[42%] The following table lists a number of tasks that must be completed in order for a crew of workers to finish a project.

Task	A	B	C	D	E	F	G	H	I	J
Time (in days)	2	2	3	1	1	2	3	4	3	3

Suppose the project starts at Apr. 9. Task A must be carried out before any other tasks can start. Task B must precede tasks E and F, and both E and F must be completed before H can begin. Tasks C and D must precede task G, which in turn must precede I. Task J must be carried out last. It is assumed that there are enough workers to carry out any number of tasks simultaneously.

(a) [5%] Draw a PERT diagram showing the relations of these tasks.

(b) [2%] Draw the critical path.

(c) [5%] We say a "conflict of tasks" occurs when two tasks violate their precedent relations. Give a suitable sequence for conducting all the tasks so that no conflict of tasks occurs in the entire process.

(d) [3%] What is the fewest number of days needed to make this product?

(e) [3%] What is the earliest date for this project to be done?

(f) [3%] If we start to do task D in Apr.14, will it affect your answer in (e)? If yes, affect how much? If no, explain why not.

(g) [5%] What is the latest date for task H to begin, in order not to affect your answer in (e)?

(h) [6%] For any two tasks x and y, suppose we define $x\mathbf{R}y$ to be true if either task x equals to task y or task x can not be started until task y is completed. What are $B\mathbf{R}J$, $I\mathbf{R}C$, $H\mathbf{R}D$?(i.e. True or False?)

(i) [4%] Following (h), let S be set of all tasks and $\mathbf{R}$ be a relation on S. List all the minimal and maximal elements of S with respect to $\mathbf{R}$.

(j) [4%] Following (h), what are the infimum and supremum of E and F? what are the infimum and supremum of G and F?

(k) [2%] Following (h), is $(S, \mathbf{R})$ a lattice? Explain your answer.(an answer without explanation gets at most 1 point)

1. [15%] Let S be a set of n elements. Answer the following questions WITH explanations. (answers WITHOUT explanation may get AT MOST 1 point)
   (a) [3%] What is the total number of relations on S?
   (b) [4%] What is the total number of reflexive relations on S?
   (c) [4%] What is the total number of symmetric relations on S?
   (d) [4%] What is the total number of antisymmetric relations on S?

2. [19%] Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. For $a, b \in A$, define aRb if and only if ab is a perfect square (i.e. the square of an integer like 1,4,9,16,25,...).
   (a) [5%] What are the ordered pairs in this relation?
   (b) [6%] For each $a \in A$, find its equivalence class $[a] = \{x \in A : xRa\}$
   (c) [8%] Explain why R is a transitive relation on A.

3. [16%] What is the tight running time (big-Θ) of the following? (i.e. a Θ notation in terms of N, M, or P) Answers without correct explanations will get zero points.
   (a) [4%]

```
1.      int count=0;
2.      for (int i=1;i<=N; i+=4)
3.              count=count+1;
```

   (b) [8%] discuss the following pseudocode for 2 cases: (b1): $M \leq N$, and (b2) $M > N$

```
1.      int count=0;
2.      int i=1, j=1;
3.      while (i<M)
4.          for (j=i;j<N;j++ )
5.              count=count+1;
6.          i=i+1;
```

   (c) [4%]

```
1.      int count=0;
2.      int i,j,k;
3.      for (i=1;i<=M;i++)
4.          for (j=1;j<=N;j*=2 )
5.              for (k=1;k<=P;k=k*3)
6.                  count=count+2;
```