

# 3D Scanning & Motion Capture

## Exercise - 2

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# Exercises – Overview

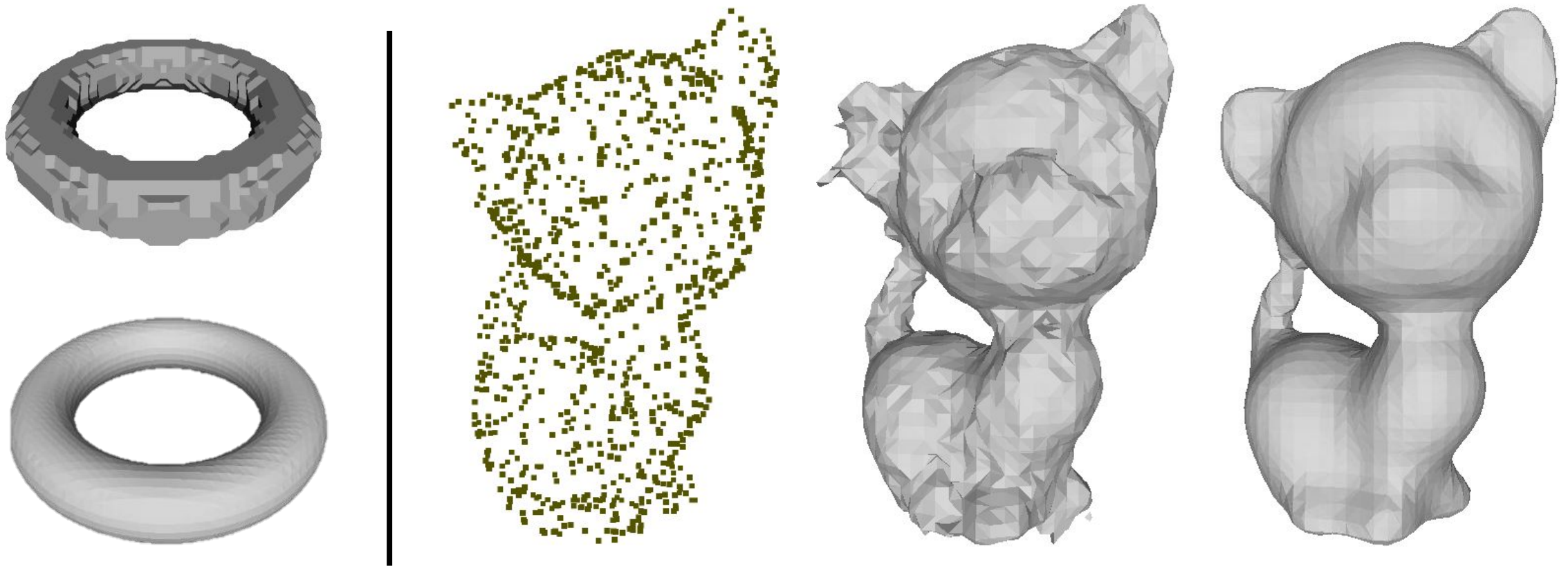
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1. Exercise → Camera Intrinsics, Back-projection, Meshes
- 2. Exercise → Surface Representations**
3. Exercise → Optimization
4. Exercise → Coarse Alignment (Procrustes)
5. Exercise → Object Alignment, ICP

# Exercises – Overview (2/5)

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## 2. Exercise → Surface Representations



