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PRECISE AUTOMATIC DIFFERENTIAL STELLAR PHOTOMETRY

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ABSTRACT

We review the factors limiting the precision of differential stellar photometry. Errors due to variable atmospheric extinction can be reduced to below 0.001 mag at good sites by utilizing the speed of robotic telescopes. Existing photometric systems produce aliasing errors, which are several millimagnitudes in general but may be reduced to about a millimagnitude in special circumstances. Conventional differential photometry neglects several other important effects, which we discuss in detail. If all of these are properly handled, it appears possible to do differential photometry of variable stars with an overall precision of 0.001 mag with ground-based robotic telescopes.

Key words: photometry–extinction–automation

1. Introduction

Differential photoelectric stellar photometry has been used for over three-quarters of a century (Stebbins 1910; Guthnick & Prager 1914) to measure variations in light as a function of time for many variable stars. Johnson & Morgan 1953 defined the broad-band *UBV* system, which remains the most widely used system for observations of variable stars. Kron & Gordon 1957 gave much practical advice on this observational technique. The methods of reduction to this system presented by Hardie 1962 are still widely used today. Hardie's approach, with somewhat simplified notation, is discussed by Hall & Genet 1988.

With reasonable care in selecting comparison stars, observations made at a good site yield a precision, including night-to-night variations, of about 0.01 mag when reduced with mean extinction coefficients. Precision is rarely better than 0.005 mag and is often poorer. Such precision is adequate for many studies but is inadequate for studying general microvariability (Frandsen, Dreyer & Kjeldsen 1989); nonlinear dynamics of pulsating white dwarfs (Auvergne & Baglin 1986); activity in solar-type stars (Lockwood & Skiff 1988); determining limb darkening and reliable values of radii, colors, etc. in eclipsing binaries (Popper 1984); searching for planetary and brown-dwarf companions of solar-type stars (Borucki & Summers 1984); and many other problems. What significantly increased precision could yield is unpredictable until it is achieved and used systematically to explore the new territory previously hidden by measurement errors. It is safe, however, to predict that important new insights will come from significantly improved precision.

In this paper we combine two strategies. The first is the analysis, by Young and others over the past quarter of a century, of the sources of errors in photometry and the consequent recommendations to improve photometric precision. The second is the development, by Boyd, Genet, and others, of robotic telescopes and observatories. These two approaches are complementary for, as we shall show, several of the key methods for improving photometric precision require automation to be fully effective in practice. While some of the means of improving precision could have been adopted in manual photometry, actual implementation seemed too difficult for most practitioners. With robotic telescopes, however, there are fewer practical barriers to adopting these recommendations. In fact, it would waste the new capabilities of robotic telescopes not to follow these precepts.

With robotic telescopes, humans have to develop the equipment and software to do precise photometry only once; when a system is operating, no additional effort is required to continue doing precise photometry, no matter how sophisticated the observing routine has become.

While much of this paper is applicable to manually operated telescopes, we emphasize methods that produce high precision by taking advantage of automated telescopes. Seven of the authors convened a year ago to determine how the two strategies could be combined, and this paper records the result of this workshop, with additions by the other four authors.

Although we concentrate on the improvement of standard techniques, we shall touch on some recent developments that promise to increase the precision of photometric measurements. We also examine in detail some topics that have not been previously considered in the astronomical photometric literature; for well-discussed topics, we refer to Young's 1974 review.

Actions that have been recommended to improve photometric precision include: (1) controlling the temperatures of photomultipliers (Young 1963) and filters (Young 1967a, 1974); (2) using telescope baffles to avoid stray light and vignetting (Young 1967b); (3) using optimal weighting functions (e.g., use of upper as well as lower discriminator levels in pulse counting) in signal detection to improve signal-to-noise ratio (S/N) (Young 1969b); (4) optimally partitioning the observing time among different types of measurements such as star and sky integrations, or low and high air-mass extinction observations (Sedmak 1973; Young 1974; Belvedere & Paterno 1976; Claudius & Florentin-Nielsen 1981); (5) choosing the best air-mass range to minimize the effects of scintillation and other noise sources (Young 1974); (6) using physically realistic and numerically accurate reduction models (Rufener 1964; Young 1974; Manfroid & Heck 1984); (7) determining time-dependent extinction (Nikonov & Nikonov 1952; Nikonov 1953, 1976; Rufener 1964); (8) using a correct treatment of bandwidth effects (King 1952; Rufener 1964; Young 1988); and (9) designing the passbands to avoid aliasing errors (Young 1974, 1988).

The initial robotic telescopes developed by the Fairborn Observatory (Boyd, Genet & Hall 1984) were originally operated at Fairborn, Ohio and Phoenix, Arizona before being relocated to the Smithsonian Fred L. Whipple Observatory on Mount Hopkins in southern Arizona (Baliunas et al. 1985). These small (25-cm) aperture telescopes could produce a high volume of fully automated differential photometry with a typical precision of about 0.01 mag (10 millimag) (Hall, Kirkpatrick & Seufert 1986). The precision of these early robotic telescopes was limited by their small size, short total integration times (10 sec), inaccuracies in their centering and tracking, lack of temperature control of their detectors, and an inability to schedule extinction observations along with the program differential observations. A newer, somewhat larger 40-cm robotic telescope began operation at Mount Hopkins in late 1987 (Hall 1989). Subject to the same limitations as the earlier 25-cm telescopes, it nonetheless succeeded in producing photometry with external errors as small as a few millimagnitudes (Hall, Henry & Sowell 1990).

The more recent robotic telescopes are larger still (75 cm) and so further reduce the effects of scintillation, use CCD cameras to provide much more rapid and accurate centering, and are fitted with temperature control to eliminate the effects of changing ambient temperature on the detectors. Furthermore, the highly versatile Automatic Telescope Instruction Set (ATIS) used on these latest robotic telescopes allows one to automatically schedule extinction observations along with the program differential observations (Boyd, Genet & Hayes 1989; Genet & Hayes 1989). While the full potential of these newer telescopes to do high-precision photometry has not yet been fully explored, preliminary results suggest it will be possible to reach a precision of 1 or 2 millimag. A more complete discussion of the performance of these robotic telescopes is found in Section 5 of this paper. The speed and efficiency of robotic telescopes, coupled with the capability of automatically scheduling and observing extinction stars along with the program differential observations, resolve a long-standing controversy in differential photometry—whether there is enough time to observe both program stars and extinction stars.

2. Measuring Extinction

The exact scheduling of extinction and other control observations throughout the night, as part of a differential photometric program, and the reduction of these observations along with the differential ones has always been possible in principle but only rarely followed in practice because it is inconvenient. Stebbins & Whitford's 1945 statement that "it is impractical to determine the extinction thoroughly and accomplish anything else" reflects the view of many (but not all) stellar photometrists. Most differential photometrists have been content to use "mean" extinction coefficients, either actually measured on occasion and applied to a season or just estimated and used at all times (Hall & Genet 1988). Others, like Nikonov 1953, 1976 and Rufener 1964, 1986 have, however, insisted on careful extinction measurements.

Accurate correction for extinction offers substantial advantages to high-precision photometry. But the significant differences in extinction coefficients of the program and comparison stars, due to differences in their spectral energy distributions, must be treated carefully (see Sections 3.3 and 3.4).

The extinction of a star of color C at air mass M is approximately (cf. Young 1974, p. 157)

$$M[A - WR(C + RM/2)] = AM - WRMC - W(RM)^2/2 \ ,$$

where A is the extinction coefficient at the passband centroid, R is a measure of atmospheric reddening, and W is proportional to the square of the optical bandwidth. Usually C is a color index and R is the difference of extinction coefficients in the corresponding bands. AM is the first-order extinction; the W terms are the "color" or second-order terms; and the term quadratic in M is usually called the Forbes effect. The magnitudes of the terms are discussed by King 1952 and Young 1974 (p. 160); W is about 0.26 for B and 0.03 for V, if the B and V extinctions and $(B-V)$ color index are used.

While A and R are the same for all stars, M and C are generally different. Thus, while the first-order extinction differences between two stars depend only on their difference in air mass, and so can be made small by choosing nearby comparison stars, the second-order differences depend on the whole air mass, multiplied by the difference in color of the stars. These color terms, which are often several hundredths of a magnitude, enjoy no cancellation in differential photometry and can be a major source of error.

3. Factors Limiting the Precision of Broad-Band Differential Photometry

In this section we enumerate and discuss the limits to precision of broad-band differential photometry. We take "broad band" to mean any band within which the extinction coefficient varies at the millimagnitude level. As our goal is to achieve a reliable precision of at worst 3 millimags and as close to 1 millimag as possible, we must consider second- or third-order effects that may be trivial in some applications but are of prime importance toward our 1 millimag goal.

A precision of 1 millimag requires each contribution to the error budget to be substantially less than a millimagnitude. Random errors, such as scintillation and photon noise, which are truly uncorrelated between observations, can be added quadratically in the error budget. However, systematic errors that affect program and comparison stars differently, such as differences in extinction coefficients due to differences in spectral distributions, do not cancel in differential photometry and must be measured carefully. As any residual systematic error is not reduced by averaging, we must add these linearly in the error budget.

The error budget is complicated by some systematic errors, such as changes in the extinction coefficient, that vary slowly with time. These are unimportant if we are interested only in very rapid stellar variations, such as the rapid pulsations in Ap stars, or the "5-minute" oscillations in solar-type stars. They enter with full force in studies of stellar rotation and spottedness and appear with nearly full force in the study of eclipsing binaries (see, e.g., Popper & Etzel 1981). For such studies we must keep these errors in the budget.

3.1 *Photon and Scintillation Noise*

First, enough photons must be detected to keep the statistical counting (Poisson) noise (see, e.g., Parratt 1961) below 1 millimag. A Poisson noise of 1 millimag corresponds to about a million counted photons. Vega (magnitude 0.04) provides about 10^{10} photons sec^{-1} m^{-1} above the Earth's atmosphere in the *B* or *V* passband (cf. pp. 15, 197, and 241 of Allen 1973). Allowing for typical detective quantum efficiencies of 0.1 in photomultipliers, we should detect a million photons from Vega—enough to achieve a RMS noise level of a millimagnitude—in 4 millisec with a 75-cm telescope (of course pulse-counting systems would have difficulties at such high count rates). Alternatively, we could get our million detected photons in 4 sec from a 7.5-mag star with the same telescope.

Actually, we need to keep the photon noise below a millimagnitude, to leave room in the budget for other errors. To reduce the photon noise to half a millimagnitude, we must detect four million photons, not just one million. This requires 16 seconds of integration for the 7.5-mag star with a 75-cm telescope. For reasons explained below, individual integrations may need to be shorter than this; 5- or 10-sec integrations are common. Then we must combine several short integrations to keep the total noise low; this limits the speed of stellar fluctuations that we can measure at the millimagnitude level. A 75-cm telescope is limited by millimagnitude photon noise near 8th magnitude with 10-sec integrations; see Figure 1. Reaching fainter stars will take longer integrations or larger apertures if 1 millimag accuracy is to be retained; see Table 1.

