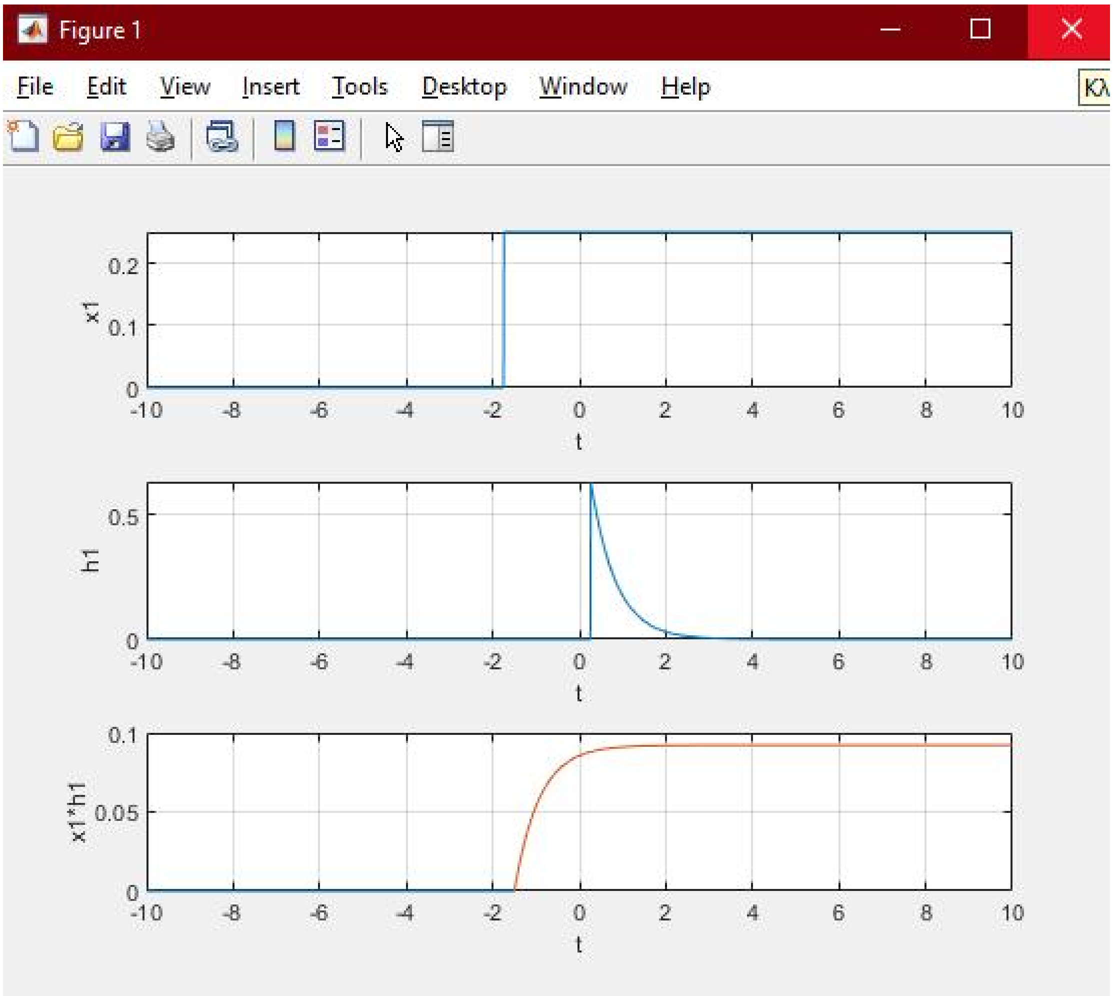
Άσκηση α

- x1*h1 = $\int_{-\infty}^{+\infty} x(\tau)\cdot h(t-\tau)d\tau = \int_{-\infty}^{+\infty} k\cdot u(\tau+m)\cdot e^{-m\cdot(t-\tau)}\cdot u(t-\tau-k)d\tau$

