Non-negative Matrix Factorization

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TSIA 206 - Speech and audio processing

Part I

Motivation and applications

Non-negative Matrix Factorization (NMF)

- Very popular tool for data analysis
- The original paper [1] has got 7000+ citations
- One of the standard tools for data mining
- Has found applications in many domains
  - Audio/Speech/Music processing
  - Computer vision
  - Recommender systems ...

[1] Lee, D., Seung, H. "Learning the parts of objects by non-negative matrix factorization." Nature vol. 401, pp. 788–791 (1999).

Applications to audio

- Audio source separation
- Audio restoration
- Music transcription
- ...

Audio source separation :

Audio source separation :

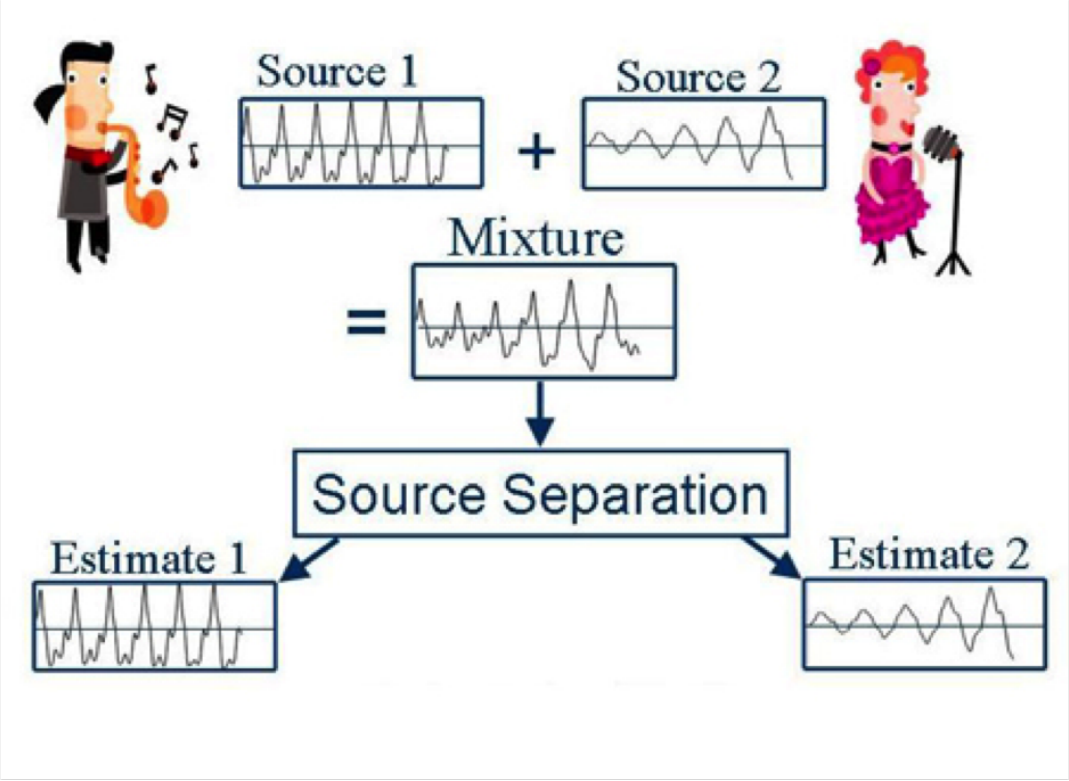


Image taken from http ://music.cs.northwestern.edu/