The RMS fluctuation due to scintillation is about 1 millimag at the zenith for a 10-sec integration made with a telescope of about 0.75-m aperture at sea level (see Table I, p. 102, of Young 1974). At the height of the robotic 75-cm telescope on Mount Hopkins (2380 m), the scintillation is about 0.75 as strong, so we need only integrate about 20 sec to reduce the RMS scintillation to 0.0005 mag. At 2 air masses, the scintillation is 3 or 4 times larger, so we would have to integrate about 200 sec. Unless we go to larger telescopes, millimagnitude photometry at 2 air masses is quite difficult.

The required integration time decreases only with the 4/3 power of the aperture because of the Kolmogorov spectrum of atmospheric turbulence (Young 1969a, 1974). As the cost of telescopes increases nearly as the cube of the aperture, this method of reducing scintillation quickly becomes expensive. Alternatively, one can use multiple telescopes, appropriately spaced. However, this introduces the problem of transforming measurements made with different instruments to a common system with high accuracy, not just high precision; Young 1974,

TABLE 1

Integration Times (Seconds) Required for 0.0005-Magnitude RMS Photon Noise. Values Greater Than 500 Sec Are Omitted

Telescope Aperture	B Magnitude 6	7	8	9	10	11	12
25 cm	36	90	200	500	-	-	-
75 cm	4	10	25	60	160	400	-
1.5 m	1	2.5	6	16	40	100	250
2.5 m	0.4	0.9	2	5	13	36	90

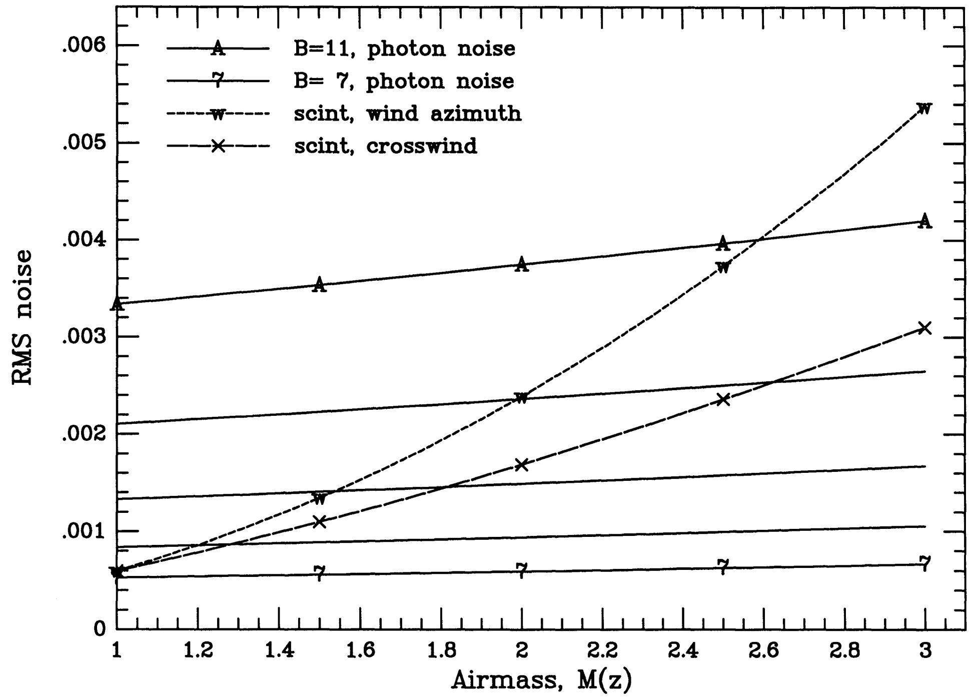

FIG. 1–RMS photon and scintillation noises in magnitudes as functions of air mass. Curves show typical values expected for 10-sec integrations with the 75-cm telescopes at Mount Hopkins and may be off in particular instances by factors of the order of 2. The solid lines show photon noise for each integer *B* magnitude from 7 (bottom contour) to 11 (top contour). The dashed lines show typical scintillation noise looking along (short dashes) or across (long dashes) the upper-atmosphere wind vector.

1988 has suggested this can be done, but not with existing photometric systems. Multiple robotic telescopes are already under serious consideration for other reasons (Genet & Boyd 1990).

Although both photon and scintillation noises are almost exactly white, neither is exactly Gaussian in distribution. Some of the errors discussed below are not even white, but have $1/f$ spectra. These properties make simple averaging and least-squares techniques suboptimal. One must be cautious in using either long integrations or the average of many short ones. Stigler 1977 points out that systematic errors are typically about half as big as the random ones. "Even the greatest scientists, exercising every ounce of their ingenuity, are unable to eliminate all bias", he says. We have no room for an extensive statistical discussion but must point out that careful statistical treatment of data, rather than blind application of least squares, is called for.

3.2 *First-Order Extinction*

The first-order extinction difference between program and comparison stars is the extinction coefficient A times their difference in air mass. The error in correcting for this is mostly due to the uncertainty in the extinction coefficient, multiplied by the difference in air mass. The air-mass difference is nearly

$$\tan z \sec z \, dz \quad ,$$

if the difference in zenith distances z is in radians. The multiplier is only 1.4 at 45 degrees; stars a degree apart on the sky at 45-degrees zenith distance differ by, at most, 0.022 air mass. Scintillation prevents us from going much further from the zenith. To keep the differential first-order extinction error down to a millimagnitude, we must know the first-order extinction coefficient to about (0.001 mag)/(0.022 air mass), or 0.045 mag/air mass. Such precision is easily reached: If the scintillation noise is 3 millimag (for a 10-sec integration at the Mount Hopkins 75-cm) at 2 air masses, combining a zenith observation with one at 2 air masses gives an extinction measurement better than 0.004 mag/air mass, far better than we need. Of course, this near-cancellation of the first-order extinction terms is the reason for doing differential photometry.

Thus, ignoring changes in the extinction with time (discussed below), we can measure the extinction about ten times better than needed to keep the first-order errors below a millimagnitude for stars a degree apart. This means that, if we measure the extinction frequently enough to measure its time variations exactly, we can use comparison stars up to 10 degrees away from the program star. In fact, there will be some residual error in the extinction coefficient interpolated to the time of an observation; but, if we can be sure the extinction coefficient is in error by no more than 0.01 mag/air mass, we can still use comparison stars 4.5 degrees from the program star.

However, we shall show below that it may be very difficult to find suitable comparison stars because of second-order (color) effects and the difficulty of finding comparison stars not variable at the 1-millimag level. We may be forced to look still further away for a good reference object. If so, we can expand the search area for candidates in some cases. For example, if we work only near the zenith, a much larger area is available without exceeding the 0.022 air-mass limit. We must then pay a price in terms of temporal coverage. If we observe only stars that pass near the zenith from a low-latitude observatory, we can extend the search area in declination without giving up time coverage. Another approach would be to pick stars within the 4.5-degree limit in altitude when the star is either east or west and relax the limit in azimuth (Milone 1967); this restricts operations to one side of the meridian.

Rufener 1986 has shown that, even at a site as good as La Silla, variations in the extinction coefficient from one night to another are frequently as large as 0.03 mag/air mass; Lockwood and Thompson 1986 found similar results at Lowell. Thus, at really good sites, mean extinction may be good enough (but just barely) to remove first-order differential extinction to better than a millimagnitude if a good comparison star can be found within a degree of the program star. At mediocre sites, mean extinction may be in error at any given time by 0.1 mag/air mass or more, and this will contribute a few millimagnitudes of error, varying from night to night. But, as the extinction is easily measured more precisely, even at poor sites, the first-order effects due to difference in air mass can be made much smaller. Thus, for reasonably close pairs measured near the zenith, this error can be made negligible.

3.3 *Second-Order Extinction*

The second-order differential extinction correction is the difference in extinction coefficients of the two stars, due to their different spectral energy distributions, multiplied by the whole air mass, or approximately

$$(WRdC)M \quad .$$

In this respect, differential photometry has no advantage over all-sky photometry. The effective extinction coefficient for each star is an average across the instrumental passband, weighted by the product of the instrumental spectral response and the spectral distribution of the star. Cousins & Jones 1976 give a clear explanation and discussion in terms of effective wavelengths, and the situation is analyzed mathematically by King 1952 and Young 1988.

The effective extinction coefficient for a star is approximately the mean of the monochromatic values at two wavelengths separated by an effective band width, weighted by the (smoothed) stellar intensities at these wavelengths. Because both the weights and the intensities differ at the two wavelengths by amounts proportional to the wavelength separation (i.e., to the bandwidth), the second-order effect is proportional to the bandwidth squared. Because the monochromatic extinction decreases monotonically with wavelength in the visible, redder stars weight the longer wavelengths more heavily and have smaller effective extinction coefficients—an effect noticed by Guthnick & Prager 1914.

Figure 2 compares the photon spectral densities of a blue (O5) and a red (K1 III) star with the *B* response function. The products of the *B* response with the photon-flux spectra of these stars are shown in Figure 3 together with typical monochromatic extinction at Mount Hopkins. As the monochromatic extinction varies by about a factor of 3 across the *B* passband, the effective extinction is considerably less for red than for blue stars. Because the difference in extinction coefficients is multiplied by the whole air mass, we must know this difference very accurately if we are to correct the differential measures to 1 millimag.

The difference of extinction coefficients is correlated with the difference in color of the two stars; this is the basis of the standard method for correcting for these effects. More precisely, the theory (Young 1988) shows that it depends on the gradient of the stellar spectrum, smoothed by the instrumental response function. A color index is only an estimate of this gradient.

The theory shows that the appropriate color index to use is the mean of the colors at the top and bottom of the Earth's atmosphere (i.e., it is the color in the middle of the atmosphere), because half the atmospheric reddening (RM) is added to the true color (C) of the star. Furthermore, this mean color is multiplied by WR, so the second-order or "color" term in the extinction depends on the actual atmospheric reddening.

Because the second-order effect depends directly on the reddening power, R, of the Earth's atmosphere—and especially because observers often do differential photometry on nights of inferior transparency with relatively high and variable reddening—we can expect changes from night to night, even at fixed air mass (near transit). If observers measure the time-dependent extinction, and use it to determine the "color term" in the extinction, this additional complication can be measured and removed. Unfortunately, Hardie 1962 neglected the dependence of the second-order extinction on the color of the atmosphere (King 1952; Young 1974, 1988) and used a cruder second-order term depending only on the color of the star.

To illustrate the size of the errors, suppose we observe two stars differing in $(B-V)$ color by 0.3 mag and neglect the aliasing due to undersampling (see below) in the *UBV* system. The *B* extinction coefficient is typically about 0.1 mag/air mass larger than that in *V*. Using values given by Young 1974 (p. 160), we see the difference in *B* extinction coefficients of the two stars is 7 or 8 millimag/air mass.

