \frac{R_2 - R_1}{R_1 + R_2}(\mathbf{V}_1 - \mathbf{V}_2)\cdot\mathbf{t}\right]. \tag{16}$$

Eq. (16) implies that the linear particle velocities contribute directly to the rolling velocity when $R_2 \neq R_1$. But it is fair to say that the formula above is only correct when $R_2 = R_1$ (in which case the third term falls away since $R_2 - R_1 = 0$). The difficulty emerges in the following two cases. First, for PS + RBR (Fig. 4, right), the particles do not rotate, and a zero rolling velocity is expected. However Eq. (16) predicts a non-zero rolling velocity from the linear particle velocities. Second, in the case of a disk sliding on a floor without any rotation, i.e. PS, we have $\Delta\theta_1 = 0$, $V_1 = 0$, $\Delta\theta_2 = 0$ and $V_2 \neq 0$. One would expect a vanishing rolling velocity. This is different from the result obtained from Eq. (16) which predicts a non-zero rolling velocity $V_r = -V_2 \neq 0$.

The reason why Eq. (16) fails when $R_2 \neq R_1$ is readily explained. Iwashita and Oda (1998) wrote: "if *da* equals *db* pure sliding occurred without any particle rotation", where *da* and *db* were the arcs $C'C_1'$ and $C'C_2'$ (or Δa and Δb in this paper). It should be pointed out that in the case of PS, $\Delta a = \Delta b$ is only valid when $R_1 = R_2$; and if $R_1 \neq R_2$, then $\Delta a \neq \Delta b$. Instead, we should use equal angles (Eq. (5)). After applying this equality as an amendment to the work of Iwashita and Oda (1998) and of Bagi and Kuhn (2004), the correct formula Eq. (11) can be obtained as described in detail below.

Iwashita and Oda (1998) used the following equations:

$$\Delta a = R_1(\Delta\theta_1 - \Delta\beta), \tag{17a}$$

$$\Delta b = R_2(\Delta\theta_2 - \Delta\beta), \tag{17b}$$

$$\Delta a = \Delta U_r + \Delta U_s, \tag{18a}$$

$$\Delta b = -\Delta U_r + \Delta U_s. \tag{18b}$$

The solutions to Eqs. (17) and (18) are:

$$\Delta U_r = \frac{\Delta a - \Delta b}{2}, \tag{19a}$$

$$\Delta U_s = \frac{\Delta a + \Delta b}{2}. \tag{19b}$$

Eq. (19a) is equivalent to Eq. (16), if Eqs. (17a) and (17b) are applied. While Eqs. (17a) and (17b) are correct, Eqs. (18a) and (18b) are not, as discussed above. Eqs. (18a) and (18b) should be replaced with the following equations (see also Eq. (5)):

$$\Delta a = \Delta U_r + \Delta U_{s1}, \tag{20a}$$

$$\Delta b = -\Delta U_r + \Delta U_{s2}, \tag{20b}$$

$$\frac{\Delta U_{s1}}{R_1} = \frac{\Delta U_{s2}}{R_2}, \tag{21}$$

$$\Delta U_s = \Delta U_{s1} + \Delta U_{s2}, \tag{22}$$

where ΔU_{s1} and ΔU_{s2} correspond to $R_1\lambda$ and $R_2\lambda$ in Eq. (4c). Solving Eqs. (17), we again derive Eqs. (9) and (10).

We prefer to define the sliding distance as $\Delta U_s = \Delta U_{s1} + \Delta U_{s2}$, rather than as $\Delta U_s = (\Delta U_{s1} + \Delta U_{s2})/2$, which is used by Iwashita and Oda (1998) and Jiang et al. (2005), because the latter leads to $\Delta U_s = V_2\Delta t/2$ in the case of a disk sliding on a flat surface (see Eq. (15)).