Audio source separation

Music remixing

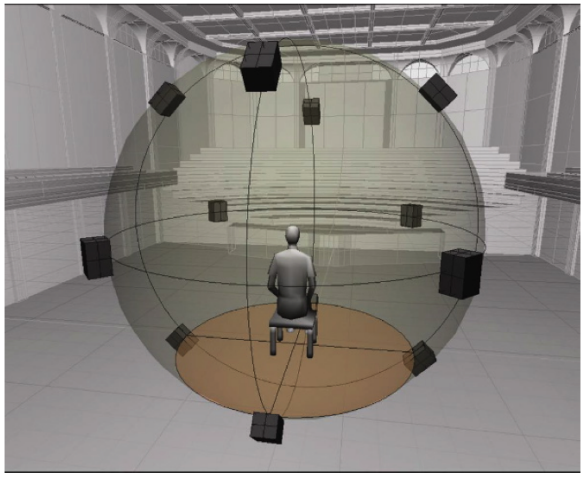
emixing and content creation

Image taken from Bryan and Sun, 2013

Audio source separation

Spatial audio up-mixing

Spatial audio and upmixing



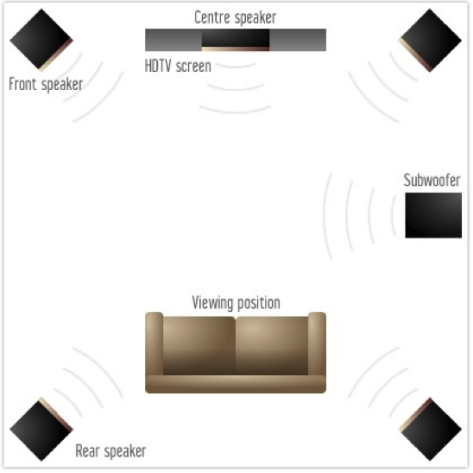


Image taken from Bryan and Sun, 2013

Audio source separation

Audio denoising

Audio restoration

Restore missing/corrupted parts of audio

Music transcription

Given the audio signal, find the musical score

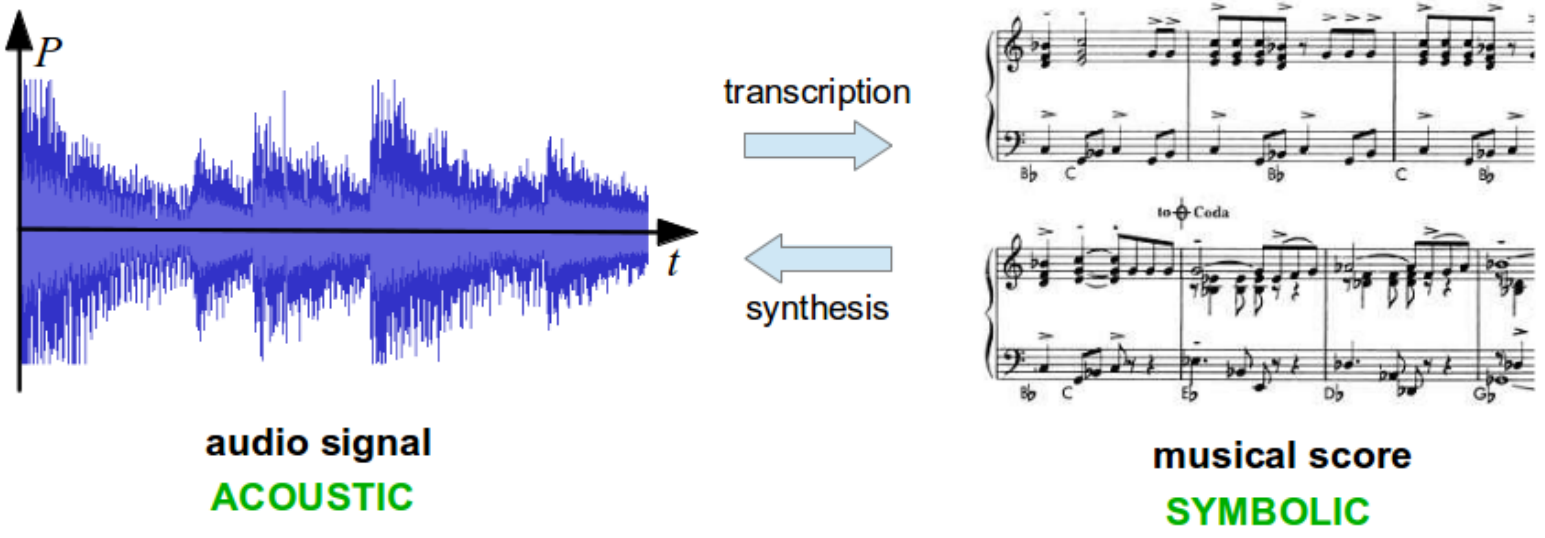


Image from http ://www-etud.iro.umontreal.ca/ boulanni/

Part II

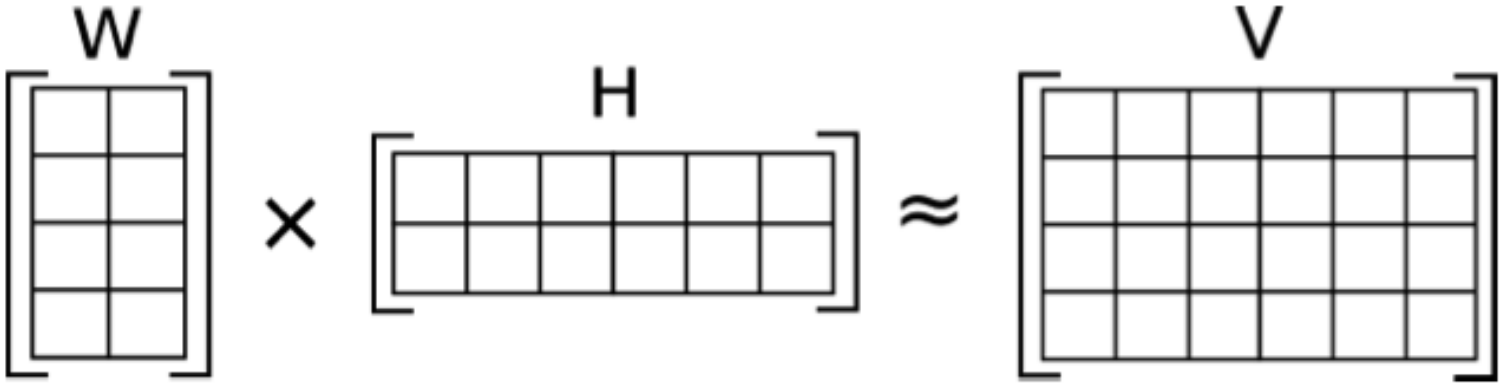
NMF

Non-Negative matrix factorization

- Aim : decompose a non-negative matrix V as :

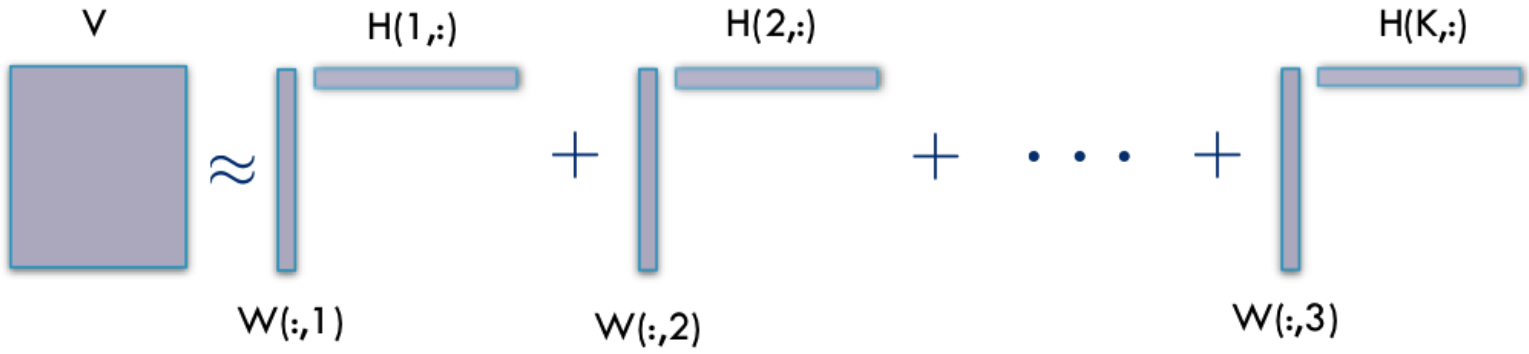
$$V \approx WH$$

- W and H are also non-negative and have lower rank than V :