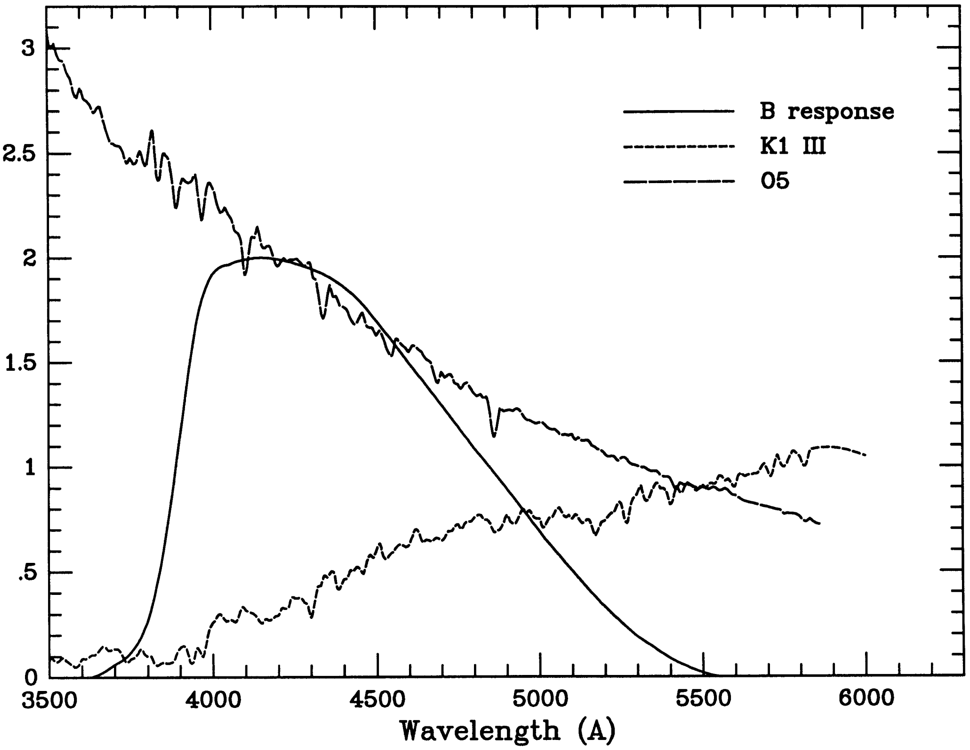

FIG. 2–The *B* response function (solid line), with spectral energy distributions for Kl III (short dashes) and O5 (long dashes) stars having similar *V* magnitudes. The vertical scale is in arbitrary units.

This part of the error can be removed by accurately measuring the extinction color term.

If we now observe on a night with aerosol extinction increased by 0.1 mag/air mass, the increase in atmospheric $(B-V)$ reddening will be about 0.02 mag/air mass, and the difference in B extinction coefficients of the two stars will increase by nearly 2 millimag/air mass. This error cannot be removed if Hardie's 1962 method is used; it will appear as a systematic shift in the differential measures by 2 millimag between the two nights, assuming an average air mass of 1.3 on both nights. As the effect is systematic, it cannot be reduced by averaging. Note that these effects arise solely from the difference in extinction coefficients of the two stars; we assume both stars have identical air masses. The effects in U are much larger; in V they are about 8 times smaller.

Although it is widely supposed that extinction observations of red and blue stars used to determine the color term must be made in pairs, this is required only if the observations are made on a single night in which the extinction varies so much that the color term in the extinction cannot be distinguished from its time dependence.

As a practical matter, the time dependence of the extinction (discussed below) is rather slow. Thus, if the colors of the extinction stars are not strongly correlated with time—that is, so long as we do not observe mostly red stars at one end of the night and blue ones at the other but mix them up fairly well—the color and time dependences of the extinction can easily be separated, provided that we solve for both effects. The separation is still better if we combine observations from several nights (Manfroid & Heck 1984), for we cannot expect the time dependence of the extinction to repeat in its details night after night.

Thus, provided only that extinction stars are well distributed in color, the color term of the extinction can be separated from time-dependent effects in a multinight solution, without making any special observational effort (such as observing red and blue stars in pairs). Even if we devote a small portion of the total observing time to extinction stars, we can beat the extinction to death with an automatic telescope and still have the great majority of our telescope time devoted to program observations.

3.4 *Aliasing Errors*

Unfortunately, we cannot determine extinction coefficients for individual stars very well. To calculate them theoretically requires knowledge of the spectral energy distributions of the stars, the instrumental spectral response functions, and the run of atmospheric monochromatic extinction, all with high accuracy. As Cousins 1969 points out, it is "impossible to make a rigorous reduction to outside the atmosphere without more information than that contained in $U-B$ and $B-V$", and Cousins & Jones 1978 show that "no equation involving $B-V$ and $U-B$ only will predict the extinction correctly for all luminosity types and different degrees of reddening. . . . Without

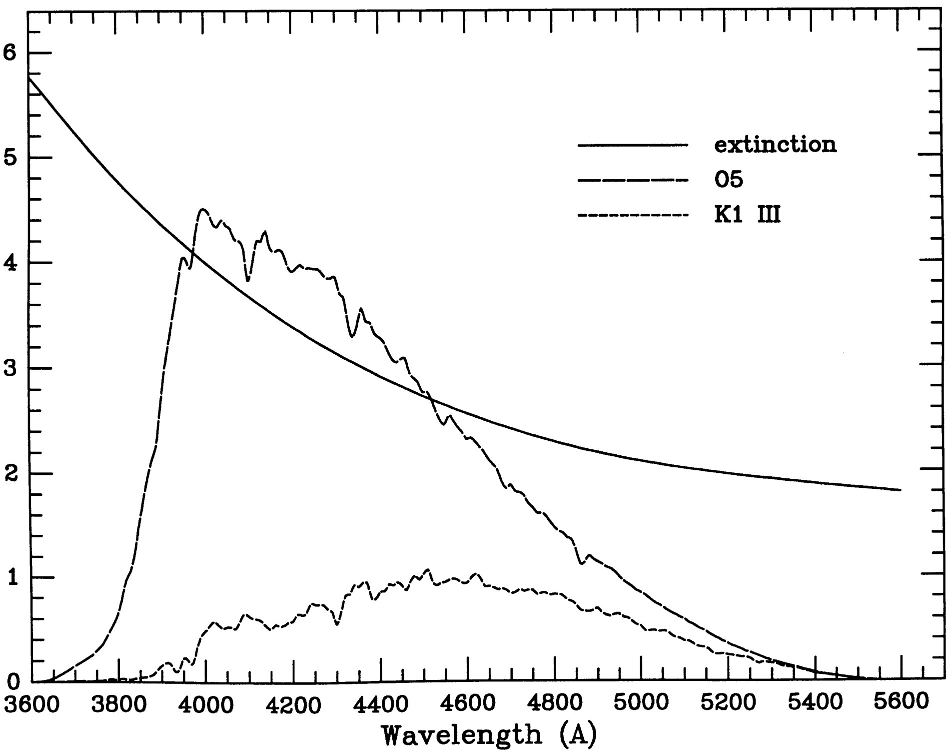

FIG. 3–Typical monochromatic extinction for Mount Hopkins in tenths of a magnitude per air mass (solid curve) with the products of the B response and stellar energy distributions from Figure 2. The O5 star contributes photons mainly around 4200 Å, while those of the K giant are closer to 4600 Å; thus, the effective extinction is smaller for the K star.

more information, direct or inferred, no rigorous color correction is possible either for extinction or for color transformation". However, Cousins & Jones 1976 suggest that the additional information can be obtained from very careful measurements of the instrumental response functions and stellar energy distributions. Whether the measurements can be made accurately enough to reach a precision of 1 millimag remains to be shown.

To measure extinction coefficients experimentally requires a way to measure the effective gradients of the smoothed stellar energy distribution and the extinction within each instrumental band (Young 1988). These gradients can be measured accurately, but only with passbands that overlap enough to satisfy the sampling theorem (Young 1974, 1988). This basic theorem of modern optics and information theory was described 40 years ago by Shannon 1949 as "a fact which is common knowledge in the communication art", but as few astrophysicists are familiar with it, a brief explanation seems appropriate here; see Bracewell 1965 for details.

The theorem shows that a function whose Fourier transform is nonzero over only a finite range can be reconstructed exactly from appropriately spaced discrete function values, called "samples"; Shannon 1949 gives a lucid proof. If we regard the samples as evenly spaced, their spacing must not exceed half the reciprocal of the cutoff frequency in transform space. We can regard the samples as providing just enough equations of condition to determine the coefficients of the sine and cosine terms in a Fourier series. If the function is "band limited", the series represents it exactly, so we can calculate the values of the function (and its derivatives) everywhere in the range of interest. Thus, the series provides exact interpolation between the samples. But if we have fewer samples, there is no way, even in principle, to determine the behavior of the function between them.

The astrophysical function we wish to determine is a low-resolution stellar energy distribution. Convolving the true stellar spectrum with a spectral resolution element (e.g., a filter passband) truncates its Fourier transform by multiplication with the transform of the filter profile, making the result band limited; thus, the sampling theorem applies. The smoother the profile, the faster its transform dies out, and the farther apart the sampling points can be. If we sample the smoothed (i.e., low-pass-filtered) stellar spectrum as prescribed by the sampling theorem, we have complete information about the stellar spectrum at the given resolution.

In particular, we know exactly (apart from experimental errors) the gradients needed to transform the spectrum observed inside the atmosphere to the system of the same instrument outside the atmosphere. Indeed, we can also transform our measurements to the system of any other instrument with the same spectral resolution, even if its passbands (sampling points) are placed completely differently from those of the instrument we used, because our samples determine the low-resolution stellar spectrum exactly. This will allow data from two or more simultaneously operated photometers to be combined, with no loss in precision or accuracy.

Young 1988 shows that stellar spectra smeared by convolution with a simple filter passband are sampled well enough to remove atmospheric extinction to better than a millimagnitude if the samples (passbands) are spaced apart about one half-width at half-peak response. Thus, critical sampling requires overlapping the passbands so that each is centered about halfway down the sides of its neighbors. The color index between the first and third bands in such a set estimates the gradient at the center of the second band very accurately, and so on through the set.

However, existing photometric systems have bands that overlap little, if at all, so the color indices measured between passbands are only crude estimators of the local gradients within individual bands. Because these systems are undersampled, local spectral features that affect the local gradients cannot be distinguished from distant features that have no influence on the effective extinction; such confusion is called "aliasing".

For example, the usual technique of correlating the extinction coefficient with the $(B-V)$ color of each star is barely good to 0.01 mag/air mass. That is, two stars with identical $(B-V)$ can have B extinction coefficients that differ by 0.01 mag/air mass (cf. Fig. 14, p. 158 of Young 1974; see also Fig. 2b of Cousins & Jones 1976). Because the UBV bands violate the sampling theorem, the $(B-V)$ color index is a poor estimator of the required derivatives (Young 1988).