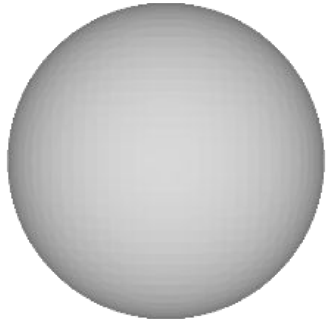
# Tasks

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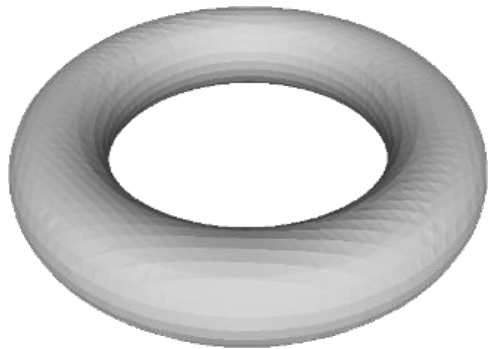
1. Project dependencies & CMake configuration
2. Implicit Surfaces
  - Sphere
  - Torus
3. Marching Cubes
  - Improve vertex positions using linear interpolation
4. Hoppe
  - Convert a point cloud to an implicit surface
5. Radial Basis Functions
  - Setup and solve system of linear equations for smoother surfaces

## Task 2) Implicit Functions – Sphere / Torus

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$$f(x, y, z) = x^2 + y^2 + z^2 - R^2$$

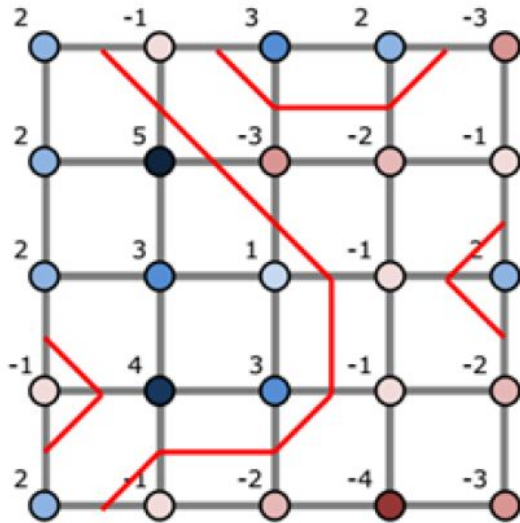


$$f(x, y, z) = (x^2 + y^2 + z^2 + R^2 - a^2)^2 - 4R^2(x^2 + y^2)$$

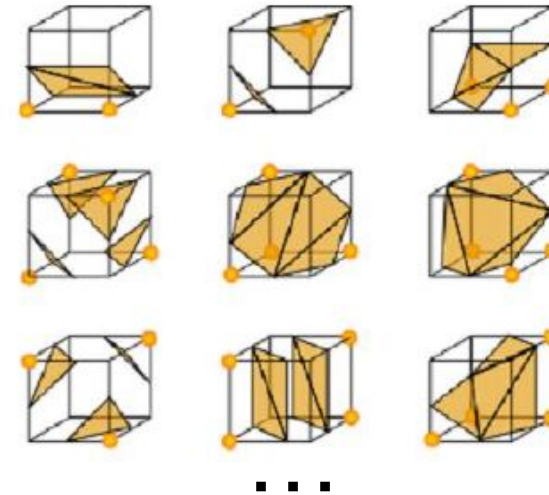
The given equations assume the sphere/torus is centered at the origin.

# Task 3) Marching Cubes

- Regular grid/volume → Extract iso-surface
  - Check for zero-crossings within each cell