Non-Negative matrix factorization

Another point of view

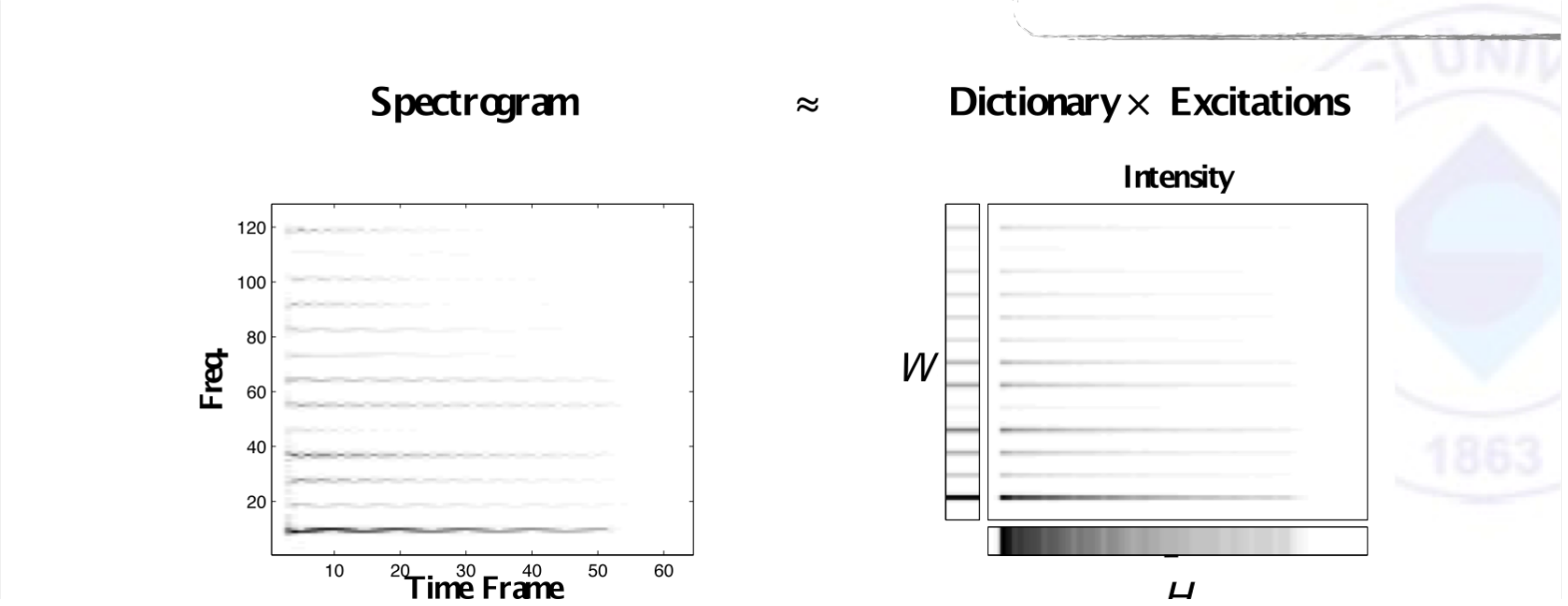


Non-Negative matrix factorization

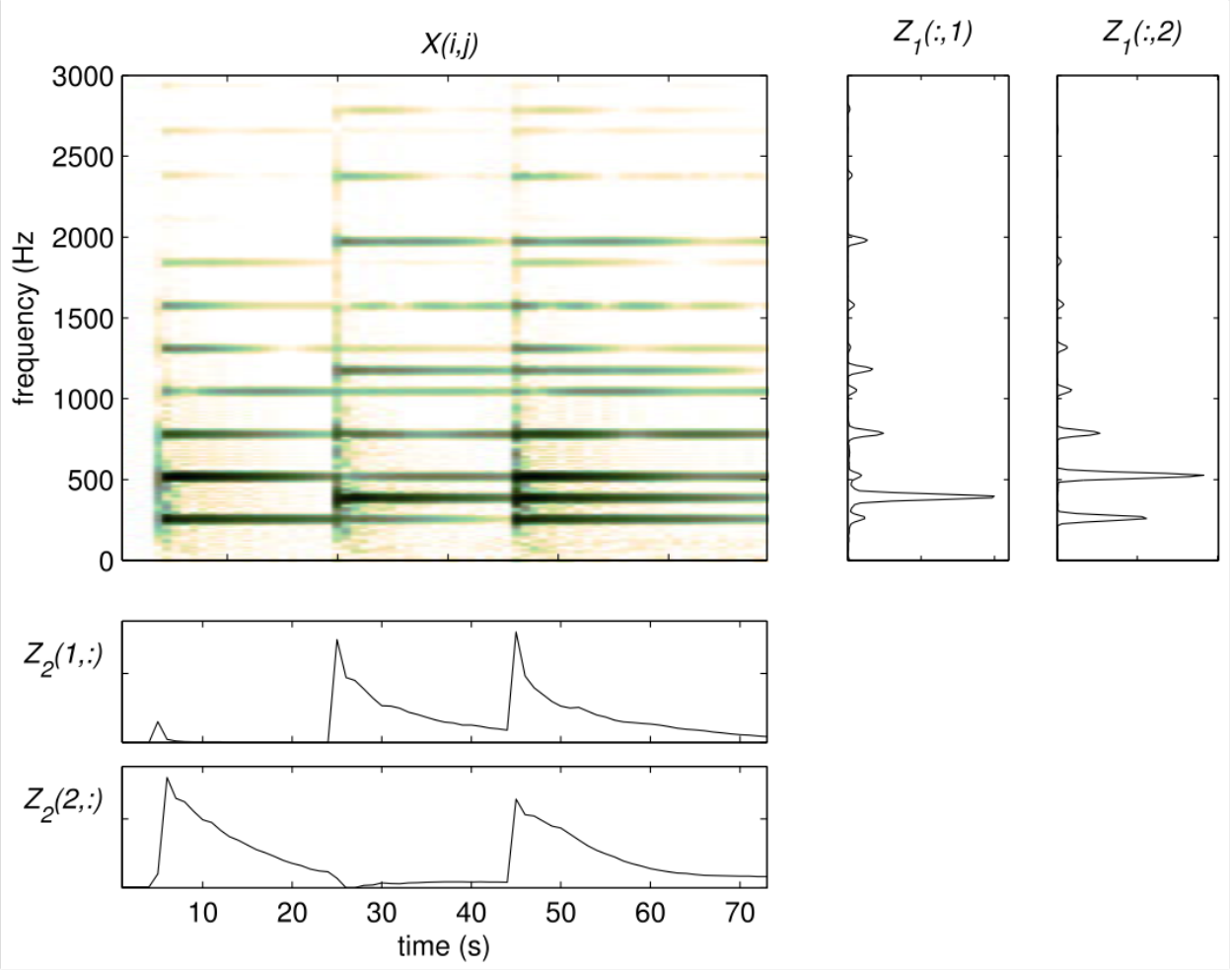
Matlab demo

NMF on audio

A simple audio example



NMF on audio


X(i,j)
$Z_1(:,1)$
$Z_1(:,2)$
3000
2500
2000
1500
1000
500
0
frequency (Hz
$Z_2(1,:)$
$Z_2(2,:)$
10
20
30
40
50
60
70
time (s)