Young 1974 and Cousins & Jones 1976 studied only a few stars, nearly all unreddened and of normal metallicity. Thus, there may be stars with similar colors but even larger differences of extinction coefficient. The differences seem to be correlated with luminosity class and probably depend on metallicity, microturbulence, and other factors that affect the strength of spectral features. A large effect due to interstellar reddening is to be expected as well. None of these effects has been investigated; a detailed study of this problem is needed.

We shall assume a range of extinction coefficients of 0.01 mag/air mass in B at a given color and a mean-square deviation of individual stars from the mean relation between extinction coefficient and color of 0.002 mag/air mass for the discussion below. As the situation is very much worse in U, we must abandon this band if we need millimagnitude accuracy.

Thus, if two stars of the same color differ by 0.01 mag/air mass in extinction coefficient, their difference in magnitude changes by 1 millimag with each 0.1 change in air mass. If the difference is only 0.002 mag/air mass, we can extend the observations over a somewhat larger range

(0.5 air mass) before this systematic error accumulates to a millimagnitude. If we are trying to keep the total error below 0.001 mag, the budget will permit only much smaller ranges of air mass. This also ignores further errors due to changes in atmospheric reddening from night to night.

If we use one such star as the "comparison star" for the other, then, even though the two have identical colors and are observed at identical air masses, the "program star" will appear to change by a millimagnitude for every change of 0.1 in air mass. If we observe this pair of stars for several hours, we can expect changes of several tenths of an air mass and, consequently, systematic changes in their magnitude difference of several millimagnitudes. The RMS error on one night is the difference of extinction coefficients, multiplied by the RMS spread in air mass of the observations, as explained by Young 1974 (p. 170).

Hence, one cannot do much better in B than the numbers mentioned above, unless one can pick a comparison star that has nearly the same effective extinction coefficient as the program star. The Chappuis ozone band diminishes the variation of extinction across V, making such problems about eight times smaller there.

Thus, conventional differential photometry makes two assumptions that are adequate for a precision of about 10 millimag but introduce significant error if one needs much higher precision. One assumption is that the differences in energy distributions of the variable and comparison stars within the observed (broad) passband can be accounted for by a single parameter, the color index (such as $(B-V)$ for the B passband). Young 1988 has shown this to be inadequate, unless the color index is based on bands that strongly overlap the band in question, which B and V fail to do. (We note that the B1 and B2 bands of the Geneva system were designed to provide this information for its B band, just as the V1 and G bands were designed to provide a gradient for the Geneva V. However, these subsidiary bands are narrower than the B and V bands and, thus, introduce additional undersampled components of the stellar spectra. Nevertheless, the high accuracy of the Geneva photometry is undoubtedly due, in part, to this pioneering attempt to measure gradients within the broad bands of the system.)

The other assumption is that the atmospheric extinction variation from night to night at most affects only the first-order extinction coefficient (k' in Hardie's notation) and that the color dependence of extinction (k'') does not vary from night to night, let alone within a night. This model fails to take account of the increase in atmospheric reddening with increased aerosol extinction. The correct theory was outlined by King 1952 and developed further by Young 1988.

We can try using observational strategies that minimize the problem. With some effort, one can try to find comparison stars that indeed have energy distributions similar to the variable star. However, if the two stars are very similar in spectra, they are probably similar in variability. For example, if one is observing a late-M-type star, or any supergiant, then any similar star is almost certain to also be variable at the millimagnitude level. Also, an eclipsing binary star with components of different spectral types will change its composite spectral type during the eclipse and, thus, cannot be matched by any single comparison star. Even a binary with similar components changes color slightly during the partial phases because of limb darkening. Under such circumstances one really would like the comparison to be of some other (nonvariable) type. The spread in individual extinction coefficients of 0.01 mag/air mass at a fixed color, together with typical changes of about 0.03 mag/air mass per magnitude interval of $(B-V)$ color, means that we can search up to 0.3 in $(B-V)$ from our program star to look for a suitable comparison. If the program star is earlier than about K0, this may succeed; if it is a late-M star, we are probably out of luck.

Another approach is to take all the observations at the same altitude, as is done by Cousins 1969, 1976 (cf. Cousins & Laing 1988). (This method effectively builds the atmosphere into the instrumental response function, so it is satisfactory only if the atmospheric extinction law does not change.) Of course, one can also make extinction observations at other air masses, including observations to determine the changes in extinction versus wavelength. This can work well (as Cousins has demonstrated) if observations of a program star once or twice a night are acceptable. This approach does limit both diurnal and seasonal coverage; it would not work well for objects that need many hours or months of continuous coverage.

This method works at excellent sites where the atmosphere is quite constant. At continental locations, even dry ones, we can expect considerable variation in the extinction coefficient. If the aerosol increases by about 0.1 mag/air mass, the atmospheric $(B-V)$ reddening will increase by about 0.002 mag/air mass; then the two stars differ in B by an additional 2 millimag even at the zenith and more at larger air masses.

Two conclusions can be drawn from this. First, if one is willing to live with fairly exacting constraints in terms of the selection of comparison stars, air mass of measurements, seasonal coverage, and measurement of extinction variations including color effects, then millimagnitude differential photometry of some stars may be possible with existing systems such as Johnson-Morgan B and V. Second, if one cannot live within these constraints but requires millimagnitude accuracy, then one has to sample the spectrum more densely to measure the energy distribution of each star more accurately.

Young 1988 shows that millimagnitude accuracy is possible, at least for solar-type stars, for observations up to 2.5 air masses if proper sampling is observed with passbands about 500 Å wide. Thus, a well-sampled system removes all the constraints discussed in this section, leaving those due to other effects such as scintillation noise at large zenith distances. If broader passbands are used, higher-order terms that have not yet been analyzed in detail may become important; but these could also be modeled correctly if the data have adequate sampling.

The obvious step of going to narrower bands reduces the mean color effect, but the existing intermediate- and narrow-band systems, like Strömgren's *uvby*, are even more undersampled than the broad-band ones like *UBV*. Furthermore, most observers assume that Strömgren photometry can be reduced as though it were monochromatic, and completely neglect the second-order extinction. But, as the Strömgren bands are only about 3 times narrower than the *UBV* bands, the second-order terms are not even an order of magnitude smaller for Strömgren than for *UBV* photometry (Tautvaišiene & Straižys 1985). In the shorter-wavelength bands, the effect can be several hundredths of a magnitude per air mass. Furthermore, the differences in extinction coefficient between stars of the same color index can exceed 0.02 mag/air mass—even worse than in *UBV*, because of the more severe undersampling.

The only full solution to this problem is to adopt bands that sample the spectrum adequately. Note that the dense band spacing required to satisfy the sampling theorem provides not only information needed for accurate data reduction but also important astrophysical information. The transformation errors of 0.01 or 0.02 mag found in reducing undersampled data to a standard system (cf. Popper & Dumont 1977; Popper 1982) are themselves aliases of much larger features in stellar spectra that are only marginally detected in conventional measurements. The underlying stellar features have amplitudes of tenths, not hundredths, of a magnitude. Thus, well-sampled data will be not only metrologically more accurate but also astrophysically richer than those obtained today.

Young 1988 has shown that the standard second-order theory is adequate to remove the atmosphere to better than a millimagnitude for bands as wide as 50 nm (FWHM), nearly as wide as *UBV*, provided that sampling is good. Note that, because fully sampled data provide complete information about the stellar spectra at their resolution, they can be used to compute samples at any lower resolution without systematic error. Thus, well-sampled data taken at 500 Å resolution could be used to synthesize lower-resolution systems, such as the *UBV* or Geneva systems, with millimagnitude accuracy. But we cannot synthesize narrow-band data like *uvby* from broad-band observations; to reproduce *uvby* perfectly requires resolution of about 200 Å.

Thus, with well-sampled bands (but not with existing systems), millimagnitude differential photometry is possible, provided that the extinction is always known to better than a few hundredths of a magnitude per air mass. On all but exceptionally good nights (which are generally not recognizable until the observations are reduced) at exceptionally good sites, this means measuring the changes in extinction with time.

If one assumes the extinction to be constant, its time dependence is aliased into the derived extinction coefficient (Nikonov 1953; Rufener 1964; Young 1974), so that the extinction of a single star appears to differ on the east and west sides of the sky. If we measure only stars in a small patch of sky, instead of interleaving rising and setting stars, we cannot detect this error. A drift of 0.01 mag/air mass/hour can easily seem to be a difference of 0.1 mag/air mass in the extinction coefficient between the east and west sides of the meridian (cf. Fig. 8, p. 143 of Young 1974), which is an unacceptable error. However, Zhilin 1977 showed that the extinction is practically constant over the whole sky, even when it varies rapidly with time. Thus, its time dependence can and must be measured.

3.5 *Short-Term Atmospheric Variations*

If time-dependent extinction is to be measured, how often must we do so? The time scale can be inferred from the temporal power spectra of the atmospheric fluctuations that cause the extinction variations. These are of two types: pressure variations that cause changes in the molecular air mass and, hence, the Rayleigh scattering, in the line of sight; and aerosol patchiness, associated with turbulence.

The pressure fluctuations are routinely studied by meteorologists using microbarographs. By far the largest effects are the diurnal and semidiurnal atmospheric tides (Gossard 1960). The amplitudes of the tides are typically one or two millibars; they are clearly visible, though not easily measurable, on the recording barographs used in ordinary weather stations. Below these two spikes, the spectrum shows a nearly $1/f$ amplitude spectrum, with changes on the scale of 5 or 10 min typically having amplitudes of a few tens of microbars. Thus, in an hour, we might see a change in air pressure of a part in ten thousand, which is well below the millimagnitude level, even near 300 nm. Clearly, only the 12- and 24-hour tides have a chance of being detectable at the millimagnitude level, even at short wavelengths. Thus, pressure variations can generally be neglected.

If the changes in extinction that are plainly observable are not due to the gaseous atmosphere, they must be due to changes in aerosol opacity. This conclusion was reached directly by Clarke 1978, 1980, first from the wavelength dependence of daytime-sky brightness fluctuations and then from the wavelength and time dependence of stellar extinction measurements.

Daytime-sky measures have a much higher *S/N* ratio than extinction data. They are, in effect, bistatic lidar, with range unresolved because the Sun is a steady source. Pulsed monostatic lidar provides more details. Hooper & Eloranta 1986 vividly illustrate the convective-plume structure of the boundary-layer aerosols during daytime; at night the atmosphere becomes stably stratified, and the surface haze layer, due to radiative cooling, shows "little apparent structure". Thus, aerosol inhomogeneities at night are much weaker than those seen in the daytime by the solar astronomers (Grec et al. 1979), typically by an order of magnitude (Post 1978). The lidar data also show better temporal (and thus, probably, angular) correlations at night than in the daytime (Kolev et al. 1988).

Because the lidar observations are spatially resolved and have much higher *S/N* ratios than attempts to measure millimagnitude fluctuations in transparency, either by day or by night, the lidar data must be given high weight. Their evidence for day–night changes (Post 1978; Kolev et al. 1988) is further supported by well-known qualitative differences between the convective daytime and the stratified nocturnal boundary layers.