<u>Για t<k-m:</u> x1*h1=0

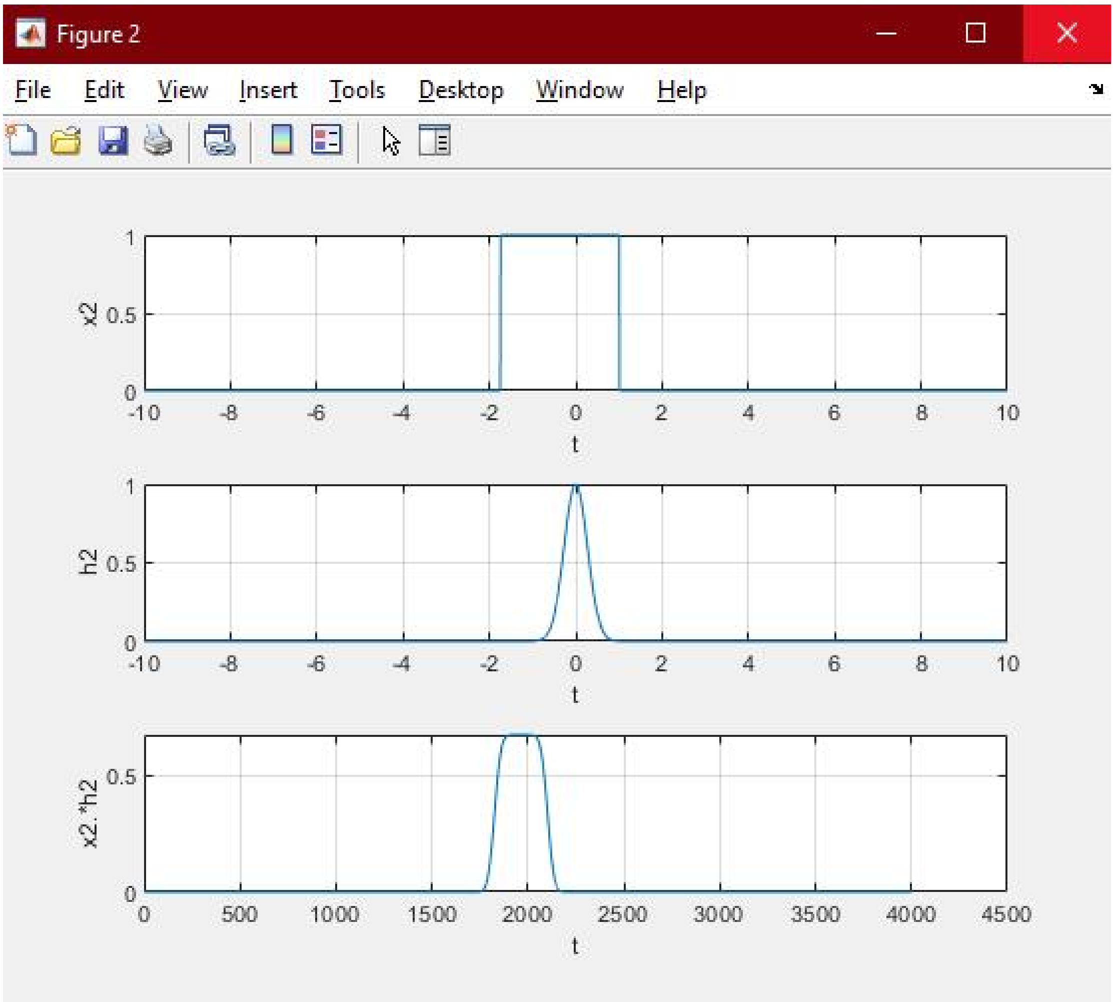
<u>Για t>k-m:</u> x1*h1 = $\int_{-m}^{t-k} k\cdot e^{-m\cdot(t-\tau)}d\tau = k\cdot e^{-m\cdot t}\cdot\int_{-m}^{t-k} e^{m\cdot\tau}d\tau = k\cdot e^{-m\cdot t}\cdot\left[\frac{e^{m\cdot\tau}}{m}\right]_{-m}^{t-k} =$

$$= \frac{k\cdot e^{-m\cdot t}}{m}\cdot\left(e^{m(t-k)} - e^{-m^2}\right) = \frac{k}{m}\cdot\left(e^{-m\cdot k} - e^{-m\cdot(t+m)}\right)$$



- x2*h2 = $\int_{-\infty}^{+\infty} x(\tau)\cdot h(t-\tau)d\tau = \int_{-\infty}^{+\infty}\left(u(\tau+m)-u(\tau-1)\right)\cdot e^{-\frac{m\cdot(t-\tau)^2}{k}}d\tau =$

$\int_{-m}^{1} e^{-\frac{m\cdot(t-\tau)^2}{k}}d\tau$ = Δεν έχει λύση



- x3*h3 =

$$\int_{-\infty}^{+\infty} h(\tau)\cdot x(t-\tau)d(\tau) = \int_{-\infty}^{+\infty}\left(k\cdot u(\tau)-m\right)\cdot e^{-k\cdot(t-\tau)}\cdot\left(u(t-\tau+2)-u(t-\tau-k)\right)d\tau$$

<u>Για t<-2:</u>

$$\int_{t-k}^{t+2} -m\cdot e^{-k\cdot(t-\tau)}d\tau = -m\cdot e^{-k\cdot t}\cdot\int_{t-k}^{t+2} e^{k\cdot\tau}d\tau = -m\cdot e^{-k\cdot t}\cdot\left[\frac{e^{k\tau}}{k}\right]_{t-k}^{t+2} = \frac{-m\cdot e^{-k\cdot t}}{k}\cdot\left(e^{k\cdot t+2\cdot k}-e^{k\cdot t-k^2}\right) =$$

$$= \frac{-m}{k}\cdot\left(e^{2\cdot k}-e^{-k^2}\right)$$

Για -2<t<k:

$$\int_{t-k}^{0} -m\cdot e^{-k\cdot(t-\tau)}d\tau + \int_{0}^{t+2}(k-m)\cdot e^{-k\cdot(t-\tau)}d\tau = -m\cdot e^{-k\cdot t}\cdot\int_{t-k}^{0}e^{k\cdot\tau}d\tau + (k-m)\cdot e^{-k\cdot t}\cdot\int_{0}^{t+2}e^{k\cdot\tau}d\tau =$$