NMF on audio

Mary had a little lamb

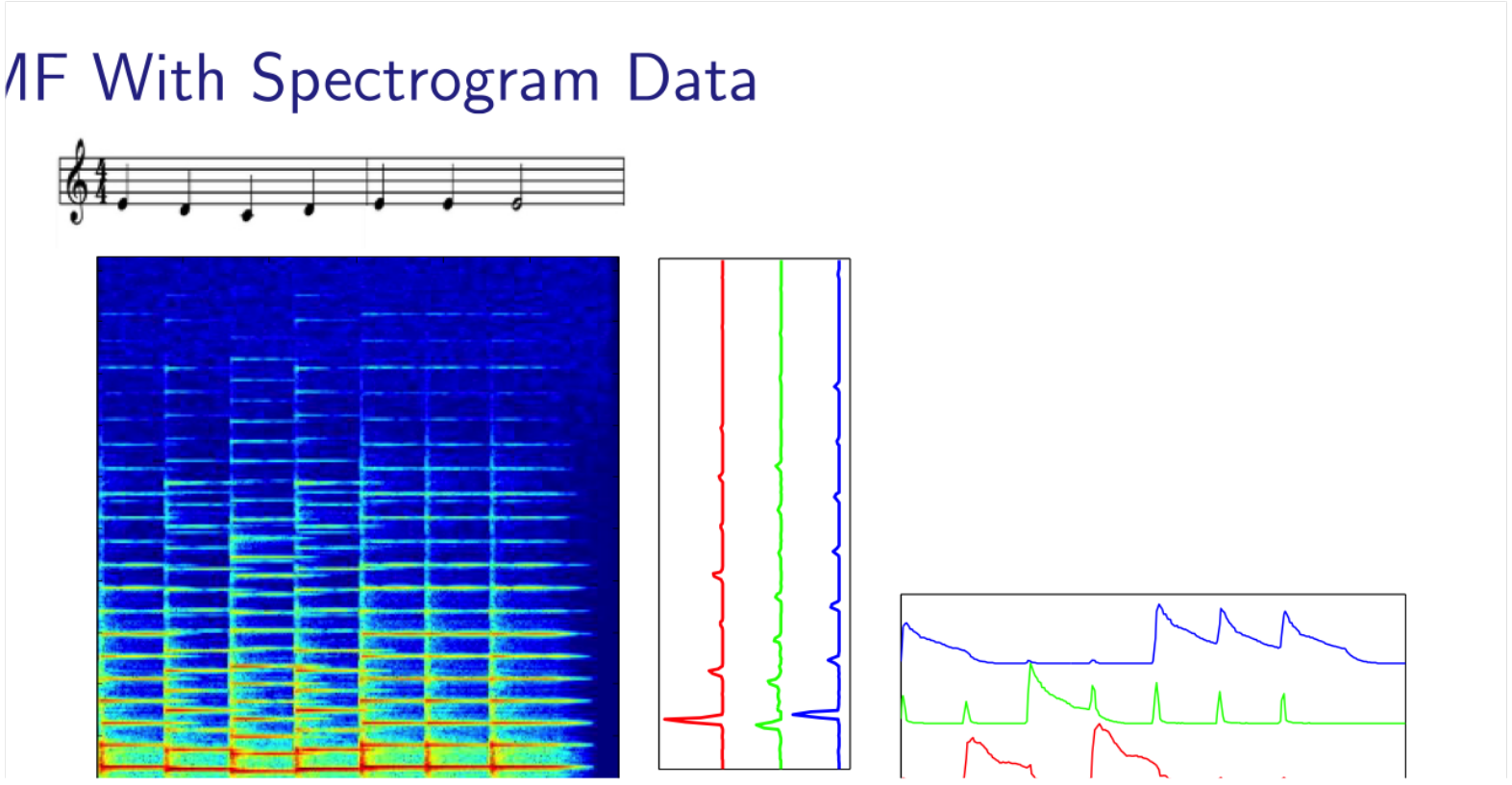


Image taken from Bryan and Sun, 2013

NMF on audio

Mary had a little lamb

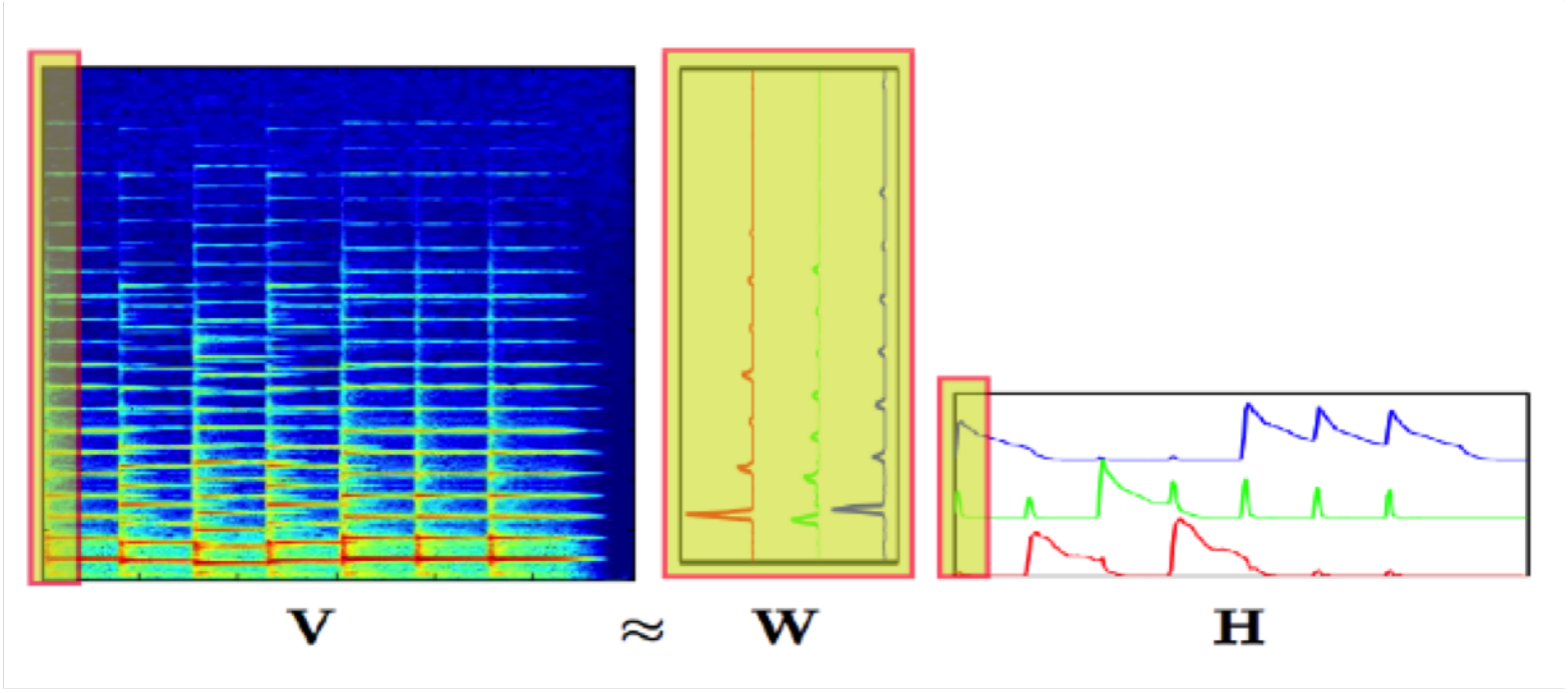
