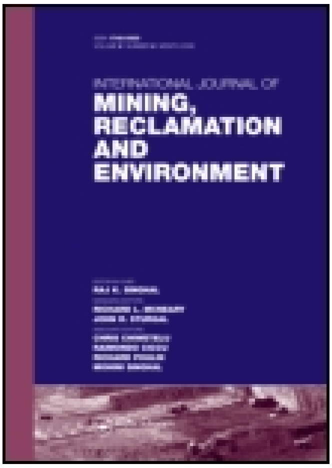

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Long-term open pit mine production planning: a review of models and algorithms

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Long-term open pit mine production planning: a review of models and algorithms

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Long-term production planning (LTPP) is a large-scale optimization problem that aims to find the block extraction sequence that produces the maximum possible net present value (NPV) whilst satisfying a variety of physical and economical constraints. The economic feasibility of a mine is highly dependent upon careful LTPP. As the mining industries extract deeper and lower grade ores, LTPP is becoming a key item that can result in ceasing operations or continuing the project. Mathematical programming models are well suited to optimizing LTPP of open pit mines. These mathematical models have been studied extensively in the literature since the 1960s. The result of this study shows that there are two approaches for dealing with LTPP problems: (1) deterministic and (2) uncertainty-based approaches. This paper first discusses the deterministic algorithms and then, after an introduction to uncertainty associated with mining projects, reviews uncertainty-based algorithms. The advantages and disadvantages of these algorithms are discussed and suggestions for future research are offered.

Keywords: Long-term production planning; Open pit mine; Deterministic approach; Uncertainty-based approach

1. Introduction

Open pit mine production scheduling can be defined as 'Specifying the sequence of blocks extraction from the mine to give the highest NPV, subject to variety of production, grade blending and pit slope constraints' (Whittle 1989). Production scheduling over a certain period of time is known as the scheduling horizon. Production scheduling typically encompasses three time ranges for decision making: long-term, medium-term and short-term. Long-term can be in

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the range of 20 – 30 years depending on the situation. This 20 – 30 year period is broken into several smaller time periods of between 1 and 5 years. A medium-term schedule has a range of 1 – 5 years. Medium-term schedules give more detailed information that allows for a more accurate design of ore extraction from a special area of the mine, or information that would allow for necessary equipment substitution or the purchase of needed equipment and machinery. The 1 – 5 year period of the medium-term schedule is further broken down into 1 – 6 month periods for even more detailed scheduling. Finally, the duration of short-term production planning is between a month and one year. Similarly, this period is divided into one-day to one-month sub-periods. This paper will focus on long-term production planning (LTPP) in open pit mines. LTPP not only determines the distribution of cash flow over the life of a mine and the feasibility of the project, but it is also a very important prerequisite for medium and short-term scheduling.

There are two approaches to solve LTPP problems. The *deterministic* approach assumes all inputs to have fixed known real values; the *uncertainty-based* approach accounts for variability in some data (e.g. ore grade, future product demand, future product price).

2. Open pit production planning problem

2.1 *Modelling the deposit*

Most of the current open pit design and scheduling processes begin with a geologic block model obtained by dividing the deposit into a three-dimensional grid of fixed size blocks, as shown in figure 1. Block dimensions are selected according to the exploration drilling pattern, ore body geology and mine equipment size. After establishing the dimensions of the block model, geological characteristics of each block (grade) are assigned using available estimation techniques such as inverse distance weighted interpolation technique, weighted moving averages, Kriging, etc. Using financial and metallurgical data, the economic value of each block is also calculated. It should be noted that this value excludes the cost of accessing the block. The economic future value of the block can then be obtained by discounting the original value to time zero, using a discounted rate.

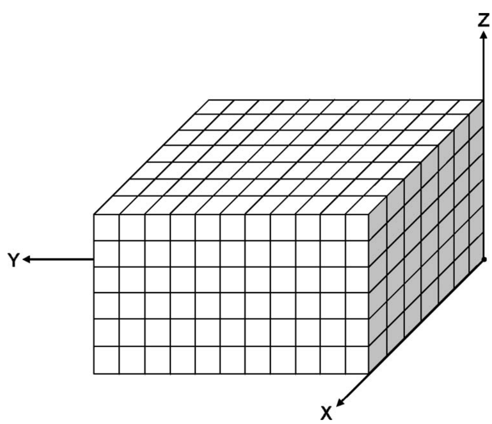

Figure 1. Isometric view of a block model.

2.2 *Problem description*

Ideally, the criterion should be maximization of the net present value (NPV) of the pit, but unfortunately, after four decades of continuing efforts, this goal could not be achieved. The reason for this problem has been simply paraphrased by Whittle (1989):

> The pit outline with the highest value cannot be determined until the block values are known. The block values are not known until the mining sequence is determined; and the mining sequence cannot be determined unless the pit outline is available.

This is a large-scale mathematical optimization problem that could not have been solved currently using commercial packages. The most common approach to the problem is dividing it into sub-problems similar to that shown in figure 2 (Dagdalen 2000).

The approach starts with assumptions about initial production capacities in the mining system and estimates for related costs and commodity prices. Then, using economic block values, each positive block is further checked to see whether its value can pay for the removal of overlying waste blocks. This analysis is based on the *breakeven cutoff grade*, which checks if undiscounted profits obtained from a given ore block can pay for the undiscounted cost of mining waste blocks. The ultimate pit limit is then determined using either a graph theory based algorithm (Lerchs and Grossman 1965, Zhao and Kim) or a network flow algorithm (Johnson and Barnes 1988, Yegulalp and Arias 1992) with the objective of maximizing (undiscounted) cash flow. Within the ultimate pit, push backs are designed so that the deposit is divided into nested pits going from the smallest pit with the highest value per ton of ore to the largest pit with the lowest value per ton of ore. These push backs can be designed using one or more of the heuristic algorithms suggested by Dagdelen and Francois-Bongarcon (1982), Gershon (1987), Whittle (1998), Wang and Sevim (1995) or Ramazan and Dagdelen (1998). These push backs act as a guide during the schedule of yearly based production planning. Before determining the extraction schedule, the cutoff grade strategy should be defined to discriminate between ore and waste during the scheduling process. Lane (1964) proposed an algorithm to determine the cutoff grade strategy that maximizes the NPV of a project subjected to mine, mill and refinery capacity constraints. Later, we will demonstrate that some algorithms can optimize cutoff grade and extraction scheduling simultaneously.

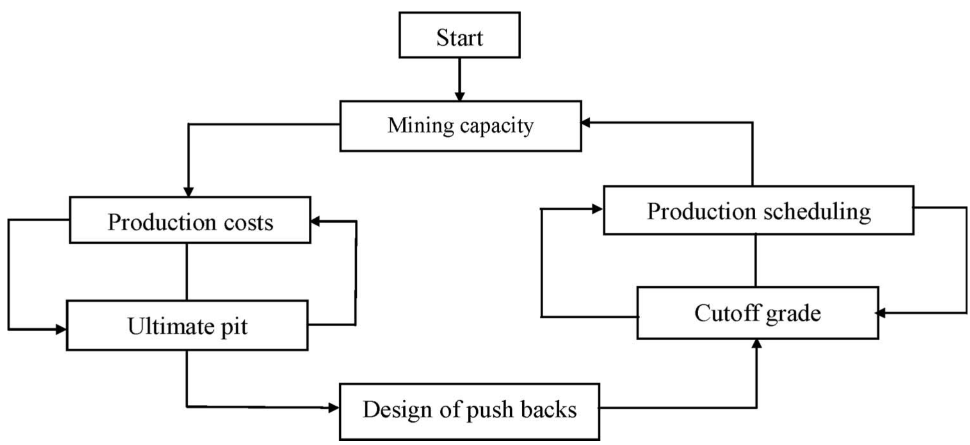

Figure 2. Open pit LTPP variables interacting in a circular fashion.

3. Long-term production planning

3.1 *Deterministic approach for LTPP*

Many researchers have worked on the LTPP problem. From 1965, several types of mathematical formulations have been considered for the LTPP problem: linear programming (LP); mixed integer programming (MIP); pure integer programming (IP); and dynamic programming (DP).