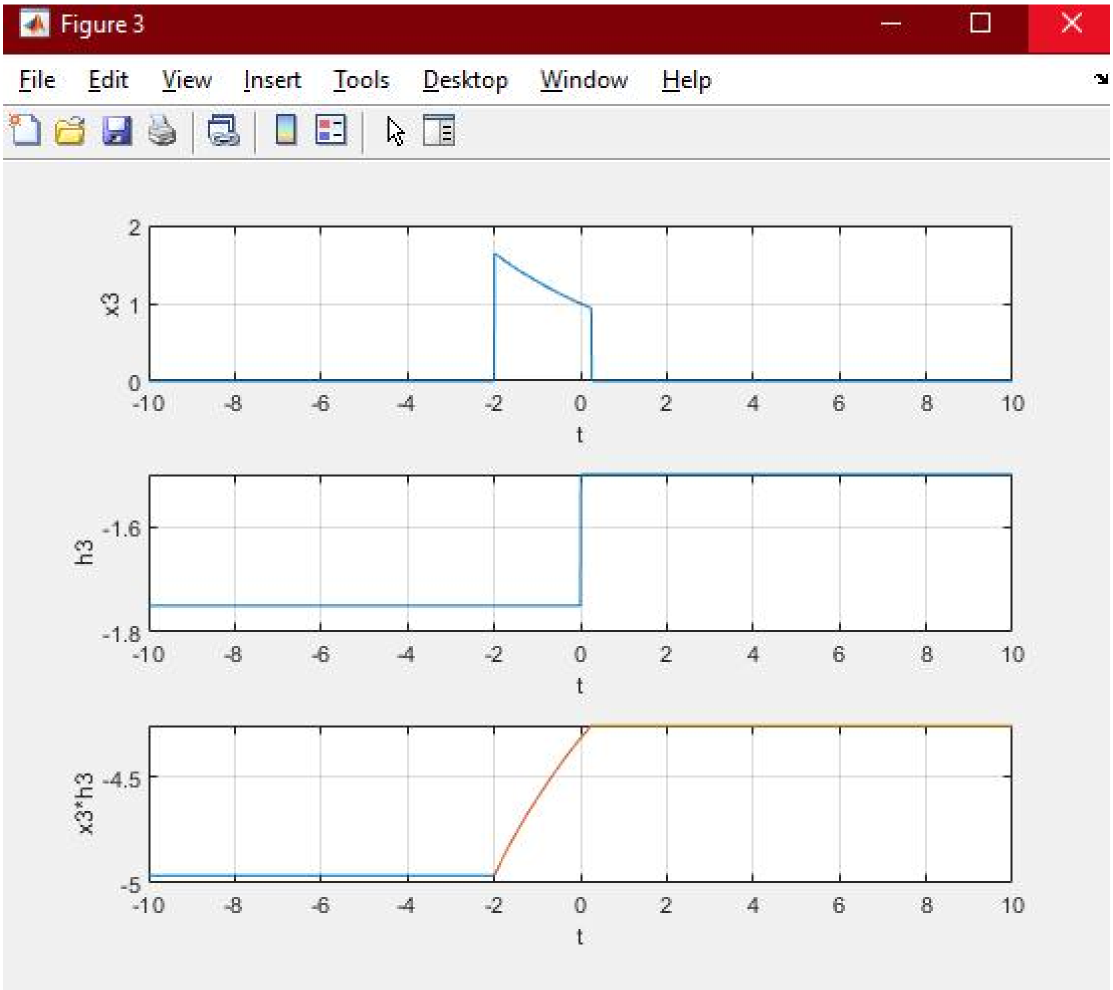
$$= -m\cdot e^{-k\cdot t}\cdot\left[\frac{e^{k\tau}}{k}\right]_{t-k}^{0} + (k-m)\cdot e^{-k\cdot t}\cdot\left[\frac{e^{k\tau}}{k}\right]_{0}^{t+2} = \frac{-m\cdot e^{-k\cdot t}}{k}\cdot\left(1-e^{k\cdot t-k^2}\right)+\frac{(k-m)\cdot e^{-k\cdot t}}{k}\cdot\left(e^{k\cdot t+2\cdot k}-1\right)=$$

$$=\frac{-m}{k}\cdot\left(e^{-k\cdot t}-e^{-k^2}\right)+\frac{k-m}{k}\cdot\left(e^{2\cdot k}-e^{-k\cdot t}\right)$$

Για t>k:

$$\int_{t-k}^{t+2}(k-m)\cdot e^{-k\cdot(t-\tau)}d\tau = (k-m)\cdot e^{-k\cdot t}\cdot\int_{t-k}^{t+2}e^{k\cdot\tau}d\tau = (k-m)\cdot e^{-k\cdot t}\cdot\left[\frac{e^{k\tau}}{k}\right]_{t-k}^{t+2} = \frac{(k-m)\cdot e^{-k\cdot t}}{k}\cdot\left(e^{k\cdot t+2\cdot k}-e^{k\cdot t-k^2}\right)=$$