Observations of extinction at good sites rarely show changes more rapid than 0.01 or 0.02 mag/air mass per hour. In the Crimean peninsula, with a climate similar to the Upper Peninsula of Michigan, changes as large as 0.2 mag/air mass have been seen in an hour, but short-term changes of 0.03 mag/air mass can be seen in 15 to 30 minutes more typically (Zhilin 1977). At the Cape, Cousins 1985 found that daytime changes of 0.01 mag/hour are "common"; changes at night, and at higher sites, should be less. This means that extinction can be followed with adequate accuracy for differential photometry at most sites by making about three good extinction determinations per hour. Although Kurtz 1982 has shown that mean extinction may be good enough at an excellent site, he has observed small variations of up to 0.01 mag/air mass on 10- to 20-min time scales; thus, even at a good site, it is prudent (and inexpensive in terms of observing time) to measure the extinction at least once an hour.

However, the change in extinction between observations of the program and comparison stars shows up in differential photometry, multiplied by the whole air mass. Thus, it is necessary to chop between program and comparison stars rapidly enough to keep these changes small. If the aerosol fluctuations have the same $1/f$ spectrum as the pressure fluctuations, a change of 0.01 mag in 10 min corresponds to a millimagnitude in 1 min. This suggests that short integrations, and rapid chopping between stars, are more useful than longer integrations, providing the overhead of resetting the telescope is small. The indicated time scale is probably too fast for good visual recentering but is possible with a computer-controlled telescope using rapid automatic centering. To keep the effects of extinction variations to a minimum, one should switch between stars faster than between filters, if the recentering time is no more than a few seconds. The indicated rule is: Get back to the comparison star at least once a minute.

Because manually operated telescopes and photometers allow quick filter changes but only slow resetting between stars, it is customary to make filter changes on a short time scale, thus chopping out most of the extinction variations between bands but not between stars. The short time scale of these extinction fluctuations, rather than the weakness of their wavelength dependence, explains why "colors" have smaller errors than "magnitudes" in conventional photometry (cf. Cousins 1985). Robotic telescopes with automatic centering can chop almost as quickly between stars as between filters, thus transferring to magnitudes the advantage previously restricted to colors.

3.6 *Centering Nonuniformities*

As spectroscopic observers know, low-frequency image motions mean that good centering cannot be done quickly. If we look at a star for just a second, and center on its average position during that time, a minute later it may be several arc seconds away from the set position. We cannot afford to spend a long time determining an average position (as the spectroscopists do), so we must be sure that image motions during the following integration time do not degrade our precision. That is, typical image motion during some tens of seconds must not cause millimagnitude errors due to field nonuniformities.

Such nonuniformities arise partly from nonuniform transmission of filters, field lenses, and windows (Kron & Gordon 1957), due chiefly to defects and dirt. Dirt can be kept from filters and lenses by sealing the photometer with a window far enough from the focal plane that the inevitable dirt on the window is not a problem.

Defects in optics are usually specified on a "cosmetic" or "scratch and dig" scale that is measured by scattered light, not by transmission losses. The "dig" number is the mean diameter of the defect in hundredths of a millimeter, so the common 80-50 surfaces can have up to a half-millimeter diameter dig. The "scratch" number is estimated visually by comparison with a set of standard scratches. Thus, the photometric implications of a quantitatively specified scratch are not obvious (M. Young 1986). If we can assume that the narrowest scratch of a given category is more nearly opaque than a wider scratch giving the same near-axis forward scattering, we can estimate the light loss on this basis. Scratches as narrow as 8 microns have been identified as number 80 scratches and, in general, the narrowest scratches seem to be about a tenth as wide in microns as the scratch number. However, much wider flat-bottomed scratches have similar numbers; presumably these are fairly transparent, and only the sides are causing the lost light. The widest scratches are about as wide in microns as the scratch number.

Now let us try to estimate the photometric consequences of a given scratch or dig. Because of the well-known anticorrelation between seeing and transparency, we must deal with seeing of five or more arc seconds on the clearest nights. Also, errors in polar-axis alignment, drive rate, and refraction corrections may cause several seconds of telescope displacement during an integration. Defects generally correspond to a few arc seconds or less in the focal plane of the telescope, so that typical image motion may move the defect completely in or out of the measuring beam. The area of a number 50 dig is 1% of the area of a 5-mm-diameter beam; thus, we cannot expect much better than 1% photometric uniformity with ordinary optics and beam sizes. If we expand the beam to 16 mm, the 50 dig is just about a thousandth of the beam area, and we have (just) reached millimagnitude uniformity. Better surface quality, such as 60-40 or 50-30, would allow some safety margin.

So much for digs. What about scratches? The question depends on the length of a scratch that can move in and out of the beam as the star image moves. If the scratch is curved to match the edge of the field, a much longer section of scratch can move in and out of the beam than if the scratch is straight. Let us assume a 1-cm length; then a number 80 scratch (8 microns wide) of this length has an area of 0.08 square mm, about equivalent to a number 30 dig. If the equivalent width were 80 microns instead, the area would correspond to a number 100 dig. Thus, scratches and digs on common surfaces are about equally important. We should probably ask for scratch numbers of 50 or smaller to be on the safe side, if we want millimagnitude photometric uniformity over a 16-mm beam.

To provide reasonable insurance and error margin, 40-20 seems a good specification for lens, window, and filter surfaces, provided that 16-mm beams can be maintained. Such high-quality surfaces can often be selected from manufacturers' stock for a modest charge. It is difficult to obtain such quality in filters (especially the Corning glass filters, which are prone to a high density of striae, bubbles, and inclusions because of the high viscosity of their Pyrex base). This problem can be alleviated by forming a large pupil image on the filter and then reimaging on the detector. The tradeoff is an extra lens or two and the exchange of spatial for angular nonuniformity (see below). There may be problems at cold-box windows, where beams are smaller. Clearly, high-quality optical surfaces and large beams are required to reach the millimagnitude level reliably.

Filter nonuniformity is another cause of field nonuniformity that can be removed by imaging the pupil on the filters. A nominal specification for parallelism of filter surfaces is 2 arc minutes. This corresponds to a relative change in thickness of a part per thousand in 3 mm across the surface of a 2-mm-thick filter. Filter wedge displaces the beam downstream from the filter, which may make the effect of photocathode nonuniformities worse if the pupil is not reimaged on both. Better parallelism is possible, but expensive. Compositional gradients in filter glass can also be important. The evaporated coatings on interference filters also have considerable spatial gradients (Schaefer & Eckerle 1984) and pinholes, which can be much more important photometrically than a dig of corresponding diameter, especially in narrow bandpass filters.

Finally, the measuring beam will not fall on the same filter area if the filter-change mechanism fails to reposition the filter exactly. Filter nonuniformities can then cause errors that depend on the precision of the mechanism. This problem can be eliminated by careful design of the mechanical system. Similar problems occur if the filters are loose, no matter how precise the mechanism is that carries them.

Similar mechanical problems can be caused by flexure in the photometer (Cousins & Laing 1988) or its mounting (Cousins 1973) and by flexure or looseness in mirror supports, particularly those of the secondary mirror. Changes in position or tilt of the secondary displace the exit pupil laterally and thus move its image on filters and/or detectors. While flexure errors should cancel out to first order in differential photometry (but possibly not to second order, owing to the wavelength dependence of the detector shading map), they appear directly in extinction measurements.

Another well-known type of centering error is due to the extended wings of the star image. These are due primarily to surface roughness on the primary mirror, not to the classical diffraction pattern of the pupil. If the star moves from the center of the focal-plane aperture, more light is shifted out of the aperture on the side toward which the star moves than is shifted in on the opposite side, so the net result is a loss of throughput. As is well-known, this effect requires large field stops (diaphragms) to achieve reproducible results—typically, at least 30 or 40 arc seconds across, and often a minute or more. The problem is that the scattered-light wings fall off only with the square of the distance from the image core; see Kron & Gordon 1957 for an example. This would make the excluded energy integral diverge logarithmically, but for the finite area of the sky. As the image motion in poor seeing and dusty or rough optics both make the variations in excluded energy larger, millimagnitude repeatability under poor seeing requires clean, well-polished optics or else very large focal-plane apertures. The latter introduce errors due to field stars and photon noise from the sky.

A further consequence of the extended image wings is measurable contamination of sky readings if the sky is not measured far enough from the star. As the size of the effect depends on the state of the mirror, it should be measured frequently by tracing a bright star across and well outside the field stop. Care should be taken to measure the sky in exactly the same place every time for each star. This also protects against varying contamination by faint stars; a star 7.5 mag fainter than a measured star will change the measurement by a millimagnitude. Even constant contamination by field stars is a problem if absolute colors are required; for eclipsing-binary light curves, the solution for "third light" removes it.

Unfortunately, these are not the only limits to spatial photometric uniformity. The field lens converts position in the field to angle of arrival at the pupil image. The pupil is usually imaged onto the photocathode to avoid problems with cathode spatial nonuniformity; but this trades spatial for angular effects, which are large for tubes with inclined cathodes, like the 1P21 and most gallium-arsenide tubes. These effects are mainly due to the variation of Fresnel reflection losses from the cathode with angle of incidence. Conservation of throughput requires that the angular excursions of the star are magnified in the ratio of the pupil image size to the full pupil—a factor of 170 if a 1-m telescope is reduced to a 6-mm spot on the cathode. Thus, a 10-arc-sec stellar motion becomes an angular excursion of about 0.5 degree, or 0.01 radian, at the cathode. With highly inclined cathodes, we can expect the photometric consequences to be of this same order of magnitude (1%, or 0.01 mag). The problem is directly proportional to telescope aperture for a fixed image size on the cathode.

Obviously, it is hopeless to expect millimagnitude photometry from moderate-sized telescopes if tubes with opaque photocathodes are used. These numbers are confirmed by the experience of Kron & Gordon 1957 who reached 5 millimag with a 1P21, using very careful centering, and concluded that "guiding is now our chief difficulty". While there is no guarantee that millimagnitude photometry is possible with end-on cathodes, they certainly seem a better choice.

Even so, if the pupil is not perfectly imaged on the detector, image motion will move the illuminated area. Some motion is unavoidable, as we have to use silica singlet field lenses to work in the ultraviolet. Thus, chromatic aberration guarantees some image motion effects at extreme wavelengths. We usually minimize this problem by adjusting the detector to find the most sensitive region; near the position of maximum response, responsivity gradients are small.

A possible solution to this problem is the use of integrating cavities rather than imaging optics in front of the detector. The price is a considerable light loss. For example, the device used by Mielenz et al. 1973 allowed a precision of a few hundredths of a millimagnitude but had an efficiency of only 20%; Mielenz & Eckerle 1975 achieved a uniformity of 0.1 millimag with 10% efficiency. Later models did better: Eckerle, Venable & Weidner 1976 describe an integrator with similar uniformity, gradients of 0.15 millimag/mm in the visible, and efficiency near 50% throughout the spectral region accessible from the ground. Blackwell, Petford & McCrea 1965 used a crude integrating cavity with less than 25% loss but gave no values for the uniformity reached. This technique may deserve further investigation for bright stars, but, as we have shown earlier, even with 0.75-m telescopes there are none too many photons for most stars.