3.1.1 Linear programming (LP) formulation. Johnson (1969) optimized mine scheduling using an LP model. The mathematical form of this kind of model can be represented as follows:

$$Maximize\ Z = \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{i=1}^{N} C_i^{tm} \cdot TB_i \cdot x_i^{tm} \tag{1}$$

subject to:

$$G_{\min}^{tm} \le \left(\sum_{i=1}^{N} g_i \cdot TB_i \cdot x_i^{tm} \Big/ \sum_{i=1}^{N} TB_i \cdot x_i^{tm} \right) \le G_{\max}^{tm} \quad \text{for } t = 1,2,\ldots,T \quad \text{and} \quad m = 2,3,\ldots,M \tag{2}$$

$$PC_{\min}^{tm} \le \sum_{i=1}^{N} TB_i \cdot x_i^{tm} \le PC_{\max}^{tm} \quad \text{for } t = 1,2,\ldots,T \quad \text{and} \quad \mathrm{m} = 2,3,\ldots,\mathrm{M} \tag{3}$$

$$MC_{\min}^{t} \le \sum_{i=1}^{N}\sum_{m=1}^{M} TB_i \cdot x_i^{tm} \le MC_{\max}^{t} \quad \text{for } t = 1,2,\ldots,T \tag{4}$$

$$\sum_{t=1}^{T}\sum_{m=1}^{M} x_i^{tm} = 1 \quad \text{for } i = 1,2,\ldots,N \tag{5}$$

$$\sum_{m=1}^{M} x_b^{tm} - \sum_{r=1}^{t}\sum_{m=1}^{M} x_l^{rm} \le 0 \quad \text{for } t = 1,2,\ldots,T \quad \text{and} \quad b = 1,2,\ldots,N \quad \text{and} \quad \forall l \in \Gamma_b \tag{6}$$

$$0 \le x_i^{tm} \le 1 \quad \text{for } t = 1,2,\ldots,T,\ \ b = 1,2,\ldots,N \quad \text{and} \quad m = 1,2,3,\ldots,M \tag{7}$$

where:

T = the maximum number of scheduling periods
N = the total number of blocks to be scheduled
i = block index (i = 1, 2,…,N)
C_i^{tm} = the NPV resulting from mining unit weight of material in block i during period t if it is considered as processing type m
x_i^{tm} = the proportion of block i to be mined in period t as a processing type m
g_i = the average grade of block i
TB_i = the total tonnes of material in block i

$G^{tm}{}_{\max}$ = maximum average grade of the material type m sent to the mill in period t
$G^{tm}{}_{\min}$ = minimum average grade of the material type m sent to the mill in period t
$PC^{tm}{}_{\max}$ = maximum processing capacity of material type m in any period
$PC^{tm}{}_{\min}$ = minimum processing capacity of material type m in any period
$MC^{t}{}_{\max}$ = maximum mining capacity in any period
$MC^{t}{}_{\min}$ = minimum mining capacity in any period
b = the index of a block considered for excavation in period t
m = material or processing type (i.e. waste = 1, ore = 2, ... = M)
l = the counter for $l \in \Gamma_b$
Γ_b = the set of block indices defined for block b. It consists of the indices of all blocks that need to be removed before extracting block b, due to the maximum pit slope angle.

This model considers the time value of money, different processing types and also the dynamic cutoff grade strategy. To solve the LP model, one first decomposes the large multi-period production planning model into a master problem and a set of sub-problems by using Dantzig–Wolf decomposition principles. Each sub-problem is then solved as a single-period problem that has the same characteristics as the ultimate pit limit problem. This can be done using a maximum network flow algorithm. After solving all sub-problems, solving the master problem is relatively simple. Although Johnson's (1969) method generates optimum results for each period individually, it does not solve the LTPP problem totally. However, its variables are linearly continuous, which is responsible for fractional block extraction. Also, this model provides situations in which some portion of a block is extracted while all the overlying blocks have not been mined. This drawback causes some percentage of overlying blocks to be suspended in air (figure 3). Another disadvantage of this model is that it has too many constraints (nine slope constraints per block), which itself limits the number of blocks that can be handled by the model.

3.1.2 Mixed integer programming (MIP) formulation. Gershon (1983) discussed an MIP model that allows for partial blocks to be mined if all precedent blocks have been completely removed. The key to this formulation is adding additional decision variables to Johnson's LP model. To do this, four different decision variables are assigned to each block:

$B_{ijk}(t)$ = zero if all precedent blocks have not been completed in period t and = 1 otherwise
$X_{ijk}(t)$ = percent of block ijk mined in period t
$C_{ijk}(t)$ = percent of block remaining at the start of period t
$D_{ijk}(t)$ = percent of precedent blocks mined at the start of period t

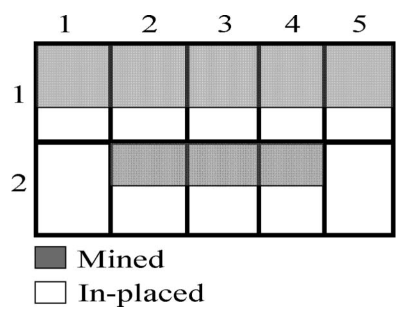

Figure 3. Problem of partial block mining of Johnson's (1969) model.

Then the slope constraints can be expressed as:

$$B_{ijk}(t) + C_{ijk}(t) - C_{ijk}(t+1) - X_{ijk}(t) \geq 0 \tag{8}$$

$$D_{ijk}(t) + \sum_{l=i-1}^{i+1} X_{l,m,k-1}(t) - D_{ijk}(t+1) - 9B_{ijk}(t) \geq 0 \tag{9}$$

This model is solved with the use of the APEX-IV software. This approach has two advantages over the LP method. First, it provides a more practical extraction sequence in mine scheduling. This follows from allowing partial block mining on the condition that all blocks preceding the partially mined block have been mined. The net result is that only one constraint per block is required.

The main disadvantage of this model is its inability to handle large problems using commercial software because it contains too many binary variables. In addition, because of increasing the size of the model, the dynamic cutoff grade concept cannot be considered.

3.1.3 Integer programming (IP) formulation. The general IP form of an open pit production planning is:

$$MaxZ = C_1X_1 + C_2X_2 + \cdots + C_TX_T \tag{10}$$

subjected to:
mining and milling constraints

$$\begin{cases} A_1X_1 \leq b_1 \\ A_2X_2 \leq b_2 \\ \vdots \\ A_TX_T \leq b_T \end{cases} \tag{11}$$

sequencing constraints

$$\begin{cases} EX_1 \leq 0 \\ EX_1 + EX_2 \leq 0 \\ \vdots \\ EX_1 + EX_2 + \cdots + EX_T \leq 0 \end{cases} \tag{12}$$

reserve constraints

$$\sum_{t=1}^{t} x_i^t = 1 \quad \forall i \tag{13}$$

and

$$X_i^t = \{0, 1\} \quad \forall i, \forall t \tag{14}$$

where:

T = the maximum number of scheduling periods
N = the total number of blocks to be scheduled

K = the number of mining and milling constraints for a given period
R = the number of overlaying restricting blocks
X_t = a column vector of N variables x_i^t
C_t = a row vector of N objective function coefficients containing c_i^t elements that represents the NPV resulting from mining block i in period t
A_t = a $K*N$ matrix of mining and milling constraints coefficients for time period t
b_t = a K element column vector of right-hand side coefficients for the mining and milling constraints
E = a $(N \cdot R)$ by N matrix with 0, 1, -1 coefficients for the sequencing constraints

Constraints (11) ensure that minimum and maximum average grades for milling operations and also milling and mining capacities are satisfied. Constraints (12) ensure that all the blocks considered in the model have to be mined once. Constraints (13) are the wall slope restrictions.

A binary IP formulation usually involves a large number of zero–one variables, which is beyond the capacity of current commercial packages. Several approaches have been proposed by researchers to solve such models.

3.1.3.1 Lagrangian relaxation approach. The first application of this approach is referred to as the Lagrangian relaxation decomposition with sub-gradient optimization method, and it was first used by Dagdelen and Johnson (1986) to solve the LTPP problem in open pit mines. Using Lagrangian multipliers, they decomposed the complex multi-period problem into smaller single-period problems that can be handled using optimum pit design algorithms (Learchs and Grossman 1965, Zhao and Kim 1992) or maximum flow algorithms (Johnson and Barnes 1988, Yegulalp and Arias 1992). This can be done easily by relaxing the mining and milling constraints into objective functions by introducing Lagrangian multipliers. Therefore, the objective function can be written as (Held *et al.* 1974, Sandi 1979):

$$MaxZ = C_1X_1 + C_2X_2 + \cdots + C_TX_T - \Lambda_1(A_1X_1 - b_1) - \Lambda_2(A_2X_2 - b_2) - \cdots - \Lambda_T(A_TX_T - b_T) \tag{15}$$

where $\Lambda = (\Lambda_1, \Lambda_2, \ldots, \Lambda_T)$; and $\Lambda_t = (\lambda_{1t}, \lambda_{2t}, \ldots, \lambda_{kt})$ are Lagrangian multipliers.

The objective function can be further simplified by substituting $C_t - A_t$ with D_t and also ignoring $A_t \cdot b_t$ as follows:

$$MaxZ = D_1X_1 + D_2X_2 + \ldots + D_TX_T \tag{16}$$

subject to sequence and reserve constraints. Next, the Lagrangian multipliers are adjusted using the sub-gradient method until the optimum schedule is obtained. At each step, a problem similar to an ultimate pit limit problem should be solved. In cases where there are no multipliers that can result in a feasible solution for the constraints, this method may not converge to an optimum solution. This problem is named the *gap problem*. Caccetta *et al.* (1998) tested this method on a real ore body with 20 979 blocks and six time periods. The schedule obtained was within 5% of the theoretical optimum. Another drawback of this algorithm is that it does not consider the dynamic cutoff grade concept during scheduling.