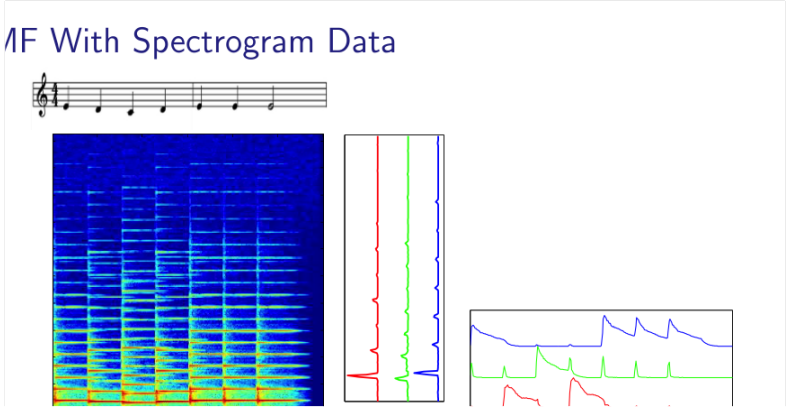


Image taken from Bryan and Sun, 2013

NMF on audio

Remember :



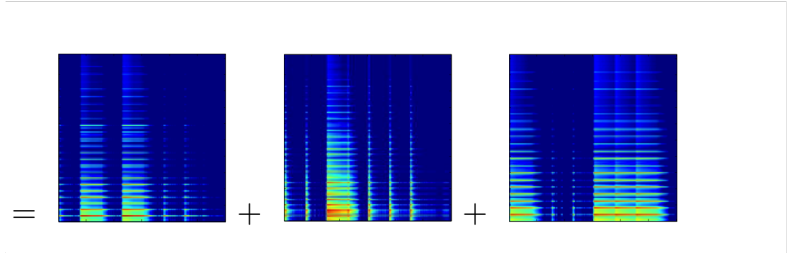
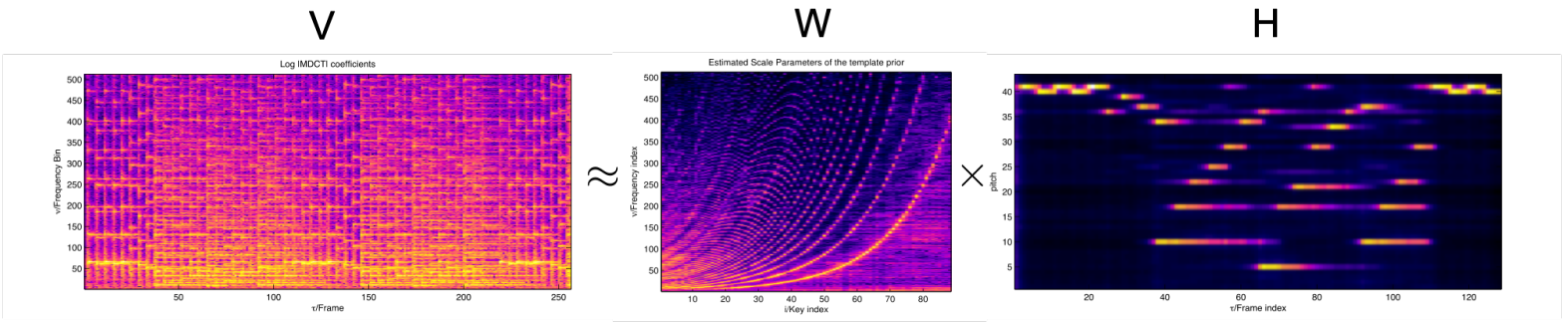