Silicon photodiodes have been suggested as a solution to this problem because of their much better uniformity. Unfortunately, present silicon diodes have spatial nonuniformities of several tenths of a per cent (Schaefer, Zalewski & Geist 1983; Campos, Corrons & Pons 1988), so some degree of image stabilization is still required. On the other hand, photodiodes have better gain stability than photomultipliers, which are more affected by temperature and magnetic fields and show short-term gain drifts or "fatigue"; see Section 3.8 below.

Fisher 1968 discussed the advantages of solid-state detectors. He examined two silicon diodes and compared their performance with S-1 photomultipliers in the visible and near-infrared, noting the much higher quantum efficiency of the diodes and their insensitivity to magnetic fields and great stability compared to photomultipliers. He also noted their extremely linear response over seven decades of intensity. However, the integration of thermal dark current changes the operating point of photodiodes and produces nonlinearity if it is a significant fraction of the saturation value (Percival & Nordsieck 1980; Schaefer et al. 1983). Budde 1979, 1983 has demonstrated linearity to 0.1% over eight decades, corresponding to 20 stellar magnitudes. According to Budde 1979 such linearity "can be, but is not always, achieved". Unfortunately, the upper end of this range is at milliwatt optical power levels and, thus, is not useful for stellar photometry (a telescope about 300 meters in diameter would be required to get a milliwatt from Vega).

In recent years silicon diodes have become popular detectors in laboratory radiometry. With sufficient care, Geist, Zalewski & Schaefer 1980 have demonstrated absolute accuracies corresponding to 0.4 millimag in the red but only 6 millimag in blue light. Because only relative measurements are needed for astronomical photometry, somewhat higher precision can be expected. Figure 4 shows some results of laboratory tests by Borucki et al. 1988 with two prototype stellar photometers. The figure shows that a recent version of the photometer can reach a precision of 0.1 millimag for a 60-sec integration.

Schaefer 1984 outlines many of the potential advantages of photodiodes. Much of this information was acquired by the National Bureau of Standards (now the National Institute of Standards and Technology) while developing silicon photodiodes as absolute radiometric standards with high quantum efficiency (Zalewski & Geist 1980; Geist et al. 1980; Geist, Liang & Schaefer 1981; Schaefer et al. 1983). These diodes are planar diffused

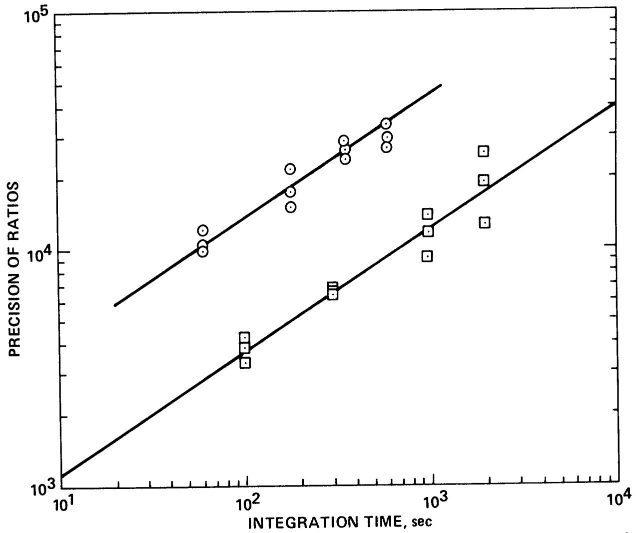

FIG. 4–Laboratory measurements of the photometric precision, expressed as reciprocal of the relative error, of silicon photodiodes as functions of integration time. The three different symbols used in the plot are for the three different photodiodes used in the measurements.

p-on-n silicon diodes with shunt resistances near 10 ohms at room temperature.

The major disadvantage of photodiodes is their large dark noise at room temperature. For diodes operated without reverse bias, most of the noise is thermal ("Johnson") noise. Typical noise values are near 4×10^{-15} watts $Hz^{-1/2}$ for a 2.4-mm-square detector at 25° C (Hamamatsu technical specifications for type 51226 PNN silicon diodes). Cryogenic operation is necessary to further reduce this noise. When cooled by liquid helium and used with amplifiers of the type described by Low 1984, the detector-amplifier unit can reach noise currents as low as 10^{-18} watts $Hz^{-1/2}$, which is nearly all thermal noise in the feedback resistance of the transimpedance amplifier. The dark noise is also reduced if smaller detectors are used, but this presents severe optical coupling problems.

For example, H. L. Johnson and his colleagues at San Pedro Martir Observatory in Mexico built and tested a stellar photometer with a cooled silicon diode detector (de Lara et al. 1977). They observed a magnitude 16 star with a 1.5-m telescope, where they obtained $S/N = 1$ with a 30 Hz bandwidth. From this they predicted that 18th magnitude could be reached with "excellent precision" if a better diode were used. However, their tests obtained a precision of only 0.025 mag, which they attributed to centering errors, as the photometer lacked a Fabry (field) lens. The detector they used was only 0.9 mm in diameter, which requires large angles of incidence at the detector (e.g., 14 degrees to cover a field of 1 arc minute in a 1.5-m telescope). Durnin, Reece & Mandel 1981 and Geist, Gladden & Zalewski 1982 show that responsivity of silicon diodes typically increases by 1% at angles of incidence near 15 degrees. Besides, such small detectors are very likely to suffer from dust contamination (cf. the discussion earlier in this section).

Similarly, charge-coupled devices (CCDs) have unsolved problems at the millimagnitude level; see Mackay 1986 for a review. King 1983 has treated their spatial aliasing problems. The intrapixel nonuniformity and small "full-well" charge require use of rather large star images for high precision. Even so, the spatial variations in spectral response frustrate exact flat fielding in broadband work. Once again, we may hope that future improvements in design and manufacturing may alleviate these problems, but CCDs are not attractive at present for millimagnitude photometry. We may add that all silicon detectors pose severe red-leak problems, if photometry is attempted at blue and UV wavelengths, because of the lack of satisfactory filters to block the red and infrared wavelengths where these detectors have their peak quantum efficiency.

Other angular errors are due to the changes in Fresnel reflection losses, and the change in path length through absorbing filters, with angle of incidence. These effects amount to a millimagnitude at 5 or 6 degrees off axis (see Fig. 9 of Mielenz 1973). Because multiple optical elements occur in the measuring beam, the actual maximum acceptable angular change is much less. Still, the half-degree mentioned above seems safe.

The shift in bandpass of interference filters is proportional to the cosine of the angle of incidence and inversely proportional to the effective spacer refractive index, which generally lies between 1.5 and 2. Neglecting this rather small correction factor, we observe that a filter centered at 500 nm shifts 0.1 nm when the incidence angle reaches about a degree. As color indices are generally based on band separations of about 100 nm, this will produce about a millimagnitude of apparent difference in brightness between two stars differing by 1 mag in color index. We must clearly try to match comparison stars to program stars as well as keeping angular shifts at filters less than a degree.

A typical field lens with a 10-cm focal length has a radius of curvature of about 47 mm, so that a 1-mm lateral displacement of the beam changes the average angle of incidence by 1/47 of a radian, or about 1.2 degrees. Thus, typical image motion may produce significant changes in the apparent transmission of field lenses due to oblique Fresnel losses. While antireflection coatings may reduce these effects by modest factors, they may also introduce instrumental instabilities due to humidity variations, which alter the effective refractive indices of the thin-film coatings.

These effects, if kept controlled, may not yet be a barrier to millimagnitude precision. Similar problems arise if reflective optics are used, as the absorption in mirror coatings depends on incidence angle and is generally larger than Fresnel losses at lens surfaces.

3.7 *Temperature Variations*

Refer again to the figure of 0.1 nm as an upper limit to acceptable passband shifts. Such shifts can also be produced by temperature changes in filters and detectors. Typical thermal shifts in absorbing filters (Young 1974, pp. 105–108) are about 0.1 nm/degree at 400 nm and increase with the square of the wavelength; clearly, we cannot tolerate 1-degree changes in filter temperatures unless program and comparison stars have very similar spectra. However, it becomes difficult to control filter temperatures much better than this—especially because the low thermal conductance of glass means that filter temperatures are determined mainly by radiative exchange with their environment.

Thermal shifts in interference filters are smaller by a factor of 4 or 5, which would make interference filters preferable, except for their unfortunate tendency to degrade by absorbing water vapor. One might choose to use interference filters but plan to replace them every few years. This represents a trade-off between long-term and short-term stability. Alternatively (and preferably) one can keep the air about the filters very dry at all times. It might be noted that, at Lowell Observatory, the same interference filters have been used with success for 15 years (Lockwood & Skiff 1988).

Of course, similar shifts also occur in the spectral response of detectors, for the same reason as the effect in absorbing filters: competition between photon and phonon energies in exciting electrons to detectable energy levels. Unfortunately, these thermal shifts are all in the same direction: passbands shift to the red at higher instrumental temperatures. As temperatures can readily be measured to 0.1 degree, it may be necessary to measure instrumental temperatures and include temperature-dependent correction terms in the reductions. Nevertheless, it would be unwise to give up trying to control temperatures.

3.8 *Gain Drift*

Gain drifts are a persistent problem. If we use dc measurements (either current or charge integration), gain changes appear directly in the data. Pulse counting can be made about five times less sensitive to drift than dc methods if an appropriate discriminator level is used. An upper-level discriminator can, in principle, cancel first-order gain changes exactly. The price is a small loss in detective quantum efficiency. Thus, if signals are well above dark noise, the main advantage of pulse counting is instrumental stability rather than signal/noise ratio.

Temperature changes affect the thresholds of discriminators and the gains of amplifiers, so they can cause gain changes in the electronics. Although gain drifts are correlated with extinction drifts in the data reduction, they can be distinguished from each other if the extinction observations are scheduled to minimize the correlation of air mass with time. Kviz 1978, 1979a,b, 1981 has discussed the analysis of such drifts. Changes in discriminator level can also affect the apparent spectral response of a photomultiplier, because the pulse-height distribution depends on many factors that depend on wavelength (see Young 1974, pp. 69, 70 for further discussion).

Because the gain of a photomultiplier is proportional to a power near 10 of the voltage across the dynode chain (see pp. 32–34 of Young 1974 for details), extremely stable high-voltage power supplies are required. Fortunately, these are commercially available. If we demand 0.1% gain stability, we need about 0.01% high-voltage stability. Short-term stability a little better than this is available. Rapid chopping between stars removes slow drifts. Running the high-voltage supply from a regulated power source may be helpful. We also expect temperature control of the high-voltage supply to be beneficial; solid-state supplies are small enough to incorporate in a temperature-regulated photometer.

A general principle for reducing the influence of temperature variations on any component, optical or electronic, is to isolate the component thermally from the environment and tie it thermally to a large mass. This guarantees a long thermal time constant, so that less-frequent chopping is required to track the resulting drifts.