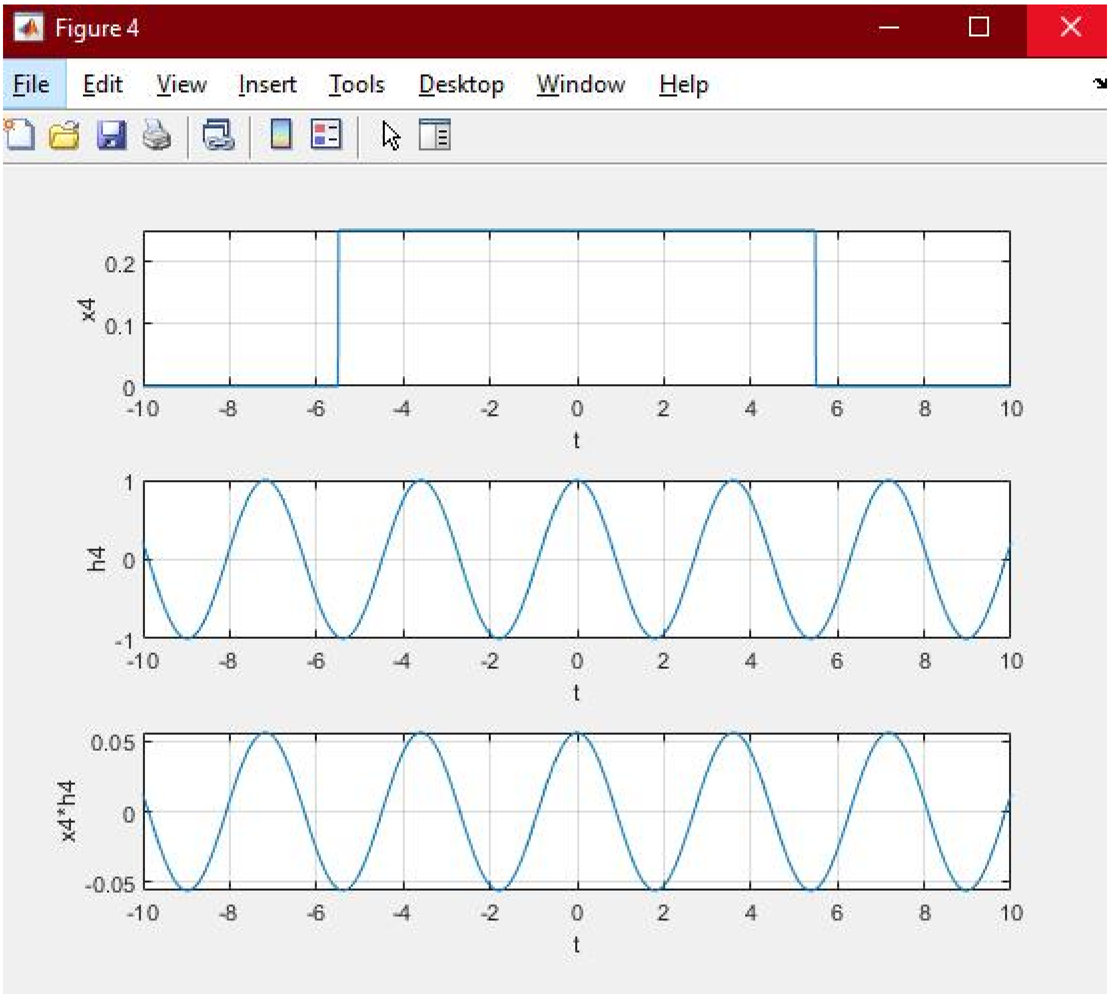
$$=\frac{k-m}{k}\cdot\left(e^{2\cdot k}-e^{-k^2}\right)$$



- x4*h4 =

$$\int_{-\infty}^{+\infty} h(\tau)\cdot x(t-\tau)d(\tau) = \int_{-\infty}^{+\infty} k\cdot\left(u(t-\tau+m\cdot\pi)-u(t-\tau-m\cdot\pi)\right)\cdot\cos(m\cdot\tau)d\tau =$$

$$\int_{t-m\cdot\pi}^{t+m\cdot\pi} k\cdot\cos(m\cdot\tau)d\tau = k\cdot\left[\frac{-\sin(m\cdot\tau)}{m}\right]_{t-m\cdot\pi}^{t+m\cdot\pi} = \frac{-k}{m}\cdot\left(\sin(m\cdot t+m^2\cdot\pi)-\sin(m\cdot t-m^2\cdot\pi)\right)$$



- x5*h5 =

$$\int_{-\infty}^{+\infty} x(\tau)\cdot h(t-\tau)d(\tau) = \int_{-\infty}^{+\infty} \left(u(\tau+1)-u(\tau-2)\right)\cdot m\cdot\left(u(t-\tau+k)-u(t-\tau-k)\right)d\tau$$