Image taken from Bryan and Sun, 2013

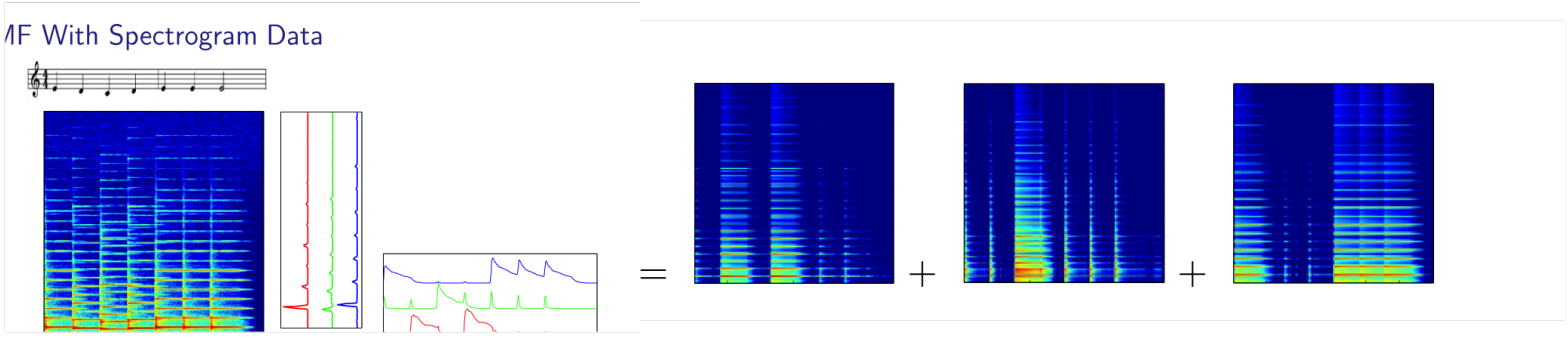
NMF on audio

Polyphonic piano music



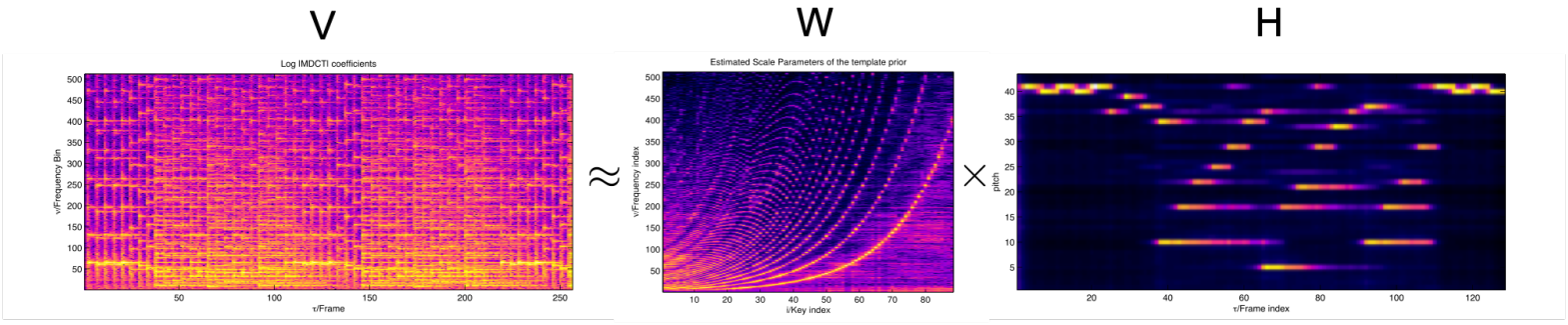
Applications revisited

Source separation : by using W and H we can estimate the sources



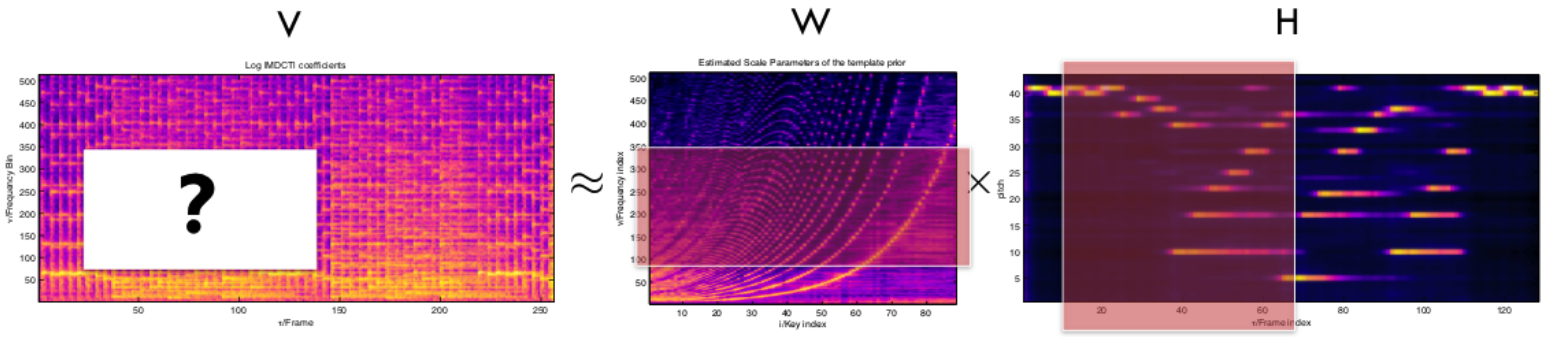
Applications revisited

Music transcription : if the columns of W correspond to different notes, H gives us an approximate transcription



Applications revisited

Audio restoration : use W and H to restore corrupted parts



Part III

How to estimate W and H?

Inference

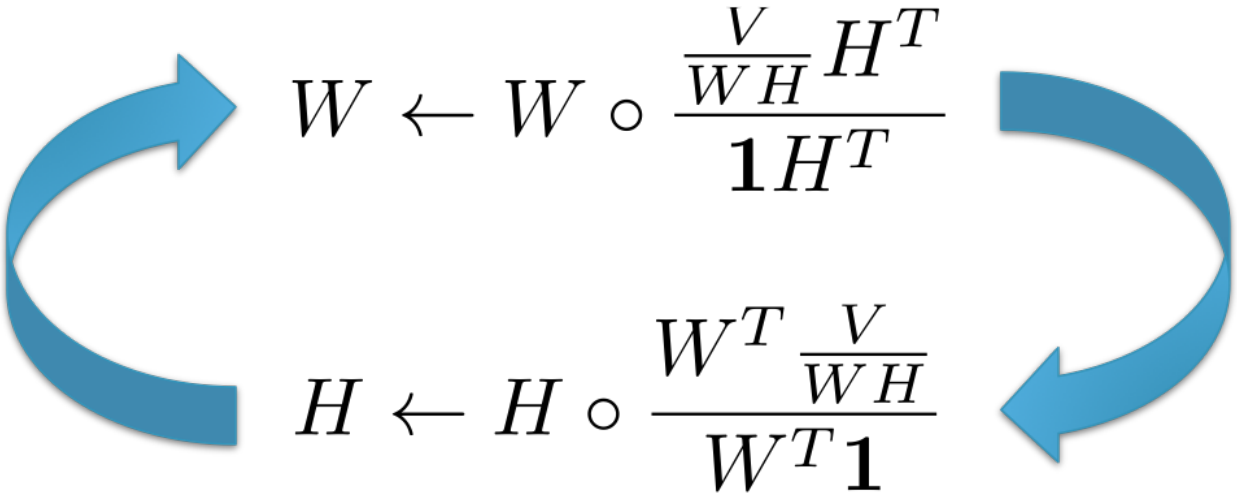
- We would like to find W and H so that some "error" metric is minimized between V and WH

$$\min_{W,H} \quad d(V||WH)$$

- $d(V||WH)$ measures the error between V and WH
  - Euclidean $\rightarrow \|V - WH\|_2^2$
  - Kullback-Leibler $\rightarrow V \ln(\frac{V}{WH}) - V + WH$ (we will use KL)
  - Itakura-Saito $\rightarrow$ also very popular in audio domain