Photomultipliers (as well as other detectors) are subject to hysteresis or "fatigue", so that the reading obtained for a star depends on the brightness of the star observed before it. Rosen & Chromey 1985 found substantial gain drifts, primarily in tubes with small dynodes, but there are numerous counterexamples. For example, Hawes 1971, Mielenz et al. 1973, Kurtz 1982, 1984, and Lockwood 1984 have looked for gain shifts at the 0.1% level and not found them. Clarke 1980 reported long-term drifts in the relative sensitivities of two photomultipliers but "no hint of fluctuations to levels of 1 part in 10^3 on time-scales of minutes".

Gain drift due to "fatigue" in photomultipliers has been reviewed by Young 1974 (pp. 49–51). It can be kept small by keeping anode currents small, and by using short integration times, so the detector does not change responsivity much during an integration. A photomultiplier used for high-precision photometry should be picked not primarily for low noise (we are, after all, looking at rather bright stars with large telescopes) but for minimum fatigue. EMI Gencom makes a PMT designed specifically for low fatigue.

A penalty paid for using pulse counting is its inherent nonlinearity, due to pulse-resolution problems. The only way to keep this effect small is to avoid bright stars, except for a few used to measure the effective dead time. Pulse pileup makes the effective dead time a function of the discriminator setting; it may also be wavelength dependent in some tubes. If the dead time is known to 10%, its uncertainty will cause millimagnitude errors when the total correction is only 1%; this makes observations of bright stars impossible, unless a quasi-neutral attenuator is used. Inconel-on-quartz "neutral" filters introduce transformation problems with undersampled photometric systems but can probably be calibrated at the millimagnitude level if a well-sampled system is used. They should be tilted slightly to keep interreflections out of the measuring beam. The effective dead time may depend on the temperatures of the tube and the electronics; such effects should certainly be looked for.

Another source of nonlinearity (see Young 1974, pp. 52–57) is loading of the dynode bleeder chain by the anode current of the tube. Although Zener diodes can be used to reduce loading effects, they introduce noise and (through a large temperature coefficient) drift. As the nonlinearity is substantially reduced by a correct discriminator setting in pulse counting, which requires low count rates (and, hence, low anode currents) to avoid dead-time nonlinearity anyway, the best policy seems to be pulse counting with strict limits on the maximum count rate and careful measurement of the nonlinearity of the system.

Although "neutral density" filters are commonly used to test for nonlinearity, no truly neutral filters exist. Screens placed over the pupil greatly enhance the scintillation noise and should be avoided (no less because they illuminate a subset of the cathode area, which has nonuniform spectral response and other properties). Probably the safest attenuator to use is the 3-polarizer type described by Bennett 1966, if calcite rather than Polaroid film is used; see also Young 1974 (pp. 59, 60, and 118) for further discussion. For the highest precision, both the linearity and the stability of the instrument should be checked by careful laboratory measurements.

As is well-known, magnetic fields affect the electron paths in photomultipliers, thus affecting both gain and spectral response (by changing the spatial weighting of the spectrally nonuniform cathode). This is a particular problem with automated instruments, as the electromechanical actuators used to move filters and other optical components can produce large external fields, especially if they employ permanent magnets. Good design and magnetic shielding are required, as even the Earth's field typically changes the gain of a photomultiplier by several percent. The shielding usually used for photomultipliers saturates and becomes ineffective at moderate field strengths; materials that maintain good permeability at higher fields should be used around motors and solenoids.

Another source of gain drift, which can be wavelength dependent, is the effect of humidity variations on the dirt on the optics. This material is deposited from the air, so it should show the same marked variations in opacity with humidity displayed by the airborne aerosols (Hänel 1976). These variations are the cause of the common observation that photometry can best be done under very dry conditions. This is an additional reason for cleaning telescope mirrors frequently. It also suggests that zero-point drifts may be correlated with local measurements of relative humidity; if the correlation is good enough, this would provide a means of removing such errors. This needs to be investigated.

3.9 *Variations in Comparison Stars*

Even at the 0.01-mag level, variable comparison stars are such a problem that observers routinely employ a second "check" star. We can expect this problem to become more severe as we approach a millimagnitude; indeed, a major astrophysical question to be answered by such work is, "Are there any constant stars?"

We have a clue in the work of Lockwood & Skiff 1988. They appear to have found pairs of solar-type stars that are indeed constant to about a millimagnitude, but these are rare. Most of their pairs show relative variations of several millimagnitudes. We should regard this as an upper limit to real stellar microvariability, as some of the errors discussed above (notably those due to aliasing in the Strömgren system) enter into the Lockwood and Skiff data.

However, if a typical relative variation of, say, 3 millimag is to be expected, we can still reduce the RMS error to 1 millimag by using a group of nine comparison stars. Such large groups can still be handled efficiently by fast robotic telescopes, though careful statistical design is required to optimize the results. But if we include this in our total error budget, we must keep the RMS error from this cause much lower. The RMS error of the mean can be kept to half a millimagnitude by using 36 stars; this is difficult and unwieldy, even for fast automatic systems.

It may not be necessary to use quite such large groups, however. The mean is a poor estimator to use when the variations are not normally distributed. As some stars in a group will be much more variable than others, we can expect a more nearly exponential than Gaussian error distribution. The optimal location estimator for such a population is the median, not the mean. Considerably more robust estimators exist that are nearly optimal. Thus, by using a suitable robust estimator, we can obtain substantially more efficient statistics.

Furthermore, some stars will vary appreciably only on time scales longer than the variation we are trying to measure. For example, solar-type stars will hardly vary within a single night and may show only small variations from one night to the next. We can treat such stars as constant comparisons within a night if we are measuring an eclipsing binary or a pulsating white dwarf. A star that varies slightly from year to year is still a good extinction star for a one-week observing run. The appropriate statistical treatment of such error sources requires allowance for their temporal correlations.

3.10 *Standard and Extinction Stars*

Although we have considered only differential photometry and photometric precision, these subjects are linked to all-sky photometric accuracy through the determination of extinction. Because we must determine time-dependent extinction, a single extinction measurement compares two stars at different air masses instead of a single star at two separated times. Thus, the accuracy with which the instrumental magnitudes of the two stars are known affects the accuracy of the extinction measurement.

If we calculate the instrumental magnitudes from standard values, then errors in the transformation from standard to instrumental system propagate into the derived extinction coefficients. As transformation errors are typically 0.01 or 0.02 mag in existing undersampled systems, errors of this order per air mass will appear in the measured extinction coefficients (Young 1974, p. 178), unless the great majority of the observing time is spent measuring extinction. As shown in Section 3.2 above, such errors are only marginally acceptable for precise differential photometry. In this case it is better to determine the relative instrumental magnitudes of the extinction stars directly in the instrumental system by combining data from several nights in the extinction solution (Manfroid & Heck 1984).

Fully sampled photometric systems will allow accurate transformation from standard to instrumental systems. This will not only greatly improve the precision of differential photometry but also allow parameters derived from this photometry, such as those of eclipsing binaries, to be related much more accurately to standard values. The need to eliminate systematic errors of this sort was emphasized by Popper 1982.

Our goal is not only to improve the photometric precision of observations at individual observatories but also to improve the precision of combined observations from networks of robotic observatories. Photometric variability sometimes occurs on time scales that require round-the-clock observations for days or even weeks, and from the Earth this can only be done with a network of telescopes at different longitudes or, with great difficulty, from near one of the poles. A global network of robotic observatories is forming. Besides several robotic telescopes at F. L. Whipple Observatory on Mount Hopkins in southern Arizona, similar robotic observatories are under construction for placement in Sicily, Australia, California, South Africa, and elsewhere. Most of these robotic observatories "speak" the same language, the Automatic Telescope Instruction Set (ATIS), which allows convenient and precise scheduling of observations of extinction and standard stars. Several of these robotic observatories may be equipped with high-precision photometers built according to the suggestions given in this paper. This could, in time, lead to a cooperative global network of high-precision automatic photometric telescopes.

We note that the Geneva Observatory has, for many years, carefully operated a tightly controlled, "closed" network of sorts. While not global in its coverage, the Geneva Observatory established a set of high-quality standards for its photometric system. As this system is somewhat undersampled, very careful control was required to make certain that filters and detectors were very closely matched and that reduction techniques were highly standardized and closely followed. The excellence of their work is witness to the fact that this was indeed accomplished.

In a fully sampled system it is possible, at least in principle, to achieve even greater accuracy in a long-term, multiobservatory sense, without having such tight control on the exact matching of filters and detectors. In other words, it may be possible to have an "open" photometric system of high accuracy. This is not to suggest that there should be large departures in filters, detectors, and reduction techniques, but only to suggest that participation could be open to all who adhere to realistic manufacturing tolerances in equipment and to adequate models and algorithms in data reduction.

As precision approaches the 1-millimag level, it will become increasingly important to develop a set of very low variability stars as a network reference. We need enough such stars that some of them could be observed within half an air mass of the zenith from observatories in both hemispheres at any time of night on any night of the year. The set must also cover a range of spectral types, luminosity classes, etc. On the other hand, the number should not be too great, as each star needs to be accessed frequently by all sites participating in networked high-precision photometric observations.

This set of reference stars will need to be refined over time. As some of the stars are found to be more variable than others, they can be replaced with stars found to be less variable. Because of photon noise, the reference stars would need to be brighter than 7th magnitude yet not so bright as to require the use of a neutral density filter (for our 0.75-m telescopes used as examples). The bright limit might be 5th magnitude.

A subset of the reference stars would establish zero points in each band. These would not cover all spectral types, luminosity classes, etc., but would be chosen to maximize stability, precision of measurement, and common accessibility. They should be stars constant at the millimagnitude level (if such stars exist), rather bright to allow low photon noise in short measurements, and spanning the celestial equator so some could be viewed from either hemisphere with ease: perhaps a star or two spaced every hour or so in R.A., for a total of a few dozen in the subset. These would be heavily observed. This will require a special observing program to find the least variable of the brighter stars.

We would hesitate to call these stars "standards" for fear that, as with the Johnson and Harris standards for *UBV*, the system would be locked in time to a fixed list of stars and values without regard to variability, measurement accuracy, etc. At any time it should be possible to improve a set of reference stars by additional measurements, more precise measurements, or eliminating some stars while adding others. Such improvements should be encouraged, not discouraged, and should be a project for community-wide involvement. However, extreme care is required to avoid drift of the system as a whole. Robotic observatories could, of course, take out most of the observational drudgery in making such improvements.

3.11 *Combining of Errors*

Finally, one must realize that all these effects are superimposed and occur simultaneously, so that an error budget must be drawn up and allowable instabilities assigned to each effect that occurs. If four uncorrelated effects dominate the errors, each must be kept below half a millimagnitude to keep the RMS error below one millimag. If nine uncorrelated errors contribute, each must be kept below 1/3 millimag. These figures appear realistic and (with good practice in design, construction, and operation of the instrument) achievable. Millimagnitude photometry can probably be reached on the ground if we use what we already know.