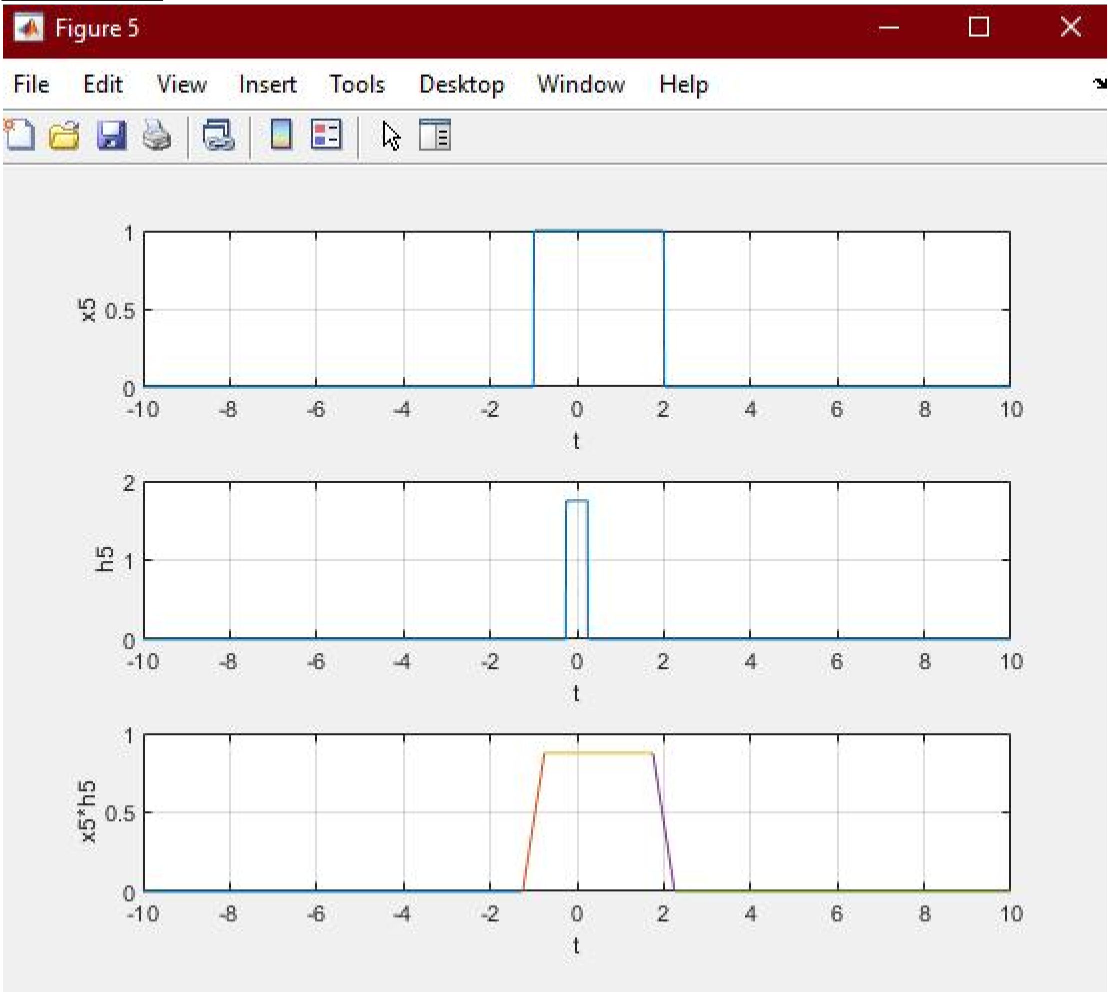
<u>Για t<-1-k:</u> x5*h5=0

<u>Για -1-k<t<-1+k:</u> $\int_{-1}^{t+k} m d\tau = m\cdot[\tau]_{-1}^{t+k} = m\cdot(t+k+1)$

Για -1+k<t<2-k: $\int_{t-k}^{t+k} m d\tau = m \cdot [\tau]_{t-k}^{t+k} = m \cdot (t+k-t+k) = 2 \cdot m \cdot k$

Για 2-k<t<2+k: $\int_{t-k}^{2} m d\tau = m \cdot [\tau]_{t-k}^{2} = m \cdot (2-t+k)$

Για t>2+k: x5*h5=0



Άσκηση β

$x(t) = \pi^2 - t^2$, $-\pi \le t \le \pi$

x(t) = $\alpha_0 + \sum_{n=1}^{+\infty}(\alpha_n \cdot \cos(n \cdot \omega_0 \cdot t) + b_n \cdot \sin(n \cdot \omega_0 \cdot t))$

$T_0 = 2 \cdot \pi$, $\omega_0 = \frac{2 \cdot \pi}{T_0} = 1$

$\alpha_0 = \frac{1}{2 \cdot \pi} \int_{-\pi}^{\pi} (\pi^2 - t^2) dt = \frac{1}{2 \cdot \pi}[\pi^2 \cdot t - \frac{t^3}{3}]_{-\pi}^{\pi} = \frac{1}{2 \cdot \pi}[\pi^3 - \frac{\pi^3}{3} - (-\pi^3 + \frac{\pi^3}{3})] =$

$= \frac{1}{2 \cdot \pi}(2 \cdot \pi^3 - \frac{2 \cdot \pi^3}{3}) = \pi^2 - \frac{\pi^2}{3} = \frac{2 \cdot \pi^2}{3}$

