

# Dinic's Sh. Aug. Path:

Layered Network: 同一长度的 sh. pa 所構成之子網路.

eg. 最短的 Aug. Path. 包含 3 段 arc. 則先將所有 3 條 arc 可連通  $s \rightarrow t$  的 path 畫出所構成的網路.  
 3  $\rightarrow$  在該網路持續送流量直至無法再送.

4  $\Rightarrow$  重新再建構最短路徑為 4 條 arc 可連通  $s \rightarrow t$  之子網路.  
 在該網路持續送流量直至無法再送.

5  $\Rightarrow$  find augmenting path  
augment flow  
 ...

(n-1)  $\Rightarrow$

Overall  $O(n \cdot nm) = O(n^2 m)$  time

最多僅能建構 (n-1) 個 Layered Network

each aug. saturates  $\geq 1$  arc  
 # augmentation:  $\frac{O(m)}{(c \cdot r)} \left. \vphantom{\frac{O(m)}{(c \cdot r)}} \right\} O(nm) \text{ time}$

relabel

$d(i)$ : l.b. on the length of Sh. pa. from  $i$  to  $t$

$$d(s) \leq n-1$$

$$d(i) \leq n-1 \Rightarrow \# \text{ relabels} : O(n) \forall i \in V. (\because \text{each relabel increase } d(i) \text{ by } \geq 1)$$

$$\text{Total \# relabels} : O(n^2)$$

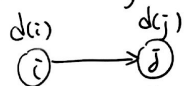
$$\text{Each relabel scan } |A(i)| \forall i \in V \Rightarrow \text{Total relabel time } O(n \sum_i |A(i)|) = O(nm)$$

augmentation

each aug takes  $O(n)$  time to identify a bottleneck arc to augment/saturate  $\geq 1$  arc

$$\# \text{ saturation} \geq \# \text{ aug}$$

each arc  $(i,j)$  in residual net is saturated at most  $O(n)$  times



$$\text{saturate } (i,j) \Rightarrow d(i) = d(j) + 1$$



$$\text{saturate } (j,i) \Rightarrow d'(j) = d'(i) + 1$$



$$\text{saturate } (i,j) \Rightarrow \underbrace{d''(i) = d''(j) + 1}_{\geq d'(j) + 1} \geq d'(i) + 2 \geq d(i) + 2$$

between consecutive saturation of  $(i,j)$

both  $d(i), d(j)$  increase at least 2 units  
(relabel) (twice)

$\geq 2$  relabels  $\Rightarrow 1$  saturation  
(for  $(i,j)$ )

$$\Rightarrow \# \text{ saturation} : O(nm)$$

$$\Rightarrow \# \text{ aug} : O(nm)$$

## Pre-flow Push

# relabel :  $O(n^2)$  次  
time to relabel :  $O(nm)$

total # saturated push = # aug. in sh. aug. pr. =  $O(nm)$  times  $(\leq \lambda)$

same as  
sh. aug. pr. Alg.

$$\text{Time: } \underbrace{\text{relabel}}_{O(nm)} + \underbrace{\text{nonsat. push}}_{O(n^2m)} + \underbrace{\text{sat. push}}_{O(nm)} \\ = O(n^2m)$$

Use Potential function  $F = \sum_{j \text{ active}} d(j)$  to bound # nonsaturated push

Each nonsat. push from an active node  $\bar{i}$  to a node  $\bar{j}$  (if  $\bar{j}$  is active)

$\bar{i}: a \rightarrow ina.$	$\bar{j}: a \rightarrow a.$	$-d(i)$
$\bar{i}: a \rightarrow ina$	$\bar{j}: ina \rightarrow a.$	$-d(i) + d(j) = -1$

Each sat. push from an active node  $\bar{i}$  to a node  $\bar{j}$  (if  $\bar{j}$  is active)

$\bar{i}: a \rightarrow a$	$\bar{j}: a \rightarrow a$	$0$
$\bar{i}: a \rightarrow a$	$\bar{j}: ina \rightarrow a$	$+d(j)$
$\bar{i}: a \rightarrow ina$	$\bar{j}: a \rightarrow a$	$-d(i)$
$\bar{i}: a \rightarrow ina$	$\bar{j}: ina \rightarrow a$	$-d(i) + d(j) = -1$

increase F by at most  $O(n)$

Overall: relabel will increase F by at most  $O(n^2)$

saturated push  $O(n^2m)$

initial F is bounded above by  $O(n^2) \because d(i) \leq n, \forall i \in N.$

each nonsat. push will decrease F by at least  $(1)$

$\therefore$  total # nonsat. push is  $O(n^2m)$  times  $(\leq \lambda)$  } nonsat. push takes  $O(n^2m)$  time  
each nonsat. push takes  $O(1)$  time