Dinic's Sh. Aug. Path:

Layered Network: 同一長度的 Sh. Pa. 所構成之子網路.

最多僅能建構 $(n-1)$ 個 Layered Network

e.g. 3 → 最短的 Aug. Path. 包含 3 段 arc. 則先將所有 3條 arc 可連通 s→t 的 path 畫出所構成的網路

在該網路持續送流量直至無法再送.

4 ⇒ 重新再建構最短路徑為 4條 arc 可連通 s→t 之子網路.

在該網路持續送流量直至無法再送.

5 ⇒ find augmenting path → $O(n)$ time

augment flow → $O(n)$ time

each aug. saturates ≥ 1 arc

\# augmentation: $O(m)$ (次)

$O(nm)$ time

⋮

$(n-1)$ ⇒

Overall $O(n \cdot nm) = O(n^2 m)$ time

relabel

$d(i)$: l.b. on the length of Sh. pa. from i to t

$d(s) \le n-1$

$d(i) \le n-1 \Rightarrow$ # relabels : $O(n)$ $\forall i \in N$. ($\because$ each relabel increase $d(i)$ by ≥ 1)

Total # relabels : $O(n^2)$ 次

Each relabel scan $|A(i)|$ $\forall i \in V$ $\Rightarrow$ Total relabel time $O(n \sum_i |A(i)|) = O(nm)$

augmentation

each aug takes $O(n)$ time to identify a bottleneck arc to augment / saturate ≥ 1 arc

- # saturation $\ge$ # aug
- each arc (i, j) in residual net is saturated at most $O(n)$ times (次)

$d(i)$ $i \longrightarrow j$ $d(j)$ saturate $(i, j) \Rightarrow d(i) = d(j) + 1$

$d'(i)$ $i \longleftarrow j$ $d'(j)$ saturate $(j, i) \Rightarrow d'(j) = d'(i) + 1$

$d''(i)$ $i \longrightarrow j$ $d''(j)$ saturate $(i, j) \Rightarrow d''(i) = d''(j) + 1 \ge d'(j) + 1 \ge d'(i) + 2 \ge d(i) + 2$

$\Rightarrow$ between consecutive saturation of (i, j) both $d(i)$, $d(j)$ increase at least 2 units (relabeled) (Twice)

2 relabels $\Rightarrow$ 1 saturation (for (i, j))

$\Rightarrow$ # saturation : $O(nm)$

$\Rightarrow$ # aug : $O(nm)$

Pre-Flow Push

relabel : $O(n^2)$ 次

time to relabel : $O(nm)$

Total # saturated push = # aug. in Sh.aug.pa. = $O(nm)$ times (次)

Same as Sh. aug. pa. Alg.

Time: relabel + nonsat. push + sat. push

$O(nm)$ $\quad O(n^2m)$ $\quad O(nm)$

$= O(n^2m)$

Use Potential function $F = \sum_{j:\text{active}} d(j)$ to bound # nonsaturated push

Each nonsat. push from an active node i to a node j (可能 active)

		$\boxed{\text{對F而言}}$
i: a → ina.	j: a → a.	$-d(i)$
i: a → ina	j: ina → a.	$-d(i)+d(j) = -1$

Each sat. push from an active node i to a node j (可能 active)

		$\boxed{\text{對F而言}}$
i: a → a	j: a → a	0
	ina → a	$+d(j)$
i: a → ina	j: a → a	$-d(i)$
	ina → a	$-d(i)+d(j) = -1$

increase F by at most $O(n)$

Overall: relabel will increase F by at most $O(n^2)$

saturated push ———— $O(n^2m)$

initial F is bounded above by $O(n^2)$ $\because d(i) \le n,\ \forall i \in N.$

each nonsat. push will decrease F by at least 1

$\therefore$ Total # nonsat. push is $O(n^2m)$ times (次) ⇒ nonsat. push takes $O(n^2m)$ time

each nonsat. push takes $O(1)$ time