**Marching Squares (2D)**  
16 configurations

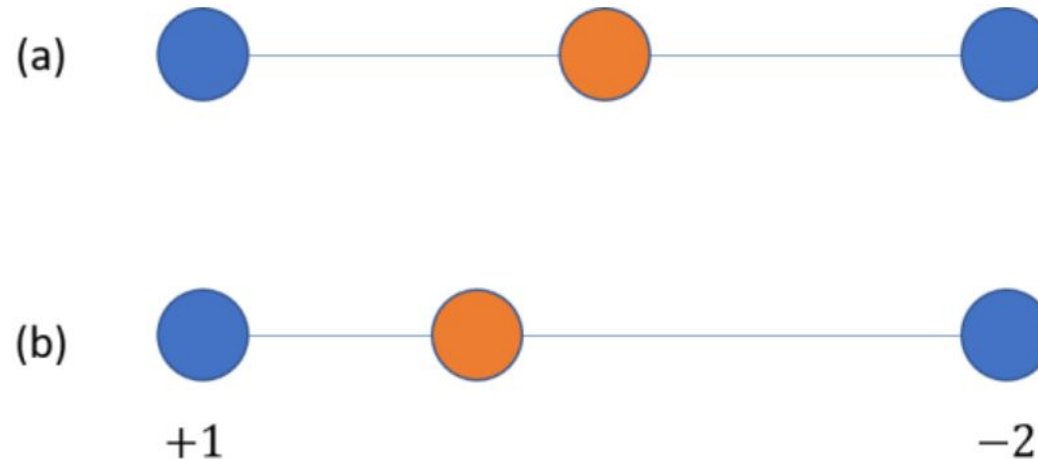


**Marching Cubes (3D)**  
256 configurations

# Task 3) Linear Interpolation

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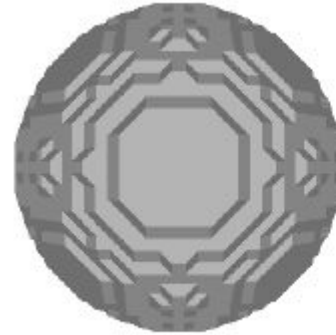
- Compute the linear interpolated point using the provided distances
  - (a) shows the basic implementation
  - (b) shows an example with *isolevel* = 0, *valp1* = +1 and *valp2* = -2 .



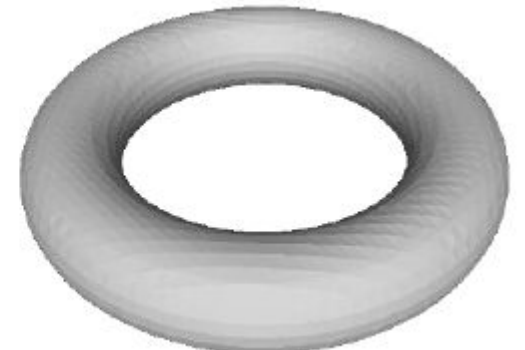
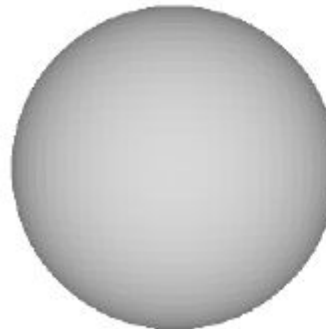
# Task 3) Linear Interpolation

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- Without linear interpolation
  - i.e. taking midpoint of each edge

