Problem 6.3.5 We have == 2-4=> 1171- (4+/c) 11 7/ 1/2112 => 1/2 - (4+/e)(1 7/ 1/2-4) be know that yis the unique solution to the Shicky comes minimsortion problem you min llowfull and as I is a constant >0 som and y, e &M. he have (4+1e) GIN as well. he therefore know that since this is the case, and there is no value smaller for 1/21-41) we have 112 - (y+/e) 11 / 1/2-y11 => 1171- (y+/e)112> 112-4112 1/2- (4+/e)11 - 1/2/12 7/6 => 11x12-2 (x, y+le) # 11y-le11 - 11x-y11270 => 1/2/1-2 (1, 4+le) + 1/4+lel1 - HATI + 2 (2,4) # 1/4/1 7/0 => -2 (x,y)-2 (x,1e) + tty112+2 (y,le)+ || lel12+2(x,y) > tty1170 => -2 (21, 1e) +2 (4, 1e) + | (hell 70 => -2 (n-y, le) + 11/ell270 => -2 (Z, le) + | | xeli 70 => 1 | leli - 21 (Z,e) 70 => 1[/1|e112-2(Z,e)]>0

Problem 6.3.5 From 1. We have 1 [1 ||ell - 2 (Z,e)] >0 => Alleli- 2 (Z. e> 70, taking the case 1=0 we have -2(z.e) 70 => (z.e) A0 Knowing floot Since YEEM, Y-EEM asuell - L(z,-e) Ma 70=> (z,e) MO (Z(e) =0 3. Given nex, yell. ZEM, and know that y= min llg-nll. Since the euclidean norm is strictly conven, we have that the Minimiser is unique for each or. In addition, we have ZEIN and 2=4+2. By Orthogonal Decomposition, we have they is the mileton and bence is ungre. 4. From 3. he have YNEX, Jy GM, ZGIM, and rey+2 : | | 1/1 = | | y + z | 1 = | | y | 2 + | | z | 1 + 2 < y , z). Gran y GM, ZEINT: = 271 (11,m) = 0 Vm EM3, he have that (4,2)=0=> |17/1= 1/4/17+ 1/2/11. Again from 3. we have that yis the projection of 2=> y= Pr 11211 = 11411 + 11211 => 11211 = 11411 + 112-411 => 12tt = 114112 - 2 (xxy> +11411 => 2(x,y) = 21/4/11 => 2(2,P2) = 2/1P/112 => 1P(1)2 = (P2,2)

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Problem 63.6
1. We know that any eGC Subspes
            Le-Peu, a-Peu) 60 for any a
he fist let new and e = ley
=> < ley -Pcx, n-lex> 60 => (a-lex, ley-lex>60
now we let y=u and e= Pcz
=> < Pcx-Pcy, y-Pcy> 60 => < y-Pcy, Pcx-Pcy> 60
Adding the two, we
=> 07 (2- Per - (y-Pey), Pey - Per)
   = < n - y - Pen - Pey, Pey - Pen>
    = - < 2 - y, Pex-Pey> + 11 Pey-Pex112
    ( >1 - y Per - Pey > > 11 Pey - Perll = 11 Pex - Peyll
   11Pcy-Pc2112 is now squaed and it must hargue be nonnegame
=> < Per-Pcy, n-y> 7/1/22-Pcy/270
     11 d 2+(1-d)41 +d(1-d)1121-411
   = ||xx||+2/2x,+(1-x)y>+ ||(1-x)y|+ d(1-x)[||x||-2x,y>+||y||2]