Akaike and Dagdelen (1999) proposed the 4D-network relaxation method. They considered a dynamic cutoff grade concept during the scheduling process. Their model also has the capability to handle the stockpile option. These two important steps were achieved by expanding the definition of the variables in equations (10) to (14) as follows:

M = the number of material or processing type (i.e. for material type 1, $m = 1\ldots$; for type M, $m = M$)
X_t = a column vector of ($M * N$) variables $x_i^{tm} \cdot x_i^{tm} = 1$ if block i is mined as type m in period t, otherwise it is equal to 0
C_t = a row vector of ($M * N$) objective function coefficients; c_i^{tm} represents the NPV resulting from mining block i in period t if it is mined as type m
A_t = a K by ($M * N$) matrix of mining and milling constraints coefficients for time period t
E = a ($N * R * M$) by ($M * N$) matrix; 0, 1, -1 coefficients for the sequencing constraints

To incorporate the stockpile option, assume that the mine life is $t_{\max}$, the total number of processing types in the model $m_{\max}$ is defined as $m_{\max} = t_{\max} + 2$. In each period t_m that is $1 \le t_m \le t_{\max}$, if we assume that there are only two processing types (ore and waste), then m is:

$m = 1$ = waste removed at period t_m
$2 \le m \le (t_{\max} + 1)$ = stockpile mined at period t_m and processed at period $t_p = m - 1$
$m = t_{\max} + 2$ = ore mined and processed at period t

To ensure that the stockpile is considered only when $t_m < t_p$, a large negative number should be assigned to c_i^{tm} when $2 \le m \le t + 1$. Akaike and Dagdelen then transformed the IP model by the use of the Lagrangian relaxation method, so that the transformed problem has the same characteristics as the final pit design problem. This problem can then be interactively solved changing the Lagrangian multipliers by using the sub-gradient method to converge it to the optimum solution of the primal problem. The authors also improved the efficiency of the sub-gradient method to reach the optimal solution much faster. The most important advantages of this algorithm are the use of the dynamic cutoff grade concept with the stockpile option with zero – one variables during the scheduling process. This will improve the NPV of a mining project. The disadvantage of this method is the possibility of a gap problem occurring, which means that it may not lead to an optimum solution.

Mogi *et al.* (2001) proposed a revised 4D-network relaxation method in order to reduce the effect of gap problem, but they could not eliminate it completely.

3.1.3.2 Clustering approach. The next approach to solve an IP model of production planning in an open pit mine is the clustering approach, which was applied by Ramazan *et al.* (2005). Clustering means classifying the large amount of data into relatively few classes of similar objects. This is the reason for complexity reduction in the considered application, which allows for improved decisions based on the information gained. Ramazan *et al.* combined ore and waste blocks together to decrease the number of binary variables in the IP model. They introduced the *fundamental tree* as any combination of blocks within the push backs, such that the blocks can be profitability mined and obey the slope constraints so that no sub-set of chosen blocks can be found that meets the above two requirements. This re-blocking (clustering) process is done using an LP mathematical formulation so that the information available for individual blocks is not lost. Figure 4 shows a 2D illustration of the block model. The three fundamental trees created by the proposed LP model can be seen in figure 5.

Tree I can be mined first; trees II and III are then feasible to mine in the suggested order. After defining the fundamental trees, their precedence relations should be determined using the cone

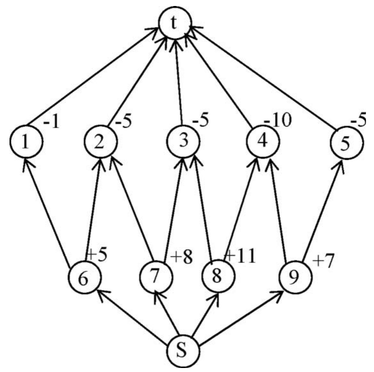

Figure 4. Network representation of 2D block model (Ramazan *et al.* 2005).

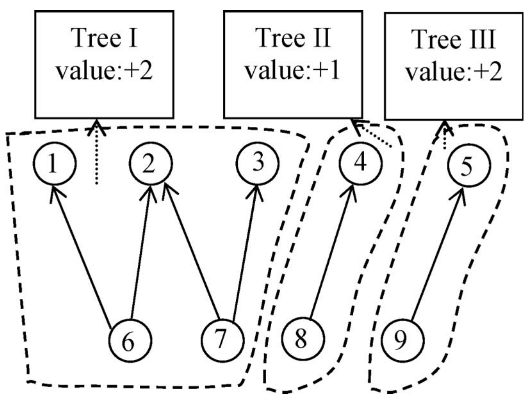

Figure 5. Three fundamental trees for 2D example (Ramazan *et al.* 2005).

template. Each fundamental tree is treated as a mining block containing a certain ore tonnage, metal content and quality parameters. Then a binary variable is assigned to each fundamental tree for each production period except the last one. In order for the IP model that uses fundamental trees to be handled by commercial software such as CPLEX*, the material within the final pit limit is divided into smaller volume by determining 3 to 5 push backs. Finally, fundamental trees should be scheduled by an IP formulation that contains all the mining and milling operational constraints and tree sequence requirements. The advantages of this method are as follows.

- The number of binary variables in the model depends directly on the number of trees generated and the number of periods in which the material in a given push back can be scheduled in the model. Consequently, reformulation of the problem using fundamental trees can result in reducing the size of the model. Much bigger ore body models can thus be handled using this method. Ramazan *et al.* showed that using this method the number of blocks requiring a binary variable for each period can decrease from 38 457 to 5512 (Ramazan *et al.* 2005).
- The gap problem is eliminated.

* CPLEX 8.0 user manual, ILOG Inc., CPLEX division, 889 Alder Avenue, Suite 200, Incline Village, NV 89451, USA.

- Results of a study carried out later with this algorithm demonstrated that this method gives a schedule with a 6% higher NPV than those predicted by the use of other software including M821V1 Scheduler Mintec, NPV Scheduler and Millawa Scheduler (Bernabe and Dagdelen 2002).

The disadvantages of this method are as follows.

- In large deposits, the number of trees to be scheduled will increase. Thus the huge number of binary variables in the IP model makes the model unsuitable for very large deposits.
- Because the fundamental trees are defined within push backs, the optimality of this method depends on the optimality of the push back determination method.
- Sometimes more than one iteration may be necessary for the LP formulation to provide an optimal solution in identifying fundamental trees.
- The complexity of the implementation of this method severely impedes its popularity.

3.1.3.3 Branch-and-cut approach. Many combinatorial optimization problems that are formulated as mixed integer linear programming problems can be solved by *branch-and-cut* methods. These are exact algorithms consisting of a combination of a *cutting plane* and *branch-and-bound* algorithms. These methods work by solving a sequence of linear programming relaxations of the IP problem. Cutting plane methods improve the relaxation of the problem to more closely approximate the IP problem. Branch-and-bound algorithms proceed by a sophisticated divide and conquer approach to solve problems. It is usually not possible to solve a general IP problem efficiently using just a cutting plane approach; it is necessary to also use branching, which results in a branch-and-cut approach (Mitchell 1999). Perhaps the best known branch-and-cut algorithms are those that have been used to solve the travelling salesman problem (TSP) (Appelegate *et al.* 1995).

Caccetta and Hill (2003) outlined their branch-and-cut procedure for solving IP models of LTPP problems. Because of the commercialization of their software, they did not provide full details of their algorithm. Explicit incorporation of all constraints (like maximum vertical depth, minimum pit bottom width and stockpile option) in the optimization procedure is a key advantage of their algorithm. Also, it can produce good solutions for medium mine production planning problems. However, obtaining optimal solutions for large problems is difficult. On a large model containing about 209 600 blocks and ten scheduling periods, they could obtain a solution within 2.5% of the optimum within four hours (Caccetta and Hill 2003). Another disadvantage of this method is that they did not optimize the cutoff grade during the optimization process. It should be noted that for large and/or hard problems, branch-and-cut methods can be used in conjunction with heuristics or meta-heuristics to obtain a good (possibly optimal) solution, and also to indicate how far from optimal this solution may be obtained.

Defining all variables for all periods as binary leads to not generating even a feasible solution for LTPP model; therefore, some variables can be defined as linear, which results in reducing the number of binary variables. For example, setting the variables of positive value blocks as binary and other variables as linear may decrease the solution time significantly. In this case the IP model converts to an MIP model. This strategy can be applied to any of the above-mentioned IP algorithms.

3.1.4 Dynamic programming (DP) formulation. In this technique the prime problem is divided into smaller problems, and for each small problem an optimal solution can be found. The theory of this method was first formulated by Bellman (1957). The main idea of the method is to search all possibilities and choose the optimum one. As opposed to other operation research techniques, this method does not have a standard mathematical formulation. The terms used in DP are listed in table 1 (Kall and Wallace 1994). Formally, the problem is described as follows (Kall and Wallace 1994):

t = the stages, $t = 1, 2, \ldots, T$
z_t = the state at stage t
x_t = the decision taken at stage t
$G_t(z_t;x_t)$ = the transition of the system from state z_t and the decision taken at stage t into the state z_{t+1} at the next stage, i.e. $z_{t+1} = G_t(z_t;x_t)$
$r_t(z_t;x_t)$ = the immediate return if at stage t the system is in state z_t and the decision x_t is taken
F = the overall objective, which is given by $F(r_1(z_1, x_1), \ldots, r_T(z_T,x_T))$
$X_t(z_t)$ = the set of feasible decision at stage t
φ_1,ψ_1 = two functions

The problem can be stated as:

$$\begin{aligned} &\max\{F(r_1(z_1,x_1),\ldots,r_T(z_T,x_T))|x_t \in X_t, t=1,\ldots,T\} = \\ &\max_{x_1\in X_1}\left[\varphi_1(r_1(z_1,x_1), \max_{x_2\in X_1,\ldots,x_T\in X_T} \psi_2(r_2(z_2;x_2);\ldots;r_T(z_T;x_T)))\right] \end{aligned} \tag{17}$$

This relation is the formal equivalent of the well-known optimality principle.

