Problem 7.1.1

assuming $\rho = 1$ we have

$$L(\Theta) = \frac{1}{2\sigma^2} \|y - \Phi\Theta\|^2 + \frac{1}{2b^2} \|\Theta\|^2$$

$$\frac{\partial L}{\partial \Theta} = \frac{\partial}{\partial \Theta} \frac{1}{2\sigma^2} \|y - \Phi\Theta\|^2 + \frac{1}{2b^2} \|\Theta\|^2 = \frac{\partial}{\partial \Theta} \frac{1}{2\sigma^2} (y - \Phi\Theta)^T (y - \Phi\Theta) + \frac{1}{2b^2} \Theta^T\Theta$$

$$= \frac{\partial}{\partial \Theta} \frac{1}{\sigma^2} \left(\frac{1}{2} y^T y - (\Phi y)^T \Theta + \frac{1}{2} \Theta^T \Phi^T \Phi\Theta \right) + \frac{1}{2b^2} \Theta^T\Theta$$

$$\Rightarrow \frac{1}{\sigma^2} \left(-(\Phi y)^T + \cancel{2}\, \Phi^T \Phi \Theta \right) + \frac{1}{b^2} \Theta = 0$$

$$\Rightarrow \frac{1}{\sigma^2} \Phi^T \Phi \Theta + \frac{1}{b^2} \Theta = \frac{1}{\sigma^2} y^T \Phi \Rightarrow \Phi^T \Phi \Theta + \frac{\sigma^2}{b^2} \Theta = y^T \Phi$$

$$\Rightarrow \left[\Phi^T \Phi + \left(\frac{\sigma}{b}\right)^2 I \right] \Theta^* = y^T \Phi$$