Alonso-Marroquin et al. (2006) gives the objective velocities of the two particles at their point of contact in terms of linear and angular velocities:

$$s_1 = R_1\omega_1 - \frac{R_1}{R_1 + R_2}(\mathbf{V}_2 - \mathbf{V}_1)\cdot\mathbf{t}, \tag{23}$$

$$s_2 = -R_2\omega_2 + \frac{R_2}{R_1 + R_2}(\mathbf{V}_2 - \mathbf{V}_1)\cdot\mathbf{t}. \tag{24}$$

In Bagi and Kuhn's papers (Bagi & Kuhn, 2004; Kuhn & Bagi, 2004a,b), the objective velocities of the two particles, given as $d\mathbf{u}^{qc}\cdot\mathbf{t}$ and $d\mathbf{u}^{pc}\cdot\mathbf{t}$, are defined as the *t*-directional translations of the contact point on the two particles. Similarly to Eq. (18), if the rolling displacement or velocity is defined as the average of $d\mathbf{u}^{qc}\cdot\mathbf{t}$ and $d\mathbf{u}^{pc}\cdot\mathbf{t}$, or s_1 and s_2, an incorrect result is obtained from Eq. (16). However, if the concept underlying Eq. (5) is considered, correct results are derived from Eqs. (9)–(12), in a similar way to Eqs. (20)–(22). Note that s_1 and s_2 in Eqs. (23) and (24) are related to Δa and Δb in this paper by $\Delta a = s_1\Delta t$ and $\Delta b = -s_2\Delta t$, respectively, with a different assignment of sign (Alonso-Marroquin et al., 2006).

Tordesillas et al. (Tordesillas & Walsh, 2002; Tordesillas et al., 2005) adopted a simple rolling distance of the following form:

$$U_r = \Delta\theta_1 - \Delta\theta_2. \tag{25}$$

This is correct in the sense that rolling depends only on angular velocities, except for the multiplicative factor found in Eq. (9). Yet this factor did not affect the calculation of rolling resistance in their papers (Tordesillas & Walsh, 2002; Tordesillas et al., 2005), as it was absorbed by the coefficient of rolling resistance in their model.

Jiang et al. (2005) obtained the following rolling and sliding displacements:

$$\Delta U_r = \frac{R_2\Delta a - R_1\Delta b}{R_1 + R_2}, \tag{26}$$

$$\Delta U_s = \frac{\Delta a + \Delta b}{2}. \tag{27}$$

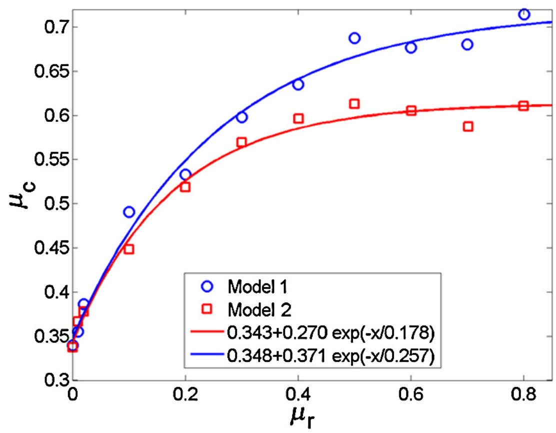

Fig. 5. Macroscopic friction versus coefficient of rolling resistance using models 1 and 2 of rolling deformation.

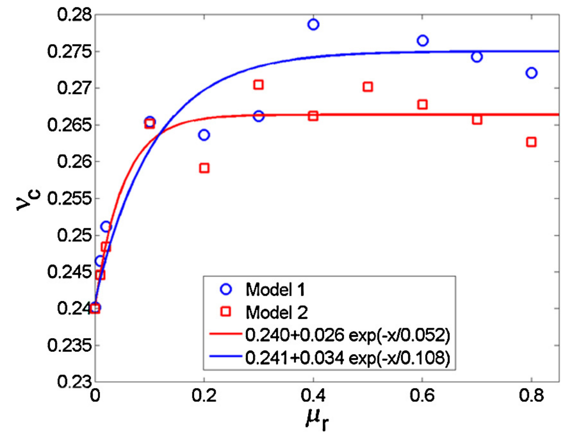

Fig. 6. Residual void ratio versus coefficient of rolling resistance for models 1 and 2 of rolling resistance.