According to equation (17), the production planning problem can be represented by a graph whose nodes represent the state of the system and arcs correspond to the action that takes the system from one state to another. Finding a path with the highest value is equivalent to solving the production planning problem. A graphic representation of the production planning problem is shown in figure 6. The thick lines in the figure represent the optimal path.

Dynamic programming was first applied in open pit production planning by Roman (1974). He integrated pit limit optimization and block sequencing. In this algorithm, the location of the last block that must be mined should be determined at the start. This block is the starting point of the sequencing process. The decision variable in this algorithm represents decisions to

Table 1. Terms used in dynamic programming (Kall and Wallace 1994).

Term	Definition
System	The problem to be optimized
Stage	Finitely sub-problems into which the master problem can be divided. These are just points in time
State	Condition of the system at any state. The state of the system is described with state variables
Decision variable	Represents decisions to build new plants
Transition function	Shows how the state variables change as a function of decision. It dictates the state that will result from the combination of the present state and the present decision
Return function	Shows the immediate returns (costs or profits) as a result of making a specific decision in a specific state
Accumulated return function	Shows the accumulated effect from now until the end of the time horizon associated with a specific decision in a specific state
Optimal accumulated return	Shows the value of making the optimal decision based on an accumulated return function or the best return that can be achieved from the present state until the end of the time horizon

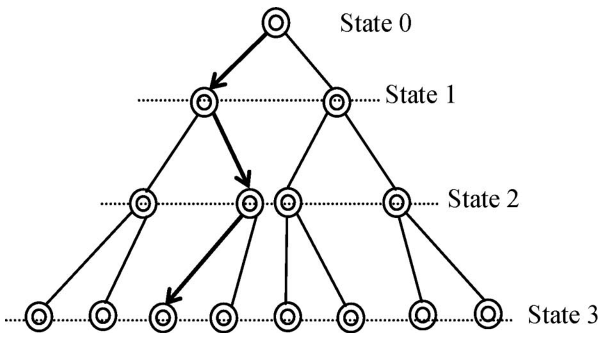

Figure 6. A graphical representation of production planning problem.

extract a new block. According to the slope constraints, all possible ways to sequence blocks above the selected block are checked and the optimum sequence is determined through a NPV calculation. The sequence with highest NPV is selected and this value is assigned to the initial pit shape. The blocks near the pit boundary should be examined whether they contribute to the positive NPV or not. Blocks that do not lead to a positive NPV are removed from the pit and a new pit sequence and NPV are calculated. This procedure continues until no block is needed to be removed from the pit. The advantage of this method is that it considers the time value of money and block sequencing to determine the ultimate pit limit. The disadvantages of this method are as follows.

- Because of the complexity associated with this method, it can not be implemented on large deposits.
- There is no guarantee that mining and milling constraints will be satisfied in each period because this algorithm starts from the bottom and extends upwards of the pit.
- The effect of pit volume on the unit cost is not considered.

Dowd and Onur (1992) and Onur and Dowd (1993) formulated the LTPP problem as DP. They showed that in the DP model of LTPP, the number of alternatives to be considered is so large that it is beyond the memory of a PC, but as a result of considering different types of constraints in production planning, the number of alternatives to be considered is reduced. This leads to the elimination of unattractive alternatives. At the beginning of the algorithm, mining is divided into periods that play the role of stages in DP. In this algorithm, the stage is the set of blocks remaining in the ore body. The program tries to find a sequence for the given time period. Then, for the next period, the program applies the discount rate. If any new added stage does not satisfy the constraints, this sequence is eliminated from further searches. When one time period defined at the beginning succeeds, the next period will start. In every period all of the user-defined conditions (operational constraints, stripping ratio control, etc.) can be applied. Finally, if some sequences result in the same pit shape, the economically most attractive sequence is selected. This procedure is illustrated in figure 7.

Assume that we want to schedule the pit shown in figure 7 in four periods and the maximum mining capacity of each period is five blocks. Therefore, one possible schedule that should be evaluated during DP is:

$$(1, 2, 3, 4, 9) \rightarrow (4, 6, 7, 8, 13) \rightarrow (10, 11, 12, 15, 16) \rightarrow (14, 17, 18, 19, 20)$$

1	2	3	4	5	6	7	8
	9	10	11	12	13	14	
		15	16	17	18		
			19	20			

Figure 7. Example of identification of four sequences.

All other feasible schedules should be evaluated by DP and the most attractive should be selected. The advantages of this method are:

- it takes into consideration all kinds of constraints, even mobility and equipment access constraints;
- the elimination of unattractive sequences as soon as they appear.

The disadvantage of this method is that a long time is needed to produce reliable results; therefore, it cannot be implemented on a PC for large ore bodies. Also, the dynamic cutoff grade concept is not considered in this algorithm.

Tolwinski and Underwood (1992) proposed a method that combines DP, stochastic optimization, artificial intelligence (AI) and heuristic rules to solve the LTPP problem. They modelled the problem as DP and then recognized that the problem was equivalent to finding a path with the largest value in a graph $G=(S, E, W)$, where S represents the set of nodes corresponding to the state of the system (state defined as a sequence of pits), E is the set of edges which represents the transition (removal of one block) and W is the set of weights associated with the elements of E (the rewards of related transition). If S_0 is the node corresponding to the initial state of the mine, then the problem of determination of an optimal production schedule is equivalent to the problem of finding a path $S_0, S_1, \ldots, S_T$ through the graph G that minimizes the total reward. In realistic mines with a large number of nodes, using DP requires complete enumeration of all the nodes (blind search method) and this problem is NP hard. To avoid complete enumeration of nodes, Tolwinski and Underwood proposed the *informed search method*, based on AI and heuristic rules. Generally speaking, the algorithm consists of a sequence of 'simulation runs', each of which generates a path through the graph, say $S_0, S_1, \ldots, S_T$. In order to eliminate states to be evaluated, state occurrence probabilities are calculated. If these probabilities can be determined then the states with low probabilities do not need to be evaluated. At the beginning, these probabilities are not known, and the system learns and makes decisions based on the probabilities as in the machine learning concept of AI. In each simulation run three state attributes are assigned to every encountered node S_t that are used in later simulation runs. Using the simulation, a sequence of paths through the graph with the corresponding path values is generated. This procedure will continue as long as the path values are improving and stops if there is no change in path values for a number of iterations. In spite of the practicality of this method in mining operations due to the consideration of all constraints issues, it suffers from the following disadvantages:

- it does not provide a mathematically proven optimal solution;
- in large ore bodies there is no guarantee that a feasible solution will be obtained.

Elevli (1995) applied Tolwinski and Underwood's method to obtain ultimate pit limit and production planning simultaneously, taking into account all types of constraints. The main advantage of this method is that it combines the ultimate pit limit with the production planning problem. The disadvantages of the method are:

- there is an existing gap problem;
- it does not provide a mathematically proven optimal solution;
- in large ore bodies there is no guarantee of obtaining even a feasible solution;
- the effect of pit volume on the unit costs is not considered.

Tolwinski (1998) and Tolwinski and Golosinski (1995) proposed a method based on the 'depth first' search technique of DP. The block model of a deposit defines a set of block attributes (block tonnages, ore tonnages, grade, etc.). This information can be used to define target variables for scheduling as:

$$TV = \frac{c_1 \cdot a_1 + c_2 \cdot a_2 + \ldots + c_n \cdot a_n}{d_1 \cdot a_1 + d_2 \cdot a_2 + \ldots + d_n \cdot a_n} \quad \text{or} \quad TV = c_1 \cdot a_1 + c_2 \cdot a_2 + \ldots + c_n \cdot a_n \tag{18}$$

where:

N = number of block attributes
$a_1, a_2, \ldots, a_n$ = attribute variables
$c_1, c_2, \ldots, c_n, d_1, d_2, \ldots, d_n$ = user-defined coefficients

These target variables include output levels, stripping ratios, blending requirements, etc. According to the attributes' outputs rates the *flow of time* is defined as:

$$t = \frac{k_1 \cdot a_1}{o_1} + \frac{k_2 \cdot a_2}{o_2} + \ldots + \frac{k_n \cdot a_n}{o_n} \tag{19}$$

where:

$o_1, o_2, \ldots, o_n$ = user-defined output rates
$k_1, k_2, \ldots, k_n$ = user-defined coefficients that take values of 0 or 1

The stages of this method are summarized as follows.