Άρα $\alpha_0 = \frac{2 \cdot \pi^2}{3}$

$\alpha_n = \frac{2}{2\pi} \int_{-\pi}^{\pi} (\pi^2 - t^2) \cdot \cos(n \cdot t) dt = \frac{1}{\pi} \int_{-\pi}^{\pi} \pi^2 \cdot \cos(n \cdot t) dt - \frac{1}{\pi} \int_{-\pi}^{\pi} t^2 \cos(n \cdot t) dt$

- $\frac{1}{\pi} \int_{-\pi}^{\pi} \pi^2 \cos(n \cdot t) = \frac{1}{\pi}[\pi^2 \cdot \frac{\sin(n \cdot t)}{n}]_{-\pi}^{\pi} = 0$, διότι sin(nπ) = 0 και sin(-nπ) = -sin(nπ) = 0

- $-\frac{1}{\pi} \int_{-\pi}^{\pi} t^2 \cdot \cos(n \cdot t) dt = -\frac{1}{\pi} \cdot [t^2 \cdot \frac{\sin(n \cdot t)}{n}]_{-\pi}^{\pi} + \frac{1}{\pi} \int_{-\pi}^{\pi} 2 \cdot t \cdot \frac{\sin(n \cdot t)}{n} dt =$

$= 0 + \frac{1}{\pi}[-2 \cdot t \cdot \frac{\cos(n \cdot t)}{n^2}]_{-\pi}^{\pi} - \frac{1}{\pi} \int_{-\pi}^{\pi} -2 \cdot \frac{\cos(n \cdot t)}{n^2} dt = -\frac{1}{\pi} \cdot [2 \cdot \pi \cdot \frac{\cos(n \cdot \pi)}{n^2} - (-2 \cdot \pi \cdot \frac{\cos(-n \cdot \pi)}{n^2})] + 0 =$

$= -\frac{1}{\pi} \cdot (\frac{2 \cdot \pi \cdot \cos(n \cdot \pi)}{n^2} + \frac{2 \cdot \pi \cdot \cos(n \cdot \pi)}{n^2}) = -\frac{1}{\pi} \cdot \frac{4 \cdot \pi \cdot \cos(n \cdot \pi)}{n^2} = -\frac{4 \cdot \cos(n \cdot \pi)}{n^2} = -\frac{4}{n^2} \cdot (-1)^n$

Άρα $\alpha_n = -\frac{4}{n^2} \cdot (-1)^n$

$b_n = \frac{2}{2\pi} \int_{-\pi}^{\pi} (\pi^2 - t^2) \cdot \sin(n \cdot t) dt = \frac{1}{\pi} \int_{-\pi}^{\pi} \pi^2 \cdot \sin(n \cdot t) dt - \frac{1}{\pi} \int_{-\pi}^{\pi} t^2 \sin(n \cdot t) dt$

- $$\frac{1}{\pi}\int_{-\pi}^{\pi}\pi^2\sin(n\cdot t)=\frac{1}{\pi}[-\pi^2\cdot\frac{\cos(n\cdot t)}{n}]_{-\pi}^{\pi}=-\frac{1}{\pi}(\pi^2\cdot\frac{\cos(n\cdot\pi)}{n}-\pi^2\cdot\frac{\cos(-n\cdot\pi)}{n})=0$$

  , διότι cos(-x) = cos(x)

- $$-\frac{1}{\pi}\int_{-\pi}^{\pi}t^2\cdot\sin(n\cdot t)dt\quad=-\frac{1}{\pi}\cdot[-t^2\cdot\frac{\cos(n\cdot t)}{n}]_{-\pi}^{\pi}+\frac{1}{\pi}\int_{-\pi}^{\pi}-2\cdot t\cdot\frac{\cos(n\cdot t)}{n}dt=$$

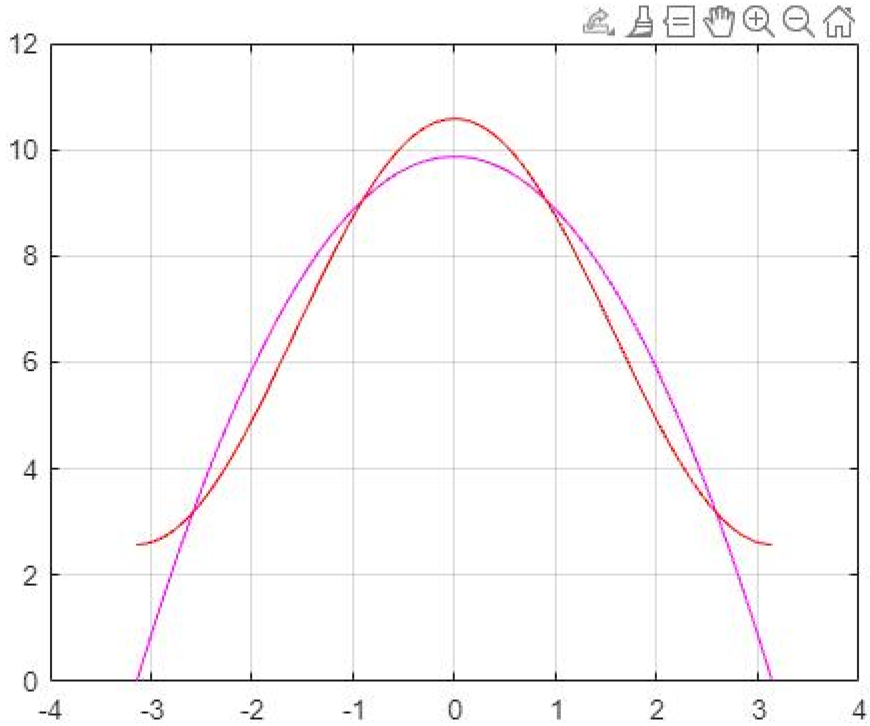
$$=-\frac{1}{\pi}\cdot(-\frac{\pi^2\cdot\cos(n\cdot\pi)}{n}+\frac{\pi^2\cdot\cos(n\cdot\pi)}{n})-\frac{1}{\pi}[2\cdot t\cdot\frac{\sin(n\cdot t)}{n^2}]_{-\pi}^{\pi}+\frac{1}{\pi}\int_{-\pi}^{\pi}2\cdot\frac{\sin(n\cdot t)}{n^2}dt=$$