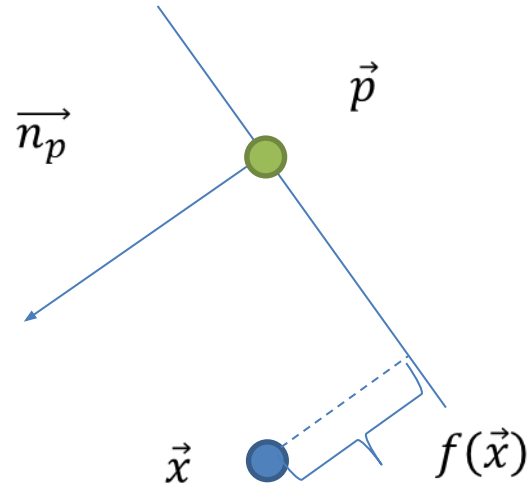
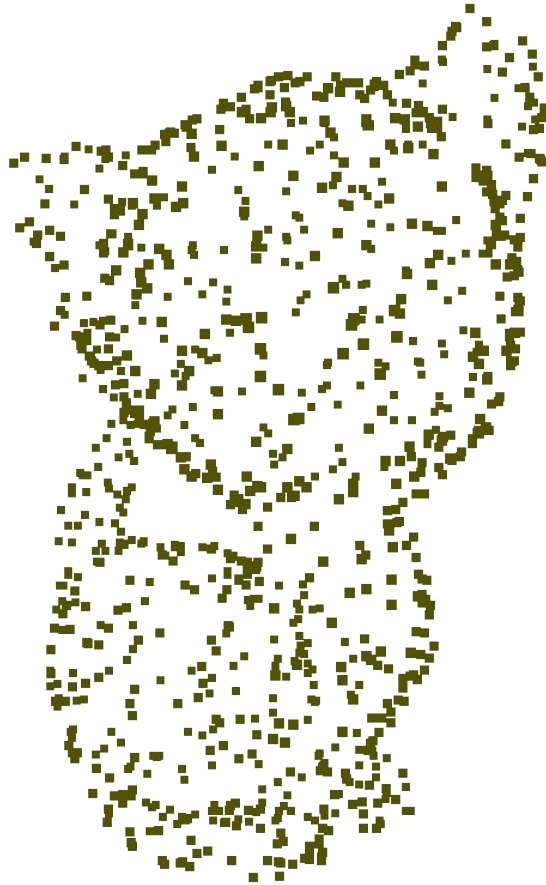