1. Determine the ultimate pit limit using the Lerchs–Grossman algorithm.
2. Create a sequence of nested pits utilizing the Lerchs–Grossman algorithm.
3. Calculate the lookahead value for each block within the ultimate pit. The lookahead value of a block is defined as a weighted average of the profit to be obtained from this block and a set of blocks contained in a downward looking cone with the vertex at the block under consideration.
4. Create an optimal mining sequence. This is done by first ordering the blocks in the highest value nested pit, then proceeding to the next one and so on until all the blocks within the ultimate pit limit are examined. This sorting is in accordance with the lookahead value.
5. Define the push backs that meet all the operational constraints.
6. The rock mass with ultimate pit limit is divided into elementary units, called atoms, which are characterized by location on the bench, the push back to which it belongs, and the

quantity of attributes that it contains. The optimization objective and constraints are defined in terms of target variables that are a function of attributes. The possible schedules that are a sequence of atoms arrange into a tree data structure. In this method two types of trees are constructed. The first tree, whose arcs correspond to the atoms, is generated using the 'depth first' search technique. Using total accumulated attributes values, a second tree is constructed in such a way that the lengths of the arcs are equal to the time unit used as a basis for scheduling. After elimination of non-feasible solutions in this tree, the optimal schedule is found by DP. The algorithm described above is part of a commercial package called NPV Scheduler.

The advantages of this method are as follows.

- It produces a schedule with a high level of applicability. This is because it takes into account all types of practical constraints in the process.
- By controlling the number of push backs that can be mined simultaneously and also the number of atoms, this method can be implemented on large size deposits.

The disadvantage of this method is that it does not guarantee optimum results in NPV maximization. This is because: (i) application of the Lerchs–Grossman algorithm to generate push backs does not guarantee optimum results in NPV maximization due to the existence of the gap problem; (ii) combining blocks into atoms may greatly reduce any possibility of getting an optimal solution depending on the size of the atoms.

Erarslan and Celebi (2001) developed a simulative optimization model to determine optimum pit limit as a function of production planning. Their method can estimate real unit costs for each new condition of pit. They used DP to solve the model, defining each extracted blocks as a state of a stage. In the model, the block extraction sequence is determined by simulating the ore production sequence using a typical mine model. When the processing unit is located at a point, the algorithm simulates block extraction at the indicated point in the three-dimensional coordinate system. As mining activity is simulated, mined ore will be sent to a stockpile and, after blending, feeding of processing plant is also simulated. In this economic model, once a processing unit is located at the starting point, its surrounding blocks are checked. Solid blocks neighbours are tested to determine if they are minable or not. Minable blocks are immediate states at that stage. Each state has successive immediate states. As mentioned before, the nature of the DP technique causes branching at each stage. The state of a stage may give rise to several states in the next branching from that stage. After some stages, the number of node points to be kept becomes excessively huge (figure 6). Branching should thus be restricted at some level, which is the termination level for branching at the same time. Therefore, to find the optimum schedule, Erarslan and Celebi used the *reduce of influence* concept as a restriction criterion for dynamic branching. The radius of influence, which is determined by a geostatistical search, shows the limiting distance through which two points have a mathematical relation. The advantages of this model are as follows.

- It solves the ultimate pit limit problem and production planning simultaneously.
- Among all open pit design and production planning algorithms this is unique because of its ability to estimate unit costs for each new pit scenario. Therefore, real optimization of pit limit and production planning can be performed.
- It considers all types of operating constraints, such as transportation, stockpiling, grade blending, plant facilities.

The disadvantage of this method is that, like many of the DP-based algorithms, it cannot be considered as an efficient tool for obtaining schedules for medium/large deposits. In these conditions, the problem of LTPP will be NP hard. Also, due to the complex structure of the algorithm, the optimality of the solution is not guaranteed in small deposits.

3.1.5 Meta-heuristic techniques. Due to the complexity associated with the open pit mining problem, some researchers have resorted to using meta-heuristic techniques. The following definition seems to be most appropriate to meta-heuristics (Voss 2001):

> A meta-heuristic is an iterative master process that guides and modifies the operation of subordinate heuristics to efficiently produce high-quality solutions. It may manipulate a complete single solution or a collection of solutions at each iteration. The subordinate heuristics may be high level procedures or a single local search, or just a construction method. The family of the meta-heuristics includes, but not limited to, Tabu search, Ant systems, Greedy Randomized Adaptive Search, Variable Neighborhood Search, Genetic Algorithms, Scatter Search, Neural Networks, Simulated Annealing and their hybrids.

Denby and Schofield (1994) and Denby *et al.* (1998) used a genetic algorithm to solve open pit design and production planning problem simultaneously. Denby and Schofield (1995a) also extended their method to underground mines. The optimization procedure can be summarized as follows.

1. Generation of random pit population with size $20-50$.
2. Assessment of fitness function, which can be used to assess the suitability of a produced solution. A typical fitness function includes: maximizing NPV, minimizing early stripping, balancing stripping and balancing ore production for multiple minerals.
3. Reproduction of pit population using probabilistic techniques.
4. Crossover of pits such that between 40 and 60% of the schedules are crossed over.
5. Mutation of pits with probability between 1 and 5%.
6. Normalization of pits to ensure that extraction constraints are not violated.
7. Local optimization of pits to improve the fitness of individual schedules.
8. Stopping condition is met when n generations (between 20 and 40) have occurred without any improvement in the best schedule.

The advantages of this method are:

- with a good definition of genetic algorithm parameters (such as size of population, cross over probability, mutation probability and fitness function) a good result in an acceptable time will be achieved;
- it is flexible;
- it solves ultimate pit limit and production planning problems simultaneously.

The disadvantages of this method are:

- it neglects the effect of pit volume on unit costs;
- results are not reproducible from one run to the next because of the stochastic nature of the method.

Thomas (1996) also noted that since little details are provided in Denby and Schofield's articles, a complete assessment of their algorithms is not possible. Table 2 summarizes the deterministic algorithms to solve LTPP problems.

3.2 *The common drawback of all deterministic algorithms*

In the above sections, all the deterministic algorithms to solve LTPP problem in open pit mine have been discussed. The optimal scenario for LTPP is affected by uncertainties related to the input parameters. These uncertainties are classified by Dimitrakopoulos (1998) as:

- *in situ* grade uncertainty;
- technical mining specification uncertainty, such as extraction capacities, slope consideration, etc.;
- economic uncertainties including capital and operating costs.

Grade uncertainty is the major source of discrepancies from planning expectations to actual production, especially in the early years of a mine's life. For example, Vallee (2000) reported that in 60% of the observed mines, the average rate of production is less than 70% that of the predicted rates in the early years of production. This is mainly due to grade uncertainty.

The common drawback of all deterministic algorithms is that they do not consider any type of uncertainty during the optimization process. The next section will discuss the algorithms that incorporate uncertainty to solve the LTPP problem.

3.3 *The uncertainty-based approach to LTPP*

Rovenscroft (1992) showed the impact of grade uncertainty on production planning. His method is based on the geostatistical technique of *conditional simulation*. The aim of geostatistical simulation is to provide alternative scenarios of the ore body by repeated simulations. Conditional simulation is a class of Monte Carlo technique that can be used to generate equally probable representations of the *in situ* ore body grade (Dowd 1994). Each simulation can be regarded as an alternative image of the deposit. Using one or other deterministic technique on each ore body alternative, the impact of grade uncertainty on production planning can be determined. These schedules should be compared with the original schedule, i.e. the one that was developed on the basis of values obtained from an optimal estimation method such as Kriging. The advantage of this method is its ability to show the impact of uncertainty on the LTPP problem. The disadvantages of this method include:

- it cannot quantify the risk of a project;
- use of repeated simulations as successive input to a mine scheduling program is too boring and time consuming;
- it does not produce an optimal scheduling solution in the presence of grade uncertainty.

Dowd (1994) proposed a framework for risk assessment in open pit mining. In this method some other variables (e.g. commodity price, mining costs, processing cost, investment required, grade, tonnages) were considered stochastically with a predefined distribution function. M simulated ore bodies and N different combinations of other inputs, selected from corresponding distribution function, are combined to produce a revenue block model. After determination of the ultimate pit limit for each model, optimal open pits are scheduled using DP. Then the distribution

Table 2. Summary of the deterministic algorithm to solve LTPP problems.

Type of model	Researcher	Year	Solution method	Advantages	Disadvantages
Linear programming	Johnson	1969	Dantzig–Wolf decomposition method	• Considers dynamic cutoff grade • It can be handled on large deposits	• Its solution is not optimal • Some block will extract fractionally • It has too many constraints • (9 constraints) per block • Some percent of overlaying blocks will be suspended
Mixed integer programming	Gershon	1983		• More practical than Johnson's model in block sequencing • Only one slope constraint per block is required	• It cannot be implemented on large deposits • Inability to handle dynamic cutoff grade concept
Integer programming	Dagdelen and Johnson	1986	Lagrangian relaxation and subgradiant	• It does not lead to fractional block extraction	• It cannot be implemented on large deposits • Inability to handle dynamic cutoff grade concept • Existing gap problem
	Akaike and Dagdelen	1999	4D-network relaxation and subgradiant	• Considers dynamic cutoff grade concept • Capability to handle stockpile option	• Existing gap problem • It cannot be implemented on large deposits
	Ramazan *et al.*	2005	Fundamental tree generation	• Reduces the binary variables • Elimination of gap problem • Generates more NPV than other scheduler softwares	• It needs to generate pushback before scheduling • More than one iteration is needed to generate fundamental trees • Its application is complicated which reduces its popularity. The optimality of the solution depends on the optimality of generated push back

(*continued*)

Table 2. (*Continued*).