$$=-0-0+\frac{1}{\pi}[-2\cdot\frac{\cos(n\cdot t)}{n^3}]_{-\pi}^{\pi}=-\frac{1}{\pi}\cdot(\frac{2\cdot\cos(n\cdot\pi)}{n^3}-\frac{2\cdot\cos(-n\cdot\pi)}{n^3})=0$$

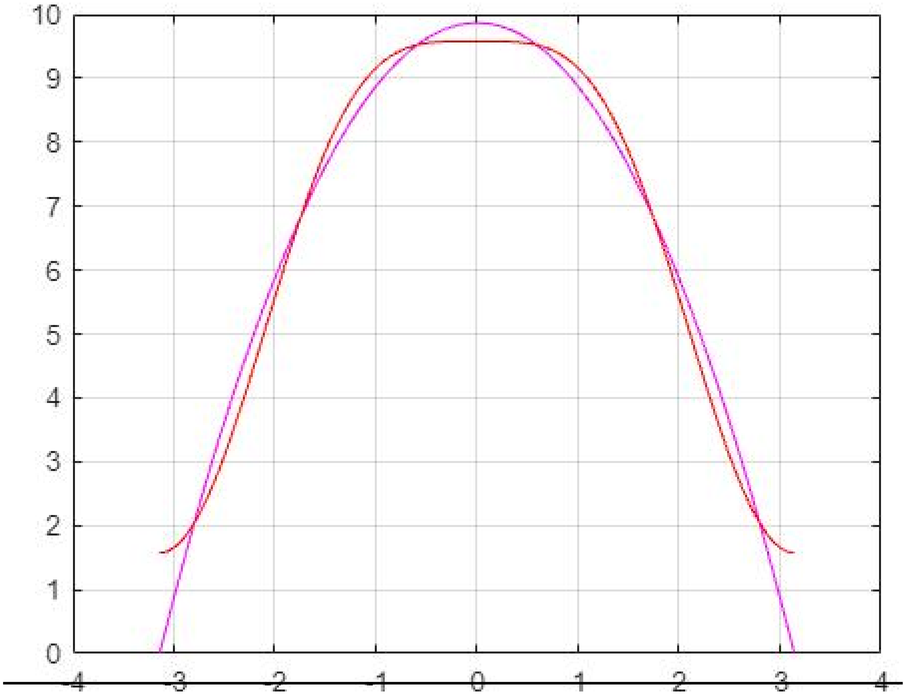
Άρα $b_n = 0$

Άρα $x(t)=\frac{2\cdot\pi^2}{3}+\sum_{n=1}^{+\infty}(-1)^n\cdot\frac{-4\cdot\cos(n\cdot t)}{n^2}$