- With linear interpolation

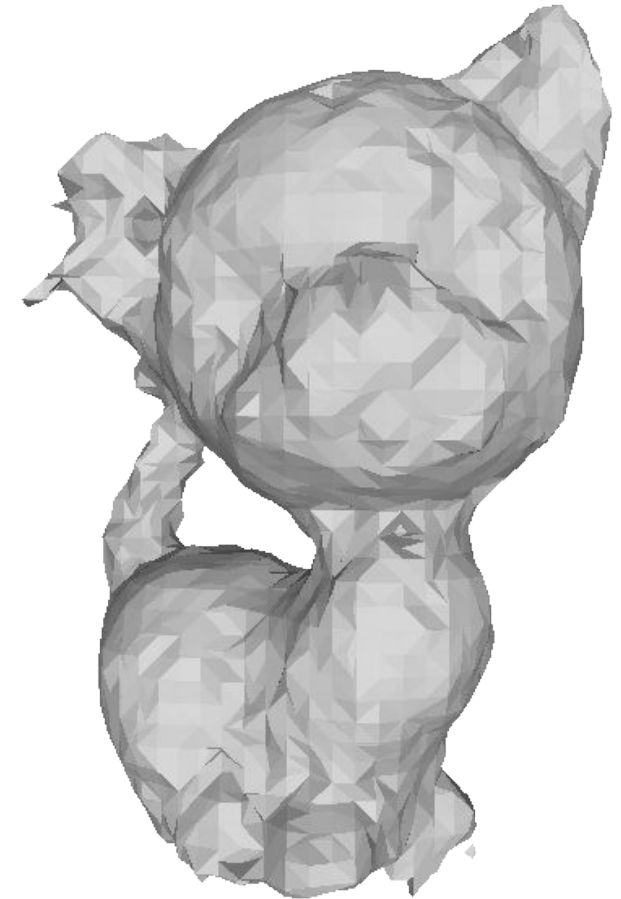




# Task 4) Hoppe

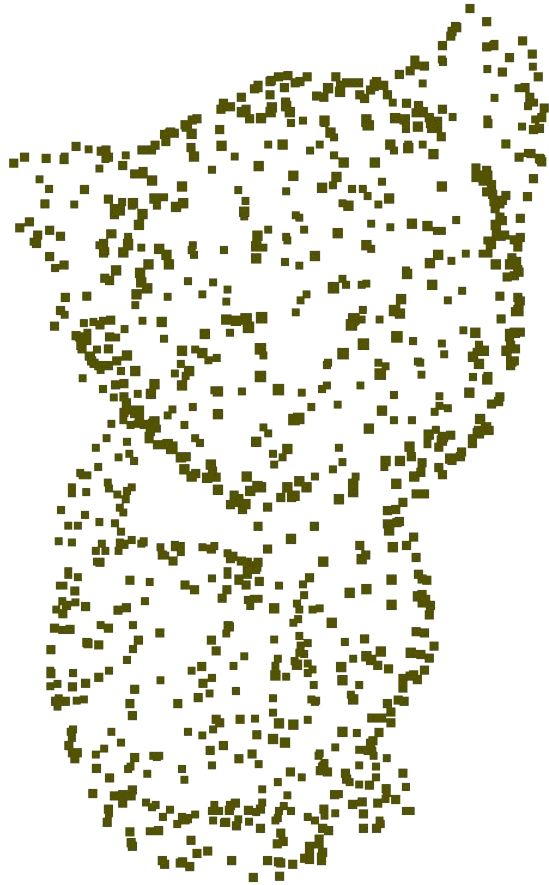


Piecewise linear surface approximation

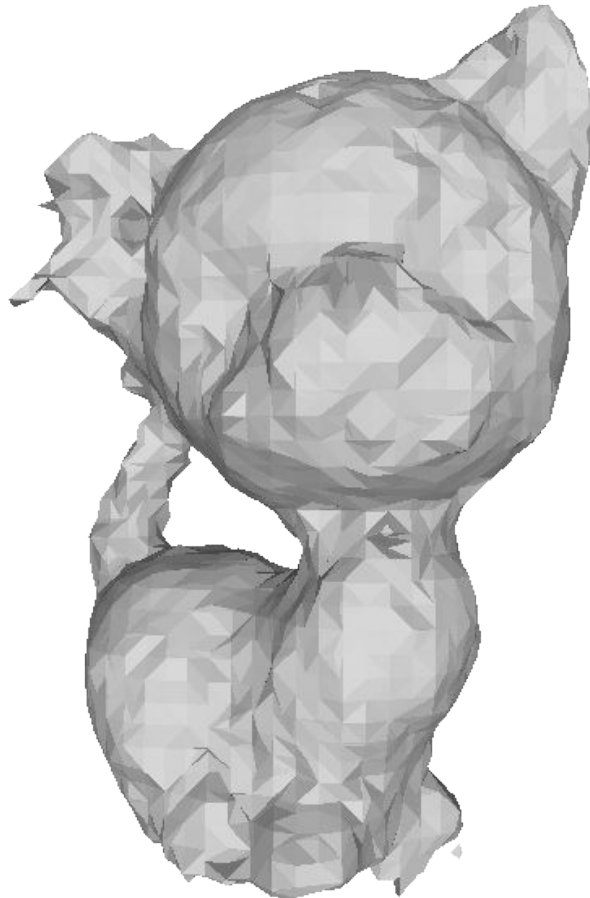


# Task 5) Radial Basis Functions (RBF)

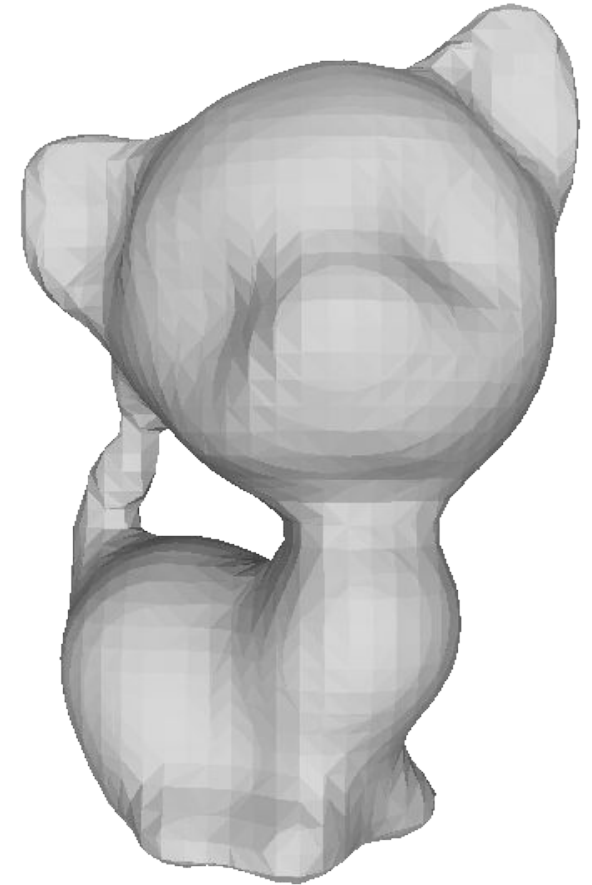
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Input Points