Type of model	Researcher	Year	Solution method	Advantages	Disadvantages
	Caccetta and Hill	2003	Branch-and-cut	• Considers all kinds of operational constraints • Obtains a good solution in medium-size deposits	• It cannot be implemented on large deposits • Inability to handle dynamic cutoff grade concept
Dynamic programming	Roman	1974	Complete enumeration	• Optimizes block sequencing and pit limit simultaneously	• It cannot be implemented on large deposits • There is no guarantee about satisfying mining and milling constraints. The effect of pit volume on the unit cost is not considered
	Dowd and Anur	1992	DP	• Considers all kinds of operational constraints • Elimination of unattractive sequences as soon as they appear	• It cannot be implemented on large deposits. • Inability to handle dynamic cutoff grade concept
	Tolwinski and Underwood	1992	DP and AI and heuristic rules	• Considers all kinds of operational constraints • Its solution is practical	• Inability to handle dynamic cutoff grade concept • Its solution is not optimal • It cannot be implemented on large deposits
	Eleveli	1995	DP and AI and heuristic rules	• Optimizes production planning and pit limit simultaneously • Considers operational constraints in the model	• Existing gap problem • Inability to handle dynamic cutoff grade concept • Its solution is not optimal • Effect of pit volume on the unit costs is not considered
	Tolwiski	1998	Depth first search technique of DP	• Considers all kinds of operational constraints, stock pile, etc. • Its solution is more practical than previous algorithms	• Inability to handle dynamic cutoff grade concept • Its solution is not optimal

(*continued*)

Table 2. (*Continued*).

Type of model	Researcher	Year	Solution method	Advantages	Disadvantages
				• It can be implemented on large deposits by controlling the input parameters like the number of atoms	
	Erarslan and Celebi	2001	Simulative optimization approach and heuristic rule	• Optimizes production planning and pit limit simultaneously • Considers all kinds of operational constraints • The effect of pit volume on the unit cost is considered	• Its solution is not optimal • It cannot be implemented on large deposits
Meta-heuristics	Denby and Schofield	1994	Genetic algorithm	• Optimizes production planning and pit limit simultaneously • A good result can be achieved in an acceptable time	• The effect of pit volume on the unit cost is not considered • Its result differs from one run to another • This method is flexible

of NPV, IRR (internal rate of return), payback period, etc. is considered to obtain the risk that is associated with these outputs. This procedure is shown in figure 8.

The advantage of this method is its ability to quantify the risk associated with a project. The disadvantages include:

- it does not give any criterion to accept or reject the risk; therefore, decision making is complicated;
- using repeated simulations as successive input to a mine scheduling process is cumbersome;
- this method does not produce an optimal scheduling solution in the presence of grade uncertainty.

Denby and Schofield (1995b) proposed an algorithm that includes grade variance in open pit design and production planning. They used the multi-objective optimization method of *maximizing value and minimizing risk*. To solve this model using a genetic algorithm, repeated

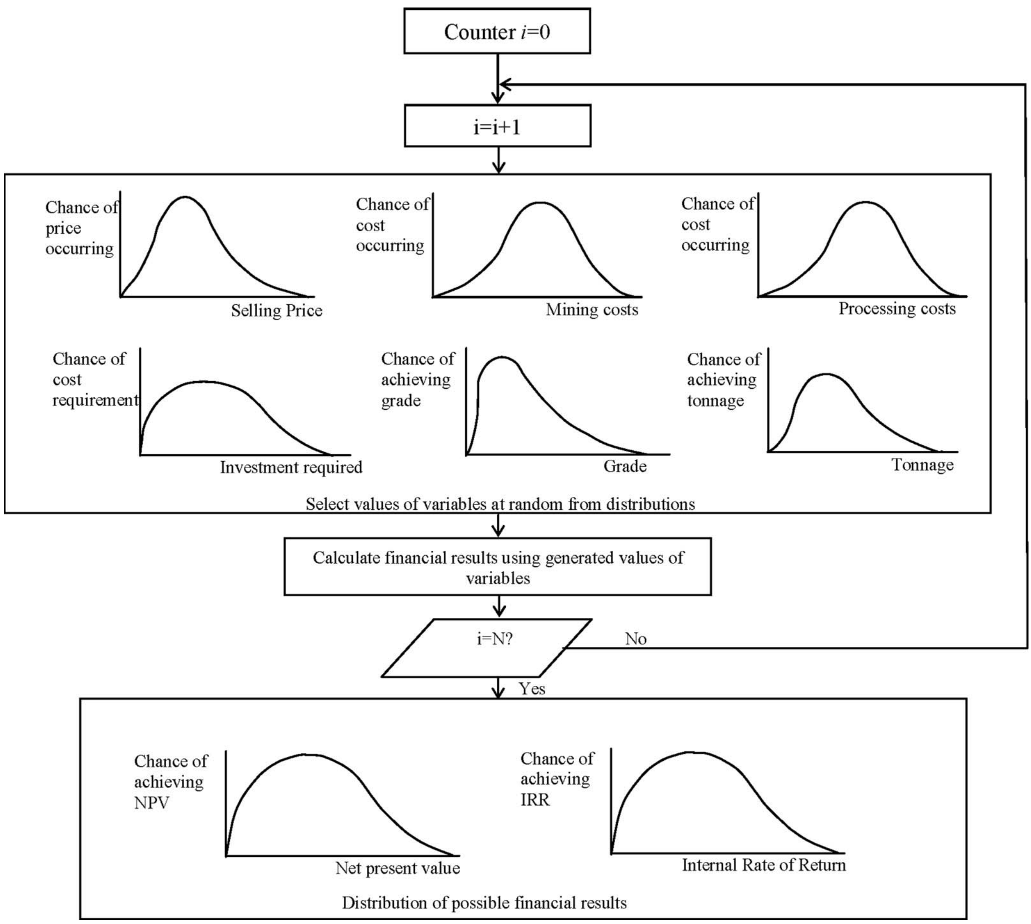

Figure 8. Procedure of risk assessment in open pit mine.

optimization on one primary objective was performed while setting constraints on the other factors. The advantages of this method are as follows.

- This model schedules risky blocks later in the extraction sequence. Thus, the measure of risk is the sum of the discounted uncertainty associated with each block in the extraction schedule.
- This algorithm converges quite quickly to a near-optimal solution.
- The model plots maximum NPV against the limit of risk that the designer wants to accept. This can enable the designer to choose interesting schedules in accordance with the accepted level of risk.
- It does not need repeated simulations as inputs to a mine scheduling process.

The disadvantages of this method are:

- the optimality of the solution cannot be guaranteed;
- geological risk is not incorporated in the optimization process explicitly.

Since little details are provided in Denby and Schofield's articles, a complete assessment of their algorithms is not possible.

Godoy and Dimitralopoulos (2003) proposed an algorithm that addresses the generation of optimal conditions under uncertainty. This algorithm consists of the following steps.

1. Produce a series of stochastically simulated, equally probable models of ore bodies using geostatistical simulation.
2. Design ultimate pit limit and push backs upon ore body models, and find a stable solution domain. The stable solution domain is generated from the cumulative graphs of ore production and waste removal from each of the simulated ore body models and ultimate pit limit and available push back. The common part of the cumulative ore and waste graphs forms a stable solution domain (figure 9).
3. Using an LP formulation, optimal mining rates for the life of mine are obtained (within the stable solution domain) with the consideration of equipment capacities (figure 10).
4. For each simulated ore body, generate a production plan using mining rates obtained in the previous stage. This can be performed using one of the deterministic methods discussed previously.
5. Combine the mining sequences to produce a single schedule that minimizes the chance of deviating from the production target. This is done using a *simulated annealing* meta-heuristic method. In this stage, an initial mining sequence is selected, where blocks with maximum probability of belonging to a given mining period are frozen to that period and not considered further in the optimization process. The optimization process is performed on non-frozen blocks. The initial sequence is perturbed by random swapping of non-frozen blocks between the candidate mining periods. Favourable perturbations lower the objective function and are accepted; unfavourable perturbations are accepted using an exponential probability distribution. The objective function is a measure of the difference between the desired characteristics and those candidates' mining sequence:

$$o = \sum_{n=1}^{N} O_n \tag{20}$$

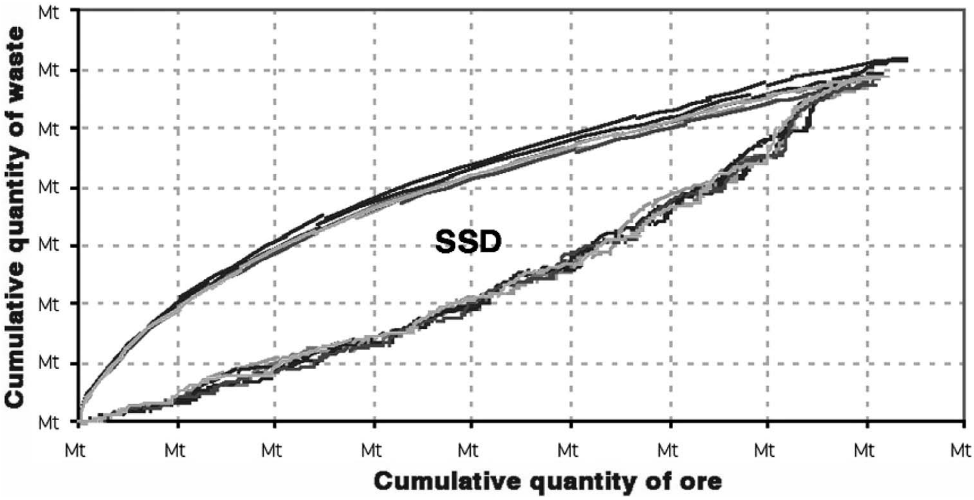

Figure 9. A stable solution domain (SSD) derived from the six simulated ore body model (Gody and Dimitrakopoulos 2003).