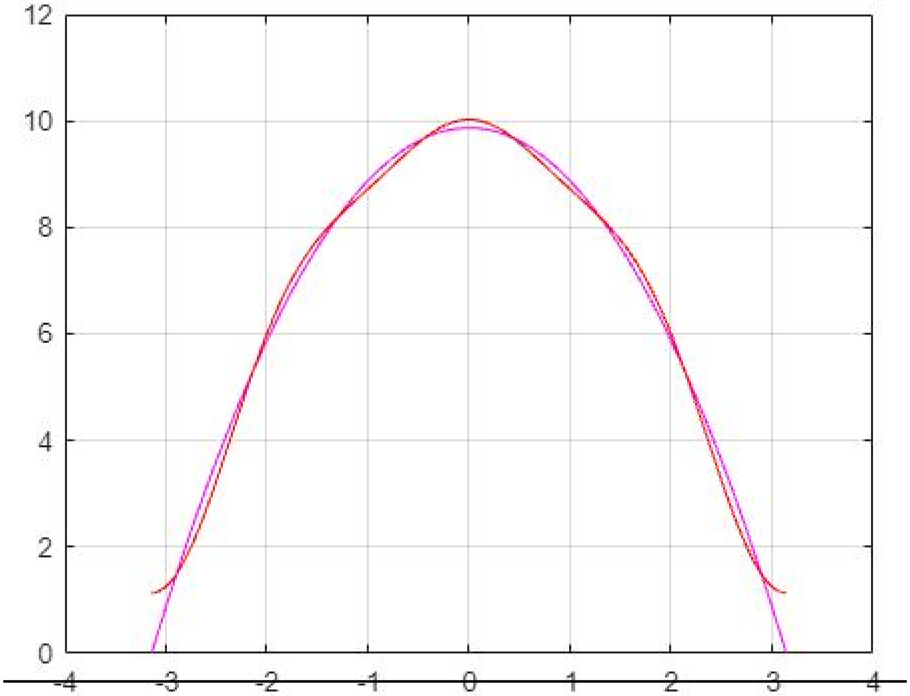
<u>Graph for n=1</u>



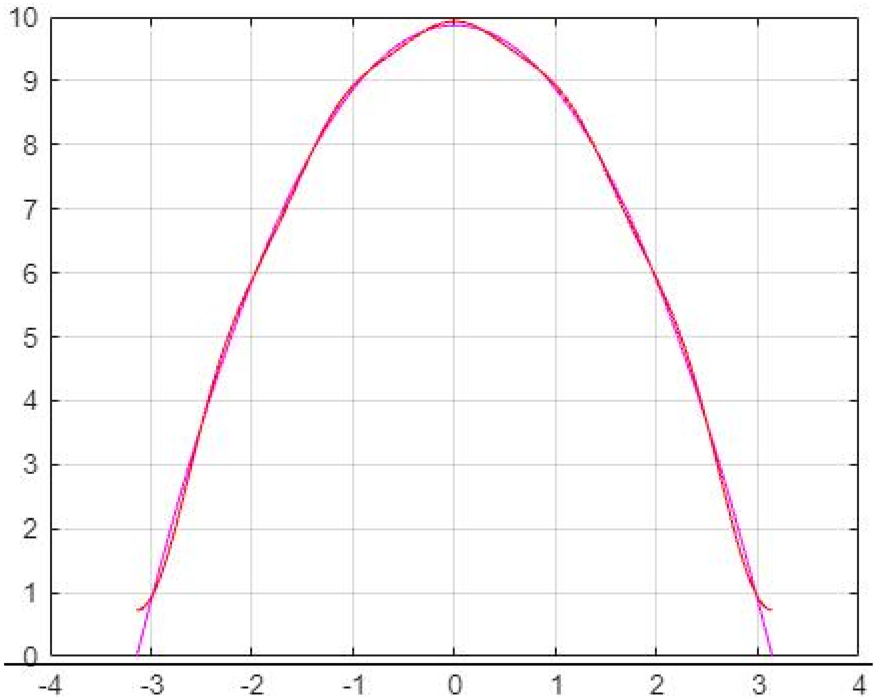
Graph for n=2



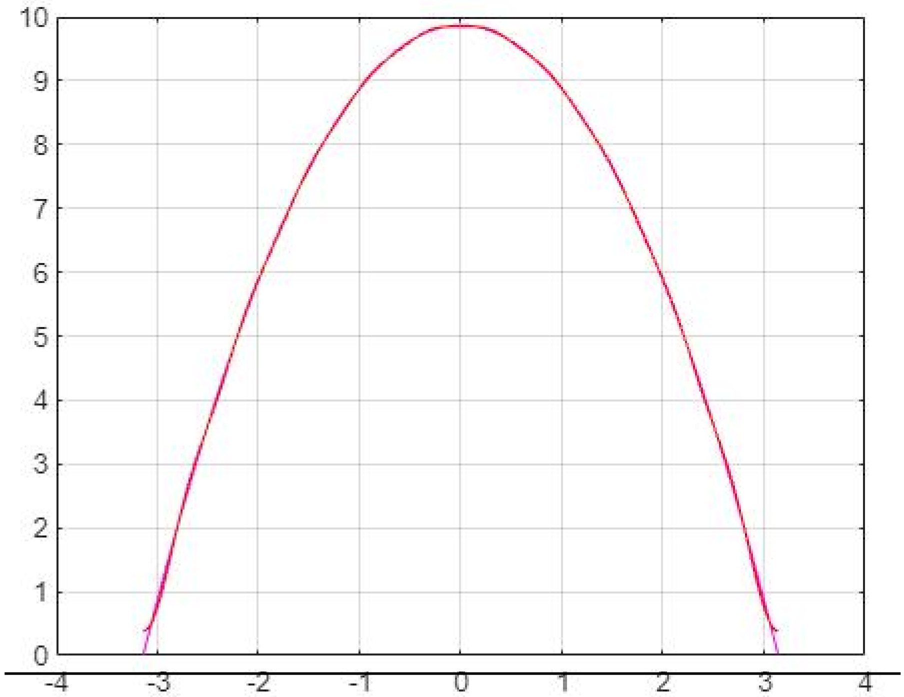
Graph for n=3



Graph for n=5



Graph for n=10



10
9
8
7
6
5
4
3
2
1
0
-4
-3
-2
-1
0
1
2
3
4