4. Recommendations for Precision Photometry

Even without an automated telescope, most photometric observers can optimize their precision by adhering strictly to a few rules.

1. Measure extinction every night, at least once an hour.

2. Choose comparison stars within 4 or 5 degrees of a variable program star, or work only near the zenith. Do not try to observe more than 45 degrees from the zenith if you need high precision.

3. Recognize that precision will always be low in the U or u bands. Lockwood & Skiff 1988 have demonstrated that high precision is possible in Strömgren b and y.

4. Choose comparison stars within 0.3 mag in $(B-V)$ color index of the variable and correct for the variation of extinction with color.

5. Use short integrations, so as to chop rapidly between variable and comparison stars (once a minute or less if possible). This may require using only one filter at a time instead of the whole set.

6. Always measure the sky at the same place for each star, at least an arc minute away from the star. Beware of faint background stars.

7. Keep dead-time corrections (with pulse-counting systems on photomultipliers) below 1%. If you must attenuate bright stars, use neutral filters rather than pupil screens, which increase scintillation noise.

8. Use a large focal-plane diaphragm.

9. Assure that the star is well centered in the diaphragm during the integration.

10. Use more than two comparison and/or check stars. Groups of four or five stars are practical with any photometric telescope.

11. Provide a temperature-controlled environment for the detector and filters.

If an automated telescope is available, further improvements are possible by following these additional recommendations.

12. Measure extinction more frequently, at least three times an hour.

13. Make use of a CCD camera centering device for quick and accurate centering.

14. Chop more rapidly between program and comparison stars.

These preceding "brute-force" methods of beating down the errors with robotic efficiency are effective.

15. Use a set of filters that satisfies the sampling theorem. The restriction on color range can then be relaxed and still result in better corrections for extinction. Sampled systems extending into the ultraviolet may allow high precision even at short wavelengths. This approach is called "work smarter, not harder".

In the above discussion, it is implicit that data only from nights of very good photometric quality (highly uniform transparency) be used. A human observer can sometimes decide, by visual inspection of the sky, to forego observing on a given poor night, but a typical robotic telescope will gather data on any night which is partially clear enough to let target stars be located, identified, and centered. Therefore, the actual procedure for excluding inferior data normally will involve an after-the-fact decision.

5. Performance of Existing Robotic Telescopes

Following is a discussion of the performance obtained over several years with four different robotic telescopes, all currently operated by the Fairborn Observatory and located at the Smithsonian Fred L. Whipple Observatory on Mount Hopkins. The telescopes include the 25-cm Phoenix-10 (Boyd et al. 1984), the 25-cm Fairborn-10 (Baliunas et al. 1987), the 40-cm Vanderbilt/Tennessee State automatic telescope (Hall 1989), and the new 75-cm Smithsonian automatic telescope (Baliunas 1989). The detectors being used by the four telescopes are an uncooled 1P21 photomultiplier, an uncooled PIN photodiode, an uncooled bialkali photomultiplier, and a thermoelectrically cooled GaAs photomultiplier, respectively. Genet & Hayes 1989 provide additional information on these four telescopes. The three smaller telescopes have not been capable of running the Automatic Telescope Instruction Set (ATIS) and so observe in the fixed sequence described by Boyd et al. 1984. The resulting data are reduced with mean extinction coefficients. One complete sequence for a given variable star on the observing program consists of eleven moves between the check star (K), sky (S), comparison star (C), and variable (V) in the following order: K,S,C,V,C,V,C,V,C,S,K. These three telescopes do follow recommendations 2 through 8 for precision photometry listed in the previous section. The 75-cm Smithsonian telescope, which is fully programmable with ATIS and makes use of CCD centering, is capable of implementing all 15 recommendations.

The Phoenix-10 began observing from Phoenix, Arizona in October 1983 and was moved to the Mount Hopkins site during the summer of 1986. During the interval from 1983 through the end of 1987, the Phoenix-10 observed 92 program stars, most of them known or suspected to be chromospherically active, for an ongoing program at Vanderbilt University. Three of the 92 program stars were constant red-blue pairs chosen to provide a check on the precision of the photometry as well as on the transformations to the standard *UBV* system. In addition, 15 of the chromospherically active stars showed no detectable light variations with respect to their comparison stars. A preliminary analysis of the first two years of data on these 18 constant stars was given by Hall et al. 1986. As these authors pointed out, the execution of the observing sequence given above results in three independent differential observations of the variable with respect to the comparison within an approximately 7-min interval of time. The internal uncertainty of a resulting mean (of the three differential observations) can be estimated from the standard deviation of the three individual observations from the mean. Also, the external uncertainty of a mean can be estimated by the standard deviation of the individual means from the overall long-term mean. As a preliminary cloud filter, the data-reduction routines for the Phoenix-10 automatically removed any group with an internal uncertainty greater than ± 0.020 mag. Hall et al. 1986 used a refinement of the three-sigma test to exclude additional means that were grossly in error and found that the external errors from the first two years of V data on the constant star pairs ranged from ± 0.008 to ± 0.012 mag with a mean external error of ± 0.010. A final analysis of all four years of data on the constant star pairs gave mean external errors of ± 0.011, ± 0.014, and ± 0.023 mag, respectively, in V, B, and U (Strassmeier & Hall 1988a) and mean internal errors of ± 0.005, ± 0.005, and ± 0.009 mag, respectively, for the pair of constant stars with the most data. Hall et al. 1990 analyzed three years of data from the Phoenix-10 on the spotted star V478 Lyrae by fitting two-spot models to the light curves. These data were first binned into 15 groups (groups 4–18 in their Table III), and each group was then fit with a spot model. External errors were determined as above but using residuals from the spot-model fits instead of from an overall mean. The results ranged from ± 0.005 to ± 0.016 mag in V with a mean of ± 0.010, confirming the size of the external error determined above from pairs of constant stars. It must be noted that any intrinsic variability in either star of the presumed constant pairs or imperfections of the spot model fits for V478 Lyr will cause an overestimate of the size of the photometric error. The internal and external errors given above can, therefore, be regarded as the lower and upper limits, respectively, to the precision of the Phoenix-10 telescope. The data on five variable chromospherically active single stars were presented and analyzed by Strassmeier & Hall 1988b and results on 49 variable chromospherically active binary systems were given in Strassmeier et al. 1989. These papers demonstrate the enormous power of the Phoenix-10 to obtain excellent results with the precisions quoted above.

Originally brought into operation in Fairborn, Ohio, the Fairborn-10 robotic telescope was moved to Mount Hopkins in 1985. Since 1986 the Fairborn-10 has observed a group of approximately 80 program stars in the V, R, and I bandpasses from Mount Hopkins for the Smithsonian Astrophysical Observatory. Baliunas et al. 1987 presented some initial results from the first year of operation and estimated a precision of 1%–2%, noting that there had been some problems with the photometer. Henry, Armour & Baliunas 1991 have analyzed several years of data from the Fairborn-10 in order to determine more accurately the precision possible with that telescope. A number of stars on the observing menu were giant stars on the observing program of the Mount Wilson H & K Project (Radick, Lockwood & Baliunas 1990, Baliunas 1990). Three of these stars (6 Canum Venaticorum, 19 Puppis, and 63 Eridani) were selected to determine the external uncertainty because they showed no detectable light variability over the five-year interval of observation. Only the final year of data on these three stars was used for the error analysis as earlier data had indicated problems with the detector and amplifier. The mean external errors for these three stars, calculated with the same procedure described above for the Phoenix-10, were ± 0.012, ± 0.013, and ± 0.014 mag, respectively, in V, R, and I. Again, these are upper limits to the accuracy of the Fairborn-10, since they assume strict constancy for those three pairs of stars. Armour, Henry & Baliunas 1990 analyzed five years of data on the semiregular variable SW Virginis, providing an example of the quality of data obtainable with the Fairborn-10.

The Vanderbilt/Tennessee State 40-cm robotic telescope has been observing approximately 100 chromospherically active stars in B and V since November 1987. Hooten & Hall 1990 analyzed photometry of 50 suspected variable stars where much of the data were taken with this 40-cm robotic telescope. For apparently constant stars or stars for which no coherent periodic variability could be demonstrated, external errors were calculated as above; for the periodic variable stars, they were calculated from residuals to best sine-curve fits. In several cases the resulting external errors were as small as $\pm$ 0.006 mag in V. In other cases the external errors were considerably larger, but Hooten & Hall 1990 attributed that to unrecognized and/or nonperiodic variability or, in the case of the variables they fit with one sine curve, to secular variation in the amplitude or departure from a strictly sinusoidal light-curve shape. The analysis of V478 Lyr in Hall et al. 1990 also included one year of 40-cm telescope data binned into four groups (19–22 in their Table III) that were fit with spot models. External errors ranged from $\pm$0.004 to $\pm$0.006 mag with a mean of $\pm$0.005. Although operated in the same way as the smaller 25-cm telescopes, the 40-cm telescope has been shown to be capable of somewhat better precision, perhaps because of its larger size.

Recent observations with the newly operational Smithsonian 75-cm automatic telescope have been made to determine the precision available with the newest generation of automatic telescopes employing CCD centering devices and full ATIS programming capability. Lockwood & Skiff 1988 have painstakingly monitored many solar-type stars with manual techniques and found that a large fraction were variable at the millimagnitude level. One of the pairs of stars they found to have the least variability was HD 124570 and HD 125451. These stars are of similar class (F6 IV and F5 IV) and brightness. To estimate the precision of the photometry of the 75-cm telescope, we monitored these two stars through a V filter approximately 50 times on every clear night for a period of 47 days. Twenty-one days of acceptable data were obtained. During each night, extinction stars were monitored to determine the extinction correction on an hourly basis. Only those data that occurred during periods in which acceptable extinction coefficients could be determined were retained. If the data varied by more than 5% during a night they were discarded, but only data from one night did not meet this selection criterion. Recommendations 1–9, 13, 14, and 16 from Section 4 were followed during these test observations, with the primary improvements over the smaller telescopes being CCD centering and frequent measurements of extinction. The mean external error for this pair of stars, calculated as outlined above for the other telescopes, was $\pm$0.0017 mag (1.7 millimag). The measurements of Lockwood & Skiff 1988 also showed an external error of 1.7 millimag for this pair, demonstrating that robotic telescopes can now produce measurement precision as good as that by careful human observers.

6. Conclusion

Preliminary indications are that the newest generation of robotic telescopes, fully programmable with the Automatic Telescope Instruction Set, equipped with CCD centering devices, and adhering to most of the recommendations for precision photometry found in Section 4, are now capable of matching the precision of the best human observers. Yet to be explored with the robotic telescopes are the full benefits obtainable by accurately controlling the temperature of the detector and filters, using multiple comparison stars, measuring extinction several times an hour, and using a set of filters that satisfies the sampling theorem (recommendations 10, 11, 12, and 15). By fully implementing all 15 of these recommendations with the newest robotic telescopes, i.e., by working both harder and smarter, it may well be possible to do ground-based photometry with an overall precision of 0.001 mag or better.

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