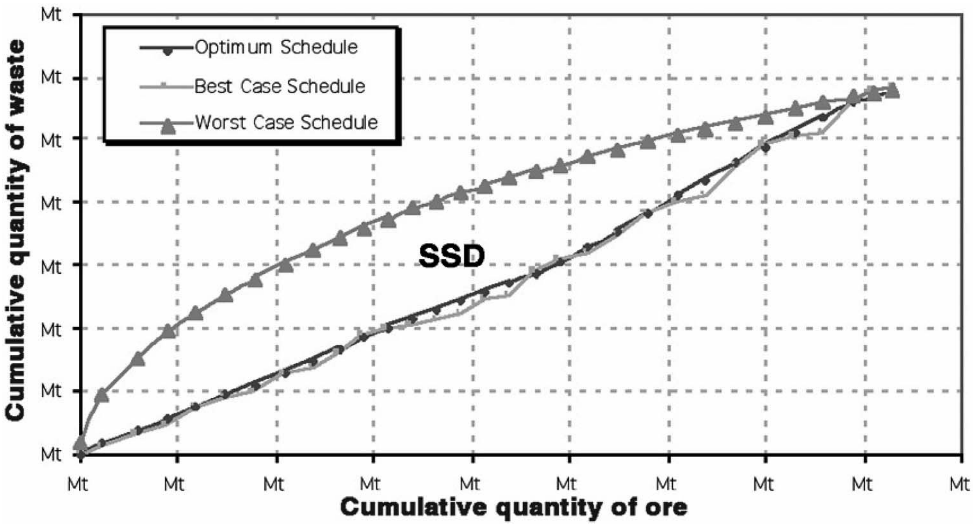

Figure 10. Optimal solution (dashed curve) obtained inside SSD (Gody and Dimitrakopoulos 2003).

where:

$N =$ the total number of scheduling periods

$O_n =$ the average deviation of ore and waste production $\dot{\theta}_n(s), \dot{W}_n(s)$ of the perturbed mining sequence from the target productions $\theta_n(s), W_n(s)$ over S simulated grade models:

$$O_n = \frac{1}{S}\sum_{s=1}^{S}\left|\dot{\theta}_n(s) - \theta_n(s)\right| + \frac{1}{S}\sum_{s=1}^{S}\left|\dot{W}_n(s) - \dot{W}_n(s)\right| \tag{21}$$

The decision to accept or reject a perturbation is based on the change to the objective function:

$$\Delta o = \sum_{n=1}^{N} \Delta O_n \tag{22}$$

The advantages of this method are as follows.

- Integration of ore body uncertainty, waste management and economic and mining considerations generate optimal mining rates.
- Production of a single production schedule in the presence of uncertainty.
- Generation of optimal mining rates for the life of the mine considering equipment issues.

The disadvantages of this method are as follows.

- Implementation of this method is complicated because it needs to generate several schedules on simulated ore bodies and requires several optimization stages to obtain the final scheduling.
- The optimality of the solution is not guaranteed.
- It does not consider equipment access and mobility constraints.
- Geological risk is not considered explicitly in the optimization process.
- Because there may be very significant local deviations between the true grade and simulated grade, especially in the situation that the drill grid is sparse and wide, a detailed mine design based on each simulation may result in generating an unrealistic scheduling in the final optimization stage.

Dimitrakopoulos and Ramazan (2003) proposed an LP model that considered grade uncertainty, equipment access and mobility constraints. This formulation is based on expected block grades and the probabilities of different element grades being above required cutoffs, both derived from simulated ore body models. Expected block grades and probabilities are integrated with equipment constraints and the practical feasibility of mining sequencing in a linear programming model. The key effect of such an approach is the possibility of extracting more certain areas of the deposit in earlier production periods. In this formulation, probabilities that are assigned to each block show the desirability of that block being mined in a given period. To consider the equipment access to each block, they defined two concentric windows around block i (figure 11).

The optimization model is supposed to mine block i together with the blocks within the inner window. If all the blocks within the inner window cannot be mined out, the percent of the tonnage of the blocks that cannot be mined is a 'deviation' (Y^t_{2i}) associated with costs (C_2) for the objective function. The mining blocks within the outer window will be mined, if possible, and again each percent of deviation (Y^t_{3i}) is assigned a cost (C_3). The smoothing formulation can ensure minimum mining width for the available equipment access and mobility. This set-up means that when block i is mined it is more desirable to mine it together with the blocks in the inner window than the blocks in the outer window. But it is even better for smoothness of mining schedules, to mine the farther blocks with block i if possible. The objective function formulation is:

$$Maximize \sum_{t=1}^{T} \left[C^t_1 * Y^t_1 + \left(\sum_{i=1}^{N} C_2 * Y^t_{2i} + C_3 * Y^t_{3i} \right) \right] \tag{23}$$

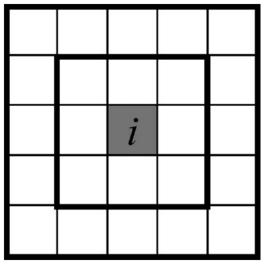

Figure 11. Inner and outer windows around block i (Dimitrakopoulos and Ramazan 2003).

subject to:

$$\sum_{i=1}^{N}(P_i - 100.0) * OT_i^t + Y1^t * \left(\frac{1}{TO}\right) = 0 \quad (24)$$

$$-\sum_{j=1}^{Nb1} K1_j * OT_j^t + K2_i * OT_i^t - Y2_i^t \leq 0 \quad (25)$$

$$-\sum_{j=1}^{Nb2} K1_j * OT_j^t + K'2_i * OT_i^t - Y3_i^t \leq 0 \quad (26)$$

where:

T = the total number of time periods for scheduling
N = the total number of blocks in the model
Y_1^t = the deviation percent from 100% probability that the material will be mined in period t would have the desired properties
C_1^t = the cost coefficient for the probability deviation in period t, such that $C_1^1 > C_1^2 > \ldots > C_1^T$
Pi = the probability of block n having a grade within a desired interval, $Pi \leq 100$
OTi^t = the ore tonnage scheduled from block i to be mined in period t
TO = a constant number representing total ore tonnage to be scheduled in period t
$K1_j = 1/TO_j$, and TO_j is the total ore tonnage available in mining block j
$K2_i = Nb_1/TO_i$ and Nb_1 is the total number of blocks within the inner window excluding the central block
$K'2_i = Nb_2/TO_i$ and Nb_2 is the total number of blocks within the outer window

The rest of the constraints (grade blending, reserve constraints, etc.) are the same as in previous LP models.

The schedule that is produced by this method is feasible in practice and the scheduling pattern does not spread over the deposit. Therefore, equipment is able to access the block to be mined in a given period and the movement of large mining equipment will be minimized. Also, this model generates the schedule that reduces the risk at early production stages without using repeated simulations as successive inputs to the mine scheduling process. However, this method does not generate maximum NPV in the presence of grade uncertainty, since NPV is not maximized explicitly in the objective function. Another disadvantage of this model is scheduling some blocks partially, which contributes to infeasibility and/or non-optimality of the design.

Ramazan and Dimitrakopoulos (2004) suggested an MIP model formulation that accommodates grade uncertainty. In this method, after obtaining simulated ore body models, scheduling patterns on each model are generated using a traditional MIP formulation (with the objective of NPV maximization). Then, using these patterns, the probability of each block to be scheduled in a given time period is calculated. The blocks with probability between 0 and 1 are considered in a new optimization model with the following objective function:

$$Maximize \sum_{t=1}^{T} \left[\sum_{n=1}^{N} (v_n^t \cdot p_n^t) \cdot x_n^t) - \sum_{m}^{M} w \cdot d_m^t \right] \tag{27}$$

where:

$T =$ the maximum number of scheduling period
$N =$ the total number of blocks to be scheduled
$v_n^t =$ the NPV to be generated by mining block n in period t
$p_n^t =$ the probability of block i to be scheduled in period t
$x_n^t =$ a binary variable, equal to 1 if the block i is to be mined in period t and 0 otherwise
$w =$ the cost of unit deviation associated with generating a smooth scheduling pattern
$d_m^t =$ the deviation from a smoothed production pattern when mining block m
$M =$ the total number of blocks with smoothness constraints

The description of this variable is the same as the previous Dimitrakopoulos and Ramazan (2003) algorithm. The first part of the objective function deals with maximizing the probability of the blocks being scheduled in the period predicted by the simulated ore body models; the second part provides the blocks to be accessed by equipment and minimizes large mining equipment movements.