Hoppe



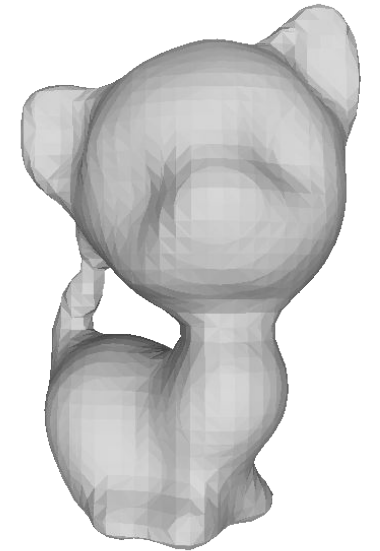
RBF

# Task 5) Radial Basis Functions (RBF)

$$f(\vec{x}) = \sum_i \alpha_i \cdot \|\vec{p}_i - \vec{x}\|^3 + \vec{b} \cdot \vec{x} + d$$

$$\begin{array}{l}
 \text{on surface points} \\
 \text{off surface points}
 \end{array}
 \left[ \begin{array}{ccccccc}
 \varphi_{1,1} & \cdots & \varphi_{1,n} & p_{1,x} & p_{1,y} & p_{1,z} & 1 \\
 \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 \varphi_{n,1} & \cdots & \varphi_{n,n} & p_{n,x} & p_{n,y} & p_{n,z} & 1 \\
 \varphi_{n+1,1} & \cdots & \varphi_{n+1,n} & p_{n+1,x} & p_{n+1,y} & p_{n+1,z} & 1 \\
 \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 \varphi_{2 \cdot n,1} & \cdots & \varphi_{2 \cdot n,n} & p_{2 \cdot n,x} & p_{2 \cdot n,y} & p_{2 \cdot n,z} & 1
 \end{array} \right] \cdot \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \\ b_1 \\ b_2 \\ b_3 \\ d \end{bmatrix} = \begin{bmatrix} h_1 \\ \vdots \\ h_{2 \cdot n} \end{bmatrix}$$

$$\underbrace{\left[ \begin{array}{ccccccc} \varphi_{1,1} & \cdots & \varphi_{1,n} & p_{1,x} & p_{1,y} & p_{1,z} & 1 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \varphi_{n,1} & \cdots & \varphi_{n,n} & p_{n,x} & p_{n,y} & p_{n,z} & 1 \\ \varphi_{n+1,1} & \cdots & \varphi_{n+1,n} & p_{n+1,x} & p_{n+1,y} & p_{n+1,z} & 1 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \varphi_{2 \cdot n,1} & \cdots & \varphi_{2 \cdot n,n} & p_{2 \cdot n,x} & p_{2 \cdot n,y} & p_{2 \cdot n,z} & 1 \end{array} \right]}_A \cdot \underbrace{\begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \\ b_1 \\ b_2 \\ b_3 \\ d \end{bmatrix}}_{\vec{c}} = \underbrace{\begin{bmatrix} h_1 \\ \vdots \\ h_{2 \cdot n} \end{bmatrix}}_{\vec{b}}$$



See you next time!

