INF280: Competitive programming

Advanced datastructure algorithms

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Sliding windows

Sliding window techniques to improve efficiency

Typical examples using a list $i_1, \ldots, i_N$ and an integer K

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Naive algorithm

Two (or three!) nested loops, recomputing from scratch for each position j.

Sliding window idea

Optimize away nested loops!

Example: fixed width (e.g. maintaining sum of K elements)

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General idea

Maintain a double ended queue where:

- you add on the right to grow
- remove on the left to shrink
- maintain some computation over the window content

Works with monotone criteria for windows!

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The deque trick to maintain min and max

Maintain the (ordered) list of elements that might become min/max.

Example: maintaining min of 5 elements)

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Updates

- Add on the right: remove everything bigger
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Candidate mins: 5, 23

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Advanced technique for sliding window

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Also works with a set... with a $O(log(n))$ penalty.

Prefix sums

Structure to compute sums in $O(1)$

Input

A list of elements $v_1 ... v_n$ over a group

Query

Compute $q(i,j) = \sum_l i \leq \ell < j v_\ell$

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Works in d dimensions!

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- Keys needs to be ordered and are typically integers,
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- values can be from any semigroup (e.g. $+$ or min over floats).

A powerful structure that supports the following queries:

- Add a key k with a value v
- Add v to the value of all keys in the range $[l; r]$
- Replace with v to the value of all keys in the range $[l; r]$
- Get the sum of all values for keys in the range $[l; r]$

Segment trees, an example

Snow problem

We have a 1D representation of the snow height and the following queries:

- maximal snow height for a span $[l; r]$
- add k centimeters of snow on the span $[l; r]$
- replace the height of snow with k for the span $[l; r]$ (snow grooming)

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A weird semigroup:

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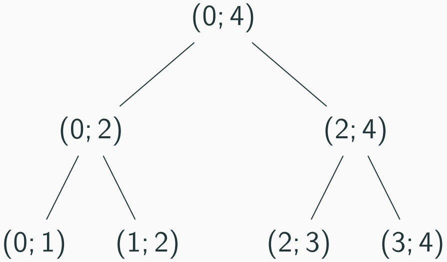
- Get the sum of all values for keys in the range $[l; r]$

Segment trees, the idea

Maintain a binary search tree on the keys

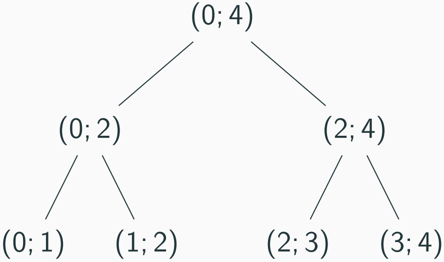


Maintain lots of information on all nodes

- the span the current node is concerned with
- the sum/max of all values from values below in the tree
- whether the values have been replaced *(lazy)*
- whether we need to add something to the children values *(lazy)*

Segment trees, the idea

Maintain a binary search tree on the keys



To update or query :

- start from the root to the leaves
- propagate any lazy value
- restore sum/max going back up

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