Imperial College London
Department of Mathematics
MSc in Mathematics and Finance
Academic year 2025–2026, Autumn term

MATH70110 Quantitative Risk Management

Assessed Coursework 1

Due date: 20 November 2025, 11:59pm

Instructions

- This assignment accounts for 20% of the total module score.
- Work in a group of no more than four members. Each member in the group will receive the same score. (Reminder: You will work in the same group for the second assignment)
- There is no formal word limit but ideally your work should not exceed 15 pages under a reasonable document layout (excluding tables, figures and other illustrations).
- You can use any programming languages you like but Python is recommended.
- You will not be penalised for "bad programming styles" provided that correct answers are generated. There is no need to provide comments to your codes or Python notebook.
- One written pdf document and relevant program file(s) to be sent via Blackboard by the deadline. Indicate all members' names and student IDs on the report clearly.

Part A: Stylised facts and GARCH modelling

This part is about statistical analysis and modelling of the *EURO STOXX 50* (symbol: SX5E) stock market index.[1] The index measures the performance of the 50 largest and most liquid Eurozone stocks. The file `QRM-2025-cw1-data-a.csv` contains its daily values at market close from 30 October 2013 to 27 October 2023. Using the index values, compute first the daily log returns (in percentage). Then:

[1] See https://en.wikipedia.org/wiki/EURO_STOXX_50.

(i) Compute the mean, standard deviation, skewness, and kurtosis of the log returns. Draw also a histogram including a kernel density estimate and a normal distribution (density) fitted to the log returns. What can you conclude?

(ii) Draw the empirical ACFs of the log returns, their absolute values, and their squares. What can you conclude?

(iii) Fit a standard GARCH(1,1) model, with constant conditional mean and standard normal innovations, to the log return data. Report the model fitting results. Plot the absolute returns and fitted volatilities. Comment on the results.

(iv) Assess the goodness of fit of the model estimated in (iii). Do the standardised residuals look like an iid sample from the standard normal distribution?

(v) Fit an ARMA(1,1)–GARCH(1,1) model with Student t-distributed innovations. Does the new specification improve the fit, compared to the model in (iii)? Is the ARMA(1,1) part necessary?

Part B: Risk measures

Bob[2] thinks it's time to short US stocks.[3] Following his belief, but unwilling to build the short position himself, he invests in the *ProShares Short Dow30* (symbol: DOG) exchange-traded fund (ETF). Using derivatives, DOG seeks to deliver a daily return that is *minus one times* the return on the *Dow Jones Industrial Average*.[4] Being a diligent, risk-conscious investor, Bob wants to monitor his daily linearised loss, relative to the value[5] of the investment, given by

$$\tilde{L}^{\Delta}_{t+1} = \frac{L^{\Delta}_{t+1}}{V_t} = -r_{t+1}. \tag{1}$$

where r_{t+1} is the log return on DOG. Daily closing prices of DOG from 30 October 2013 to 27 October 2023 are provided in the file `QRM-2025-cw1-data-b.csv`.

Help Bob out by implementing and evaluating *one-day-ahead VaR and ES forecasts* for the loss (1), at 95% and 99% confidence levels, from an appropriate starting date until 27 October 2023. Compute the forecasts using each of the following three methods:

[2] a fictional character.....

[3] This does not constitute financial advice.

[4] See https://www.proshares.com/our-etfs/leveraged-and-inverse/dog.

[5] which he prefers not to disclose

(i) *Historical simulation* (HS) — use a rolling window scheme, whereby for each forecast you use the data over the past 500 trading days to compute the HS forecasts of VaR and ES (see Part 2, slide 54 - 55).

(ii) *Filtered historical simulation* (FHS) with *exponentially-weighted moving average* (EWMA) — generate first volatility forecasts for the entire time series by applying the EWMA scheme with $\alpha = 0.06$ and $\hat{\mu}_t = 0$ for all t (see Part 3, slide 97), and with a reasonable starting value $\hat{\sigma}_0$. Use the formulae in Part 3, slide 101 to compute the VaR and ES forecasts. Determine the distribution F_Z by applying historical simulation to the standardised EWMA residuals, within a rolling window over the past 500 trading days.

(iii) *FHS with GARCH* — use a rolling window scheme, whereby you estimate a GARCH(1,1) model, with constant conditional mean and standard normal innovations, every day using the data over the past 500 trading days. Determine the distribution F_Z by applying historical simulation to the standardised GARCH residuals and use then the formulae in Part 3, slide 101 and the one-day-ahead GARCH volatility forecast to compute the VaR and ES forecasts.

Present the results, for each risk measure and confidence level, by drawing a figure where the actual losses and forecasts, using (i), (ii), and (iii), are plotted.

Finally, *backtest* all forecasts; in the case of VaR, use both unconditional coverage and joint coverage–independence tests. Report and interpret your results - in particular, which forecasting method would you recommend to Bob?

Other tips, recommendations and best practices

- When fitting a time series model, you don't need to estimate the model yourself "by hand" but instead feel free to use any available packages (provided that you mention the packages used in your report). Examples include the arch[6] package for Python, and the rugarch/fGarch[7] packages for R.
- Only include a table/figure in the report if it is discussed. If it is never mentioned in the report, then it shouldn't be there in the first place.

[6] See https://pypi.org/project/arch/.

[7] See https://cran.r-project.org/web/packages/rugarch/ and https://cran.r-project.org/web/packages/fGarch/index.html.

- Label all the tables and figures, provide a caption for each of them, and cite them by their labels when they are mentioned in text. Avoid phrases like "The figure above shows that..." but say instead "Figure 2 shows that...".
- Whenever possible, all axes in a graph should be labelled.
- Export the graphical outputs from your programming software properly (as pdf, png, jpg, etc...) and then import them to the report. Avoid using screen-shot.
- When reporting numerical results, use judgement to determine a sensible number of digits to be displayed.
- If you are using ideas or sentences from other sources, cite them properly and include a list of references at the end of your report.