   = 110x11+2(dx, (1-x)y)+11(1-x)y11+d11111-x11111-d(1-x)2(x,y)
     + d(1-d) | | | |
   = 2(dx, (1-d/y) + ||y||2 ((1-d)2+d(1-d)) + 4||71||2-d(1-d)2(2)
   = 2 (xx, (1-x)y) + ||y|2 [1- 2x+2+x-2] + x ||x|| - x (1-x) 2 (2xy)
   = 2d(1-d) (ny) -2d(1/a) (ny) + llyl12 (1-x) +d|1/11
   = 2/12/11 + (1-x) /14/12
```

Problem 6.3.6 3. From 2. ar have 11 drit (1-0) yl td(1-0) 1121-yl = 2/12/11 + (1-0) 1/4/11 if we let d= 2, 21 = T21-Ty, y = 21-y, we have 11 2 (Tx - Ty) - (2-4) 112-211 (Tx - Ty) - (2-4)11 = 211 Tx-Ty/12-112-112-112 => 1/2(Tx-Ty) - (2-y)11=2/17x-Ty/1-1/2-1/12-1/1-1/2)1+2/1(Tx-Ty)-(x-y)11 11122-12412=> 112T2-2-2Ty+411= 11(2Tg2-2Ty)-(2-y)11 = 112(Tg2-Ty) - (21-4)11 => 21172-Tyl2-1121-41 + 211(Tx-Ty)-(2-4)11 by the enghession above => 21172-Tyll-1121-41172-2-7y+y11 30 21172- [4]1- 112-4/11 + 211 (T-I)2 -4 67-4/ (T-I)411 = 2/172-74/12-112-4/12+2/1 (I-T)2-(I-T)4/1

Publim 6.3.6

4 a b

We have $||T_{1}-T_{1}||^{2}+||(T-7)_{1}-(T-7)_{1}||^{2}\leq||T_{1}-y||^{2}$ => $2||T_{1}-T_{2}||^{2}+2||(T-7)_{1}-||T-7)_{1}||^{2}\leq|T_{1}-y||^{2}$ => $2||T_{1}-T_{2}||^{2}-||T_{1}-y||^{2}+2||(T-7)_{2}-(T-7)_{2}|\leq||T_{1}-y||^{2}$ From 3. LHS = $||(2T-T)_{1}-(2T-T)_{2}||^{2}\leq||T_{1}-y||^{2}$ => $||(2T-T)_{1}-(2T-T)_{2}||^{2}\leq||T_{1}-y||^{2}$ => $||(2T-T)_{1}-(2T-T)_{2}||\leq||T_{1}-y||^{2}$ we have $||T_{1}-T_{1}||\leq||T_{1}-y||$ we have $||T_{1}-T_{1}||\leq||T_{1}-y||$

Problem 6.3.6 a. 6600 ne have 2T-I is non-engansme Tis firmly hon - engansie => |(T2-Tyll+ |1 (I-T)21-(I-T)41 \le |121-411 => ||Tn-Tyle = ||n-yle - || (I-T)n - (I-T)y||2 => 1172-Tyll = 1121-Trell + 2 < 21-72, y-Ty) Tolly-Tyll => 1/72-Tyli < 1/2/2 /2/2 + Hyli - HATT + 2 < 2, Ta> -11 TAIL + 2 < 2-Ta, y-Ty> - Hyli+ 2 < 1/4 y , Ty > - 11 Ty 11' => 11T2-Tyll < 2 < 2, T2) - 2 < 2, y> +2 < 2, T2) +2 < y, Ty) - 11Tyll - 11 => 11 Tri-Tyll < 2 < 2 < 2, Tri) - 2 (51, y) + 2 (51,y) - 2 (51, Ty) - 2 (y, Tri) + 2 (Tx, Ty) + 2 (y, Ty) - 11 Tx/1 - 11 Ty/12 => 11Tx-Tyli <2 <x, Tx> - 2 <x, Ty> - 2 <y, Tx> +2 <Tx, Ty> +2 <y, Ty> - 11 Tali - 11 Tyli => 11 Tx-Tyli < 2 < 2, Tx> - 2 < x, Ty> - 2 < y, Tx > + K Tx, Ty> + 2 < y, Ty> a-11 Tre-Tyl =>12/172-Tyli < 2 (2, Tx) - 2 (2, Ty) - 2 (y, Tx) + 2 (y, Ty) => 11 Tri-Tyli 4 (x, Tri) - (y, Ta) - (2, Ty) + (y, Ty) => 11T2-Tyli < (T2-Ty, 2-4) As the norm squared is non-negative => (T2-Ty, 1-4) > 11 T2-Tyli2 > 0