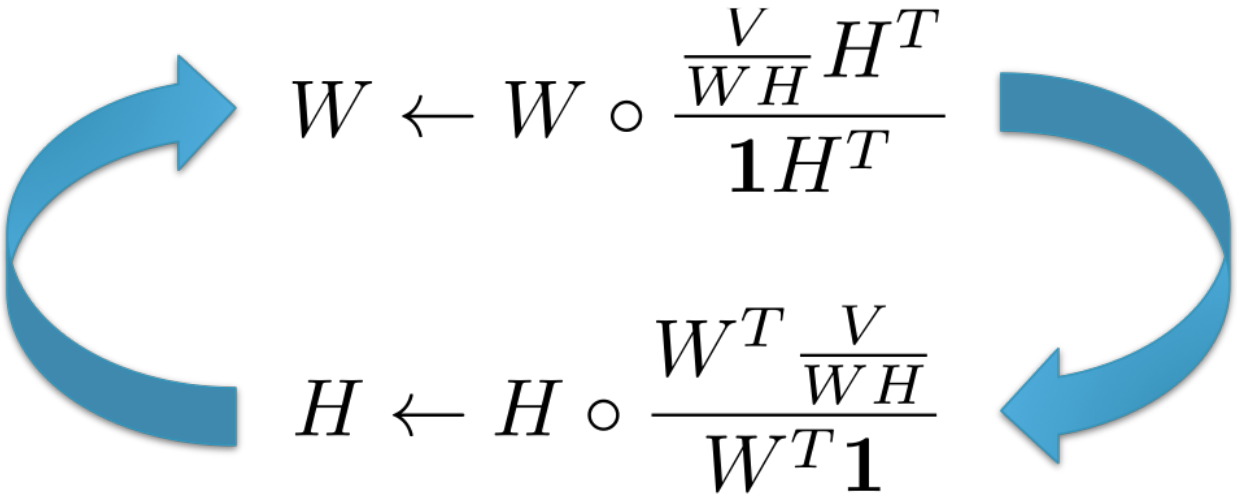
Multiplicative update rules (MUR)

- Easy to code : two lines in MATLAB
- Iteratively apply "multiplicative" updates to the factors (W and H)



Multiplicative update rules (MUR)

Derivation on the board



Multiplicative update rules (MUR)

Matlab demo

NMF on audio

- Recipe for audio applications
  1. Compute the STFT X
  2. Compute the magnitude spectrogram $V = |X|$
  3. Run the multiplicative update rules on V

Multiplicative update rules

Matlab demo

Part IV

How to reconstruct the sources by using W and H?

Reconstructing the source

- For source separation applications, are we done after estimating W and H ?

NO!

- Wiener filtering, lossless reconstruction
- Reconstructing the STFT of source i :