As can be seen from the proposed model, it can maximize NPV explicitly with the consideration of equipment movements and block access in such a way as to produce a schedule pattern that is less risky than the traditional methods. Furthermore, partial block mining is eliminated in this approach. The disadvantages of this method are as follows.

- The generation of several scheduling patterns on simulated ore bodies is complicated and costly.
- The direct integration of grade uncertainty in production planning has not been done. This contributes to the stochastic nature of the grade that in turn leads to violating some constraints some periods of time. As a matter of fact this method does not give the best profitable schedule with the minimum possible geological risk.
- Because there may be very significant local deviations between the true grade and simulated grade, especially in the situation that the drill grid is sparse and wide, detailed mine design on each simulation may result in generating an unrealistic scheduling in the final optimization stage.
- Like all other MIP models for LTPP problems, it cannot be implemented on large deposits.

Table 3 summarizes the uncertainty-based algorithms to solve LTPP problems.

Table 3. Summary of the uncertainty-based algorithm to solve LTPP problems.

Type of model	Researcher	Year	Solution method	Advantages	Disadvantages
Risk analysis using deterministic algorithms	Rovenscroft	1992	Conditional simulation technique and deterministic LTPP algorithms	• Shows the impact of grade uncertainty on LTPP	• It cannot quantify the risk of a project • It does not optimal solution in presence of grade uncertainty • Use of repeated simulations as successive input to a mine scheduling process is cumbersome
Risk analysis using dynamic programming	Dowd	1994	Conditional simulation and DP	• Its ability to quantify the risk associated in a project	• It does not give any criteria to accept or reject the risk • Use of repeated simulations as successive input to a mine scheduling process is cumbersome • Does not produce optimal scheduling solution in presence of grade uncertainty
Linear programming	Dimitrakopoulos and Ramazan	2003	Linear goal programming	• Generates the schedule that reduces the risk at early production stages • Considers equipment mobility and block access in production planning • Does not need repeated simulations as successive input to a mine scheduling process is cumbersome	• Will schedule some blocks partially • It does not generate maximum NPV in the presence of grade uncertainty
Mixed integer programming	Ramazan and Dimitrakopoulos	2004	Linear goal programming	• Maximizes NPV explicitly with the consideration of	• Generation of several scheduling pattern on simulated

(*continued*)

Table 3. (*Continued*).

Type of model	Researcher	Year	Solution method	Advantages	Disadvantages
				equipment mobility and block access	ore bodies is complicated and costly • The direct integration of grade uncertainty in production planning has not been done, due to the stochastic nature of the grade • It cannot be implemented on large deposits
Meta-heuristics	Denby and Schofield	1995a, b	Genetic algorithm	• The generated schedule extracts risky blocks later in the extraction sequence • The algorithm converges quite quickly to near-optimal solution • Makes a plot of maximum NPV against the limit of risk that designer wants to accept	• The optimality of the solution cannot be guaranteed • The algorithm does not guarantee that all mining and milling constraints will be satisfied in the presence of uncertainty • Geological risk is not incorporated explicitly in the production planning process
	Gody and Dimitrakopoulos	2003	Conditional simulation and simulated annealing	• Integration of ore body uncertainty, waste management and economic and mining consideration to generate optimal mining rates • Produces a single production planning in the presence of uncertainty • Generates optimal mining rates for the life of the mine considering equipment issues	• Implementation of this method is complicated • The optimality of the solution cannot be guaranteed • It does not consider equipment access and mobility constraints • Geological risk is not incorporated explicitly in the production planning process

4. Limitation of current approaches and future research direction

4.1 *Inability to solve LTPP and ultimate pit limit problems concurrently*

Until now, the effect of time on ultimate pit limit has not been considered properly. Although some researchers like Denby and Schofield, Eleveli, Erarslan and Celebi, etc. proposed algorithms to solve this problem, their algorithms were unable to derive the ultimate pit limit and production planning concurrently. Therefore, a practical and efficient algorithm is necessary to determine the ultimate pit limit considering the time of extraction of blocks in such a way as to incorporate the effect of pit volume on the unit cost. Certainly, using meta-heuristics like genetic algorithms, simulated annealing, Tabu search, etc., can be helpful in this regard.

4.2 *Considering the uncertainty related to input data*

The uncertainty related to input parameters can increase the difference between calculated and realized NPV. Considering the uncertainty of data during the scheduling process will decrease this difference. Of the known uncertain parameters, ore block grade is a major contributor to large discrepancies between what is mathematically optimal and what is practically achievable. Therefore, explicit integration of uncertainties, especially grade uncertainty, into production planning process can result in producing a schedule that maximizes the NPV of a project with a high degree of confidence. Using stochastic programming to achieve this goal in conjunction with meta-heuristics is a fruitful area of research (Gholamnejad *et al.* 2006).

4.3 *Practicality of the produced schedule*

Equipment access and mobility constraints issues are ignored in most proposed production planning algorithms. They are necessary for making changes in optimum production schedule that are intended to be practically implemented. These changes cause either some constraints to be violated or the schedule to be sub-optimal. Effective integration of these issues is a necessary area of research.

4.4 *Dynamic cutoff grade concept in LTPP*

Long-term production planning should have the capability to determine the processing type for each block as part of the optimum solution (dynamic cutoff grade concept). Production planning and cutoff grade strategy influence each other interactively because the cutoff grade depends on the current and future states of the mine simultaneously. Solving these two problems simultaneously will improve the NPV of a mining project. Some of the models mentioned in the preceding sections such as Johnson's and Akaike's models considered this approach, but their models suffered serious shortfalls. Consequently, developing an efficient algorithm for simultaneously optimizing production planning and cutoff grade is also a good research area. This idea can be extended to multi-metal deposits. Many researchers have tried to solve the optimum cutoff grade determination problem in multi-metal deposits without considering production planning. Some methods such as the equivalent grade method (Staples 1995, Wahrton 1996, Osanloo and Ataei 2003a), the grid search method (Osanloo and Ataei 2002), genetic algorithms (Osanloo and Ataei 2003b) and the golden section search method (Osanloo and Ataei 2003c) can be used to address this issue.

4.5 *Extending the clustering approach*

Ramazan *et al.* (2005) opened a new horizon in the field of production planning optimization. They used clustering analysis to combine ore and waste blocks. The aim of a cluster analysis is to partition a given set of data or objects into clusters (subsets, groups, classes). This partition should have the following properties:

- homogeneity within the clusters, i.e. data that belong to the same cluster should be as similar as possible;
- heterogeneity between clusters, i.e. data that belong to the different cluster should be as different as possible.

The concept of 'similarity' has to be specified according to the data. Improving this approach in production planning problems can reduce the number of variables to be used in the mathematical model such that larger models can be handled by commercially available software.

4.6 *Considering short-term production scheduling issues in the LTPP problem*

Long-term production planning is considered as the basis for medium and short-term production scheduling. In some cases, for achieving the objectives of short-term production scheduling, some deviation from a given long-term plan may occur and this can result in sub-optimal exploitation of the ore throughout the entire life of the mine. To avoid this shortcoming, LTPP should be obtained in such a way that the objectives of short-term scheduling are also satisfied.

4.7 *Using meta-heuristics methods in open pit design and scheduling*

Using meta-heuristic methods like Tabu search, genetic algorithms, simulated annealing, etc. for solving LTPP problems is an interesting area of research. These methods have been shown to be effective for large size NP-hard problems, especially in the broader field of production planning and scheduling (Karimi *et al.* 2001, Torabi *et al.* 2005, 2006).

5. Conclusions

Production planning through the use of different kinds of algorithms to solve LTPP problems has been described. Traditional and uncertainty-based methods to solve this problem have been studied by many and the proposed methods have been discussed and categorized in this paper. Deterministic and uncertainty-based techniques to solve LTPP problems utilize one of the following strategies.

1. First, the ultimate pit limit is determined and then production planning is obtained utilizing mathematical programming in order to maximize the NPV. In large pits, after determination of the ultimate pit limit, a series of push backs can be derived using one of the heuristic algorithms. The extraction process is then scheduled within the limit of push backs using mathematical techniques. Most of the mentioned algorithms follow this strategy.
2. Ultimate pit limits and production planning is determined simultaneously. Roman (1974), Elevli (1995), Erarslan and Celebi (2001) and Denby and Schofield's (1994) genetic algorithms utilize this strategy.

Despite the progress made to date, the need still exists for a more efficient and practical solution approach to the LTPP problem. *In situ* grade uncertainty is an important issue that should be incorporated explicitly in LTPP optimization. Due to the fact that these considerations reduce the discrepancies between what is mathematically optimal and what can be practically achieved, the realized NPV of the project will increase. Considering equipment access and mobility constraints in LTPP model results in producing a more practical schedule with the least equipment movements. In an efficient LTPP model, block destination and the block extraction sequence should be determined concurrently. This approach (dynamic cutoff grade concept) is capable of improving the NPV of the mining project. An LTPP solution should ensure that the short-term production scheduling objectives are achievable. Otherwise, some deviation from optimal LTPP may result and this can result in decreasing the NPV of a project. Incorporation of the above issues into an optimization model certainly leads to enlarging the derived mathematical model into one that cannot be handled using exact solution methods. Decreasing the number of variables using block clustering (aggregation) methods and also implementing heuristic and meta-heuristic methods are considered powerful tools to solve LTPP problems effectively.

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