It would be straightforward to show that Eq. (26) is equivalent to Eq. (9). Although the derivations by Jiang et al. are somewhat complicated, their definitions of PR and PS are conceptually correct, except that the factor of 1/2 in Eq. (27) should be removed.

Luding (2008) defined a rolling velocity directly based on the principle of objectivity. Although no further explanation is given, his formula is identical to Eq. (11). Luding's rolling velocity is the only correct and explicit rolling velocity published thus far.

DEM simulations with different rolling models

To compare the different rolling velocity models, we performed biaxial test simulations using a DEM model with circular particles implemented in the SPOLY software (Alonso-Marroquin et al., 2013). The model includes both the contact forces (Alonso-Marroquin et al., 2013) and rolling resistance, which is calculated from:

$$T_r^e = k_r \Delta x_r, \quad k_r \Delta x_r \le \mu_r F_n, \tag{28}$$

where k_r is the rolling resistance, and Δx_r is the rolling displacement which is incremented by $V_r \Delta t$ at each time step Δt whenever $k_r \Delta x_r \le \mu_r F_n$ is satisfied, where μ_r is the rolling coefficient, and F_n is the normal force. The sample consists of 400 circular particles with a uniform distribution of radii between 0.76 and 1.52 mm. In the biaxial tests, the confining pressure in the lateral walls $\sigma_2 = 160$ kN/m is kept constant and the horizontal walls are moved at a constant rate of 10^{-3} mm/s, inducing deviatoric stress.

Now we investigate how the rolling coefficient μ_r affects the macroscopic friction and void ratio. We adopt two rolling deformation models: model 1 from Eq. (11) of this paper and model 2 from the paper by Alonso-Marroquin et al. (2006). Fig. 5 shows the dependency of the macroscopic friction on the coefficient of rolling resistance. We observe the same trend for both models, i.e., a residual friction for small values of rolling resistance is given by the combination of microscopic friction and interlocking. For large values of rolling coefficient, the global friction is not very sensitive to changes in the coefficient of rolling resistance. Note that for low values of the coefficient of rolling ($\mu_r < 0.2$) the macroscopic friction is almost the same for the two models. This is because the macroscopic resistance is governed by microscopic friction and interlocking. On the other hand, if the rolling resistance is large ($\mu_r > 0.2$), model 1 predicts a larger macroscopic friction than model 2. This is because model 2 leads to zero rolling resistance when the objective velocities at the contact point have different signs, whereas in model 1 the rolling velocity is different from zero.

Fig. 6 shows the residual void ratio versus rolling resistance. The initial void ratio in all samples was 0.2, so the samples exhibit dilatancy in all cases. The results are consistent with the previous macroscopic friction results, showing that high macroscopic friction is related to high values of void ratio at the critical state. Model 2 predicts a lower dilatancy, which is ~4% smaller than that of model 1. We could not determine whether or not the correct rolling velocity of model 1 produces more accurate and reasonable results. These simulations, however, at least show that the macroscopic properties (such as strength and void ratio) of the granular materials depend on the rolling velocity models on which the rolling resistance is calculated.

Concluding remarks

In this paper we revisited previous studies on rolling and sliding in 2D discrete element models. We presented the derivations of rolling and sliding in a simple and concise way. First, we developed unique definitions for RBR, and for PR and PS deformation between two particles. Second, based on these simple definitions, the correct rolling and sliding decomposition was obtained by a one-step deduction.

As a core result, we extended the Iwashita-Oda's rolling model by defining PR and PS for two particles of different sizes. This allowed us to resolve inconsistencies in rolling velocity models found in the literature. After correcting the definitions of PR and PS for particles of different sizes, the correct result can be achieved using Iwashita-Oda's and subsequent derivations. This leads us to conclude that there is indeed a unique way to define the rolling displacement, and that rolling and sliding are completely decomposable.

The numerical results show that the strength of granular materials depends on the model for rolling velocity that we use to calculate the rolling resistance. We still require empirical evidence to demonstrate that the corrected formula for rolling resistance better fits experimental data. We are now identifying suitable experimental biaxial tests using different coefficients of rolling resistance that will ultimately decide which rolling velocity model is most suitable for taking into account internal moments in the deformation of granular materials.

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