$$X_i = X \circ \frac{W(:,i)H(i,:)}{WH}$$

Multiplicative update rules

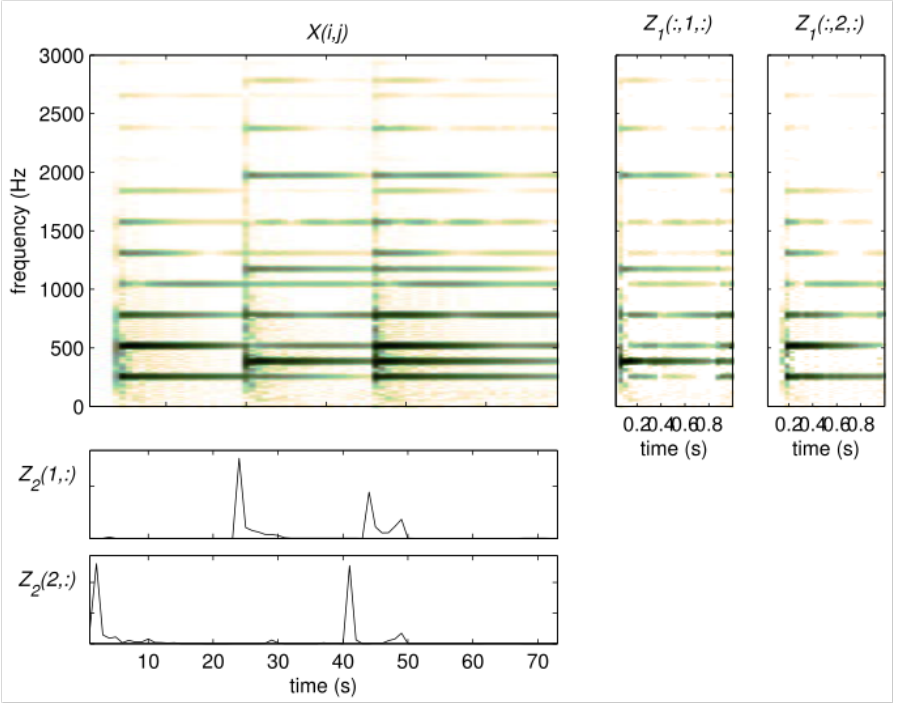
Matlab demo

Part V

More on NMF

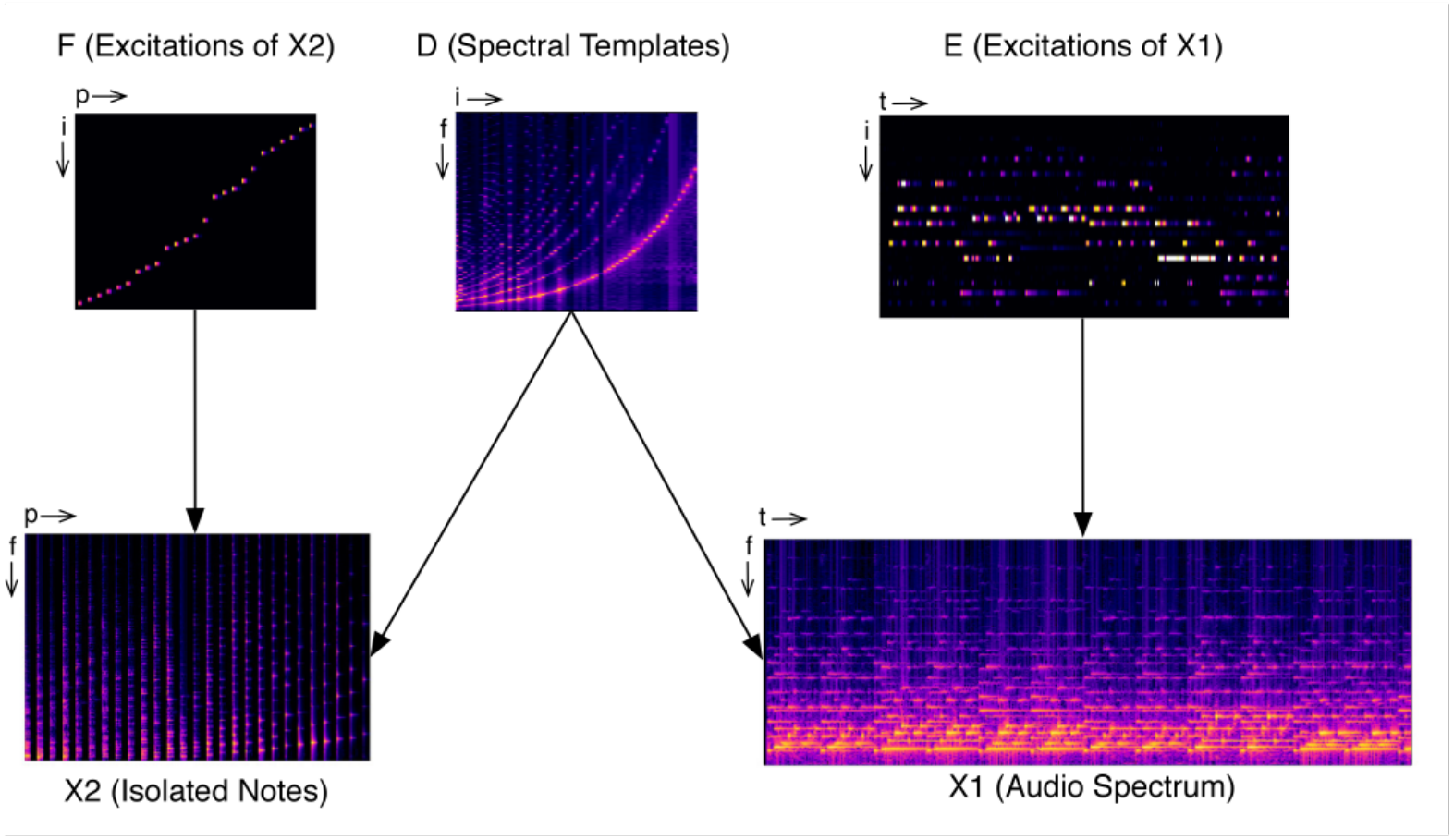
More on NMF

- Supervised vs Unsupervised
- Temporal information $\longrightarrow$ Convolutive models, non-negative dynamical systems



More on NMF

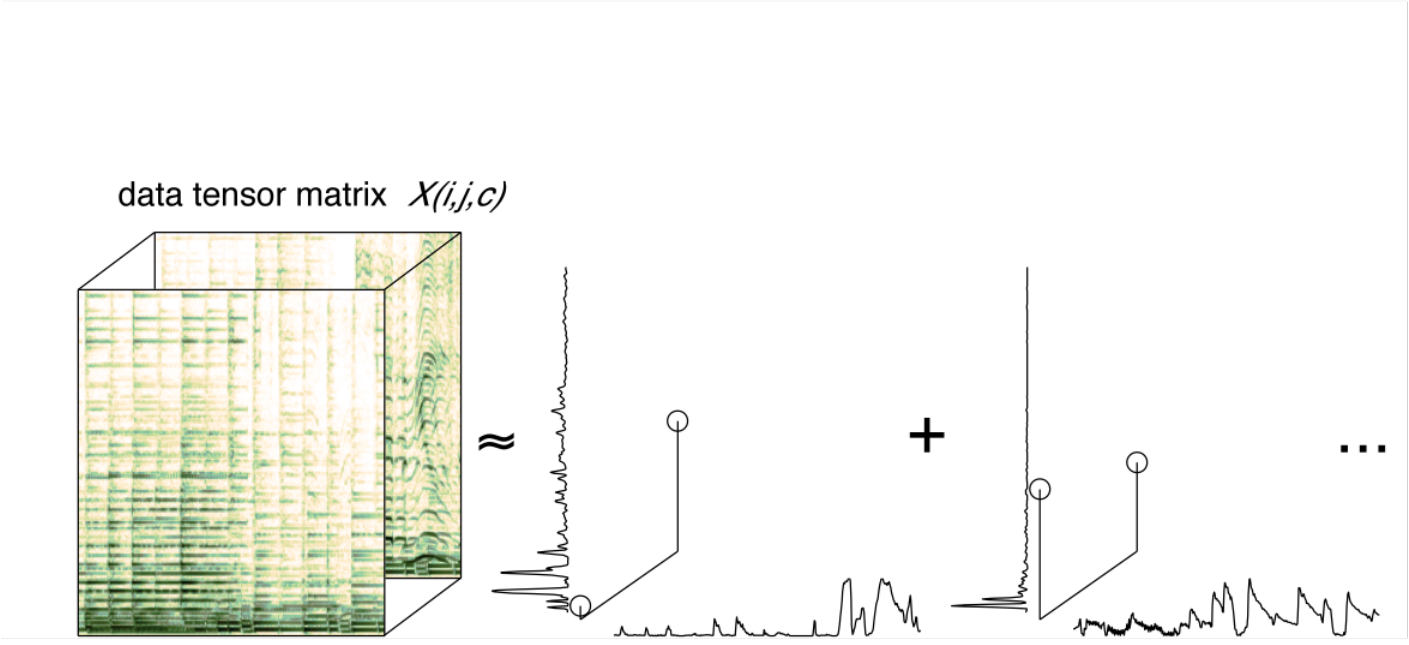
Side information $\longrightarrow$ Coupled factorization models



More on NMF

- Single channel vs Multi-channel
- Data is no longer a matrix, but rather a tensor

- Tensor factorization models



Part VI

Summary

Summary

- Applications of NMF (source separation, restoration, music transcription, etc.)
- Model definition
- MUR for estimating the factors
- Wiener filtering for reconstructing the source signals